CHAPTER 2

Review of approximation techniques

2.1 Introduction

Optimization problems in engineering design are characterized by the following associated features:

- the objective function and constraints are implicit functions evaluated by a numerical method which usually involves a large amount of computer resources (e.g. the finite element method),
- function values present a certain level of noise and can only be estimated with a finite accuracy, especially when some iterative techniques are applied because of the limited accuracy of calculations or for physical reasons (Toropov et al. 1996).

An alternative to such prohibitive computational costs is the use of approximation concepts which are much less expensive to compute (Schmit and Farshi, 1974). This chapter reviews the basic approximation techniques (Barthelemy and Haftka, 1993). Depending on the range of their applicability, the approximations can be classified as local (valid in a vicinity of a design point), global (valid in the whole region defined by side constraints) and mid-range (also called multipoint approximations) which is the combination of the two basic approaches. Attention is focused on the possibility of the use of approximation functions for filtering out the numerical noise.

2.2 Local approximations

Local approximations are valid in the vicinity of the point at which they are generated. These functions are variations on the Taylor series expansion and, therefore, based on the derivatives of the objective function and constraints with respect to the design variables.

The simplest form is the linear approximation defined as follows:

$$F(\boldsymbol{x}) = F(x_0) + \sum_{i=1}^{N} (x_i - x_{0i}) \left(\frac{\partial F}{\partial xi} \right)_{x_0} \tag{2.1}$$

A disadvantage of expression (2.1) is the possible inaccuracy for points even close to x_0. It was noticed that in many structural optimization problems better accuracy was achieved with an alternative formulation called the *reciprocal approximation*, defined as

$$y_i = \frac{1}{x_i}; \quad F(\boldsymbol{x}) = F(x_0) + \sum_{i=1}^{N} (x_i - x_{0i}) \frac{x_{0i}}{x_i} \left(\frac{\partial F}{\partial x_i} \right)_{x_0} \tag{2.2}$$

In the case of truss elements where the design variables are the cross sectional areas, expression (2.2) makes the constraints less nonlinear.

The combination of expressions (2.1) and (2.2) gives a new approximation called the *conservative approximation* (Starnes and Haftka, 1979). Barthelemy and Haftka (1993) give the following definition for the conservative approximation:

$$
\begin{aligned}
F(x) &= F(x_0) + \sum_{i=1}^{n} F_i \,(x_i - x_{0i})\left(\frac{\partial F}{\partial x_i}\right)_{x_0} \\
F_i &= \begin{cases} 1 & \text{if } x_{0i}\,(\partial F/\partial x_i) \leq 0 \\ x_{0i}/x_i & \text{otherwise} \end{cases}
\end{aligned}
\tag{2.3}
$$

If $F_i = 1$, expression (2.3) is identical to the linear approximation (2.1), and if $F_i = x_{0i} / x_i$ expression (2.3) is identical to the reciprocal approximation (2.2).

The most popular local approximation techniques also include CONLIN (Fleury, 1989) and MMA (Svanberg, 1987).

Higher order approximations are rarely used because the higher order derivatives are very difficult to obtain. For example, a quadratic approximation based on the Taylor series expansion is:

$$
\begin{aligned}
F(\boldsymbol{x}) &= F(x0) + \sum_{i=1}^{n} (x_i - x_{0i})\left(\frac{\partial F}{\partial x_i}\right)_{x_0} + \\
&\quad + \sum_{i=1}^{n}\sum_{j=1}^{n} (x_i - x_{0i})(x_j - x_{0j})\left(\frac{\partial^2 F}{\partial x_i\,\partial x_j}\right)_{x_0}
\end{aligned}
\tag{2.4}
$$

It requires calculation of the elements of the matrix of second-order derivatives.

Local approximations help to reduce the complexity of the problem, but due to their local characteristics, they lack a global perspective of the problem with the risk of converging to a local optima. Also, these approximations do not address the issue of numerical noise in the function value.

2.3 Global approximations

Global approximations are valid in the whole design space. They allow the study of the region for the location of optimum points. Most typically, global approximation techniques include the response surface methodology (RSM), neural networks and the design and analysis of computer experiments (DACE).

The performance of global approximations was reviewed by Roux et al. (1996) and Sobieszczanski-Sobieski and Haftka (1997) among others.

2.3.1 Response surface methodology

Response surface methodology (RSM) is a method of constructing approximations of the system behaviour using results of the response analysis carried out at a series of points in the design variable space. The approximation functions are obtained by the least-squares method. The strength of the technique is in application to problems where the design sensitivity information is difficult or impossible to obtain, as well as in cases where the response function values contain some level of computational noise. A complete description of RSM is given in Chapter 3.

2.3.2 Design and analysis of computer experiments (DACE)

The polynomial approximations used in RSM were originally developed for responses obtained from the design of physical experiments, which involves random errors due to noise and human errors (Box and Draper, 1987). Later, these techniques migrated to the field of computer experiments where there is no random error, although this point is subject to debate (van Campen et al., 1990, Schoofs et al., 1997, Simpson et al., 1997, Toropov et al., 1999c).

Sacks et al. (1989) proposed a methodology to approximate deterministic responses based on interpolation models. The idea is that, as opposed to classical design of experiments, replicated runs at the same settings with the same inputs will be identical. The approximations are found by kriging models (Lewis, 1998) evaluated with Latin hypercube sampling. Applications are reviewed in the papers by Sacks et al. (1989), Booker (1998), Bates et al. (1998), Torczon and Trosset (1998) and Giunta and Watson (1998) among others.

Figure 2.1 illustrates the difference between deterministic and non-deterministic curve fitting (Simpson et al., 1997).

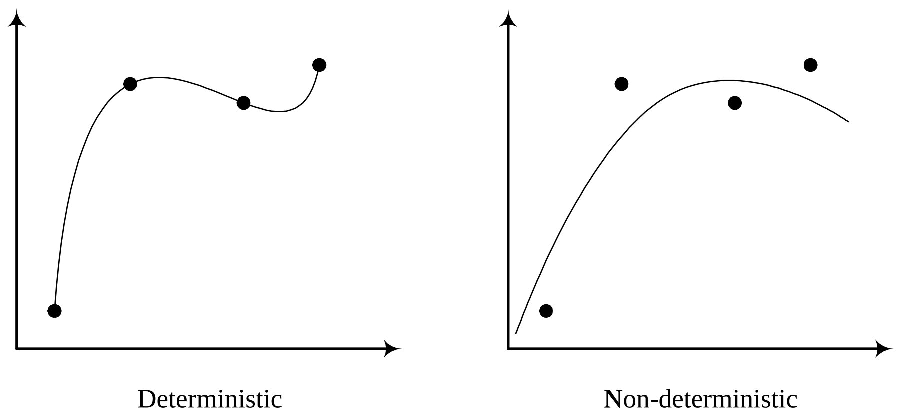

Figure 2.1 Deterministic and non-deterministic curve fitting

2.3.3 Neural network

Many applications use a neural network to realise self-learning models. It works by adjusting weights between pairs of input/output. Once trained, the neural network can replace complex analysis procedures. Therefore, neural networks present an alternative approach to global function approximation.

The major disadvantage of neural models is that input/output is approximated by a black box approach and no understanding of the underlying relationship can be gained.

2.3.4 Genetic Programming

Genetic Programming (Koza, 1992) uses the same principles as genetic algorithms (Goldberg, 1989) with a different representation of the solutions in the form of computer programmes.

The ability of genetic programming to evolve symbolic solutions is investigated in this thesis for the selection of the structure of global approximations. Chapter 4 describes the methodology of genetic programming and Chapters 5 and 6 show applications to simulated and experimental responses.

2.4 Mid-range approximations

Global approximations allow the construction of explicit approximations valid in the entire design space, but as the number of design variables grows, they require too many function evaluations to build.

An alternative approach based on variations of local approximations has been suggested with information about objective function and constraints calculated at more than one point. The purpose is to give an enhanced accuracy and expanded applicability. Such applications can be classified as mid-range approximations.

Early work in multipoint approximations concentrated in constructing approximations along a line search defined by values of the constraints (Haftka and Gurdal, 1993). Further developments used data from several optimization iterations to find approximations in the entire design space. Usually they are based on two or three points (Barthelemy and Haftka, 1993).

A different approach combines response surface methodology (RSM) with multipoint approximations. In this way, the approximations are obtained by least-squares procedures instead of interpolation between design points. Toropov replaced the original optimization problem by a succession of mid-range approximations of the corresponding original functions. This technique employs move limits to define a new sub-region around the current optimum point. Each approximation is evaluated by a weighted least-squares method using the original function values (and their derivatives, when available (Toropov et al., 1993)) at several points of the design variable space. Applications are discussed in Toropov and Carlsen (1994), van Keulen and Toropov (1997,1998) and Markine (1999). Recent developments are presented in Toropov et al. (1999c), including an implementation in a parallel computing environment (van Keulen and Toropov, 1999).

Other work on multipoint approximations can be found in Fadel et al. (1990), Etman (1997), Schoofs et al. (1997) and Xu and Grandhi (1999). An extensive literature review is given by Venter (1998).

2.5 Conclusion

The basic approximation methodologies used in design optimization have been reviewed. Local approximations lack a general perspective of the design space, which has several implications, e.g. the risk of convergence to local optima or an insufficient overview of the problem. Also, the convergence can be seriously affected by the presence of numerical noise. Generally, global and mid-range approximations are preferred.

The main difficulty in the current global approximation techniques based on RSM is the necessity to specify a structure for the approximation function. In contrast, the genetic programming methodology is the only technique that does not assume the structure of the model in advance, but suggests a solution for both the structure and the coefficients of the model.

Therefore, it is proposed to investigate the use of the genetic programming methodology to obtain high quality approximations based on RSM. RSM will be described in Chapter 3 and genetic programming in Chapter 4.