# Using Genetic Algorithm as An Automatic Structural Design Tool S-Y.Chen<sup>1</sup> and S. D. Rajan<sup>2</sup> Department of Civil Engineering Arizona State University Tempe, AZ 85287

## ABSTRACT

The focus of this paper is on the development and implementation of a methodology for automated design of discrete structural systems. The research is aimed at utilizing Genetic Algorithms (GAs) as an automated design tool. Several key enhancements are made to the simple GA in order to increase the efficiency, reliability and accuracy of the GA methodology for code-based frame design. The AISC-ASD design code is used to illustrate the design methodology. Small as well as large-scale problems are solved. Simultaneous sizing, shape and topology optimal designs of structural framed systems subjected to static and dynamic loads are considered. Comparisons with results from prior publications and solution to new examples show that the enhancements made to the GA do indeed make the design system more efficient and robust than a simple GA.

Keywords: Genetic algorithm, optimal design, AISC, frame design, design automation.

#### INTRODUCTION

The simple GA while powerful, is perhaps too general to be efficient and robust for structural design problems. First, function (or, fitness) evaluations are computationally expensive since they typically involve finite element analysis. Second, the (feasible) design space is at times disjointed with multiple local minima. Third, the design space can be a function of boolean, discrete and continuous design variables. The use of GA to find the optimal solution(s) of engineering design problems is still an open research area. Experience with GA has indicated that more often than not, tuning the GA strategy and parameters can lead to more efficient solution process for a class of problems. Researchers have proposed modifications, such as parameters-pace size adjustment and adaptive mutation for continuous problems<sup>1</sup>, which focus on refining the searching space adaptively, niching genetic algorithms that emphasizing on repeating the fitter individuals<sup>2</sup> and special modification for construction time-cost optimization problems<sup>3</sup>. Research has also made it possible to combine genetic algorithms and gradient-based techniques for handling constraints for aerodynamic shape optimization problems<sup>4</sup>.

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In this paper, the proposed improvements to the simple GA are discussed. The basic terms related to any methodology are explained first.

*Efficient*: A methodology is defined as being efficient if it finds an acceptable solution with minimal computational effort.

*Reliable*: A methodology is defined as being reliable if it finds an acceptable solution regardless of the problem nuances or the starting point used.

Accurate: A methodology is defined as being accurate if it finds the best possible solution to a problem.

Robust: A methodology that is generally efficient, reliable and accurate.

#### FORMULATION OF THE DESIGN PROBLEM

The design of three-dimensional frames can be stated as follows.

Find	$\mathbf{x} = \left[ {}^{b} x_{1},, {}^{b} x_{nb}; {}^{i} x_{1},, {}^{i} x_{nd}; {}^{s} x_{1},, {}^{s} x_{ns} \right]$					
to minimize	$f(\mathbf{x})$					
subject to	$g_i(\mathbf{x}) \leq 0$	<i>i</i> = 1,, <i>ni</i>				
	$h_j(\mathbf{x}) = 0$	j = 1,,ne				
	${}^{b}x_{p} \in \{0,1\}$	p = 1,,nb	(1)			
	${}^{i}x_{q} \in \{x_{q}^{1}, x_{q}^{2}, \dots, x_{q}^{nq}\}$	q = 1,,nd				
	${}^{s}x_{r}^{L} \leq {}^{s}x_{r} \leq {}^{s}x_{r}^{U}$	r = 1,,ns				

where  $\mathbf{x}$  is the design variable vector,  $f(\mathbf{x})$  is the objective function, ni is the number of inequality constraints, ne is the number of inequality constraints, nb is the number of boolean design variables, nd is the number of discrete design variables selected from a list of nq values, and ns is the number of continuous design variables. All structural design problems do not lend themselves to a simultaneous consideration of all of the above-mentioned constraints and design variables. Design problems are usually categorized as sizing, shape or topology design problems or combinations thereof.

This paper deals with the solution to the above-mentioned problem. The specific tasks, methodologies and GA enhancements are discussed next.

## GENETIC ALGORITHM AS A DESIGN AUTOMATION TOOL

#### Adaptive Penalty Function for Constraints

GAs were developed to solve unconstrained optimization problems. However, engineering design problems are usually constrained. They are solved by transforming the problem to an unconstrained problem. The transformation is not unique and one possibility is to use the following strategy.

find: 
$$\mathbf{x} = \begin{bmatrix} {}^{b} x_{1}, \dots, {}^{b} x_{NBDV}; {}^{i} x_{1}, \dots, {}^{i} x_{NIDV}; {}^{s} x_{1}, \dots, {}^{s} x_{NSDV} \end{bmatrix}$$
  
minimize:  $f(\mathbf{x}) + \sum_{i} c_{i} \cdot \max(0, g_{i}) + \sum_{j} c_{j} \cdot \left| h_{j} \right|$ 
(2)

where  $c_i$  and  $c_j$  are penalty parameters used with inequality and equality constraints. Determining the appropriate penalty weights  $c_i$  and  $c_j$  is always problematic. We propose an algorithm here where the penalty weight is computed automatically and adjusted in an adaptive manner. First the objective function is modified as

$$f(\mathbf{x}) + c_a \cdot \left( \sum_{i} \max(0, g_i) + \sum_{j} \left| h_j \right| \right)$$
(3)

The following rules are used to select  $c_a$ .

- (1) If there are feasible designs in the current generation,  $c_a$  is set as the minimum *f* among all feasible designs in the current generation. The rationale is that for the design with minor violations and smaller objective value, the probability of survival is not eliminated. If, on the other hand, the maximum *f* among all feasible designs is used, infeasible designs will have a smaller probability to survive even if the constraint violations are small.
- (2) If there is no feasible design,  $c_a$  is set as the *f* that has the least constraint violation. The motivation idea has the effect of both pushing the design into feasible domain as well as preserving the design with the smallest fitness.

## Improving Crossover Operators Using the Association String

As discussed by some researchers<sup>5,6</sup>, the one-point crossover is preferred for continuous domains, and the uniform crossover for discrete domains. However, schema representation still plays a pivotal role in the efficiency of the GA. If one uses a one-point crossover then it is obvious that the ordering of the design variables is an important issue. Since the characteristic of one-point crossover is that the shorter schema has a better chance to survive**Error! Bookmark not defined.**, if two variables that have less of an interdependency are placed adjacent to each other, or two variables with a strong relationship are placed far away from each other, the crossover operation will make it more difficult for the GA to search the design space efficiently. To

implement this strategy, we introduce an additional string called the *association string*. The details of this scheme can be found in a previous publication [Chen and Rajan<sup>5,7</sup>]. Results show that the association string improves the robustness of the solution process.

#### Mating Pool Selection

The selection scheme (for generating the mating pool) together with the penalty function dictate the probability of survival of each string. While it is very important to preserve the diversity in each generation, researchers have also found that sometimes it may be profitable to bias certain schema<sup>8</sup>. However, results from most of the selection rules, like roulette wheel, depend heavily on the mapping of fitness function.

In this paper, the tournament selection<sup>9</sup> is used. There are at least two reasons for this choice. First, tournament selection increases the probability of survival of better strings. Second, only the relative fitness values are relevant when comparing two strings. In other words, the selection depends on individual fitness rather than ratio of fitness values. This is attractive since in this research, the fitness value contains the penalty function and does not represent the true objective function.

#### Elitist Approach

The elitist approach was proposed by De Jong<sup>10</sup>. Research<sup>5,6</sup> has shown GA with the incorporation of the elitist approach can be more reliable and efficient than the ones without. This approach is used in the current research.

#### Repeating Chromosome

It is found that, during the evolutionary process, the same chromosomes at times are repeatedly generated<sup>11</sup>. Since the fitness evaluation in structural design involves finite element analysis, a computationally expensive step, all generated chromosome and the associated fitness information are saved in memory. In this way, if a chromosome is repeated, a finite element analysis is not necessary. Saved chromosomes may also be helpful for further processing of the design history.

#### Population Size and Stopping Criteria

In the first section we suggested that the initial population should contain uniformly distributed alleles. By this, it is meant that no chromosome pattern should be missed. Each chromosome is represented by n bits with each bit being either 1 or 0. If the distribution of 1's in each bit location is to be uniform, the initial population size should be at least n. During the evolution, it is expected that that the chromosome converges to some special pattern with the (0-1) choice decided for n locations.

Assume that the choice of each bit is independent of all the other bits. Since the population size is n in each generation, after every generation from the statistical viewpoint we can expect to learn about at least one bit. Ideally then after n generations, one can expect to learn about all the n bits forming the chromosome. However, since each bit is not independent of the others, more than n generations are perhaps necessary to obtain a good solution. This suggests that the population size and the number of generations should be *at least* n. Previous work suggests that using population and generation size of 2n leads to reasonable results efficiently<sup>5,10</sup>.

#### The Improved GA Optimizer

As mentioned before selective improvement can be made to obtain a more robust solution methodology for a class of problems. The primary focus in this research is to make the GA a powerful and reliable optimizer for structural optimization. Table 1 shows the proposed improvements.

	Traditional GA	Proposed GA
Penalty Function	ad hoc	Automatic
Schema	ad hoc	Ordered
Cross-over Probability	ad hoc	Adaptive
Population/Max Generation Size	ad hoc	Suggested as 2n

Table 1 Differences Between Traditional and Proposed GA

The optimizer is written in ANSI FORTRAN. The programming interface is full modularized, and requires little effort in meshing the program with a simulation program. A total of 21 combinations of the options have been proposed for testing in the previous publication. While the combinations are not exhaustive, they provide most of the major ones. The complete data can be found in previous publication<sup>5,7</sup>. The results show that the proposed algorithm performance better than all others in efficiency, accuracy and reliability<sup>5,7</sup>.

#### CODE-BASED DESIGN

The Ninth Edition of Allowable Stress Design procedure from AISC<sup>12</sup> is used. There are primarily two reasons for this choice. First, this is the newest ASD code from AISC. Second, since it is computationally expensive to carry out plastic analysis of structural systems and given the fact that linear elastic analysis is carried out in this research, using the ASD design code is a natural choice. It should be noted that the ASD code still enjoys widespread use in the industry.

For the purpose of code checks, the finite element analysis is performed first. The member force vector for each member is calculated. Using the cross-sectional property, the axial

stress  $f_a$ , bending stress along major axis  $f_{bx}$ , bending stress along minor axis  $f_{by}$ , shear stress in the major and minor axis directions ( $f_{vx}$ ,  $f_{vy}$ ) are calculated. The allowable stress is then obtained as per code provisions, and the Code-based constraint equations are used. For each member, whenever appropriate, these checks are carried out at three internal points in addition to the ends of the member.

## Basic Constraint Equations

Axial Compression and Bending: For the member in axial compression and bending, the normal stress of a beam should be proportioned appropriately. In general the requirements are as

follows. If 
$$\frac{f_a}{F_a} > 0.15$$
 then  

$$\begin{cases} \frac{f_a}{F_a} + \frac{C_{mx} \cdot f_{bx}}{\left(1 - \frac{f_a}{F_{ex}}\right) \cdot F_{bx}} + \frac{C_{my} \cdot f_{by}}{\left(1 - \frac{f_a}{F_{ey}}\right) \cdot F_{by}} \le 1.0 \\ \frac{f_a}{0.6 \cdot F_y} + \frac{f_{bx}}{F_{bx}} + \frac{f_{by}}{F_{by}} \le 1.0 \end{cases}$$
(4)

$$\frac{f_a}{F_a} + \frac{f_{bx}}{F_{bx}} + \frac{f_{by}}{F_{by}} \le 1.0$$
(5)

$$F_{ex}^{'} = \frac{12 \cdot \boldsymbol{p}^{2} \cdot E}{23 \cdot \left( \begin{matrix} K_{x} \cdot l_{b} \\ / r_{b} \end{matrix} \right)^{2}}$$

$$F_{ey}^{'} = \frac{12 \cdot \boldsymbol{p}^{2} \cdot E}{23 \cdot \left( \begin{matrix} K_{y} \cdot l_{b} \\ / r_{b} \end{matrix} \right)^{2}}$$
(6)

 $C_m = 0.85$  with sway

$$C_m = 0.6 - 0.4 \frac{M_1}{M_2}$$
: braced against sway, no transverse loading.  
 $C_m = 0.85$ : braced against sway, with transverse loading, no rotations, both ends  
 $C_m = 1.0$  braced against sway, with transverse loading, with rotations, both ends  
 $C_m$  can be conservatively taken as 1.0 (7)

Eqn. (7) defines the reduction factor, which is taken from the code. However, some conditions are not well-defined. (For example, for members braced against sway, with transverse loading, with rotation on one end and without on the other end, the factor is not defined.) *Axial Tension and Bending*: For the member in axial tension and bending

$$\frac{f_a}{F_t} + \frac{f_{bx}}{F_{bx}} + \frac{f_{by}}{F_{by}} \le 1.0$$
(8)

In the above equations,  $f_a$  is the normal stress caused by axial stress,  $f_{bx}$  is the normal stress caused by bending about local x axis (major axis of the cross-section),  $f_{by}$  is the normal stress caused by bending about local y axis (minor axis of the cross-section), and  $F_a$  and  $F_b$  are the allowable axial and bending stress, which is defined in the code.

Shear Stress: For the shear stress

$$\frac{f_{vx}}{F_{vx}} + \frac{f_{vy}}{F_{vy}} \le 1.0$$
(9)

where  $f_{vx}$  is the shear stress in the local x (major axis of the cross-section) direction, and  $f_{vy}$  is the shear stress in the local y (minor axis of the cross-section) direction. The code provides a complex procedure to arrive at the allowable normal stress for various sections under different stress and loading conditions. A flow chart outlining the checks is available<sup>7</sup> to explain the program flow.

## The Effective Length Factor

One of the most intriguing portion of the code deals with the effective length factor, K. The factor is heavily used not only where hot-rolled steel members are used, but also in the use of cold-formed steel members such as those governed by the AISI-LRFD<sup>13</sup> design manual. Typically, engineers use the Alignment Charts<sup>14</sup>. Some researchers have raised important questions about the applicability of these charts <sup>15,16</sup>. Research aimed at deriving the equation for K-Factor based on the end restraint conditions of each member<sup>17,18,19</sup> under certain circumstances have been carried out. In Johnston's book<sup>14</sup>, the use of the K-Factor is described for two commonly encountered situations. First, it is used to predict the buckling of an axial-loaded column. Second, it is used as an amplification factor, for considering the *P*- $\Delta$  effect in eccentrically loaded beam-column. That is,

$$\boldsymbol{d}_{m} = \boldsymbol{d}_{0} \cdot \left[ \frac{1}{1 - \frac{P}{P_{cr}}} \right]$$
(10)

where  $d_0$  is the deflection without  $P-\Delta$  effect, and  $d_m$  is the deflection accounting for the effect. This equation implies that, as the load approaches the critical load, the deflection tends to infinity. Furthermore, it is also stated that in a complex structure the K-Factor depends on the final buckling mode of the structure. Therefore, an alternative way is to find the member capacity is to carry out the elastic buckling analysis. A linear elastic buckling analysis of the structure is described by the following equation.

$$\mathbf{K}_{E} + \mathbf{I} \cdot \mathbf{K}_{C} = \mathbf{P} \tag{110}$$

Here  $\mathbf{K}_{E}$  is the elastic stiffness matrix,  $\mathbf{P}$  the load vector and  $\mathbf{K}_{G}$  the geometric stiffness matrix, which is a function of the axial load for each element. To implement the computations associated with Eqn. (21), a linear static analysis is first performed with the  $\mathbf{P}$  vector to obtain the axial force in each member. Equation (21) is then recast to obtain a nontrivial solution by setting  $\mathbf{P}$  to zero. This requires an eigenvalue analysis and  $\mathbf{I}$  is the lowest eigenvalue as well as the critical load factor for the system. The results from a linearized buckling analysis can be converted to equivalent K-Factor in each member as

$$K = \frac{\mathbf{p}}{l} \sqrt{\frac{EI}{\mathbf{l} \cdot P}}$$
(12)

where *P* is the axial force in the member. Note that the factor should be calculated for bending about both major ( $K_x$ ) and minor ( $K_y$ ) bending axis using the appropriate *I* value for each axis.

#### SIZING, SHAPE AND TOPOLOGY OPTIMIZATION OF SPACE FRAMES

The structural optimization problem involving sizing, topology and shape parameters has always been a difficult problem to handle. Some of the design variables are discrete, the design space is disjoint and traditional gradient-based methods cannot be employed. The design problem of a three-dimensional frame can be stated as shown in Eqn. (1).

Researchers working in this area have divided the existing algorithms for discrete variables into three types - branch and bound, approximation, and ad-hoc methods<sup>20</sup>. The solution techniques such as approximation methods<sup>21</sup>, branch and bound methods, and ad hoc strategies of adapting continuous design variables in NLP techniques<sup>22,23</sup> suffer from several drawbacks. These methods either are inefficient, or do not really converge to the optimal solution or can be used under very restrictive conditions. For example, the approximation method allows the candidate solution to be discrete, but still require the whole design domain to be differentiable and continuous.

In the case of topology optimization, approximation methods and branch and bound techniques cannot be applied since the methods cannot handle the presence or absence of members as design variables. Instead, approaches like the homogenization methods have been widely discussed<sup>24,25</sup>. Researchers have also used simultaneous analysis and design method to solve the topology design problem<sup>26</sup>. However, these methods consider only the minimization of the compliance of the structure instead of handling the problem described by (1). Furthermore, it is not clear how the final structure is formed once the material distribution is obtained. The design problem can be solved more easily using GAs since they can be adapted to work with discrete and boolean design variables.

#### Design Variable Linking

As shown in Eqn. (1), GAs essentially can handle three types of design variables – discrete or integer, real, and boolean. These design variables capture all the possible structural design parameters. The sizing design variables considered in this dissertation are either cross-sectional dimensions or available cross-section. The former can be described using continuous design variables since these dimensions can vary continuously. The latter is described in terms of integers (an integer index that points to a row in a table of available cross-sections). The table search is carried out by using a table of ordered available cross-sections with the lower and upper bound candidate cross-sections specified by the user. The shape design variables are the nodal locations. These are real design variables. The topology (boolean) design variables can be structural parameters such as the presence or absence of members, and presence or absence of fixity conditions at supports or connections.

Optimization	Physical Meaning	Design	Note
		variable Type	
		in GA	
Topology	Element Existence	Boolean	
Sizing	Cross-sectional selection	Integer	Search through a given
			table
Shape	Nodal Coordinates	Real	Varies between upper and
			lower bounds

Table 2 Linking of Design Variables and the Physical Meaning

## Special Considerations

When topology design is considered, several problems should be handled very carefully.(i) There may be elements not connected to the structure during design, if topology design is performed. This can be detected by examining the singularity of the stiffness matrix.

(ii) There may be "null" nodes during the design. A null node is one to which no element is attached. Such nodes need to be suppressed (from the finite element analysis) in order to find the response of the remaining structure.

(iii) Sometimes, crisscrossing members are not allowed in frame structures. This situation is detected by testing the possible intersection of a member with all other members. It should be noted that handling such a constraint by traditional (gradient-based) optimization approach can be very challenging.

#### NUMERICAL EXAMPLES

Two numerical examples are solved in this paper. The purpose of solving these examples is to show the robustness of the proposed algorithm. The first example is taken from prior research publications. The second example is a larger problem and involves a building frame. All tests are performed on a Pentium Pro 180 PC running Windows NT. For the cases where our GA methodology is used, the crossover probability is 0.9 and mutation is 0.03.

#### Roof Frame Design

This example is taken from Grierson and Lee 's paper<sup>22</sup>. The structure is shown in Figure 1. The dead, live and wind load intensities define the service load level. The material properties and other design data of the original publication are listed in Table 3.

The original publication used assumed Kl/r factors, and the allowable stress is calculated on the assumed values. In the current research, the linearized buckling analysis is used to compute the slenderness factor of each member. The values of material properties as used in the current research are listed in Table 3. Figure 2 through Figure 6 show the layout of the five different load cases considered in the design.

Table 4 lists the load values for the five load cases. In addition to the stress constraint, displacements in the Y-direction at node 8 and 11 are limited to 4 inches. The problem is formulated as

Find:	Cross – section x of each member				
To minimize:	Weight o	f the structure			
Subjected to:	$\boldsymbol{s} \leq \boldsymbol{s}_{a}$	$oldsymbol{s}_{a}$ defined by AISC - ASD	(13)		
	$ \mathbf{u}  \leq \mathbf{u}_{a}$	$u_a$ the allowable displacements			
	$\mathbf{x} \in \left\{ b_i \middle  i = \right.$	1,295, $b_i$ is AISC standard section}			

	Grierson ar	nd Lee	Current Research		
	<b>Rafter and Chord</b>	Web	Rafter and Chord	Web	
Density	0.283 lb/in3	0.283 lb/in3	0.283 lb/in3	0.283 lb/in3	
Young's Modulus	30,000 ksi	30,000 ksi	30,000 ksi	30,000 ksi	
Yield Stress	44 ksi	36 ksi	44 ksi	36 ksi	
<b>Ultimate Stress</b>	N/A	N/A	60 ksi	58 ksi	
Allowable Stress	26.4 ksi	21.5 ksi	AISC	AISC	
KI/r	Assumed	Assumed	Buckling Analysis	Buckling Analysis	

## Table 3 Material Properties and Design Data

Table 4 Load Values for the Five Load Cases

Units = k/in							
W1	0.04783	W6	0.01179				
W2	0.02873	W7	0.03586				
W3	0.00783	W8	0.01344				
W4	0.01792	W9	0.00698				
W5	0.00931						

Grierson and Lee consider only sizing design variables. The members are divided into three property groups. The first group consists of the rafters, top chords and bottom chords. CISC W sections are used as rafters. The top and bottom chords are structural T's positioned appropriately. The second group is the vertical web member, and inclined webs form the third group. In the second and third groups, CISC Standard Double Angle (DL) sections are used.

We consider two test cases - TEST1 and TEST2. Only sizing design variables are used in TEST1. However, sizing and topology design variables are used in TEST2. In addition, we consider only AISC W sections (a total of 295 AISC standard W sections are considered). The design variables used in TEST1 and TEST2 are listed in Table 5. For each test, two GA strategies labeled F and D are considered. The choice is based on our prior work<sup>5</sup> The F operator denotes the proposed algorithm, and D the traditional one-point crossover<sup>5</sup>. The design results are shown in Table 6.**Error! Reference source not found.**. The chromosome length for TEST1 and TEST2 are 36 and 44 respectively. The population size and number of generations is taken to be 72 for TEST1 and 88 for TEST2.

	Grierson and Lee (1984)		TES	T1	TEST2		
	Sizing DV	Section	Sizing DV	Section	Sizing DV	Topology	Section
Rafter	1	CISC W	1	AISC W	1	N/A	AISC W
Top Chord	1	CISC WT	2	AISC W	2	N/A	AISC W
Bottom Chord	1	CISC WT	2	AISC W	2	N/A	AISC W
Vertical Web	2	CISC DL	3	AISC W	3	1~4	AISC W
Inclined Web	3	CISC DL	4	AISC W	4	5~8	AISC W

Table 5 Design Variables Linking

	Grierson	TEST1-F	TEST1-D	TEST2-F		TEST2-D	
	Section	Section	Section	Section	Exist	Section	Exist
Rafter	W460x61	W6X25	W8X24	W8X24	ALL	W8X24	ALL
Top Chord	WT230x30.5	W12X14	W12X14	W12X14	ALL	W12X14	ALL
Bottom Chord	WT230x30.5	W12X14	W12X14	W12X14	ALL	W12X14	ALL
Vertical Web	DL100x90x6	W12X14	W6X9	W6X9	25~29	W6X9	25~29
Inclined Web	DL 55x35x4	W6X9	W6X9	W14X74	NONE	W18X50	NONE
Weight (lb)	2918.5	2445.2	2319.6	1818.1		1818.1	
CPU Time (sec)		669	865	1147		1309	
Function Evals.	N/A	3279	4326	6145		7101	

Table 6 Final Design Results

The results are encouraging. With only sizing design variables, the final weight is about 20% less than those reported in the earlier publication. With the addition of topology design variables the savings are even greater – about 40%. It should also be noted that in TEST2 the proposed GA (operator F) uses much less computation time and function evaluation (about 13% less) than the traditional GA (operator D), with similar results. The final topology of TEST2 is shown on Figure 7.

#### Ten-Story Frame

The structure has four bays in both directions and is twenty story in height. The details of the frame are shown in Figure 8, Figure 9 and Figure 10. A linear, elastic, small displacement finite element analysis is carried out to compute the structural response. All connections are assumed to be rigid. The base of each column is assumed to be rigidly supported. For each story, the members are divided into five groups - corner column, outer column, inner column, outer beam and inner beam. The material properties of steel are listed in Table 7.

The loading on the frame includes the dead load (in the negative Z direction) on each floor, and the wind load on left and right sides of the building (in the positive X direction). The dead load is taken to be 0.75 pound per square inch on the bottom nine stories, and 0.347 pounds per square inch on the roof. The inward wind load (acting on the left side of the building) is 0.07 pound per square inch, and the outward wind (acting on the right side of the building) is 0.04375 pound per square inch. The distributed loading on the floors is transferred as equivalent line distributed load on all members surrounding the loaded area.

The (sizing) design problem is formulated as follows.

Find	Cross-section x of each member	
to minimize	Weight of the structure	
subjected to :	$\boldsymbol{s} \leq \boldsymbol{s}_{a}$ with $\boldsymbol{s}_{a}$ defined by AISC - ASD code	(14)
	$\mathbf{x} \in \{b_i   i = 1,, n, b_i \text{ is an AISC standard section}\}$	

To make the overall design process systematic and efficient, the design is carried out in two steps. In the first step, only forty-nine candidate AISC W sections are considered for each beam or column group. The candidate AISC standard W sections for the first level design are listed in Table 8, and the design variable linking for each story is shown in Table 9. With this definition of the design variables, the chromosome length is 300. The population size and number of generations are both taken to be 300. The results for this first level design are shown in

Table 10 and Table 11.

	Steel
Young's Modulus	29,000
Yield Stress	36
Ultimate Stress	58
Poison's Ratio	0.3

Table 7 Material Properties of Steel (ksi)

Table 8 Candidate AISC W Sections in the First Step Design

Rank	Section								
1	W4X13	11	W10X45	21	W14X82	31	W18X311	41	W30X581
2	W5X19	12	W10X112	22	W14X132	32	W21X57	42	W33X169
3	W6X16	13	W12X22	23	W14X426	33	W21X93	43	W33X619
4	W6X25	14	W12X35	24	W14X730	34	W21X402	44	W36X256
5	W8X15	15	W12X50	25	W16X31	35	W24X62	45	W36X848
6	W8X21	16	W12X58	26	W16X57	36	W24X103	46	W40X183
7	W8X28	17	W12X336	27	W16X100	37	W24X492	47	W40X655
8	W8X67	18	W14X26	28	W18X46	38	W27X129	48	W40X328
9	W10X19	19	W14X38	29	W18X71	39	W27X539	49	W44X285
10	W10X30	20	W14X53	30	W18X119	40	W30X148		

Table 9 Design Variable Linking for Each Story

	DV Type	Lower	Upper
Corner Column	AISC W	1	49
Outer Column	AISC W	1	49
Inner Column	AISC W	1	49
Outer Beam	AISC W	1	49
Inner Beam	AISC W	1	49

	1st Story	2nd Story	3rd Story	4th Story	5th Story	6th Story
Corner Column	W27X129	W12X35	W8X28	W14X426	W24X103	W16X31
Outer Column	W30X148	W12X58	W40X183	W27X129	W18X71	W10X45
Inner Column	W14X132	W18X311	W18X119	W36X256	W44X285	W16X100
Outer Beam	W33X169	W18X46	W16X57	W18X46	W14X132	W18X71
Inner Beam	W12X50	W18X46	W21X93	W10X112	W12X35	W12X35

Table 10 Results for First Step Design, Story One to Six

Table 11 Results For First Step Design, Story Seven to Ten

	7th Story	8th Story	9th Story	10th Story
Corner Column	W16X100	W12X336	W5X19	W36X256
Outer Column	W21X93	W18X71	W40X328	W30X148
Inner Column	W16X100	W18X71	W21X93	W6X25
Outer Beam	W8X28	W14X53	W12X50	W8X28
Inner Beam	W27X129	W14X53	W12X35	W40X183

Based on these results, the candidate member sections are refined. Only 8 AISC W sections were considered as the possible choices for each design variable. These sections were selected based on the final results from step 1 - using the order from AISC ASD W section tables, four sections above the final design, three sections below, and the final section from step 1. Table 12 lists a few examples of the refined search. For this stage of the design, the chromosome length is 150; the population size and generation numbers are both taken as 150. The results of the second level design are shown in Table 13 and Table 14, with Table 15 showing the comparison between the two design steps in terms of the objective function value and the computational effort.

Table 12 Candidate Design Variables for Second Level Design

Previous								
Result	W27X129	W30X148	W14X132	W33X169	W12X50	W12X35	W12X58	W18X311
	W27X194	W30X235	W14X193	W33X263	W12X72	W12X53	W12X87	W21X62
	W27X178	W30X211	W14X176	W33X241	W12X65	W12X50	W12X79	W21X57
Ч.	W27X161	W30X191	W14X159	W33X221	W12X58	W12X45	W12X72	W21X50
arc	W27X146	W30X173	W14X145	W33X201	W12X53	W12X40	W12X65	W21X44
Se	W27X129	W30X148	W14X132	W33X169	W12X50	W12X35	W12X58	W18X311
ler	W27X114	W30X132	W14X120	W33X152	W12X45	W12X30	W12X53	W18X283
Irth	W27X102	W30X124	W14X109	W33X141	W12X40	W12X26	W12X50	W18X258
Fu	W27X94	W30X116	W14X99	W33X130	W12X35	W12X22	W12X45	W18X234

	1st Story	2nd Story	3rd Story	4th Story	5th Story	6th Story
Corner Column	W27X102	W12X35	W8X35	W14X342	W24X76	W16X36
Outer Column	W30X124	W12X72	W40X149	W27X146	W18X60	W10X39
Inner Column	W14X99	W18X258	W18X119	W36X260	W44X198	W16X77
Outer Beam	W33X130	W18X60	W16X40	W18X35	W14X99	W18X55
Inner Beam	W12X45	W18X46	W21X73	W10X77	W12X35	W12X53

Table 13 Result For Second Level Design, Story One to Six

Table 14 Result For Second Level Design, Story Seven to Ten

	7th Story	8th Story	9th Story	10th Story
Corner Column	W18X50	W14X30	W6X15	W36X210
Outer Column	W21X93	W18X65	W44X198	W30X148
Inner Column	W16X77	W18X76	W21X73	W8X18
Outer Beam	W8X24	W14X38	W12X40	W8X24
Inner Beam	W27X94	W14X38	W12X35	W40X149

Table 15 Comparison of the Two Steps

	Weight (kips)	Function	Time (hrs)
		Evaluations	
1st Step	1,005	89,700	19
2nd Step	823	22,312	5

Clearly for this problem it was possible to separate the candidate sections into several groups, so that one can compute a rough design (step 1) first, and then refine the search.

## CONCLUDING REMARKS

In this research, a genetic algorithm based design optimization methodology is developed and implemented for sizing, shape and topology optimization of discrete structural systems subject to either strength considerations or code provisions. AISC ASD design checks are carried out systematically and enforced during the design process. Enhancements have been made in making the GA robust and efficient. New stopping criteria, penalty function, crossover operator and schema representation have been developed and implemented. Particular attention is paid to reducing the number of user-input optimization parameters. Basic theoretical considerations are developed and used to arrive at minimum acceptable values for the population size and number of generations to consider. As evidenced by the results from several numerical experiments the developed methodologies show promise in terms of efficiency, reliability and accuracy.

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Figure 4 Load Case 3



Figure 7 Final Topology for Both Operators



Figure 8 Ten-Story Building, Top View







Figure 10 Ten-Story Building, 3D View

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