UNIVERSITY OF ILLINOIS
DIGITAL COMPUTER

AUXILIARY
LIBRARY ROUTINE V 16 - 326

TITLE: Irregular Spherical Bessel Functions (Spherical Neumann Functions) (DOI or SADOI)

TYPE: Closed, standard entry, with one program parameter

NUMBER OF WORDS: 29

TEMPORARY STORAGE: 0, 1, 2, 3, 4

DURATION: 19.6 + 3.7N milliseconds

ACCURACY: 7 decimal digits

PRESET PARAMETERS:

S3: during readin of this routine, location 3 must contain the address where the table of functions $n_0(x)$, $n_1(x)$, ... $n_N(x)$ is to be stored.

S4: address of the first word of a closed sine-cosine subroutine which computes 1/2 sin x (usually routine T5-157 will be used).

DESCRIPTION: To compute the Spherical Neumann Functions $n_0(x)$, $n_1(x)$, ... $n_N(x)$, enter this routine with the scaled argument $2^{-5}x$ in the accumulator and with the orders

	-- NF	$1 \leq N \leq 15$
p	50 pF	$0 < x < 32$ (see below).
	26 qF ,	
p+1	-- -	

where q is the location of this routine. When control is returned to the right hand side of p+1, $n_k(x) \cdot 2^{-12}$ will be found in location kS3. The functions n_0 and n_1 are always computed.

METHOD: The functions are computed using the formulas

$$n_0(x) = -\frac{\cos x}{x}$$

$$n_1(x) = -\frac{\cos x}{x^2} - \frac{\sin x}{x}, \text{ and}$$

$$n_{k+1} = \frac{2k+1}{x} n_k - n_{k-1} \text{ for } k \geq 1.$$

Because of the scaling used, all function values must be less than 4096 in absolute magnitude. Since $n_k(x) \to -\infty$ as $x \to 0$, x must be greater than the value given below in order to compute the functions up to a given N:

If N is	Then x must be greater than
1	0.01563
2	0.091
3	0.25
4	0.49
5	0.79
6	1.16
7	1.57
8	2.03
9	2.52
10	3.04
11	3.60
12	4.17
13	4.77
14	5.38
15	6.01

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LOCATION	ORDER		NOTES PAGE 1 V 16
	00K(V16)		
0	40 3F		Store $2^{-5}x$ at 3
	K5 1F	by 14', 15'	
1	42 18L		Plant link
	10 19F		N x 2^{-38}
2	42 26L		Plant 2N x 2^{-39}
	19 5F		
3	50 3F		
	74 28L		
4	00 5F		(x/π)(mod 2), rounded
	40 4F		
5	LJ 4F		
	50 5L		jump to sin-cos subroutine
6	26 S4		for 1/2 cos x
	10 16F		
7	66 3F		$\frac{2^{-17}\cos x}{2^{-5}x} = 2^{-12}\frac{\cos x}{x}$
	S1 S3	by 13'	
8	40 S3		Store $n_0(x)$
	41 27L		clear k count to zero
9	L5 4F		
	50 9L		jump to sin-cos subroutine
10	26 S4		for 1/2 sin x
	10 11F		
11	L0 S3		
	10 5F		$2^{-17}[\frac{\cos x}{x} + \sin x]$
12	66 3F		$\div 2^{-5}x$
	S1 F		
13	40 1S3		store $n_1(x)$
	L5 7L		set addresses
14	42 22L		plant n_{k-1} address
	L4 L		
15	42 20L		plant n_k address
	L4 L		
16	42 24L		plant n_{k+1} address
	F5 27L		step k count

LOCATION	ORDER		NOTES PAGE 2 V 16
17	40 27L		
	F4 27L		
18	L0 26L		2k+1-2N
	32 [ ]F	by 1	exit via link
19	L4 26L		
	50 27L		clear Q
20	10 5F		$(2k+1)\ 2^{-5}$ in Q
	75 [ ]F	by 15	$2^{-17}\ (2k+1)\ n_k$
21	40 F		save MSP
	S5 F		LSP to A
22	50 3F		
	70 [ ]F	by 14	$-[x\ n_{k-1}]\ 2^{-17}$
23	L4 F		$\frac{(2k+1)n_k - xn_{k-1}}{x}$
	66 3F		
24	S5 F		
	40 [ ]F	by 16	Store n_{k+1}
25	F5 22L		Advance addresses
	26 14L		and loop
26	00 F		
	00 [ ]F	by 2	$2N \times 2^{-39}$
27	00 F		
	00 F		$k \times 2^{-39}$
28	28 3046F		
	0J 2964F		$1/\pi = .318309886184$