UNIVERSITY OF ILLINOIS

DIGITAL COMPUTER

AUXILIARY
LIBRARY ROUTINE E 8 - 307

TITLE: Integration by Weddle's Rule

TYPE: Closed

NUMBER OF WORDS: 30

TEMPORARY STORAGE: 0, 1

DURATION: 3.4 millisec.

DESCRIPTION: This routine uses Weddle's Rule to approximate an integral:

$$\frac{1}{b-a}\int_a^b f(x)\,dx \approx \frac{1}{20}\left[f_0 + 5f_1 + f_2 + 6f_3 + f_4 + f5_5 + f_6\right]$$

where

f_n $f(a + nh)$

is the value of $f(x)$ at $x = a + nh$ where $h = \frac{b-a}{6}$ and is tabulated at memory location $t + n$. There are seven function values in all.

ENTRY: This routine is entered by means of the instructions

p	-- tF
	50 pF
p+1	26 --
	-- --

where t is the location of the first tabulated function value f_0.

When control is returned to the right side of p + 1, the approximation to the integral will be in both the accumulator and quotient registers.

ACCURACY: The main error term in Weddle's rule is

$$-\frac{1}{140} h^7 f^{(6)}(\xi),$$

so that results will be exact for a polynomial of fifth degree or less.

EXAMPLE: Use of this routine to compute

$$\int_0^1 \frac{1}{2} \sin \pi x \, dx = \frac{1}{\pi} = .318309886$$

gave the result
.31830127.
Using the same ordinates, integration by Simpson's Rule (code E - 2) gave a value of .31844727, with about 15 times the error of this routine.

NOTE:
1. Because the coefficients in Weddle's Rule are small integers, the intermediate terms in the calculation can be generated without using multiplication or division with the result that this routine is at least three and one-half times faster than the Simpson Method used in code E-2 for the same number of ordinates. The main advantage of this routine is thus its speed.
2. The limits a and b are not needed, since the result is independent of a linear change of variable.
3. Because of the method used, accuracy may be lost for very small integrands.

DATE July 5, 1960
PROGRAMMED BY John Ehrman
APPROVED BY J. N. Snyder

n.

LOCATION	ORDER	NOTES PAGE 1 E 8
	OOK	
0	K5F	
	42 28L	Plant link address
1	46 8L	
	L4 22L	Plant f_0 address
2	46 17L	
	L4 22L	Plant f_1 address
3	46 12L	
	10 20F	Plant f_2 address
4	L4 8L	
	42 23L	Plant f_3 address
5	L4 8L	
	42 9L	Plant f_4 address
6	L4 8L	
	42 18L	Plant f_5 address
7	L4 8L	
	42 13L	Plant f_6 address
8	L5 F	
	10 1F	
9	40 F	
	L5 F	
10	10 1F	
	L4 F	
11	10 1F	
	40 1F	$1/4 \left[f_0 + f_4\right]$
12	L5 F	
	10 1F	
13	40 F	
	L5 F	
14	10 1F	
	L4 F	
15	10 1F	
	L4 1F	
16	10 2F	
	40 1F	$1/16 \left[f_0 + f_2 + f_4 + f_6\right]$

LOCATION	ORDER	NOTES PAGE 2 E 8
17	L5 F	
	10 1F	
18	40 F	
	L5 F	
19	10 1F	
	L4 F	
20	10 1F	
	40 F	
21	10 2F	
	L4 F	$5/16 \left[f_1 + f_5\right]$
22	L4 1F	
	10 1F	$1/32 \left[f_0 + 5f_1 + f_2 + \ + f_4 + 5f_5 + f_6\right]$
23	40 1F	
	L5 F	
24	10 1F	
	40 F	
25	10 1F	
	L4 F	$3/4\ f_3$
26	10 2F	
	L4 1F	$6/32\ f_3$
27	J0 20L	
	66 29L	clear Q (roughly), divide by 5/8
28	S5 F	
	22 F	exit via link
29	50 F	
	00 F	constant = 5/8