

UNIVERSITY OF ILLINOIS
DIGITAL COMPUTER

AUXILIARY
LIBRARY ROUTINE E 1 - 40

TITLE: Integration Routine for $f(x)$ Given at Equal Increments of x
(SADOI or DOI)

TYPE: Closed

NUMBER OF WORDS: 45

TEMPORARY STORAGE: 0 fixed, 42, 43, 44 relative

ACCURACY: Error $< 10^{-7}$

DURATION: 22 + 2n milliseconds

READ AROUND: Adjacent to temporary storage 34
Adjacent to routine 17
Within routine 8

DESCRIPTION: The function, $f(x)$, must be given in a sequence of n values (at least 15) for equal increments, Δx , of the argument, x , and must be such that $-1 \leq f_i < 1$ for all i , $1 \leq i \leq 15 \leq n$ where $f_i = f(x_i)$. Furthermore the range x_1 to x_n must be such that $|x_n - x_1| : |f_i| < 2$.

$\int_{x_1}^{x_n} f(x) dx$ is approximated by the formula*

$$\frac{1}{2} \int_{x_1}^{x_n} f(x) dx = \Delta x [a_1(f_1+f_n) + a_2(f_2+f_{n-1}) + a_3(f_3+f_{n-2}) + a_4(f_4+f_{n-3}) + a_5(f_5+f_{n-4}) + a_6(f_6+f_{n-5}) + a_7(f_7+f_{n-6}) + 1/2(f_8+\dots+f_{n-7})]$$

where the a 's are such that $f(x)$ is approximated by a 6th degree polynomial.

Entry to the subroutine is made via orders

j-1 L5 n-14 L5 A

j L5 B L5 j

j+1 26 m

where $n \geq 15$ is the number of tabulated function values,

A is the address of the first of these, f_1 , B is the address of the increment, $\Delta x = x_{i+1} - x_i$, and m is the address of the first word of the integration routine. Control is returned to the right hand side of j+1 with $1/2 \int_{x_1}^{x_n} f(x)dx$ in R_1 .

*G. Bland and I. Rhodes, "Seven-point Lagrangian Integration Formulas", Math. Table MT 25, U.S. Department of Commerce. 1949

RT: 3/24/60

DATE July 16 1954

CODED BY Clutterham

CHECKED BY R. King

APPROVED BY J.P.Nash

LOCATION	ORDER	NOTES	PAGE 1	E 1
	OOK(EL)			
0	46 30L		Plant link	
	46 11L			
1	46 19L		Plant Δx address	
	10 34L			
2	42 3L			
	14 10L			
3	42 33L			
	15 F	By 2'		
4	42 9L		Plant f_1 address	
	46 44L			
5	10 20F		Plant (h-14)	
	42 44L			
6	15 7L			
	46 9L			
7	15 35L		Initial address for 9	
	41 43L			
8	19 6F			
	40 42L	From 16		
9	50 F	By 6'		
	7J F	By 4		
10	40 2F			
	50 2F			
11	7J F	By 0'	7	
	14 43L		$\sum_{i=1} x (a_i f_i)$	
12	40 43L			
	15 9L			
13	14 34L			
	46 28L			
14	40 9L			
	42 19L			
15	15 42L			
	00 1F			
16	32 8L			
	15 44L	From 23'		

LOCATION	ORDER		NOTES	PAGE 2	E 1
17	LO 34L 32 18L				
18	26 24L 40 44L	from 17'			
19	50 F 7J F	by 14', 23			
20	10 1F 14 43L				
21	40 43L 15 19L				
22	42 28L 14 34L				
23	42 19L 22 16L				
24	19 6F 40 42L	from 18 from 32			
25	19 38F 14 28L				
26	42 28L 10 34L				
27	46 28L 26 28L				
28	50 F 7J F	by 13', 27 by 22, 26			
29	40 OF 50 OF				
30	7J F 14 43L	by 1'			
31	40 43L 15 42L				
32	00 1F 32 24L				
33	15 43L 22 F	by 0			

$$\sum_{i=1}^7 \Delta x (a_i f_i) + 1/2 \sum_{i=8}^{n-7} \Delta x f_i$$

$$\sum_{i=1}^7 \Delta x (a_i f_i) + 1/2 \sum_{i=8}^{n-7} \Delta x f_i$$

$$\sum_{i=1}^7 \Delta x a_i f_i + 1/2 \sum_{i=8}^{n-7} \Delta x f_i + \sum_{n-6}^n \Delta x a_i f_i = 1/2 \int_{x_1}^x f(x) dx$$

Link

LOCATION	ORDER		NOTES	PAGE 3	E 1
34	00 1F 00 1F				
35	00F 0015 2112 2685 19J		l_0 a_1		
36	40F 00 2301 9279 8942 J		a_2		
37	00F 00 2267 3197 7513 J		a_3		
38	40F 00 2357 1428 5714 J		a_4		
39	00F 00 3696 9659 3915 J		a_5		
40	40F 00 0412 3677 2487 J		a_6		
41	00F 00 4943 1630 2910 J		a_7 End of tape		
42	00 F 00 F	Temporary	i		
43	00 F 00 F	Storage	Σ		
44	00 F 00 F	by 3,4,5	$(n-14)_0$		