TITLE   Solution of a Set of n Simultaneous Linear Algebraic

Equations using Magnetic Drum Storage   (DOI or SADOI)

TYPE   Open

NUMBER OF WORDS   162

TEMPORARY STORAGE   9 words at 0 thru 8

$n+1$ words at r_0, r_1, $\ldots$, r_n

$n+2$ words at w_0, w_1, $\ldots$, w_n, w_{n+1}

$\dfrac{n(n+5)}{2}$ words at k_{00}, k_{01}, $\ldots$, k_{on}, s_0

k_{11}, $\ldots$, k_{1n}, s_1

$k_{n-1,n}$, s_{n-1}

Where k_{ij} $(i \leq j)$ is the location of a_{ij}, an element of
the triangularized matrix

and s_i is the location of the sum check of the $(i+1)$'st row
of the triangularized matrix

CAPACITY   143 equations

ACCURACY   Depends on the conditioning of the equations.

PARAMETERS   S3, S4, $\ldots$, SS. During input of the routine, the following
parameters are required

3:      00F 00 r_0F

4:      00F 00 r_nF

5:      00F 00 nF

6:      00F 00 k_{00}F

7:      00F 00 iF

8:      00F 00 w_0F

9:      00F 00 w_nF

K: 00F 00 eF

S: 00F 00 dF

where r_0, r_1, $\ldots$, r_n are used for the results x_0, x_1,
$\ldots$, x_n (see description);

k_{00} is the first location of the reduced triangular matrix
in the electrostatic memory;

i is the address of the first order of the input subroutine;

w_0, w_1, . . ., w_n, w_{n+1} are used for temporary storage;

e is the location to which control is transferred when the routine is finished.

d is the first location of the drum to be used.

DESCRIPTION: This routine will read a set of equations, reduce them to triangular form, and then calculate the solution vector x_0, x_1, . . ., x_{n-1}. If the triangular matrix exceeds location 989, then the successive elements are stored on the drum beginning at d. x_n is placed in location $r_n = r_0 + n$. x_n is the scale factor (a power of ten) such that the roots are x_0/x_n, x_1/x_n, . . ., x_{n-1}/x_n. Although the scale factor will be a power of 1/10 one may alter this property of the routine. If the word in 147, relative, is made 1/r instead of 1/10 the scale factor will be power of 1/r.

SCALING THE DATA: To solve the equations

$$\sum_{j=0}^{n-1} a_{ij} x_j + a_{in} = 0, \quad i = 0, 1, . . ., n-1$$

each of the elements a_{ij} must be scaled to be less than one-half in absolute value. Scaling of the a_{ij} matrix by rows and columns should be done so as to make the magnitudes of the elements as large as possible consistent with having them less than 1/2.

DISCUSSION OF THE METHOD: The method used here is the one used in library routine L1. Several slight changes have been made to keep the maximum amount of precision. First, when an element of a row exceeds one-half, the elements are all multiplied by 4/10 rather than by 1/10 as in L1. Secondly, before a row becomes part of the triangular matrix, the row is scaled up so that the largest element in the row will become greater or equal to a fourth but less than a half.

SUM CHECKS: If a row has been read from the drum inaccurately during the

triangularization, then the program will stop on an FF
instruction. The right hand address will be $(26)_{16}$. If
the sum check fails during back substitution, the right
hand address of the FF instruction will be $(27)_{16}$. The
back substitution sequence can be begun again by flipping
the white switch.

lgr

LOCATION	ORDER		NOTES
	00 K(L6)		
0	41 3F		
	41 6F		Clear Counters
1	41 8F		
	41 4F	From 86	Set i = 0
2	50 S3		
	50 2L		Input row m, $(a_{m0}, a_{m1}, \ldots, a_{mn-1})$ into
			$(r_0, r_1, \ldots, r_{n-1})$
3	26 S7		
	L5 150L		m = 0, 1, ..., n-1
4	40 74L		
	L5 133L		
5	40 21L		
	41 7F	from 82,84	
6	L5 135L		Set initial values
	L4 4F		$w = w_0$
7	42 73L		(for triangularization)
	46 33L		
8	42 22L		
	42 35L		
9	46 39L		
	46 43L		
10	42 47L		
	42 67L		
11	42 69L		
	46 19L		
12	42 28L		
	49 2F		
13	L5 136L		Set initial values $r = r_0$
	L4 4F		
14	40 58L		(for triangularization)
	42 33L		
15	46 45L		
	46 37L		
16	42 34L		
	42 39L		

LOCATION	ORDER		NOTES
17	42 41L		
	L5 134L		Set both addresses in interchange
18	42 43L		order to 1
	46 42L		
19	41 (w)F	by 11	
	L3 8F		
20	50 1023S4		— If last row, jump to 33L
	36 33L		Read row of triangularized matrix
21	J0 F	From 27	into $w_0 \quad w_1, \ldots, w_{n-1}$
	L5 (a_{00})F	by 5,23,25	
22	40. F		
	40 (w)F	by 8,26	
23	F5 21L		
	40 21L		
24	L0 137L		
	36 26L	←	Test to see if storage will exceed location 989
25	L5 138L		in electrostatic memory
	40 21L		
26	F5 22L	from 24	
	40 22L		
27	L0 152L		
	36 21L		
28	L5 F	from 31	
	L0 (w)F	by 12,30	
29	40 F		
	F5 28L		
30	40 28L		
	L0 161L		
31	36 28L		
	L3 F		
32	36 33L		If sum check fails, then stop
	FF 38F		
33	L3 (w)F	by 7, 46 from 20,32	
	L6 (r)F	by 14	Test $\lvert N(r_k) \rvert - \lvert N(w_k) \rvert$
34	36 37L		
	L5 (r)F	by 16	

LOCATION	ORDER		NOTES
35	50 6F		
	66 (w)F	by 8	Divide and arrange
36	47 42L		no interchange
	22 40L		
37	50 (r)F	by 15,92; from 34	
	S3 F		
38	36 40L		Test for zero
	50 140L		Take care of case when $w_i = r$
39	75 (w)F	by 9	Divide and arrange
	66 (r)F	by 16	
40	43 43L		
	S1 F	from 36	
41	40 5F		Store ratios = -k in 5
	L5 (r)F	by 17,51 from 55	
42	40 (1)F	by 18,36	
	50 5F		
43	L5 (w)F	by 9,54	
	40 (1)F	by 18,40	
44	7J 1F		
	L4 F		
45	40 (r)F	by 15,52,93	
	L4 2F		if new r_i is $< -1/2$ or $\geq +1/2$, this fact is recorded by clearing address of 33.
46	36 47L		
	47 33L		
47	L5 1F	from 46	$w_i \rightarrow w_i$ (no interchange)
	40 (w)F	by 10,53	or $r_i \rightarrow w_i$ (interchange)
48	L3 1F		Determine largest element of row
	L6 7F		
49	32 50L		
	L7 1F		
50	40 7F		
	F5 41L	from 49	Step various addresses
51	42 41L		dependent on i, and loop until
	00 20F		after i = n has been done.
52	46 45L		
	F5 47L		

LOCATION	ORDER	NOTES
53	42 47L	
	00 20F	
54	46 43L	
	L0 139L	
55	32 41L	
	43 68L	
56	L5 33L	
	L0 48L	
57	32 61L	
	50 141L	from 61
58	7J (r)F	by 14,60
	40 (r)F	by 14,60
59	L5 58L	
	L4 134L	
60	40 58L	
	L0 142L	
61	32 57L	
	L5 42L	from 57
62	L0 132L	
	36 153L	Determine if interchange has taken place.
63	50 6F	
	F5 68L	from 66
64	42 68L	
	L7 7F	
65	00 1F	
	40 7F	
66	32 63L	
	41 1S9	
67	00 1F	
	L5 (w)F	by 10, 71,73 Scale up elements
68	10 2F	
	00 ( )F	by 55,64
69	32 69L	
	40 (w)F	by 11,72
70	L4 1S9	Compute sum check
	40 1S9	

LOCATION	ORDER	NOTES
71	F5 67L	
	40 67L	
72	42 69L	
	L0 143L	
73	32 67L	
	L5 (w)F	by 7,78; from 79
74	(J0 F) (40 F)	by 4,75,77,155,157 Store row if interchange has taken place.
75	F5 74L	
	40 74L	
76	L0 144L	
	36 78L	
77	L5 145L	
	40 74L	
78	F5 73L	from 76
	42 73L	
79	L0 151L	
	32 73L	
80	L5 4F	from 158
	L4 134L	
81	40 4F	
	L0 3F	
82	36 83L	
	22 5L	
83	40 8F	from 82
	L3 8F	
84	32 5L	
	L5 3F	
85	L4 134L	
	40 3F	
86	L0 146L	
	32 1L	
87	L5 147L	from 115
	40 S4	from 121 $x_n = 1/10$
88	19 37F	
	40 6F	

LOCATION	ORDER	NOTES
89	F4 21L	
	40 7F	
90	L5 20L	
	46 127L	
91	L5 70L	
	LO 134L	
92	LO 134L	from 131
	46 37L	
93	46 126L	
	46 45L	
94	46 124L	
	L5 120L	
95	42 117L	
	L5 115L	
96	46 117L	
	L5 7F	
97	FO 6F	
	LO 138L	
98	32 99L	
	F4 148L	
99	26 100L	
	L4 138L	from 98
100	40 7F	from 99
	40 103L	
101	L5 47L	
	LO 6F	
102	42 110L	
	42 104L	
103	(85 11F)	from 109
	(00 F)	by 100,105,107
104	40 F	
	40 (w)F	by 102
105	F5 103L	
	40 103L	
106	LO 137L	
	36 108L	
107	L5 138L	
	40 103L	

LOCATION	ORDER	NOTES
108	F5 104L	from 106
	42 104L	
109	LO 152L	
	36 103L	
110	L5 F	from 113
	LO (w)F	by 102,112
111	40 F	
	F5 110L	
112	42 110L	
	LO 161L	
113	36 110L	
	L3 F	
114	36 116L	
	FF 39F		Test sum check
115	50 S9		If sum check has failed, back substitution can
	26 87L		begin again by flipping white switch.
116	50 F	from 114,123
	S5 F	
117	50 (w)F	by 96,122	Evaluate scaler product
	74 (r)F	by 95,122	$\sum\limits_{s=i+1}^{n} w_s x_s$
118	L4 F	
	40 F	
119	L4 2F	
	32 121L	
120	50 147L	from 125	If we are liable to exceed capacity
	7J S4		reduce x_n by 10
121	22 87L	
	L5 117L	from 119
122	LO 134L	
	40 117L	
123	LO 37L	
	32 116L	
124	L3 (w_i)F	by 94
	L6 F	
125	36 120L	
	L5 F	

LOCATION	ORDER		NOTES
126	66 (w_1)F	by 93	
	81 F		
127	40 (r_1)F	by 90,128	
	L5 127L		
128	L0 134L		
	46 127L		
129	L0 149L		
	36 SK		link
130	F5 6F		
	40 6F		
131	L5 37L		
	26 92L		
132	N0 1F		
	00 F		
133	J0 F		
	L5 86		
134	00 1F		
	00 1F		
135	L7 88		
	L2 88		
136	7J 83		
	40 83		
137	50 F		
	L5 990F		
138	85 11F		
	00 88		
139	N0 189		
	00 F		
140	7L 4095F		
	LL 4095F		
141	00 F		
	00 4000 0000 0000J		
142	LJ 184		
	40 184		
143	80 1F		
	L5 189		

LOCATION	ORDER	
144	50 F	
	40 990F	
145	86 11F	
	00 SS	
146	80 S5	
	00 S5	
147	00 F	
	00 1000 0000 0000J	
148	J0 F	
	L5 989F	
149	N0 S3	
	L5 F	
150	J0 F	
	40 S6	
151	S2 67L	
	L5 2S9	
152	N0 F	
	40 2S9	
153	L5 159L	from 62
	L0 4F	
154	10 20F	
	L4 74L	
155	40 74L	
	L0 144L	
156	36 158L	
	L4 160L	
157	L4 145L	
	40 74L	
158	26 80L	from 156
	00 F	
159	00 2S5	
	00 2S5	
160	80 F	
	00 F	
161	75 F	
	L0 1S9	