HANDBOOK OF AUTOMATION COMPUTATION AND CONTROL

VOLUME 3

HANDBOOK OF AUTOMATION, COMPUTATION, AND CONTROL

Volume 3

SYSTEMS AND COMPONENTS

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HANDBOOK OF AUTOMATION, COMPUTATION, AND CONTROL

Volume 3

SYSTEMS AND COMPONENTS

Prepared by a Staff of Specialists

Edited by

EUGENE M. GRABBE

SIMON RAMO

DEAN E. WOOLDRIDGE

Thompson Ramo Wooldridge Inc.

Los Angeles, California

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CONTRIBUTORS

G. S. AXELBY, *Westinghouse Electric Corporation, Baltimore, Maryland* (Chapter 22)

C. W. BAILEY, *Consolidated Electrodynamics Corporation, Pasadena, California* (Chapter 24)

E. V. BERSINGER, *Radio Corporation of America, Van Nuys, California* (Chapter 19)

P. S. BUCKLEY, *E. I. du Pont de Nemours and Company, Inc., Wilmington, Delaware* (Chapter 7)

N. COHN, *Leeds and Northrup Company, Philadelphia, Pennsylvania* (Chapter 17)

M. E. CONNELLY, *Massachusetts Institute of Technology, Cambridge, Massachusetts* (Chapter 1)

P. E. A. COWLEY, *Shell Development Company, Emeryville, California* (Chapter 9)

R. O. DECKER, *Westinghouse Electric Corporation, Pittsburgh, Pennsylvania* (Chapter 25)

D. P. ECKMAN, *Case Institute of Technology, Cleveland, Ohio* (Editor, Part D)

L. J. FOGEL, *Astronautics Division, General Dynamics, San Diego, California* (Chapter 2)

A. S. FULTON, *Thompson Ramo Wooldridge Inc., Canoga Park, California* (*Editor, Part F; Chapters 18 and 23*)

E. F. HOLBEN, *Conoflow Corporation, Philadelphia, Pennsylvania* (Chapter 8)

E. M. GRABBE, *Compagnie Europeénne d'Automatisme Electronique, Paris, France* (Chapter 14)

T. R. JAMES, *General Mills, Inc., Minneapolis, Minnesota* (Chapter 3)

D. L. JOHNSTON, *Urwick-Orr and Partners, Hertfordshire, England* (Chapter 4)

R. E. KALMAN, *International Business Machines, Baltimore, Maryland* (Chapter 12)

A. G. KEGEL, *Westinghouse Electric Corporation, Baltimore, Maryland* (Chapter 22)

C. G. LASPE, *Thompson Ramo Wooldridge Products Company, Beverly Hills, California* (Chapter 11)

I. LEFKOWITZ, *Case Institute of Technology, Cleveland, Ohio* (Chapter 13)

J. LYMAN, *University of California, Los Angeles, California* (Chapter 2)

R. O. MAZE, *Minneapolis-Honeywell Regulator Company, Minneapolis, Minnesota* (Chapter 21)

J. M. MOZLEY, *The Johns Hopkins Hospital, Baltimore, Maryland* (Chapter 7)

G. F. PITTMAN, JR., *Westinghouse Electric Corporation, Pittsburgh, Pennsylvania* (Chapter 25)

J. E. RIJNSDORP, *Royal Dutch Shell, Amsterdam, Holland* (Chapter 10)

J. ROSENBERG, *University of California, Los Angeles, California* (Chapter 6)

J. S. SABY, *General Electric Company, Cleveland, Ohio* (Chapter 26)

J. A. SARGROVE, *Automation Consultants and Associates, Ltd., London, England* (Chapter 4)

A. J. SCHENK, *Jervis B. Webb Company, Detroit, Michigan* (Chapter 5)

M. A. SCHULTZ, *Westinghouse Electric Corporation, Pittsburgh, Pennsylvania* (Chapter 16)

W. E. SHOUPP, *Westinghouse Electric Corporation, Pittsburgh, Pennsylvania* (Chapter 16)

L. M. SILVA, *Beckman Instruments, Inc., Anaheim, California* (Chapter 15)

A. P. STERN, *General Electric Company, Syracuse, New York* (Chapter 27)

M. E. STICKNEY, *Lockheed Aircraft Corporation, Sunnyvale, California* (Chapter 20)

T. M. STOUT, *Thompson Ramo Wooldridge Products Company, Beverly Hills, California* (Chapter 11)

J. WALKER, *Librascope, Burbank, California* (Chapter 24)

W. G. WING, *Sperry Gyroscope Company, Great Neck, New York* (Chapter 28)

FOREWORD

The proliferation of knowledge now makes it most difficult for scientists or engineers to keep ahead of change even in their own fields, let alone in contiguous fields. One of the fields where recent change has been most noticeable, and in fact exponential, has been automatic control. This three-volume *Handbook* will aid individuals in almost every branch of technology who must constantly refresh their memories or refurbish their knowledge about many aspects of their work.

Automation, computation, and control, as we know them, have been evolving for centuries, but within the last generation their impact has been felt in nearly every segment of human endeavor. Feedback principles were exploited by Leonardo da Vinci and applied by James Watt. Some of the early theoretical work of importance was contributed by Lord Kelvin, who also, together with Charles Babbage, pointed the way to the development of today's giant computational aids. Since about the turn of the present century, the works of men like Minorsky, Nyquist, Wiener, Bush, Hazen, and Von Neumann gave quantum jumps to computation and control. But it was during and immediately following World War II that quantum jumps occurred in abundance. This was the period when theories of control, new concepts of computation, new areas of application, and a host of new devices appeared with great rapidity. Technologists now find these fields charged with challenge, but at the same time hard to encompass. From the activities of World War II such terms as servomechanism, feedback control, digital and analog computer, transducer, and system engineering reached maturity. More recently the word automation has become deeply entrenched as meaning something about the field on which no two people agree.

Philosophically minded technologists do not accept automation merely as a third Industrial Revolution. They see it, as they stand about where the editors of this *Handbook* stood when they projected this work, as a manifestation of one of the greatest *Intellectual Revolutions in Thinking* that has occurred for a long time. They see in automation the natural consequences of man's urge to exploit modern science on a wide front to perform useful tasks in, for example, manufacturing, transportation, business, physical science, social science, medicine, the military, and government. They see that it has brought great change to our conventional way of thinking about the human use of human beings, to quote Norbert Wiener, and in turn about how our engineers will be trained to solve tomorrow's engineering problems. They even see that it has precipitated some deep thinking on the part of our industrial and union leadership about the organization of workers in order not to hold captive bodies of workmen for jobs that automation, computation, and control have swept or will soon sweep away.

Perhaps the important new face on today's technological scene is the degree to which the broad field needs codification and unification in order that technologists can optimize their role to exploit it for the general good. One of the early instances of organized academic instruction in the field was at The Massachusetts Institute of Technology in the Electrical Engineering Department in September 1939, as a course entitled Theory and Application of Servomechanisms. I can well recollect discussions around 1940 with the late Dr. Donald P. Campbell and Dr. Harold L. Hazen, which led temporarily to renaming the course Dynamic Analysis of Automatic Control Systems because so few students knew what "servomechanisms" were. But when the GI's returned from war everybody knew, and everybody wanted instruction. Since that time engineering colleges throughout the land have elected to offer organized instruction in a multitude of topics ranging from the most abstract mathematical fundamentals to the most specific applications of hardware. Textbooks are available on every subject along this broad spectrum. But still the practicing control or computer technologist experiences great difficulty keeping abreast of what he needs to know.

As organized instruction appeared in educational institutions, and as industrial activity increased, professional societies organized groups in the areas of control and computation to meet the needs of their members to tell one another about technical advances. Within the past five years several trade journals have undertaken to report regularly on developments in theory, components, and systems. The net effect of all this is that the technologist is overwhelmed with fragmentary, sometimes contradictory, redundant information that comes at him at random and in many languages. The problem of assessing and codifying even a portion of this avalanche of knowledge is beyond the capabilities of even the most able technologist.

The editors of the *Handbook* have rightly concluded that what each technologist needs for his long-term professional growth is *to have a body of knowledge that is negotiable at par in any one of a number of related* *fields for many years to come.* It would be ideal, of course, if a college education could give a prospective technologist this kind of knowledge. It is in the hope of doing this that engineering curricula are becoming more broadly based in science and engineering science. But it is unlikely that even this kind of college training will be adequate to cope with the consequences of the rapid proliferation of technology as is manifest in the area of automation, computation, and control. Hence, handbooks are an essential component of the technical literature when they provide the unity and continuity that are requisite.

I can think of no better way to describe this *Handbook* than to say that the editors, in both their organization of material and selection of substance, have given technologists a unified work of lasting value. It truly represents today's optimum package of that body of knowledge that will be negotiable at par by technologists for many years to come in a wide range of disciplines.

GORDON S. BROWN
Dean, School of Engineering
Massachusetts Institute of Technology

PREFACE

Accelerated advances in technology have brought a steady stream of automatic machines to our factories, offices, and homes. The earliest automation forms were concerned with doing work, followed by the controlling function, and recently the big surge in automation has been directed toward data handling functions. New devices ranging from digital computers to satellites have resulted from military and other government research and development programs. Such activity will continue to have an important impact on automation progress.

One of the pressures for the development of automation has been the growing complexity and speed of business and industrial operations. But automation in turn accelerates the tempo of whatever it touches, so that we can expect future systems to be even larger, faster, and more complex. While a segment of engineering will continue to mastermind, by rule of thumb procedures, the design and construction of automatic equipment and systems, a growing percentage of engineering effort will be devoted to activities that may be classified as *problem solving*. The activities of the problem solver involve analysis of previous behavior of systems and equipment, simulation of present situations, and predictions about the future. In the past, problem solving has largely been practiced by engineers and scientists, using slide rules and hand calculators, but with the advent of large-scale data processing systems, the range of applications has been broadened considerably to include economic, government, and social activities. Air traffic control, traffic simulation, library searching, and language translation are typical of the problems that have been attacked.

This *Handbook* is directed toward the problem solvers—the engineers, scientists, technicians, managers, and others from all walks of life who are concerned with applying technology to the mushrooming developments in automatic equipment and systems. It is our purpose to gather together in one place the available theory and information on general mathematics, feedback control, computers, data processing, and systems design. The emphasis has been on practical methods of applying theory, new techniques and components, and the ever broadening role of the electronic computer. Each chapter starts with definitions and descriptions aimed at providing perspective and moves on to more complicated theory, analysis, and applications. In general, the *Handbook* assumes some engineering training and will serve as an information source and refresher for practicing engineers. For management, it will provide a frame of reference and background material for understanding modern techniques of importance to business and industry. To others engaged in various ramifications of automation systems, the *Handbook* will provide a source of definitions and descriptive material about new areas of technology.

It would be difficult for any one individual or small group of individuals to prepare a handbook of this type. A large number of contributors, each with a field of specialty, is required to provide the engineer with the desired coverage. With such a broad field, it is difficult to treat all material in a homogeneous manner. Topics in new fields are given in more detail than the older, established ones since there is a need for more background information on these new subjects. The organization of the material is in three volumes as shown on the inside cover of the *Handbook*. Volume 1 is on *Control Fundamentals*, Volume 2 is concerned with *Computers and Data Processing*, and Volume 3 with *Systems and Components*.

In keeping with the purpose of this *Handbook*, Volume 1 has a strong treatment of general mathematics which includes chapters on subjects not ordinarily found in engineering handbooks. These include sets and relations, Boolean algebra, probability, and statistics. Additional chapters are devoted to numerical analysis, operations research, and information theory. Finally, the present status of feedback control theory is summarized in eight chapters. Components have been placed with systems in Volume 3 rather than with control theory in Volume 1, although any discussion of feedback control must, of necessity, be concerned with components.

The importance of computing in research, development, production, real time process control, and business applications has steadily increased. Hence, Volume 2 is devoted entirely to the design and use of analog and digital computers and data processors. In addition to covering the status of knowledge today in these fields, there are chapters on unusual computer systems, magnetic core and transistor circuits, and an advanced treatment of programming. Volume 3 emphasizes systems engineering. A part of the volume covers techniques used in important industrial applications by examining typical systems. The treatment of components is largely concerned with how to select components among the various alternates, their mathematical description, and their integration into systems. There is also a treatment of the design of components of considerable importance today. These include magnetic amplifiers, semiconductors, and gyroscopes.

We consider this *Handbook* a pioneering effort in a field that is steadily pushing back frontiers. It is our hope that these volumes will not only provide basic information on new fields, but will also inspire work and further research and development in the fields of automatic control. The editors are pleased to acknowledge the advice and assistance of Professor Gordon S. Brown and Professor Jerome S. Wiesner of the Massachusetts Institute of Technology, and Dr. Brockway McMillan of the Bell Telephone Laboratories, in organizing the subject matter. To the contributors goes the major credit for providing clear, thorough treatments of their subjects. The editors are deeply indebted to the large number of specialists in the control field who have aided and encouraged this undertaking by reviewing manuscripts and making valuable suggestions. Many members of the technical staff and secretarial staff of Thompson Ramo Wooldridge Inc. and the Ramo-Wooldridge Division have been especially helpful in speeding the progress of the *Handbook*.

EUGENE M. GRABBE
SIMON RAMO
DEAN E. WOOLDRIDGE

August 1961

CONTENTS

G. DESIGN OF COMPONENTS

SYSTEMS ENGINEERING

Systems Design

M. E. Connelly

1. SCOPE OF CONTROL SYSTEM APPLICATIONS

A *control system* is defined as an integrated complex of devices that governs or regulates a process or an operation. In many cases, it is difficult to delineate sharply between the system being controlled and the control system. Often the two are so interdependent that they must be designed as a composite unit, in which case the distinction becomes academic. Control systems may or may not require human participation. In addition, they may or may not be responsive to the state of the process or operation under control.

The scope of control system applications is extremely diversified and is expanding rapidly as more industries become aware of the possibilities of control techniques. These possibilities may be listed briefly as follows:

Reduction in manpower required.
Greater production capacity.

Increased production flexibility.
Lower production costs, higher efficiency.
Improved quality control, product standardization.
Shorter lead times, inventory reduction.
Safety.
Elimination of monotonous human operations.
Improved performance: power amplification, fast response, accuracy, rapid coordination of multiple factors.
Operation under adverse conditions.
Increased equipment utilization.
Easier production control.

In some applications, such as the control and guidance of high-speed missiles, there is no alternative to the use of automatic devices if the required performance is to be achieved. When faced by a multiplicity of operations or the need for rapid response, human operators simply do not measure up to the task. In other cases, operations or processes have been automatized because it was the most satisfactory or the most efficient way to achieve a given result. The introduction of control techniques has in some measure freed production from the limitations of the human operator and has opened new possibilities for product and process simplification.

To indicate the wide variety of fields in which control systems are being utilized, Table 1 lists a few representative applications. Several complex systems are treated in detail in the chapters that follow (Refs. 1 to 9).

TABLE 1. REPRESENTATIVE CONTROL SYSTEM APPLICATIONS

Automatic Machines. Numerically controlled milling machines, automatic electronic assembly lines, self-regulated rolling mills, engine block production lines, program-controlled lathes, automatic inspection and quality control devices, material-handling automata, packaging and bottling machines

Communications. Dial telephone systems, test range communications

Transportation. Automatic railroad freight-sorting yards, pipeline controls, power distribution control, air traffic control systems, autopilot and landing devices, navigation aids, ship stabilizers

Process Control. Chemical plants, nuclear controls, petroleum refineries, distilleries

Military. Fire-control systems (airborne, shipboard, and ground-based), missile stabilization and guidance, air defense control systems, training simulators

Research and Development. Diffraction grating rulers, x-ray positioners, iron-lung regulators, synthetic human organs (heart, kidney), automatic spectrometers

2. EDUCATIONAL REQUIREMENTS

In order to cope with the control system problems that arise in such fields as those listed in Table 1, the system designer must master a variety of skills. Since it involves the techniques of a number of the engineering and scientific disciplines, control system design demands a broad understanding of basic physical principles and a thorough working knowledge of practical components. To emphasize this requirement, a list of representative topics that might be included in the training of system designers is presented in Table 2. The breadth of these studies and the extensive

TABLE 2. REPRESENTATIVE BACKGROUND FOR CONTROL SYSTEM DESIGN

Mathematics	*Engineering*
Vector analysis	Circuit theory and network synthesis
Laplace transform and Fourier analysis	Applied electronics
Functions of a complex variable	Feedback control
Differential and integral equations	Energy conversion
Probability and statistics	Hydraulics
Numerical analysis	Pneumatics
Advanced algebra	Principles of radar
Information theory	Machine design
Operations research and game theory	Chemical engineering
Basic Science	Measurement and instrumentation
Classical and statistical mechanics	Switching circuits
Thermodynamics and heat	Digital computing techniques
Optics	Analog computing techniques
Electromagnetic theory	Pulse circuits
Atomic, molecular, and nuclear physics	Nonlinear mechanics
Geophysics	Aerodynamics
Astrophysics	Metallurgy
Acoustics	Heat engineering
	Solid state devices

scope of control system applications illustrate that, in order to do even a very little in the field, one must know a great deal. Moreover, this strong academic background must be supplemented by a high degree of practical, mechanical ability.

In general, however, each control system problem is unique and the background demanded of the designer varies accordingly. It is hardly likely that any one control engineer is expert on all the subjects listed in Table 2.

3. FORMULATION OF THE DESIGN PROBLEM

Design procedures for control systems vary from problem to problem and any suggested approach, such as the one that follows, can be treated only as a rough guide that must be modified to suit specific control situations. Procedural patterns in control work recur frequently enough, however, to warrant the presentation of a generalized design procedure.

Problem Definition. The first task facing the designer is to define his problem precisely or even to perceive that a problem exists. The statement of the problem may be specific or may be so indeterminate that it can be expressed only in statistical terms. For example, the problem might be to perform a fixed set of operations, as in a bottling machine, or to maintain a sequence of specified conditions, as in a chemical process. Other control systems are called upon to adapt themselves to a variety of changing circumstances, in which case the statement of the problem involves the determination of the range of these conditions. In many cases, future, as well as present, requirements must be specified. The planning of military systems is extremely difficult in this respect in that every weapons system requires an estimate of what the enemy capabilities will be several years in the future. The problem, in this case, is a matter of speculation.

Most nonmilitary control problems can be formulated with some degree of precision, although even here it is not uncommon for design specifications to be based on estimated requirements. The capacity of an automatic freightyard, for example, would depend on the railroad's expected future traffic situation.

Typical of the data that the designer tries to establish at the outset are inputs, outputs, overall performance requirements, environment, economic factors, and time schedules. These are the basic ingredients of the problem.

Operations Research. The relatively new discipline of operations research can be used to advantage at this stage of the planning, particularly in translating a vague, functional requirement into quantitative terms. As an illustration, in designing an air traffic control system for a metropolitan area one would naturally have to specify the capacity of the system (see Ref. 1). From aircraft manufacturing data, Federal Aviation Agency route plans, military and airline traffic estimates, and from current airport operational data, an estimate could be made of the future traffic situation. If the expected average rate of aircraft arrivals to the area is Q_A, and the average rate at which the airport facilities can land planes is Q_L, it is possible to compute the probability P_n that n aircraft will be waiting to land when servicing has reached an equilibrium. By the queueing theory of operations research (see Vol. 1, Chap. 15, Operations Research, Sect. 5, Waiting Time Models)

$$P_n = \left(\frac{Q_A}{Q_L}\right)^n \left(1 - \frac{Q_A}{Q_L}\right), \qquad \frac{Q_A}{Q_L} \leq 1. \tag{1}$$

The mean number of planes waiting to land will be

$$W = \sum_{n=0}^{\infty} nP_n = \frac{Q_A/Q_L}{1 - (Q_A/Q_L)}. \tag{2}$$

Figure 1 shows the variation of the mean number of planes waiting to land, W, with the ratio Q_A/Q_L. Before undertaking such an analysis, one might intuitively assume that a landing capacity Q_L equal to the average rate of arrival Q_A would be adequate. However, from Fig. 1 it is clear that a much greater landing capacity is required to prevent the incoming traffic from saturating the system. In cases such as this, a quantitative analysis can often rescue the intuition from major blunders. Unfortunately, the converse is occasionally true. A poorly conceived analysis may also lead common sense astray.

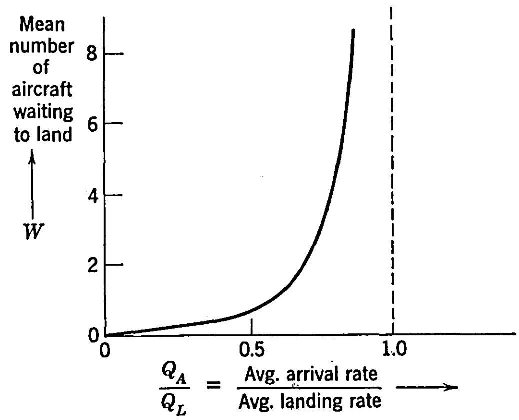


FIG. 1. Queued aircraft as a function of the ratio of arrival rate to landing capacity.

Setting Limits. In formulating a problem, care must be exercised to avoid expanding it beyond its efficient limits. In lieu of a thorough study of the real requirements for a system, there is also a temptation to set excessively stringent specifications in the hope that all possible contingencies will be adequately covered. On the other hand, a more serious error is to understate the problem. Similarly, the partial treatment of a problem often has only limited usefulness. For example, the design of a traffic control system to coordinate the arrival of 200 aircraft into an area per hour would be of little use if a landing system having a capacity of 20 planes per hour were retained. These two problems must be treated as an integrated whole. In fact, the modern emphasis on the overall systems approach to complex problems originated in the proven inadequacy of piecemeal attacks.

Importance. It would be difficult to overemphasize the importance of a well-conceived statement of the problem in control system design. Often this statement more or less completely determines the nature of the design, the cost, and the ultimate effectiveness of the system. In many cases, additional effort spent on this initial planning can prevent a control system from being stillborn.

4. SYSTEM FUNCTIONS

Simple Sequence Control. Having defined the problem, the designer next outlines the operations necessary to cope with it. In some applications, where the problem might consist simply of a sequence of functions to be performed, these two steps are closely related.

EXAMPLE. A typical functional sequence can be listed for the automatic machine tool shown in Fig. 2. This machine automatically loads, rough

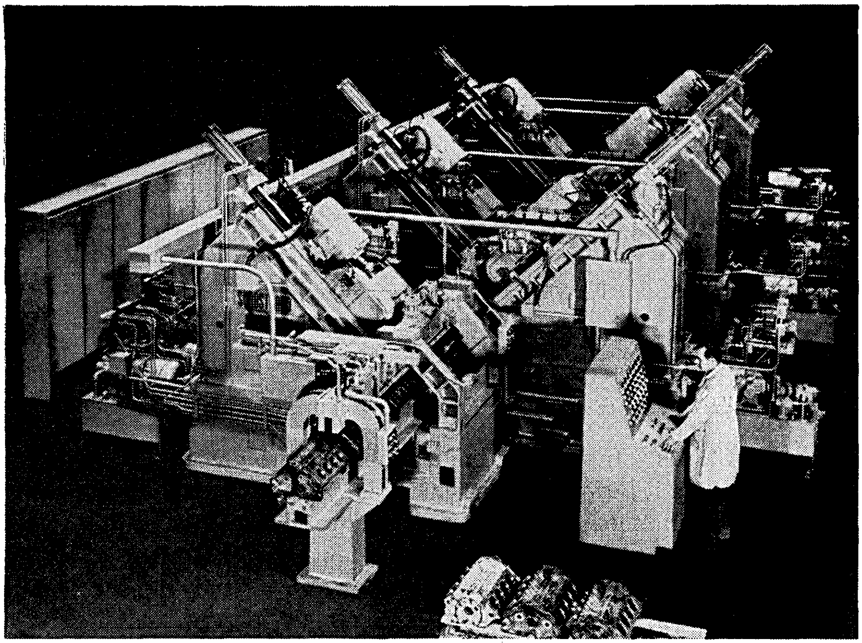


FIG. 2. Rough boring unit for engine blocks. (Courtesy T. C. Cameron, Sundstrand Machine Tool Co.)

bores, chamfers, transfers, and unloads engine blocks. At the same time it performs the auxiliary functions of lubrication and chip removal. The functional cycle is as follows:

1. The transfer bar lowers to engage work.
2. The transfer bar advances and moves each part to the next station.
3. The locating pins in each fixture rise.
4. The clamps lower to secure the part.

5. The transfer bar raises, then returns; simultaneously all heads start rapid approach.
6. Heads feed individually.
7. Heads return rapidly individually.
8. The locating pins drop, and the clamps rise.
9. The cycle is repeated if a new part is available and the finished part has been removed from the unload station.

A system of limit switches, solenoid valves, clamps, locating pins, and transfer devices positions the engine blocks in sequence and actuates the feed and withdrawal of the machine heads. A limit switch is required at the end of each motion and at any point in the cycle where a machine member stops, starts, or changes rate. From the time sequence of these functions, the designer can draw up a cycle diagram showing the order in which the operations take place. Figure 3 illustrates such a diagram for the rough bore machine (see Ref. 6).

Control Logic. Although the rough bore cycle can be interrupted by malfunctions or by manual intervention, this machine generally illustrates a large class of special purpose control systems for which the operation is a simple sequence of specified steps. The logic controlling such machines can be considerably more complex than the elementary example just cited, and operations based on position, time, and arbitrary combinations of conditions can be instrumented by using switching circuits. Control systems can even be designed with the ability to choose between alternate modes of operation depending on the circumstances. In Boolean notation, one can express a typical decision as follows. (See Vol. 2, Chap. 17.)

$$(A + B)\cdot C = D, \tag{3}$$

$$(A + B)\cdot \bar{C} = E. \tag{4}$$

In words, these equations state that if condition A *or* condition B exists *and* if condition C also exists, then response D will be activated. However, if condition A *or* condition B exists *and* condition C does *not* exist, then response E will be activated. The switching circuit for implementing this decision is shown in Fig. 4. When complex logical nets are built up using basic *and-or* elements, these switching circuits can often be greatly simplified by algebraic manipulation of the Boolean equations. (See Vol. 2, Chap. 17 for a table of Boolean equivalences.) To illustrate this point, note the simplification of the following Boolean equation.

$$A\bar{B} + AB = A(\bar{B} + B) = A. \tag{5}$$

The corresponding switching circuits are also shown in Fig. 4.

Programmed Control. More flexible control systems than the fixed

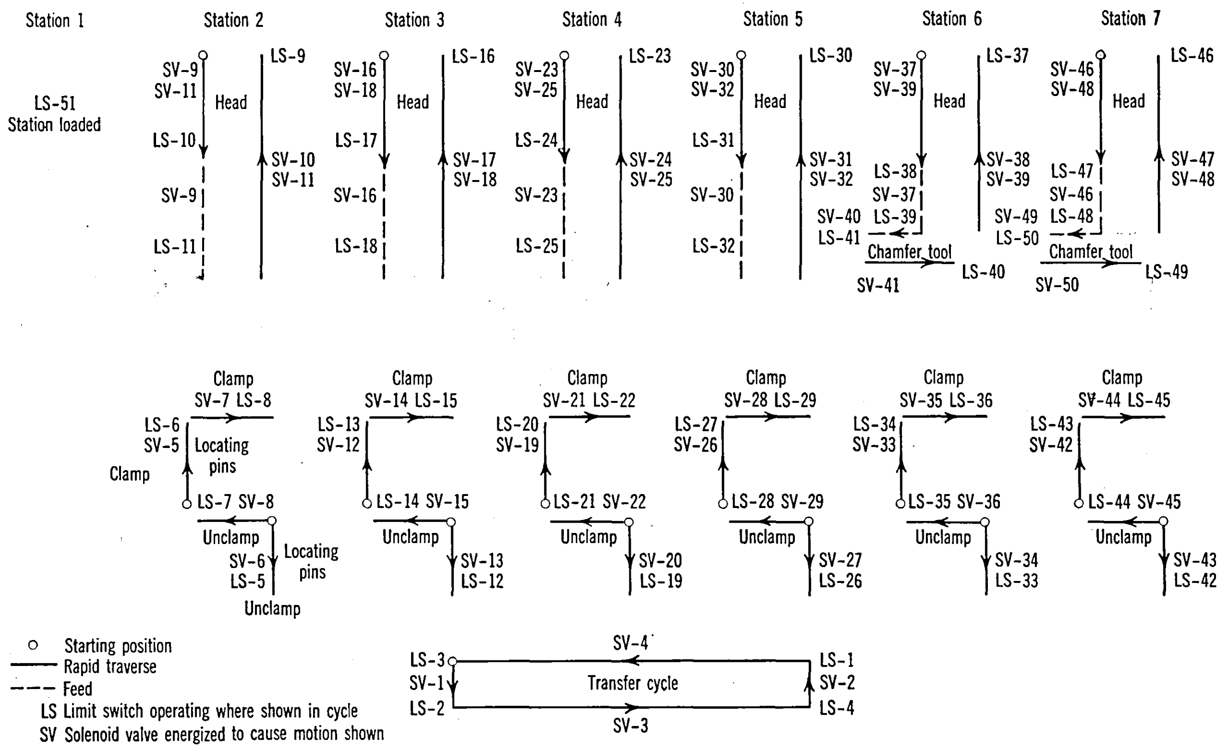


FIG. 3. Cycle diagram showing machine sequence controls. (Courtesy T. C. Cameron, Sundstrand Machine Tool Company.)

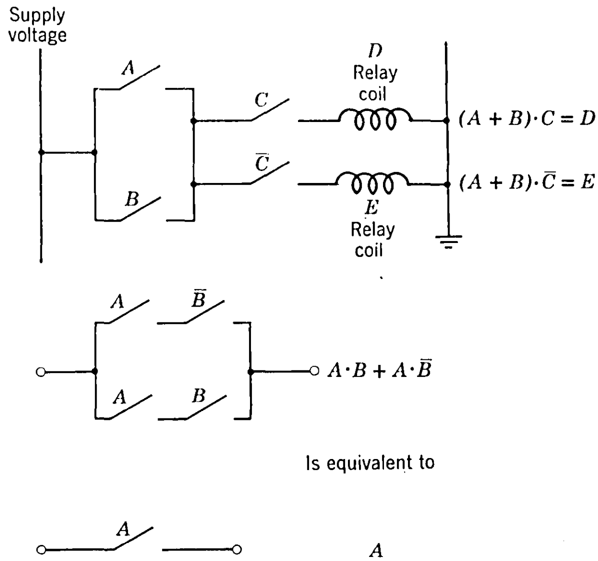


FIG. 4. Simple switching logic circuits.

logic machines just discussed are possible if the sequence of operations is controlled by a program of instructions which can be read or set into the system. In these cases, the system must be designed to accommodate a range of instructions and performance requirements. By simply revising the program, one can change the functions of the system.

Continuous Control. In contrast to the fixed logic and programmed control systems above, there is a more sophisticated class of controls that continuously and automatically adjust themselves to the state of the process or operation being controlled. Process controls in which the state of the process is monitored and in which these data are used to regulate the operations of the process are typical of this class. An autopilot that must stabilize the orientation of an aircraft in space under such conditions as atmospheric turbulence is a second illustration. In this case, the autopilot function is to detect deviations of the aircraft heading and orientation from the desired state and to actuate the control surfaces of the aircraft so as to reduce this deviation to zero.

In the process industry, the basic functions have been classified, and Table 3 lists these so-called *unit operations* and *unit processes*. It is common practice to exercise control over each of these operations individually, rather than attempt the integrated control of multiple functions. See Chap. 3, Automatic Machines; Chap. 7, Instrumentation Systems; and Chap. 10, Single and Multiple Loop Controls.

TABLE 3. UNIT FUNCTIONS IN PROCESS CONTROL (Ref. 7)

Combustion	Esterification	Diazotization and coupling
Oxidation	Reduction	Fermentation
Neutralization	Ammonolysis	Pyrolysis (cracking)
Silicate formation	Halogenation	Aromatization
Causticization	Sulphonation	Isomerization
Electrolysis	Hydrolysis	Acylation
Double decomposition	Alkylation	Oxo reaction
Calcination	Friedel-Crafts	Mechanical separation
Dehydration	Condensation	Size reduction
Nitration	Humidification and cooling	Size enlargement
Fluid flow	Drying	Mixing
Heat transmission	Adsorption	High-pressure techniques
Evaporation	Solvent extraction and dialysis	Movement and storage of materials
Distillation and sublimation	Polymerization	
Gas absorption		

Criteria for Control. In the design of many complex control systems, the desired functioning of the system may not be at all obvious. In the case of the air traffic control problem previously cited, for example, the designer must decide what the most efficient sequence of functions would be, let us say, to maximize the rate of landing aircraft. Confronted by a problem of this complexity, intuition alone is usually inadequate, and recourse to the formal mathematical techniques of operations research may be necessary. The mathematical model here would have to consider incoming and outgoing routes, altitudes, aircraft speeds and endurance, local geographic features, Federal Aviation Agency regulations, and waiting procedures, all under a variety of weather and traffic conditions. A single functional sequence would be inadequate under such circumstances, and the control system would have to be capable of several alternate modes depending on the situation.

In every case, stating the functional requirements for a system implies a quantitative specification of how well these functions must be performed. These functional specifications are the basis for the unit and component specifications to be established later in the design.

5. DETAILED SYSTEM DESIGN

System Block Diagram

Having established what the control system is to do, the designer next translates this concept into a system block diagram. This is essentially the creative stage in the history of the design. There are innumerable ways to solve a given control system problem, and the designer must decide which of these approaches seems to offer the simplest means of achieving the required performance. There is an element of truth in the statement that almost any result can be achieved by the brute force deployment of hardware, hence to some extent the success of the designer can be measured by the relative simplicity of his design. Practically, from the point of view of reliability alone, an unnecessarily complex system often defeats its own purpose.

Block diagrams exist at various levels of detail. They can be used to divide the functions of the system into logical subsidiary operations, to indicate the flow of information throughout the system, or to represent the system dynamics schematically. A typical preliminary system block diagram for the numerically controlled milling machine developed at the Massachusetts Institute of Technology is shown in Fig. 5 (see Ref. 8). Other block diagrams are presented throughout this volume.

The system block diagram is normally in a state of constant evolution and becomes more detailed and specific as the unit designs develop. Each diagram incorporates a host of major engineering decisions, particularly with respect to the alternative techniques for performing various operations.

System Simplifications. In addition to specifying the essential features of the design, the system block diagram reduces a single, large problem to a set of simpler unit problems, each of which can be assigned to an individual or group for solution. In this fashion, the detailed design of the various units can proceed in parallel, and the design responsibility can be shared by a number of engineers. The interrelation between these units and the setting of unit design criteria consistent with the overall system specifications remain the responsibility of the systems designer.

Design Decisions. In the selection of the key techniques and components for the system, the designer had best have on hand what can only be described as an ample bag of tricks. A file of catalogues of commercial equipment and a library of technical books and journals constitute only part of the requirement. There is no substitute for ingenuity and actual experience with the techniques for performing a multitude of operations (see Ref. 9). Typical decisions that might be made at this stage of the system design are listed in Table 4.

Economics. Generally speaking, all control systems must pay their own way, that is to say, the benefits must be worth the cost. Even military systems must demonstrate that they produce more offensive or defensive capability per dollar spent than alternative systems. Normally, these economic factors are considered before a design is initiated, but quantitative data on the system are usually sketchy at the beginning, and a reevaluation of the system concept and the economic factors is often

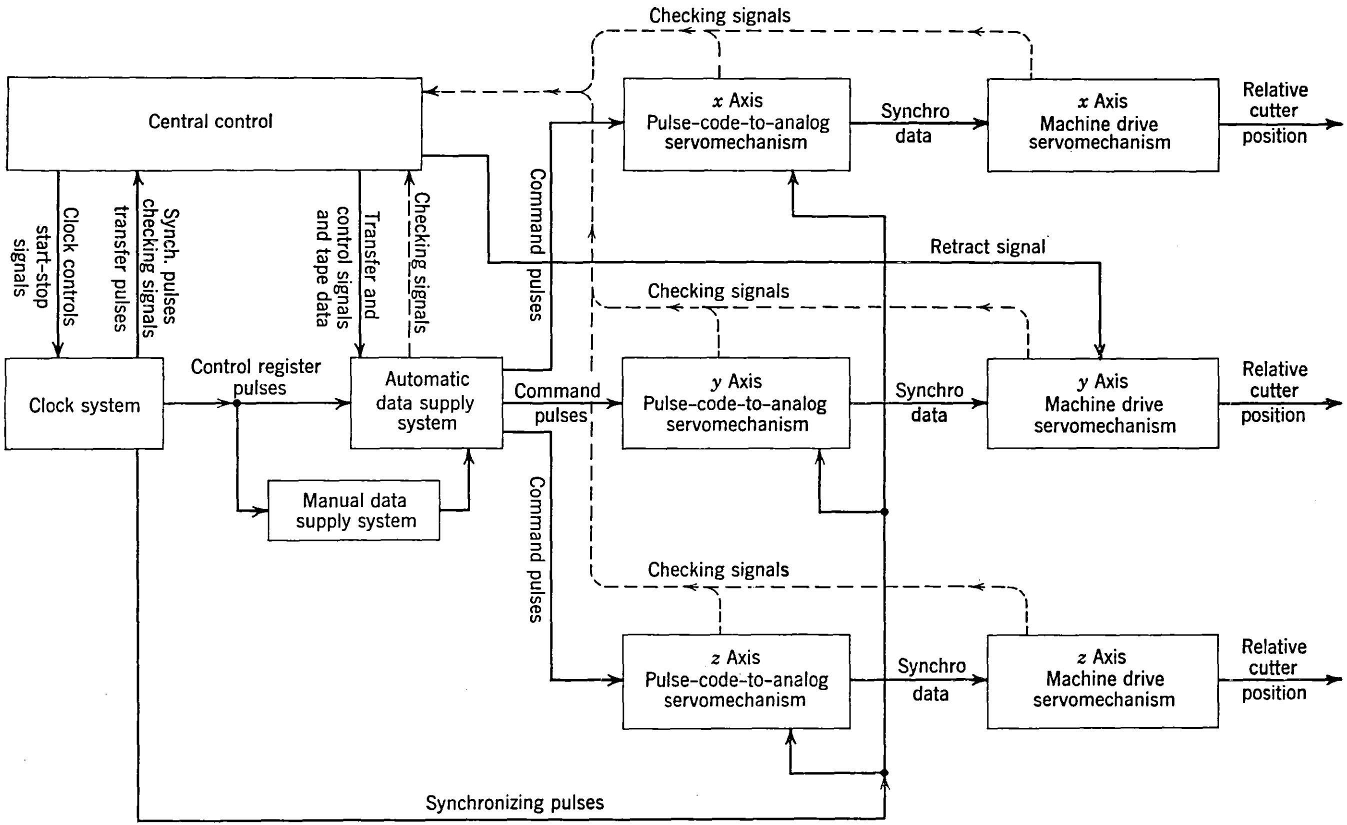


FIG. 5. Numerically controlled milling machine—simplified block diagram.

TABLE 4. REPRESENTATIVE DESIGN DECISIONS

System Type.

Manual	Automatic	Semiautomatic
Analog	Digital	Analog-digital
Continuous data	Sampled data	
Continuous control	Discontinuous control	
Fixed function	Adjustable function	Programmed
60 cps	400 cps	dc

Data Representation. Voltage, phase, frequency, current, charge, magnetic state, pulses, visual indication, aural indication, count, time interval, impedance, force, torque, density, radioactivity, volume, deformation, flow, electromagnetic intensity, pressure, temperature, displacement, velocity, acceleration, angular rotation, angular velocity, angular acceleration, relay state

Power levels, impedance levels, scale and conversion factors

Components.

Electromechanical devices	Electronic devices	Mechanical devices
Pneumatic devices	Hydraulic devices	Electrical devices

Regular, miniature, sub-miniature tubes, transistors, magnetic circuits

Transducers	Analog-digital converters	Indicators

System plate, filament, bias, and reference power supplies

Input-Output Devices. Manual, typewriter, plugboard, punched tape, magnetic tape, punched cards, film, models, magnetic drums, cathode ray tubes, meters, neon tubes

Digital Computer Design. Memory capacity, access time and type, serial or parallel, synchronous or asynchronous, radix, word length, fixed point or floating point, single address or multiple address, coding, operating speeds, standard program orders, number range, marginal checking, parity checks

Formulation of Dynamic Equations. Mathematical model, choice of variables, axis system, approximations.

Optimum filtering in the presence of noise

System Configuration (example for fire-control system). On or off carriage, gun-drive or antenna-drive tracking, gun-line or tracking-line computation, rectangular or polar coordinates, stabilized or nonstabilized platform

Grounding and shielding system, cable and connector diagrams, fuses and circuit breakers, clock frequencies, carrier and modulation frequencies, pulse repetition rates, compatibility with existing systems, malfunction detectors, space, weight, power allowances, pulse timing sequences and waveforms, optimum coding, specifications of realistic unit design criteria

Communications. Messages, data, remote control

Process Control Design. Batch or continuous production, yield, flow diagrams, material and energy balance, quality control, specifications, disturbances (type, location, magnitude), choice and location of measurement devices and controllers, choice of measured and controlled conditions, waste disposal, corrosion protection, transfer lags, buffer storage requirements, ambient conditions, starting and shutdown procedures, process monitors, economic factors, requirements for fuel, power, water, and raw materials

General. Schedules, personnel, deliveries, costs, reports, contracts in order after the initial systems design has been completed. At this stage, estimates of cost, performance, manpower requirements, and depreciation have considerably more authority than estimates made at the outset of the design.

This is the proper time also, before extensive development has started, to decide which features of the design are worthwhile and which are superfluous. In addition, the basic compromises that must be made are more apparent when the initial system design is available. One desired requirement, say high accuracy, may have to be traded off against another requirement, such as fast response. Cost, maintenance, and manpower requirements are constraints that affect every system feature. To illustrate a typical compromise of engineering economics, one can consider a process for which the yield increases with operating temperature, but so does the rate of deposit formation. The designer must decide whether the plant should be designed for high yield and short life, or lower yield and longer life.

Dynamic Analysis

When the principal units of a system have been chosen and their characteristics established, it is then possible to carry out a dynamic analysis of the system. Frequently major components are fixed beforehand, and the remaining components must be selected for compatibility. In many cases, the dynamic idiosyncrasies of the fixed elements must be compensated for in the characteristics of the auxiliary equipment. For example, the aircraft to be used with a given autopilot design may be specified, consequently the autopilot parameters must be adapted to the dynamics of this specific aircraft. Achieving satisfactory system dynamics is difficult enough, but when the principal dynamic element is fixed, as it is in the case of the autopilot-aircraft combination, a high degree of analytical skill is required.

Dynamic Block Diagrams. In dynamic analysis, a form of the system block diagram that exhibits the dynamic features of the system is very helpful. Such a diagram for the lateral autopilot of an airplane is shown in Fig. 6. Note that the transfer function of each dynamic element has been expressed in Laplace transform notation. Usually it is necessary to write the integro-differential equations describing an operation or process and to convert these equations into transfer functions for use in the block diagram.

To complete the illustration of the autopilot system, the differential equations describing the lateral motion of the aircraft itself are necessary. For small perturbations about a level, steady-state flight condition, these can be written as follows (see Ref. 10).

(6) Roll $\dot{p} = C_{l_{\delta A}} \dfrac{\rho V^2 Sb}{2I_{xx}} \delta A + C_{l_p} \dfrac{\rho V S b^2}{4I_{xx}} p + C_{l_\beta} \dfrac{\rho V^2 Sb}{2I_{xx}} \beta$

$$+ C_{l_r} \frac{\rho V S b^2}{4I_{xx}} r + C_{l_{\delta R}} \frac{\rho V^2 Sb}{2I_{xx}} \delta R,$$

(7) Yaw $\dot{r} = C_{n_{\delta R}} \dfrac{\rho V^2 Sb}{2I_{zz}} \delta R + C_{n_\beta} \dfrac{\rho V^2 Sb}{2I_{zz}} \beta + C_{n_r} \dfrac{\rho V S b^2}{4I_{zz}} r$

$$+ C_{n_p} \frac{\rho V S b^2}{4I_{zz}} p + C_{n_{\delta A}} \frac{\rho V^2 Sb}{2I_{z'}} \delta A,$$

(8) Y force $V r_w = g \sin \phi + C_{y_\beta} \dfrac{\rho V^2 S}{2m} \beta + C_{y_{\delta R}} \dfrac{\rho V^2 S}{2m} \delta R,$

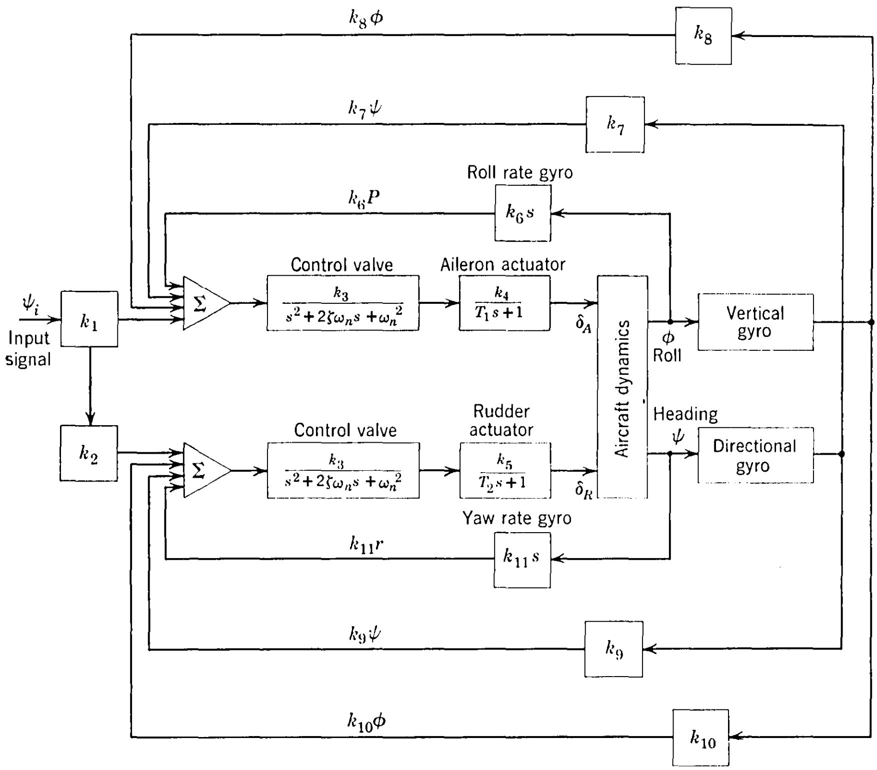


FIG. 6. Autopilot dynamic block diagram.

(9) Euler heading $\dot{\psi} = r,$

(10) Euler roll $\dot{\phi} = p,$

(11) Sideslip $\dot{\beta} = r_w - r,$

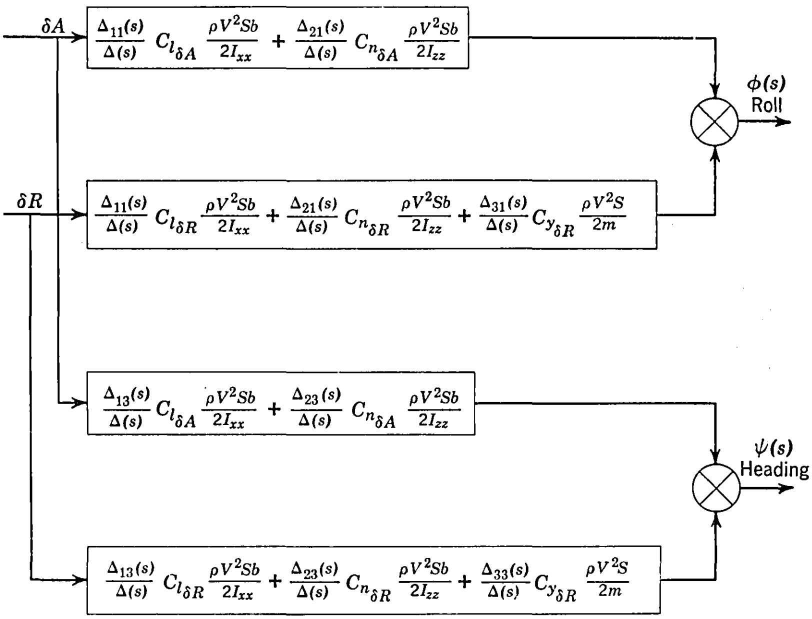
where the symbols used in the equations may be defined as follows:

p = roll component of body axis angular rate,
r = yaw component of body axis angular rate,
r_w = yaw wind axis component of wind axis angular rate,
V = aircraft velocity,
S = wing area,
b = wing span,
I_{xx}, I_{zz} = moments of inertia about roll and yaw axes, respectively,
ρ = air density,
m = aircraft mass,
β = sideslip angle,
δA = aileron deflection,
δR = rudder deflection,
ψ = Euler heading angle (wind axes),
ϕ = Euler roll angle (wind axes),
C = Aerodynamic coefficients (usually nonlinear functions of one or more variables).

The equivalent block diagram for the aircraft dynamics alone is shown in Fig. 7. Combining Fig. 6 and Fig. 7 gives the overall system block diagram from which it is possible to analyze the response of the system to representative inputs under a variety of noise conditions.

Techniques for Dynamic Analysis. Several theoretical techniques are available for the dynamic analysis of systems of moderate complexity. The designer should be particularly familiar with the following techniques, all of which are treated in detail in Vol. 1: Nyquist plots, Bode asymptotes, Nichols charts, root locus plots, signal flow diagrams, pole-zero analysis, correlation functions and spectral density, Laplace transforms, Fourier analysis, classic differential equation theory, phase plane analysis, describing functions, and sampled data analysis using the z transform (see Vol. 1, Part E, Feedback Control, and Ref. 11).

Simulation. For a system as complex as the autopilot-aircraft combination, however, the simulation of the dynamic equations on an analog or digital computer is the most practical approach. Although specific and simplified characteristic modes of a system's operation might be analyzed through hand calculations, most complete system analyses require a computing facility—the more complex the problem, the larger the facility. Such simulation studies can be used to determine the optimum



where

$$\Delta(s) = \begin{vmatrix} s^2 - C_{l_p} \frac{\rho V S b^2}{4 I_{xx}} s & -C_{l\beta} \frac{\rho V^2 S b}{2 I_{xx}} \frac{1}{s} & C_{l\beta} \frac{\rho V^2 S b}{2 I_{xx}} - C_{l_r} \frac{\rho V S b^2}{4 I_{xx}} s \\ -C_{n_p} \frac{\rho V S b^2}{4 I_{zz}} s & -C_{n\beta} \frac{\rho V^2 S b}{2 I_{zz}} \frac{1}{s} & C_{n\beta} \frac{\rho V^2 S b}{2 I_{zz}} - C_{n_r} \frac{\rho V S b^2}{4 I_{zz}} s + s^2 \\ -g & V - C_{y\beta} \frac{\rho V^2 S}{2m} \frac{1}{s} & C_{y\beta} \frac{\rho V^2 S}{2m} \end{vmatrix}$$

s = complex variable

$\Delta_{jk}(s)$ = cofactor of $\Delta(s)$

j = row

k = column

FIG. 7. Block diagram for lateral dynamics of an aircraft.

system parameters and to indicate the need for auxiliary compensation to correct for undesirable dynamic features. Actual system components often are incorporated in the simulation setup. The techniques of simulation are treated at length in Vol. 2, Chap. 2, Programming and Coding; Chap. 17, Logical Design; and Chap. 23, Nonlinear Electronic Computer Elements.

Alternate Formulations of Dynamic Equations. Not all formulations of a given dynamic problem are of equal complexity. The lateral equations just given, for example, employ an axis system fixed to the aircraft for roll and yaw and an axis system fixed by the relative wind for y force. Alternative formulations could be devised employing other axis systems, but these are generally more complicated than the one given. The designer should select the axis system, the variables, and the form of the equations that express the essential dynamic features of the system, yet offer the greatest computational simplicity. Dynamic analysis can be a complicated business and every effort must be made at the outset to eliminate superfluous terms. In the aircraft lateral equations, for example, it was possible to neglect cross-coupling and product of inertia terms because motion was restricted to small perturbations. Under such constraints, numerous trigonometric approximations may also be allowable.

Importance of Dynamic Analysis. The dynamic analysis of a system is helpful in determining realistic design criteria for the various units making up the system. The unit designer must know not only the inputs and outputs of his unit, but also the allowable static and dynamic errors under representative signal-noise conditions. In this regard a system simulation or analysis can determine the unit dynamic response necessary to achieve satisfactory overall performance. The situations chosen may represent the worst cases that the system is expected to encounter, or may represent a statistical sample of representative cases.

Process Control Dynamics. A few comments may be made at this point concerning process control dynamics, a subject which is in a relatively primitive state because of the great difficulty in describing the unit operations analytically. Designs are largely carried out on an empirical basis, with past experience playing an important role. Transfer characteristics are approximated as simple lags for the most part, in lieu of more complete knowledge of the process or plant dynamics.

Rapid measurement of certain quantities, such as the homogeneity of a mix or the exact chemical composition of the product, may be difficult or impossible to achieve. As a consequence, these quantities may not only have to be measured indirectly but also controlled indirectly. Dynamically a process control system is often considered successful if it can keep deviations within prescribed bounds for severe disturbances. The time scale of these phenomena may be several orders of magnitude away from that encountered in other control fields with time constants measured in hours or even days (see Ref. 12).

System Error Study

Error Criteria. At the outset of a design, a decision must be made on what constitutes an acceptable system error. This specification might take the form of a probability plot such as the normal distribution shown in Fig. 8. This error criterion implies a large statistical sample of systems and situations.

The designer assigns an error specification to each unit of the system such that the overall error distribution at least meets the designated standard. Two common error distributions utilized in unit specifications

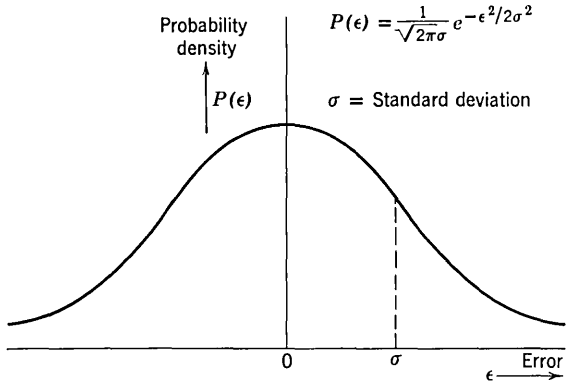


FIG. 8. Normal error distribution.

are the normal plot, shown in Fig. 8, and the rectangular distribution, shown in Fig. 9 (see Ref. 1).

If the total system error can be taken as the sum of the output errors due to each unit, the total error due to n units would be

$$\xi_T = \xi_1 + \xi_2 + \cdots + \xi_n. \tag{12}$$

If the errors are independent, the *standard deviation* of the total error distribution is equal to the square root of the sum of the squares of the individual standard deviations. That is

$$\sigma_T = (\sigma_1^2 + \sigma_2^2 + \cdots + \sigma_n^2)^{1/2}. \tag{13}$$

This relation is true regardless of the nature of the individual distributions as long as these errors are independent. For most systems, the total error ξ_T approaches a normal distribution having a standard deviation σ_T. The

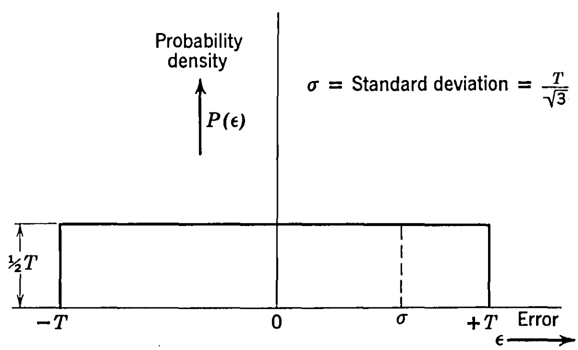


FIG. 9. Rectangular error distribution.

standard deviation of a probability density function $P(\xi)$ having a zero mean value is defined by

$$\sigma = \left(\int_{-\infty}^{\infty} \xi^2 P(\xi)\, d\xi \right)^{1/2}. \tag{14}$$

Assignment of Errors. In assigning error specifications to subsidiary units, the system designer is bound by two constraints:

1. The square root of the sum of the squares of the unit standard deviations must not exceed the desired standard deviation for the system error.
2. The assignment of errors must show a decent regard for what can reasonably be achieved within the constraints of the existing state of the art, cost, and development time. It is not facetious to state that a prime object is to minimize the grumbling of the unit designers.

To illustrate this technique, Table 5 assigns errors to various units of a hypothetical fire-control system whose overall error was specified to be less than five mils standard deviation. Special care must be exercised in deciding which errors are biases and which are random distributions.

TABLE 5. ASSIGNMENT OF UNIT ERRORS IN A FIRE-CONTROL SYSTEM

Error	Distribution	Standard Deviation, mils
Tracking error	Normal	1
Range error, converted to equivalent fire-control error	Rectangular	2
Computer errors	Normal	3
Ballistic correction error	Rectangular	2
Servo dynamic errors	Normal	1
Alignment and data transmission errors	Rectangular	1
Ammunition dispersion	Normal	2

Total standard deviation $= [1 + 2^2 + 3^2 + 2^2 + 1 + 1 + 2^2]^{1/2} = 4.9$ mils

Optimizing. For systems of moderate complexity in which noise dominates the selection of appropriate system dynamics, analytic techniques are available for optimizing these dynamics. Generally, a mathematical statement of the expected information signal as well as the noise input must be formulated, and an appropriate optimizing criterion, such as the minimization of the root-mean-square error, must be selected. The principal difficulty in such analysis is to select a correct mathematical model for the situation and to select the optimum optimizing criterion. Neither of these steps can be described as straightforward or unambiguous for most cases (see Ref. 11).

Special System Problems

In the course of the detailed system design, a number of problems arise that must be treated on a systems basis. Typical of these is the problem of interaction between units of the system. A common source of trouble is the effect of power or reference supply loading by one unit on the operation of other units using the same supplies. The familiar motorboating of audio amplifiers is a simple example of such interaction.

Complex Systems. In a complex system, innumerable opportunities exist for interaction. Radiation from power-level signals frequently is picked up by associated circuits, particularly if impedance levels are high. The manner in which units are interconnected is also a major design problem. Long connecting leads can introduce phase shift, time delays, pickup, and ohmic loss if care is not exercised by providing low impedance driving sources and adequate shielding. Mechanical deflections and vibrations caused by one part of a system can adversely affect the performance of other units, as in the case of tube microphonic effects.

Grounding System. In many control systems, the haphazard design of the grounding system and failure to pay attention to the ordinary decencies of shielding and circuit location have led to interminable difficulties. Several common errors are:

1. Use of a common ground buss for power, plate supply, and signals.
2. Indiscriminate use of the chassis as a ground.
3. Insufficient or indiscriminate shielding.
4. Creation of ground loops.
5. Poor location of circuits.
6. Inattention to impedance levels.

Figure 10 illustrates some of these faults in practice.

6. DETAILED UNIT DESIGN

Specifications. The unit designer must translate the specifications for his unit into a practical piece of equipment. The form of these specifications may be such that they have to be converted to more usable parameters like bandwidth, velocity constant, damping ratio, maximum slewing rate, and maximum torque before the actual design can proceed. A careful study of the accuracy requirements placed on the unit is also important at the outset, for these will affect the choice of components.

Practical Problems. The basic theories of feedback control and digital design have been treated in Vol. 1 and Vol. 2, and no repetition of this material is necessary here. However, a few of the practical problems of unit design can be discussed with advantage at this point.

The first step in unit design is customarily the formulation of a block

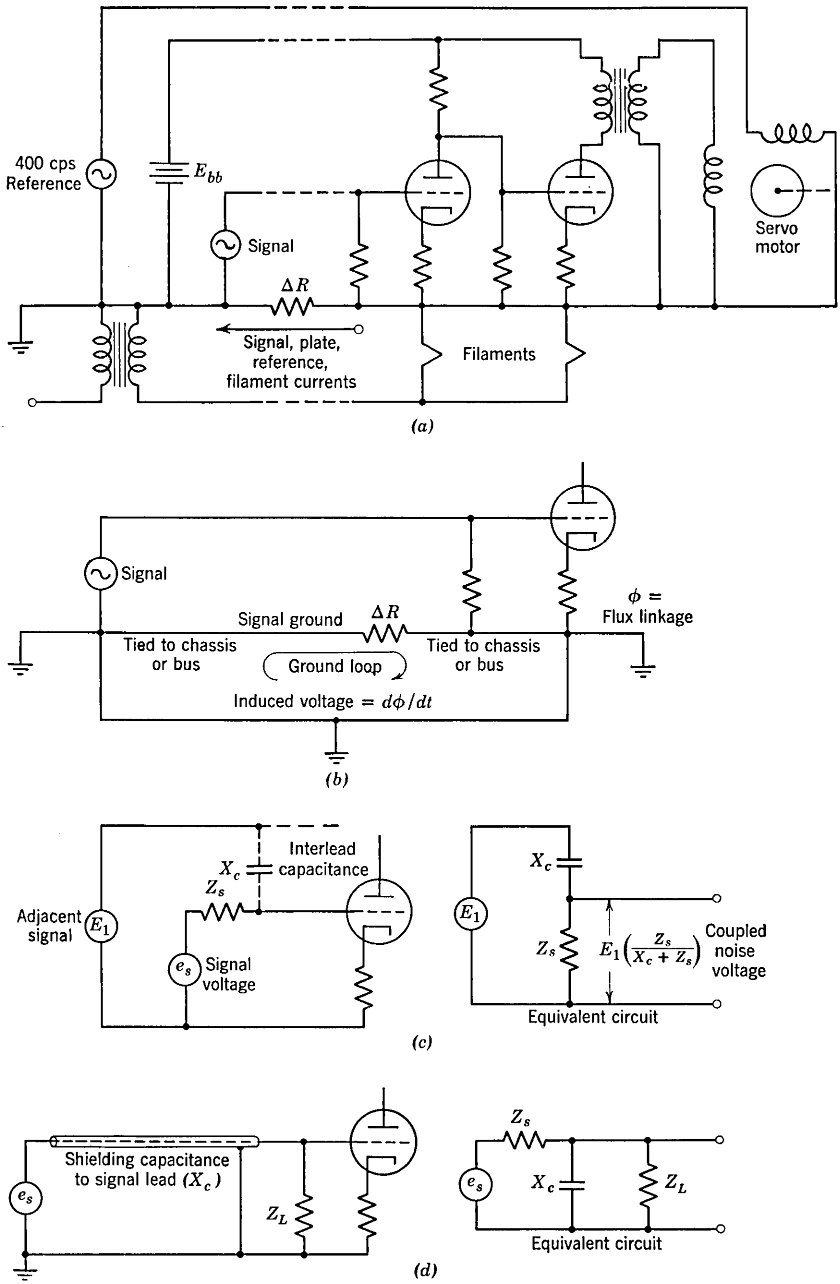


FIG. 10. Common interconnection problems: (*a*) common ground, (*b*) ground loop, (*c*) high-impedance levels, (*d*) phase shift introduced by shielding.

diagram indicating the basic techniques by which the operation is to be performed. Next, the principal components such as transducers, servo motors, and power devices are selected. This choice usually involves a thorough survey of commercially available components of the desired type and even comparative testing of these components in the laboratory. It is important to choose these major items early in the design because extensive delivery delays are possible.

In many respects, the procedure for unit design corresponds to that for system design on a smaller scale (see Ref. 13). A representative list of problems that must be considered by the unit designer is presented in Table 6.

TABLE 6. REPRESENTATIVE UNIT DESIGN PROBLEMS

Amplifier Design. Saturation levels, gain, feedback, stability, power and voltage levels, phase, tube and transistor selection, impedance levels, coupling, drift, quadrature rejection, automatic gain control, noise, balancing, magnetic circuit design, decoupling

Choice of servo motors, tachometers, potentiometers, synchros, gyroscopes, transducers, resolvers, relays, choppers, valves, indicators

Synthesis of compensation networks and filters
Unit ground system and shielding

Power Supply Requirements. Capacity, regulation, ripple

Static and dynamic analysis

Nonlinearity Effects. Backlash, coulomb friction, potentiometer wire stepping, stiction, hysteresis, cogging, saturation, potentiometer loading, motor characteristics

Design of Mechanical Assemblies and Automata. Layout, detailing, checking, supervision of machine shop, inspection, assembly

Pulse Circuit Design. Multivibrators, flip-flops, blocking oscillators, delay circuits, gates, pulse shapers, comparators, counters, diode logical circuits, boxcar generators, sweep circuits, frequency dividers, sampling circuits

Selection of Motors and Transmissions. Single-phase, two-phase, three-phase, dc, series, shunt, compound, armature-controlled, field-controlled, induction, synchronous, Ward-Leonard, amplidyne, rototrol, hydraulic, pneumatic

Switching circuit design
Marginal checking, test points, test instruments, alarms, panel indicators
Manual control provisions
Fusing and circuit breakers, interlocks, fail-safe devices
Design of modulators and demodulators
Noise
Hydraulic and pneumatic pressures, relief valves
Component tolerances, component tests
Unit schematics, electrical and mechanical layouts, parts lists, reports

7. UNIT AND SYSTEM TESTS

Every experienced engineer is acquainted with the utter perversity of nature. For this reason, unit and system designs are usually verified experimentally in the laboratory or in pilot plant operation before the final system is produced. Almost inevitably, a host of shortcomings appears in the course of these tests, many of which originate in incompatibilities and interactions between units of the system. A deliberate attempt should be made at this stage not only to determine the basic operating characteristics of each unit and the system as a whole but also to subject the equipment to a wide variety of severe conditions. A unit that gives weak performance or a unit for which the adjustments are critical should be redesigned. In addition, a systematic simplification of the various units is often attempted during the breadboard tests. Generally, the probability of successful system operation increases with a decrease in the number of components employed, although redundant components are sometimes deliberately added with the express purpose of improving reliability.

Static and dynamic performance can be established during these tests, and the system and unit parameters adjusted for optimum performance, although in some cases optimum performance may be difficult to define. In refinery operations, for example, a variety of crude oil types and catalysts may be utilized, with products ranging from aviation gas and fuel oil to wax and asphalt. The significant parameters of the process may number in the hundreds. Specifying optimum performance for such a system is rather difficult.

A chronic hazard in control system design is over-optimism in estimating the time, care, and patience necessary to put even a well-designed system into working order. In many cases, the test and evaluation of a system is an operation comparable in magnitude to the design, and the test facilities, as in the case of missile programs, may be far more elaborate than the system itself. The problems of data instrumentation and data reduction for large-scale systems tests are extensive. A modern trend in this regard is to employ statistical methods in the design and analysis of test experiments (see Ref. 1).

Standard test and calibration procedures for the system can also be evolved at this stage of the development. In the final system, built-in test equipment tailored to these procedures can often save substantial maintenance and checkout time.

8. FINAL DESIGN

The construction and test of a breadboard system is fundamentally intended to establish the basic soundness of the system concept. In the interest of expediency, the execution of such systems is generally informal. The final unit, however, must make its way in the humid, vibrating and fungus-laden world, consequently more sophisticated packaging is required. The final system must generally incorporate a multitude of essential virtues ranging from rustproofing to gopher shields. To indicate the scope of the packaging problem, a partial list of such considerations is presented in Table 7.

TABLE 7. REPRESENTATIVE PACKAGING PROBLEMS

Military specifications (MIL specs)
Mounting, mechanical strength, vibration and resonance
Space allocation
Ventilation, lighting, heating
Ease of operation and maintenance, accessibility
Facilities for personnel
Test equipment, test points, name plates
Junction boxes, system wiring, color codes, terminal strips, connectors
Special Packaging. Mobile, airborne, underwater, explosion-proof
Graphic instrument panels (process controls)
Automatic data logging
Intercommunication circuits
Electrical outlets
Malfunction Indicators. Excess error, alarms, fuse lamps
Environment Factors. Ambient temperature, shock, humidity, altitude, attitude, accelerations, pressure
Human engineering (matching machine to operator)
Safety
Reliability
Standardization of parts, interchangeable plug-in units, spares
Finishes, appearance
Rustproofing, fungus-proofing, weather-sealing, dustproofing
Instrumentation
Noise levels and acoustics
Insulation
Lubrication
Preliminary mockups
Weight
Cost
Tolerances

When completed, of course, the final system must be thoroughly checked for performance under a variety of conditions and any new deficiencies must be corrected. The ultimate user will most likely require a field test or demonstration of the system before acceptance, as well as complete operating and maintenance manuals, schematics, and parts lists.

On complex systems, field service personnel may remain with the unit for months after delivery for maintenance purposes, additional debugging, and training of customer personnel. Some control systems are purchased with provisions for permanent field service.

A record of system malfunctions kept during the development phase and during the first months of system operation will aid in the design of succeeding models by uncovering poorly designed or unreliable components. To achieve reliable operation with a large system demands exceptional reliability from the individual components. Many of today's control systems, such as the air defense complex, demand a degree of reliability per operation several orders of magnitude better than that of a dial telephone system.

The system records should also indicate the economics of the system's operation, if possible. This involves maintenance and operating costs, spoilage, down-time and productivity. Such a study will either demonstrate the economic virtues of the design or will warn the designer not to make the same mistake again.

9. CONCLUSION

The advantages that result from the application of control techniques to industrial and military problems have been summarized and the broad scope of such applications indicated. The background required for successful control systems design has been suggested, and a generalized design procedure presented. Practical problems frequently encountered in such designs have been listed in tabular form.

Although the control system design procedure has been presented as a step-by-step sequence, it must be emphasized that the various stages of design and test interact with each other in innumerable ways. Design is itself a feedback process, and some steps may be repeated several times before a satisfactory system results. In particular, the designer often discovers that the original realistic design criteria are unrealistic and must be modified.

The remainder of Vol. 3 will examine specific control system designs in detail and will present further information on components frequently employed in control work, including the ubiquitous human operator.

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The Human Component

J. Lyman and L. J. Fogel

1. GENERAL COMPARISON OF HUMANS AND MACHINE COMPONENTS

Human Operations. The unique ability which makes the human operator particularly suited to control operation as part of a servo system is that he can program and reprogram his computation while the process is in progress to account for transient and nonstationary characteristics in the perceived data.

This reprogramming may be looked upon as a decision which occurs at a particular level within a hierarchy of decisions relevant to the intended purpose. This hierarchy can be grossly characterized as follows:

1. Decision that determines the general computation program which will be used to process the received data so as to accomplish the intended purpose in an optimum manner, e.g., a flight plan.

2. Decision that selects data from the information displayed in the environment.

3. Decision that determines the state of the system from the perceived information.

4. Decision that determines the manner of control action which will minimize some function of anticipated error.

These classifications are not mutually exclusive, but they are distinctly ordered. They must be performed in an ordered sequence to lead to effective system control. Each decision in this sequence depends on the relationship between the human and equipment components of the task. The nature of this relationship is shown in Fig. 1.

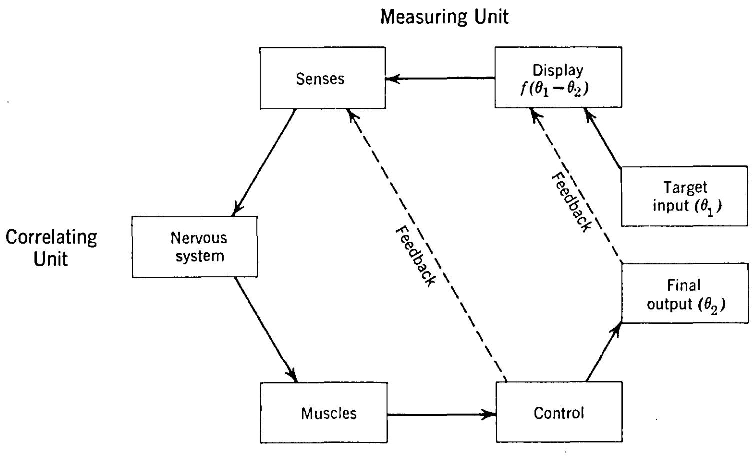

FIG. 1. Human and technical links in a control cycle.

Humans and Machines. The successful operation of the total system depends on establishing the highest possible degree of compatibility between those parts of the task under design control and the functional properties of the human over which the designer has little influence. As an initial step it becomes necessary for the engineer to know what some of these properties are in terms of functions in which machines excel as compared with those in which humans excel. He can then select the proper part of the task for the man and avoid assigning duties to him that a machine can do better, recognizing any compromise he is making. To assist in this process a comparison of the functions of men and machines is presented in Table 1. It should be emphasized that as technical sophistication improves, history has shown that machines can economically take over more of the functions in which man appears to excel. Accordingly, the designer must be constantly alert to developments that can be substituted for human functions.

TABLE 1. FUNCTIONAL ADVANTAGES AND DISADVANTAGES OF MEN AND MACHINES

Data Sensing

Man	Machines
Can monitor low-probability events for which, because of the number possible, automatic systems would not be feasible.	Program complexity and alternatives limited so that unexpected events cannot be adequately handled.
Under favorable conditions absolute thresholds of sensitivity in various modes are very low.	Generally not as low as human thresholds.
Can detect masked signals effectively in an overlapping noise spectrum on displays such as radar and sonar.	May not be useful when noise spectra overlap detection of signal.
Able to acquire and report information incidental to primary activity.	Discovery and selection of incidental intelligence not feasible in present designs.
Not subject to jamming by ordinary methods.	Generally subject to disruption by various interference and noise sources.

Data Processing

Man	Machines
Able to recognize and use the information, redundancy (pattern) of the real world to simplify complex situations, e.g., recognition of airport through stages of ground contact, approach, and landing.	Little or no perceptual constancy or ability to recognize similarity of pattern in either the spatial or temporal domain.
Reasonable reliability in which the same purpose can be accomplished by different approach (corollary of reprogramming ability).	May have high reliability at increased cost and complexity. Particularly reliable for routine repetitive functioning.
Can make inductive decisions in situations not previously encountered; can generalize from few data.	Virtually no capacity for creative or inductive functions.
Computation is weak and relatively inaccurate; optimal theory of games strategy cannot be routinely expected.	Can be programmed to use optimum strategy for high-probability situations.
Channel capacity limited to relatively small information throughput rates.	Channel capacity can be made as large as necessary for task.
Can handle variety of transient overloads and some permanent overloads without disruption.	Transient and permanent overloads may lead to disruption of system.
Short-term memory relatively poor.	Short-term memory and access times excellent.

(*continued*)

TABLE 1. FUNCTIONAL ADVANTAGES AND DISADVANTAGES OF MEN AND MACHINES—(*Continued*)

Data Transmitting

Man	Machines
Can tolerate only relatively low imposed forces and generate relatively low forces for short time periods.	Can withstand very large forces and generate them for prolonged periods.
Generally not good at tracking though may be satisfactory where situation requires frequent reprogramming; can change to meet situation. Is best at position tracking with changes under 3 radians per second.	Good tracking characteristics may be obtained over limited set of requirements.
Performance may deteriorate with time; usually recovers with rest.	Behavior decrement relatively small with time; wear maintenance and product quality control necessary.
Relatively high response latency.	Arbitrarily low response latencies possible.

Economic Properties

Man	Machines
Relatively inexpensive for available complexity and in good supply; must be trained.	Complexity and supply limited by cost and time; performance built in.
Light in weight and small in size for function achieved; low power requirement, less than 100 watts.	Equivalent complexity and function would require radically heavier components and enormous power and cooling resources.
Easy to maintain with minimum of "in task" extras.	Maintenance problem becomes disproportionately serious as complexity increases.
Nonexpendable and interested in personal survival; emotional.	Expendable and unconscious of personal existence; will perform without distraction from problems arising outside of task.

2. DESIGN PROBLEMS SPECIFIC TO HUMAN COMPONENTS

Once the task has been defined, it is necessary to consider the environment in detail to insure that the assignment can and will be fulfilled. The important physical aspects are shown in Fig. 2, but the environment has not been completely specified without the information input and output coupling to and from the man (Ref. 20). The task must be analyzed into decisions, and assurance provided to the operator that the required information can and will be received as it is needed. He must also have

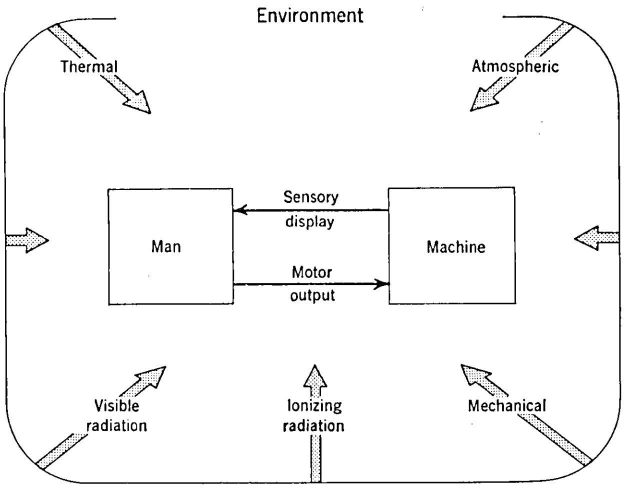

FIG. 2. Environmental and machine links to the human.

efficient means for coupling his control actions to the system. Specific hardware design problems requiring data on humans fall into the following categories:

1. Problems of the general working environment such as optimum heating, lighting, and ventilation in relation to human physiological parameters.
2. Problems of size, shape, and arrangement in which human characteristics limit the physical form of controls and spaces.
3. Problems of information input in which human encoding properties limit channel capacity in terms of speed and sensitivity of response and the nature of the stimulating energy which can be applied to the senses.
4. Problems of motor output (including voice) in which human neuromuscular characteristics limit the static and dynamic properties of control movements and the useful power that is available.

In this presentation attention will be directed primarily to categories 3 and 4.

3. INFORMATION INPUTS TO THE HUMAN COMPONENT

Displays and instruments are devices which enable adequate control by virtue of the information they transmit. The operator is a channel for processing and transmitting this information from the display to the controls. The controls are monitored by means of an information channel from the operator to the machine. The channel capacity of the human and its optimum utilization through proper encoding of the information to be transmitted by the human to the controls is an orienting viewpoint that aids in making appropriate design decisions.

Vision

Perceptual Capacity. Vision is the most important and widely utilized input channel. In considering the visual acuity and physical characteristics of the human eye, it has been estimated that about 4.3 million bits per second can be received by the retina (Ref. 16). However, this is not *perception,* the usable acceptance of data. Empirical studies have revealed that the maximum amount of information which can be accepted by a human observer when he attempts to locate a point on a straight line (e.g., an indicator scale) is about 3 bits (Ref. 9). As the number of coordinates is increased, there is an increase in the message carrying capacity of the stimulus, but this does not appear to be linear. Thus two coordinates of a dot on a plane transmit about 4.4 bits and the eight coordinates of one-to-four dots on a plane about 7.8 bits (Ref. 17). The importance of the type of encoding is further suggested by the fact that apparently maximum perceptual capacity is approached during the reading of printed English text. This figure is about 50 bits per second (Ref. 3).

Other Limitations. The time operating characteristics of the eye impose certain communication channel limitations. Dark and light adaptation (Ref. 13) of the light-sensitive material of the retina must be considered in any evaluation of the communication constraints caused by the physical environment. Furthermore, the stimulus brightness is perceived in relation to its previous intensity level. This function is approximated logarithmically over the usual range of values (Refs. 23, 28). The eye scans the visual field in a series of irregular discrete steps. This saccadic motion limits the fixation time to about 90% of the observation time interval (Ref. 3). This motion is not random and, although little is known about the causal relations, it appears quite efficient as a preliminary pattern discrimination filter (Ref. 19).

Filter Action. Perceived data are recognized as part of a time-series and experiments have shown (Ref. 10) that the human operator apparently cannot perceive data without some reference to previous and possible future data. The observer appears to postulate a structure on the incoming message and proceeds to verify or deny the hypothesized pattern. The hypothesized pattern introduces a certain amount of redundancy and thus can only diminish the channel capacity. His perceptual operation thus becomes a filter operation which examines the redundancy of the received data in comparison to that of the hypothesis. For example, target detection would result when the amount of the particular type of redundancy became sufficiently close to that of the estimated message structure as the latter is programmed in the human's memory.

Averaging. The human operator has an even more subtle filter action, he apparently performs a moving time average on the received data. This corresponds to a spectral transformation which varies as a function of the interval of averaging so that he can select the shape of the desired signal spectrum and increase the signal-to-noise ratio even if the signal and noise spectra overlap (Ref. 6).

To illustrate, consider the simplified example, where equal importance is attributed to data over the entire range of the averaging time. (Actually the importance weighting is probably exponential with maximum importance given to the latest received data.) Let the signal and noise spectra be independent and added together to form the received message. Such a linear moving time average may be shown to perform a spectral transformation of the form

$$T(\omega) = \frac{\sin^2 (\tau\omega/2)}{(\tau\omega/2)^2}, \tag{1}$$

where ω is the radian frequency and τ is the time interval of averaging observation. This transformation may be plotted for values of ω as shown in Fig. 3. Note that instantaneous averaging introduces a unitary transfer, no modification; while as τ approaches infinity, the transfer function approaches a unitary impulse. The signal-to-noise ratio of the averaged

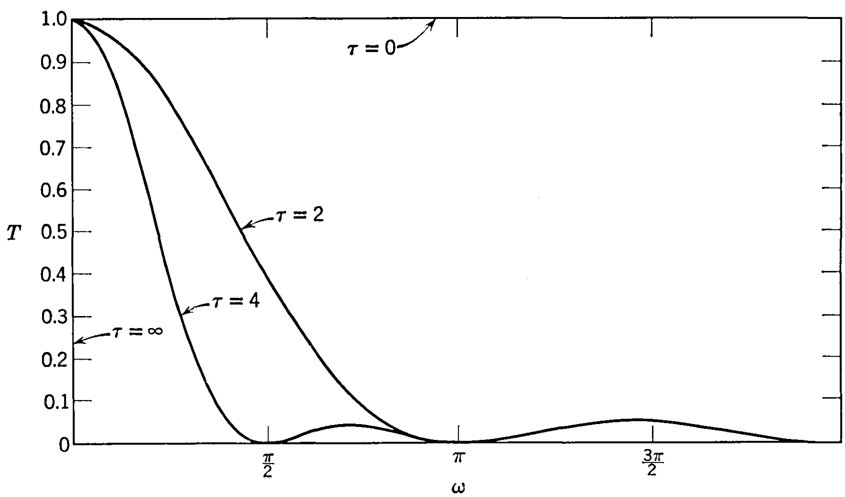

FIG. 3. Spectral transformation of linear moving time average.

signal may be written in the following form

$$S/N = \frac{\int_0^\infty T(\omega)S_s(\omega)\,d\omega}{\int_0^\infty (1 - T(\omega))S_s(\omega)\,d\omega} \tag{2}$$

so that the maximum signal-to-noise ratio is achieved when the shape of the transformation best approximates that of the signal spectrum S_s as shown in Fig. 4. This corresponds to an optimal interval of time averag-

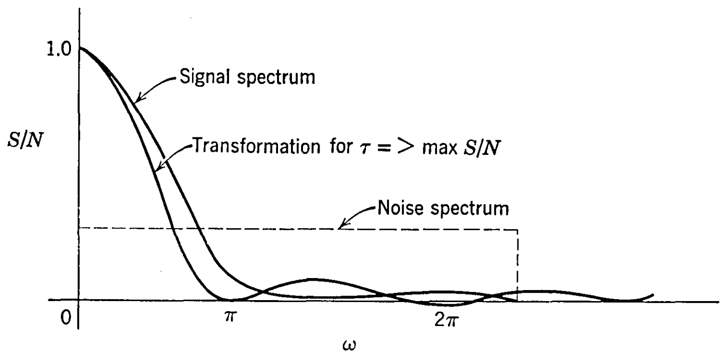

FIG. 4. Spectra and transformation of signal and noise.

ing. Some displays already perform such an averaging process and practice allows the human to take this into account by a modification of the signal "inertia." A cathode ray tube screen provides this kind of noise filtering of input data. Too large a persistence would filter out the signal as well as the noise and thus would greatly reduce the value of the display.

Audition

Next to vision, audition is the main primary input channel. It is temporal more than spatial in its perceived dimensionality and being omnidirectional for input signals it is particularly suited to applications where a high "attention getting" value is required by the task.

The physical information capacity of the ear has been estimated to be of the order of tens of thousands of bits per second (Ref. 15). However, as in vision the maximum rate of information perception is probably quite small, being nearer to dozens than thousands of bits per second. Experiments (Ref. 22) on the identification of stimuli on the auditory dimensions of pitch and loudness show about three bits per single dimensional stimulus (e.g., frequency) and up to about five bits for multiple stimuli in both dimensions. Binaural localization (Ref. 18) of the direction of a sound source can provide a small amount of additional information along another coordinate.

The time resolution of the received data limits the information rate of auditory perception. Consider the identification of a single frequency tone. The spectrum of an instantaneous impulse is flat and provides no pitch identity. As the reception time proceeds, the equivalent spectrum gathers in a more and more sharply defined peak at the appropriate frequency, until it is identified. The listener appears to accrue the individual cycles of energy and compare the period to that stored in the memory for the expected frequency. As this process proceeds, the level of confidence increases with the corresponding level of the redundancy until the required significance level is reached and identification takes place. This suggested mechanism is heuristic and appears to agree with the observed empirical evidence (Ref. 25) that indicates increased pitch loss for sounds of low frequency displayed for the same small interval as those of higher frequency. Further, it seems to help account for the decreased reaction time of a listener when presented some higher frequency (Ref. 7).

Other Senses

Kinesthetic. The human is equipped with a kinesthetic channel which displays information generated by the vestibular canal and the many proprioceptors distributed in the muscles, tendons, and tissues in and around joints throughout the body. This force and motion sensing system becomes an important consideration in relation to control operation for it gives a spatial reference for the relation of the operator's body and its parts to the location and reactive forces ("feel") of the control. During accelerations of the controlled system (e.g., flight) the information from this source as to the state of the system may become unreliable and be inconsistent with that from other sources, tending to disorient the operator (Ref. 26). Its relative discrimination of spatial position is less accurate than vision, and it may be overridden when more accuracy of control is required.

Olfactory. The human operator associates a general meaning with each of the sensory channels. This inherent meaning is maximum for channels where the diversity of data meanings is minimum. For example, the initial meaning carried by any new odor inside an aircraft cockpit is—emergency warning. Only after the smell has been properly identified can this inherent meaning be rejected. Practical use has been made of this by adding artificial odor to illuminating gas to ensure identification of a dangerous leak in the home. Although the olfactory sense channel is not suited to a high average information rate (Ref. 24), it can carry a large amount of information at particular times when it is excited by the low probability occurrence it monitors.

Taste. The taste sense channel is closely linked to the olfactory sense. Its initial activation period is short, but it adapts rapidly and returns to its original state relatively slowly. This prevents a rapid information flow rate. There are four basic taste dimensions: sweet, bitter, sour, and salty. This can again be utilized to form an attribute space within which redundancy comparison and identification can take place.

Skin Senses. The skin senses, consisting of touch, heat, cold, and pain can provide effective communication channels, e.g., Braille. In the usual servo control loops, the human operator uses the tactile sense to identify the meaning of a particular control by the shape of a knob or handle when vision is occupied elsewhere. Various codings have been suggested for "blind feeling" (Refs. 12, 27). As with olfaction these senses seem well suited to utilization for appropriate high-surprise value data.

4. CONTROL OPERATION

In terms of decision by the operator the most important reason for displaying information is to tell the operator how to apply force and move controls—which one, what direction, how much, and for how long in order to maintain some criterion state in the system. The aim of good display design practices is to minimize computations by the operator and supply only the required information and not more. In transferring this information to the control the interaction of operator and the physical control characteristics determines the net effectiveness of input information utilization. Friction, inertia, and compliance of the control members as well as control-display amplification ratio and control-to-display transfer function are all matters which the designer must consider. Assuming that optimum values for these physical factors are possible in a given system design, the limitations that the operator imposes for transferring information to the controls are determined by his storage capacity, his motor output capacity, and the effects of overloading channel capacity.

Storage Capacity. A control operation decision requires comparing the redundancy of input information with a recalled pattern. It has been estimated that total human storage capacity falls somewhere in the range of 10^8 to 10^{15} bits (Refs. 8, 21), but the maximum amount is not what is relevant to a control decision; it is the effectiveness of access that is important. If the large storage capacity is considered, the access time for a human is relatively rapid, apparently of the order of a few milliseconds, and seems to result from the memory of conditional probabilities between events rather than an address to the separate events themselves.

Motor Output Capacity. Once the inputs have been processed through the operator's central correlational processes his output to the controls is determined by the performance capacity of his muscles and the associated visual and proprioceptive feedback mechanisms. Empirical data indicate the following approximate output rates of information transmission by the human (Ref. 1): Piano playing, 22 bits/sec; typing, 17 bits/sec; impromptu speaking, 26 bits/sec; reading aloud, 24 bits/sec. On the basis of known data it appears probable that humans are not capable of transmitting more than about 26 bits/sec. Optimum performance seems to be somewhat less than this and 10 to 12 bits/sec has been suggested as the information handling capacity that is optimum for a variety of motor tasks (Ref. 5).

Channel Overloading. Irrelevant as well as relevant information is transduced by the operator and his properties as a living organism are such that any increase in the amount of information tends to take its toll in terms of distractions, fatigue, inaccuracy and the imposition of a stressful condition which makes the operator introduce subjective noise and clutter into the displayed data (Ref. 4). When functioning near channel capacity, it is usual to find that each error the operator makes tends to set off a train of succeeding errors on account of the additional data provided by recognition of the first error and a consequent further reduction in remaining channel capacity. This effect further emphasizes the importance of designing the task environment so as to minimize unnecessary loads on the channel capacity of the operator.

5. HUMAN TRANSFER FUNCTIONS

A great deal of effort has been devoted to finding an adequate mathematical model for the human operator in a simple closed-loop system. The universally recognized nonlinearity and time varying characteristics of the human have made this a formidable task. The effective reprogramming property that characterizes the human permits him to modify his transfer function and alter his gain to suit the control task with which he is confronted, integrating or differentiating as required. The type of information encoding in the display, and the degree of information transformation or reencoding necessary by the operator determine the extent of load on the operator's channel capacity and thus his effectiveness in the system.

As a practical matter it has been pointed out (Ref. 2) that in the simple tracking situation the transfer function required should be as simple as possible and whenever practical, the operator should act only as a simple amplifier and never have to deal with a bandpass greater than 3 radians/sec. Since this is often not feasible and, if it is, the human may as well be replaced by an amplifier, it has become customary to treat the problem with some linear time invariant approximate transfer function that will help account for at least some of the empirical facts. One such approximation for the control of visual displacement ratios in the complex frequency domain is as follows:

$$\frac{C(s)}{U(s)} = \frac{Ke^{-\tau s}(1 + T_L s)}{(1 + T_I s)(1 + T_N s)}, \tag{3}$$

where $e^{-\tau s}$ = the reaction time delay with τ having values ranging from 0.2 to 0.5 sec for random stimuli. (Reaction time appears to be approximated by 0.3 ln $(n + 1)$, where n is the number of equiprobable choice possibilities in each control decision.) If the perceived stimulus becomes predictable, the human operator may begin to generate an output which replicates the input and is synchronized with it. When such is the case τ becomes negligible. Any phase discrepancy is not due to the reaction time delay. On the other hand, τ may be greater than 0.5 sec, depending upon the interpretation complexity of the perceived data.

T_N = the neuromuscular lag. T_N is normally between 0.1 and 0.16 sec for the arm.

T_L = the lead time constant and has been observed to have values between 0.25 and 2.5 sec; however, these values are not the limit of its range. This constant is a function of both the dynamic response of the controlled system and the bandwidth of the visual stimulus. This linear factor in the numerator provides a 6 db/octave rise in the gain characteristic from the break point identified by $\omega = 1/T_L$ which may be looked upon as the added importance the higher frequency components receive as they imply imminent "anticipatory" information.

T_I = the system lag time with observed values between 5 and 20 sec; it can have any value, dependent upon the dynamics of the controlled system and the stimulus bandwidth. This "integrating" factor provides a smoothing of the input data so as to allow the output spectrum generated to approximate better the response spectral characteristic of the physical system. The closer this term approximates pure integration, the greater relative importance the operator has attributed to the "drift components" of the stimulus.

K = the gain, adjusted by the human operator to allow proximity to the point of marginal stability. For tasks requiring greater sensitivity and accuracy, he would raise the gain.

6. PRACTICAL HUMAN FACTORS DESIGN

The foregoing treatment of the human as an information channel is greatly oversimplified in terms of man's complexity. No attempt has been made to deal with individual differences, motivational factors, and the learning process, all of which must be included in practical design considerations. In spite of these limitations, however, three general principles of practical importance emerge from what has been said:

1. The task must be analyzed in detail to assign the human component where he will be most effective in terms of functions which he can perform better than machines.

2. Information at each stage of the process from display to control must be encoded so as to minimize reencoding steps, that is, displays and controls should be "compatible" in the sense that inputs and outputs through the human link are similar in their pattern characteristics, e.g., if an indicator turns clockwise the control knob should also turn clockwise.

3. The information transmitted through the human should be limited to only that which is essential to his assigned function.

Steps in Human Factors Design Problems

Because the sources of human data are manifold and relatively unfamiliar to the engineer, it is necessary that some systematic approach be set up to formulate this aspect of a given engineering problem and permit a solution in terms of actual hardware. Steps which parallel those of other engineering considerations are as follows:

Step 1. Answer these questions:

(*a*) How is the information the human must receive encoded? (i.e., words, pictures, warning signals, etc.)

(*b*) Through what sensory channels is the information to come?

(*c*) What kind of perceptual decisions must be made (i.e., "yes-no" type, qualitative "plus or minus" type, or quantitative "read a number" type; simple or complex judgments)?

(*d*) Through what motor channels do the responses to the information occur?

(*e*) What kind of motor outputs must be made (i.e., fine movements, coarse movements, simple or complex coordinations, relative importance of speed and accuracy, etc.)?

(*f*) What is the general situation in which the design will be used (i.e., illumination level, etc.—a general description of the environment)?

(*g*) What is the general condition of the human component in the normal operation of the design (i.e., state of health, age, sex, length of time expected to operate, position during operation, etc.)?

These questions, specifically answered, will set up the problem with respect to the human factors in most cases.

Step 2. Go to some general reference covering the area in which you are interested (see reference list below). Use the index and bibliography in these references to lead you to the specific material which has bearing on your problem. As this field of technology is new and is developing at a rapid pace in some cases you may not find what you want and it may be necessary to consult with human factors specialists. They will frequently be able to indicate the status of information which is not yet in general reference works.

Step 3. After you have gathered the information which seems to apply and are ready for the design stage, lay out a tentative design and check it in relation to specifications imposed by the human factors you have discovered. You may, of course, be required to make a number of compromises before you have a workable design. It almost goes without saying that the compromises will have to be in the direction of either eliminating human elements or improving the extra-human components so that the human "bottleneck" can function at a more efficient level. Selection and training of the human components will probably help to overcome some of the factors that require compromise, but cannot be counted on as a way to correct mistakes of judgment during the design phase.

Step 4. Where circumstances permit, before going into full scale production of a design, a pilot model should be built (just as in cases where the human factors are not specifically considered) and thoroughly tested under conditions as close as possible to those of normal use. Here auxiliary tests using as many humans as many times as possible will allow you to make the final modifications that will yield an optimally effective design.

Annotated List of Basic Data Sources

Baker, C. A., and W. F. Grether. Visual presentation of information. WADC Technical Report 54-160, Wright-Patterson AFB, Dayton, Ohio.

Detailed and comprehensive design data for visual displays. Available from Office of Technical Services, Department of Commerce, Washington 25, D. C.

Chapanis, A., W. R. Garner, and C. T. Morgan. *Applied Experimental Psychology.* Wiley, New York, 1949.

The first textbook of "engineering psychology," this book still retains its position as a standard source book; it is recommended both as a reference and as a readable introduction to the field.

Dallavalle, J. M. The industrial environment and its control. Pitman Publishing Corp., New York, 1948.

A general text covering the thermal atmospheric, illumination, and noise environments from the standpoint of industrial hygiene. Many data for engineering control of these environments are given.

Ely, Jerome H., *et al.* Design of controls. WADC Technical Report 56–172, Wright-Patterson AFB, Dayton, Ohio, November 1956.

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McCollum, I. N., and A. Chapanis. *A Human Engineering Bibliography.* San Diego State College Foundation, San Diego, 1956.

A coverage of human engineering material organized by subject with a system of cross references.

McCormick, E. J. *Human Engineering.* McGraw-Hill, New York, 1957.

A recent textbook which is valuable for becoming acquainted with the field and which contains many useful techniques and data.

McFarland, Ross A. *Human Factors in Air Transport Design.* McGraw-Hill, New York, 1946.

A detailed coverage of both physiological and psychological factors.

Tufts College. Institute of Applied Experimental Psychology, Report No. 199-1-2, *Handbook of Human Engineering Data for Design Engineers,* 1953.

An extensive coverage of the experimental literature in the field. This is a basic reference work which is recommended for use as an aid in evaluating the applicability of the results of specific experiments to practical situations.

Woodson, Wesley, E. Human engineering guide for equipment designers. University of California Press, Berkeley, 1954.

A practical handbook which is strong on human sizing data; contains a bibliography of several hundred titles.

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MANUFACTURING PROCESS CONTROL

B. MANUFACTURING PROCESS CONTROL

Automatic Machines

T. R. James

1. TYPES OF PROCESSES

The emphasis in this chapter will be on automatic mechanisms for material shaping and assembly processes.

Definitions. *Batch Process.* An operation in which a quantity of material or parts undergoes a chemical and/or physical change taking place in one operation throughout the quantity of material under treatment. Steps are fill, operate, and discharge. *Example.* Deburring parts in a tumbler.

Continuous Process. An operation or series of operations in which the material or parts are fed in at one point; move through the equipment, undergoing the chemical and/or physical change, the condition of the material or parts being related to its position in the equipment; then are discharged at the end of the treatment channel. *Example.* Baking bread in the type of oven equipped with a traveling deck.

Machine. A piece of equipment for processing material or parts, having moving elements to facilitate the processing. *Example.* Sewing machine.

Automatic Machine. A machine which processes material or parts without routine human assistance. *Example.* Nail making machine.

In many machines, a portion of the operation is automatic. A sewing machine automatically produces the stitch, but the work must be guided by hand. Automation of a process machine usually takes place a few steps at a time for economic reasons.

Advantages and Disadvantages of Continuous Processing.

1. *High volume* is possible with less labor and equipment. Production planning and quality control are usually simpler. Both are therefore less costly. If a bottleneck occurs, the effect is seen at once and corrected. Better daily and monthly forecasts can be made when the rate of output is constant. In chemical or physical processing where heating or cooling is needed, process time can usually be reduced. Continuous processing usually produces a better product.

2. *Automatic continuous processing presents some obstacles;* however, long runs of the same product are needed to pay for expensive machines. A substantial change in the product may mean rebuilding or scrapping the whole process line.

Much equipment must be operated simultaneously. Startups may require skillful and rapid adjustment of the machinery to avoid wasting material. In some processes, a satisfactory yield of an acceptable product cannot be obtained until the system reaches an approximate equilibrium.

One problem is too great a variety of products for the use of single purpose machines or tools. However, in some cases, machines can be made more versatile with certain types of readily changeable automatic programming control.

3. *Failures.* A shortcoming of continuous processing is the loss caused by the breakdown of process machinery or its control system. One failure stops the whole machine. However, this hazard can be held to a minimum with good design which is simple and substantial. Instruments to detect incipient trouble should be used with automatic correction, if possible, and operator warning. Along with good design in the first place, a preventive maintenance program is needed to assure success. This means (1) anticipating troubles, (2) keeping the equipment in top condition, and (3) training crews in the operation, inspection, and repair of all machinery, equipment, and control devices.

4. Too large a *number of operations,* even with the most favorable conditions, should not be attempted on a single machine. A line of ma-

chines is usually more practical with some storage between to take care of short interruptions automatically without loss of time by all machines.

5. *A general comparison* of automatic and batch processes is given in Table 1. (This list represents only an average as not strictly true in all cases.)

TABLE 1. AUTOMATIC CONTINUOUS *vs* BATCH PROCESSES

	Batch	Continuous
Ease of starting	Good	Difficult
Operation labor	High	Low
Automatic control	Practical	Practical
Product control	Fair	Good
Minimum investment	Moderate	High
Investment per unit of capacity	Moderate	Low
Automatic feed and discharge	Expensive	Practical

2. CLASSIFICATION OF AUTOMATIC MECHANISMS

Automatic System. An automatic system is an arrangement of automatic mechanisms so that a process operation can be performed with a minimum amount of hand labor and mental strain. The ideal automatic system consists of:

1. A device for accepting the raw material in bulk and feeding it into the machine properly oriented and at a controlled rate.
2. Mechanisms for performing operations on the material to give useful results.
3. Transfer mechanisms to move the material from one operation to the next and finally to discharge the product from the system.
4. Means for properly programming the feeding device, the operation and transfer mechanisms.
5. Controls that compensate for effects caused by unusual deviations in the raw material and/or the motions of the machine, or that notify the operator that manual intervention is required.

Automatic Assembly. This process will include packaging as well as fastening parts together, since there is a similarity in the mechanisms used.

Classification of Operations. In automatic processes, the various operations can be divided into two main groups: (1) those that transport and position the material or parts, and (2) those that perform the operations that furnish the desired results. The latter operations either change the shape of the parts or fasten them together.

3. TRANSPORTING AND POSITIONING MECHANISMS

Continuous Material Feeding Devices. In many cases, the material used is in the form of bar stock, wire, or narrow sheet stock called strip, which usually is handled in rolls. For feeding material of this type, a device that will engage the surface of the material and move it the required amount each cycle is used. A pair of spring-loaded rolls intermittently driven by a ratchet and an adjustable crank is the most usual device for sheet stock. However, this method is not readily applicable to feeding bar stock to rotating machines. Usually, the feeding of automatic lathes is accomplished by advancing the stock with a chuck that revolves with the machine and is capable of being given an adjustable motion of the stock lengthwise.

Feeding Devices for Individual Parts. In other cases, the material may be in the form of castings or partially finished pieces from rolled material. The steps in feeding parts are usually (1) the separation of single parts from the general mass, (2) the orientation of parts, and (3) the passing of the parts at the desired time to the operation. Often, the same device performs more than one of these steps.

Separation and Orientation. Successful means for orientation have usually caused a random motion of the parts, but allow parts that happen to be properly oriented to fall into grooves or pockets. Rotational or oscillating movement of the equipment is generally used. By the use of gravity, mechanical motion of the pockets, or vibrational conveyance, the parts are usually separated in the same operation. The principal of vibrational conveyance is to cause the friction between the parts and the surface on which they rest to be greater than the acceleration forces in the direction of travel, but less than these forces in the opposite direction.

Orientation is usually accomplished by:

1. Holding one end of the object while the other end continues to progress.
2. Moving pockets the shape of the parts under a mass of the objects so that only those happening to be caught in the right orientation will be caught by the pockets, or moving the mass over and having the opening so shaped that only those parts having the right orientation will fall through, or moving the parts along a linear trough and allowing the excess parts to fall off.
3. Conveying a round piece by gravity or vibration until a flat side or projection prevents rotation and the part now slides. A trough formed by two rollers may be used.
4. Turning the part after picking it up for the final feeding operation until a lug strikes a stop or an indentation is engaged by a pin, the grip then slipping during the rest of its rotation.

Parts that need orientation only along one axis, such as resistors and paper capacitors, are easiest to handle. Parts such as nuts and screws are not overly difficult as the heads can be used for orientation. Objects that require orientation for electrical reasons should not be symmetrical with regard to a centerline at right angles to the direction of orientation.

Transfer to Operation. In some cases where belts, gravity chutes, or vibratory conveyors are used, all that may be required to feed parts to the operation is a timed gate.

1. The feasibility of the *simple gate feeder* can be determined by building a mockup of the proposed chute or conveyor, closing the exit end, filling with the parts, and opening the exit. If the conveying means does not jam, the first problem has been solved.
2. The next problem is the design of a *conveying means* that permits the feeding of a single part at the entrance with the conveying means empty without jamming. If this can be done, the problem is eliminated. Figure 1 shows three electronic component chutes as a further explanation.
3. Another problem is the effect on the *orientation device* if the chute overfills. This problem usually occurs at the entrance of the chute. If the parts are oriented in the same relation to their direction of travel as in the feed conveying means, backup into the orientation device is not likely to cause trouble. If parts are oriented in another direction, and the answer is not clear cut, build a mockup of the gravity chute for conveying the parts. Test by repeated overfilling and emptying. If the parts empty freely, a backup into the orientation device will probably do no harm. If parts do not empty freely, jamming will almost certainly occur at the conveying means entrance. Funneling of the entrance will only make matters worse.

If the first two problems mentioned above have been dealt with satisfactorily but the last problem has presented difficulties, means of stopping and starting the orientation device in response to the quantity in the gravity chute will clear up the feeding problem.

If the first and last problems have been solved, the second problem can be avoided by operating the chute full, stopping the feed gate by electronic means if the chute starts to empty, and starting again by the same means when the chute fills again.

Feeding Devices for Flowable Material. Granular or powdered material can be fed by opening and closing slides, a rotary valve, a vibratory conveyor, a screw, a belt conveyor, a chain conveyor, or a rotary conveyor.

In most instances, discrete uniform quantities are required at either exact or approximate time cycles. In some cases, measurement by volume,

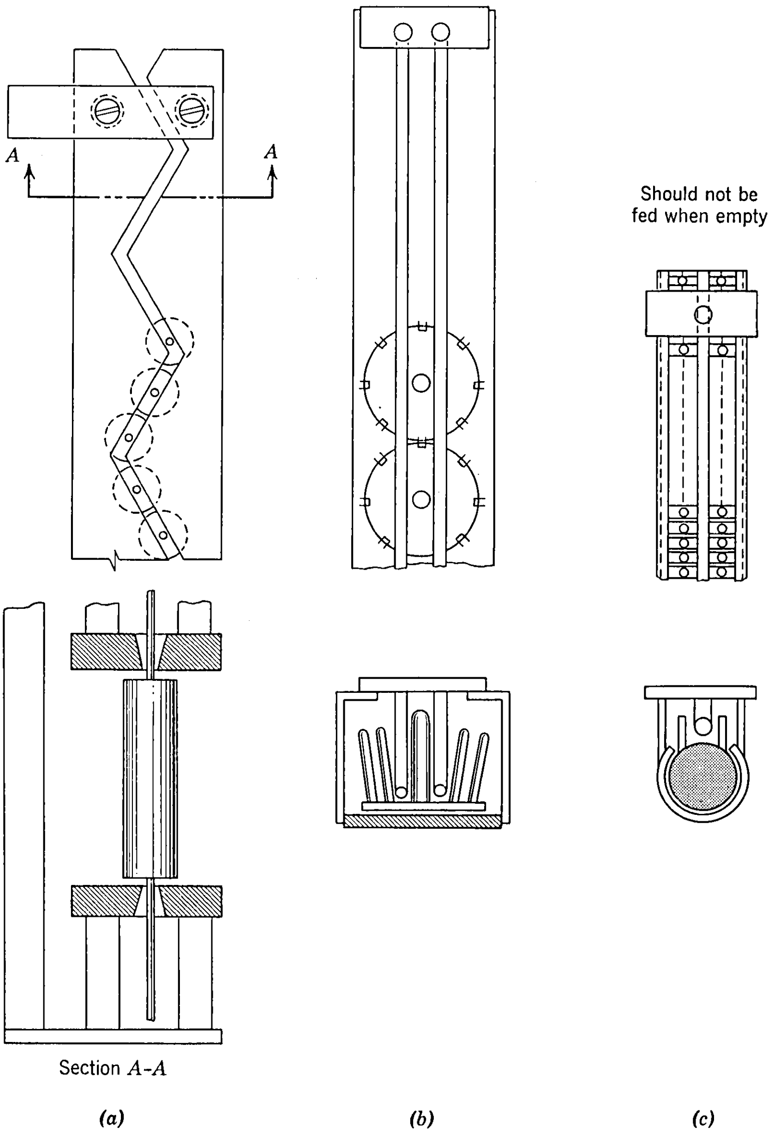


FIG. 1. Electronic component chutes: (*a*) resistors, (*b*) sockets, (*c*) capacitors (ceramic, button type).

such as the space between two slides, or the amount to fill a pocket, will suffice. Sometimes the final container itself is used to measure the amount. Vibration, plunger pressure, and oscillation of the air pressure between atmospheric and a partial vacuum, or combination of any of the above, are often used to secure a uniform fill. Scale weights, either manual or automatic, are used as a check, and the volume is readjusted in accordance with average weights.

In some cases, especially where quantities are large, the first fill is by volume and is slightly under the required amount, and the remainder is made up at one to two additional succeeding stations by being dribbled in slowly while the receiver is on a scale that will cut off the flow at a predetermined weight.

If good accuracy is required, especially if the quantities are small, weight alone is generally used. The weighing may be done either in a hopper type scale or in the container. In either case, the rate of flow is high at first and is slowed to a dribble to complete the weighing.

Liquids, even where accuracy is necessary, can always be fed by volume. Piston displacement will measure even highly viscous liquids or plastic materials, such as bread dough and thermoplastics, with sufficient accuracy for most purposes. As in the case for volume feeders of granular material, occasional weighing of samples is used as a check on the volume feed.

Transfer Mechanisms. The three general types of transfer mechanisms are listed below:

1. A mechanism which intermittently or, in some cases, continuously advances all parts in a process an equal distance during any given time interval.
2. A mechanism which transports parts from one operation to the next operation as fast as received, with a continuous motion.
3. A mechanism which operates in synchronism with the preceding and/or following operation and actually grips the part as an individual piece when moving it to the next operation, or to either of the two transfer mechanisms described above.

Intermittent Cycle Transfer. In some operations, the transfer mechanism need be only a trough of suitable construction. This is the case where parts have at least one fairly flat surface at right angles with two other opposite part surfaces, and the parts are strong enough so that one can be pushed with another. A reciprocating plunger is then used to push the parts along. A continuously moving belt, with fingers to stop the pieces at each station, is another example of this type of transfer mechanism. An intermittently moving chain carrying flights may be used instead of a belt.

Accurate positioning in the horizontal plane, if required, may be obtained by one surface of a part being pushed against a side rail, while a portion of one of the end surfaces rests against a retractable stop. Another method is to use two locating holes in the part. Then, at the station the transfer mechanism inserts a pin in each of these holes. Parts not easily aligned or kept in alignment by guides are sometimes put on a special adapter or pallet for conveying and positioning by this type of transfer mechanism.

A bar on which pushing flights are mounted is also used as this type of transfer mechanism. This bar is given a lengthwise reciprocating motion to move parts, and either a sidewise or angular motion to move the flights clear of the parts on the return stroke. Good positioning accuracy without stops is obtained by this method if the speed is kept low enough to prevent coasting of the parts after the conveyor stops. Figures 2–4 illustrate these linear types of transfer mechanisms.

An intermittently driven rotary table makes a transfer device requiring only accurate positioning of the table to position accurately at all stations. The table is at a disadvantage from the standpoint of inertia when compared with a chain conveyor. The chain or other straight type of conveyor also has the added advantage of accessibility to both sides of the line for the feeding of parts and materials and for adjustment and maintenance. Nevertheless, rotary tables are often the best solution where accurate positioning is needed.

For very heavy work, the parts are mounted on cars which are pulled from station to station and held in position by locking pins at the station while the operations are in progress.

Continuous Transfer. These may be belts, chains with flights that form a flat surface, or gravity chutes. In some cases, parts are blown through tubes with air. These conveyors should maintain the orientation of the parts, but synchronism of delivery is not required. Such conveyors are generally used between machines in a line and, should the machine to which the parts are being fed be stopped to correct trouble, a bank of parts is built up in the conveyor, rather than shutting down the line. Since the parts are already oriented, only a simple gate feeder is required.

Single Operation Transfer. Transfer mechanisms of this type may take either a positive grip on the article or hold it by partial encirclement for the required time.

Single paddles or fingers, either reciprocating or mounted on a revolving shaft are used to remove articles from an operation and feed them to the next one.

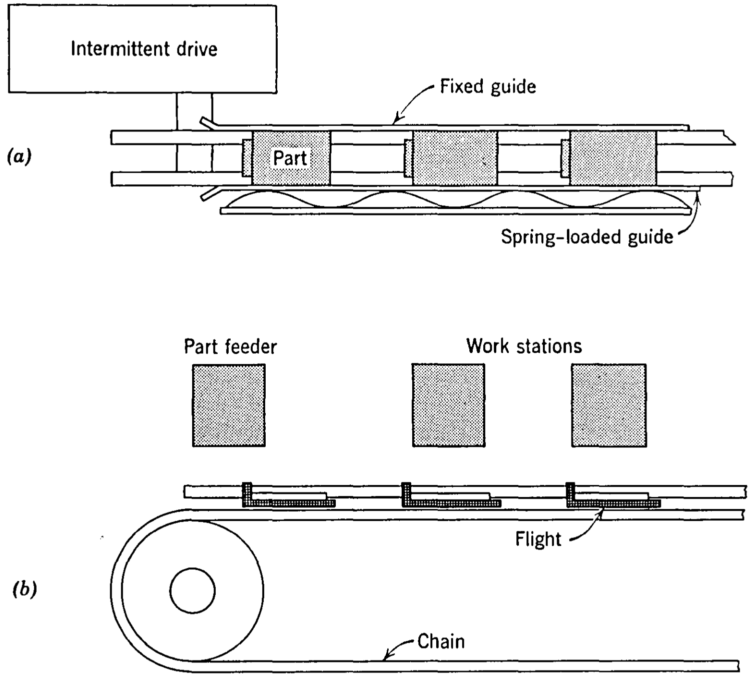


FIG. 2. Transfer mechanism, fixed cycle type: (*a*) top view, (*b*) side view.

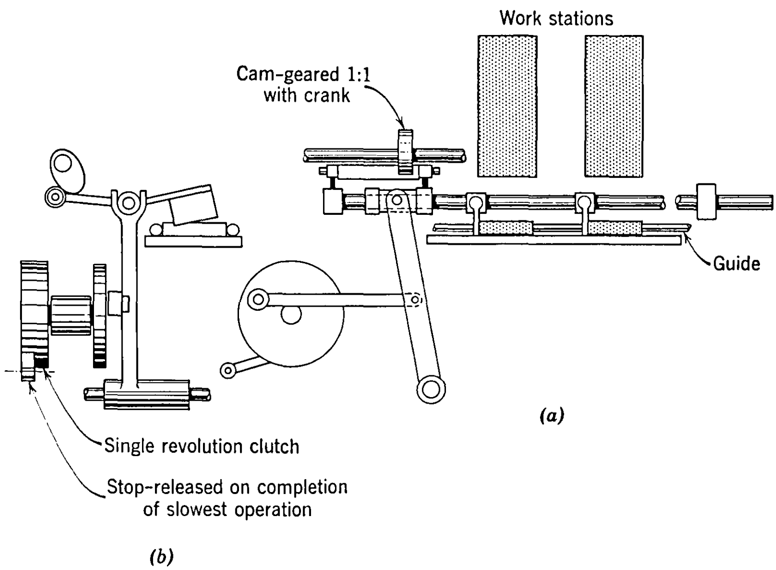
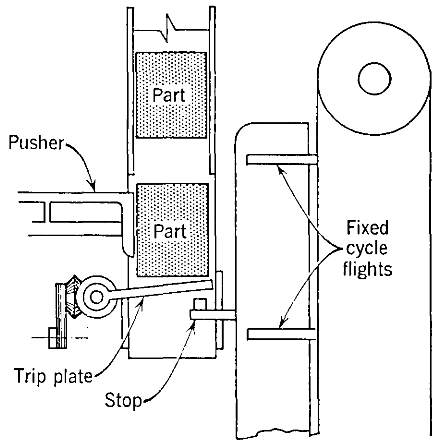


FIG. 3. Transfer mechanism, variable cycle type: (*a*) top view, (*b*) side view.

(a)



(b)

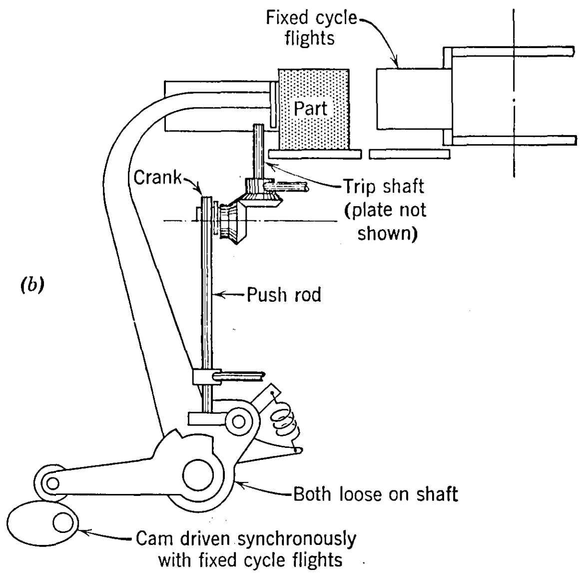


FIG. 4. Transfer mechanism, single station type: (*a*) top view, (*b*) side view.

In other cases, a positive grip on the part is used. One company markets a machine employing an arm with a swing and lift motion to pick the pieces from a rotary table and place them in a die. For light-weight pieces with a flat surface, a vacuum cup is used to grip the piece. Pieces that cannot be handled by vacuum are gripped by vacuum-operated fingers. These fingers, two stationary and one movable, are arranged so that their positions can be adjusted to fit the work piece. Figure 5 is an illustration of this machine. For large work difficult to

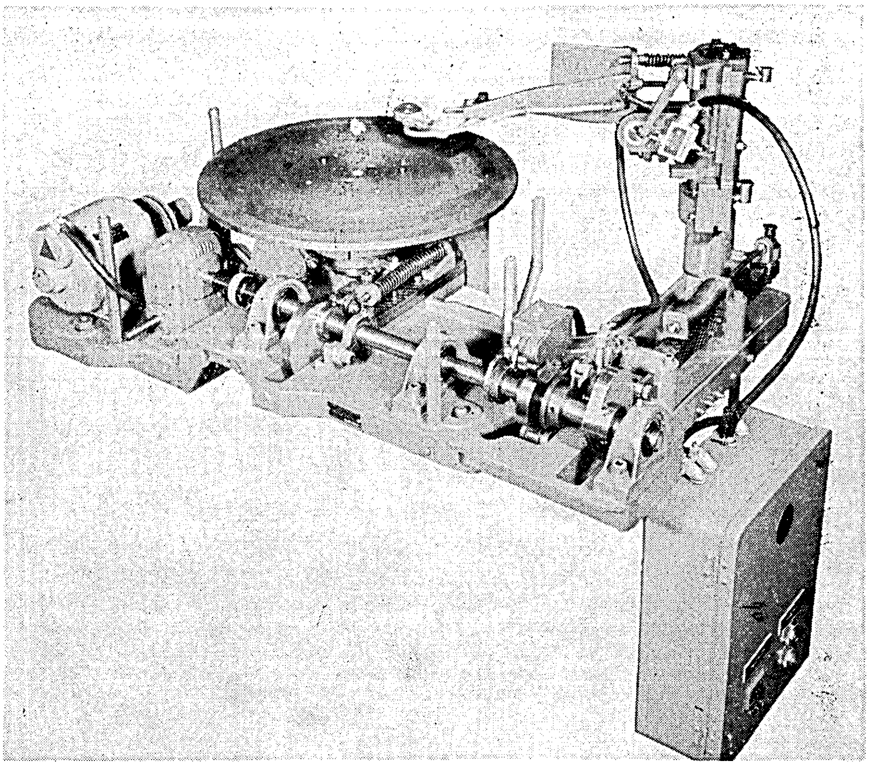


FIG. 5. Punch press feeder, vacuum pickup.

remove by hand or by any of the foregoing methods, an air-operated mechanical arm is available that can be preprogrammed to grip the piece, remove it from the press, turn it over, or change its orientation if desired, and then place it on a conveyor belt. Figure 6 is an illustration of this arm.

Storage Devices for Processes Requiring Extra Time. Some parts of automatic processes such as cooling for sand casting, proofing of bread dough, and drying and curing of plastics may require too long a time at a station or series of stations, without resulting in an impractical machine.

In these cases, the situation can often be saved by the use of a long belt conveyor between machines. Sometimes if parts are to be held together for an adhesion process, an additional upper belt weighted with rollers is used. To save floor space, it is usually best if the shortest dimension of the part can be parallel to the direction of travel and the belt speed low enough so that the parts almost touch each other. If further reduction of floor space is needed, a wider belt may be used and the parts

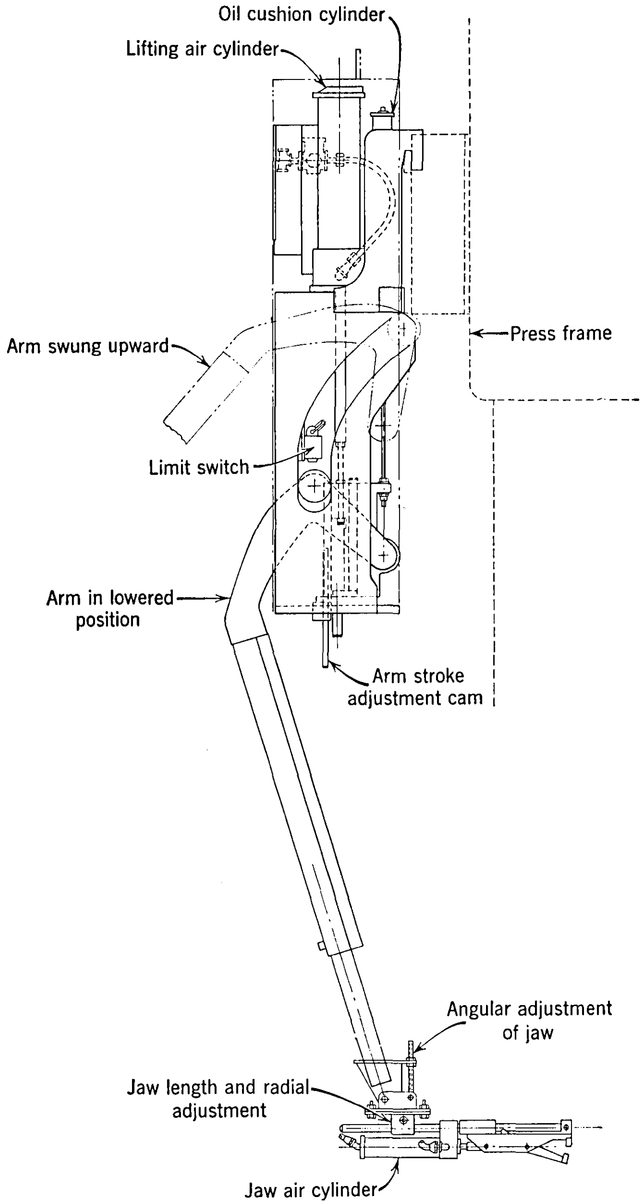


FIG. 6. Sahlin iron hand.

may be placed several abreast during the operation. If the latter plan is needed, it will simplify the transfer if the belt is at right angles to both the preceding and following lines. Parts can then be allowed to accumulate until the desired number abreast is obtained and then pushed by a single stroke onto the belt.

Storage Devices to Minimize Production Loss Due to Down Time. In the operation of a line of machines, minor malfunctioning or tool replacement causes a certain amount of down time on the individual machines. Unless there is some storage between machines, all machines in the line must shut down when one is down. If there are more than two or three machines in the line, the loss of production is often considerable. Automatic storage between the machines is usually desirable to reduce these short down-time losses of single machines. Here the problem is different from that for process storage, as the parts in storage should travel quickly between machines if the receiving machine has no supply.

Parts that are not damaged by a belt sliding against their lower side are usually simply held back by a gate on a belt conveyor ahead of the receiving machine. To utilize this kind of storage, the parts must also be of such shape that they will not jam between the guides of the conveyor or override preceding parts. A vibratory conveyor may in some cases be less likely to jam than a belt conveyor, and its smooth metal bottom may be less damaging to the parts. In other cases, an inclined gravity chute may be the best answer, even if the parts must be elevated before entering the chute. In the case of small parts, it is sometimes most practical to have a hopper and another feeder between machines. This type of storage will allow one machine to be down for a longer time than a chute or a conveyor without shutting down the line.

4. WORK PERFORMING MECHANISMS

Some operations may be performed in series at a single station as on an automatic turret lathe. In other instances only a single operation may be performed at a station, as in packaging where a work station is required for each step as unfolding a box blank, gluing the bottom, and filling.

Machining Equipment

Commonly used equipment includes lathes, milling machines, drills, boring machines, gear cutting machines, and grinders.

Lathes. Three types used for automatic machining operations are:

1. *Single Spindle.* The single spindle is used for one or two specialized operations. There may be several of these in line.

2. *Single Spindle with Turret Support Tools.* Several operations may be performed without moving the piece being machined. The tools are

installed in the turret and are brought in and out of action, step by step.

3. *Multiple Spindle.* These spindles move from work station to work station. Here, only a few tools are used at each station.

The tools used on automatic lathes are the same type used for manually controlled operations, but more care must be taken in selecting long-wearing materials and shapes. It is also possible to supply a means of changing tools without making adjustments each time to compensate for tool length.

Milling, Drilling, and Boring. These operations are usually at stopping points of a transfer system in adjustable positions and at angles which will allow the transfer to be in the direction of line of travel to keep the system simple. Again, the tools are conventional, but are selected for long-wearing qualities. Carbide-tipped drills are often used even on softer metals for durability.

Gear Cutting. Gear blanks are generally formed in a separate operation, since they may be made at higher speeds than the teeth can be cut. Cutting teeth is a special operation of milling or shaping. The equipment may be basically automatic, but it requires not more than two stations for forming gear teeth. At the first station, most of the cutting is completed. The second station is reserved for the finishing operation, which is usually of a type termed "shaving."

Grinding. Two types of surfaces are ground, circular and flat. Circular surfaces may be ground by methods similar to those used in lathes or, if of uniform diameter, between two grinding wheels. Flat surfaces are generally ground as in a milling station arrangement, although two parallel sides are often ground at once between the flat faces of two wheels. Parts for grinding are semifinished previously, and generally this operation does not involve high cutting forces, which makes it feasible to simplify the feeding and holding of parts. Circular parts with only one diameter to be ground accurately may be passed between two wheels, the work being supported by a single track. If the parts are comparatively short, a gravity chute, curved to become level as it passes between the wheels, is a satisfactory feeding means. Parts with two parallel surfaces to be ground may be pushed between two surface grinding wheels, with each part pushing the piece in line ahead of it through the machine.

Stamping and Forming. Tools for automatic punch press and stamping operations are conventional. Standard dies are often used, since even for manual operation the dies are designed for a large number of repeat operations. When many operations are to be performed, the strip stock is fed into the initial station and the parts are moved from station to station by transfer devices installed between stations. Frequently, the parts in press work must be removed from the transfer device at a station and placed in the dies.

Automatic forging operations are more complicated than those for cold forming. Progress has been made, though, by using electric induction heating of the parts. A machine is now offered which automatically feeds blanks and turns out small finished forgings. Certain types of forming operations, such as rolling, drawing, and extrusion lend themselves readily to automatic, continuous operation without the need of intermittent transfer devices. For rolling and drawing operations, the drawing rolls may be considered to be the transfer devices and the dies and forming rolls the work stations. For extrusion, a plunger or screw is the transfer device, with the die serving as the work station.

Casting and Molding. *Automatic sand mold forming* is usually done in several steps. At the first work station, the flasks are filled with sand by gravity. In the following stages, the flask is rammed or vibrated, then parted, and in the next step the pattern is removed.

Sand and permanent mold castings are poured by gravity. Die castings are made in metal molds which are pressure filled.

Molding plastics may readily be done automatically, particularly when large quantities are used. This is true for both thermoplastic and thermosetting. The raw material is usually granular in form and requires only moderate temperatures and pressures.

(*a*) Thermoplastics are fed from a hopper or bin into a melting chamber. The molten plastic is injected into the dies with a plunger. A multiple cavity die is easily handled. The metal dies are good heat conductors, so it is possible to open the die for removal of the part in a very short length of time.

(*b*) Thermosetting material is fed in measured quantities into the dies which are mounted on a turntable at the first work station. In successive stations, the filled dies are closed and subjected to heat and pressure for a given length of time. Since the parts become rigid during the heating process, it is possible to eject the parts immediately after the heating cycle is completed.

5. MACHINE PROGRAMMING

Automatic machines must, of course, be designed so that the operations are performed in the proper sequence, and the correct time must be allowed for each operation.

Synchronous Method. The machine is designed either with all moving parts directly and positively driven from the same power source or, if more than one power source is used, all sources are controlled by one central mechanism to synchronize the operations. Mass production automatic machines performing operations not requiring an allowance for time variation are usually of this type, as it is the simplest kind of control mechanism.

If, however, some of the operations are likely to vary in time because of variations in the material or tools, it may be wise to start an operation by a signal given on completion of the preceding operation. For the purpose of brevity in discussion, this will be called the *sequential method.*

Recent advances in the art of automation to make it applicable to moderate, rather than mass production quantities, have been accomplished by the application of much additional engineering skill to make automatic machines more versatile and more readily changeable from one product to another. In some cases, a mere change in the size of parts of the machine, either by adjustment or interchange, has multiplied the use to which a machine could be put at only a moderate increase in the cost of the machine.

Flexible Programming. To make an economic success of automatic machines when their scope was enlarged to include items of which only moderate quantities were produced, a greater variety in products with frequent changes in programming was needed, resulting in machines with variable programming readily selected by the user. This type of programming is treated in later sections.

Synchronous Programming. A typical automatic machine using this type of programming consists of a transfer mechanism of the intermittent type and work stations located at approximately equal intervals along the travel of the transfer mechanism.

For operation of the work stations by the synchronous programming method, a direct positive drive by the means of gearing, chain drives, and shafting from the same power source that operates the transfer mechanism may be used. In other cases, the work stations may be operated electrically, pneumatically, or hydraulically, these means being programmed by cams positively driven by the motive source of the transfer conveyor.

The transfer mechanism, whether in the form of a rotary table or straight-line chain, may be driven and programmed automatically by one of several different means: (1) ratchet and pawl, (2) Geneva drive, (3) variable pitch worm drive, and (4) cam and differential gear drive.

Servo Transfer Mechanisms. Intermittent transfer may also be secured by controlling the drive mechanism from a central machine control by a cam-operated valve or switch. Often compressed air or hydraulic means are used for such transfer mechanisms because the resulting mechanism is simpler than if an electric motor were used. This type of transfer is not suitable for a large number of operations per minute as some allowance has to be made for variations in transfer time.

Sequential Programming. For sequential programming, transfer mechanisms are of the servo type except that instead of being operated by a central control, the cycle is started by a switch or valve moved by the previous operation.

An example of this type of programming is used in the filling and closing of flour bags. Since an operator places the bags on the filler tube, it is desirable not to depend on a predetermined cycle time for the filling operation. Therefore, the filling cycle is initiated by tripping an electric switch as the bag goes on the tube. In the next step, the time required for the auger to compress the desired amount of flour into the bag varies considerably, not only with the type of flour but also with the length of time the flour has stood in the bin since being milled. Therefore, the next operation, which consists of adding a small amount of flour slowly until the correct weight is reached, is also initiated by the filled bag tripping a switch as it passes onto the weighing scales.

Because of the variability of the time cycle of the filling and weighing operations, the machine stations that close the tops of the bags after filling must also be designed to accept bags on irregular time schedule.

Flexible Programming. While nearly all automatic machines in the high production field can be changed to accommodate some variation in the size of the product, and many can be varied to produce a variety of products, these changes generally require both the changing of parts and some time-consuming adjustments before being ready to produce the new product. However, much progress has been made in this direction and some machines are now available in which a template change or a new punched card or tape in the control device is all that is needed to reprogram the machine.

Flexible Control by Mechanical Adjustments. In order to be economically feasible in the small lot field, a type of programming that can be quickly changed is needed.

If the machine performs only one simple operation such as boring a single hole, a convenient means of adjusting the bore diameter and feed rate will serve the purpose. Interchangeable cams are used if several operations are needed. However, some of the more complex automatic lathes now have sufficient adjustability so that special cams are not needed for each product. Pieces with several surfaces to machine, however, still require considerable time to set up for a new part.

Flexible Control by Template. For some operations, the following of a template, which may be, in many cases, an accurately made sample part, is often the best answer.

Milling. Contour milling in either two or three dimensions can be accomplished by connecting the template follower mechanically to the milling machine carriages. For two-dimensional milling, the work carriage usually is moved. In some machines, connections can be made to rotate cylindrical work. Only one motion in the horizontal plane is then needed, this being along the line of the cylinder's axis.

For three-dimensional milling by direct mechanical connection, generally only the tool head is moved. Three motions are required for this type of operation, but these need not be straight-line motions or at right angles to each other. In fact, arcuate motions often simplify the design and reduce guidance friction.

A follower of the same radius as the milling cutter is invariably used to simplify the making of the template.

Similar mechanisms can be used for lathe work on soft materials like wood but, in general, because of the high tool pressure and because of difficulty in controlling depth of cut, this method has not been used for metal turning.

Even in the case of milling, especially in metals, the tool pressure is high enough to cause operator fatigue and enough deflection in the mechanical connections to make high accuracy impossible. Therefore, servomechanisms, usually with electric sensors, are used which maintain a certain pressure on the template follower. The follower is supported on the stationary part of the machine along with the tool mount. The template and work are both attached to the movable part of the machine. Figure 7 explains the follower and servo operation.

Lathe Work. For lathe work, template-actuated servos have been quite successful. Generally for this kind of operation, the longitudinal feed is preset and remains the same for the whole operation, the servo controlling only the depth of cut. Cuts, at right angles to the centerline of the work, can be secured either by stopping the longitudinal feed, or by setting the cross-feed at an angle of perhaps 45° and withdrawing the cross-feed enough faster than the longitudinal feed to produce a right angle cut. By this latter means, cuts at any angle can be followed. The same means also serve for following curved contours. One manufacturer uses an electronic tracing finger similar to that used in milling. Contacts in this finger control both the tool cross-feed carriage and the main tool carriage through magnetic clutches.

In the case of lathes, direct hydraulic sensing has proved practical. A differential valve operated by the follower is used. The valve follower assembly is supported on the lathe tool carriage. A very minute difference in the valve plunger position is sufficient to operate the hydraulic carriage drive in the direction required to null the valve. Accuracies of within 0.0001 in. of the template diameter are claimed for this type of servo.

An air gage sensing element, based on the change in back pressure as

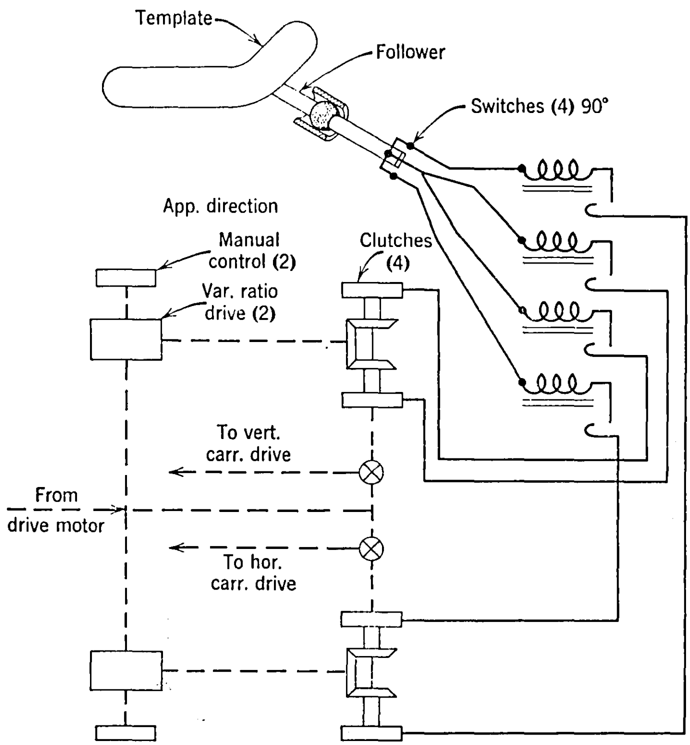


FIG. 7. Follower and servo diagram for milling machine.

the open end of a tube through which air is flowing adjacent to the surface of the template, has also proved practical for copying lathes. This air gage in turn controls a hydraulic valve of the diaphragm-operated type. This valve controls the tool carriage position. As in the case of the full hydraulic servo, the air gage is mounted in the tool carriage, forming a closed-loop servo. Figure 8 illustrates this servo.

Flexible Control by Digital Input Controlled Servos. Means are now available for programming automatic machinery by digital inputs. The simplest operation of this type is positioning for drilling or boring operations inasmuch as no close rate control is needed. However, most machining operations need a high degree of accuracy and the problem is to obtain an error signal that correctly represents the true deviation of the machine from the required position with a high degree of sensitivity.

This type of control is treated in Chap. 6, Numerical Control of Machines.

Overriding Controls. Even the best designed automatic machines make errors. Unless a control steps in and causes a deviation from the

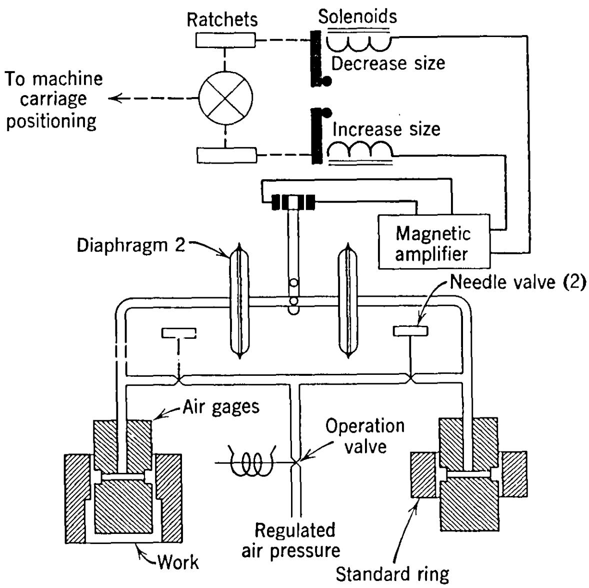


FIG. 8. Air gage servo diagram.

programmed operation, some decidedly disastrous results can happen. The following are several examples:

1. In a machining operation, a drill may become dull, break off and stick in the workpiece. If this drill is followed by a reaming or tapping operation without controls, more broken tools and perhaps broken machinery can result.
2. A defective paper flour bag may leak during the filling operation. Unless there is a limiting control, the weighing scale will continue to call for more flour until turned off by the operator.
3. On high-speed packaging machinery, one or two broken boxes will do the machine no harm, but if boxes continue to break on this operation, the accumulated mass will usually damage the machine, and it may require considerable time to pry it loose from the ways of the machine.
4. In the case of attaching electronic components to printed circuit boards, a loose or partially inserted component on the board may result in this component interfering with a succeeding attaching operation and may perhaps break the circuit board.

In addition, some controls also serve as inspection devices and automatically stop the machine if it is doing unsatisfactory work.

Automatic Stopping in Case of Malfunction. In packaging machinery, it is usually customary to locate electrical sensing fingers so that a part of a package which is out of its normal position will stop the machine by striking a finger. These fingers can be located on a moving arm as well as on a stationary part of the machine. Another plan to prevent damage to the machine is to use a spring operating against a cam to power the working stroke. This prevents damage to the parts in case of jamming. It also allows the insertion of a switch that will shut down the machine unless it is opened at the proper time at the extreme forward position of the stroke.

In the *automatic assembly of electronic components* of a printed circuit board, improper operation can be made to stop the machine in case one or more of the terminals fails to appear on the underside of the board. Switches on the terminal-clinching mechanism sense the lack of a projecting terminal by not closing.

In *automatic drilling machines,* dulling of drills causes the principal control problem. The torque required to turn the drill and the pressure required to maintain a preset rate of feed are the most readily used means of control. Single drills driven by individual motors can be easily made to give automatic notice of approaching need of tool change by power consumption increases, an amperage-operated device being sufficient.

Power consumption has also been successfully used to detect dull drills in multiple head spindles, where a small number of spindles is used, e.g., not more than six, and where drills have only small variations in size. To accomplish this, the current consumption is integrated by a small servo motor. The speed of the motor is proportional to operational current consumption for a normal drilling cycle as compared with that of a sharp drill cycle. When current consumption exceeds the sharp drill cycle by a certain amount of tool wear, an alarm is given.

Since the power consumption method does not show which drills are too dull, and since it is not reliable for multiple spindles with a large number of drills, the thrust pressure applied to the drill is generally used for multiple head drills. While strain gage or carbon pile electrical resistance methods are suitable, a liquid-filled metal bellows which operates the alarm switch directly is a simpler and, in general, a more practical solution. A diagram of this arrangement is shown in Fig. 9.

Direct measurement of torque, although possible, has the disadvantage of requiring either electrical slip rings or differential gearing for each drill and generally is not used.

Automatic lathes and grinders rarely jam if fed with the correct size parts. Automatic gaging of incoming parts and rejection of unusable pieces appear to be the best answers to this problem. Also, automatic gaging of the output for quality control, connected with a device for stop-

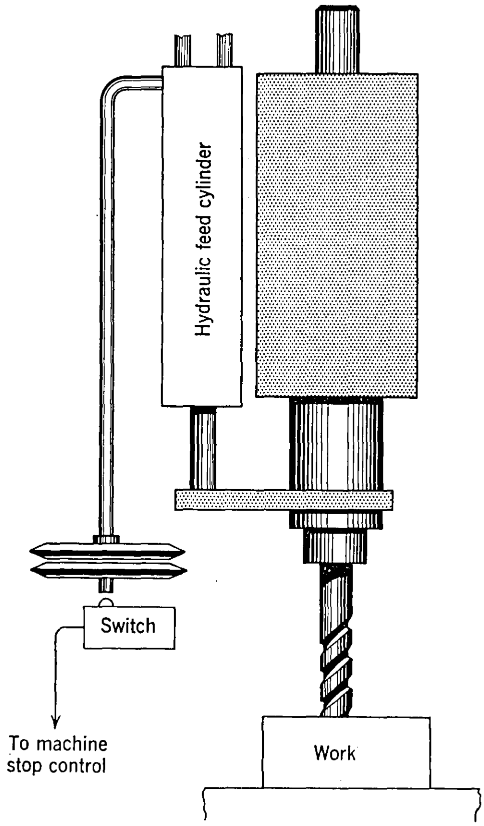


FIG. 9. Drill pressure signal diagram.

ping the machine, is desirable. It is generally customary to arrange this device to stop the machine only after two out-of-tolerance pieces have been made in succession.

6. AUTOMATIC INSPECTION (See also Chap. 4, Automatic Inspection and Control.)

Automatic weighing scales are a familiar device. Much recent progress has been made in automatic gaging. Both the air and electronic gages have been applied to measure automatically machined dimensions. The air gage is generally favored because of its simplicity and reliability.

The invention of radiation thickness gages have made possible automatic thickness control in steel mills and paper mills.

Most of the successful gages for automation have no rubbing contact with the material. Although this is not an absolute necessity for a gage used for automatic control, it is desirable in that it eliminates the need for frequent recalibration due to wear of gage points, requires no control of the pressure applied to a gaging finger, and eliminates loss of accuracy due to friction in a mechanical leverage system.

Automatic Adjustment by Inspection. Automatic weighing devices make possible the automatic adjustment of fillers by the use of servo controls on either volume or weight fillers. Such control will often make a simple and fast volume filler accurate enough to replace a slower weight-operated filler. Figure 10 is a diagram of such a device.

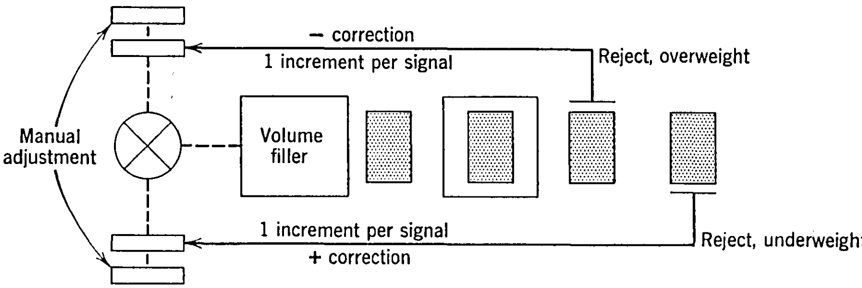


Fig. 10. Volume feeder with automatic weight adjustment.

Some boring and turning operations protect against the manufacture of poor quality parts by making use of an automatic gaging operation to reset the tools. In order to keep this control system simple, the following scheme is generally used.

1. If the diameter approaches either tolerance limit closer than a predetermined amount, a very small increment correction is made.
2. If the diameter becomes out of tolerance, the same small correction is all that the mechanism makes.
3. If two successive pieces are made exceeding the tolerance limit, the machine stops automatically and waits for manual readjustment or new tools if needed.

In some of the modern steel mills radiation gages through servo motors reset the distance separating rolls in steel mills so as to produce the correct thickness. In the paper industry, these gages are making it possible to regulate automatically the flow of pulp onto the screen on which the wet sheet is first formed.

Automatic Rejection of Products. It is desirable to protect automatic machines by rejecting unusable parts and pieces of scrap that may be in the product fed to it. Automatic gaging at the end of the machine making the parts may, in general, prevent the receipt of over- and under-sized parts, but mistakes do happen and occasionally pieces of scrap may get into a parts box. Since a mistake of this sort might result in a costly breakdown of the machine, a rough sizing operation is often included in the feeding device. To illustrate: when gear blanks are being fed by a vibratory hopper, these blanks are flat on the conveying shelf. At one point on the shelf, a diagonal finger at the correct height sweeps off oversize pieces to rejection. The next finger allows undersize pieces to go under, but sweeps off those of correct size to feed the machine.

Much development work is yet needed to devise a fairly simple means to reject parts that may jam an automatic machine. In electronic assembly, means are needed to reject components with badly bent leads. In automatic packaging, damaged boxes and bags should be rejected to prevent spilling of the product and possible machine damage. While these off-grade items are few in terms of per cent of the feed, the per cent down time caused will usually average from 10 to 100 times the per cent of such items based on number of pieces fed. For example, a machine handling 2000 pieces an hour may receive only one faulty part in 1000 (0.1%). However, the two faulty parts per hour are likely to shut the machine down a minimum of thirty seconds each, resulting in about 2% loss of operating time. One bad jam during an hour's operation that takes six minutes to remedy will mean a 10% loss of operating time.

7. TYPICAL EXAMPLES

Bemis Bag Packer

This machine will handle granular and powdered material of average flowability such as wheat flour. Paper bags are generally used but bags of other material of moderate stiffness could be handled. Figure 11 is an illustration of this machine.

The bags are automatically fed from two magazines, which must be arcuate in shape because of the extra thickness at the bottom of the folded bag. The magazines are manually filled. On one side of the bag, the top has a fairly long but narrow portion cut out so that fingers can readily engage only one side of the bag first and open it slightly, before being inserted to a greater depth to complete the opening.

The material is preweighed, six automatic scales being used, as the weighing operation is a slow one because of the need of final scale balancing without overweight. Two fillers operating alternately are used, as a bag top folder and a closer will handle the flow from two fillers. The six automatic scales are operated in equally spaced time sequences, so that each scale has a time allowance of six times that of the top folder and gluer. They are also operated so that the scale dumps are alter-

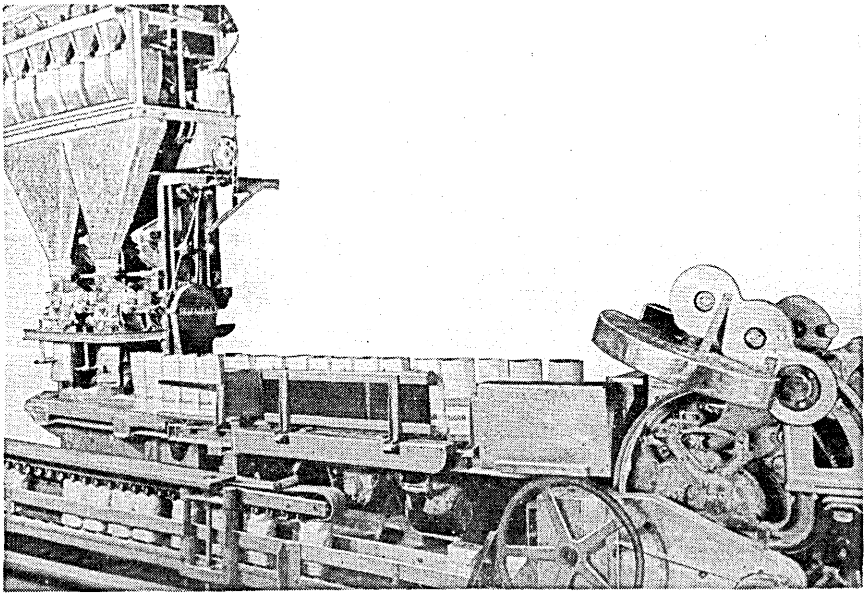


FIG. 11. Bemis bag filler and closer.

nated between the two filling hoppers, the filling operation being at one-half the folding and gluing operation rate.

After filling, the bags are pushed onto a conveyor belt extending to the top folding mechanism. Along the belt, between the filling and top folding operation, is a device for compacting and leveling the material in the bags. This is done by vertical vibration and by patting the bags on the broader side with the conveying guides.

The top folding and gluing mechanism is of the ferris wheel type. The bags go in open side up at the top of the wheel and are delivered below upside down onto the drying belt, only 180° being used for operations. The wheel moves continuously, all operations, including feeding and discharge being done without stopping the wheel.

Feeding is accomplished by a paddle that pushes the bags into the wheel in synchronism with the flights of the wheel. However, this paddle operates only if a bag is in proper position for feeding. This is accomplished through bag contact with a switch, which causes a solenoid-operated pawl to engage the paddle with the pushing mechanism at the correct time for the next feeding cycle. The pushing mechanism is always in positive gear with the "ferris" wheel.

A series of rollers is used to put the glue on the bags, one roller dipping into the glue and one or two rollers transferring it to the final roll that spreads it on the bag.

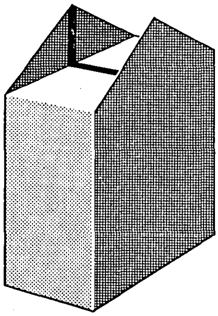


FIG. 12. Partly folded bag.

Glue is first applied, while the bag is in motion, to the inside surfaces of the bag where needed. Fingers then fold the bag to the extent shown in Fig. 12. Glue is then applied to the upright flaps, and these are folded down. All the motions used in folding and gluing are rotary and yet are so designed that all the operations are done as the bag is in motion. No cams are required for these operations. The bags are then delivered upside down to a drying belt conveyor. This conveyor also has an upper belt, weighted for extra pressure on the bags during drying.

This bag packing line is adjustable over a continuous range of sizes, with only a few changes in parts.

Automatic Gear Production Line

This line is for the production of small pinions 1.08 in. diameter by 5/8 in. thick. It will not be described in detail because of its complexity, but certain important features will be discussed.

The raw products for the line are semifinished turned blanks from eight-spindle automatic lathes. The gear production line performs the following operations in the sequence given; storage, feeding, and transfer are omitted.

1. Inspect for O.D., length, and bore.
2. Centerless grind outside diameter.
3. Inspect O.D.
4. Semifinish machine bore; finish one face and chamfer.
5. Inspect bore.
6. Finish other face; chamfer and finish bore.
7. Inspect bore and thickness; hone undersize bores and return to line.
8. Cut teeth (Fellows gear shapers).
9. Inspect for irregularity of cut and pitch diameter.
10. Remove burrs.
11. Finish machine teeth by "shaving."
12. Inspect for pitch diameter and helix angle.
13. Wash for chip and oil removal.

Parts are bulked in baskets and sent to heat treating furnaces. They are then returned in baskets to the line for the following operations.

14. Grind tooth chamfers.
15. Grind oil grooves.
16. Hone the bore.

17. Operation noise inspection.
18. Final inspection of bores.

The interesting and important features of this line are:

1. The feeding and distribution system.
2. The inspection and quality control system.
3. Waste removal. Automatic waste removal is an essential labor-saving feature of the line. The chips and waste coolant are piped to a filtration system, and the coolant is returned to the machines. Waste from machines using the same coolant is handled by a central filtration system.

Since the capacity of the machines to perform the large variety of operations varies widely, from one machine for O.D. grinding to fifteen for cutting the teeth, a complex distribution system is needed.

In general for feeding, vibratory feeders of the circular bowl type are used. These also serve for storage between operations, the 24 in. bowls holding 500 parts.

A belt is used to distribute the parts to the 15 gear teeth shapers. As soon as the feed chute on the first machine is filled, the parts pass to the next machine's feed chute. When all feed chutes are full, they bypass to the vibratory feeder bowls, one at each machine, to build up a reserve supply should the preceding machines in line be down. Should one of the shapers be down, the line operator, by the push of a button, can reduce the number of pieces going to the shaper distribution belt and cause the excess to be stored until all shapers are again operating.

The vibratory feeders are operated intermittently, filling the feed chutes, and then stopping until the chute is partially empty.

Inspection and Quality Control. The work of each operation, with the exception of a few noncritical dimensions, is automatically inspected by air gages immediately after the operation.

If an inspection shows that tolerance limits are being approached, a definite single increment of automatic adjustment of the cutting tool is made to correct the operation. If parts are beyond tolerance, they are rejected. In general, three consecutive out-of-tolerance parts will automatically shut down the machine.

Parts rejected are sorted into two classes, salvageable and scrap. In some instances, salvageable parts are automatically reworked and returned to the line.

Automatic Malleable Iron Foundry

Automation of a foundry presents some interesting problems because of the wide range of time requirements for the operations. Making

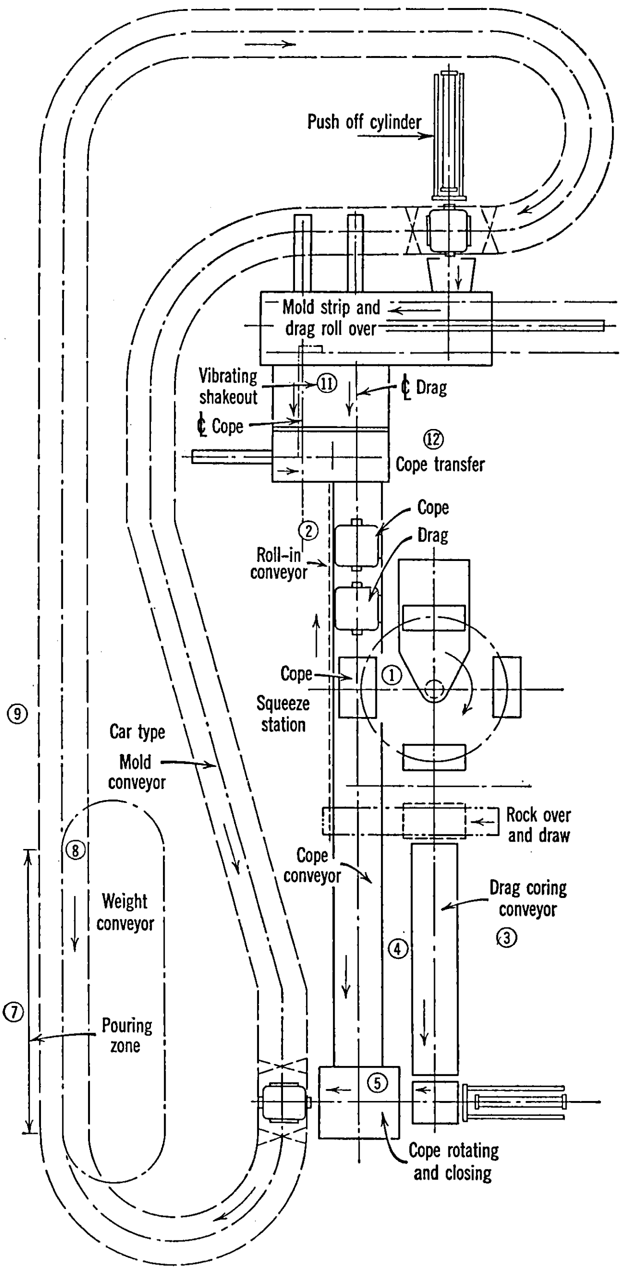


FIG. 13. Osborn automatic foundry plan.

molds, when done by machine, requires only twelve seconds including transfer time for some of the operations. The machine adopted by the foundry given as an example had a nominal rate of five molds per minute. Cooling of the castings in the mold requires a time period of fifteen to twenty minutes, or about eighty times that of the mold-making operations. There is also the problem of return of the flasks and of sand recovery and return.

Figure 13 gives a plan view of the foundry. The filling machine at station 1 is fed alternately cope and drag (upper and lower) mold flasks by roll in conveyor 2. The flasks are fed along the conveyors by a pusher with a stroke equal to the length of a flask, one flask pushing the other along the line.

After filling and with the pattern in place, the flasks go to the molding station where the sand is rammed. As they pass from the molding station, the patterns are withdrawn, and the flasks again are placed on a conveyor. The drag flasks are next removed from this conveyor and placed on conveyor 3 where cores are placed in the drag, if needed. The drag flasks, which are turned at a right angle at station 5, the closing machine, are now on a lower level than the cope flasks on conveyor 2. The cope is also turned 90° by the closing machine and placed on the drag, thus closing the mold.

The molds now go to the pour station 7, where they are picked up by pallets. Weights are added to hold down securely the cope at station 8. The metal is now poured and the molds are transferred to the cooling conveyor, 9. This conveyor is amply wide to hold a number of flasks abreast. The flasks are elevated at the end of the cooling conveyor to the automatic shake out machine, 11, where screens separate the flasks, castings, and sand. The flasks are now passed along the conveyor to indexing mechanisms 12, which return the flasks to the conveyor, 2, feeding flasks to the line.

This foundry requires 7200 square feet and is said to produce as much as a conventional foundry of four times this floor area. It has the further advantage of not requiring transportation of the hot melted metal.

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Automatic Inspection and Control

J. A. Sargrove and D. L. Johnston

1. PURPOSE

Automatic inspection is more than a substitute for the human inspector for, in this class of repetitive operation, human ability is of a low order. A mechanistic approach can yield an altogether higher order of performance and, because of this basic difference, a process control system based on inspection should be completely *rethought* when it is made fully automatic. Certain of the principles to be followed are set down here, with an indication of the detailed method of approach.

2. LIMITATIONS OF HUMAN INSPECTOR

The work of Belbin in England, at McGill University, and elsewhere, (Refs. 1–5), has established the very low capacity of a human being to detect occasional faults, such as foreign objects in bottles in a bottle washing plant, faults in sheeting materials, and imperfections in enrobbed chocolate confectionery and in ball bearings.

The task of observing a uniform flow results in a *hypnotic effect* on the subject, which increases as the frequency of the unusual events sought diminishes. A similar experience occurs in driving on a straight road amidst monotonous terrain or in flying over the open sea. In the extreme case, hallucinations are experienced which may lead to illogical and dangerous reactions.

The control engineer in charge of a process control plant is exposed to these circumstances and, however complete the instrumental display provided, the operator cannot be relied upon to detect marginal conditions that may occur. *Automatic alarms* should be provided for this purpose. A watch-keeping routine is desirable, as on a ship, and can include the duties of second line of defence, by checking at intervals the alarm devices and working levels of significant factors as plotted by recorders.

In certain forms of human inspection, as for surface flaws in ball bearings, a proportion as high as 20% of the faults may pass a first inspector, and this ratio will worsen with each successive inspection. *Thus, even with triple inspection more than one fault in 100 will remain undetected;* this is a serious matter in such end uses as aircraft.

Mechanical Inspection. Thus a mechanical and instrumental method of inspection is inherently superior, and its reliability is predictable and does not diminish with repeated application. Some research and ingenuity may be necessary to determine the best method of automatic inspection, particularly for a qualitative fault such as in the following example.

EXAMPLE. *Surface flaws in a ball-bearing ball.* In such a case the object should obviously be rotated through the full range of solid angles, either progressively or by random motion, and the local character of the surface observed by a transducer. The physical effect used in the transducer should be so chosen that it gives the simplest and most elegant solution. Methods that might be considered are: (*a*) local irregularity of eddy current skin effect (Ref. 7), (*b*) scatter of optical reflection, and (*c*) irregularity in free-rolling trajectory or friction characteristics.

3. CHARACTERISTICS OF FAULT STATISTICS

Despite human limitations in a repetitive task, an inspector exercises a degree of intelligence and judgment that may not readily be duplicated by instrumental devices. It is therefore important to distinguish the different characteristics likely to be found in the statistical data from an inspection operation.

Models typical of practical conditions as shown in Fig. 1 are: (*a*) stable flow processes subject only to slow trenditional changes; (*b*) steady processes subject to dangerous conditions near to the tolerance limits;

(*c*) batch or semibatch processes where end effects are significant; (*d*) processes subject to repetitive changes, possibly at a relatively high frequency; and (*e*) processes with sporadic or statistically random variations, which would be dealt with by an accept/reject mechanism.

The simplest inspection and control systems are those applicable to (*a*) and (*e*), and it is desirable to reexamine the process to eliminate if possible the nonlinear element in (*b*) or the discontinuities in (*c*) and (*d*).

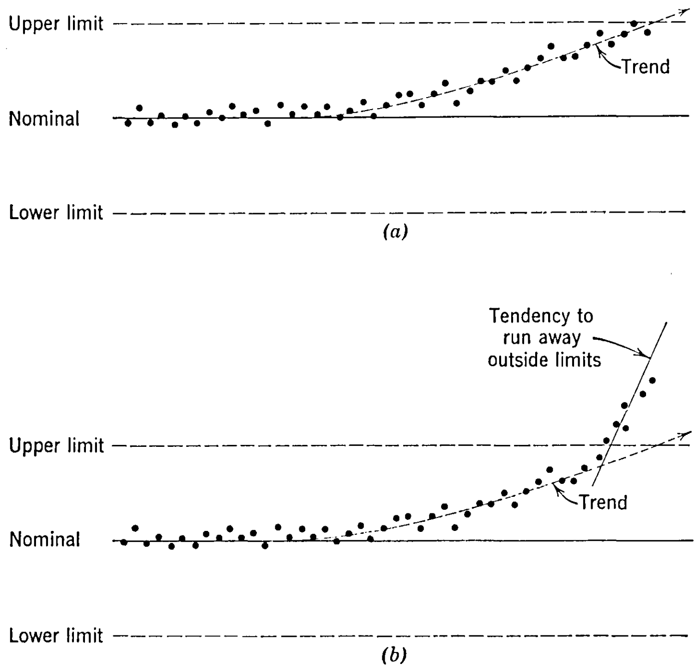

FIG. 1. Characteristics of the statistical variations experienced in the five classes: (*a*) trenditional changes, stable; (*b*) trenditional changes, unstable limits; (*c*) semibatch process, with end effects; (*d*) repetitive changes; (*e*) sporadic variations.

For *example,* the discontinuity between successive batches in a process can be eliminated simply by arranging that the conveyor system merges the end of one batch with the beginning of the next. Repetitive changes can be reduced by some reservoir or smoothing function (Fig. 2).

Feedback Control. Where a process is unavoidably nonlinear or rapidly varying, the *inspection-feedback control system* must be designed from the point of view of the Nyquist stability criterion. The gaging or sensing mechanism must be rapid in action, and a derivative control will be necessary to prevent hunting. The inspection point must be as close as possible to the process in terms of time scale. In these circum-

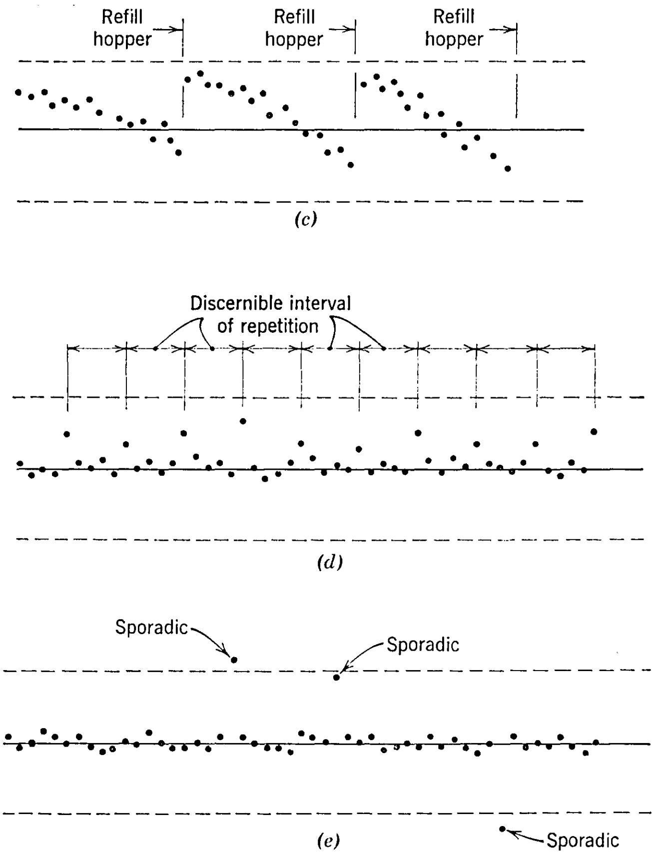

FIG. 1—(*Continued*)

stances a high-velocity low-mass process is more readily controlled than a slow ponderous one.

There are a number of interesting *special cases* that arise.

Asynchronous Faults. A multihead production machine might develop a single faulty station and, rather than reject every piece produced, one can design the inspection system so that it causes this one faulty station to be put out of use automatically or its identity signaled, the production being permitted to continue from the other stations.

Legal Limits. A normal gaussian distribution about a mean value is normally acceptable for technical requirements, but legal definitions such as weights of commodities, are based on the concept of a minimum value (Ref. 5). It is then desirable to arrange the process for a very narrow gaussian distribution (which may require a high speed of apparatus response) and possibly skew the distribution curve (Fig. 3) or recirculate rejects for readjustment.

Flukes Ignored. When an average error signal is fed back, it may be advisable to exclude the extreme values by rejection before permitting the computing apparatus to calculate the average; similarly there will be an optimum averaging number or time constant for any particular numerical distribution.

4. SENSING ELEMENTS FOR INSPECTION

The operation of inspection may depend on quantitative factors, and also on the qualitative factors already mentioned. For the former the whole range of classical measuring techniques is available for adaptation (Fig. 4), but the latter may require careful study of alternatives and some ingenuity in application.

For the quantitative measurements it is not sufficient to take a laboratory-bench test instrument and put it into continuous service. Many such instruments depend on frequent setting-up and zero-setting adjustments, and it may be necessary to modify the basic method of operation in order to obtain a stable mechanism that will not require adjustment over long periods.

The principal requirements of the sensing instrument can be listed as follows:

1. Stability and absence of the need for recalibration and zero adjustments.
2. Robustness, of a much higher order than for laboratory instruments, preferably with fully sealed housing.
3. Reliability by designing conservatively for long life. Ultimately magnetic amplifiers and transistor circuits will be preferred to tube circuits.
4. Modulated a-c systems or differential systems always to be preferred above d-c amplifiers.
5. Ease of servicing, using miniature plug-in "packages" easily exchangeable by maintenance staff, but factory-serviced.
6. Compatibility with other devices, for ease of interconnection and building up of systems.

5. INSPECTION AND CONTROL SYSTEM DESIGN

In most applications the magnitudes of the parameters to be inspected are fixed, and it is sufficient to deal only with the first derivative or percentage error. For *example,* if it is required to gage an object nominal length L, limits $\pm 5\%$, significant gaging error $\frac{1}{2}\%$, the gaging system need resolve only $\frac{1}{2}\%$ in $\pm 5\%$ or one part in 20 provided the total

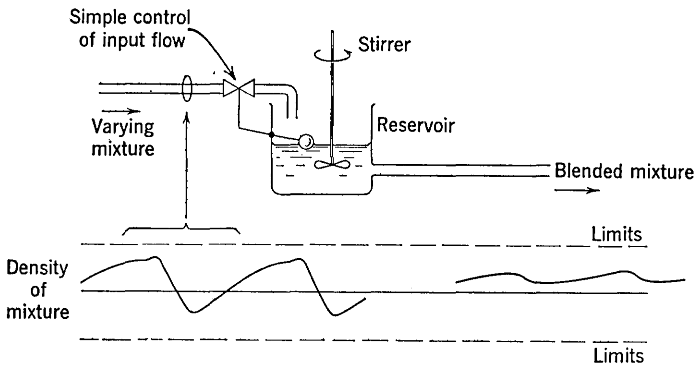

FIG. 2. Smoothing to eliminate excessive variations. Smoothing by reservoir or other means eliminates excessive variations due to repetitive changes, where phase lags are not objectionable.

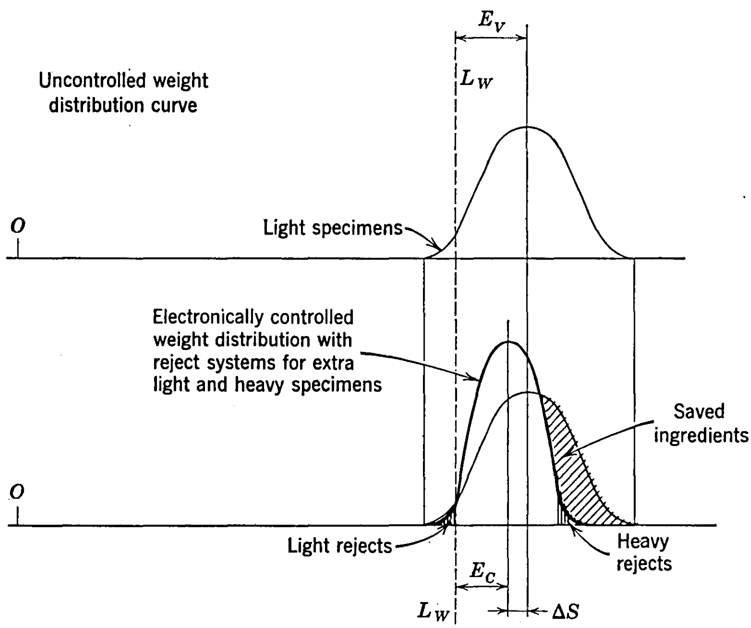

FIG. 3. Normal gaussian curve showing symmetrical limits, and also the asymmetrical limits that occur with "legal minimum" legislation; here a skew gaussian distribution is preferred. Effective spread of weight distribution histogram of uncontrolled dough divider (top curve). Reduced spread of weight distribution of dough pieces (bottom steep curve) after combined effect of electronic computer weight control and pass-and-reject gate action eliminating the heavy and light weight dough pieces. L_W = legal minimum weight. Note shift in mean weight ΔS and effective saving in material used.

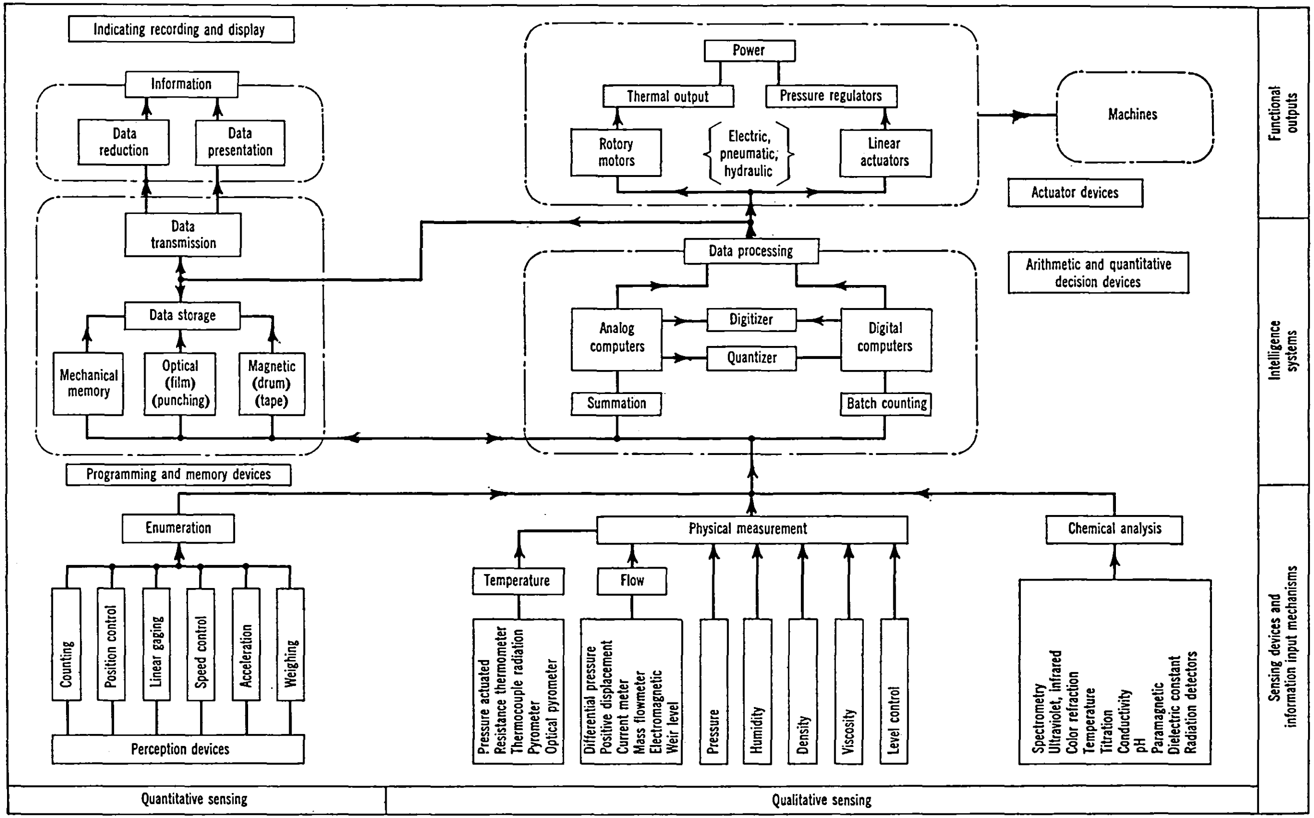

FIG. 4. Classification chart of inspection sensing techniques and decision-making and actuating mechanisms.

magnitude of the main dimension A is constant. This is a very moderate specification. If a simple computation such as density D is to be made from the parameters length L, area A, width W, and weight M, we have $D = M/(L \times A \times W)$, but if one is only interested in the fractional error in density δ_D, one needs measure only the three fractional errors δ_A, δ_L, and δ_W, and approximately $\delta_D = \delta_M - \delta_A - \delta_L - \delta_W$ for small magnitudes of error.

Thus for relatively narrow limits of error, the *numerical computation system* can be a very simple one, using summation operations only, without the need for an operation of multiplication or division.

It is desirable that the system be constructed physically as a group of predesigned "black boxes," chosen for their compatibility, so that one may be connected to another without the need for any transformation or conversion. Unfortunately such coordination does not exist between all classes of commercially available automation components.

In the density gaging system of Fig. 5 it is a considerable simplification to employ the same pattern of differential transformer positional-transducer for all four functions of A, L, W, and M, and to carry out the latter weighing process by detecting the deflection of a standard cantilever. The gaging and computation system of Fig. 5 operates in the following manner:

1. The molded objects are moved from station to station on the conveyor.
2. At successive stations, variations in height, diameter, and weight are sensed by variable inductance heads. Height transducer gives function δ_L; diameter transducer gives "area" function δ_A.
3. The information is indicated in d-c meters and used to modulate an a-c carrier in each channel, and recorded as a voltage analog on a slowly rotating magnetic drum.
4. Information is read out time-displaced, so that data on a particular object are available simultaneously. It is demodulated, and the d-c signals are used to actuate limit circuits and for a simple analog computation of density.
5. Limit circuits control rejection of individual objects, and a quality control chart is plotted.
6. An average signal is fed back as error correction to the molding process.

Similarly, in an air gaging system, pneumatic controller components would be used throughout, up to the point where an electrical command output becomes necessary.

Conditional gaging systems are similar to the numerical computing

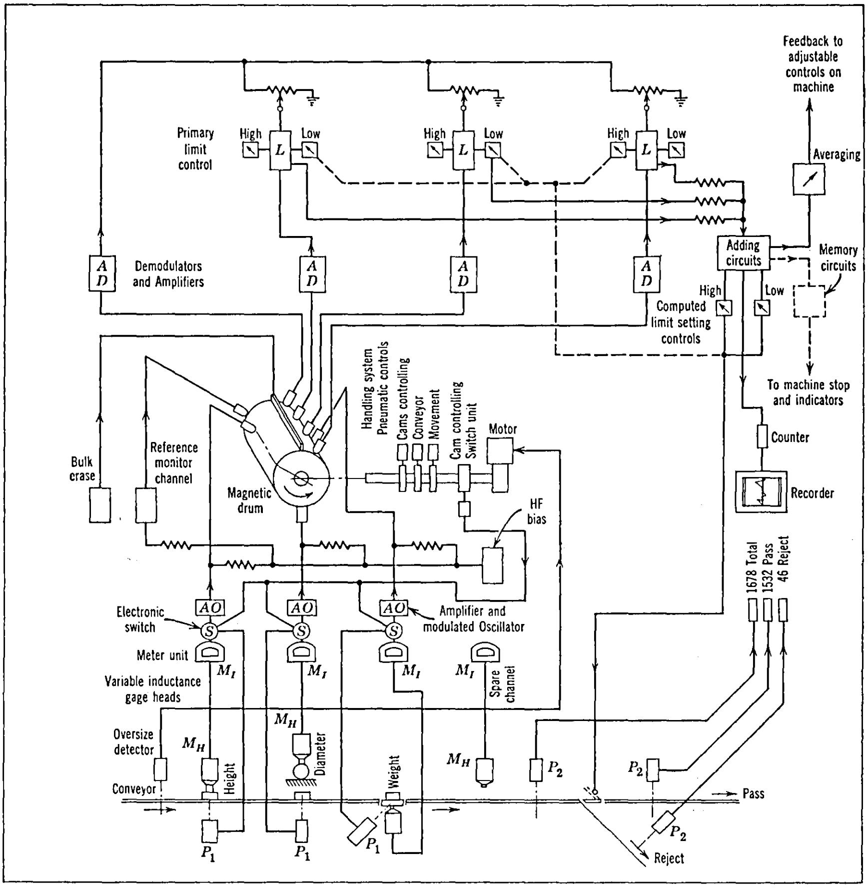

FIG. 5. Schematic arrangement of a simple gaging and computation system for inspection of density of molded objects over a small range of variation.

P_1	Photocell (presence indicator)	S	Switch unit
P_2	Photocell (counters)	AO	Amplifier and oscillator
M_H	Magna-gage head	AD	Amplifier and detector
M_I	Magna-gage indicator	L	Primary limit setting control

ones outlined, except that the acceptance limits for one or more parameters are conditioned by the magnitudes of the other parameters.

6. MANIPULATION OF TIME SCALE

With multistation transfer machines it is usually convenient to perform only one operation at each station: for *example,* it would be difficult to both weigh and gage simultaneously. Some mechanism for automatic recording or register is then necessary.

A simple form is a system of pegs or markers associated with each position on the conveyor system to enable an accept/reject or other instruction to proceed synchronously with the test object to a later station where it will be acted upon.

More elaborate arrangements of this sort can be made with conventional digital storage techniques, up to any speed or accuracy likely to be required in practical cases.

As already discussed, a resolution of one part in 20 is adequate in most inspection applications. It is possible to register the magnitude as a voltage analog and then to read out simultaneously data measured at intervals of time at several successive stations.

The same approach is useful in giving a temporary identity to articles while on a conveyor, so they may be registered and dealt with individually at a later stage in the process, either on an accept/reject basis, or by a correction operation, bringing the object to a standard weight, etc. For such analog memory systems a slow-running magnetic tape or drum is adequate.

7. DISPLAYS AND RECORDING SYSTEMS

When it is necessary to involve a human being in an otherwise automatic inspection and control system, the principle to be followed is that of "management by exception." By this procedure the steady operation is regarded as the normal or quiescent state, and the human supervisor's attention is called only to extreme conditions or such trends that may approach the extreme limits.

A complete "mimic" display of the system is usually provided by the apparatus designer for convenience of the operator in setting the process in motion. This "mimic" is usually diagrammatic with visual indicators or recorders for each key parameter and integrating or counting equipment displaying accept/reject data. In addition summary displays are provided for those administratively interested, usually in the form of moving displays, electric counters, or printout by electric typewriters.

Such displays can be provided at reasonable cost, with standard equipment, provided that a degree of compatibility exists through the whole system.

Presentation of data in this way can usually be shown to recover its equipment cost by a saving in capital on inventory, as all stages of production can thus be closely geared to demand. Process control by feedback from inspection data reduces the cost of rejects and confers on the product the character of greater uniformity which is highly regarded by the user.

8. ELECTRICAL COMPONENT TESTING

Incentives. To ascertain the future life behavior of a particular electrical component, the life testing to destruction of other similar components gives only indirect evidence. Where electric circuits become more and more complex, it has, therefore, been necessary to attempt nondestructive testing of even the simplest components prior to their insertion in more complex systems if the final life reliability of the whole complex system is to be of a very high order.

Apart from the requirements of the scientific testing of electrical devices as single items or as a network, there have been three main incentives to evolving systems of automation in electrical component testing.

1. The continuous supervision of a long component, such as an insulated wire component, prior to its incorporation in a more complex device so as to eliminate an incipient fault before it can cause expensive trouble. An example is a submarine cable.
2. The testing of circuit networks prior to inserting more costly components, such as electronic tubes, transistors, etc.
3. The desirability of large scale testing of components for the purposes of "type approval."

EXAMPLE. An automatic component testing equipment was designed for the automatic testing of batches of up to 1000 resistors (Ref. 9). The resistors can be tested under full load over a wide range of environmental conditions from $-80°$ to $+100°C$ with controlled humidity variable from very dry to full precipitation. The resistors are automatically measured, and the results are recorded on a continuous chart, containing 1000 discrete areas; the testing sequence is controlled by a five-dot code punched paper tape. The system is flexible and up to four months continuous testing can be programmed by this tape.

To implement a decision to carry out a program of mass testing of components the following steps have to be taken:

1. The program sequence of climatic and electrical parameters has to be decided upon and then punched onto the five-hole code tape. (About one hour.)
2. The jig block has to be loaded with the 1000 components, all of which are mounted on separate insulated pillars. (Achieved in approximately one day.)
3. The program tape unit is then loaded, also the various chart recorders, including the continuous 1000-chart recorder (the ease of rewind flow of paper is checked). (About one hour.)

4. The jig block is then mounted within the climatic cabinet. (About 15 minutes.)

5. After general supervision of all the equipment the program tape unit is started. (The equipment will refuse to start unless all the appropriate doors on dangerous power units, etc., etc., are closed and safe.) After this, the equipment will start its own automatic test run program. (May last up to 4 months or be as short as a few weeks.) All measurement readings (50,000–100,000) are sorted by the apparatus into the 1000 discrete areas of the chart, giving the life history of each component separately.

6. At the end of the test program which has been prepunched on the tape (whether it is a few weeks or months from start) the equipment gives an automatic indication that it has accomplished its task. The whole equipment switches itself off.

7. At this point the doors can be opened in complete safety as all the high voltage equipment, etc., is also dead.

8. The resultant charts can now be examined (after their removal from the chart spools) for the best test behavior of the component, and any conclusion can be derived from these test results.

9. Another test can be in progress while doing (8), at leisure.

Improvements in this system can be made by punching the results on paper tape rather than using the 1000-chart recorder system. The output tape could then be fed into an electronic computer programmed to analyze the results.

Conclusions. Automatic component testing is an essential step forward in automation of a highly complex but none the less wearisome and boring task. If one wished to carry out such exhaustive test proceedings on 1000 components in conventional climatic equipment, one would probably have to envisage using twenty or more people. Surplus laboratory personnel just do not exist and are usually engaged on the normal research program. By the aid of automatic equipment two or three people can carry out the task. In fact, the two or three people who would normally be engaged on a small scale testing program can now carry out the statistically valid large scale testing program. Such automatic testing system equipment might well become common in the future for modular single slab functional circuits.

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Materials Handling

A. J. Schenk

1. CONVEYOR SYSTEMS

The meaning of the term "materials handling" has expanded tremendously in the last few years. It includes such diverse equipment as lift trucks, skid platforms, hand-pushed monorail, gravity roller conveyors, and many types of powered product movers. This chapter will discuss only a few types adaptable to automatic control for integration into more or less complex systems and commonly called *conveyors.*

Applications. Conveyors are used for movement and/or storage of parts or products between manufacturing operations, through processing, assembly, testing, and packaging and to warehouse or shipping point. Conveyors can control production rate by simply varying speed of system to compensate for manpower available, etc. They provide the tie between automatic machines to make an *automated system.* Conveyor applications can be broken down into four rough classifications:

1. *Bulk handling.* Materials such as coal, sand, ores, chemicals, and grains. Using troughed belts, buckets, pneumatic, and similar types of conveyors.

2. *Package handling.* Cartons, boxes, tote pans, and similar packages for products. Using gravity roller and wheel conveyors, flat belt, live roller, pusher bar, slat, chutes, and similar types of conveyors.

3. *Trolley and chain conveyors* based largely on rivetless type chains. Includes trolley, floor pusher, power and free, and similar types.

4. *Special designs for handling products between machining operations* including reciprocating transfers, turnovers, rollovers, and lifts operated by air or hydraulic cylinders.

Design of Automatic Controls for Conveyors. In general, there is no such thing as a "standard" conveyor. There are standard components, but every conveyor is tailor-made to perform a certain function in a specific place. System design requires:

1. A thorough knowledge and analysis of devices controlled. This includes their limitations as well as potentialities. Equipment that is suitable for precision machine tools may be totally inadequate for less precise conveyors without expensive added operating means.

2. Close cooperation is required between mechanical parts designer and control designer to assure that control elements can be incorporated in correct positions in basic design. The control requirements frequently dictate elements in the mechanical design. Too frequently, the control design is left until too late, the designed mechanism is inflexible, and expensive rework is required to make the machine operate.

3. Design of controls is a logical step from a complete description of mechanical sequence of functions required. Simple sequence functions using limit switches, wipers, pushbuttons, relays, solenoid valves, and motor starters are used to control most conveyors.

4. When more units are coordinated together, centralized panels with provisions for starting, stopping, speed changing, and condition signaling for each unit may be required. The special requirements for multiple drivers, synchronization for automatic transfer between several conveyors, and selective dispatching require more challenging consideration.

2. PROBLEMS OF CONVEYOR CONTROLS

General. Application of electrical controls for operations of conveyors poses problems which are not present in the control of precision machines. Even large transfer machines, while consisting of a large number of parts, will occupy relatively small, compact areas. Conveyors often operate over very large areas. Individual conveyors are seldom less than 100 ft and are frequently 2000 to 10,000 ft or more long. Conveyor systems may extend into several buildings and on more than one floor. Large portions of systems may be overhead in otherwise unused space and are relatively inaccessible.

Conveyors are seldom assembled and tested except in their permanent location. Once a conveyor is installed, the plant must get into production quickly so there is a minimum opportunity to make changes or adjustments. This requires careful engineering to ensure immediate operation. Frequently adjustments must be made at expensive overtime costs.

Mechanical precision is not a general characteristic of conveyors. Basic designs have been fixed by usage and proved adequate before the advent of special controls now added. Many conveyors are designed around drop-forged rivetless chains. These chains have least weight and cost for their strength. Most are heat treated while the larger sizes may be made of alloy steel for greater strength.

Few chains are made to precise pitch. Dimensions of drop forged chain change slightly as forgings and trim dies wear. Normal runout is about 1¾ in. per 10 ft of #458 chain, the most commonly used size. Rough spots wear down quite rapidly during first weeks of operation. Then wear and elongation changes remain about constant at a slower rate. Chains can elongate as much as 5% before they need replacing.

Attachments for loads can normally be spaced only at multiples of twice the chain pitch. Special provisions may be made for multiples of pitch spacing at higher cost. If load spacing is important, it may be necessary to select a different type of chain.

All parts of these conveyors have loose fits and are normally not guided closely. Load carriers are seldom exactly alike and can hang at various angles and be out of line horizontally or vertically or both. This creates problems when attempting to operate limit switches and signal devices from conveyor parts.

Overload protection for conveyor drives must be provided when they are driven by electric motors. Those driven by pneumatic or hydraulic power can usually stall safely without damage.

The most effective method utilizes a *floating drive*. The drive machinery is mounted on a platform that rests on wheels in a fixed frame. Chain pull is counteracted by springs so that floating frame position is a measure of force exerted. If chain pull exceeds designed value, a limit switch is operated to stop the conveyor. Drive frame can still travel farther to absorb energy of drive parts without damage. This method is independent of speed.

Fixed drives can use an adjustable slip clutch with underspeed switch to indicate stall, overload cutout with parts to separate and operate a limit switch, or overcurrent relays. Signals from motor current are unreliable if a mechanical variable-speed device is used between motor and speed reducer.

Slack chain control must be provided since any chain will elongate from wear. If a rivetless chain is permitted to run too loose, pins may fall out or center links slip and lock crossways and cause jams on turns and drives.

A *takeup* is usually a 180° turn mounted movably so that effective track length can be changed. Movement is controlled by screws, screws with springs for manual adjustment periodically, or automatically by spring, counterweight, or adjustable pressure air cylinders.

Travel of takeups is limited by the necessity of carrying chain and loads across a slip joint between the fixed and movable tracks, and providing sufficient strength in a limited space. Minimum travel must be sufficient to take out at least two pitches of chain and still permit chain coupling.

When equal load spacing must be maintained, provisions are made to move the whole takeup frame bodily each time the limit of travel is reached. It is usually necessary to cut the track and insert new sections to fill the gap. Eventually one complete space will be removed and the process started over again. A conveyor must be out of production while changes are made.

Location of takeups is important, particularly for multiple drive conveyors. They are usually located at points of lowest tension or elevation. For example, a point past a drive exit between a dip and a rise would be ideal.

Automatic takeups must have sufficient power to keep a chain tight under any variable conditions of loading. This means that there may be several hundred to more than 1000 lb initial tension in a chain. Since each horizontal turn or vertical bend adds 2 to 10% to chain tension on entering side, excessively high chain loads can develop.

Conveyors passing through ovens should have takeups located nearby so that when the heat is turned off and the chain contracts on cooling, it can be released to the oven with little force. Oven turns have been pulled down or damaged by lack of attention to this point.

Conditions of loading of a conveyor can affect selection of control elements. Most assembly conveyors are uniformly loaded. Those passing through units for chemical treatment, painting, etc., are frequently cleared each day and reloaded the next day. Storage types at times are heavily loaded in sections only.

The problem is greatest when a conveyor passes through ovens and/or has many vertical bends and high lift loads. Chain pull lift load at the top of a vertical bend due to loads on the incline is equal to the live load per foot of conveyor times vertical height of lift. The difference between the lift load from empty carriers and that from loaded carriers is frequently much more than friction load for the entire conveyor.

Under some conditions there may be runaway forces tending to overspeed drives. If a conveyor must be stopped, drive brakes are required.

Improper lubrication can double or triple normal drive pull requirements. This is important when variable-speed, constant-torque motors are specified.

Most conveyors can be readily rearranged, shortened, lengthened, or combined. Model and method changes usually require conveyor rearrangements. Loads may increase in size, weight, and spacing. Controls and components should be selected for best adaptation to change as well as for standardization.

Adjustable speed requirements affect control means. Single drive conveyors usually use a variable-speed pulley or a variable-speed transmission either adjusted manually by handwheel on the drive or remotely by speed-changing motors.

Conveyors with multiple drives or those which must run at precise speeds or in synchronization with other units require more elaborate controls.

3. MULTIPLE DRIVE CONVEYOR REQUIREMENTS

Long or heavily loaded conveyors require more than one drive to keep chain tensions within allowable limits. For long life and reliability it has been found to be best to keep working tensions below 4 to 6% of ultimate strength. EXAMPLE. For a #458 chain with ultimate strength of 48,000 lb the maximum load is 2500 lb. Higher loading may be used with slow speed, few turns, and little change in elevation.

Multiple drives permit use of lighter, standard construction and practically unlimited length. Surge, which would cause trouble through paint spraying, for example, is reduced by low chain pull and by strategic location of drives.

The common problem in all multiple drive applications has been control of slack chain without building up excess tensions and overloads. Conveyors passing through ovens must have means to take up elongation of chain from heating and to relax when chain shrinks on cooling. Also chains continually wear. For *example,* a typical 7800-ft conveyor will lengthen by 1 in. every 8 hours.

The classic method requires that drives have high-slip characteristics, that is, will slow down under increased load. Any difference in speed will develop an increased load on the faster running drive and decreased load on the slower. This causes the faster drive to slow down and/or the slower drive to speed up.

Constant Speed. If chain can be kept taut at all times, even standard a-c motors will divide loads. Constant-speed conveyors can utilize high-slip (8 to 13%) a-c motors. Tests show the load on each motor will be equal even though chain tensions at the drive may be widely variable or different. High-slip motors work best with moderate drive pulls.

Variable speed, particularly with remote control of speed changes, becomes a more complicated problem. Drives must divide loads but not necessarily equally. Drives can seldom be located at ideal points with equal loading because of clearances, ovens, process equipment, and other interferences.

Control Methods. Change of drive speeds in response to chain pull variations may be made (1) by developing excess forces at drive or (2) by using a feedback signal from a control takeup in conveyor path. The first method has been used most frequently but requires manual adjustment of elements for correct results. The second can automatically compensate for variations in load, speeds, and components automatically. The method selected depends on types of variable speed devices used on the drives.

Driving means used have been as follows:

1. Constant-speed, normal torque, a-c motors with variable speed transmissions.
2. Variable-frequency, high-slip a-c motors.
3. Direct-current motors with high-slip characteristics, either by compound winding or armature dropping resistors and shunt field control.
4. Eddy current clutch motors with electronic control with torque limiting and adjustable slip characteristic features.

Balancing by Force

Constant speed a-c motors with variable-speed transmissions are used with floating drive frames. Where speeds are changed only infrequently, a mechanical rigging between the transmission adjusting screw and fixed frame causes speed to vary with load owing to speed differences. Hand wheels can disconnect the balance rigging so that a base speed can be set on each drive simultaneously. Drives are adjusted for best operation. Mechanical balancing makes necessary speed adjustments.

Floating drive frame position can also be used to operate a potentiometer slider to provide a signal for servo motor electronic control of variable-speed transmission to change speed in response to varying chain loads and also by remote control. One drive is made a master whose speed is changed by manual switch. Tachometer generators on variable-speed shafts of drives provide velocity signals. Follower drives match speed and preset drive load relationships to master drive.

High-slip (8 to 13%) a-c squirrel cage motors supplied by a variable frequency 3-phase alternator may be used. The alternator is driven by mechanical speed changer from a constant-speed motor. Voltage varies approximately with frequency. Standard 220-volt, 60-cycle motors can be operated over a range of 20 to 100 cps. All motors are connected in parallel with alternator through thermal overload relays. Conveyor starts and stops with alternator.

Drives must be located for nearly equal loading. Care is required in matching motor size to load. Best results are obtained when motors are nearly fully loaded.

Direct-current motor drives require a motor generator set and operator's panel for each conveyor to supply variable armature voltage. Motors are compound wound for 10 to 20% slip or are shunt wound with armature voltage dropping resistors to vary speed with load.

Excitation for generator and motor field is supplied by belt-driven generator, electronic tube, or dry type rectifier. The generator provides for safe stopping on power failure when dynamic braking is used.

Motor armatures are connected in parallel with each other and in series with generator armature and d-c contactor contact. A thermal overload relay and ammeter is provided for each motor.

Motor shunt fields are connected in parallel to the exciter. Each is provided with a series vernier rheostat for adjusting drive balance. Increasing field resistance causes motor to tend to run faster, forcing it to take more load to hold its speed down to that of the other drives. Ammeters show drive loads and show operator when correct adjustment is made. Experience will determine if conveyor runs best with equal or unequal drive loads.

Conveyor speed is controlled by a rheostat in series with the generator field. This controls armature voltage, and speed is approximately proportional. When more exact speeds are required, generator field may be controlled electronically.

Motor and generator sizes must be carefully considered. Motors operate in the constant-torque range below base speed theoretically, but when shunt field rheostats are used, base speed increases and torque decreases. This can be compensated for by increasing ratio of V-belt drive between motor and speed reducer. In calculating drive speed ratio use motor base speed plus 10 to 15% when conveyor travels at maximum design speed. This will assure ample torque and permit slow down to minimum speeds.

Note that motor manufacturers may indicate an 8 to 1 speed range but that continuous operation at minimum speed and full load is not recommended. At extremely slow speeds, regulation is very poor and heating becomes a problem. Best results are obtained when normal probable speed is near base speed.

Since cost of motors and generators rises rapidly with size, there is sometimes pressure to keep them as small as possible. This leads to overloading and poor operating conditions develop. Many companies now specify that motor generator and control be large enough to supply at least one additional drive. Otherwise if conveyor requires an extension, new equipment would be required.

Eddy current clutch motors with electronic excitation can be used. Electronic control matches output speed as indicated by builtin tachometer generator with reference voltage from manually set potentiometer. Torque-limiting and sensitivity circuits permit matching drive loads as indicated by motor current ammeters. Each control is kept electrically separate. Reference potentiometers for the drives are ganged. Flanking potentiometers provide individual adjustment.

One control is adjusted for close speed regulation. Other drives are set with lower sensitivity. Two sets of control wiring are required. One is used for motor starters with interlocks with electronic control panels. The other, interlocked with control panel time delay relays, controls the on-off energization of clutches.

Balancing by Feedback

Control takeups following all but one of several drives on a conveyor provide means to operate a feedback signal to synchronize the drives. Any difference in speed between two drives will cause the takeup between to move. This movement, coupled to a potentiometer or rheostat slider, provides signal to correct the speed of the drive feeding chain into the takeup to stop movement.

A rheostat in series with the motor shunt field can be used with d-c motors. A potentiometer, in parallel with an adjustable tap resistor in control panel, provides a signal between slider and tap for electronic servomotor speed control with mechanical speed changer. Movement of control takeup need not be more than 3 or 4 in.

Limit switches operated by extreme travel of takeup are recommended in drive control circuits and are provided with signals on a central panel. This ensures stopping the conveyor and trouble point indication before damage if a conveyor jams or a control component fails.

A long travel takeup should be used after the one uncontrolled drive. This is called a slack chain takeup and accepts all the elongation of chain due to wear or heat expansion. It must have sufficient travel to compensate for movement of control takeups to synchronize drives, temperature change length, and wear elongation for a reasonable time.

Location of control takeups are important to ensure that movement will be due to speed difference only. Takeups may be operated by counterweights or air cylinders with adjustable pressure regulator. Takeup tension need only be sufficient to keep chain stripped from drive. As long as control takeup is neither fully open or closed, tension is just right. The control takeups may be built into either 90° or 180° turns.

This method is fully automatic and controls drive speeds regardless of variations in speed or load. Minimum chain pull is developed since each drive pulls its section of conveyor only. Drive loads can be unequal and different sized drives may be synchronized. Since no manual adjustment of individual drives is required, production supervisors can be permitted to make speed changes.

Follower drives must be capable of running both faster and slower than the master at extremes of production speed. Rheostats for d-c drives have value to raise base speed of motors by 20%. The master drive has manual field rheostat to raise its base speed by 10%. Control rheostats, by field control, can make follower drives run up to 10% faster or more slowly.

The layout of some types of conveyors do not permit use of control takeups. These conveyors then must use one of the force methods with manual adjustment.

4. BASIC ELECTRICAL CONTROLS

Controls for conveyors and automation should be designed with emphasis on the requirements of the men who must keep the systems operating. The best approach is to have a system that will automatically stop safely in case of failure, and provide signals to pinpoint the area of the trouble. Usually more time is required to locate the cause of the failure than to correct it.

Modern factory equipment is becoming more complex and diversified. Conveyors add to the complexity. It is difficult to recruit and train enough qualified maintenance people. If stoppages are frequent, maintenance men learn exactly what to do. The more reliable a piece of equipment is, the more important it is to spot troubles quickly. If maintenance personnel changes frequently, or if the regular maintenance man is away, the foreman or some other mechanic must take over. Then it is an advantage to have controls that are easy to understand, to maintain, and to service.

The cost of down time in modern plants is prohibitive. The savings of even a few minutes in restoring production operations will pay for a great deal of wiring and extra signals. It is sometimes difficult to sell this idea to purchasing departments or those with little experience in operating production machinery. Conveying systems cannot be installed in duplicate as is possible with machine tools. A stoppage at one point stops the whole system and all related operations.

For example, an automobile final assembly line may be set for 60 jobs an hour. One minute lost means one car that may not be built that day. Union rules generally do not permit speedup to make up for lost time or production.

Joint Industry Conference (J.I.C.) Standards

These standards provide an accepted guide for Materials Handling Control. The prime purposes of the standards are to:

1. Reduce maintenance and increase reliability.
2. Increase safety for operating and maintenance workers.
3. Reduce down time by making control functions easier to trace, understand, and repair.

Most companies try to standardize on only a few makes of motors and control components. This reduces their spare parts inventory. It is important to determine, in writing, just what will be acceptable in a particular plant.

Enclosures must be semi-dust-tight with gasketed openings. There can be no knockouts to open accidently or openings through which liquids or dust can enter. Since most control panels are custom designed, this requirement adds little to the cost. Disconnect switches must open both power and control circuits.

Enclosure size and layout of components should permit installation later on of at least 15 or 20% more relays and terminals. This gives the engineer an opportunity to take care of inevitable errors or the requirement of future changes in the field. Extra contacts on relays are good insurance. Relays with contacts interchangeable for either normally open or normally closed reduce costs.

Control voltage is usually specified as 110 volts a-c. There are many more components readily available and stocked for 110 volts a-c than any other voltage. This voltage is high enough to ensure good contact and relay operation even with long control leads. Standard incandescent lamps can be used for local signals at emergency stop pushbuttons.

Panel Wiring. All external wires must come to numbered terminals in control panels. All internal wires must be marked with numbered labels on each end. All wires must be stranded and use acceptable crimped-on terminals. Color coding of wires for different functions is required. Fiber wiring channels with removable covers are preferable to laced cables for ease of installation, change, and tracing. Relays, pushbuttons, and other similar components in a control panel should be marked with labels corresponding to diagrams.

Relays should preferably be identical. Four pole relays will take care of nearly all normal conditions. If more poles are required, it is better to parallel added relays. Generally, industrial type relays have given little trouble, and provisions for plug-in types are not necessary. The principal difficulty with relays has been dirt in normally closed contacts.

Signals. *Limit switches* operated by machine parts or products are generally used for signals to control functions.

The control engineer must work with the mechanical designer to be sure that limit switches can be located at the proper points. Frequently some mechanical part is already in the way when the field electrician starts to adjust limit switches. Then a compromise must be worked out either to change the original part or control diagram or both.

It is recommended that limit switches be mounted on clamp-on brackets or in other ways to allow for plenty of adjustment. Switches should be designed so that accidental reversal cannot cause damage to the operating arms or plungers. Timing of operations is usually based on the position of the limit switches.

Standard machine-tool type limit switches either require too much movement to operate the contacts or have too little overtravel, or the operating arms are too short to work directly off conveyor parts. This requires additional operating arms and cams in special mountings. Some limit switches are designed especially for conveyor work. These have (*a*) heavy operating shafts turning in ball bearings, (*b*) extended arms up to 15 in. long, (*c*) shorter travel to operate, and (*d*) up to 90° total travel. This permits the arms to be made cam shaped so that switch operation will be the same regardless of the direction of travel of parts past the arm.

It is desirable to restrict the number of types of limit switches used on a system. Also, it is preferable not to use more than one normally open and one normally closed contact on a switch. If more contacts are required, use relays operated by limit switches. Each switch can be wired exactly the same with a 4-conductor cord and lock type cap so that prewired switches can be stocked. A permanently wired receptacle located near the switch position is wired for the correct connection used.

Limit switches are more likely to fail mechanically than electrically. In case of failure, it is only necessary to remove and replace the faulty switch, plug it in, and adjust the operating arm to get the machine back into operation.

"Wiper" circuits have been developed to reduce problems of limit switches. Standard 110-volt relays are used in series with the primary of a specially designed transformer. The secondary of the transformer can be wound for 5 to 24 volts. With proper values in the transformer, its impedance, with the secondary open, will prevent the relay from picking up. When the secondary is shorted, its impedance is lowered, and the relay is energized. See Fig. 1.

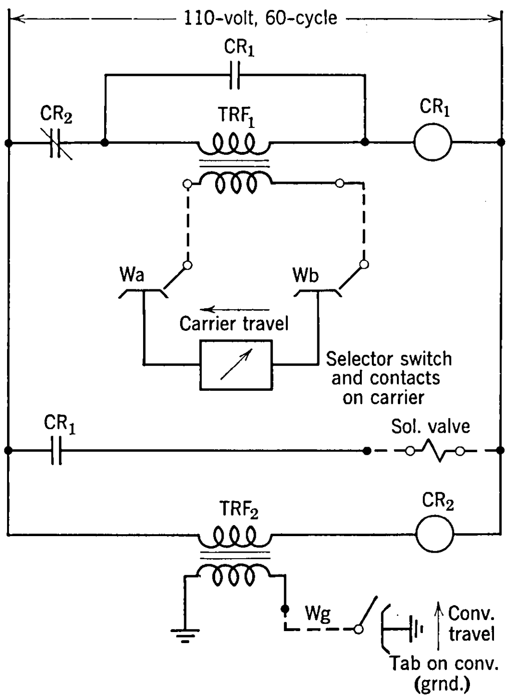
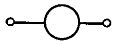
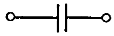
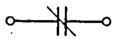
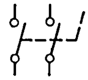
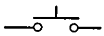
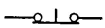
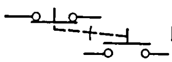
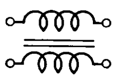
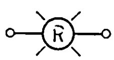
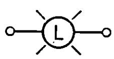


FIG. 1. Typical "wiper" circuit selector for conveyor carriers. Short circuiting secondary of transformer causes relays to energize. See Table 1 for symbols.

The secondary is connected to insulated wipers which are contacted by the conveyor parts. Two or more wipers can be connected by selectively controlled patterns of contacts for different combinations of signals. Or one side of the secondary can be grounded and one wiper can contact any metal part of the machine.

Wipers are plated steel springs supported at an angle of about 45° to the direction of travel by pivoted arms which lift clear if parts reverse. No movement is required for operation. This permits a variation of

TABLE 1. SYMBOLS USED IN FIGS. 1 AND 2

	Relay, starter operating coils
	Normally open relay contact
	Normally closed relay or thermal overload contact
	Solenoid coil
	Disconnect switch
	Normally open limit switch
	Normally closed limit switch
	Normally open, momentary contact pushbutton
	Normally closed, momentary contact pushbutton
	Maintained contact pushbutton
	Fuse
	Control or voltage transformer
	Signal light, letter indicates color: amber, green, red, etc.
	Local light, adjacent to pushbutton or other equipment
————	Solid line to show panel wiring
– – – –	Broken line, connections external to panel
CR_1, CR_2, M_a, etc.	Contacts and coils of same name operate together

plus or minus ½ in. of contact alignment for positive contact. Voltage at the wipers is less than 24 volts, and it is safe to use exposed contacts. Wipers can be connected with simple open wiring. This reduces cost of installation.

This type of circuit is particularly adapted to automatic loading and unloading of overhead trolley conveyors and power-and-free conveyors. Wipers have long life and are cheap and easy to replace. The greater latitude for variations in operating positions reduces problems of field adjustment. Combinations for shorting the transformer are infinite.

Relay contacts and manual selector switches can be added in the secondary circuits.

Other means of providing signals include photoelectric relays, with or without modulated light, proximity limit switches using radio-frequency circuits and operated by absorption of energy by a passing metallic part, and magnetic pickup devices. These are usually more expensive and subject to failure of tubes and lamps. The elimination of physical contact warrants their use with proper safeguards in same applications.

Safety circuits should be designed to prevent damage to personnel or equipment. Hazards should be analyzed for the worst conditions. Emphasis should be on keeping machine running, and not stopping until the last possible moment. It may not be necessary to stop so often. This requires the machine to stop quickly on signal and indicates drive motor brakes. If a machine stops before parts jam, it is usually easy to clear. If parts are damaged, repair will require a costly down time.

5. CONVEYOR CONTROL CIRCUITS

Separate circuits should be used for distinguishing between overload or jam conditions, and production stops. The first type usually requires the assistance of a maintenance electrician or a mechanic. The trouble must be located and cleared, and then a reset pushbutton is operated to restore service. See Fig. 2.

Production stops are made with maintain-contact type pushbuttons so that the system will start immediately after the button is returned to the run position. Local indicating lights, with about 25-watt off-white lamps, can be located above and adjacent to pushbutton stations. They should be located so that they may be seen from a distance.

Circuit Checking. It is desirable to be able to monitor circuits from a central point. One method is to use the back contacts on emergency run-stop pushbuttons and limit switches to operate signal lights. This does not guarantee, however, that the actual operating contacts are functioning. Also, a large number of signal lights with nameplates is required.

At least one wire from a junction of each pair of contacts in a circuit can be brought back to the central panel. A tap switch with the center terminal connected to a small signal lamp can be used to sample each contact. A name card identifies each point. To check the circuit, the tap switch is rotated until the light shows. Then the next contact in the series is the one which is open. Operating circuits are checked quickly.

Jams on conveyor systems must usually be cleared by reversing the drives. Jams may be caused by parts falling off, damaged carriers, or malfunctioning of a component. Drives can be reversed by manually operated reversing switches with extra contacts to block out holding circuits. A motor should run in reverse only as long as a jog pushbutton is held in. Reversible motor starters may also be used, with proper precautions.

Decentralized Versus Centralized Systems

Location of controls will have a bearing on their design. As a general rule, control panels should be located so that a man working on the controls can see the operation controlled.

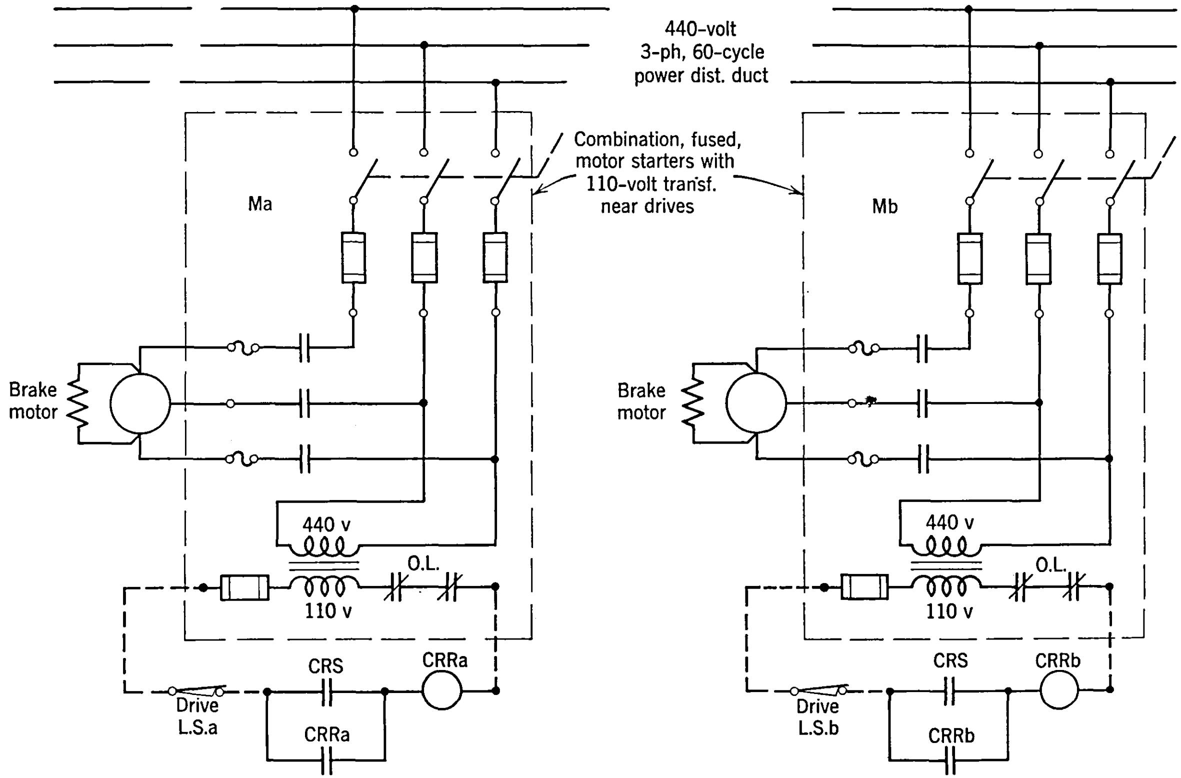
Wiring installation costs are usually reduced with a number of small control panels at correct locations. Each unit has its own fused disconnect switch for isolation. Control components on a conveyor can be marked to correspond with the diagram in the panel. Frequently, mechanical breakdowns can be located only by means of the electric circuits. Visual check of conditions at the point of stoppage will frequently show the difficulty so it can be corrected.

With a *decentralized system,* the power supply for various motors may be taken from the nearest distribution duct. All control circuits should be taken from the same 110-volt transformer source and distributed to the various unit panels. This permits fewer wires to transmit signals. Branch circuits should be adequately protected with fuses or circuit breakers to permit isolation of faults.

A central panel is frequently provided with means for starting and stopping various conveyors in the system and showing area operating condition. A map of the system may be included. It is not necessary to show all the individual points of stoppage but only which local panel has trouble. This permits the maintenance electrician to go directly to the area involved.

A centralized system would have all controls grouped in one area remote from conveyors controlled. Some customers seem to prefer this method. This is particularly true when using d-c equipment with motor generator sets. This involves longer wiring for large systems and problems of communication with the areas in difficulty.

A telephone system can be readily installed with a jack in each panel. The maintenance electricians carry portable telephones with a jack to plug in at any point. The telephone wires can be run in the same conduits with the control wires. This permits electricians at various locations to communicate or to call for help from the maintenance foreman. A signaling device preferably should be included with the telephone circuits.



(a)

FIG. 2. Typical circuit for two-drive conveyor with automatic transfer controls, showing means to separate normal operating stops from trouble stops and interlock of power for drive motors with control. See Table 1 for symbols.

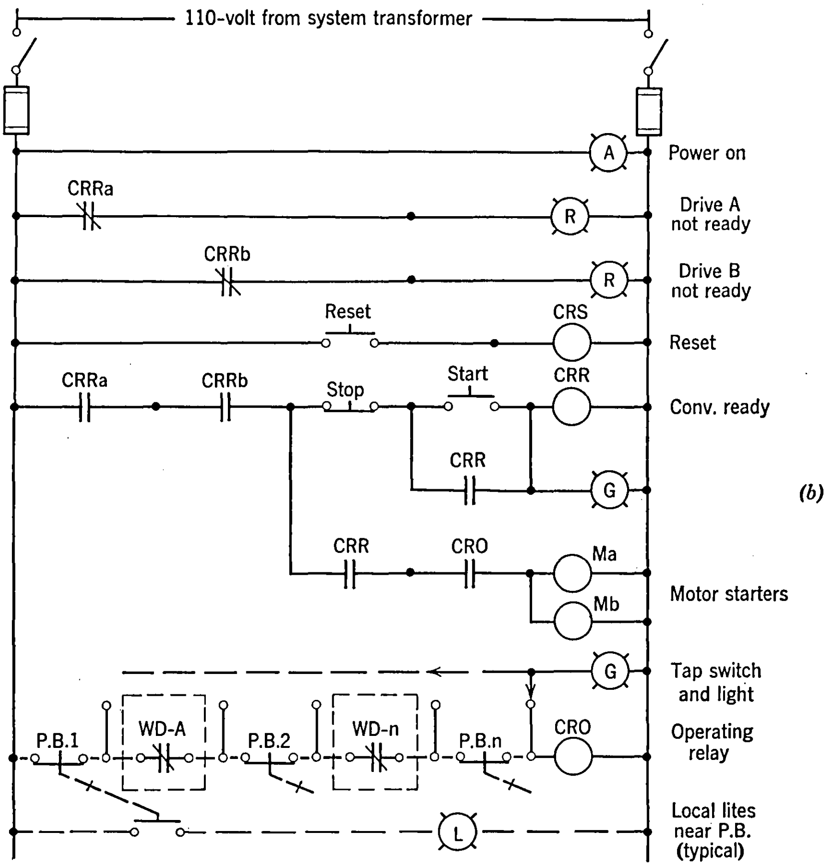


FIG. 2—(*Continued*)

6. SYNCHRONIZED CONVEYOR SYSTEMS

A number of individual conveyors may be synchronized to permit manual or automatic transfer of product or product supporting carriers between them. Conveyors may be of different types operating at the same rate but at different speeds and load centers. A number of synchronized, separate conveyors are used, rather than one long conveyor, for one or more of the following reasons.

1. Separate periodic operation of one or more conveyor units. Chemical surface preparation processes require that parts must not be left in them when system stops more than a short period. When some types of paint are used, sprayed parts must pass into or through an oven before stopping for the night.

This requires means for separate operation of the conveyor units before or after main system stops and storage for the parts removed.

2. The conveyor section subjected to chemical actions, paint, etc., should be as short as possible.

3. Different load spacing may be required. A paint spray operation requires that loads be separated but the following oven conveyors require closer spacing for economy.

4. Different types of conveyors. Parts might be suspended from an overhead conveyor and deposited on castered trucks drawn by a floor conveyor or vice versa. Conveyor position is synchronized longitudinally, and transfer is effected by changes in elevation of one or both conveyors.

5. Feeder conveyors bringing special parts to an assembly line may be synchronized with main conveyor to assure matching colors, for example, at a correct time. Transfer of parts would normally be manual.

Requirements for Synchronized Conveyor System

1. *Equal load spacing* must be maintained on each conveyor. Special provisions must be made so that no slack chain may be removed unless a complete space is taken out.

2. Drives must be provided with means for *remote control speed changing* and with brakes for quick, uniform stopping.

3. Controls must provide for *interlocking* all units to run and change speed together. They must set the speed of individual conveyors to maintain position relationships, permit manual emergency operating and starting, and provide signals to show malfunction or local stoppage. Components of the control will vary with the types of conveyor drives used.

4. When *transferring* between two conveyors end to end, provisions must be made for positively pushing the carrier across the gap between them. The delivering conveyor pushes the carrier beyond the point where receiving conveyor can pick up the load, and then disengages. The carriers stand momentarily until picked up by the receiving conveyor. Clearances between the carriers determine the allowable tolerance for conveyor synchronization to prevent either jamming of the carriers or missing the take-away pusher.

Use of Synchros

Position signals are obtained by synchros driven by individual conveyors. They should preferably be located adjacent to transfer or junction points to minimize effects of chain length change.

The *master unit* can be one of the conveyors or a separately driven, interlocked unit. A synchro transmitter is used on the master with synchro control transformers on the *followers*. Some conveyors may be controlled as followers and act as submasters for other units.

Synchros may be driven by small caterpillar chain units engaging main conveyor chain. This permits location at the most advantageous points. Gearing is provided to make the synchro turn exactly one or more revolutions per conveyor space. See Fig. 3. The number of revolutions of synchros per space is determined by length of space and relation-

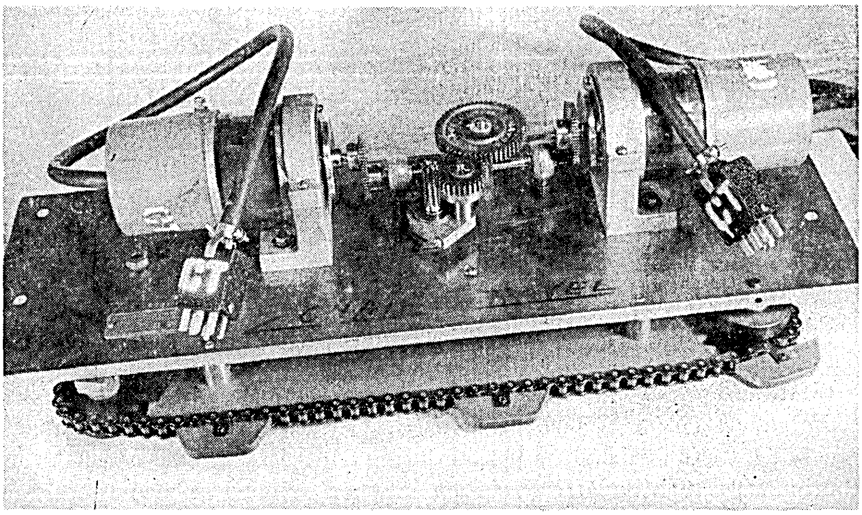


FIG. 3. Caterpillar drive for synchros with change gears to permit respacing of conveyor loads. A similar unit may be used to drive multiple cam switches and may also be used for synchronized conveyors. (Courtesy Jervis B. Webb Company.)

ship to the desired accuracy and practical sensitivity of the control means. In some systems, particularly with very slow speed (3 to 24 inches per minute), better results may be obtained by higher synchro speeds or more revolutions per space. Much depends on the dynamics of the system used.

Most synchronized systems have been used with loads in all spaces. This means that control cannot be permitted to find its own locked-in synchronizing point by moving ahead or back one space. Some means is required to indicate out of synchronization and also exact synchronizing.

A *sensitive relay* with adjusting rheostat and rectifier placed across the output of the control transformer is energized when the error is more than a preset amount. It has little effect on the electronic circuit at normal operating error. However, there is zero voltage at both 0° and 180°. If used with several revolutions of synchro per space, the number of like indications increases.

A second set of synchros driven at one-half revolution per space and connected to a similar signal relay provides an exact signal. Contacts on two relays in series provide a signal only when both are at zero voltage. Relays can be provided with extra backlash requiring higher voltage to pick up and will drop out closer to true zero position. We term these "junction relays." They provide contact for the interlock circuit to stop the system if for any reason it goes out of synchronization, and also to signal for manual jogging of units back into step. Also they can be used to stop conveyors automatically in synchronization after separate operation of one or more units.

Correct transfer between conveyors can be assured only when units are synchronized between certain limits. This means that the system should run automatically only when synchronized. Out-of-step condition can be caused by mechanical jams, failure, misadjustment of control components, etc. This condition should stop the system and indicate the point of trouble so that the electrician can find it quickly. A signal should indicate whether the offending unit is ahead or behind. A manual control with a pushbutton can be used to move the conveyor into position and restart the system. The electrician can then watch the operation to determine the cause of the trouble without interfering with production.

Continual starting and stopping of new systems has been a considerable source of difficulty. When a new system is installed, or model changes are made, there is usually a period during which the system will not run continuously. Frequently a system may be stopped several times per space. It is difficult to make all conveyors stop in the same length of time. If, after starting, a conveyor is permitted to run long enough for the controls to correct the position, there is no trouble. Continual jogging will eventually cause enough error to trip the junction relays and require manual repositioning of units.

It is possible to add a discriminator circuit and necessary relays to permit first the starting of units that are behind, and the starting of the others at the correct time. This adds to cost.

7. CONTROL SYSTEMS FOR SYNCHRONIZATION

The two systems most used have been (1) a modified Ward-Leonard d-c system with motor generators and (2) drives with electronically controlled mechanical variable-speed transmissions and a-c motors.

Direct-Current Motor Systems

Control Methods. A separate motor generator set is usually used with each conveyor to be synchronized. Speed is controlled by varying the generator field completely or in part electronically. A master timing control with feedback from a synchro on the conveyor sets the pace. Three common methods use (1) an electronic rectifier to supply all the generator field, (2) double windings with a manual rheostat in one (the other winding is controlled by a buck or boost amplifier), and (3) an electronic circuit to operate a motor-driven rheostat in series with a manually set rheostat. The last two methods permit limiting the amount of speed change that can be made to bring the conveyors into synchronization. Conveyor speed should not change more than about 10% above or below the preset speed.

When the individual conveyors in a group can run together always, it is practical to use one motor generator set for them all. Shunt-wound motors are used, and individual speeds are varied by shunt field control. A rheostat is used in the master conveyor motor field to raise its base speed so that the drive motors of the follower conveyors can run slower than the master. The shunt fields of the follower drive motors use electronically controlled separate field power supply. The master conveyor has its speed controlled by reference to the master timer. The junctions between the master and follower conveyors are equipped with synchros to give position signals for control of the follower drives. Changing the generator field voltage raises or lowers the speed of the system as a whole.

Manual operation in emergencies should be provided. The standard excitation of the motor generator set can be connected through selector switches and manual rheostats to replace the electronic controls. This permits the maintenance electricians to switch quickly to manual control to keep the conveyor operating while the trouble is located and repaired. It is common practice to provide a duplicate motor generator set with throwover switches for quick change in emergencies.

Production Speed Control

A master timer with adjustable speed can be used to set the conveyor speed. This is a small, synchronous speed motor with change gears or accurate variable-speed transmission to drive a synchro transmitter at one revolution per job space. A synchro control transformer driven by the conveyor is geared to make one revolution per job space. Tachometer generators, driven by the master timer and a conveyor drive, provide for velocity feedback. Synchros provide accumulated error signals for an electronic control for a motor-generator field. Changing the speed of the master timer sets the production rate. See Fig. 4.

Controls for the master timer and the conveyor are interlocked so that they run and stop together. Provisions should be made for manual

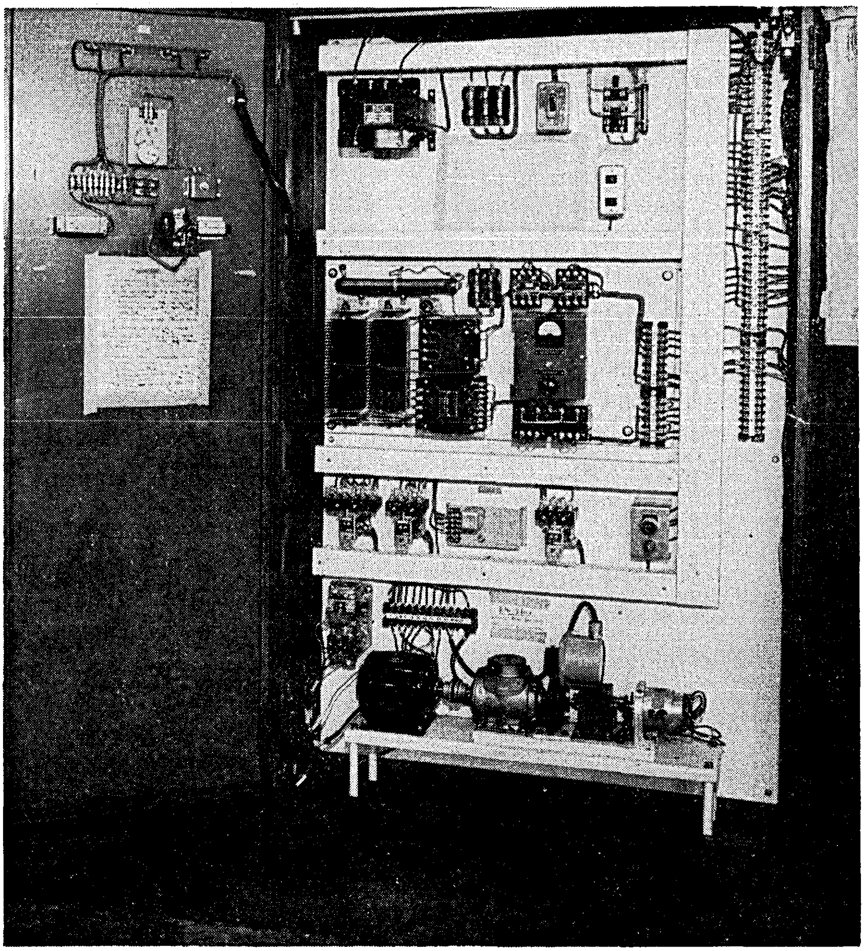


FIG. 4. Typical control for d-c motor generator set with adjustable speed master timer in lower section. Magnetic amplifier controls generator field to synchronize conveyor to master. (Courtesy Jervis B. Webb Company.)

adjustment of conveyor speeds in case of failure of components of the master timer.

The systems are usually used with high unit value production such as automobile assembly. Requirements usually are that the production rate can be set to a variation of one-tenth to one-half job per hour in a range from 20 to 80 jobs per hour. When change gears are used, a very large number are required. The Graham transmission provides sufficient power for driving, and is infinitely variable. Once adjusted, it maintains precise speed range ratio for a long time.

Alternating-Current Motor Systems

Systems using mechanical variable-speed transmissions are well suited to decentralized controls. Near each drive is a standard, combination, reversible motor starter. Servomotors drive the adjusting screw of the transmissions to vary the speed. The master conveyor speed is set by manual operation of a reversing switch on central control.

The *follower drives* are controlled through amplifiers with feedback from tachometers on the drives and synchros on the conveyor. A master timer is usually not required. Servomotors may also be operated by fast-slow switches.

Amplifiers for controling follower servomotors are located adjacent to the motor starters. This permits observation of the speed control from the panel. A selector switch marked "Auto-Off-Hand," with forward and reverse jog buttons, is mounted in the control panel.

Zero center meters on the control panel can be used to indicate whether a follower conveyor is running faster or more slowly, and ahead or behind, the master.

Limits of synchronization are set by the use of a sensitive relay called a "junction" relay across the output of the control transformer. During normal operations, the output of the control transformer is near zero and the relay does not pick up. If conveyors get too far out of step, the junction relay is energized to stop the conveyor system and show a signal light. An electrician inspects the conveyor or control signals meter, turns the selector switch to "Hand," and jogs the conveyor in the correct direction until the signal goes out and the conveyor is synchronized. The selector switch can then be returned to "Automatic," and the system again operates as a unit.

8. SELECTIVE DISPATCHING SYSTEMS

A selective dispatching system provides means for automatically transporting products or load carriers from one or more originating points and discharging to a number of destinations, along one or more main line conveyors.

Selection means may be external to conveyors but synchronized with them, or may be carried by product carriers. Signals from selection means may initiate operation of deflectors, switches, or unloading devices directly or only may warn operators that approaching units are to be removed manually.

Conveyors Types and Applications

Belt and live roller conveyors with power-operated deflectors or switches. Conveyors usually run at fixed speeds, from 40 to 150 fpm.

Selection means usually consist of timing "memories" with separate channel for each destination point and driven in synchronization with conveyors. All conveyors and timing means in a system must start and stop together. Packages may be located at random and not in any fixed relationship to the conveyor.

A typical system would handle cartons from one or more central points to various points in a warehouse, or to a number of shipping areas for truck or railroad car loading. Products are usually accumulated in trains which travel as units. All parts of the train go to the same destination. Trains tend to increase the capacity of such a system as a space of 10 or more feet is required between trains to clear switches or deflectors.

Selection of the destination is performed manually by an operator at a central point. As the head of a train passes a given point, the operator presses a destination button which starts a timing element. Length of timing element travel is directly proportional to the distance between the starting point and the destination. A signal is given to a deflector or switch control just before the train of packages arrives at its destination.

Considerable development work has been done in marking packages with coded reflectors to operate photoelectric relays at deflector points. This is most satisfactory when a particular kind of package always goes to the same destination.

Timing memories have been built with separate motor-driven timers for each station or special multichannel mechanical timing units driven from a common motor, or punched and magnetic tape, or wheels and movable pins set by solenoids and operating limit switches. A recent development by several companies is a magnetic drum providing readout signals even at zero speeds.

Packages in this type of system normally do not recirculate. If discharge station becomes full, the complete system must stop until cleared. Timing elements are reset to starting position at end of cycle for reuse by another train. Memory systems have been developed which allow the packages to recirculate.

Deflectors or switches normally remain in the position last used until it is necessary to change position either to clear or discharge the next passing train. This permits setting up a predetermined path for as long a train as desired. Accumulated errors are no problem as each dispatch starts a new cycle. In a similar manner, individual packages may be dispatched from a central station to a number of destinations. Usually this is used for relatively small capacity operations since there must be spaces between packages.

Trolley conveyors with automatic loading and unloading stations have been used to distribute parts in baskets and pans, and cartons. Normally there is a separate loading station for each part which always goes to the same destination.

Carriers on trolley conveyors carry *coded contacts* indicating which stations they must discharge into. A repeating series of carriers passes each station. When an empty carrier of correct destination approaches a loading station with a load waiting, loading devices transfer the load to the carrier. The loading device is blocked out if the carrier is not empty. A selector switch can be provided to change the destination from any loading station by selecting properly coded empty carriers.

Unloading stations remove any package from any carrier with a corresponding code. If the unloading station is filled, a package remains on the carrier and recirculates until the next time around or until the station is ready to receive. If desired, an unloading station can be shut off and the loads may stay on the conveyor and form a recirculating "bank."

Loading and unloading stations usually have space for holding several units, means for separating units and for moving units into or out of the path of carriers. Most are air operated, electrically controlled. Design of the carriers and stations depends on the shape and size of parts or units to be handled.

Controls are usually decentralized with a control panel for each station or group of closely spaced stations. Means are required at loading stations to indicate that a unit is ready to be loaded, to show the approaching carrier classification, whether loaded or unloaded, to retract loader when carrier is in position to receive load and safety circuits to stop the conveyor in case of malfunction of parts. Unload stations require means to indicate that the receiving station is not filled and to indicate a load on the approaching carrier and its classification, and safety circuits to stop the conveyor if the condition is not correct.

Other coding methods. Another application could have carriers manually loaded and automatically unloaded at a number of points, with manual selectors set by the operator who loads the carrier. Or selection can be made by pushbuttons to a timing memory device synchronized but separate from the conveyor, as carriers pass a central point. Or selector devices on a carrier may be set automatically from punched cards placed on the carrier with the package, removed automatically at a central point, decoded, and signals used to set the selection code.

Selective dispatching has been used mostly with power-and-free conveyors or overhead trolley conveyors. Contact making devices can be disposed in various physical patterns to provide combinations. Each signal station has a unique pattern of limit switches or wipers which make contact simultaneously. Only the correct combination will complete the circuit to initiate a cycle.

The *selecting means* is determined by the number of selections required; if carriers are required to pass over more than one conveyor, signal systems are mounted on the carriers. See Fig. 5.

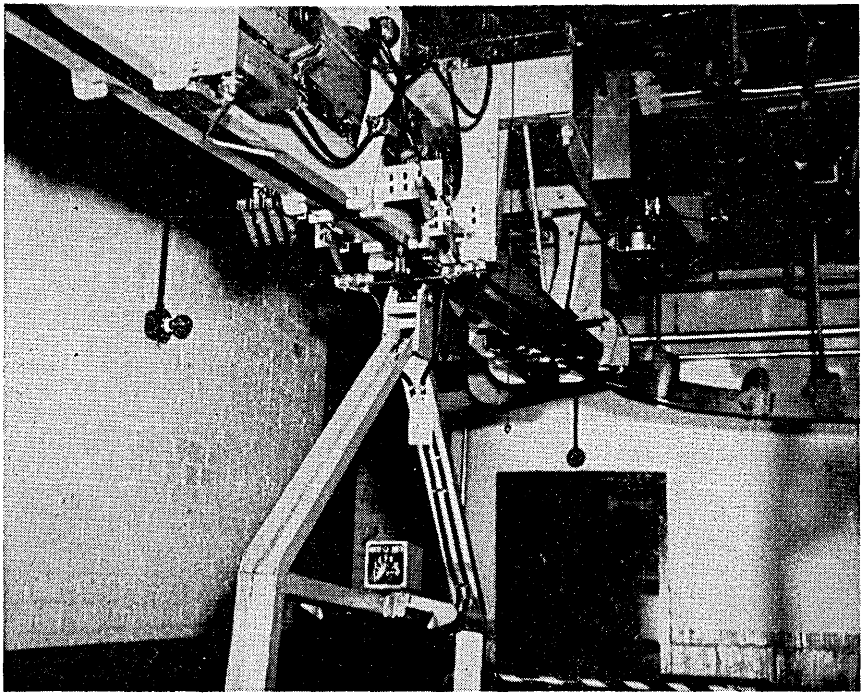


FIG. 5. Typical power and free carrier with selector switch and contacts for fourteen combinations. Photo shows unloading track switch leading to free track to the right. Limit switches and "wipers" provide signals to control track switch. (Courtesy Jervis B. Webb Company.)

By the use of wiper circuits, a change in signals can be accomplished by using tap switches with several decks to connect contacts in various combinations. A selector switch nameplate can be lettered or numbered to correspond to operation names. The operator does not have to remember combinations. Selections can be made by moving contact devices into various paths parallel to the travel or by changing connections between contacts. A cam contact can be made to stand in one of two positions and be set mechanically by means of a solenoid-operated plunger as a carrier passes a selector setting station.

Overhead trolley conveyors and power-and-free can be coded by tabs in various combinations attached to trolley brackets. Each conveyor load-carrying point has the possible tab positions of "high," "low," "left," "right," "forward," "rear."

A recent development permits the use of various radio frequencies for signals. Each signal station has an oscillator tuned to a frequency below 500 kilocycles. Carrier signal device consists of a coil tuned by capacitors through selector switch to match a station frequency. When the coil passes a station with corresponding frequency, energy is absorbed to provide a signal to operate a relay. This can provide a visual or audible signal or start an automatic cycle.

Automation conveyors between machine operations usually have separate controls. Interlocks with machine controls should be kept completely electrically separate. Limit switches to be operated by the parts handled or machine elements provide the signals; if relays are used, there is always a possibility that the control of one part may be shut off and the signal lost.

Indexing conveyors require rapid travel and accurate positioning. Heavy loads create problems of acceleration and deceleration to prevent shifting of positions.

Electric clutch brakes permit control of torque by simple electrical adjustments. They require no mechanical adjustments, and torque remains constant during life of friction surfaces.

Accurate positioning requires uniformity of time of stopping. A signal for stopping must be the same every time. A *multiple cam switch* driven by the conveyor without backlash will have a uniform signal. If limit switches are operated by parts of the conveyor that are not exactly uniform, the timing will be different.

Positioning of loads which tend to run ahead by gravity can be controlled by an electronically controlled eddy-current clutch brake unit. This permits the use of preset speed steps for slow down before stopping. A solenoid brake makes final stop and holds in position.

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Numerical Control of Machines

J. Rosenberg

1. TYPES OF CONTROL SYSTEMS

Conventional machine tools in present day use fall naturally into two major categories.

a. Positioning. This type performs operations only at discrete points in its traverse. For such machines position control systems, which determine the location only of end points, have been developed. *Examples* are drilling machines, boring machines, punching machines, welding machines, and riveting machines. Table 1 gives the characteristics of typical position control systems.

b. Contouring. These machine tools remove metal continuously and are usually required to generate a solid surface in space. *Examples* are milling machines, profilers, lathes, grinding machines, and broaching machines. The slides of such tools must be under continuous control, in order that proper synchronization for the generation of the complete con-

TABLE 1. CHARACTERISTICS OF TYPICAL POSITION CONTROL SYSTEMS

System	Storage Medium	Code	Feed Rate (in./min)	Accuracy (in.)	Type of Transducer	Comments on Controls
A	4-in.-wide punched tape	Decimal	100	±0.0005	Decimal shaft position encoder driven by ball screw	
B	1-in. P.T., IBM, or R-R cards	BCD, decimal	60	±0.0005	Syncros driven by rack or ball screw	
C	1-in. P.T.	Decimal	80	±0.0002	Stepping motor	Open loop
D	1-in. P.T.	BCD	100	±0.0002 in./ft	Linear or rotary syncro	
E	$3\frac{7}{16}$-in. punched tape	Decimal	Multispeed 3-loop control	±0.0001	Differential transformer plus potentiometer	Final approach from one direction
F	1-in. punched tape	BCD	100	±0.0002	BCD shaft position encoder plus resolver	
G	1-in. punched tape	BCD		+0.0001	Optical grating, incremental	
H	$\frac{3}{4}$-in. steel tape	Analog	90	±0.001	Indentation on steel tape, limit switch	Mechanical followup system
I	$4\frac{5}{8}$-in. punched tape	Binary	50	±0.0005	Binary shaft position encoder	
J	IBM cards	Decimal	144	±0.0001	Gage rods plus limit switch	Mechanical followup system

tour may be assured. The characteristics of path control systems that have been developed to satisfy the requirements of this class are shown in Table 2.

Control Characteristics. Position controls are characterized by a small volume of input data, simple control logic, simple nonlinear, intermittent servo actuators, relatively low costs, and modest demands on the operator. Path control, by its nature, requires large volumes of command data, high-speed control logic, linear servo characteristics, fairly high equipment costs, and a considerable amount of intelligence and training on the part of the user. Machine tools under continuous control remove much more material per unit time than the tools which function only at discrete points, usually involve servo actuators of higher power capacity, and possess greater versatility. Most machines with path control can be so programmed as to perform operations only at specific locations. Thus position control can be considered inherent in path control systems, although the reverse is not true.

The most sophisticated and reliable control systems of both types have been conceived with closed loop or true servo control characteristics. However, open loop actuators have been incorporated by designers of each type of control. Open loop controls offer the advantages of simpler logic, fewer components, and appreciably lower costs. There also exist samples of hybrid systems, wherein two or more control loops exist, one of the nonservo type, the second with true feedback characteristics. Later in this chapter typical examples of controls of several types will be briefly described.

The Operator. All numerical control systems, whether for position or path control, may be best understood if considered as man-machine combinations. Instead of requiring less judgment than manual machines, they demand more judgment on the part of the human who programs (prepares instructions) for the control system, since commands are obeyed without further human intervention. This concept is a natural result of the elimination of judgment on the part of the machine tool operator, by virtue of the presence of a complete series of machining commands on the permanent storage medium which serves as the input to the machine tool. The operator of conventional machining systems is expected to exercise judgment and make many decisions, and it is differences in judgment which distinguish the good from the average machinist. In the numerical control cycle, the human who prepares the program, a man likely to be a specially trained tool engineer, must be thoroughly familiar with tooling, fixturing, and all the parameters which are now decided by two, three, or more individuals responsible for the total machining process. In the new scheme of things, the programmer passes final judgment on all these

TABLE 2. CHARACTERISTICS OF TYPICAL PATH CONTROL SYSTEMS

System[a]	Type of Path Interpolated	Interpolator Input Data	System Resolution (in.)	Interpolator Separate from Control	Type of Comparator	Type of Transducer	Type of Actuator
A	Straight line segments of tool center	$\Delta X, \Delta Y, \Delta Z,$ segment time	0.0002	No	Reversible binary counter	Linear optical grating	Hydraulic valve and linear or rotary actuator
B	Straight line segments of tool center	$\Delta X, \Delta Y, \Delta Z,$ clock pulse rate	0.0002	No	Reversible binary counter	Rotary electromagnetic grating	Hydraulic valve and motor
C	Straight line and parabolic segments of tool center	Two points (line) three points (parabola)	0.001	No	Analog voltage amplitude	Toroidal transformer, rotary switch, induction potentiometer	Hydraulic valve and motor
D	Straight line and circular segments of tool center	$\Delta X, \Delta Y, \Delta Z$ (line), P_1, P_2, and center (circle)	0.001	Yes	Reversible decimal counter	Rotary electromagnetic grating	60-cps static magamp and a-c servo motor
E	Straight line segments of tool center	$\Delta X, \Delta Y, \Delta Z,$ segment time	0.000125	Yes	Analog voltage phase	Rotary synchro	Rotary or magnetic amplifier and d-c motor

[a] Letters identify same systems as those in Table 3.

variables; the machine operator can intervene only by interrupting the cycle.

In order that the differences between position and path control may be more readily understood, a generalized block diagram of each type of system is shown in Figs. 1 and 2.

Position Control System. Figure 1 illustrates a typical control system of the closed loop or servo type. The steps in the operation are as follows.

1. *Programming Sheet.* As in present day shop practice, the entire operation begins with the part drawing. This is the medium by which the designer of the part defines as concisely and unambiguously as possible the finished workpiece. From the drawing the tool engineer or programmer prepares an organized chart containing all the locations for each slide at which machine operations are desired. He also enters other instructions for the machine tool, such as the initiation of a drilling or punching cycle, the indexing of a multispindle drill turret, the turning on or off of coolant, and other details which may be considered auxiliary machine functions. The programming sheet should contain this information in the sequence with which it must be conveyed to the numerically controlled machine.

2. *Storage on Punched Tape.* The next step in the manufacturing cycle is the conversion of programming data to a storage medium. This is done by inserting the digital or numerical instructions into a keyboard which actuates a tape punching mechanism. The keyboard may be that of a typewriter, or an adding machine, or may be designed for the specific purpose of numerical programming. As a block of data representing a slide coordinate in inserted into this keyboard, it is permanently recorded, line by line, on a section of perforated tape. Important tape parameters, such as code, format or sequence, and mechanical size will be discussed later in this chapter. Step by step, all the information from the programming sheet is converted into correlated blocks of perforated information on a roll of punched tape.

3. *Verification.* As soon as the programming phase has been completed, the operation should be verified. This may be accomplished either by comparing a simultaneously printed record with the original program sheet, by playing back the punched tape into a printer, or by visual inspection of the perforated tape, line by line.

4. *Memory Readout.* The completed and verified tape instructions are then inserted into the punched tape reader, ready to assume command of the machining operation. Since one line across the punched tape is likely to contain a maximum of eight bits of digital information, a block is composed of several lines of tape data. Lines of tape data must be scanned sequentially and put into temporary (buffer) storage in such a

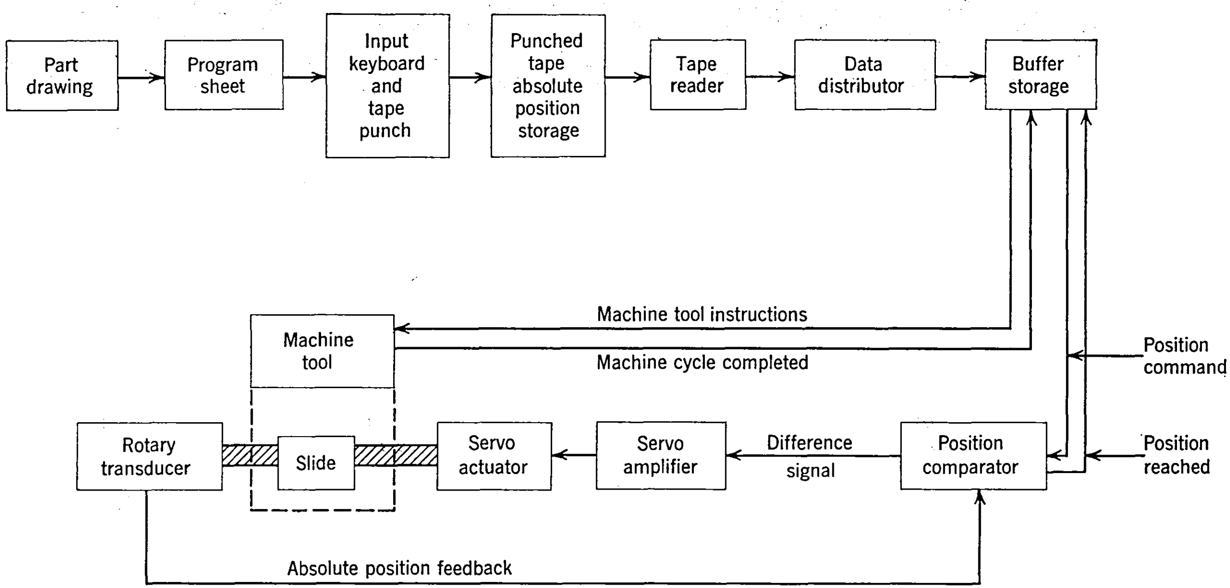


Fig. 1. Typical closed loop position control block diagram.

manner that the entire block will be available simultaneously to the control system. The function of the box labeled "Data Distributor" is to transmit the line data from the reader to the appropriate regions of the temporary storage register. A block end signal on the punched tape instructs the reader to cease operation, and initiates the control cycle.

5. *Machine Control.* The complete position command is presented continuously by the temporary storage register to the position comparator. The other input to the comparator, which may be of the analog or digital type, is obtained from the rotary transducer, permanently attached to the lead screw which displaces the slide. The difference between commanded and actual position is continuously presented to the box titled "Servo Amplifier," whose output causes the servo actuator to rotate the lead screw until the difference has been reduced to zero. At this point the slide will be at the intended location, and the comparator will signal that the command has been satisfied. The temporary storage register, which in addition to position commands also contains machine tool instructions, will cause the tool to perform its machining cycle.

6. *Cycle Control.* As soon as the cycle has been completed, a signal from the machine tool instructs the tape reader to proceed with the reading of the next block of tape data. The entire operation has been simplified so that only the principles are illustrated. In most actual cases, two, three, or more blocks of position commands representing the location of two, three, or more movable slides, must be processed and satisfied prior to each machining operation. As may be seen from the above outline, the operation of a position control system is an intermittent affair. At any given time either the tape reader will be in operation, the slide will be moved, or the machining operation will be in process. The three phases in general will not occur simultaneously.

Path Control System. A generalized block diagram of a path control system for a milling machine may be seen in Fig. 2. Table 3 shows the data flow and codes in typical systems. The steps in the operation are:

1. *Program Manuscript.* Again the operation begins with a part drawing. Because in this case a considerable amount of processing must be performed on the drawing data, an organized form usually termed a manuscript is prepared. This manuscript generally lists the important dimensions of the finished part, along with other instructions pertinent to the preparation of the final storage medium for the machine tool control. *Examples* of nondimensional data include tolerance, cutting feed rates, spindle speeds, type and diameter of cutting tool, rate of material removal, type of coolant, and instructions to the machine operator for the change of cutting tools or holding fixtures.

2. *Computing and Storage of Computed Data.* At this point the

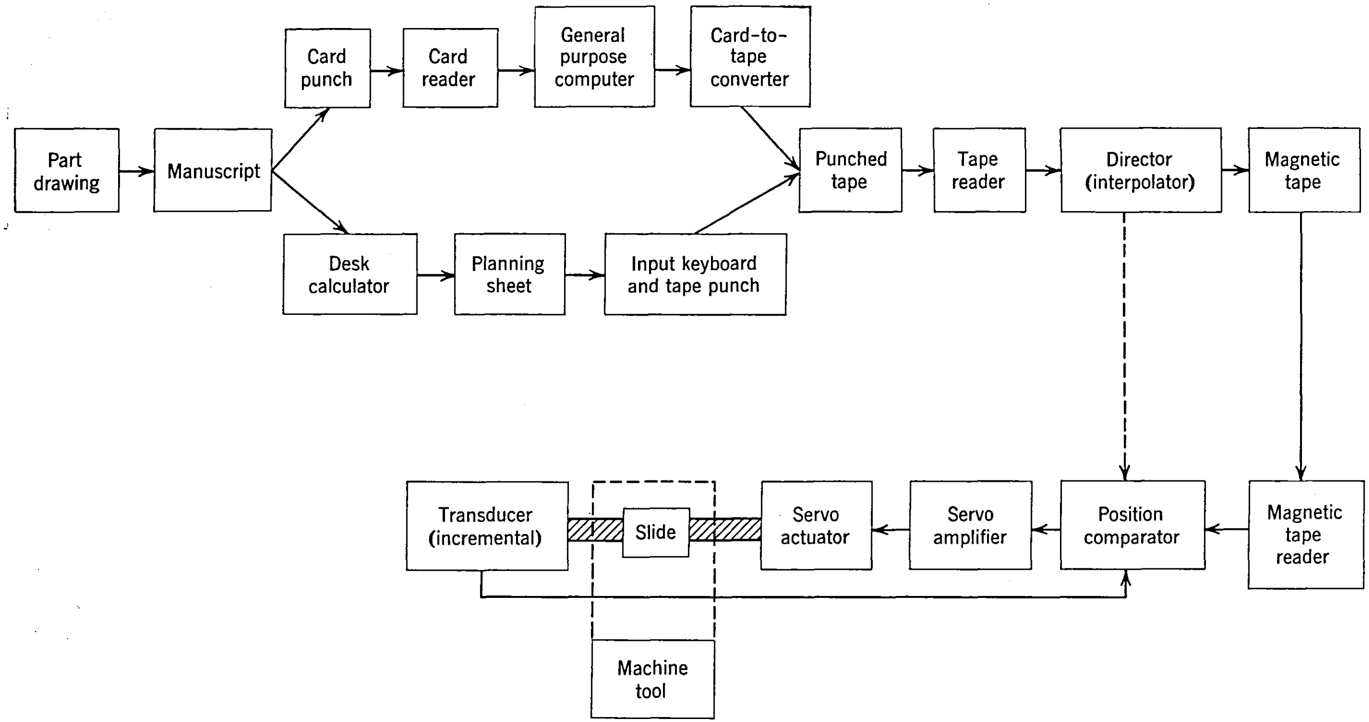


FIG. 2. Typical closed loop path control block diagram.

TABLE 3. DATA FLOW AND CODES IN TYPICAL PATH CONTROL SYSTEMS

System [a]	Storage Medium into Interpolator	Code	Type of Buffer Store in Interpolator	Code	Interpolation Code	Interpolation Storage Medium	Control Code	Transducer Code
A	7-channel, 0.5-in. magnetic tape or 8-channel, 1-in. punched tape	Binary with parity check; binary with parity check across tape	Flip-flops	Binary	Binary	None	Binary	Binary (incremental)
B	7-channel punched tape	Binary along tape; parity check across tape	Magnetic core shift registers	Binary	Binary	None	Binary	Binary (incremental)
C	Punched cards (Rem-Rand)	Decimal	Stepping switches	Decimal	Analog	None	Analog voltage amplitude	Analog voltage amplitude
D	8-channel, 1-in. punched tape	Modified BCD (6, 4, 2, 1)	Relays	Modified BCD (6, 4, 2, 1)	Decimal	8-channel, 0.5-in. magnetic tape, binary (incremental) code	Decimal	Binary (incremental)
E	7-channel, $\frac{7}{8}$-in. punched tape	BCD with parity check across tape	Magnetic core shift registers	Modified BCD (5, 2, 1, 1)	Binary	14-channel, 1-in. magnetic tape, phase modulated	Analog	Analog

[a] Letters identify same system as those in Table 2.

process may take either of two paths: (*a*) manual computing and storage on punched tape, or (*b*) automatic computing and storage on punched tape.

Manual Method. If the part to be machined is a simple one, the main task to be accomplished is that of determining the path of the tool center offset by the tool radius from the finished part. This may be performed manually by the use of a desk calculator. Locations of the breakpoints in the tool center path are then entered, in the proper sequence, along with feed rate instructions and machine tool auxiliary commands, on another form frequently called a planning sheet. The organized data from the planning sheet are then entered line by line on the input keyboard of a tape punch. This keyboard also produces a printed copy of the entered data, for verification against the planning sheet.

Automatic Method. If, on the other hand, the desired part is complex, automatic data processing by means of a general purpose (GP) computer is in order. Data from the manuscript are entered into the keyboard of a card punch, which produces a sequence of punched cards for input to the computer. Besides calculating the tool offset path, the computer may also approximate the desired contours of the finished part with the minimum number of chords or circular arcs which will satisfy the tolerance and surface finish requirements stated in the drawing. Other typical tasks for the GP (general purpose) computer may be the calculation of tool offset for a ball nosed cutter, or other shaped cutting tool. Routines for the automatic cleanup of pockets may be included, and in some cases automatic compensation for characteristics of the machine tool or servo system may be required. Tool deflection may be compensated for, and a series of step velocity commands inserted, to prevent overshoot, excessive dimensional errors, or actual information loss during the machining operation. When the computer is employed, its output data are usually converted by automatic means to punched tape which serves as input to the interpolator. The translation of computer information directly into punched tape is accomplished by the use of a converter, translating either from magnetic tape or punched cards into punched tape.

Contained in the punched tape are coordinate data specifying the terminal points of each feed for a single segment of the path of the center of the cutting tool, along with feed rate or slide-velocity instructions, programmed stops (points at which the machine is brought to a halt for operator intervention), and auxiliary machine tool functions. If the part is so simple that it can be machined by a sequence of four straight line segments, the punched tape is likely to contain only four blocks of data. If, on the other hand, a sequence of sophisticated, empirical curves must be produced with a fine surface finish, hundreds or thousands of individual path segments may have been calculated by the GP computer, and hundreds or thousands of data blocks have been recorded on the punched tape.

3. *Director* (*interpolation*). Since the punched tape contains block data defining each segment in the cutting tool path, while the machine tool servomechanisms require simultaneous, continuous, coördinated commands to produce accurate paths in space, still another form of data processing must take place. The conversion of discrete dimensional information into continuous coordinated command data for the machine servos is defined as interpolation, and is performed by a special purpose computer usually termed a *director.* To prevent intolerable errors which analog interpolation would likely introduce during large traverses, interpolation is commonly performed in a digital manner with digital output.

4. *Storage on Magnetic Tape.* Data emerging from the director will therefore usually be in the form of simultaneous trains of incremental pulse commands, a separate train for each slide to be displaced during the interval. The large volume of information represented by these pulse trains may be conveniently recorded on separate channels of magnetic tape. The precomputed magnetic tape serves as a final storage medium, one which instructs the machine tool each time the part must be machined. Some control systems avoid the use of magnetic tape, feeding the pulse trains directly into the position comparator by incorporating the interpolator into the control unit.

5. *Machine Control.* The servo portion of the control system appears almost identical to that given in Fig. 1 for position control. Incremental motion commands are fed, either from the director or from magnetic tape, into a position comparator, which also receives displacement signals from transducers monitoring slide motion. The difference between command and feedback is continually fed from the comparator to the servo amplifier which generates the power to drive the servo actuator. The actuator displaces a ball lead screw or rack to produce slide motion.

Differences between Positioning and Path Control. Although the logic appears the same, details of design and operation in path control are vastly different from those in position control. In the case of path control, information flows continuously from the magnetic tape into the position comparator, as the machining process takes place. Data input and slide motion occur simultaneously on two or three channels. Thus the storage medium is read, the slides are actuated, and cutting takes place simultaneously, instead of sequentially as mentioned under position control. Other differences are high storage bandwidths, high data processing rates in the position comparator and transducer, and linear servo amplifiers and servo actuators. The servo systems must be matched from one slide to another, in order that accurate paths can be achieved. Faster response, higher bandwidth servos are generally involved. Transducers producing incremental output information are usually employed to mate with the incremental character of the input instructions to the comparator.

2. INFORMATION REQUIREMENTS

Selection of the types of storage media for each type of control system has been influenced largely by the economics and convenience of use for each application.

Position Control. Such systems operate in an intermittent fashion, the three phases being storage readout, slide positioning, and then machine operation. Assuming that the maximum displacement of any slide is 99.999 in., and that two-axis or X-Y motion is involved, a total of ten decimal digits of dimensional information must be read between successive machining cycles. If a few extra pieces of information, such as identification of slide or selection of one of several machine functions, are included, and if a decimal digit consists of some combination of four binary digits, for each machine operation approximately 50 binary bits must be unloaded from the storage medium.

For inexpensive punched tape readers, the scanning of 50 bits will require approximately one second. The machining operation will usually require one second or more (drilling or punching), and the actuation of the slide over a small traverse will also take at least one second. Thus three seconds is a minimum interval for the completion of a single machining cycle; during the one-second interval when the storage is interrogated the data output rate is 50 bits per second. This relatively low rate is well matched to the characteristics of punched tape and punched tape readers. The low cost and high reliability of tape and tape reading equipment has lent added emphasis to the selection of these components in most current position control systems.

The low sampling rate required of the transducers monitoring slide motions, together with the advantages of absolute position readout, has led to the use of shaft position encoders as the feedback elements. Shaft encoders utilizing brushes and printed circuit commutator segments have been found to be of adequate reliability and life expectancy in this connection.

Path Control. Information requirements for path control are vastly greater than for position control. Accurate generation of paths in space require that each of the synchronized servos receive continuous, accurate input data. Although a line in space contains an infinite number of points, for practical purposes a finite number of incremental pulse commands has been found adequate, especially in view of the smoothing action of servo systems. For a skew line in space, representing the diagonal of a cube one inch on edge, to be traversed at 60 in. per minute along each axis in a typical contouring system, data input to each servo in present systems will vary from 1000 to 5000 bits per second. The incremental value ranges from 0.001 to 0.0002 in. If the total requirements for the three slides are included, the director or tape must generate between 3000 and 15,000 bits per second. The storage bandwidth may therefore be 300 times greater than that in position control. At the present time, magnetic tape offers the only practical solution to such requirements.

Such unusual data input needs for control systems have influenced the incorporation of directors into most path control systems. For smooth and accurate motion, the servos require periodic information simultaneously on each channel. GP (general purpose) digital computers are organized to perform operations on only one word of information at a time, and provide output in a similarly discontinuous manner. Thus the computer is poorly mated to the data requirements of the servos. Furthermore, the furnishing of 15,000 new pieces of data per second would strain the capacities of even the fastest presently available computing instruments, to say nothing of the high costs of such machines (upwards of one million dollars). It is for these reasons that the digital interpolators or directors have been developed. Accepting relatively little data at their input, they provide at fairly low cost simultaneous high-rate periodic output trains, properly synchronized among the channels.

As a consequence of the high pulse rates required for input to the position comparators, the incremental feedback transducers evolved for contouring systems also process data at high rates. Commutator type position encoders have been found noisy and of limited life. Noncontact transducers, of the rotary or linear magnetic or optical grating type, are universally employed in such applications, even though they provide much smaller output signals. The high mechanical and electrical noise environment has led to sophisticated shielding and amplifying measures in order to preserve information reliability in this very critical, low-energy level portion of the control system.

3. NUMERICAL CODES AND THEIR SELECTION

A comparison of the block diagrams of Fig. 1 and Fig. 2 shows that numerical codes are likely to differ. In both cases, economic and technical factors govern the decisions. Information bandwidth requirements, storage media, available transducers and their codes affect code selection. In any single position or path control system, it is common for one code to be used in the first storage medium, another in the GP computer (if one is employed), another in the second storage medium, another code employed in the control itself, and still another present in the feedback transducer. Not only codes but also format (location of data in the memory medium) play an important part in system design.

Point Positioning Code Selection

Because of the low information bandwidths required, most point positioners utilize a punched tape input medium. Punched cards or magnetic tape are less frequently found, since handling equipment is more expensive. The punched tape is usually of the teletype variety, with a width of $\frac{7}{8}$ or 1 in., depending on the number of hole positions or channels across the tape. In order to make an entire position location available simultaneously across one line of tape, a few systems employ a very wide, nonstandard multichannel plastic tape. In return for the advantage of simultaneous information availability, the designer must accept the disadvantages of difficult line registration, special and rather expensive tape punching and reading equipment, and slow reading capabilities. For description of equipment, see Vol. 2, Chap. 5, Equipment Description, and Chap. 20, Input-Output Equipment for Digital Computers.

Punched Card Codes. The Hollerith code is shown in Fig. 3. Although this code utilizes card space fairly inefficiently, it has the advantage of being standard on all IBM card reading and punching equipment, computers, and many special data converters. The code used with Remington Rand equipment is shown in Fig. 4.

Punched Tape. Where position control systems have been designed to utilize existing keyboard and tape punching apparatus, Flexowriters and the Flexowriter code and format are usually found (see Fig. 5*a*). Since the Flexowriter has an alphanumeric keyboard, one line across the tape must accommodate any alphabetic or numeric symbol. Flexowriter punched tape has an eight-channel capacity, with one channel used for parity checking, or error sensing. Since one line transmits a single decimal digit of the intended table position, a typical Flexowriter tape control will employ six lines to command an address in one axis, and twelve lines for a two-axis location. As this tape is read, line by line, and entered into buffer storage (usually relays) in the control unit, the Flexowriter numerical code (binary coded decimal) is often converted into one more convenient for comparator or storage purposes. At least one position control utilizes decimal relay storage, converting from the Flexowriter code as it is being read. In this case, the decimal relay storage acts as a switch or commutator to set up from the digital input an unambiguous carrier phase instruction, since the transducers and comparator are analog in nature.

For maximum tape density, a straight binary code would be optimum.

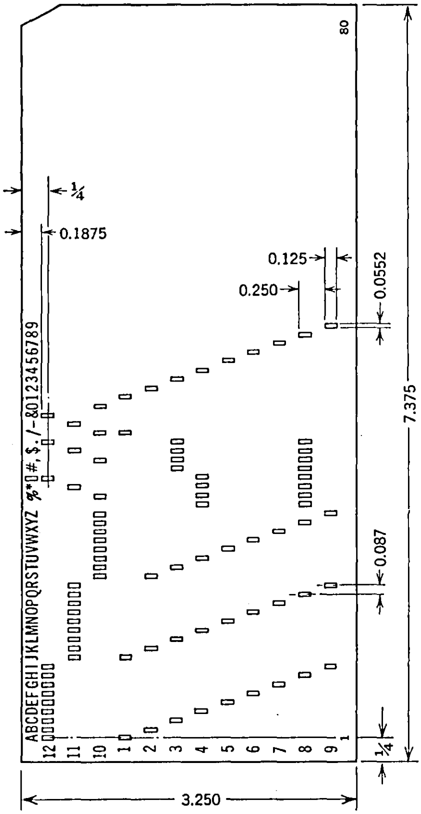


FIG. 3. IBM card.

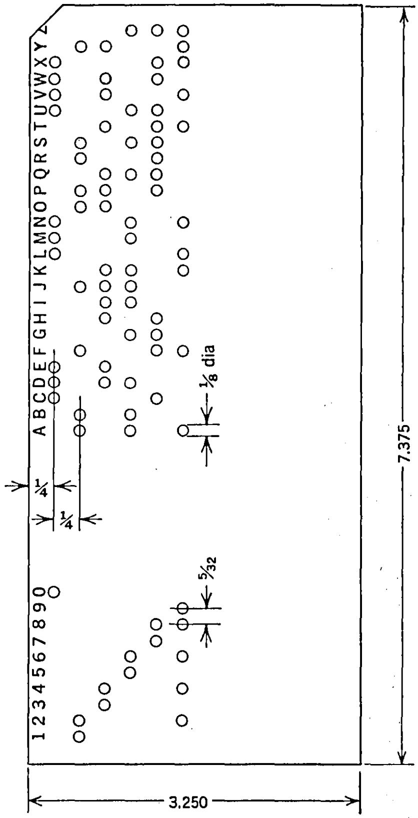
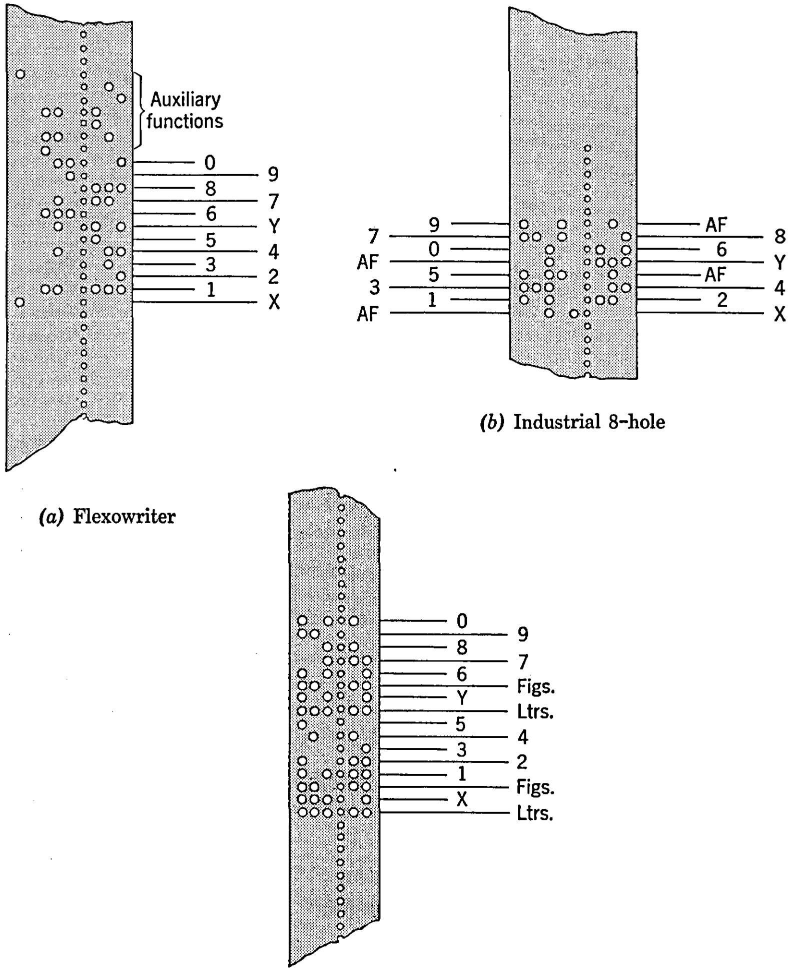


FIG. 4. Sperry Rand card, Model 2 Type 203 or 204, 45 column.

However, this might require a binary keyboard, which would create a formidable programming burden. The alternative is almost equally undesirable, an expensive input machine with a decimal keyboard and a decimal-to-binary code converter. The most practical approach to good tape packing is the use of standard 8-hole, 1-in. punched tape, where one line across the tape contains two groups of four bits, each representing a decimal digit, as shown in Fig. 5*b*. A binary coded decimal or a variation is usually employed; the code which minimizes comparator complexity is chosen. Figure 5*c* shows the Teletype code.

Analog to Digital Converters. In transducers, which are generally of the shaft encoder or absolute type, a reflected code (Gray code) is necessary to prevent ambiguities if a single brush pickoff is used, since only one brush changes its output signal at any one time. If double brush logic is available, binary coded decimal may be used, with the choice of



(*a*) Flexowriter

(*b*) Industrial 8-hole

(*c*) Teletype

Fig. 5. Punched tape codes and format.

brushes determined by the direction of motion. Analog transducers may be potentiometers, resolvers, or synchros. The last may operate into carrier phase or amplitude detectors. In the digital sense, they possess no code characteristics. Table 4 shows several binary codes.

Codes in Contouring

Since contouring systems require the services of a general purpose computer for the programming of complicated parts, the computer input code is used in the preparation of instructions from manuscript. Most

TABLE 4. BINARY CODES

Decimal	Gray	Binary	Special Transducer [a]
0	000000	000000	100010
1	100000	100000	110010
2	110000	010000	010010
3	010000	110000	011010
4	011000	001000	001010
5	111000	101000	001110
6	101000	011000	011110
7	001000	111000	010110
8	001100	000100	110110
9	101100	100100	100110
10	111100	010100	100111
11	011100	110100	110111
12	010100	001100	010111
13	110100	101100	011111
14	100100	011100	001111
15	000100	111100	001011
16	000110	000010	011011
17	100110	100010	010011
18	110110	010010	110011
19	010110	110010	100011
20	011110	001010	100001

[a] Modified binary coded decimal.

current users of contouring controls rely on some type of IBM equipment and therefore employ the Hollerith-coded IBM punched cards. Remington Rand punched cards are used in the Univac series of computers, while some smaller computing systems utilize Flexowriter or other punched tape codes (see Table 3).

It is possible to become specific only when dealing with codes for storage input to the interpolator in the control system. One typical system employs a standard Flexowriter tape as input. A modified binary coded decimal (5, 2, 1, 1) is used for interpolation. Another system uses punched tape with straight binary code, the binary number representing the displacement for an axis during a given interval being arrayed lengthwise along the tape. This system also performs interpolation and control in binary code.

Still another control system accepts Remington Rand punched cards in code, at its input, and interpolates by a combination of digital and analog techniques. In hybrid systems, a formal discussion of coding becomes somewhat irrelevant. One system employs an adding machine keyboard for data input and tape punching service, utilizing 8-hole, 1-in. tape with two modified binary coded decimal (BCD) (6, 4, 2, 1) numbers across a line of tape. This particular BCD was selected to minimize cost of equipment in the buffer relay register. Interpolation in this system takes place in decimal (one of ten positions) code, and is achieved with high-speed decimal magnetron beam switch tubes. The output is recorded in incremental (binary) form on magnetic tape, since absolute position tape storage would require excessive storage bandwidth. As information is fed into the comparator of the control system, it becomes decimal again.

Practically all transducers utilized in path control systems are of the incremental type. These transducers are either rotary electromagnetic (noncontact) gratings or linear optical gratings of high resolution.

Standards. Technical committees of two large organizations, the Aircraft Industries Association (representing a large group of equipment users) and the Electronic Industries Association (representing controls manufacturers) are in the process of evolving standards for codes, format, and physical media used in control systems. While the original effort was devoted to punched tape for position controls, these standardization activities have been broadened to cover punched and magnetic tape standards for position and contouring control systems.

Summary. The single most important factor in selection of codes is equipment cost. Next in importance comes convenience to the operator, such as the availability of standard Flexowriter or IBM apparatus for functions other than numerical control. The next most important factor is the information rate required. There are other considerations, but of relatively minor importance. Reliability is usually considered, but is assumed to coincide with minimum control logic and minimum equipment cost.

4. STORAGE MEDIA APPLICABLE TO NUMERICAL CONTROL

Three different storage media are commonly employed in numerical control systems. These media, punched tape, punched cards, and magnetic tape, are also those universally employed in computing and data processing systems. Volumetric efficiency, cost, and information output rate dictate the selection of storage for various control systems and at different points within a given control system. Punched cards and punched tape are used almost exclusively for storing absolute positions or their equivalent. One or the other is therefore found in almost all position control systems and at those points in contouring systems where position storage is mandatory. Where continuous path storage is inherent in the philosophy of a contouring system, magnetic tape is found almost exclusively. Its high packing factor, modest cost, and very high bandwidth potential (it can be used for the recording of wideband video signals, if desired) make it the best choice. For details of storage media see Vol. 2, Chap. 5, Equipment Description, and Chap. 20, Input-Output Equipment for Digital Computers.

Punched Cards and Tape. Punching and reading equipment for cards and tape have been highly developed by applications which considerably antedate numerical control. The advent of this new technology has therefore caused very little change in handling equipment. Perhaps the only significant effect has been the development of punched tape readers which can simultaneously present 40 or 50 bits of output data. Special, very wide tape punches and readers (up to 12 in. in width) have been evolved by some controls manufacturers. Also, field readers for standard 1-in. tape, which sense simultaneously 10 or 20 codes (lines) of tape, are now beginning to appear on the market. If developed sufficiently to become competitive in cost with standard single-code readers, such apparatus could have an appreciable influence on control system design, since it permits the elimination of rather expensive data distribution and buffer storage equipment.

Magnetic Tape. Magnetic tape has been in widespread use for telemetry and digital computing equipment. Recording and playback apparatus, optimum operating parameters, and tape durability and pulse reliability generally leave much to be desired. The problems have been (*a*) recording and reading heads which will assure intimate contact between tape and the head pole pieces, (*b*) homogeneity of oxide deposition on the tape to prevent dropouts and nodules, and (*c*) preparing the oxide surface of tapes so that oxide migration to the playback heads will be greatly reduced or eliminated. Magnetic tape is used for complete path storage in both digital and analog phase applications. It is probably the weakest link in the entire data processing chain, yet it offers the systems designer such a great economic advantage by permitting separation of interpolator from machine control unit that its use for commercial controls seems almost inevitable. Table 5 lists the problems in using magnetic tape.

5. INCREMENTAL AND ABSOLUTE CONTROL LOGIC

Since data transmission rates required for positioning controls are relatively low, absolute position storage has been generally adopted because of the very high reliability it affords. A minority of position controls employs incremental logic, wherein only the displacement from the previous location is given. In contouring applications, however, incremental data transmission is almost mandatory, because of the tremendous appetite of such controls for input data. Most path systems operate on

TABLE 5. MAGNETIC TAPE PROBLEMS

Problem	Solution
Reliability	
Tape-head intimacy	Metal-faced heads, polished surfaces Polished tape surface Sandwich tape construction
Dropouts	Quality control of tape base and oxide mix Pretesting of tape
Crosstalk, signal amplitude	Tight dimensional control of tape width, width stability, careful design of tape guides and tension control devices Better interchannel magnetic shielding Precision tape reels Improved reel storage and transportation containers
Bandwidth	Improved head manufacturing techniques and materials, permitting narrower head gaps, better channel-to-channel alignment Higher tape speeds
Stability	
Dimensional	Higher modulus tape bases, prestressed (tensilized)
Magnetic	Development of high remanence, stable magnetic coatings

incremental digital data; a minority employs analog data storage and interpolation.

Comparators. As for comparator logic, position controls working with absolute data possess either true parallel digital subtractors, or less complex comparators which provide an error signal of proper sign but not necessarily proportional to displacement error. Comparators designed for analog followup systems are usually simpler. However, they place requirements for high component accuracy, linearity, and stability on the digital-to-analog converters which are employed between the buffer storage and the comparator logic.

Path control digital comparators are usually reversible counters, which contain the instantaneous difference between command and feedback information. Complex logic, including storage capacity, is necessary ahead of the counter to guarantee that all command and feedback pulses reach the counter sequentially, since it can process only one pulse at a time without error. Comparators in analog systems are either phase or amplitude sensitive, and of high linearity and stability.

Interpolators. In the interpolator portion of path control systems, a variation of the digital binary rate multiplier is commonly found. (See Vol. 2, Chap. 30.) This offers a very economical means of producing multiaxis coordination for linear paths. Somewhere between the original manuscript and the interpolator a decimal-to-binary conversion must take place automatically.

One digital interpolator performs linear and circular interpolation. Where curved surfaces are frequent, this capability can greatly reduce the amount of preliminary data processing and the data volume held in position storage. A form of frequency division, which permits a continuously changing relationship between pulse frequencies on different channels, produces a high-speed, all-decimal digital differential analyzer.

As an understanding of the capabilities of numerical contouring control increases throughout the metal cutting industry, and more sophisticated part designs are introduced to such systems, it is likely that second degree interpolation will become more important in the future. While parabolic interpolation offers the greatest flexibility to the mathematician, for metal removal circular interpolation appears more practical. The interpolated path is that of the center of the cutting tool; the contour produced is offset by a constant, the tool radius, from the tool center path. Whereas the path offset to a circle is another circle, one offset by a constant from a parabola is not another parabola, but an equation of eighth degree.

6. TRANSDUCERS

Position Measurement. The parameter whose dimension is of greatest interest is the machined surface of the desired part. In position controls, the ideal can be closely approximated, since the position of the part on the moving table and the center of a drilled hole would coincide within the eccentricity tolerance of the hole drilling mechanism. Positioning systems commonly employ precision lead screws to drive digital shaft position encoders with a resolution of 0.001 in. or better. If positioning speeds are high or resolution is well beyond 0.001 in., it is desirable to lift the brushes on the lowest order digits until the slide nears the home position, to extend the life of the encoder. For analog servos rotary potentiometers, resolvers, or synchros are either connected to a precision lead screw or geared to a rack and pinion mechanism. There are examples of absolute rotary position encoders based on optics and photosensitive pickup elements. A rather sophisticated extension of the conventional synchro transformer is a linear synchronous transformer based on a linear inductive grating. Utilizing etched circuits on glass plates and high audio frequency carriers, this device is capable of extremely high linearity and resolution and, when geared to lower speed synchros, can give an unambiguous position indication.

Independent measuring elements are generally absent in open loop systems, whether of the position or contouring type. The prime mover may also play a part in measurement. Examples of open loop devices are stepping motors, synchronous variable-speed motors, mechanical gage rods, and in the simpler systems limit switches.

Path Control Measurement. Path control transducers for closed loop systems are fewer in type, probably because the higher system development costs have produced fewer demonstrable contouring systems to date. Because the point of tangency between the rotating cutting tool and the finished surface of the workpiece changes with direction of cut on a milling machine, direct measurement of the workpiece dimensions has thus far not been reduced to practice. The next most desirable measurement is that of actual slide position.

Linear Optical Gratings. The most sophisticated linear position transducer now in use is an optical grating with a resolution of 0.0002 in. Originally developed for light diffraction measurements in physical laboratories, such gratings offer position mensuration independent of mechanical elements such as lead screws, racks, and gears. Attaining sufficient linearity and high resolution requires tight control of ruling and grating reproduction, and good dimensional stability over the normal range of operating temperatures and humidity. At least two advanced contouring systems employ linear optical gratings.

Rotary Converters. For many applications, lead screws employing hardened, precision ground threads with split nut ball bearing races for backlash elimination can be manufactured with sufficient linear accuracy. Where traverse is so long as to make the moment of inertia of a rigid screw too great for the servo system, precision racks with dual preloaded pinions have been developed. In either case, a rotary incremental transducer of the electromagnetic grating type has been found to be a convenient solution. The lead screw or rack plus rotary grating can offer an overall accuracy as high as ± 0.0005 in. in a stroke of 72 in. This exceeds the accuracy of slide straightness and squareness, concentricity of cutting tools and their spindles, and the deflection characteristics of cutting tools at reasonable side cutting loads. Based on the amplitude modulation of an approximately 1-megacycle carrier, this rotary transducer offers the advantages of appreciably lower cost, simplicity of installation, shielding from contaminants, and greater electrical noise isolation from the machine tool environment.

Possessing similar advantages are analog output rotary transducers, such as synchro transformers, resolvers, and potentiometers. While they furnish higher output signal power, and therefore greater noise immunity, they often have a more limited information rate capability than do rotary digital transducers. At least two examples of the digital type can be found in contouring systems, and two of the analog type.

Machine Tolerances. At the present time, the accuracy of measurement equipment exceeds that of the machining equipment. This is probably due to the fact that machine tools now employing numerical controls are of conventional, pre-1956 design. It is likely that machine tools, designed specifically for servo controls, will possess tolerances compatible with the better present day transducer equipment. Regular contouring accuracies of ± 0.002 in. can now be achieved by several machine control systems. Users are now expressing interest in systems with an overall accuracy in the order of ± 0.0002 in. Although this figure is probably within the reach of present day control technology, a good deal of machine tool evolution and metallurgical research will likely be necessary to make this dream a reality.

7. SERVO SYSTEM CONSIDERATIONS

This section will examine parameters germane to closed loop systems, since the word "servo" by definition excludes open loop control. In earlier sections, storage media, interpolators, comparators, and transducers were discussed. Elements of the forward portion of the servo systems include servo amplifiers, servo actuators, and slide driving mechanisms.

Servo Amplifiers. The role of a servo amplifier is to receive the error output signal from the comparator, at a fairly low power level, and to perform amplification so that sufficient wattage is available to power the servo actuator.

Relay Amplifiers. The simplest form of power amplifier is a relay. With an input signal of a few milliwatts, its contacts can deliver hundreds of watts. The output is of course discontinuous, having no intermediate output levels, only zero or full power output. Relay amplifiers are used occasionally in the forward loop of some point positioning systems. Since only the ultimate location is of importance, the On-Off (bang-bang) nature of their motion may be acceptable.

In some instances, combinations of relays to provide several velocity steps have been designed as improvements over the basic two-condition servo. Since a relay amplifier will cause a slide to be actuated over a traverse directly proportional to the on-time, such servos are considered to be pulse-width-modulated. A d-c motor normally serves as the prime mover driven by the relay amplifier. An important consideration in the selection of an amplifier-actuator combination is its response time, or the length of time required to accelerate the slide from zero to full velocity. Since a relay can be operated within a few milliseconds of time, the response time of this combination is determined by the acceleration time of the motor itself. A conventional d-c shunt motor, with poor torque-to-inertia characteristics, requires hundreds of milliseconds to reach full speed from standstill.

Proportional Amplifiers. The next step in amplifier sophistication beyond the relay is the proportional amplifier. Examples of proportional, though not necessarily linearly proportional, amplifiers utilized in positioning controls are thyratrons, rotary magnetic amplifiers (such as amplidynes), which are in reality special motor-generator combinations, hydraulic valves, and magnetic clutches. Thyratrons can respond within one cycle of the carrier or power supply frequency, and are therefore relatively fast. They are used to drive either d-c variable-speed shunt motors or a-c servo motors. Again, the acceleration time of the motor controls the response of the total system. The total for the thyratron-motor combination may vary from 50 msec for a true low-inertia d-c or a-c motor driven by a 60-cycle thyratron amplifier up to 0.5 sec in the case of a high-inertia d-c motor. The thyratron offers the advantages of moderate price, high speed, and a very high-power amplification factor. Its disadvantages are fairly short life expectancy and characteristics which are affected by the ambient temperature.

Rotary Amplifiers. Rotary magnetic amplifiers, developed during World War II for the actuation of large loads such as gun mounts, searchlights, and radar platforms, have a long record of successful performance. One notable characteristic, a very high power gain, has led to the use of this device in the current drive to the electromagnetic coils of cyclotrons, wherein the current must be controlled to within 0.01% or better for successful particle acceleration. Rotary amplifiers are generally used to drive variable-speed d-c motors. Besides high gain, they offer fairly long life and insensitivity to mechanical shock. Their disadvantages are rather high price, slow response, and frequent maintenance which is typical of rotating electrical machinery involving brushes and commutators or slip rings. Rotary amplifiers are considerably slower than thyratrons. A rotary amplifier–d-c motor combination may require 0.3 to 0.5 sec to accelerate from standstill to full speed.

Hydraulic Amplifiers. Another device with a considerable history is the hydraulic actuator, driven by a precision hydraulic valve. Since valves can provide a very high mechanical power gain, they are usually excited by vacuum tube amplifiers operating at a level of 2 or 3 watts. Valves functioning at a fairly high pressure, such as 2000 or 3000 psi, can easily drive actuators to an output of 5 hp or more. In positioning systems the compliance of a large column of hydraulic fluid produces no great disadvantage; therefore an inexpensive hydraulic cylinder is often found as the actuating element. Hydraulic valves possess the desirable attributes of high power gain, high output power capacity, and very high bandwidth (short response time). The valve-cylinder combination can be designed to provide full speed output from standstill within 10 or 15 msec. Undesirable features are fairly high price (intermediate between thyratrons and rotary magnetic amplifiers), instability of characteristics due to accumulation of gas in the hydraulic reservoir or foreign particles in the valve elements, and a short life expectancy due to wear by contaminants on the moving spool, which must generally be ground to a tolerance of one or two ten-thousandths of an inch. The small size and low weight, which make hydraulic valves and actuators so indispensable to aircraft servo systems, are of very little importance in machine tool applications.

Differential Hysteresis Magnetic Clutch Actuator. This relatively new actuator in the position control field is a proportional device of rather large power gain, requiring only several watts of input excitation. Other advantages are moderate cost (about the same as thyratron amplifiers), long life expectancy, and high response speed. Driven by a constant-speed motor, a magnetic clutch can accelerate its load to full velocity in 50 or 75 msec. Its acceleration time is therefore better than any proportional actuator except hydraulic prime movers. The disadvantages are relatively little history and limited maximum output (about 0.5 hp at present). Another point in its favor is the fact that nonprecision reduction gears can be used, since the working clutch can operate against a slight drag produced by the inactive clutch, thus eliminating backlash.

Amplifier-actuator combinations which have found application in contouring controls are those which are fairly rapid in response, and have a linear relationship between error input and power output. Rotary amplifiers and d-c variable-speed motors have been in use for a number of years. Hydraulic systems, with valves feeding rotary hydraulic actuators, have also been used extensively. A more recent innovation is the combination of static magnetic amplifiers and two-phase a-c servo motors. Not to be confused with the fairly slow saturable reactor, magnetic amplifiers were developed for military use during the early 1950's; later models employ new materials and manufacturing techniques to give outputs of 1 hp or greater.

Amplifier-Actuator Selection. All three combinations, rotary amplifiers and d-c motors, hydraulic systems, and magnetic amplifiers and a-c motors, are rather high in price, so the selection is based on other characteristics.

1. *Bandwidth* is a parameter of great importance. Hydraulic systems, which employ rotary motors instead of linear actuators to avoid objectionable compliance, can provide a bandwidth, including the mechanical load, of 30 or 40 cps. Very rapid slide acceleration can thus be effected. Magnetic amplifier–servo motor packages can develop a bandwidth, including load, of 5 to 8 cps when excited by a 60-cps power source. If the

inconvenience of a 400-cps generator can be accepted, the bandwidth may be doubled. The rotary amplifier–d-c motor combination can provide a useful response of 3 to 5 cps.

2. *Stability* and *service* form another basis for comparison. The greatest freedom from maintenance is offered by the static magnetic amplifier, since it contains no moving parts. The long-term electrical stability is also excellent, with intervals of a year or two between readjustments already a reality. The life and stability records of rotary amplifiers and d-c motors are fairly good. There may be bearing problems in the rotary amplifier, but they are not greater than that of any rotating machine. Since they are d-c machines, the brushes and commutators produce electrical noise and are subject to wear. The same is true of the d-c motor. Service intervals are likely to be less than one year.

3. *Reliability.* Hydraulic systems, with appreciably faster response and greater peak power outputs, require that their oil reservoirs be kept meticulously clean and free from gas. In an industrial environment, oil replacement (or cleansing) and removal of entrapped gas by purging may be required at intervals of 30 to 60 days. In addition, the stability of the very precise valve may be adversely affected by the accumulation of foreign material on the critical spool assembly or wear by hard foreign particles. While considerable progress has been made in improving the reliability of high-performance servo valves, the current record indicates the valve is the least stable amplifier element of the three herein discussed.

Purely Mechanical Elements in the Servo Loop. Since all three commonly used actuators have a rotary output, a reduction gear box is the first element in the mechanical chain. Next comes the rotary-to-linear converter, which is generally a lead screw for short and medium strokes, and a rack and pinion for large traverses. The slide and its bearings comprise the driven element, while the reaction of the cutting tool on the workpiece is still another source of loading on the actuator.

In relative order of importance to the servo engineer are the backlash, friction, and moment of inertia of the total load as seen by the rotary servo actuator.

Backlash. Backlash can be virtually eliminated by the use of ball-bearing lead screws with two preloaded nuts and rack drives containing two separate preloaded pinions. Such drives can be manufactured to a very high degree of precision, and with a long life expectancy, but at considerable cost. A precision screw or rack costing $5000 per axis is not unusual.

It is more difficult to fabricate gear boxes of high efficiency and low backlash. With precision spur gears located on accurately bored bearing centers, a backlash figure referred to the input shaft as low as $\frac{1}{4}$ or $\frac{1}{8}$ degree can be achieved. Since an appreciable reduction ratio is often necessary, a gear box with at least three meshes to minimize moment of inertia will be found. Although split, preloaded antibacklash gears are common in instrument drives, their use in power drives for machine tools is yet uncommon. In order to be effective, very high preload torques would be necessary, with attendant gear wear.

Friction. The art of precision gear cutting and train assembly is being intensively explored at the present time, and improvements in precision can be expected. Another approach, less acceptable to the servo engineer, is the practice of installing oversized gears, so that at least initially the train is free of backlash. The very high static and running friction introduced by this expedient may create a servo stability problem, however. The highest quality spur gears are first hardened, then ground or hobbed.

Inertia. Since gear reduction ratios are generally greater than 10:1, most of the inertia in the mechanical system is contributed by the rotor of the drive motor and the moment of the first one or two gears. These elements generally contribute at least 50% of the total system moment of inertia. Where a long stroke or a high thrust is necessary, a large-diameter lead screw can introduce a noticeable moment of inertia. It is mainly for this reason that strokes in excess of 6 ft usually utilize a rack and pinion drive. The moment of the lead screw may contribute 30% of the total system inertia.

Lastly, the mass of the slide and the workpiece should be considered. Although the inertia may not be appreciable, the mass may react upon the lead screw in such a way as to cause significant end motion due to compressibility of the retainer bearings, and almost certainly the static and running friction of the slide bearings will be affected by the weight of the slide and its load.

Stiction. Next to backlash, stiction (static friction) can generate the greatest obstacles to servo stabilization. Especially where stiction exceeds running friction by 20% or more, it will be difficult to produce slide motion which is smooth over the entire range of feed rates, from maximum feed (rapid traverse) down to a creep feed or under 1 in. per minute. Usually loop gain, and therefore acceleration, is compromised in an effort to overcome pulsating slide velocity at the very low and very high feed rates. In an effort to improve the situation, the more progressive machine tool manufacturers are designing antifriction bearings for their slides. This usually involves roller or ball bearings riding linearly on hardened ways. Since the slides must be restrained against unwanted motion in many directions, antifriction mechanisms can become very involved, very expensive, and difficult to adjust and maintain. It is not unusual to find a slide which has been initially adjusted to provide extremely smooth velocity becoming jittery because of a small change in friction resulting from wear or vibration. Plastic sliding bearings were formerly employed but were found unsatisfactory.

Looseness of slide bearings can build up owing to the compounding of two or three slides upon one another, and produce very undesirable vibrations. Although it does not represent servo instability, the vibration produces very poor machined surface finishes and can also result in the premature breakage of very expensive, hard, brittle cutting tools. In machine tool parlance this shortcoming is called "fishtailing."

Cutting Thrust. Another important detail is the reaction of the cutting tool on the servo system. Very few machine tool manufacturers have ever made cutting measurements to provide the servo engineer with load information. Generally the thrust provided by the actuator at the slide is far in excess of that introduced by the cutting tool. In most cases the friction of the slide, screw or rack, and gear box completely masks the cutting thrust. However, there are situations, such as slab cutting with a large face mill, where the load introduced is 10 or 20% of the total actuator horsepower. Thus, final stabilization should be done under actual operating conditions. In milling machines, climb milling (work moving in the same direction as the cutting edge of the tool) should be investigated, since the spindle motor feeds power into the slide (instead of opposing it). The servo system must here act as a brake.

8. PROGRAMMING (PREPARATION OF CONTROL TAPES OR CARDS)

Position Control. Computation is seldom necessary in reducing part drawing data to the form necessary for entry to the input keyboard. Since machining takes place only at discrete points, after the table has reached the intended position and come to rest, calculation of tool offset is not involved; the table location and center of the cutting tool (punch, drill, boring tool, spot welder, rivet) coincide.

A program sheet, listing input data in the same code and sequence as required for the control function, must be prepared from the part drawing. Most control systems function on an absolute basis, with all positions referred to a fixed origin on the compound table. Drafting practices are inconsistent with this philosophy; not only are part details dimensioned from a variety of references (hole centers, external surfaces, corners) but also permissible tolerances are inconsistent and often cumulative. Data reduction is chiefly concerned with redimensioning of the part to the proper reference, and itemization of the locations and machine operation commands in the most efficient sequence.

Drawings are almost without exception dimensioned in decimal code; decimal input is also common to most controls. In those cases where binary input is required, the conversion is usually performed prior to preparation of the program sheet. Manual code conversion, with the aid of tables, may be practical; automatic conversion by electromechanical or computer means can be arranged. In either case, it amounts to addition of powers of two until the total equals that of the decimal number.

Path Control. Several operations must be performed on drawing data to provide input information for interpolators in path control systems.

a. Redimensioning. All significant locations on the part, such as hole and radius centers, breakpoints (where different lines or curves intersect), points through which faired curves must pass, and other critical data, must be dimensioned with respect to a single origin. This reference should be off the part, so that all dimensions are positive.

b. Tool Planning. A tool engineer should choose the machine tool, specify the holding fixtures and cutting tools, decide in what sequence the material will be removed, how deep a cut will be made at each pass, and from his experience call out the optimum tool tooth loading by specifying the spindle speed (rpm) and feed rate of the part relative to the cutting tool. The last decisions will determine the accuracy of the final part, since they control the tool loading and therefore the elastic deflection under load, and also the surface finish of the product.

c. Part Programming. The tool engineer's decisions must be converted to a detailed list of operations. A point on the part holding fixture is selected as a secondary reference, from which the cutting tool starts and ends its motions. The gross points in the outline produced by the rotating tool as it roughs out and finishes cutting of the final part must be called out. Usually the redimensioned part drawing is amended to show the paths followed by the tool during cutting and noncutting segments of motion. Tool changes, interruptions for inspection, and other machine operator's instructions must be itemized.

d. Data Processing. All the breakpoints in the path described by the center of the cutting tool must be accurately calculated. Since most interpolators perform only linear interpolation, the start and end of each straight line segment in the entire tool path must be determined, and the displacement of each axis during each segment (ΔX, ΔY, ΔZ) derived by subtraction. Tool offset corrections must be included.

Where curved contours are required, the tool path is broken down to the minimum number of linear segments which will meet the accuracy and surface finish specified. In processing data for interpolators which can describe circular or other second degree equations, the curved contour is also reduced to the least number of segments. Here the quantity of segments is appreciably fewer (by a factor of 10 or more).

In addition to the above operations, data reduction must also include derivation of specific commands to the interpolator relating to slide feed rates. In most cases the feed rate specified by the tool engineer must be converted to either a clock pulse rate or total time interval for segment interpolation. The vector velocity of the part is broken down into its components along each axis, and each component related to output command rate. Some interpolators receive feed rate vector commands directly, and perform this computation automatically.

Still other functions which may be performed under this category are controlled slide acceleration and deceleration (to accommodate for servo limitations), overshoot prevention (on inside corners), offset compensation for shaped cutting tools (tools other than flat end mills), and automatic pocket cleanout.

If the services of a GP (general purpose) computer and computer programmer are available, all data processing can be channeled through this facility, and accomplished with ease. Large-scale users of NC (numerically controlled) equipment follow this practice. However, it is feasible to perform data reduction manually, on desk calculators, for simple and even complex part designs. Where the director is capable of first and second degree interpolation, tool offset and curve fitting computation can be accomplished manually with little difficulty by a trained operator.

The first large-scale installation of NC systems took place during 1958 in aircraft manufacturing plants. Several plants organized a cooperative mathematical programming effort to prepare a library of subroutines for the IBM 704 GP computer, specifically for NC data processing. This program is titled APT-2 (Automatic Programmed Tools).

e. Interpolator Input Preparation. The final step is preparation of the storage medium which commands the interpolator. Where a GP computer has been employed for data processing, an automatic converter to prepare the punched tape or cards from computer information is the most logical solution.

If a converter is unavailable, or manual data processing has occurred, then the part programmer prepares a Planning Sheet (it may be titled Process Sheet or Manufacturing Outline) listing all data in the precise sequence for entry into the input keyboard. A typical part will require at least 100 lines of entry on a Planning Sheet and may exceed 500 lines. Next, a trained operator will enter these data into the keyboard, and produce the card or tape storage medium. Verification of this step, either by comparison of two independently punched tapes (or cards) or by printout of the data from an automatic reader, constitutes the final step in the programming chain.

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CHEMICAL PROCESS CONTROL INSTRUMENTATION

Instrumentation Systems

P. S. Buckley and J. M. Mozley

1. TRENDS AND LIMITATIONS IN SYSTEMS ENGINEERING

In the process industries, as exemplified by the chemical and petroleum industries, the use of instrumentation and automatic controls has tended to follow certain typical patterns (Ref. 4). The best practice today consists of transmitting process data from the plant to a central control room where there are data recorders, indicators, controllers, and command devices for setting the level of those process variables which are to be controlled. From the control room command signals are sent back out to the plant to final control elements, usually control valves. Usually, the process variables are controlled separately and individually; in a plant with a high degree of automatic control this sometimes leads to interactions between control systems.

This chapter has been organized and presented for use in a systems engineering approach to the design of process control systems. The design methods and principles are covered in Chap. 1, Systems Design, and in Vol. 1, Part E, Feedback Control. To use these procedures, the engineer must have certain data on the static and dynamic behavior of processes as well as data on the static and dynamic characteristics of instruments and control components. Equations for and data on process dynamics are now becoming available (see Ref. 16). In using the material in this section one must keep in mind certain facts about present day process control equipment and its applications.

1. The quantitative techniques of designing control systems developed since about 1940 for military purposes are just now beginning to penetrate into the process industries where control system design has usually been qualitative.

2. Quantitative methods of designing control system components for specific dynamic behavior have not been widely used in the design of typical process instruments.

3. The commonest process control instruments are pneumatic. As a result of (2), they are often less than optimum with respect to such factors as impedance matching, power supply, saturation, and dynamic nonlinearity.

4. Measurement problems are much more severe in chemical process operations than in standard electrical and mechanical operations. Highly corrosive fluids, fluids containing solids and gummy materials, high temperatures, and high pressures often require that measurement devices be protected from the environment whose properties they are trying to measure. Both static and dynamic accuracy may suffer, and the questions of reliability and maintenance may be serious.

5. Partly as a result of (4), process control systems are almost always designed with provision for manual control in case of emergencies.

6. A process control system is usually a single variable control system (such as temperature or pressure control), and it is rare for systems to have numerically identical parameters. This means that it is hard to justify for each problem the extensive research and engineering that goes into mass-produced, identical control systems.

7. The objectives of process control system design are quite different from those of servomechanism design. In typical servo designs, performance is maximized, size and weight are limited, and cost is usually not a major consideration. In typical process control systems, cost is minimized for a certain lower limit on performance, and size and weight are usually not important. The process control system is usually a regulator, while the servomechanism is a followup system. It should be noted that the same control system may be required to function both as a regulator and

as a servomechanism. The distinction is that a *regulator* keeps the value of the controlled variable constant in the face of disturbances, while a *servomechanism* makes the output of the controlled system follow the input.

In view of the above, although special purpose control systems are sometimes designed, application of the systems engineering philosophy is usually directed toward effective utilization of commercially available process instruments. By providing better power supplies, by improving the impedance match between components (see Sect. 2), and by careful attention to installation practices, it is sometimes possible to achieve phenomenal improvement in system dynamics. Often, too, a simpler, less expensive, and more readily maintainable system results.

In the sections which follow, the term "system" is usually used in the restricted sense of applying only to instrument components; the process is not included except in the early part of Sect. 2. A comprehensive discussion of available components was not possible, and the ones chosen for discussion are typical only. Neither approbation nor condemnation of any manufacturer's equipment is intended or implied.

Other important aspects of process control and process control hardware are discussed elsewhere in this handbook. To handle most effectively the mass of data from a big refinery or chemical plant, data loggers, which include scanning, monitoring, and interlock functions, are being used to an increasing extent. These are discussed in Chap. 14, Data Processing. To tie together local or individual control loops into an overall process control system, process control computers have been developed (see Chap. 13, Computer Control). These computers are making it possible to optimize automatically process economics.

2. CONTROL FUNCTIONS

Introduction

A simplified schematic diagram of a typical process control loop is given in Fig. 1. For purposes of clarity, none of the normally provided subsidiary features, such as manual-automatic transfer stations, fail-safe devices, safety interlocks and alarms, and indicating and recording equipment, have been included. The operation of the control loop may be qualitatively described as follows. When disturbances act upon the process, they cause a change in the measured variable, which has been selected to be most representative of the desired process condition. The measured variable actuates the transmitter which relays a signal representative of the magnitude of the measured variable to the controller. As indicated in Fig. 2, the controller compares the transmitted value of the

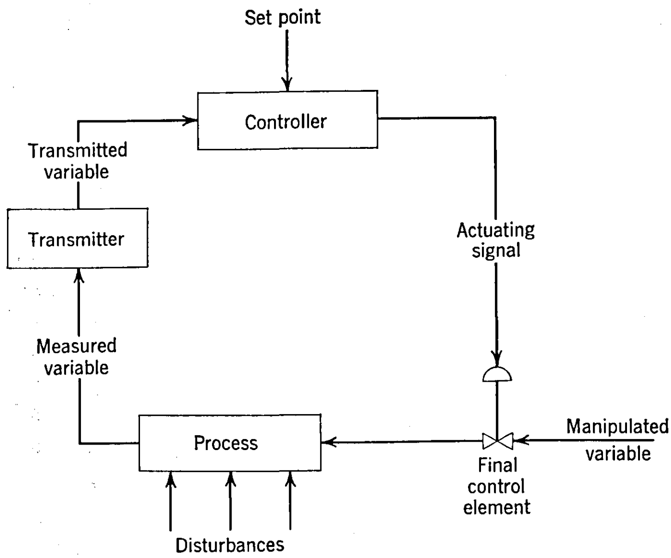


FIG. 1. Simplified process control loop.

measured variable θ to the desired value of the measured variable θ_S which is stored in the controller as a set-point adjustment, and produces an error signal θ_E, equivalent to the difference between the transmitted and desired values of the measured variable. The error signal is operated on by the controller mechanism to produce the controller output P, an actuating signal of sufficient power to operate the final control element. The final control element adjusts the flow of energy or material (manipulated variable) entering or leaving the process in the proper direction so as to force the error to zero. The functional relationships developed between P and θ_E by the controller mechanism are known as the control functions or control modes. The control functions may be continuous or discontinuous.

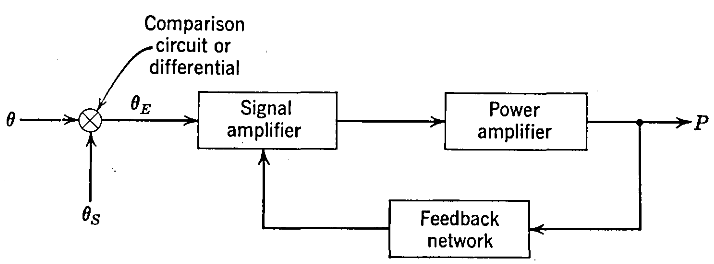


FIG. 2. Generalized controller block diagram.

Continuous Control Functions

Although the number of possible continuous control functions which might be used is very large, only three are used to any great extent in process control. These are (*a*) proportional action, (*b*) automatic reset or proportional plus integral (floating) action, and (*c*) rate or derivative action.

Proportional Control. In the proportional control mode, the controller output P is proportional to the control error θ_E:

$$P = K\theta_E$$

where K = proportional gain.

In process control the more common expression for the proportional factor is proportional band or throttling range, defined by the following:

$$\text{Per cent proportional band} = \frac{100}{K}.$$

The frequency response amplitude characteristic of a proportional controller is not perfectly flat as indicated by the defining equation above, but has some dynamic features dependent upon the particular controller design and the controller load.

Reset Action. A controller having only integral or floating action is not too common. Usually proportional and floating action are combined. Ideal proportional plus automatic reset action may be defined in the time domain as

$$P = K\theta_E + K_R \int \theta_E \, dt.$$

Laplace transforming leads to the equation

$$\begin{aligned} P(s) &= \left(K + \frac{K_R}{s}\right)\theta_E(s) \\ &= \theta_E(s)\,\frac{K_R}{s}\left(\frac{K}{K_R}s + 1\right) \\ &= \theta_E(s)\,\frac{K}{s(K/K_R)}\left(\frac{K}{K_R}s + 1\right), \end{aligned}$$

or

$$\frac{P(s)}{\theta_E(s)} = \frac{K_R}{s}(\tau_R s + 1) = \frac{K}{\tau_R s}(\tau_R s + 1)$$

where $\tau_R = K/K_R$ = Automatic reset time constant.

The frequency response plot of this idealized control function is given in Fig. 3. The effect of automatic reset is to give greater controller gain

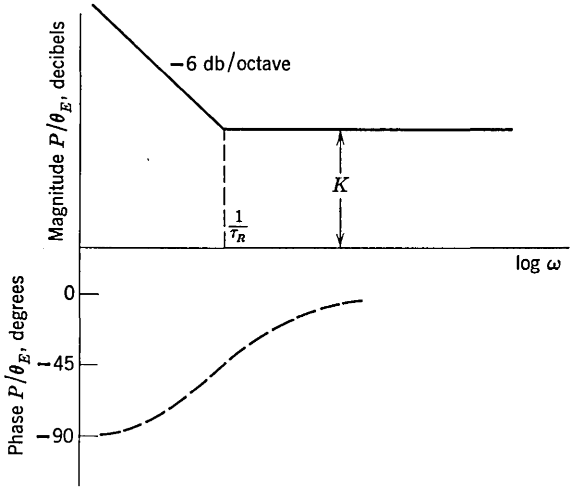


FIG. 3. Frequency response, ideal proportional reset action.

at the low frequencies, starting at the corner frequency, $1/\tau_R$, and increasing at a rate of 6 db/octave as frequency decreases. An increased phase lag is also associated with this action. It is common in process control to refer to the amount of automatic reset action by the magnitude of $1/\tau_R$ expressed as repeats/unit time (equivalent to radians/unit time).

The idealized automatic reset action described above is never actually obtained in a practical controller because of the expense involved in its mechanization. The reset action most often realized is similar in performance to a lag network having the transfer function

$$\frac{P(s)}{\theta_E(s)} = \alpha K \frac{\tau_R s + 1}{\alpha \tau_R s + 1}.$$

The frequency response plot of this control function is given in Fig. 4. The reset gain α ranges from zero to 500 depending upon the controller design and sometimes upon the value of the proportional gain K. This interaction between the reset gain and proportional gain is not generally desirable but cannot be avoided in certain controller designs.

Rate Action. The rate or derivative control mode is never used alone in a process controller. It most commonly appears in conjunction with proportional or with proportional-reset action. Ideal proportional rate action may be characterized in the time domain by

$$P = K\theta_E + K_D \frac{d\theta_E}{dt}.$$

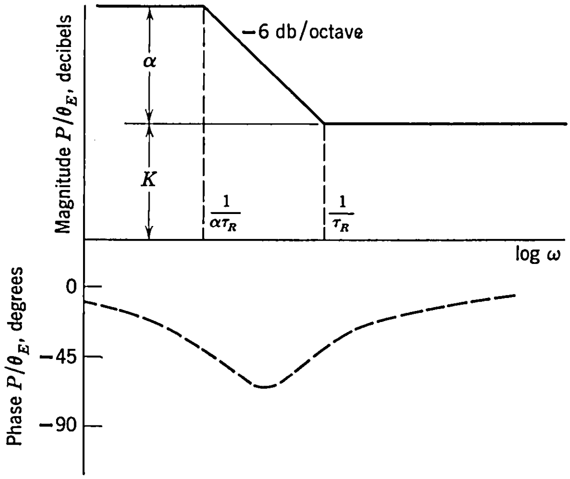


FIG. 4. Frequency response, realistic proportional reset action.

By Laplace transformation,

$$\frac{P(s)}{\theta_E(s)} = K\left(\frac{K_D}{K}s + 1\right) = K(\tau_D s + 1)$$

where $\tau_D = K_D/K$ = Derivative time constant.

The frequency response plot of this idealized proportional rate control function is given in Fig. 5. The effect of derivative action is to give phase

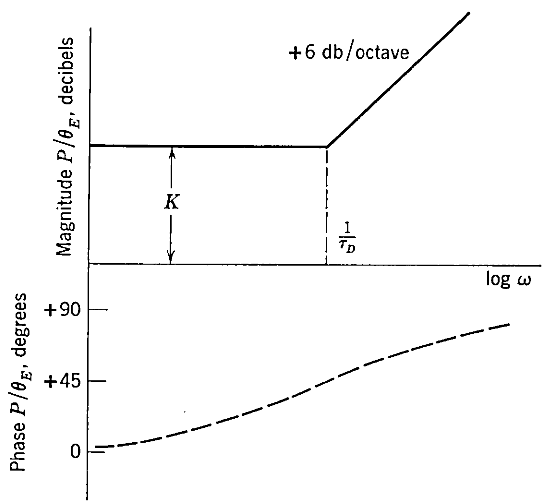


FIG. 5. Frequency response, ideal proportional rate action.

lead. Also associated with this desired phase lead is an inescapable increase in controller gain at the higher frequencies. In process control, the amount of derivative action is expressed as rate time in units of time, equivalent to the magnitude of τ_D.

In a practical controller, it is physically impossible to achieve ideal derivative action. In fact, such action would render the controller useless, since it would amplify process "noise" (which usually predominates at the higher frequencies) and would saturate the controller output. Therefore, the practical implementation of rate action is very similar to a lead network having the transfer function

$$\frac{P(s)}{\theta_E(s)} = K\left[\frac{\tau_D s + 1}{(1/\beta)\tau_D s + 1}\right]$$

The frequency response plot of this more practical control function is given in Fig. 6. The value of the rate time is τ_D. The rate gain β ranges

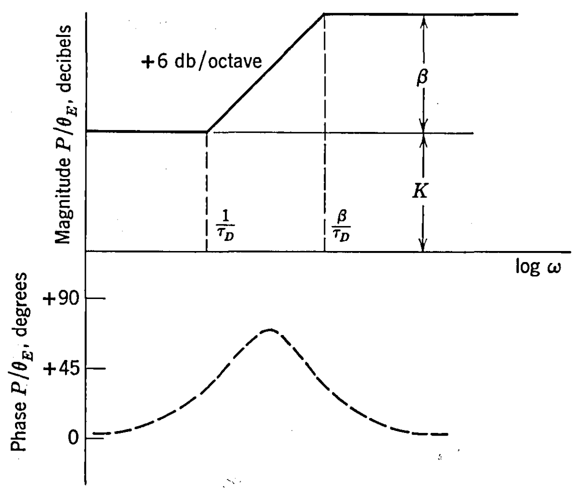


FIG. 6. Frequency response, realistic proportional rate action.

from 0 to 50 depending upon the controller design and sometimes upon the value of the proportional gain.

Controller Mechanisms. Mechanization of control functions can be accomplished in a variety of different ways, electronically, pneumatically, hydraulically, mechanically, or by any combination thereof. The task of quantitatively analyzing all these specific devices, indeed even of qualitatively describing most of them, is next to impossible. Therefore, the reader is referred to the voluminous controller manufacturers' literature for these specific details.

However, two important types of controllers will be discussed by means of selecting examples—the electronic controller (see Sect. 7, Electric and Electronic Components) and the pneumatic controller (see Sect. 6, Pneumatic Components). The latter is important since pneumatic controllers are by far the most commonly used type in the chemical and petroleum industries. The former is important because it represents a new trend and is being applied more and more frequently in industrial process control systems.

Discontinuous Control Functions

A great many types of controllers operate in a discontinuous fashion. In one class of discontinuous controllers, the corrective action is a discontinuous function of the measured variable. In this class are the off-on or two-position controllers, which are used widely in industrial process control and which will be described briefly here. In another class of discontinuous controllers, the corrective action and/or the error sampling are discontinuous functions of time. Members of this class are discussed in Chap. 12, Sampled-Data Control, and Vol. 1, Chap. 26, Sampled-Data Systems and Periodic Controllers, and will not be treated here.

The off-on controller is used primarily because of its simplicity of design and construction and its correspondingly low cost. Its successful use is restricted to processes which are characterized by one predominantly large first-order time constant and small dead time.

The off-on controller action is given in Fig. 7. When the controlled variable is outside the differential gap in one direction, the corrective action is maximum or on; when the controlled variable is outside the

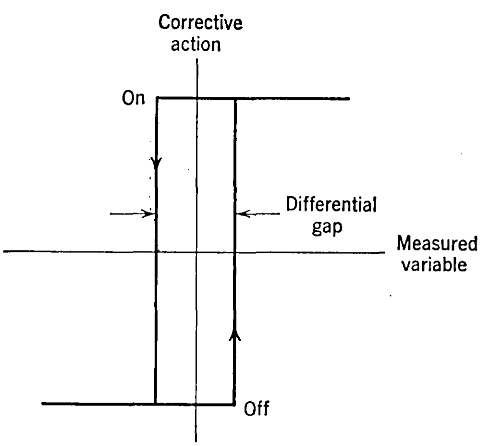


FIG. 7. Off-on controller action.

differential gap in the other direction, the corrective action is minimum or off. The purpose of the differential gap is to increase the switching period so as to reduce wear.

Analysis of the off-on controller alone is fruitless and must be done in conjunction with the process which it is to control. Use of analog computers is highly recommended for this type of problem. However, some powerful analytical methods are available where access to an analog computer is inconvenient. Oldenbourg and Sartorius (Ref. 9) have analyzed the off-on controller with processes having dead time and a single first-order lag, and have developed charts for predicting the period and maximum amplitude of the controlled variable. Kochenburger has developed a describing function technique suitable for analysis and synthesis of off-on control systems (Ref. 8). The powerful phase plane technique, useful in analyzing systems of lower than third order, has been applied to off-on control systems by Eckman (Ref. 5).

3. PNEUMATIC CONTROL SYSTEMS

Historically, pneumatic and hydraulic devices antedated the development of electronics. For reasons of cheapness and safety (no spark hazard, no combustible hydraulic fluid), pneumatic equipment took and maintained an early lead in the petroleum industry, which until World War II was more advanced than any other process industry in its use of automatic control.

Board-Mounted Controller

The commonest arrangement of pneumatic devices in a pneumatic control system has both the process variable transmitter and the final control element in the plant with other equipment located on or behind an instrument panel (control board) in the central control room. As shown by Fig. 8, this system may be cut into three noninteracting segments for testing or for system design.

Transmitter Input to Controller Input. It is assumed that the transmitter input impedance is high in comparison with the signal source impedance. This is sometimes not true, however, as for example when a pressure or differential pressure transmitter is connected by long, small-diameter impulse lines to the signal source. Displacement type level transmitters also have a low input impedance. This necessitates consideration of the interaction between the process and the transmitter. The frequency response of a typical transmitter plus 250 ft of ¼-in. o.d. (0.180-in. i.d.) tubing is shown on Fig. 9 (Ref. 4). This is valid for pressure or differential transmitters; for temperature and liquid level the frequency response of the input circuit must be added in. (Note that P has the units of lb/ft^2.)

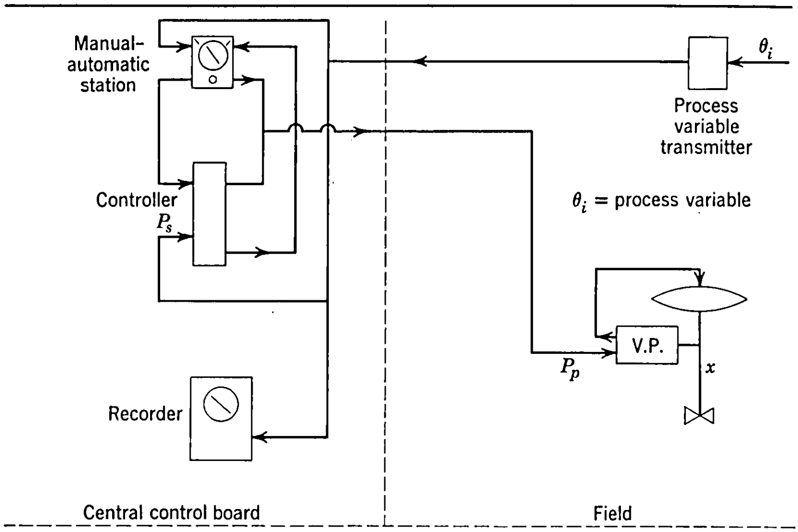


FIG. 8. Typical pneumatic control system.

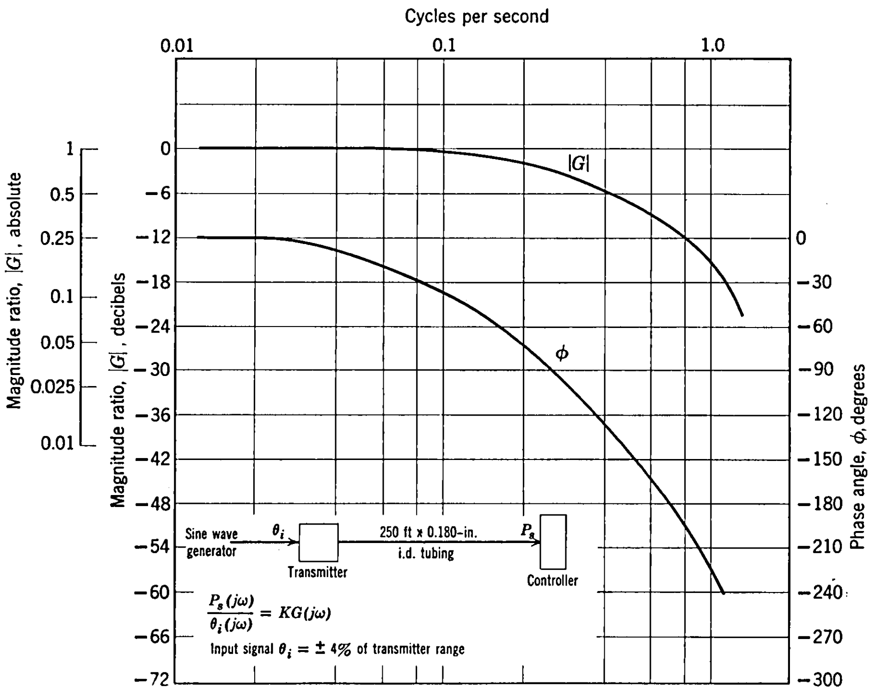


FIG. 9. Frequency response of typical pneumatic transmitter plus long line.

Controller Input to Valve Input. Controller input volume and mechanical compliance are usually small, which means that the controller usually has a high input impedance. The controller output goes through the manual-automatic station and on to the valve input. By valve input is meant (1) the receiving element of a valve positioner or booster relay, or (2) the dome of a spring-and-diaphragm valve. The frequency response of a typical proportional controller plus 250 ft of ¼-in. (0.180-in. i.d.) tubing is shown on Fig. 10 (Ref. 4). The effect of adding automatic reset and derivative action is shown in Sect. 2.

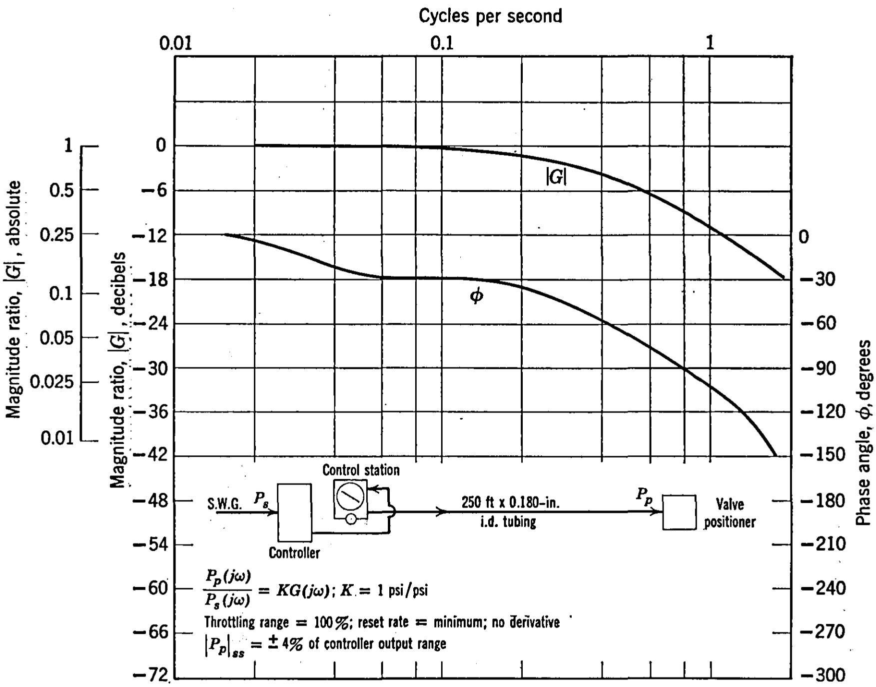


FIG. 10. Frequency response of typical pneumatic controller plus long line.

Valve Input to Valve Stem Position. It is usually permissible to measure or compute the transfer function x_v/P_p without concern for dynamic axial, valve stem forces originating in the process line, although this is beginning to be a consideration with large valves (over 6 in.) and fast processes. The frequency response of typical control valves (up to and including 6-in. valves) is shown on Fig. 11 (Ref. 3).

Field-Mounted Controller

To minimize control system lags due to transmission line length the controller is sometimes located in the field (see Fig. 12). As before, the

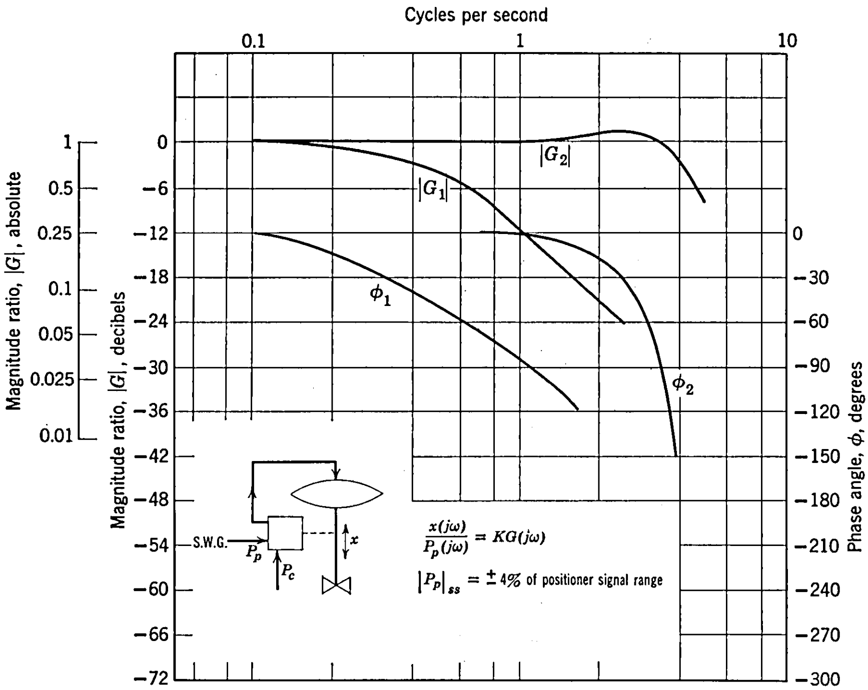


FIG. 11. Frequency response of typical pneumatic control valve: (1) positioner only; (2) positioner plus booster.

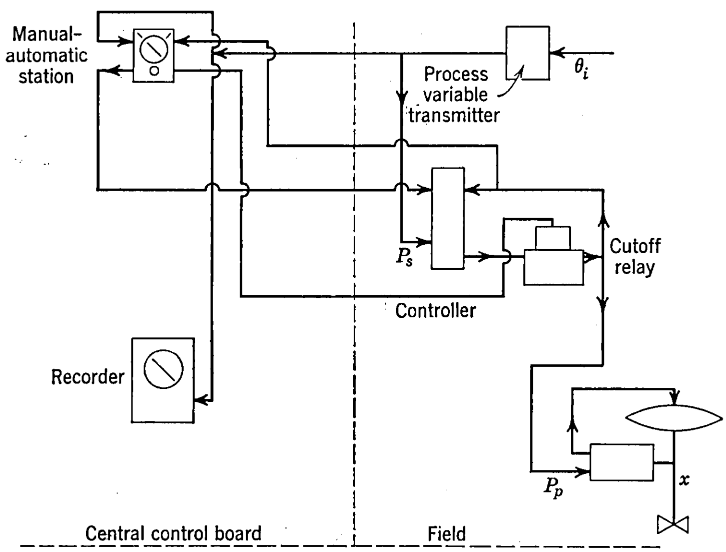


FIG. 12. Field-mounted controller in pneumatic control system.

instrument configuration may be cut into three segments for testing or for system design.

Transmitter Input to Controller Input. Here the transmitter is close coupled to the controller; the distance is usually less than 25 ft. A branch line, however, goes from the transmitter output back to the control board. This has the effect of lowering the transmitter load impedance. The use of a 1:1 pneumatic relay located in the branch line close to the transmitter output (see Fig. 13) isolates the branch, raises the

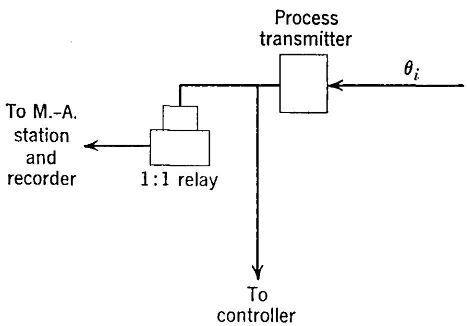


FIG. 13. Isolation of transmitter output line to main control board.

transmitter load impedance, and speeds up signal transmission to the controller. The frequency response of a typical transmitter plus a short line with and without a branched load is shown on Fig. 14 (Ref. 4).

Controller Input to Valve Input. The controller is close-coupled to the valve input with a branch line from the cutoff relay output back to the main control board. The controller load impedance may be raised by isolating the branch line with a 1:1 relay (cannot be used if system ever is to be put on manual control), by using a small diameter branch line, or by putting a restriction in the branch line (see Fig. 15). The frequency response of a typical proportional controller plus a short line with and without a branched load is shown on Fig. 16 (Ref. 4). Again, the effect of adding automatic reset and derivative action is shown in Sect. 2, Control Functions.

Valve Input to Valve Stem Position. The same considerations apply here as for the board mounted controller.

Factors Affecting Pneumatic System Performance

Transmission Line Diameter. The most commonly used pneumatic transmission line is ¼ in. o.d. (0.180 in. i.d.). The use of ⅜-in. o.d. (0.305-in. i.d.) tubing improves speed of response but sharply lowers the device load impedance. For some devices with low pilot valve capacity

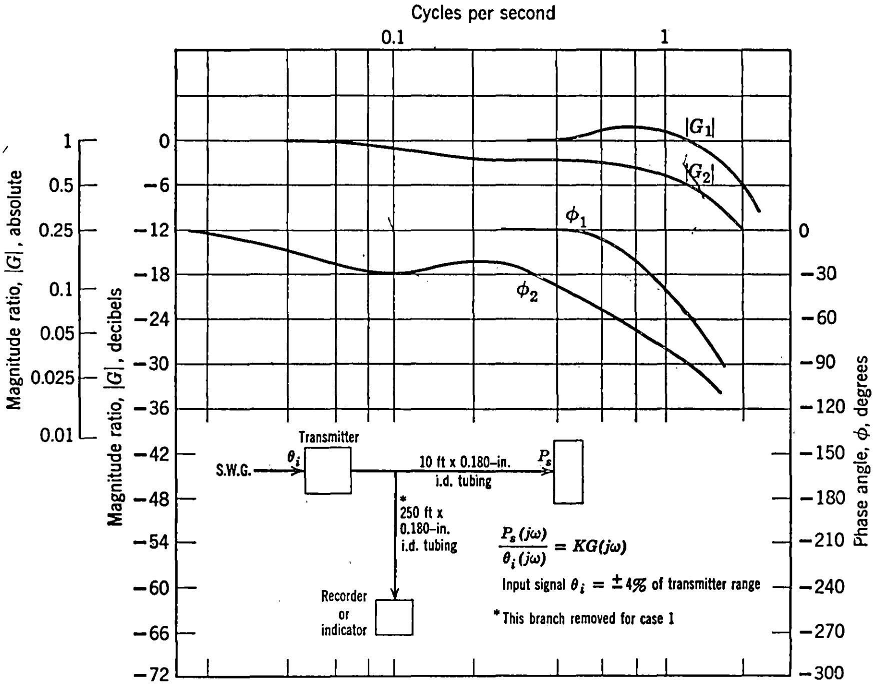


FIG. 14. Frequency response of typical pneumatic transmitter plus short line: (1) with short line load only; (2) with short line plus long branched line.

(high internal impedance) this is equivalent to a short circuit, and distortion and poor performance result.

Length of Transmission Line and Branched Loads. For reasons which are discussed earlier it is not easy to draw simple generalizations about the effects of either transmission line length or branched loads. As line length increases, the input impedance approaches the characteristic impedance of the line. For line lengths of 250 ft and greater there is little change in the loading of the device driving the line. As line length approaches zero, the input impedance approaches the line termination impedance. Generally, speed of transmission is proportional to line length. See Figs. 20 and 28 in Sect. 6.

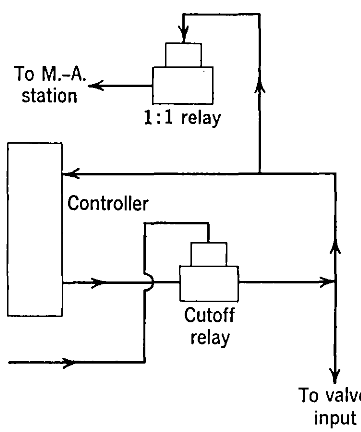


FIG. 15. Isolation of controller output line to main control board.

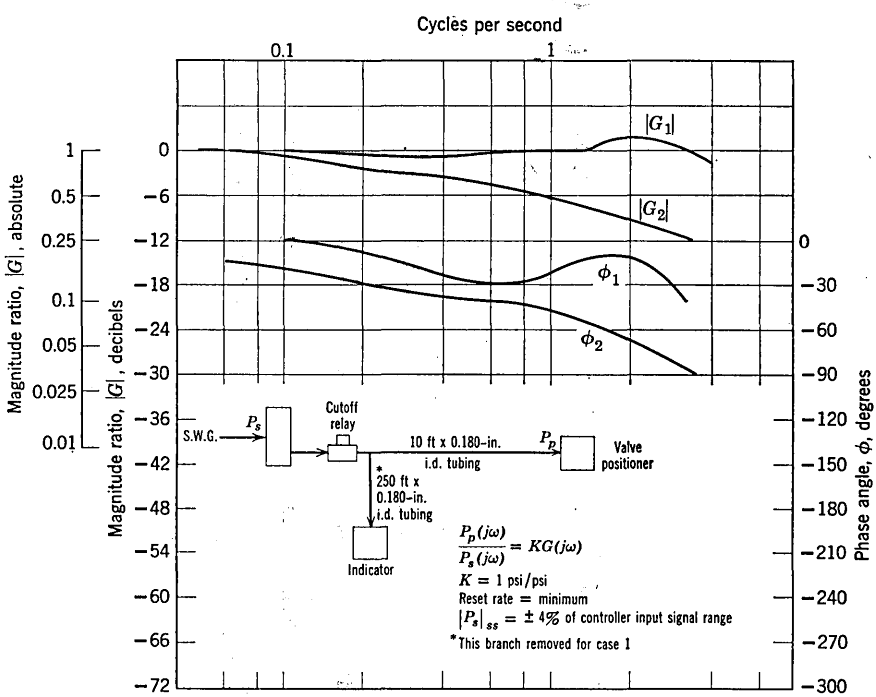


FIG. 16. Frequency response of typical pneumatic controller plus short line: (1) without branched load; (2) with branched load.

Branched loads, as shown by Figs. 14 and 16, can have quite a detrimental effect on performance.

Impedance of Transmission Line Termination. The termination is almost always a volume—pure capacitance. For very small volumes the transmission line acts as though it is terminated in an open circuit, while for large volumes, such as the dome of a spring-and-diaphragm valve, the line may act as though it is short-circuited.

Signal Level. The Instrument Society of America and the Scientific Apparatus Manufacturers Association have standardized on pneumatic transmission systems with a range of 3–15 psig. Additional ranges of 3–27 psig and 6–54 psig have recently been made standard. The higher the signal level, the faster is transmission.

Signal Amplitude. Most pneumatic devices are decidedly nonlinear in a dynamic sense. For very small input signals, say less than 0.1 psi peak to peak, the effects of internal stiction and hysteresis are noticeable. For signals of 0.5–1.0 psi peak to peak most devices are fairly linear provided they are terminated with a high impedance. Signals much above 1.0 psi peak to peak usually cause saturation and clipping; also a shift in the output d-c level occurs because of unbalanced action of the loading and exhaust pilot valves.

Air Supply Impedance. If the pneumatic power supply has high internal impedance—and this is often the case—device and system performance are affected adversely. The usual trouble is that the supply line is too long or too small, but sometimes the supply regulator cannot supply enough air. Generally speaking, the supply regulator should have an impedance of no greater than 0.1 psi/scfm over the desired airflow range. Instrument header pressure is commonly 46–60 psig, which means supply lines to and from the regulator should not be less than 3/8 in. o.d. (0.305 in. i.d.), and the line from the regulator to its load should not be more than 10 ft long. If more than one device is to be supplied from a single regulator it may be necessary to "decouple" each device by insertion of a volume (1 cu ft or greater) in the supply line just ahead of the device. For devices such as boosters or positioners which require lots of air but do not require regulated air pressure, it is best to omit the supply regulator and connect the device directly to the air header by a line at least 3/8 in. o.d. (0.305 in. i.d.).

Nature of Restrictions in Transmission Systems. An increasing number of pneumatic systems use plug-in connectors with internal check valves to prevent air leakage during disconnects. These connectors and the pneumatic switches in the manual-automatic station sometimes have such small openings relative to the tubing cross-sectional area that effectiveness of transient signal transmission is seriously reduced.

4. ELECTRIC AND ELECTRONIC CONTROL SYSTEMS

Unlike pneumatic control systems, electric and electronic control systems do not fall into well-defined patterns. Both a-c and d-c transmission systems are used, and manufacturers employ a wider variety of components and techniques to accomplish given measurement and control functions than do manufacturers of pneumatic gear. The use of impedance-matching techniques is standard and system performance is not as critically dependent on component location and arrangement as for pneumatic systems. Usually the electric system is not, however, entirely electric. The valve-actuating mechanism is most often pneumatic, although self-contained electrohydraulic positioners are beginning to be used.

An interesting feature of present electric control systems is that they are analogs of conventional pneumatic systems. The inherent flexibility of electronics as exploited for computational and control functions in aircraft and military applications has as yet not been employed in process control systems.

A typical electronic process control system is shown on Fig. 17. At

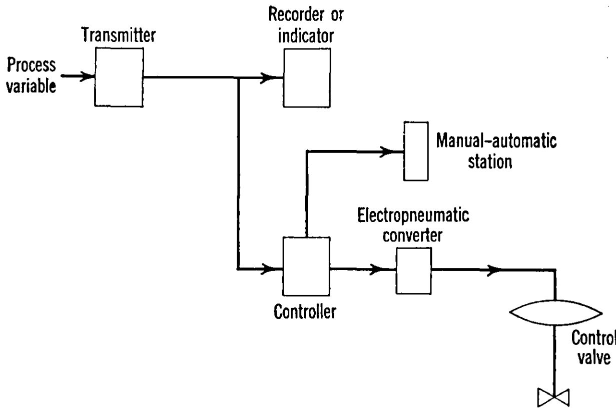


FIG. 17. Typical electronic process control system.

present there is no uniformity among manufacturers as to the mode of signal transmission. Swartwout uses a hybrid transmission system: (*a*) ac from the transmitter to the controller, (*b*) dc from the controller to the electropneumatic converter, and (*c*) pneumatic transmission from the converter to the valve. Robertshaw-Fulton uses all d-c transmission, and the controller output normally goes to an electropneumatic or electrohydraulic positioner rather than to an electropneumatic converter.

5. HYDRAULIC CONTROL SYSTEMS

Hydraulic controls have been most commonly used for heavy duty service in steel mills, coke ovens, etc. They are characterized by extremely durable construction for long, maintenance-free service under difficult operating conditions. Usually they are located in the field rather than in a central control room. Typical applications include turbine and engine control, flow and pressure control, gas holder level control, and gas mixing control. Hydraulic control systems are not usually divided into separate components such as transmitters and controllers, but commonly have many of these functions combined in one apparatus.

A hydraulic flow control system made by General Precision Equipment is shown on Fig. 18. The differential-pressure detector, set point, summing circuit, and hydraulic amplifier constitute a regulator package. When used in conjunction with a piston actuator as shown, the total comprises a system with integral (floating) control action.

6. PNEUMATIC COMPONENTS

Pneumatic Transmission Systems

Pneumatic Circuit Elements. Pneumatic circuits may be analyzed and designed in a manner analogous to electric circuits except that careful

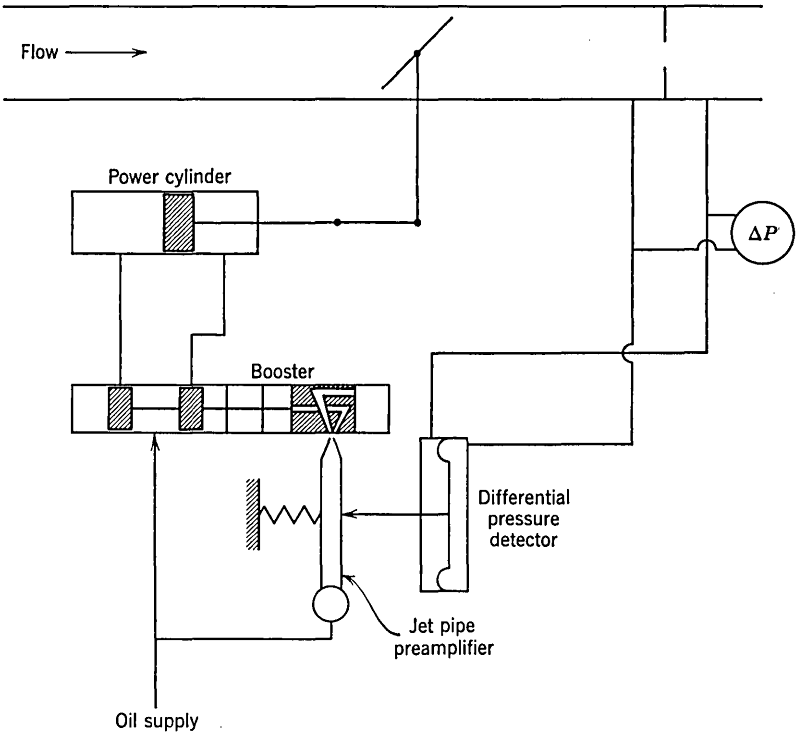


FIG. 18. Typical GPE hydraulic flow control system.

attention must be given to nonlinear and distributed effects (Refs. 10 and 11). In the ensuing discussion nonlinearities are linearized by standard methods.

Pneumatic Capacitance.

a. Pneumatic capacitance is defined by the expression

$$\frac{P(s)}{Q(s)} = \frac{1}{Cs}$$

where s = Laplace transform variable,
Q = air flow, ft^3/sec,
P = air pressure, lb/ft^2 abs,
C = capacitance, ft^5/lb.

For a simple volume filled with air,

$$C_{\text{adiabatic}} = \frac{0.720\,V}{P_{\text{av}}}$$

where V = volume, ft^3,

$$C_{\text{isothermal}} = \frac{V}{P_{\text{av}}}.$$

b. If the volume is not simple but has some additional form of compliance associated with it, the expression above must be modified. Consider, for example, the capacitance of a bellows or topworks of a spring-and-diaphragm valve:

$$Q(s) = s\left(C + \frac{A^2}{K}\right) P(s)$$

where A = average cross-sectional area of bellows or diaphragm, ft^2,
K = spring constant, lb/ft,
C = capacitance due to average volume.

c. The capacitance per foot of ¼-in. o.d. (0.180-in. i.d.) tubing is

$$C' = \frac{0.000177}{P_{av}} \text{ (ft}^5\text{/lb)/ft} \quad \text{(isothermal value).}$$

The capacitance per foot of ⅜-in. o.d. (0.305-in. i.d.) tubing is

$$C' = \frac{0.000506}{P_{av}} \text{ (ft}^5\text{/lb)/ft.}$$

If the tubing does not have rigid walls, as for example if rubber or plastic tubing is used, it may be necessary to add in the effect of the compliance of the tubing. Then

$$C' = V'\left(\frac{1}{P_{av}} + \frac{D}{bE}\right) \text{ (ft}^5\text{/lb)/ft}$$

where V' = volume per ft of tubing, ft^3/ft,
P_{av} = average pressure, lb/ft^2 abs,
D = mean diameter of tubing, ft,
b = tubing wall thickness, ft,
E = bulk modulus of tubing wall material, lb/ft^2.

Pneumatic Inertance. Inertance is the hydraulic or acoustic analog of inductance. It may be defined as follows for tubing:

$$P(s) = sLQ(s)$$

where

$$L = \frac{l\rho}{32.2A}, \text{ lb sec}^2\text{/ft}^5,$$

where l = tubing length, ft,
ρ = density of air, lb/ft^3,
A = tubing cross-sectional area, ft^2.

For ¼-in. o.d. (0.180-in. i.d.) tubing

$$L' = 6.18 \times 10^{-3} P_{av} \text{(lb sec}^2\text{/ft}^5\text{)/ft.}$$

For $\frac{3}{8}$-in. o.d. (0.305-in. i.d.) tubing

$$L' = 2.15 \times 10^{-3} P_{av} \text{ (lb sec}^2/\text{ft}^5)/\text{ft}.$$

Pneumatic Resistance. Pneumatic resistance for tubing or valves may be defined as

$$R = \left.\frac{\partial P}{\partial Q}\right|_{Q=Q_{av}} \text{ lb sec/ft}^5.$$

Laminar Flow. For $\frac{1}{4}$-in. o.d. (0.180-in. i.d.) tubing,

$$R' = \frac{R}{l} = 301 \text{ (lb sec/ft}^5)/\text{ft}.$$

For $\frac{3}{8}$-in. o.d. (0.305-in. i.d.) tubing,

$$R' = \frac{R}{l} = 36.6 \text{ (lb sec/ft}^5)/\text{ft}.$$

Plus-Minus Flow. Most pneumatic equipment operates in such a fashion that the average or steady-state flow through the pneumatic transmission line is zero. This is necessarily so since the load is usually a bellows, spring-and-diaphragm topworks, or some other purely reactive load. If we assume that a large sine wave of pressure is applied at the input of the transmission line, then within a half-cycle flow will be laminar, transitional, turbulent, transitional, and laminar. The pattern of this flow, which we have chosen to call "plus-minus" flow, negates use of the techniques commonly employed for linearizing resistance when the amplitude of flow variations is small compared to the average flow. If, of course, the amplitude of the driving pressure sine wave is never large enough to force the flow out of the laminar range, then the value for laminar flow resistance may be used.

To handle this problem it is necessary to take into account the magnitude of the driving signal and the length of tubing. Figure 19 presents a correlation which has been found to give fairly good results. Strictly speaking, however, it is necessary by trial-and-error calculation to find out at each frequency of driving signal just how much of the signal appears across the resistance in the line. This value of pressure drop per foot is then used on Fig. 19. At very low frequencies the flow will be laminar throughout the line. At high frequencies there will be turbulence at the inlet to the line which, however, will die down within a short distance so that overall flow may be treated as laminar.

Pneumatic Transmission Lines. *Simplified Theory for Short Lines.*

a. Low-Impedance Termination. It often happens that a pneumatic line is terminated by a large capacitance such as that due to the top-

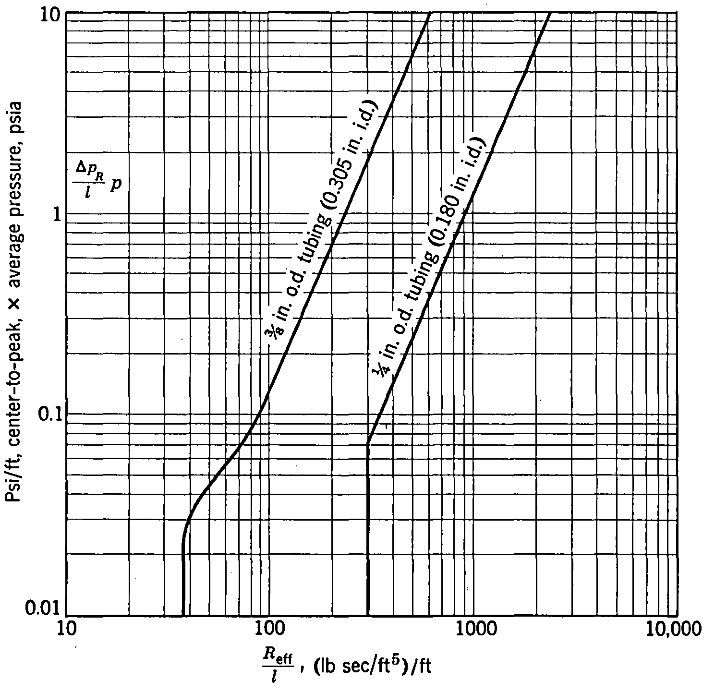
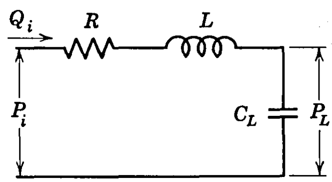


FIG. 19. Effective resistance of air lines at 25°C at any pressure level.

works of a valve. In this event the capacitance of the line may be neglected as being small by comparison. The equivalent circuit is then



R = total line resistance
L = total line inertance
C_L = capacitance of load

The output-input pressure ratio is

$$\frac{P_L(s)}{P_i(s)} = \frac{1}{LC_Ls^2 + RC_Ls + 1}.$$

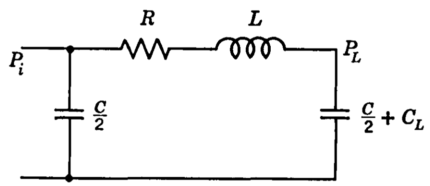
The driving point impedance is

$$Z_i = R + Ls + \frac{1}{C_Ls} = \frac{LC_Ls^2 + RC_Ls + 1}{C_Ls}.$$

This analysis gives rough checks with experimental data for lines from 5 to 10 ft in length which are terminated by the topworks of a spring-and-diaphragm valve. For longer lines it gives a fair check for amplitude but not phase.

b. High-Impedance Termination. It is often true that a pneumatic line is terminated by a high impedance, such as the input chamber of a controller or the input bellows of a valve positioner.

The equivalent circuit is



where C = capacitance of line. The output-input pressure ratio is

$$\frac{P_L(s)}{P_i(s)} = \frac{1}{L\left(\frac{C}{2} + C_L\right)s^2 + R\left(\frac{C}{2} + C_L\right)s + 1}$$

and the driving-point impedance is

$$Z_i = \frac{L\left(\frac{C}{2} + C_L\right)s^2 + R\left(\frac{C}{2} + C_L\right)s + 1}{\left(\frac{C}{2} + C_L\right)s + \frac{C}{2}s\left[L\left(\frac{C}{2} + C_L\right)s^2 + R\left(\frac{C}{2} + C_L\right)s + 1\right]}.$$

This has been found to give excellent results, both for magnitude and phase, for lines up to 30 ft in length with small volume terminations.

Distributed Treatment of Pneumatic Transmission Line. Because of distributed capacitance, a long pneumatic line and its load cannot be treated as a lumped circuit. Instead, it must be handled as a distributed system, and one may employ an analysis very similar to that of electric transmission lines. Since a full treatment of this subject is well covered in electrical engineering literature, remarks here will be restricted to a brief discussion of transmission line parameters and important transmission line relationships in terms of frequency response.

Let us consider first the transmission line parameters:

a. Series Impedance, Z_s

$Z_s = R' + j\omega L'$ = complex series impedance per unit length of line (L' and R' are on a per foot basis).

b. Shunt Admittance, Y_s

$Y_s = G + j\omega C'$ = complex shunt admittance per unit length of line (C' and G, the conductance, are on a per foot basis).

c. Propagation Factor, γ

$$\gamma = \sqrt{Z_s Y_s} = \sqrt{(R' + j\omega L')(G + j\omega C')}$$ For fluids in conduits with impervious walls $G = 0$.

d. Characteristic Impedance, Z_k

$$Z_k = \sqrt{\frac{Z_s}{Y_s}} = \sqrt{\frac{R' + j\omega L'}{G + j\omega C'}}$$

At low frequencies, the ratio of Z_k for ¼-in. o.d. (0.180-in. i.d.) tubing to Z_k for ⅜-in. o.d. (0.305-in. i.d.) tubing is 4.9; at high frequencies it is 2.9.

From these parameters, certain important transmission line relationships may be derived:

a. Driving Point Impedance, Z_i

$$Z_i = Z_k \frac{Z_L + Z_k \tanh \gamma l}{Z_k + Z_L \tanh \gamma l}$$

where Z_L = transmission line termination or load impedance,
l = line length, ft.

b. Output-Input Pressure Ratio, P_L/P_i

$$\frac{P_L}{P_i} = \frac{1}{\dfrac{Z_L + Z_k}{2Z_L} e^{\gamma l} + \dfrac{Z_L - Z_k}{2Z_L} e^{-\gamma l}}$$

$$= \frac{1}{\cosh \gamma l + \dfrac{Z_k}{Z_L} \sinh \gamma l}$$

c. Output Pressure–Input Flow Ratio, P_L/Q_i

$$\frac{P_L(j\omega)}{Q_i(j\omega)} = \frac{2Z_L Z_k}{(Z_L + Z_k)e^{\gamma l} - (Z_L - Z_k)e^{-\gamma l}}$$

$$= \frac{Z_k Z_L}{Z_k + Z_L \tanh \gamma l \cosh \gamma l}$$

Pneumatic Transmission Line Data. The frequency response of several lengths of ¼ in. o.d. (0.180 in. i.d.) is shown on Fig. 20. A comparison between the frequency response of ¼-in. o.d. and ⅜-in. o.d. tubing for 200 ft. of tubing is also shown.

Optimum Transmission of Pneumatic Signals. Consider pneumatic transmission systems such as shown in Figs. 21 and 22. Optimum trans-

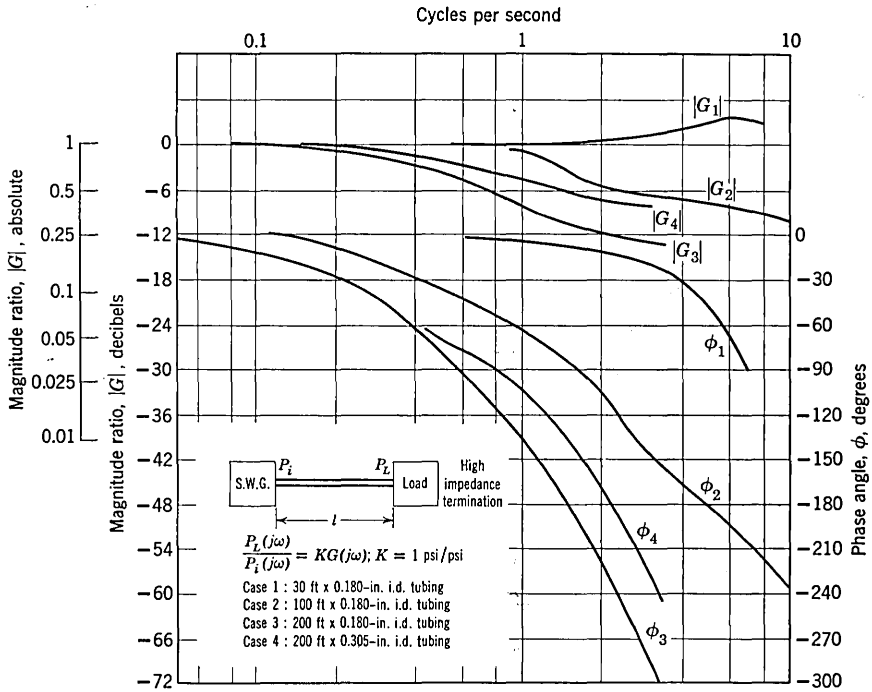


FIG. 20. Frequency response of pneumatic transmission lines.

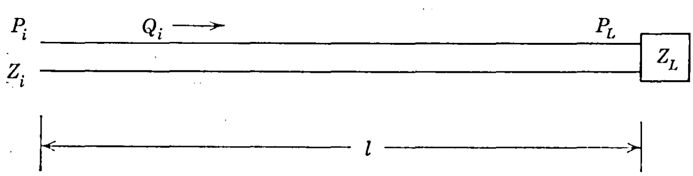


FIG. 21. Passive transmission system.

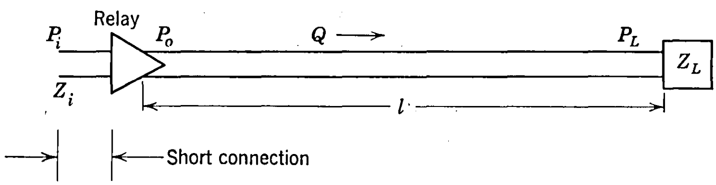


FIG. 22. Transmission system with active elements.

mission is obtained when $P_L(j\omega)/P_i(j\omega) = 1$ and phase shift is zero over the widest possible range of frequencies. It is not possible to accomplish these objectives over an infinite range of frequencies, but much can be accomplished by suitable modification of a "passive" transmission system or by insertion of "active" elements or both, as shown by the following sections.

Passive Transmission System. It is apparent by inspection of Fig. 21 that if P_L/P_i is to be constant over a wide range of frequencies, then Z_L must be very much larger than the impedance looking back from Z_L over the range of frequencies of interest. This may be accomplished by (a) lowering the impedance between P_i and P_L and (b) raising Z_L. By referring to the transmission line relationships above we note that

$$\frac{P_L}{P_i} = \frac{1}{\cosh \gamma l + (Z_k/Z_L) \sinh \gamma l}$$

$$= \frac{Z_L}{Z_L + Z_k \tanh \gamma l} \times \frac{1}{\cosh \gamma l}$$

and
$$Z_i = Z_k \frac{Z_L + Z_k \tanh \gamma l}{Z_k + Z_L \tanh \gamma l}$$

$$Z_k = \frac{\sqrt{R' + j\omega L'}}{j\omega C'}; \quad \gamma = \sqrt{(R' + j\omega L')j\omega C'}.$$

It may be seen that $P_L/P_i \to 1$ as $Z_k \tanh \gamma l \to 0$ and as $\cosh \gamma l \to 1$. This will be brought about, however, only if γl can be made to approach zero. If it is assumed that line length is fixed, then γ would have to approach zero. If, now, the tubing diameter is increased so that $R' \to 0$, then $Z_k \to \sqrt{L'/C'}$, $\gamma \to j\omega\sqrt{L'C'}$, and $\tanh \gamma l \to j\omega l\sqrt{L'C'}$. Increasing line diameter can therefore accomplish only so much; Z_L must be increased.

Since Z_L is usually of the form $1/C_L j\omega$, it is necessary to reduce the terminal volume or mechanical compliance or both. The input to a valve positioner, booster, or controller usually has as small a volume as is practical and accomplishes this end. A valve topworks, on the other hand, usually has a rather large volume. In this case, Z_L may be increased greatly by use of a booster or valve positioner. Consider now what happens to Z_i as $R' \to 0$ and $Z_L \to \infty$. In this event

$$Z_i \to Z_k \frac{1}{\tanh \gamma l} \to \sqrt{\frac{L'}{C'}} \times \frac{1}{j\omega l\sqrt{L'C'}} = \frac{1}{j\omega l C'}.$$

Since C' increases as R' decreases, the net effect is to lower Z_i.

If, now, Z_i is being driven by a transmitter or controller, the effect of improving the transmission characteristics of a line and its load is a tendency to short-circuit the driving device. It might seem at first thought that this is an unfortunate state of affairs, but so far the transmission system has been treated as composed only of passive elements.

Transmission System with Active Elements. If, as shown by Fig. 22, an active network such as a booster relay is inserted close to P_i, this raises Z_i to a large value. If the booster is so designed that it has a static gain $\left|\frac{P_o}{P_i}\right|_{ss} = 1$ and if its dynamic gain G could be so designed that

$$\left|\frac{P_o}{P_i}\right| \times \left|\frac{P_L}{P_o}\right| = \left|\frac{P_L}{P_i}\right| = 1$$

over a wide band width, then in effect transmission line lag would be largely canceled out (Refs. 4 and 15). It is shown under General Pneumatic Transmitter Design how this may be accomplished.

General Pneumatic Transmitter Design (Ref. 4)

The general transmitter or relay (see Fig. 23) contains the following elements:

1. *Comparison circuit* where the input signal is compared to the feedback signal to produce the error signal.
2. *Error amplifier circuit* which is usually a flapper-nozzle combination.
3. *Power amplifier* which is usually a spring-loaded, three-way, pilot valve.
4. *Feedback circuit* which provides a path for the signal at the immediate transmitter output to feed back to the comparison circuit.
5. *The load,* which consists of tubing, single or branched paths, valves or other restrictions, and terminations.

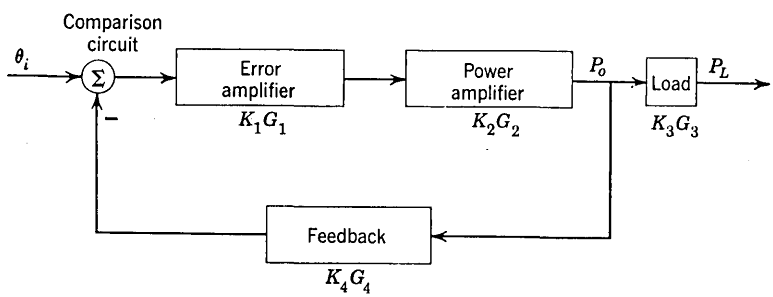


FIG. 23. Signal flow diagram for general transmitter and load.

Some devices have only one stage of amplification. The Kendall booster, for instance, has no flapper-nozzle, whereas some flow transmitters have the flapper-nozzle but lack a power stage.

Relationship between Feedback and Transmission Line. The signal flow diagram for the general transmitter is shown on Fig. 23. The performance may be measured as the response, P_L, to an input signal, θ_i.

$$\frac{P_L}{\theta_i} = \frac{K_1G_1 \times K_2G_2 \times K_4G_4}{1 + K_1G_1 \times K_2G_2 \times K_4G_4} \times \frac{1}{K_4G_4} \times K_3G_3.$$

As a general rule, all dynamic elements within the device itself will be fast in comparison to the load. This has led to a widespread opinion that pneumatic transmission line lags constitute a fundamental limitation in pneumatic control systems. This is true, but, as shown below, it is not nearly as critical as commonly believed.

Improvement of Transmission System Response. Figure 24 represents a rearrangement of Fig. 23. It shows that the relationship between

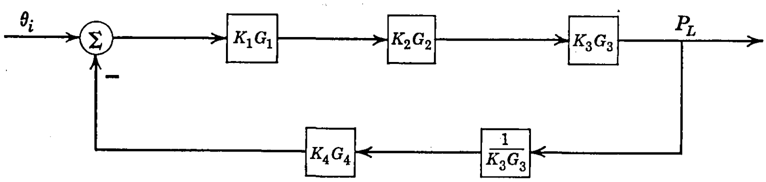


FIG. 24. Rearranged signal flow diagram for general transmitter and load.

θ_i and P_L may be represented by a single loop. P_L/θ_i is the same as in the first expression given above. If, now, the feedback dynamics K_4G_4 were made equal to the transmission line dynamics K_3G_3, the feedback path would contain no dynamics. Then

$$\frac{P_L}{\theta_i} = \frac{K_1G_1 \times K_2G_2 \times K_3G_3}{1 + K_1G_1 \times K_2G_2 \times K_3G_3}.$$

This is mathematically equivalent to closing the loop *around* the transmission line, which greatly enhances performance. An examination of some transmission line data indicates that by this method it should be possible to widen the bandwidth P_L/θ_i by 500 to 1000%. To achieve this, however, it would probably be necessary to increase greatly the pilot valve capacity of the average transmitter or relay.

Pilot Valve Design. Pilots used in pneumatic devices fall into two important classes: (*a*) three-way or bleed type and (*b*) dual valve or nonbleed type.

Three-Way Pilot Valve. A good example of this type is that used extensively by Taylor throughout their line of transmitters and controllers. This pilot is shown on Fig. 25. It may be visualized as the pneumatic equivalent of a potentiometer or voltage divider circuit. Motion of the pilot valve plug causes one resistance to increase while the other is de-

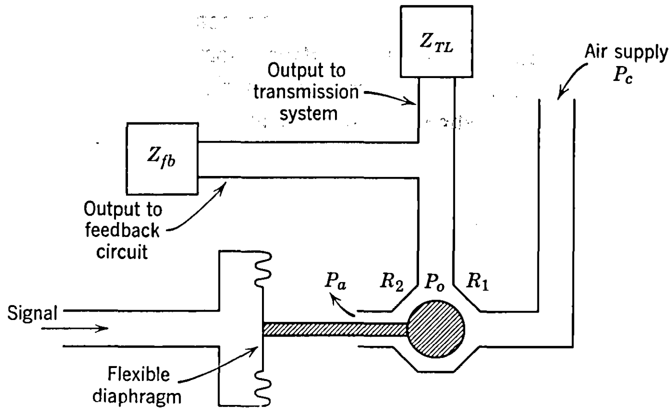


FIG. 25. Schematic diagram of three-way pilot valve.

creasing. By linearizing the supply and exhaust resistances R_1 and R_2, one obtains

$$\frac{P_o(s)}{R_2(s)} = \frac{P_c - P_a}{(R_1 + R_2)_{\text{av}}} \times \frac{X_{fb} \times Z_{TL}}{Z_{fb}Z_{TL} + k_2(Z_{fb} + Z_{TL})}$$

where P_c = supply pressure, lb/ft^2,
P_a = exhaust pressure, lb/ft^2,
Z_{fb} = transmitter feedback impedance, lb sec/ft^5,
Z_{TL} = transmission system input impedance, lb sec/ft^5,
k_2 = pilot valve impedance, lb sec/ft^5,
$$= \frac{(R_1)_{\text{av}}(R_2)_{\text{av}}}{(R_1)_{\text{av}} + (R_2)_{\text{av}}},$$
R_1 = supply valve resistance, lb sec/ft^5,
R_2 = exhaust valve resistance, lb sec/ft^5.

Note. At midscale,

$$\frac{P_o(s)}{R_2(s)} = \frac{P_o(s)}{R_1(s)}, \quad \text{and} \quad k_2 = 0.5(R_2)_{\text{av}} = 0.5(R_1)_{\text{av}}$$

If, as is usually the case, $Z_{fb} \gg Z_{TL}$, then

$$\frac{P_o(s)}{R_2(s)} = \frac{P_c - P_a}{(R_1 + R_2)_{av}} \times \frac{Z_{TL}}{Z_{TL} + k_2}.$$

To guarantee that P_o/R_2 would be independent of Z_{TL}, it would be necessary for $(R_2)_{av}$ and $(R_1)_{av}$ to approach zero. The pilot valve would then have a very high bleed rate. But this is economically unacceptable, so pilots of this type are therefore limited in air-handling capacity, and P_o/R_2 is dependent on Z_{TL}.

Dual Valve Pilot. To minimize air consumption under quiescent conditions and to provide high capacity under transient conditions some manufacturers use a dual valve or nonbleed pilot. An example is that used in the Kendall booster, Model 20, which is shown in schematic form on Fig. 26.

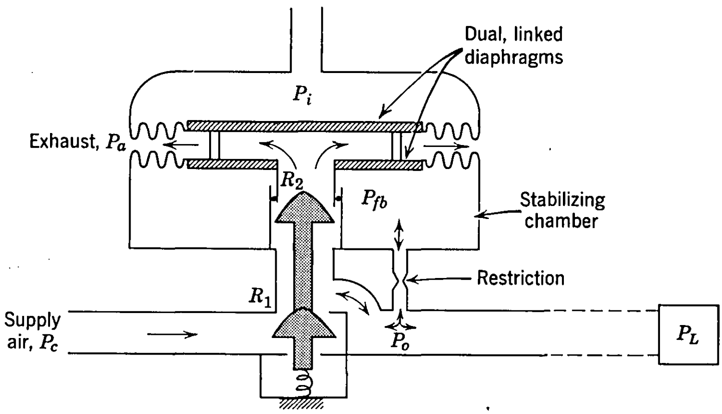


FIG. 26. Schematic diagram of Kendall booster, Model 20.

The action of this pilot differs significantly from that of the three-way valve type. Under quiescent conditions both valves are closed, or nearly so. Then in response to a signal the appropriate valve opens. This is accomplished by having one fixed valve seat, R_1, and a spring-loaded movable stem with valve plugs at each end, and a movable valve seat, R_2, at the opposite end of the stem from R_1.

During the loading half-cycle the exhaust valve is closed and R_2 is infinite, whereas during the exhaust half-cycle the supply valve is closed and R_1 is infinite. The action of the pilot is therefore quite nonlinear, but a linearized analysis nevertheless gives useful insight into pilot dynamics.

When the booster is loading,

$$\frac{P_o(s)}{R_1(s)} = -\left(\frac{P_c - P_o}{R_1}\right)_{av} \times \frac{Z_o}{(R_1)_{av} + Z_o}.$$

When the booster is exhausting,

$$\frac{P_o(s)}{R_2(s)} = +\left(\frac{P_o - P_a}{R_2}\right)_{\text{av}} \times \frac{Z_o}{(R_2)_{\text{av}} + Z_o}$$

where $Z_o = \dfrac{Z_{fb}Z_{TL}}{Z_{fb} + Z_{TL}}$.

It should be noted that for zero signal both R_1 and R_2 are infinite. The pilot gain, P_o/R_2 (or P_o/R_1), is therefore a function of signal size, since this affects R_2 (or R_1). For small signals, pilot gain is low, whereas for large signals it is high.

Commercial Pneumatic Transmitters. Commercial pneumatic transmitters fall into two general categories insofar as principle of mechanical design is concerned: (*a*) force balance and (*b*) non-force balance. In each category an input signal is detected as a force on a diaphragm or free end of a bellows or a bourdon tube. In the force-balance type any change in force due to a change in input signal is opposed by a force from the transmitter feedback circuit in such a way as to maintain the diaphragm or bellows in the same position. The required feedback force or pressure is then a measure of the input signal. This arrangement has several advantages: (*a*) it raises the transmitter input impedance, (*b*) it provides a linear output-input relationship, (*c*) it minimizes the effect of stiction and hysteresis in the input circuit, and (*d*) it provides greater sensitivity.

A schematic diagram of a typical force-balance transmitter is shown on Fig. 27. Not shown are zero and bias adjustments, damping circuits (such as dashpots), and range (gain) adjustments. The force beam and the nozzle constitute the comparison circuit and error amplifier, and the booster pilot is the power amplifier. The manner in which various signals are converted into input forces acting on the force beam will be shown in succeeding sections.

It may be shown that the overall dynamics of the transmitter may be represented by an expression of the form:

$$\frac{P_o(s)}{\theta_i(s)} = K_2 \times \frac{K_1G_1(s) \times [P_o(s)/R_2(s)] \times [P_{fb}(s)/P_o(s)]}{1 + K_1G_1(s) \times [P_o(s)/R_2(s)] \times [P_{fb}(s)/P_o(s)]} \times \frac{1}{[P_{fb}(s)/P_o(s)]}$$

where $K_1G_1(s)$ = dynamics of transmitter due to the mass and compliance of its moving parts. Usually, manufacturers make Z_{fb} much larger than

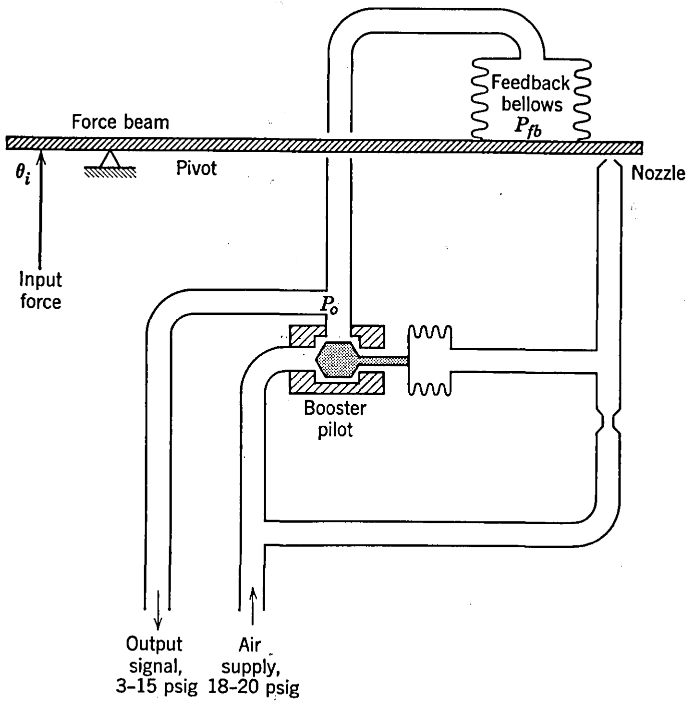


FIG. 27. Schematic diagram of typical force-balance transmitter.

Z_{TL} and $P_{fb}(s)/P_o(s) = 1$. Then

$$\frac{P_o(s)}{\theta_i(s)} = K_2 \times \frac{K_1G_1(s) \times \dfrac{P_c - P_a}{(R_1 + R_2)_{\text{av}}} \times \dfrac{Z_{TL}}{Z_{TL} + k_2}}{1 + K_1G_1(s) \times \dfrac{P_c - P_a}{(R_1 + R_2)_{\text{av}}} \times \dfrac{Z_{TL}}{Z_{TL} + k_2}}$$

The influence of pilot impedance, k_2, and Z_{TL} on transmitter performance is now apparent. If k_2 were made very small compared to Z_{TL}, transmitter performance would be independent of Z_{TL}. In practice, however, k_2 is usually quite appreciable relative to Z_{TL}. Since under field conditions Z_{TL} varies widely from installation to installation (due to different lengths of tubing, different diameter tubing, branched loads, etc.), this means that transmitter performance can be defined only in terms of specified values of Z_{TL}. The frequency response, $P_o(j\omega)/\theta_i(j\omega)$, of a typical pneumatic transmitter is shown on Fig. 28.

To obtain overall performance from transmitter input to controller input we must multiply $P_o(s)/\theta_i(s)$ by the transmitter load transfer function.

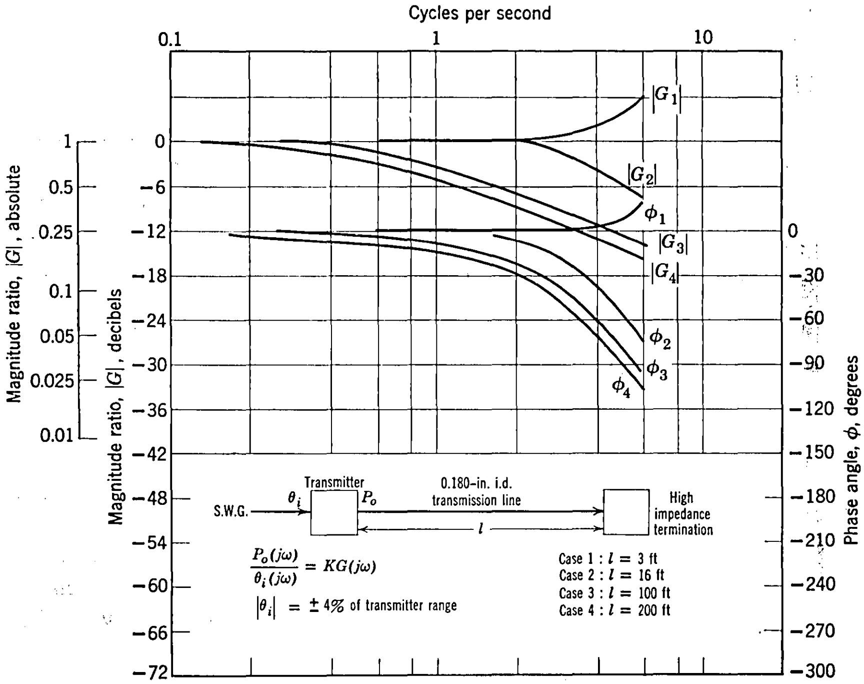


FIG. 28. Frequency response of typical pneumatic transmitter with various loads.

For transmission distances of 250 ft or over $P_L(s)/P_o(s)$ is usually so much slower than $P_o(s)/\theta_i(s)$ that the latter is negligible.

A basic method of compensation for transmission line lag is shown earlier in this section. From the general expression above for $P_o(s)/\theta_i(s)$, it may be seen that if

$$\frac{P_{fb}(s)}{P_o(s)} = \frac{P_L(s)}{P_o(s)},$$

then transmission line lag is largely canceled out. To do this it is necessary to make $Z_{fb} = Z_{TL}$. Then

$$\frac{P_L(s)}{\theta_i(s)} = \frac{P_o(s)}{\theta_i(s)} \times \frac{P_L(s)}{P_o(s)}$$

$$= K_2 \times \frac{K_1G_1(s) \times \dfrac{P_c - P_a}{(R_1 + R_2)_{\text{av}}} \times \dfrac{Z_{TL}}{Z_{TL} + 2k_2} \times \dfrac{P_L(s)}{P_o(s)}}{1 + K_1G_1(s) \times \dfrac{P_c - P_a}{(R_1 + R_2)_{\text{av}}} \times \dfrac{Z_{TL}}{Z_{TL} + 2k_2} \times \dfrac{P_L(s)}{P_o(s)}}$$

The effect of pilot impedance is now twice as great as it was before.

In the non-force-balance type, the motion of the diaphragm or bellows or bourdon tube is not nulled out but is detected by a flapper-nozzle circuit and converted into an equivalent output air pressure signal. The advantages of this type are (*a*) simpler construction, (*b*) lower price, and (*c*) occasionally more rugged construction.

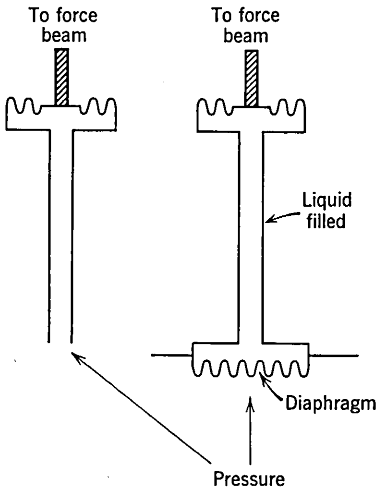


FIG. 29. Input circuits for force-balance pressure transmitters.

Pressure Transmitters. Two typical input circuits for force-balance pressure transmitters are shown on Figure 29. In the first, the pressure to be detected appears directly on the force diaphragm. The second is of the so-called volumetric type. The pressure is detected by a diaphragm of light construction and is transmitted hydraulically through a liquid-filled tube to the force diaphragm. This type has the advantage that the transmitter is isolated from process fluids which are corrosive or which would plug the nonvolumetric type.

Differential Pressure Transmitters. The commonest method of measuring flow in the process industries is by measuring pressure drop across an orifice. Differential pressure is also used as a measure of liquid level and specific gravity. A typical input circuit for a force-balance transmitter for these applications is shown by Fig. 30.

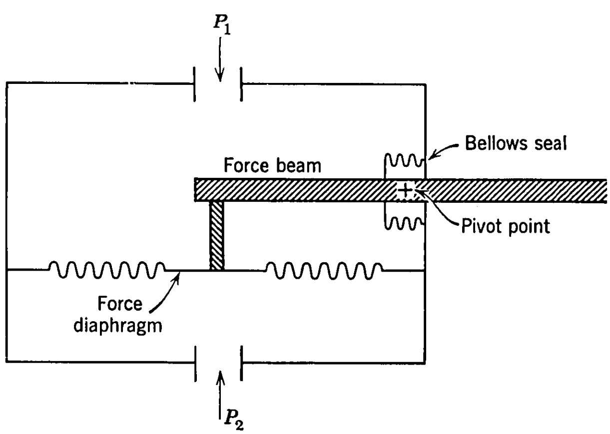


FIG. 30. Input circuit for force-balance differential pressure transmitter.

Most manufacturers make the central part of the force diaphragm rigid and provide shoulders or stops to prevent excess differential pressure from rupturing the diaphragm.

Temperature Transmitters. The input circuit for a force-balance, temperature transmitter is shown on Fig. 31. The bulb which is inserted into the process fluid or into a protective thermowell is of relatively stiff construction to minimize pressure errors. Gas is usually chosen as a filling medium. A bulb for this service may be ⅜ in. or ½ in. in outside diameter and several inches long. Bulb dynamics may be approximated by

$$\frac{T_a(s)}{T_o(s)} = \frac{1}{\tau s + 1}$$

where T_a = gas temperature in bulb,
T_o = temperature of medium outside bulb,
$\tau = VPk_g/hA$, min,
V = volume of gas in bulb, ft^3,
ρ = gas density, lb/ft^3,
k_g = gas specific heat, pcu/lb °C or Btu/lb °F,
A = bulb area, ft^2,
h = coefficient of heat transfer, outside medium to gas in bulb, pcu/°C ft^2 min or Btu/°F ft^2 min.

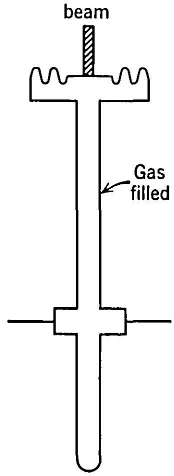


FIG. 31. Input circuit for force-balance temperature transmitters.

If the bulb is inserted in a thermowell, the above formula will still work fairly well if care is taken to provide good contact between bulb and thermowell. Crimped metal foil serves well for this purpose.

Since $h \sim v^{0.8}$ where v is velocity of outside medium past the bulb, the bulb should always be inserted into the process at a point of high fluid velocity if this can be done without introducing excessive dead time (velocity-distance lag).

Liquid Level Transmitters. Differential pressure transmitters are sometimes used to measure level, particularly where good accuracy is required as, for example, inventory measurements. Level is more commonly measured, however, by displacement type transmitters or ball-float transmitters, both of which are of the non-force-balance design.

Displacement Type Level Transmitter (see Fig. 32). Displacement type transmitters are usually sold and used as proportional controllers rather than as transmitters. They commonly have both set-point and proportional band adjustments. Sometimes they have dual pilots, one of which is used as a controller, the other as a transmitter. Typical ranges run from 14 in. to 7 ft or more.

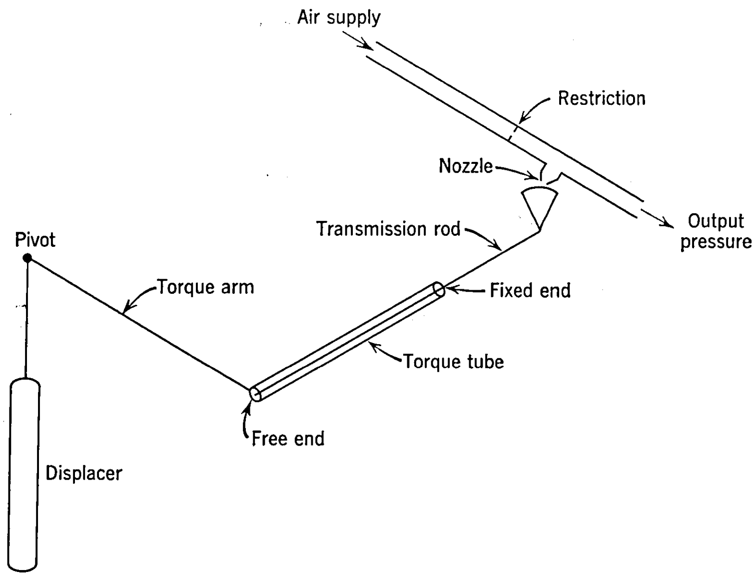


FIG. 32. Schematic diagram of displacement type level transmitter.

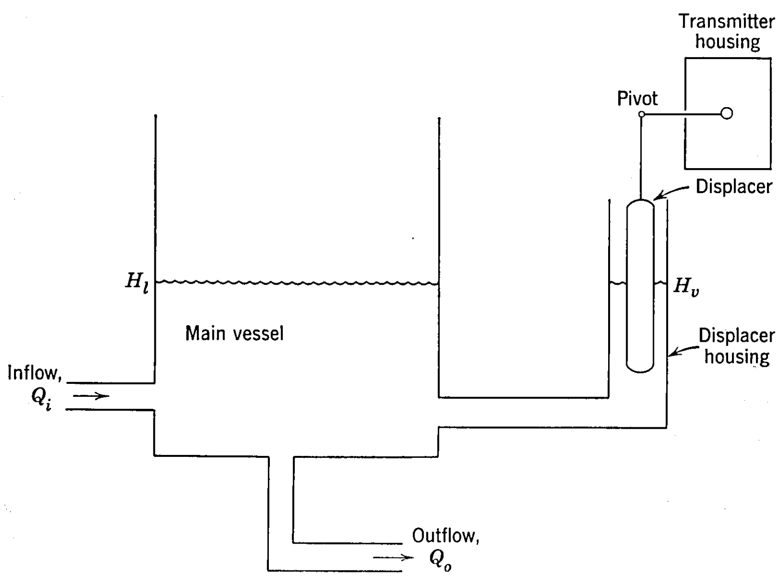


FIG. 33. Typical installation of displacement type level transmitter.

The principle of operation is as follows. A cylindrical displacer with a specific gravity somewhat heavier than that of water is partly immersed in the liquid whose level is to be measured. The displacer is suspended from a torque arm which is connected to a torque tube. Support for the displacer then comes partly from the buoyancy of the liquid and partly from the spring action of the torque tube. A rod inside the torque tube is fastened to its free end. An eccentric cam or lever on the other end of the rod acts as a flapper to a fixed nozzle. Angular motion on the free end of the torque tube is thereby converted to a proportional displacement between nozzle and flapper. This results in an output air pressure approximately proportional to liquid level. An adjustment of the nozzle position serves to shift the output air pressure range, and thereby functions as a set-point adjustment for automatic control.

Figure 33 shows a typical installation involving an external housing. It is important to note that there are two modes of resonance which the system designer must consider carefully. The first is that which exists between the two liquid levels, level in the main vessel, H_l, and level in the housing, H_v (Ref. 13). For most installations,

$$\frac{H_v(s)}{H_l(s)} = \frac{1}{\dfrac{L_T A_a}{\rho_l} s^2 + \dfrac{R_T A_a}{\rho_l} s + 1}$$

where L_T = total inertance, lb sec^3/ft^5, between the two liquid levels,
ρ_l = liquid density, lb/ft^3,
R_T = total hydraulic resistance, lb sec/ft^5, between the two liquid levels,
A_a = annular area, ft^2, between displacer and housing.

In practice, such installations are sometimes afflicted with severe resonance. The leg between the main vessel and the displacer housing must be kept as short as possible and it is occasionally necessary to insert resistance, usually in the form of a valve, to raise the damping ratio, $\dfrac{R_T}{2}\sqrt{\dfrac{A_a}{L_T \rho_l}}$, to a satisfactory value.

The other mode of resonance is that due to the mass of the displacer and the torque tube spring constant:

$$\frac{H_F(s)}{H_v(s)} = \frac{K_l}{K_l + K_{TT}} \times \frac{1}{\dfrac{M}{K_l + K_{TT}} s^2 + \dfrac{b}{K_l + K_{TT}} s + 1}$$

where H_F = displacer motion, ft,
M = displacer mass, lb sec^2/ft, = W_F/g,
b = viscous friction, lb sec/ft, between liquid and displacer,
K_{TT} = torque tube spring constant, lb/ft,
K_l = liquid "spring constant," lb/ft,
= $\rho_l\ (\pi D_F^2/4)$,
D_F = displacer outside diameter, ft.

When used with liquids whose specific gravity approaches that of water, commercial transmitters have resonant frequencies in the range of 2–3 cps and are underdamped, with damping ratios less than unity. When the pilot output is terminated in a low-impedance load such as the top-works of a large spring-and-diaphragm valve, this resonance may be fairly well damped out. If, however, the pilot output is terminated in a high-impedance load such as a valve positioner, resonance may be severe. In this event it may be necessary to remove it with a suitable filter circuit inserted in the output air line.

Ball-Float Transmitters. Where it is desired to measure or control level within narrow limits, say 0.5 in. or less, a ball-float transmitter is often used. This usually has a fixed output pressure range with an adjustment to the nozzle or flapper functioning as a set point.

One design (Moore Products Company) is packless and uses a flexible float arm (see Fig. 34). The analysis of this device is similar to that of the displacement type transmitter.

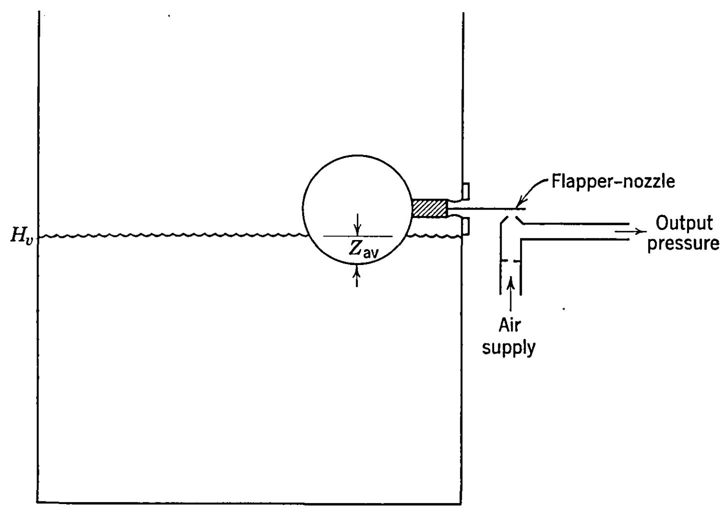


FIG. 34. Typical installation of ball-float level transmitter.

$$\frac{H_F(s)}{H_v(s)} = \frac{K_l}{K_l + K_{fa}} \times \frac{1}{\dfrac{M_F}{K_l + K_{fa}} s^2 + \dfrac{b}{K_l + K_{fa}} s + 1}$$

where K_{fa} = float-arm spring constant, lb/ft,
K_l = liquid spring constant, lb/ft,
$= \rho_l\, [\pi D_F Z_{\text{av}} - \pi Z_{\text{av}}^2]$,
Z_{av} = average submergence of float, ft,
D_F = diameter of float, ft.

Again, this type of device tends to be underdamped and it is sometimes necessary to insert an appropriate filter in the output air line.

Flow Transmitters. Flow is most commonly determined by measuring the pressure drop across a fixed orifice:

$$Q = kA\sqrt{\Delta P}$$

where A is the orifice area.

An alternate procedure, however, is to hold the pressure drop fixed by suitably varying the orifice area. The usual way of doing this is to insert a bob into a tapered, vertical tube (see Fig. 35*a*) such that the bob is supported by fluid entering at the bottom. As the flow increases, the bob moves upward, thereby increasing the annular area between the bob and tube. The pressure drop meanwhile remains constant. An instrument of this type is called a *rotameter* and has an advantage in that flow is linearly related to bob position.

The earliest versions of the rotameter had glass tubes and were used as indicating devices only. More recently, however, there has been a trend to various methods of transmitting bob position and a trend to metal tubes, particularly for high-pressure service. Although a number of electrical methods have been developed to detect bob position, the pneumatic transmitter has been more popular. One method, used by Fischer & Porter Company, provides an extension to the float. The top of this extension has mounted on it a magnet whose position is tracked by an external magnetic follower yoke (see Fig. 35*b*). This yoke is connected to a precision pneumatic circuit which converts position to a proportional air pressure.

Pneumatic Controllers

In order to show one example of how the control functions described under Continuous Control Functions, Sect. 2 might actually be achieved in a practical pneumatic controller, an analysis of the Taylor Tri-Act controller will be made. This example is based on the analysis developed by Bigliano (Ref. 1).

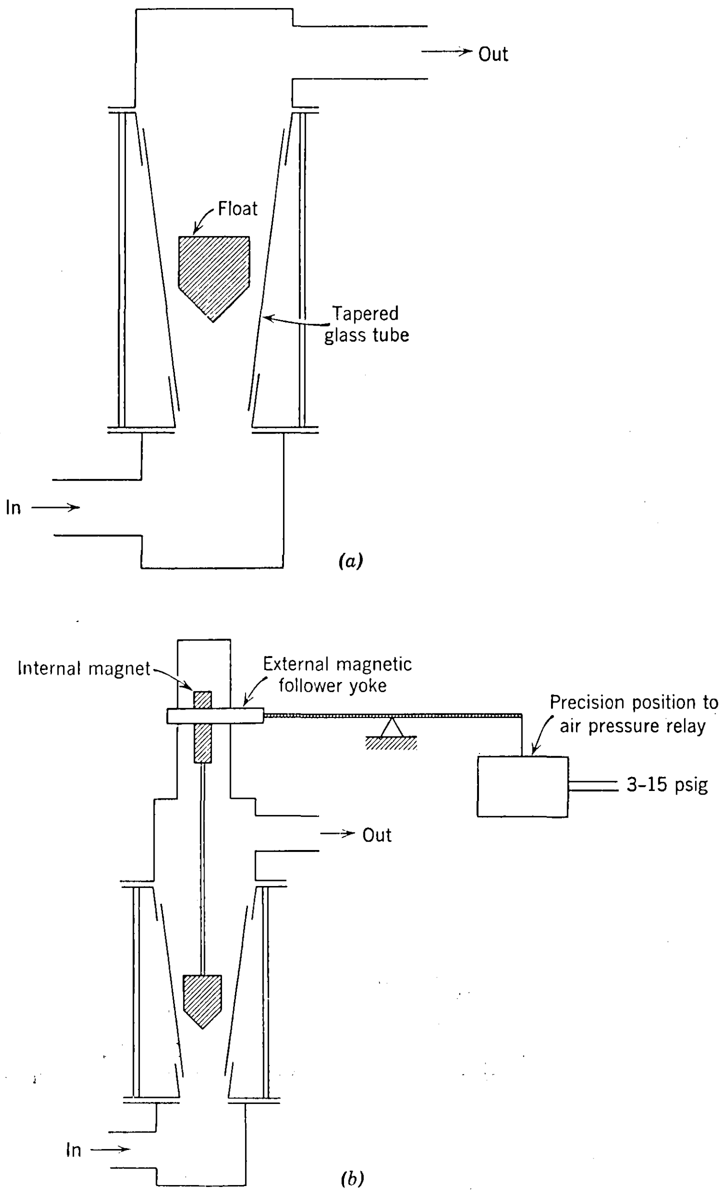


FIG. 35. Rotameter design: (*a*) indicating rotameter with glass tube; (*b*) transmitting rotameter with pneumatic output.

Starting with a cut-away pictorial drawing of the controller as shown in Fig. 36, the block diagram may be built up as given in Fig. 37. The symbols used are defined as follows:

$A_1 - A_2$ = effective diaphragm area at controller input, in.^{2},
k_d = spring constant of rate circuit, in./lb,
K_d = rate nozzle sensitivity, psi/in.,
R_1 = adjustable restriction in gain circuit,
R_2 = fixed restriction in gain circuit,
q_1 = feedback flow in gain circuit,
q_2 = input flow in gain circuit,
ΔP_1 = pressure change at input to gain stage, psi,
ΔP_2 = pressure change at output of gain stage, psi,
ΔP_g = pressure change at output of gain circuit, psi,
A_3 = area of diaphragm in gain circuit, in.^{2},
k_g = spring constant of gain circuit, in./lb,
K_g = gain nozzle sensitivity, psi/in.,
$A_3 - A_4$ = effective diaphragm area at input to reset circuit, in.^{2},
k_r = spring constant of reset circuit, in./lb,
K_r = reset nozzle sensitivity, psi/in.,
K_p = pilot relay sensitivity, psi/psi,
ΔP_E = controller input pressure, psi,
ΔP_d = output pressure of rate circuit, psi,
ΔP_o = controller output pressure, psi,
τ_d = rate circuit time constant, sec,
τ_r = reset circuit time constant, sec.

In this analysis it is assumed that no load is on the controller output and that each stage does not load the preceding one.

Rate Circuit

$$\frac{\Delta P_d}{\Delta P_E} = \left(\frac{k_d K_d}{1 + \dfrac{k_d K_d A_1}{\tau_d s + 1}}\right)(A_1 - A_2)$$

$$= k_d K_d (A_1 - A_2)\left[\frac{\tau_d s + 1}{\tau_d s + 1 + k_d K_d A_1}\right]$$

$$= \frac{k_d K_d (A_1 - A_2)}{1 + k_d K_d A_1}\left[\frac{\tau_d s + 1}{\left(\dfrac{\tau_d}{1 + k_d K_d A_1}\right)s + 1}\right].$$

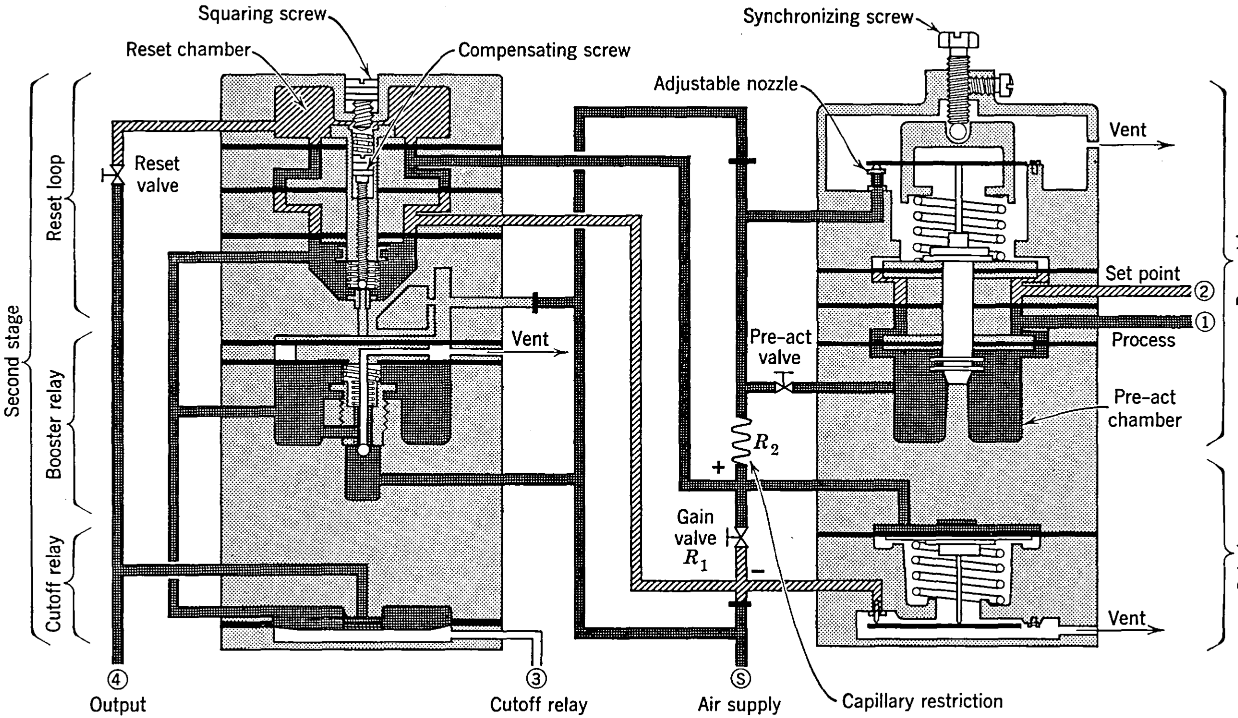


FIG. 36. Schematic diagram of Taylor Tri-Act controller. (Courtesy of Taylor Instrument Companies.)

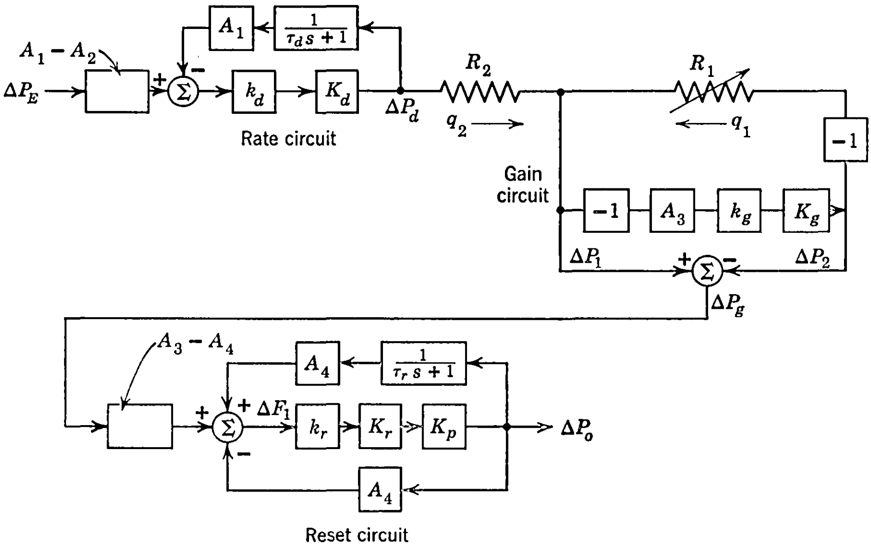


FIG. 37. Block diagram of Taylor Tri-Act controller.

According to manufacturers' data,

$$k_d = \tfrac{1}{170} \text{ in./lb,}$$

$$K_d = 6300 \text{ psi/in.,}$$

$$A_1 = \pi \text{ in.}^2,$$

$$A_2 = \pi/2 \text{ in.}^2,$$

$$\frac{\Delta P_d}{\Delta P_E} = \frac{(\pi/2)(37)}{1 + \pi(37)} \left[\frac{\tau_d s + 1}{\left(\dfrac{\tau_d}{1 + \pi(37)} \right) s + 1} \right]$$

$$= 0.5 \left[\frac{\tau_d s + 1}{(\tau_d/117)s + 1} \right].$$

Gain Circuit

$$\Delta P_1 = \Delta P_d - R_2 q_2,$$

$$-\Delta P_2 = A_3 k_g K_g \Delta P_1,$$

$$\Delta P_2 - \Delta P_1 = R_1 q_1,$$

$$-A_3 k_g K_g \Delta P_1 - \Delta P_1 = R_1 q_1,$$

$$q_1 = -\frac{\Delta P_1}{R_1} (A_3 k_g K_g + 1),$$

$$q_1 + q_2 = 0; \quad q_2 = -q_1,$$

$$\Delta P_1 = \Delta P_d - R_2 \left[\frac{\Delta P_1}{R_1} (A_3 k_g K_g + 1) \right],$$

$$\Delta P_1 \left[1 + \frac{R_2}{R_1} (A_3 k_g K_g + 1) \right] = \Delta P_d,$$

$$\Delta P_1 = \frac{\Delta P_d}{1 + (R_2/R_1)(A_3 k_g K_g + 1)},$$

$$\Delta P_2 = \frac{-A_3 k_g K_g \Delta P_d}{1 + (R_2/R_1)(A_3 k_g K_g + 1)},$$

$$\Delta P_g = \Delta P_1 - \Delta P_2 = \frac{\Delta P_d}{1 + (R_2/R_1)(A_3 k_g K_g + 1)} (1 + A_3 k_g K_g)$$

$$= \Delta P_d \frac{R_1(1 + A_3 k_g K_g)}{R_1 + R_2(A_3 k_g K_g + 1)}.$$

According to manufacturers' data:

$$k_g = \tfrac{1}{110} \text{ in./lb},$$

$$K_g = 35{,}000 \text{ psi/in.},$$

$$A_3 = 3 \text{ in.}^2,$$

$$\frac{\Delta P_g}{\Delta P_d} = \frac{R_1(1 + 950)}{R_1 + R_2(950 + 1)} = \frac{951 R_1}{951 R_2 + R_1}.$$

Reset Circuit

$$\Delta F_1 = \Delta P_g(A_s - A_4) + \Delta F_3 - \Delta F_2,$$

$$\Delta F_3 = \Delta P_o A_4 \frac{1}{\tau_r s + 1},$$

$$\Delta F_2 = \Delta P_o A_4,$$

$$\Delta P_o = k_r K_r K_p \Delta F_1,$$

$$\Delta F_1 = \frac{\Delta P_o}{k_r K_r K_p},$$

$$\frac{\Delta P_o}{k_r K_r K_p} = \Delta P_g(A_3 - A_4) + \frac{\Delta P_o A_4}{\tau_r s + 1} - \Delta P_o A_4,$$

$$\frac{\Delta P_o}{\Delta P_g} = (A_3 - A_4) \left[\frac{k_r K_r K_p(\tau_r s + 1)}{(1 + A_4 k_r K_r K_p)\tau_r s + 1} \right].$$

According to manufacturers' data:

$$A_3 = 2\pi/3 \text{ in.}^2,$$
$$A_4 = \pi/3 \text{ in.}^2,$$
$$k_r = \tfrac{1}{80} \text{ in./lb},$$
$$K_r = 15{,}000 \text{ psi/in.},$$
$$K_p = 1.0 \text{ psi/psi},$$
$$\frac{\Delta P_o}{\Delta P_g} = 196\left[\frac{\tau_r s + 1}{196\tau_r s + 1}\right]$$

Complete Controller

$$\frac{\Delta P_o}{\Delta P_E} = \left(\frac{\Delta P_d}{\Delta P_E}\right)\left(\frac{\Delta P_g}{\Delta P_d}\right)\left(\frac{\Delta P_o}{\Delta P_g}\right)$$
$$= 0.5\left[\frac{\tau_d s + 1}{(\tau_d/117)s + 1}\right]\left[\frac{951R_1}{951R_2 + R_1}\right]\left[\frac{196(\tau_r s + 1)}{196\tau_r s + 1}\right],$$
$$K_c = \text{Controller gain setting} = \frac{(0.5)(951R_1)}{951R_2 + R_1},$$
$$\frac{\Delta P_o}{\Delta P_E} = K_c \frac{196(\tau_r s + 1)(\tau_d s + 1)}{(196\tau_r s + 1)[(\tau_d/117)s + 1]}.$$

According to actual measurement, the maximum rate gain is 55 compared to the calculated value of 117, and the maximum reset gain is 200 compared to the calculated value of 196. This discrepancy is probably due to some uncertainty in the values of k_d, K_d, k_r, and K_r. Because of assumptions regarding loading, the transfer function derived here is not accurate at signal frequencies higher than 1 cps.

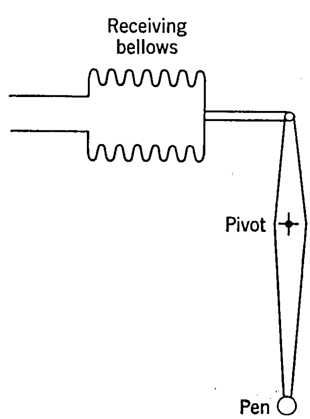


FIG. 38. Basic pneumatic recorder mechanism.

Pneumatic Recorders

Nearly all nonelectrical data recorders work on the same principle as shown on Fig. 38. The free end of a bellows, bourdon tube, or other receiving element is attached to a pivoted arm. The other end of the arm has a pen attached to it. Various mechanisms are used to linearize and adjust input-output motion ratio. Recorders fall into two general categories as far as type of chart is concerned, circular chart and strip chart. The former is usually of the 24-hour type whereas the latter may provide as much as two weeks to thirty days chart supply.

It should be noted that while the signal to the recorder is often a transmitted pneumatic signal, it may come directly from the process. The receiving element may, for example, be connected to process pressure by an impulse line or to the capillary tubing from a thermal element.

Pneumatic Control Stations

The trends toward unitized process instruments and smaller control panels has led to the development of compact control stations. These stations incorporate the following features: (*a*) manual-automatic switching, (*b*) indication or recording of process variable, (*c*) indication or recording of signal pressure to control valve, (*d*) control point adjustment, and (*e*) remote manual operation of control valve. This multiplicity of functions requires complex hardware, and the cost of modern control stations sometimes exceeds that of controllers.

A typical indicating control station is shown on Fig. 39. The control knob functions as a control point adjustment while the system is on automatic, and as a remote manual loader for the valve while the system is under manual control. The small index pointer normally functions as indication of the control point, but when the transfer switch in the upper left corner is in the valve position, it indicates signal pressure to the valve. On manual control, the index reads manual signal pressure to the valve.

To achieve "bumpless transfer," that is, manual-automatic switching without upsetting the process, it is necessary to have circuitry to provide a means for equalizing controller output pressure and manual signal pressure. When the manual-automatic switch is in the seal position, the valve pressure is sealed in. The control knob is then adjusted until the manual signal pressure is the same as the valve pressure. The system may then be switched to either manual or automatic without upsetting the process. It is important to note that in order to prevent the integral action of the reset circuit of the controller from driving the controller output to one extreme or the other while the system is on manual control, the valve signal pressure is continually fed back to the reset circuit.

It is also important to note that the controller output does not go directly to the valve, but first passes through the manual-automatic switch. In some manufacturers' stations the ports in this switch are so small and offer such high resistance as to seriously attenuate controller-valve transmission system performance.

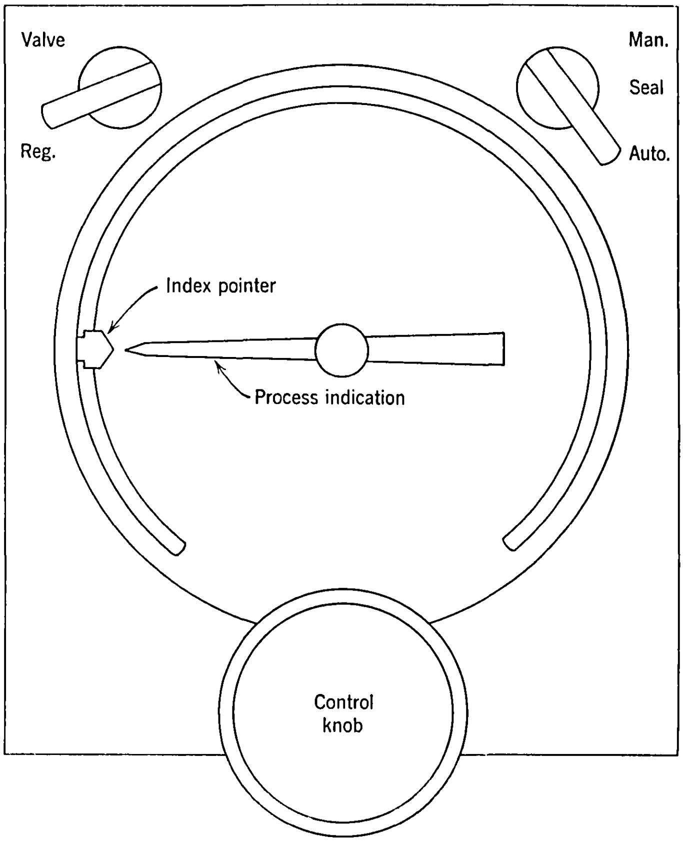


FIG. 39. Control station.

Control Valves and Actuators

Irrespective of what type of measurement is involved or whether control is pneumatic, electric, electronic, or hydraulic, a process control system usually has a valve as its final control element. For most system studies a control valve may be treated as a variable, nonlinear resistance.

Control Valve Flow Formulas (see Ref. 7). Control valve manufacturers rate their valves by the valve flow coefficient C_v. This is defined as the number of gallons per minute of water which will pass through a given flow restriction with a pressure drop of 1 psi. The following formulas relate C_v to flowing conditions.

$$\text{Liquid flow:} \quad C_v = \frac{V\sqrt{m}}{\sqrt{(p_1 - p_2)}}$$

Gas flow: $$C_v = \frac{\phi\sqrt{m}}{61\sqrt{(p_1 - p_2)p_2}} \quad \text{if } p_2 > \frac{p_1}{2}$$

$$= \frac{\phi\sqrt{m}}{30.5p_1} \quad \text{if } p_2 \leqq \frac{p_1}{2} \quad \text{(critical flow)}$$

where V = liquid flow, gpm,
ϕ = cfh gas at 14.7 psia and 60°F,
p_1 = pressure, psia, at upstream side of valve,
p_2 = pressure, psia, at downstream side of valve,
m = liquid or gas specific gravity (air and water 1.0).

Reliable methods do not exist for estimating C_v for highly viscous liquids or for liquids which tend to flash on passing through the valve. For these cases individual manufacturers sometimes do have empirical data which apply to their own valves. It is also known that C_v decreases at high pressure drops, but, again, quantitative data in general form are lacking.

From the above we may derive formulas which relate changes in pressure to changes in flow. These formulas represent the hydraulic or acoustic resistance of a valve. It should be noted that the resistance for a change in upstream pressure is not always the same as for a change in downstream pressure.

For liquid flow:

$$R_{v1} = \frac{\partial P_1}{\partial Q} = \frac{5.78 \times 10^7 Q_{av} m}{(C_v)_{av}^2} = \frac{2(P_1 - P_2)_{av}}{Q_{av}}$$

$$R_{v2} = \frac{\partial P_2}{\partial Q} = \frac{-5.78 \times 10^7 Q_{av} m}{(C_v)_{av}^2} = \frac{-2(P_1 - P_2)_{av}}{Q_{av}}$$

For gas flow, $p_2 > p_1/2$:

$$R_{v1} = \frac{\partial P_1}{\partial Q} = 466 \left[\frac{P_1(P_1 - P_2)}{T_1 q}\right]_{av}$$

$$R_{v2} = \frac{\partial P_2}{\partial Q} = 466 \left[\frac{P_2^2(P_1 - P_2)}{T_2(P_1 - 2P_2)_q}\right]_{av}$$

For gas flow, $p_2 \leqq p_1/2$:

$$R_{v1} = \frac{\partial P_1}{\partial Q} = 466 \left[\frac{P_1^2}{T_1 q}\right]_{av}$$

$$R_{v2} = \frac{\partial P_2}{\partial Q} = 0$$

where P_1 = upstream pressure, lb/ft^2 abs,
P_2 = downstream pressure, lb/ft^2 abs,
T_1 = upstream temperature, °K,
T_2 = downstream temperature, °K
Q = ft^3/sec at flowing conditions.

Since under industrial conditions flow through a valve is almost always turbulent, the average resistance of the valve will be

$$R_{\text{av}} = 2\left(\frac{\Delta P}{Q}\right)_{\text{av}} \text{ lb sec/ft}^5$$

where ΔP = pressure drop across the valve, lb/ft^2,
Q = flow, ft^3/sec.

For liquid flow,

$$R_{\text{av}} = 12.9 \times 10^4 \left(\frac{V}{C_v^2}\right)_{\text{av}} m$$

For gas flow,

$$R_{\text{av}} = 5.50 \times 10^3 \left(\frac{p_1}{p_2 C_v^2 T}\right)_{\text{av}} m \quad \text{if } p_2 > \frac{p_1}{2}$$

$$= 1.67 \times 10^5 \left(\frac{p_1}{TC_v}\right)_{\text{av}} \sqrt{m} \quad \text{if } p_2 < \frac{p_1}{2}.$$

Control Valve Trim Characteristics. The trim of a control valve consists of a stem with plug, and an orifice or seat. The plug is so contoured as to obtain a specific relationship between valve stem position and C_v. The stem position (or lift or travel) is measured relative to its position with the valve closed, i.e., plug fully seated. Stem travel is then limited to some maximum value which will prevent damage to the plug or valve body.

a. Linear Trim (Figs. 40 and 41). Trim is said to be linear if the following relationship holds:

$$C_v = \frac{(C_v)_{\max}}{X_{\max}} X$$

where X = stem position or travel or lift.

b. Equal Percentage Trim (Figs. 40 and 41). The most commonly used trim in the process industries is the equal percentage type. For a certain change in stem position there is the same percentage change in C_v regardless of the valve position. Theoretically such a valve would never shut off. Practically, the manufacturers choose a ratio of maxi-

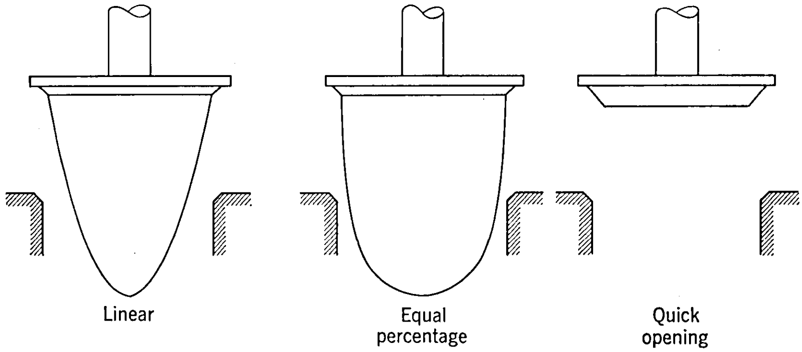


FIG. 40. Control valve trim.

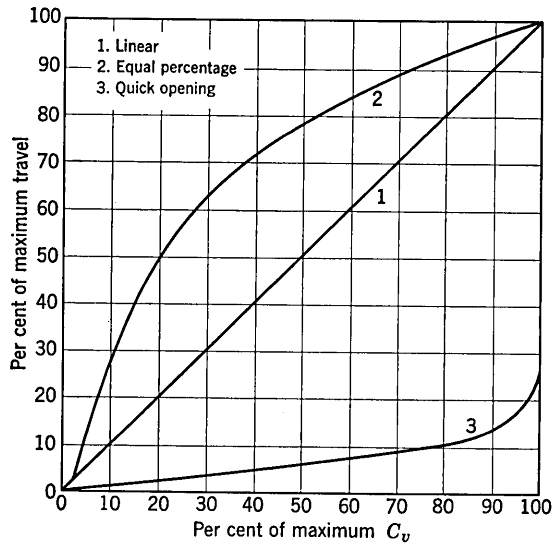


FIG. 41. Flow capacity curves for control valves.

mum C_v to minimum C_v, usually 40 or 50, then put a shoulder on the plug for tight shutoff. Mathematically,

$$C_v = (C_v)_{\min} \left[\frac{(C_v)_{\max}}{(C_v)_{\min}} \right]^{X/X_{\max}}$$

For small variations about $X = X_{av}$:

$$\frac{C_v(s)}{X(s)} = \frac{\partial C_v}{\partial X} = (C_v)_{av} k_{EP}$$

where
$$k_{EP} = \ln \frac{(C_v)_{\max}}{(C_v)_{\min}} \times \frac{1}{X_{\max}}.$$

c. Quick-Opening Valve (Figs. 40 and 41). Occasionally there is a need for a control valve to operate either fully open or fully closed and to do so quickly. For such service a valve usually has a short, beveled disk plug, and is known as a quick-opening valve.

Valve Body Design. *a. Double-Seated Valve.* To a great extent valve body design must be considered along with valve actuator design. The double-seated valve with spring-and-diaphragm actuator is by far the commonest combination used today in the process industries. As shown by Fig. 42*a*, the two ports are so arranged that part of the flow

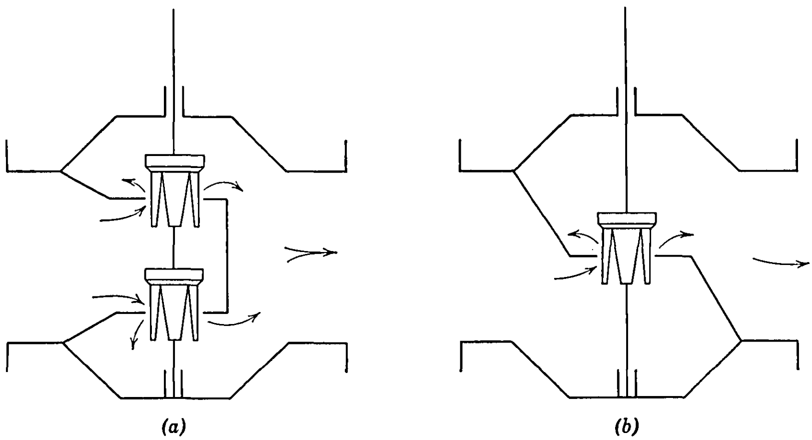


FIG. 42. Valve body design: (*a*) double-seated valve with v-port plug; (*b*) single-seated valve with v-port plug.

is up against the upper plug, and tends to force the stem upward, while the remainder of the flow is down over the lower plug, and tends to force the stem downward. It is the objective of this design, therefore, that the coercive axial stem forces be zero. This would permit the use of a low-powered actuator of simple design with no offset in stem position (see discussion on actuators). In practice, perfect balance is not achieved, and unbalance forces are sometimes quite significant (Refs. 2 and 12).

To minimize coercive axial stem forces further, manufacturers often use skirted plugs (see Fig. 42*a*). The slots, usually V-shaped, in the skirt may be so cut as to achieve either linear or equal percentage characteristics. An unfortunate characteristic of skirted plugs, however, is a tendency to develop coercive torsional stem forces, although these may be minimized by careful design.

b. Single-Seated Valve. Aside from the fact that double-seated valves are difficult to build in small sizes, the advent of more powerful positioners, particularly those with valve stem position feedback, has led to more extensive use of single-seated valves. The single-seated body, shown in Fig. 42*b*, is simpler and cheaper to build than the double-seated design. It is also potentially more efficient hydraulically (Refs. 6 and 7), although commercial designs do not always achieve this potentiality.

c. Butterfly Valves. For controlling gas flow in large diameter pipes where moderate or low pressure drops are involved, butterfly valves are often used. As shown by Fig. 43, a butterfly valve consists of a disk in a line. The disk may be rotated through an angle of 90 degrees although a 60-degree angle usually provides nearly full range. The flow characteristics are similar to those of an equal percentage valve although the ratio of maximum flow to minimum flow is smaller.

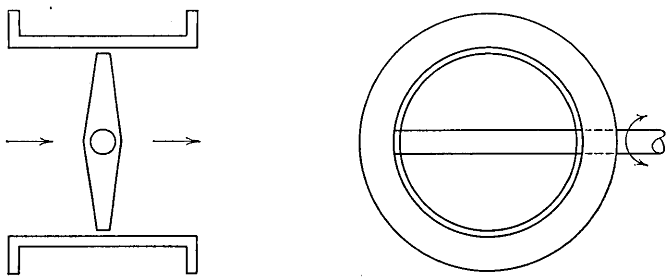


FIG. 43. Butterfly valve.

Many mechanisms have been devised to rotate the disk of the butterfly valve. Hydraulic piston actuators are often used in the steel industry whereas the spring-and-diaphragm actuator is more common in the chemical and petroleum industries.

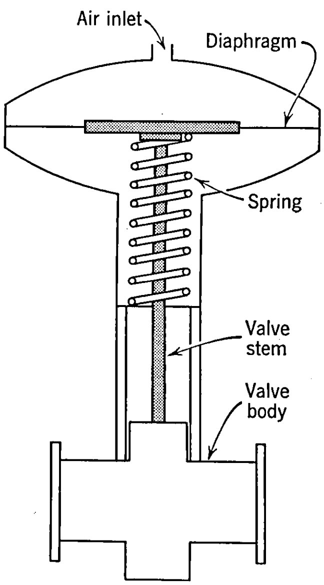


FIG. 44. Control valve with spring-and-diaphragm actuator.

Control Valve Actuators. *a. Spring and Diaphragm.* If one end of a spring is fixed and a force is applied to the other end, the motion or displacement is proportional to the force and the spring rate. In this case the force is generated on a diaphragm by applied air pressure (see Fig. 44). To obtain a specified valve travel to correspond to the standard 3–15 psig range used for pneumatic systems, the spring must be slightly preloaded. The spring and diaphragm are sometimes arranged so that an increase in pressure opens the valve (air-to-open), and sometimes so that an increase in pressure closes the valve (air-to-close). In either case, if there are no coercive axial stem forces, there exists a fixed relationship between air pressure and valve flow coefficient, C_v. If, however, as is usually the case, there is some stem thrust due to the fluid flowing through the body, the valve actuator will no longer have a 3–15 psig range, although the span will still be 12 psi. For example, an air-to-open valve with a 100-in.^{2} diaphragm is subjected to a 200-lb downward stem thrust, the effective actuator range will be 5–17 psig.

The transfer function for a valve with spring-and-diaphragm actuator is approximately

$$\frac{X(s)}{P(s)} = \frac{A/K}{\dfrac{W}{gK}s^2 + \dfrac{b}{K}s + 1}$$

where X = stem motion, ft,
P = diaphragm pressure, lb/ft^2 abs,
A = diaphragm area, ft^2,
W = weight of moving parts, including stem and plug, lb,
K = spring rate, lb/ft,
b = coefficient of viscous friction between valve stem packing, lb sec/ft,
g = 32.2 ft/sec^2.

The equation is useful for showing that a fast, stable valve should have a high spring rate and low mass of moving parts. Experience has shown that large valves (nominal size 8-in. or larger) with solid plugs sometimes have a tendency to instability. The reason is that the damping ratio, $\zeta = (b/2)\sqrt{g/(WK)}$, is too small because W is too large. For valves of nominal 6-in. size or smaller, the resonant frequency, $f_r = (1/2\pi)\sqrt{gK/W}$, is typically 15–30 cps.

In the equation above there is some error in treating friction as being viscous. There is always some static friction. This plus diaphragm hysteresis leads to an overall hysteresis which under shop conditions is rarely less than 0.25–0.33 psi. Under plant conditions overall hysteresis may change drastically and under extreme conditions may be as high as 4–5 psi.

b. Spring-and-Diaphragm Valve with Booster. To get faster response, particularly from valves with large topworks, a 1:1 booster relay may be inserted in the controller-valve transmission line just ahead of the valve. The free volume in a valve topworks is commonly 100 in.^{3} or more whereas the input volume of a booster is usually 1–2 in.^{3}. The use of a booster at the valve therefore raises the controller-valve, transmission-line terminal impedance as well as the transmission line driving-point impedance. Speed of response between controller and valve is therefore much faster. Furthermore, boosters usually have much greater air handling capacity than controllers, are less likely to saturate with a low-impedance load, and can therefore fill or dump the dome of a valve faster than an unaided controller.

c. Spring-and-Diaphragm Valve with Positioner. To an increasing extent positioners (valve stem positioner controllers) are used to position spring-and-diaphragm valves. The advantages are: (*a*) very little stem position offset due to coercive, axial stem forces, (*b*) More precise positioning of the valve, and (*c*) better impedance match between controller and valve. As an example, consider the Mason-Neilan Series 7400 valve positioner (see Fig. 45). The signal pressure from the controller, P_p, is

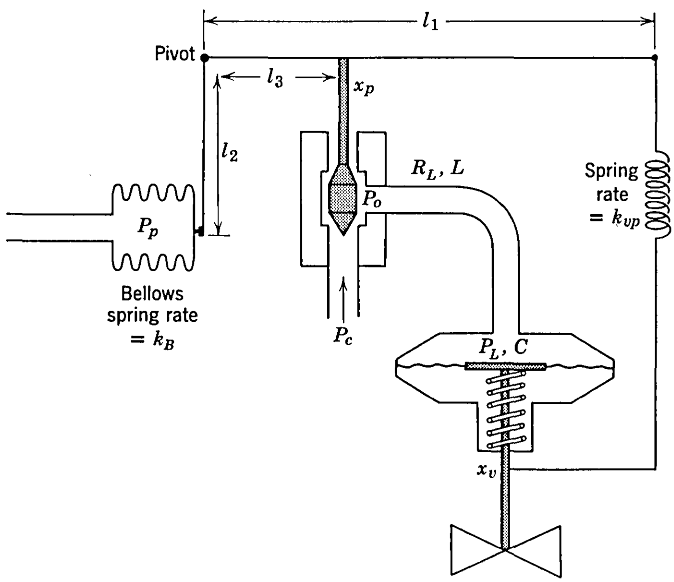


FIG. 45. Spring-and-diaphragm valve with positioner schematic.

multiplied by the end area, A_B, of the input bellows to create a signal force P_pA_B on one end of a lever. This moves the lever about the fulcrum or pivot, thereby deflecting the pilot valve. The pilot valve is of the three-way, or bleed type. Air is admitted to or exhausted from the valve topworks and causes the valve stem to go up or down. This motion varies the tension of the positioner spring in such a way as to create a balancing force at the opposite end of the lever from the input bellows.

d. Positioner-Operated Spring-and-Diaphragm Valve with Booster.

The ability of a conventional positioner to drive at high speed a valve with a moderate or large size topworks is limited by pilot valve capacity. As mentioned previously a three-way or bleed type pilot cannot have high capacity without a high bleed rate. This has led to various positioner-booster combinations for high-speed stroking of the control valve.

A typical hookup which has been popular is shown in Fig. 46. The bypass from the booster input to the booster output is for the purpose of stabilization. The farther open the needle valve, the greater is the damping. The reasoning which leads to this arrangement is as follows.

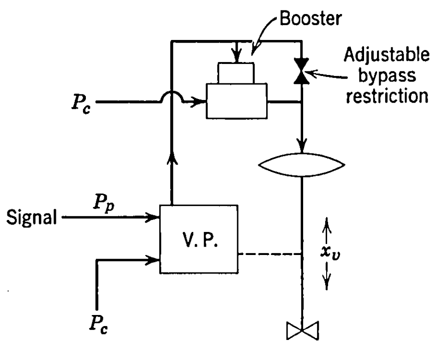


FIG. 46. Schematic arrangement of spring-and-diaphragm valve driven by positioner plus booster.

A pneumatic valve positioner normally is loaded by a low impedance—that of the connecting tubing and valve topworks. The positioner-valve loop gain, under these conditions, is set to give reasonable stability. The insertion of a booster, however, presents to the positioner output a relatively high impedance. Furthermore, the input-output pressure transfer function of the tubing from the positioner to the booster is apt to be highly resonant. The result is that the static loop gain, which is unchanged, is too high, and the system is oscillatory. Since there is no convenient way to alter the static loop gain, one must modify the loop dynamics. Reducing the booster input impedance, as by providing a bypass around the booster, will accomplish this. This booster-positioner combination permits performance which is five to six times faster than that obtained with a positioner only.

e. Piston-Operated Valve with Integral Positioner. In recent years the piston-operated valve with integral positioner has become a serious competitor to the spring-and-diaphragm operated valve. The following discussion refers to the actuator and valve made by the Annin Company (see Fig. 47) but, with minor modifications, may be applied to those made by several other manufacturers.

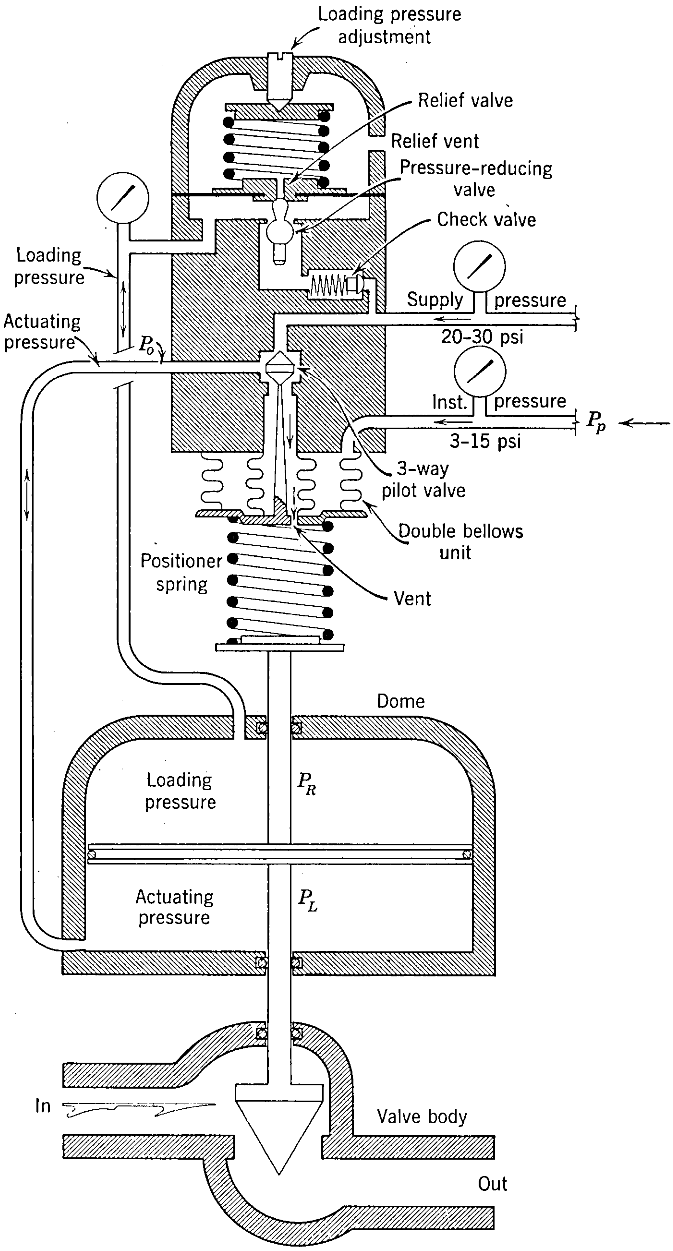


FIG. 47. Schematic diagram of domotor. (Courtesy of Annin Company.)

The signal pressure P_p from the controller or manual loading station enters a double bellows unit. The free end of the bellows is fastened to a plate which is also attached to the upper side of the positioner spring. In addition, the stem of a three-way (bleed type) pilot valve is fastened to the plate. The output P_o from the pilot goes to the lower side of the power cylinder.

The upper side of the power cylinder is maintained at a constant loading or reference pressure, P_R, which is supplied from a built-in regulator. The driving force for the piston is therefore furnished by P_L. As it varies and moves the power piston, the compression on the positioner spring, whose lower end is attached to the power piston, also varies. This is so arranged that an increase of P_p in the double bellows unit is balanced by an increased compression force in the positioner spring.

For high performance, piston actuators have an advantage in that higher differential pressure may be applied across a piston than across a diaphragm, which might rupture.

f. Piston-Actuators with Integral Positioners and Boosters. To get maximum performance out of a piston actuator with positioner it is necessary to add two boosters, one in the loading line, the other in the reference pressure line (see Fig. 48). An alternate which provides somewhat greater

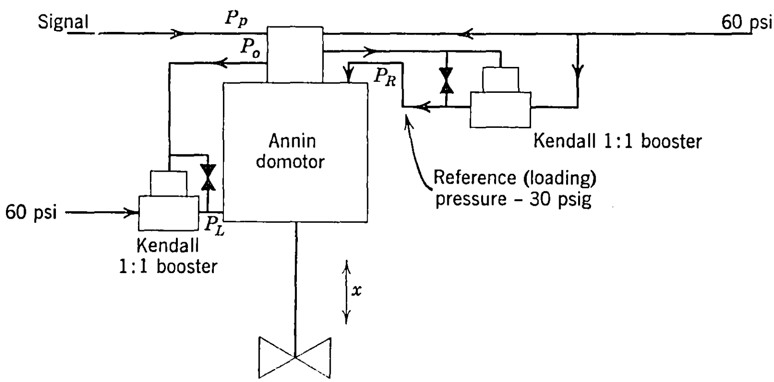


FIG. 48. Annin valve with dual 1:1 boosters.

speed and power is to use one booster and one reversing booster (see Fig. 49). This has the effect of doubling the driving force across the piston for a given signal.

Miscellaneous Pneumatic Devices

Booster Relays. Booster relays are commonly used in pneumatic circuits for either or both of two functions: (*a*) isolation amplifier and

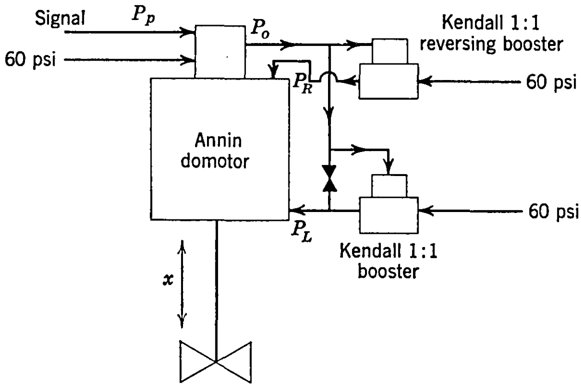


FIG. 49. Annin valve with booster and reversing booster.

(b) power amplifier. In either case the basic design is that of the general transmitter (see General Pneumatic Transmitter Design).

As an isolation amplifier, a booster relay is designed to maintain an accurate static output-input ratio (usually 1:1 within $\pm 0.1\%$), and to have low hysteresis, less than 0.1%. Air-handling capacity is a secondary consideration and does not exceed 1–3 scfm. An example of this kind of device is the Moore Products Company Model 61F, Fig. 50.

The power amplifier type of booster relay is used to improve the impedance match between a relatively high-impedance signal source and a low-impedance load. The most frequent application is for increasing the speed of response of control valves (see Control Valve Actuators). The best devices of this type maintain static gain and hysteresis within 0.5%. Air-handling capacity may be as much as 10–40 scfm. An example of this kind of booster is that made by the Kendall Corporation (Fig. 26).

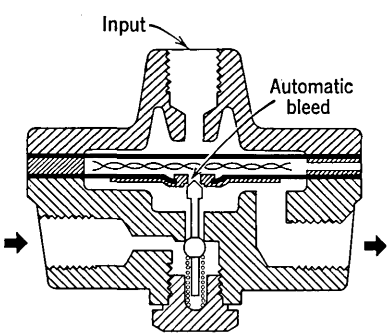


FIG. 50. Moore reducing relay, Model 61. (Courtesy of Moore Products Company.)

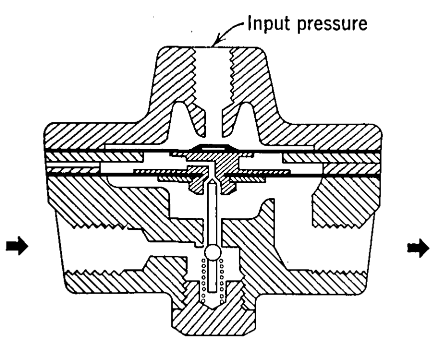


FIG. 51. Moore reducing relay, Model 66BR. (Courtesy of Moore Products Company.)

Some boosters have reversing action; a given increase in signal produces a corresponding decrease in output pressure.

Amplifying and Reducing Relays. It is sometimes desirable to insert into a pneumatic circuit an isolation amplifier which amplifies or attenuates by a nominal, fixed ratio. If high precision is not required, relay design can be simple. An example of this kind of relay is the Moore Products Company Model 66 (Fig. 51). Note that output-input ratio is determined by the ratio of the input diaphragm area to the output feedback diaphragm area. If the input diaphragm is the smaller, the relay attenuates or "reduces." Ratios up to 6:1 or 1:6 may be attained.

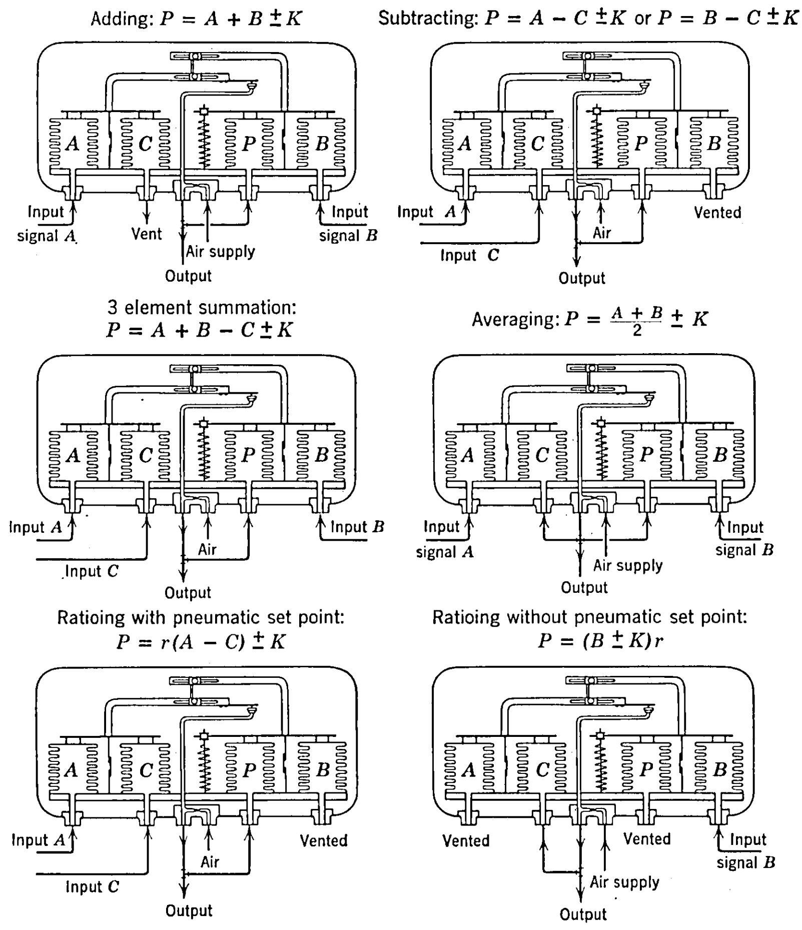


FIG. 52. Taylor computing relay, Model 348RF1. (Courtesy of Taylor Instrument Companies.)

Computing Relays. To achieve more precision and flexibility than are obtainable from the above devices, as well as to accomplish additional functions such as addition, subtraction, integration, and differentiation, a different class of relays is employed. These are aptly called computing or multifunction relays. They are made in a variety of different models, depending on which groups of functions are desired.

Two different approaches to the design of computing relays are shown in Figs. 52 and 53. Moore uses a stacked diaphragm construction while

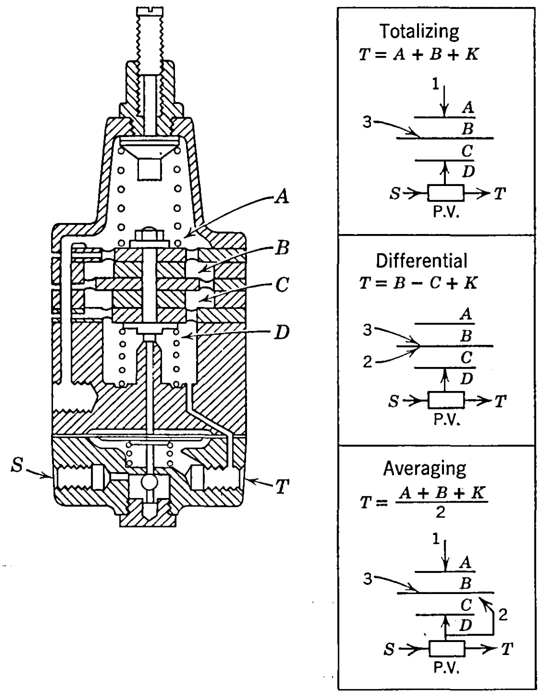


FIG. 53. Moore M/F (multifunction) relay: three examples of different functions produced by the Model 68 relay. In the formulas, K is the suppression, // indicates a vent to the atmosphere, and A, B, C, and D are the pressure chambers. (Courtesy Moore Products Company.)

Taylor uses bellows. Only a small number of the computing functions achievable with these relays is shown.

Snap-Acting Relays. Pneumatic on-off action is sometimes required for interlock circuits, alarms, or sequential operation of control valves used for batch or cyclic process operation. An example of a snap-acting relay with adjustable set point is the Moore Model 67 (see Fig. 54).

Limiting and Selector Relays. Limiting relays are used to limit the output from a controller. Some models have either a high limit stop or a low limit stop; others have both high and low stops (see Fig. 55).

Selector relays compare two pressures and transmit only one of them. A high-pressure selector passes the higher of two pressures while a low-pressure selector passes the lower of two pressures (Fig. 56).

Pulsation Dampers for Pneumatic Circuits. In principle, the full range of filter theory as developed for electric circuits may be applied to the attenuation or reduction of undesired signals in pneumatic circuits. One may use either active or passive filters. In practice, pneumatic filters are usually single section, low-pass *RC* filters. The resistance *R* is commonly an adjustable needle valve, whereas the capacitance may be only that of the tubing or else a volume pot is added.

An exception to the above is the Taylor pressure pulsation damping unit. As shown by Fig. 57 it is a ten-section, low-pass *RC* filter. It may be used for both gas and liquid service but is more effective on the former.

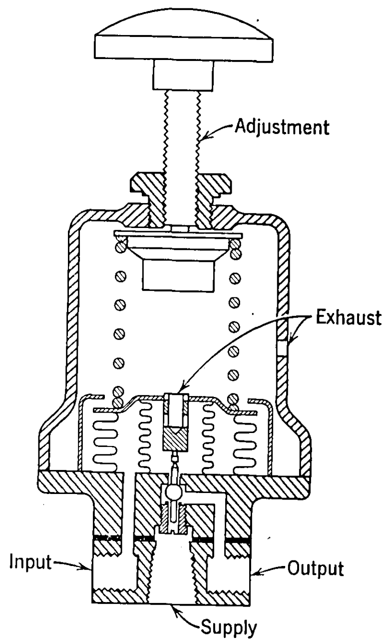


FIG. 54. Moore snap acting relay, Model 67. (Courtesy of Moore Products Company.)

7. ELECTRIC AND ELECTRONIC COMPONENTS

In general, most of the primary measuring elements which can be used with pneumatic transmitters also can be used with electric or electronic circuits to produce electric signals. Conversely, primary measuring elements which are inherently electrical in nature can, by translation, be used to produce pneumatic signals.

Transmitters

Differential Pressure and Pressure. The Swartwout pressure and differential pressure transmitters consist of a measuring diaphragm across which the pressure or differential pressure appears, a mechanical linkage and bias spring, and a differential transformer. Minute movements of the diaphragm are thereby converted to motion of the transformer core and hence to proportional a-c voltage output changes. A coarse span

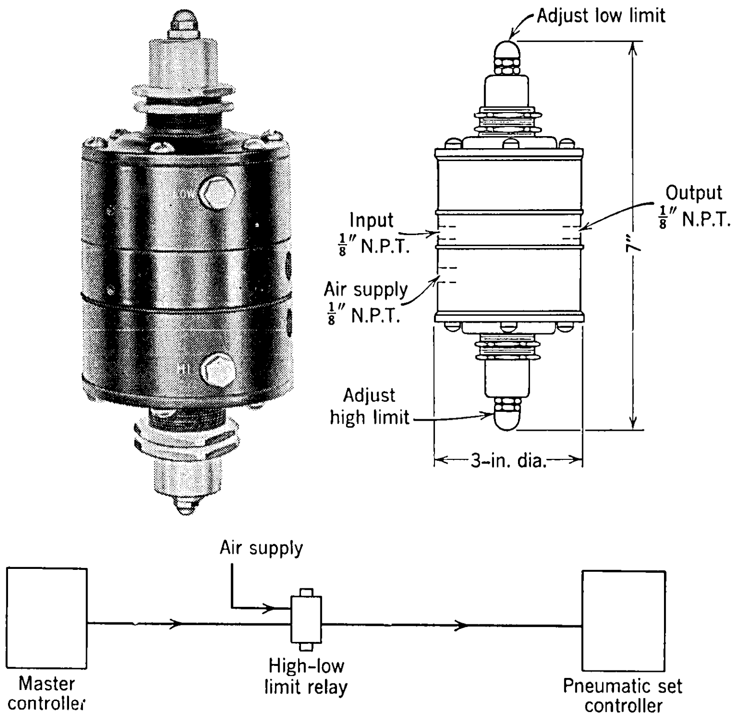


FIG. 55. Taylor high- and low-limit stops relay. (Courtesy of Taylor Instrument Companies.)

adjustment is provided by a linkage ratio adjustment while a fine span adjustment is accomplished electrically.

The Robertshaw-Fulton pressure transmitter works on a considerably different principle. The pressure detecting element is a bourdon tube or bellows whose free end is connected to a spring which in turn is connected to the beam of an electromechanical amplifier (Microsen balance). As the loading on the beam changes, the end of the beam moves toward or away from an oscillator coil. This changes the amplitude of oscillation and the amplifier puts out a direct current proportional to beam motion. Part of the output is fed back to a balance coil and magnet to offset the initial torque.

Another method converting pressure or differential pressure to electrical signals is by means of strain gages. The Baldwin SR-4 fluid pressure cell is a commercial instrument of this type (see Fig. 58). It consists of a metal block with a hole drilled in one end and Baldwin SR-4 strain gages bonded to the block. An increase in pressure causes the block to expand, thereby stretching the metal filaments of the strain gages and changing their electrical resistance. The strain gages are connected to-

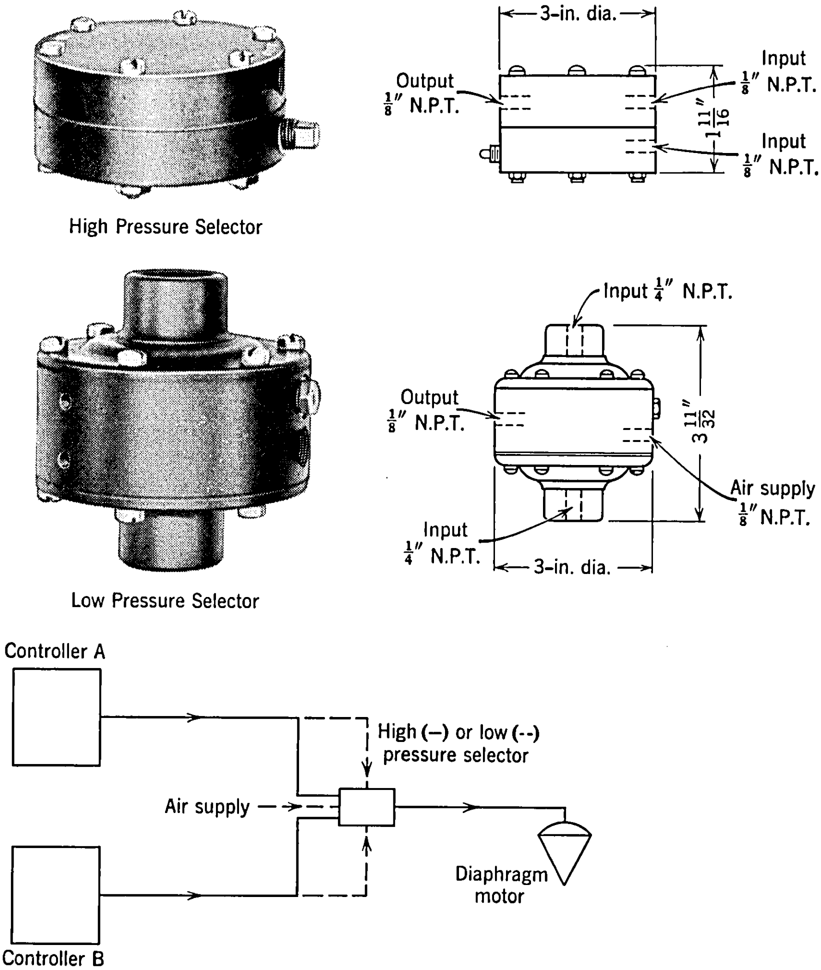


FIG. 56. Taylor high- and low-pressure selectors. (Courtesy of Taylor Instrument Companies.)

gether in a Wheatstone bridge arrangement and have a nominal full-scale output of 1 mv per volt of bridge supply. This type of instrument may be used for pressures up to 50,000 psi and is capable of following pressure pulsations with frequencies up to several thousand cycles per second.

Temperature. A common means of measuring temperature is by means of thermocouples. Since thermocouples provide low-level, d-c voltages proportional to the difference in temperature of the hot and cold junctions, it is necessary to amplify these signals. For a control system such as Swartwout's, it is also necessary to convert from dc to ac. As shown by Fig. 59, this is done with two d-c to a-c conversions; the first

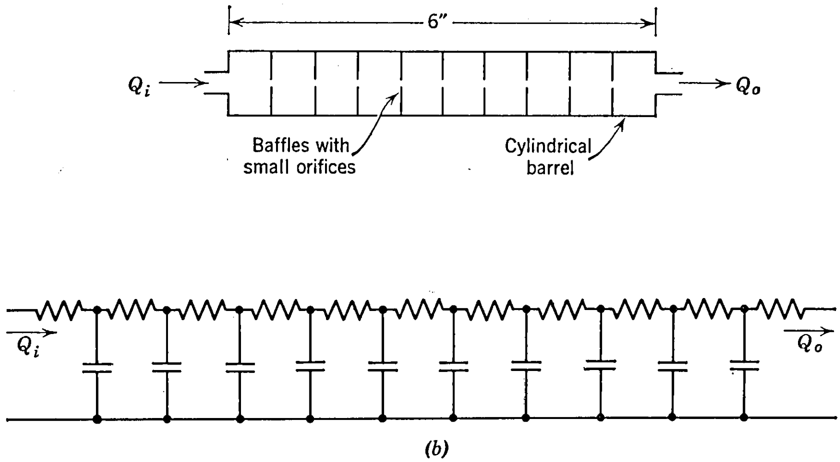


FIG. 57. Taylor snubber (pulsation damping unit): (*a*) schematic arrangement; (*b*) equivalent circuit.

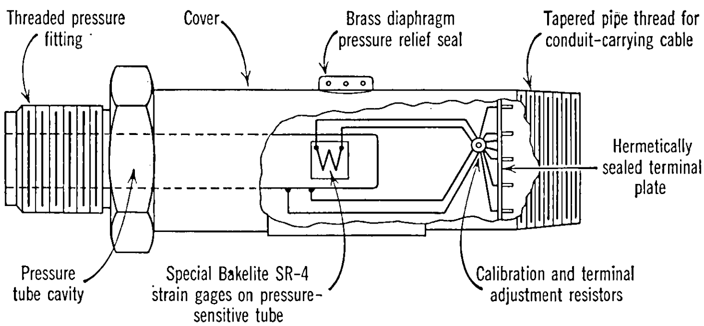


FIG. 58. Baldwin SR-4 fluid pressure cell. (Courtesy of Baldwin-Lima-Hamilton Corporation.)

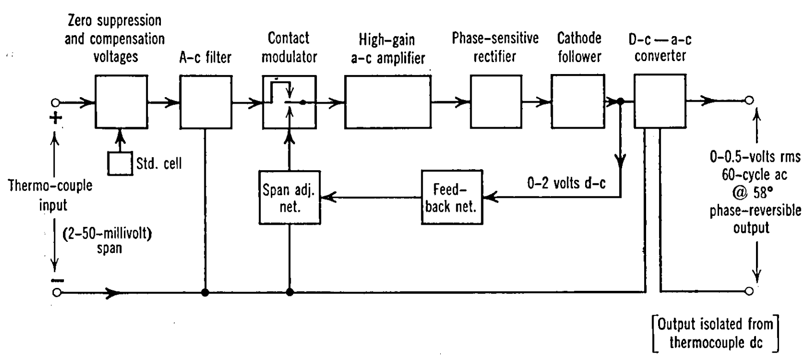


FIG. 59. T2C thermocouple converter block diagram. (Courtesy of Swartwout Company.)

provides voltage gain while the second provides conversion and isolation from the thermocouple circuit.

In one type of Robertshaw-Fulton temperature transmitter, the thermocouple is used to generate current rather than voltage. This current flows through an input coil wound around a magnet core. A change in temperature causes a change in input current flow and the electromagnet deflects one end of a pivoted beam. The other end of the beam moves in the field of an oscillator coil (see Pressure Transmitters and Differential Pressure Transmitters in Sect. 6), and causes an amplifier to put out a proportional direct current. Part of this current is fed back to another coil on the same core as the input coil, thereby providing a torque balance on the beam.

Liquid Level. *a. Capacitance Bridge.* Although either differential pressure or displacer type instruments described earlier may have an electrical instead of a pneumatic output, there are ways of measuring level which are inherently electrical in nature. The capacitance bridge is an example.

If a rod is inserted into a vessel partly filled with liquid, the rod may function as one plate of a capacitor while the vessel itself serves as the other plate. Since air and most gases have dielectric constants much smaller than those of most liquids, a rise of liquid level increases the capacitance. The rod is usually protected from the liquid by suitable insulation.

In the Telstor, manufactured by the Robertshaw-Fulton Controls Company, the bridge circuit is supplied with an RF voltage modulated by a 60-cycle power supply. Two inductance coils and two capacitors constitute the basic bridge circuit. One capacitor serves as a zero adjustment while the other is the vessel-probe combination. Bridge unbalance is detected by a rectifier; the d-c output is then applied to an indicating meter or recorder.

b. Nuclear Radiation. Several methods of measuring liquid or solids level by nuclear radiation have been devised. One of these is based on a cell manufactured by the Ohmart Corporation. A typical model cell is a small cylinder 7 in. long by 4 in. in diameter. It may be regarded as a type of ionization chamber with two chemically dissimilar electrodes, and therefore requires no external potential to maintain a signal current. It generates 2×10^{-12} amp when exposed to a field of one milliroentgen per hour from radium.

To measure level, the cells are stacked to an appropriate height and a source is installed so that the material in the tank or bin partially absorbs the radiation which would otherwise fall on the cells (Figs. 60 and 61).

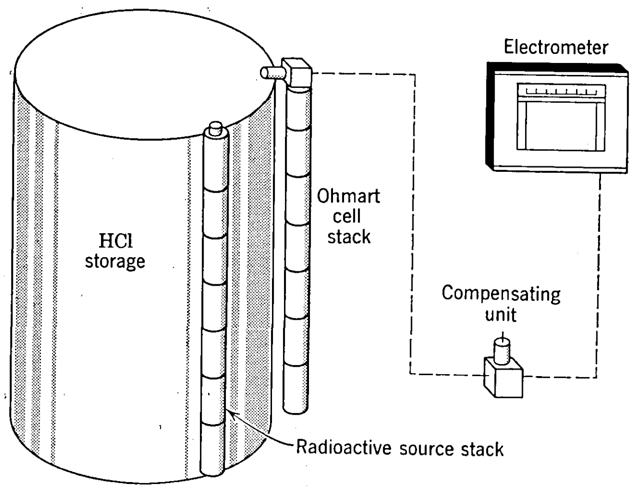


FIG. 60. Measurement of liquid level is across the chord of the tank with all components exterior to the tank. (Courtesy of the Ohmart Corporation and Minneapolis-Honeywell Regulator Company.)

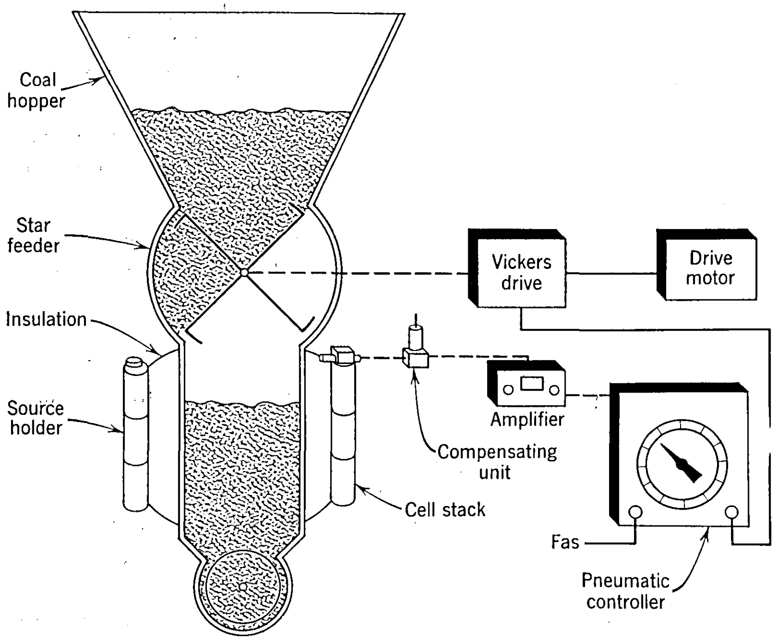


FIG. 61. Application of Ohmart cell and ElectroniK recorder to control level of crushed coal fed to a pulverizer. Note: Fas = filtered air supply. (Courtesy of the Ohmart Corporation and Minneapolis-Honeywell Regulator Company.)

Flow. *a. Turbine Flow Meter.* The turbine flow meter has the characteristic that speed of rotation of the turbine is linearly proportional to volumetric liquid flow rate. By magnetizing the blades or by inserting a magnet in the rotor, one may cause an a-c voltage to be induced in a pickup coil external to the conduit containing the flow. The frequency of this voltage is proportional to flow rate. This voltage may then be discriminated to obtain flow rate, or it may be treated as a train of pulses which may be totalized to obtain a measure of total flow. A sectional view of the turbine element made by the Potter Aeronautical Company is shown in Fig. 62. A feature of this particular design is that for turbine

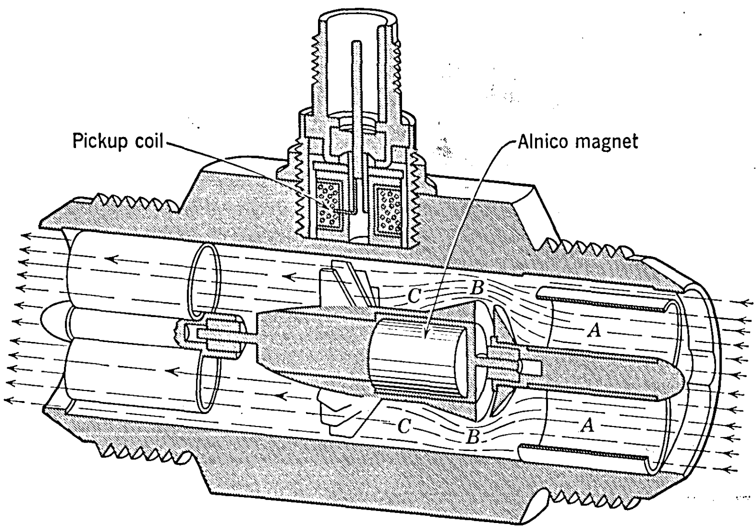
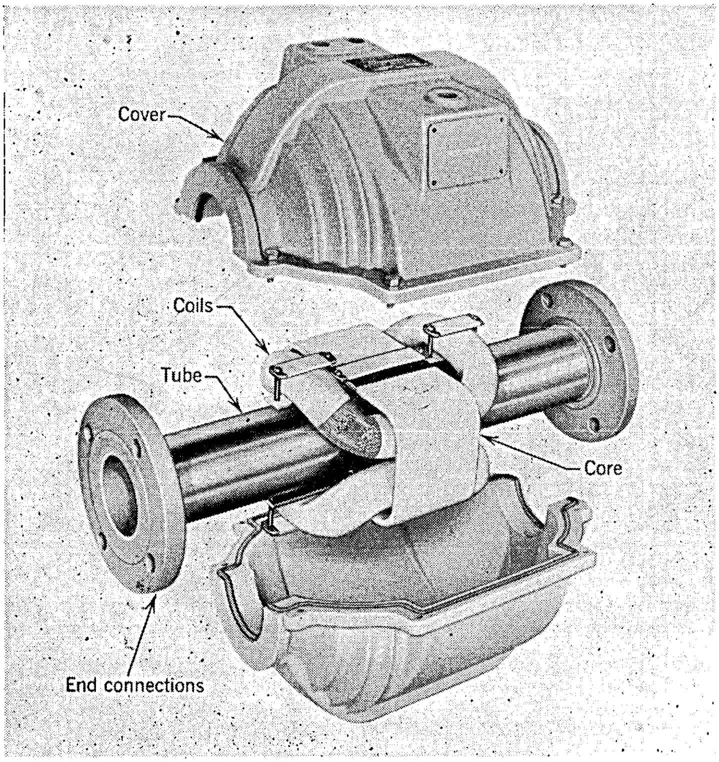


FIG. 62. Potter turbine type flowmeter. (Courtesy of Potter Aeronautical Company.)

speed above a certain minimum the rotor "floats" without slippage or thrust friction, and thereby eliminates the need for thrust bearings.

b. Electromagnetic Flow Meter. Most flow meters depend on the use of either fixed or variable restrictions in the conduit. To avoid the use of any obstructions, an electromagnetic flow meter may sometimes be used. Only the so-called a-c type will be discussed here.

This flow meter is based on Faraday's law of electromagnetic induction. If a conductor moves through a magnetic field, there is induced in it a voltage proportional to its velocity. A conductive fluid flowing through a conduit may be regarded as a series of conductors. If, then, a magnetic field perpendicular to the direction of flow is imposed on the conduit and its contents, a voltage will be generated which is perpendicular both to the magnetic field and the direction of flow. This voltage which is directly



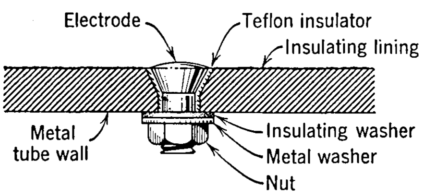


FIG. 63. Foxboro electromagnetic flowmeter. (Courtesy of Foxboro Company.)

proportional to volumetric flow rate may be picked up by a pair of electrodes which just barely protrude through the walls of the conduit.

An example of this kind of flow meter is that manufactured by the Foxboro Company. The transmitter, consisting of tube, coils, core, electrodes, cover, and end connections, is shown in Fig. 63. The overall circuitry, which requires a special Foxboro Dynalog receiver, is shown in Fig. 64.

Controllers

A number of electronic controllers are now on the market, but their designs are quite different and no one of them may be considered typical.

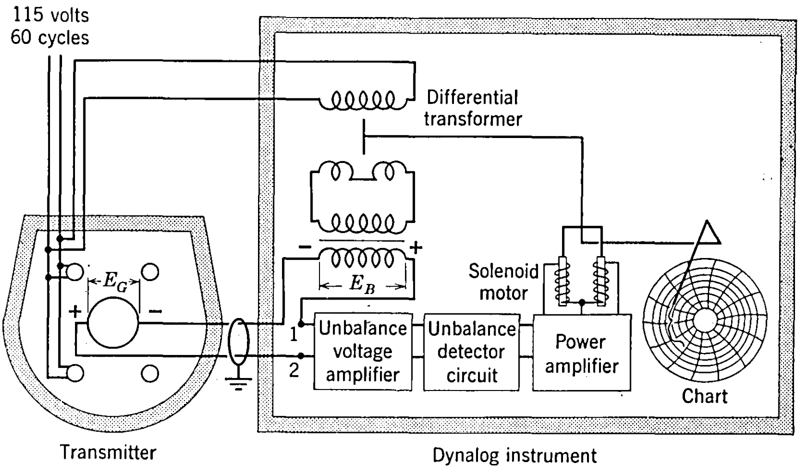


FIG. 64. Foxboro electromagnetic flowmeter circuit arrangement. (Courtesy of Foxboro Company.)

In Fig. 65 is presented a simplified block diagram of the Swartwout Autronic controller. The phase-sensitive rectifier stage between the proportional circuit and the rate circuit is not shown, since it is not pertinent to the analysis to be presented. The transfer function of each stage and of the complete controller may readily be formulated as follows (Ref. 14).

Proportional Circuit

$$K_pG_p = \frac{K_1G_1}{1 + K_1G_1K_2G_2} = \frac{K_1}{1 + K_1K_2},$$

since G_1 and $G_2 \cong 1$.

If K_1 is very large,

$$K_pG_p = \frac{1}{K_2}$$

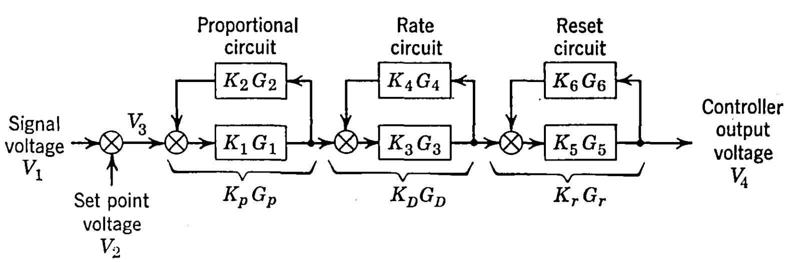


FIG. 65. Block diagram of Swartwout Autronic controller.

Rate Circuit

$$K_D G_D = \frac{K_3 G_3}{1 + K_3 G_3 K_4 G_4} = \frac{K_3}{1 + \dfrac{K_3}{1 + \tau_D s}}$$

(since $G_3 \cong 1$ and $K_4 G_4 = \dfrac{1}{1 + \tau_D s}$)

$$= \frac{K_3(1 + \tau_D s)}{1 + K_3 + \tau_D s}$$

$$= \frac{K_3}{1 + K_3} \frac{1 + \tau_D s}{1 + \left(\dfrac{\tau_D}{1 + K_3}\right) s}.$$

Reset Circuit

$$K_r G_r = \frac{K_5 G_5}{1 + K_5 G_5 K_6 G_6} = \frac{K_5}{1 + \dfrac{K_5 \tau_r s}{1 + \tau_r s}}$$

(since $G_5 \cong 1$ and $K_6 G_6 = \dfrac{\tau_r s}{1 + \tau_r s}$)

$$= \frac{K_5(1 + \tau_r s)}{1 + (1 + K_5)\tau_r s}.$$

Thus for *complete controller:*

$$\frac{V_4}{V_3} = K_p G_p K_D G_D K_r G_r,$$

$$\frac{V_4}{V_3} = \left[\frac{K_5}{K_2} \cdot \frac{K_3}{1 + K_3}\right] \left[\frac{1 + \tau_D s}{1 + \dfrac{\tau_D}{1 + K_3} s}\right] \left[\frac{1 + \tau_r s}{1 + (1 + K_5)\tau_r s}\right]$$

$$= K \left[\frac{1 + \tau_D s}{1 + \dfrac{\tau_D}{1 + K_3} s}\right] \left[\frac{1 + \tau_r s}{1 + (1 + K_5)\tau_r s}\right].$$

According to manufacturers' data:

$$K_3 = 15,$$

$$K_5 = 200,$$

$$\frac{V_4}{V_3} = K \left[\frac{1 + \tau_D s}{1 + \left(\dfrac{\tau_D}{15}\right) s}\right] \left[\frac{1 + \tau_r s}{1 + (200)\tau_r s}\right],$$

K is adjustable from 0.5 to 38,

τ_D is adjustable from 0.003 to 10 min,

τ_r is adjustable from 0.026 to 10 min.

The frequency response of the Swartwout Autronic controller is given in Fig. 66. The cutoff frequency ω_c results from the bandwidth limitation of the phase-sensitive rectifier and usually has a value of approximately 7 cps.

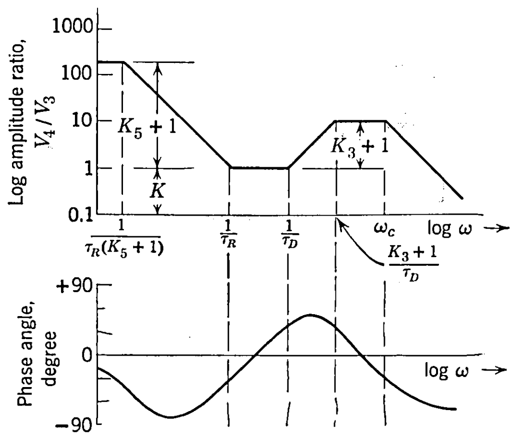
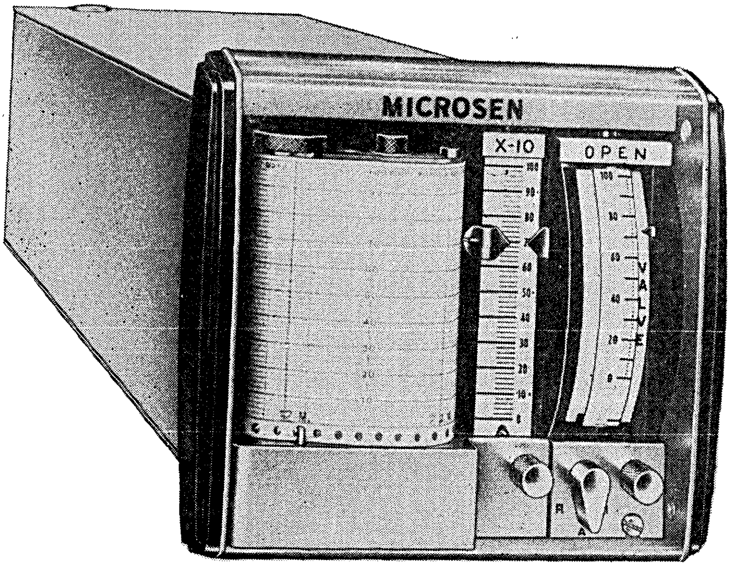


FIG. 66. Frequency response diagram for Swartwout Autronic controller.

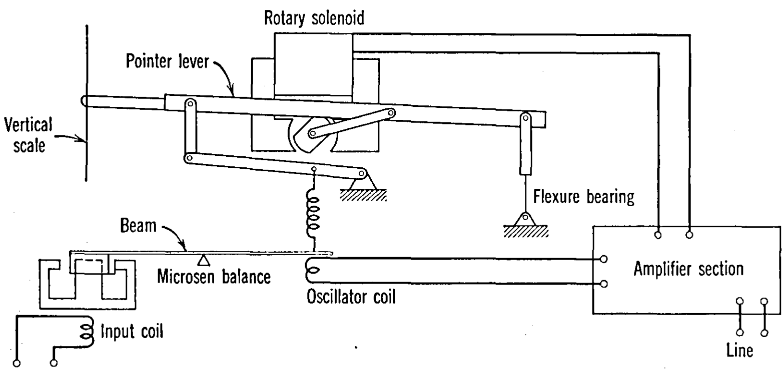
Electronic Recorders

One of the earliest applications of electronics to process control was the use of the potentiometer recorder for thermocouples. As developed by Minneapolis-Honeywell and by Leeds and Northrup, two of the chief manufacturers of this equipment, it consists of a potentiometer input circuit followed by what is essentially a position servo system. The thermocouple signal and a feedback signal are combined in a d-c bridge circuit. Bridge unbalance is amplified electronically, causing a two-phase servomotor to rotate in a direction determined by the polarity of the unbalance. Rotor position is fed back to the input circuit by a variable resistor whose slider is mechanically linked to the rotor. The recorder pen is also linked to the rotor. The pen is sometimes replaced by a rotating printing mechanism, and the recorder becomes multipoint by a switching circuit which selects in turn each of a number of input signals.

Several miniature electronic recorders are now available which work on the same basic principle outlined above. Figure 67 shows the Microsen recorder made by Robertshaw-Fulton.



(a)



(b)

FIG. 67. Miniature electronic recorder: (a) miniature recorder with controller installed; (b) schematic diagram of recorder. (Courtesy of Robertshaw-Fulton Controls Company.)

Valve Actuators

Electropneumatic Valve Actuators. An electropneumatic valve positioner, such as those manufactured by Evershed & Vignoles or by Robertshaw-Fulton, uses electricity primarily for signal transmission. The positioner is otherwise quite similar to a conventional pneumatic positioner. As shown by Fig. 68, the input circuit compares a force

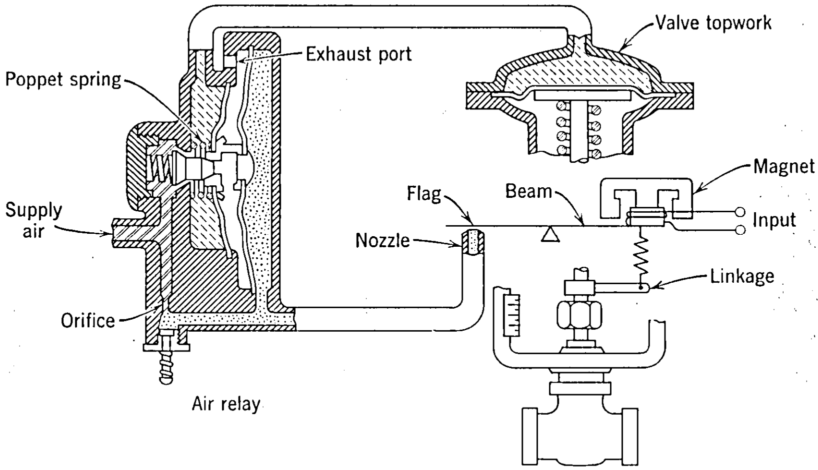
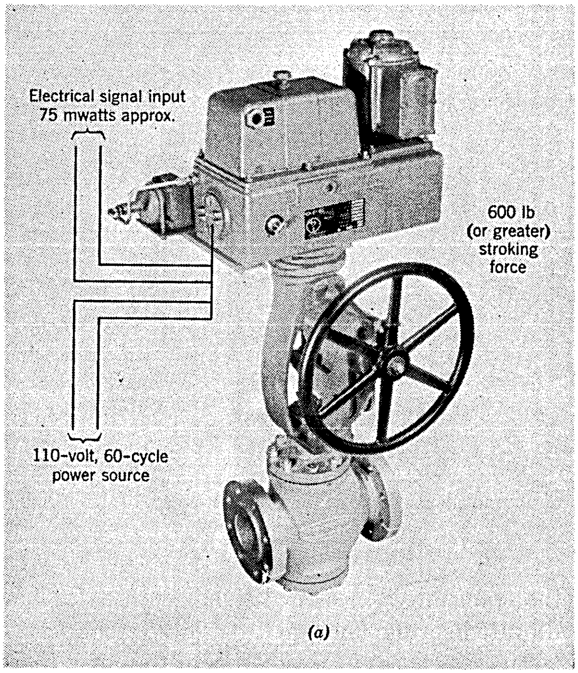


FIG. 68. Electropneumatic valve positioner. (Courtesy of Robertshaw-Fulton Controls Company.)

produced by the transmitted electrical signal with a feedback force produced by a calibrated spring which detects valve stem position. Any unbalance in these forces causes a pivoted beam to move, thereby changing the displacement between a flapper and a nozzle. The corresponding change in nozzle back pressure actuates a pneumatic booster stage which increases or decreases the pressure in the dome of a spring-and-diaphragm valve actuator. This pressure change in turn repositions the valve stem until there is no unbalance force in the positioner input circuit.

Electrohydraulic Valve Actuators. For military purposes, high-performance (50–100 cps bandwidth) electrohydraulic servos have been developed. In principle these may be adapted to process control valves, but in practice their complexity and expense are prohibitive for all except a few applications which require maximum performance. Several manufacturers have, however, produced highly simplified versions which are competitive in price with pneumatic actuators when the savings in instrument air facilities are taken into account.

An example is the Askania electrohydraulic valve actuator (see Fig. 69). The input circuit compares a force produced by the transmitted electrical signal with a feedback force produced by a spring which detects valve stem position. An unbalance of these forces deflects the jet pipe hydraulic preamplifier. This causes the power piston to move until the feedback force balances the electrical force, and thereby causes the jet pipe to return to its neutral position. The only power requirements are 110 volt, 60 cycle.



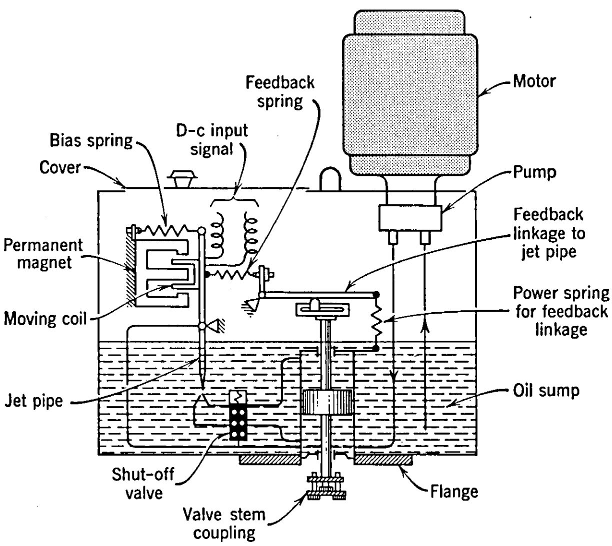


FIG. 69. (*a*) GPE electrohydraulic valve actuator and (*b*) diagram showing its operation. (Courtesy of GPE Controls Inc.)

8. SELF-ACTUATED CONTROLLERS

When control requirements are not tight and when it is desired to minimize first cost, self-actuated controllers are often used. As a class, these devices are rugged, dependable, and simple in design. In comparison with the control equipment previously described, they are more specialized, less flexible, less precise, and require no external source.

Temperature

An example of a self-actuated temperature controller is one made by the Leslie Company (Fig. 70). A vapor-filled temperature-sensing bulb is inserted in the process and causes a force to be exerted on the diaphragm of a spring-and-diaphragm actuator which is proportional to

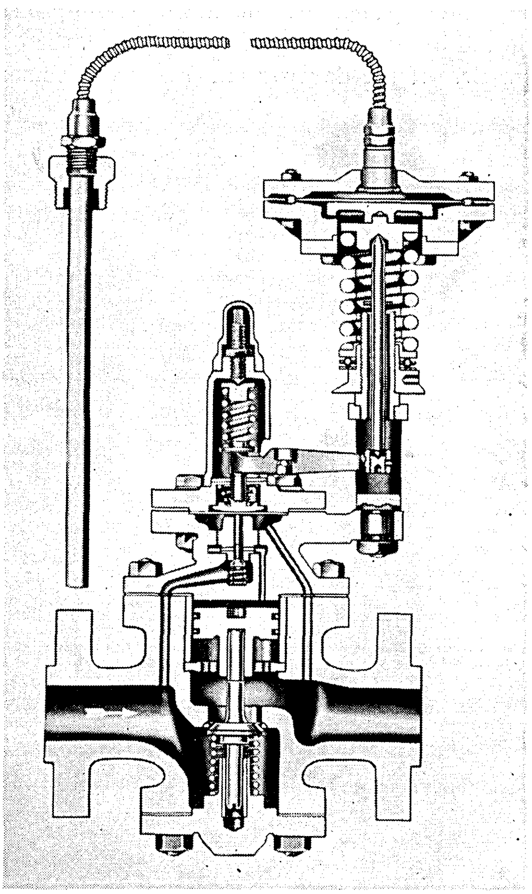


FIG. 70. Self-actuated temperature controller. (Courtesy of the Leslie Company.)

temperature. The actuator stem moves a distance proportional to the applied force and, through a linkage, moves the stem of a pilot valve connected to the high-pressure side of the process valve. The position of the pilot valve then determines how much pressure is applied to the piston of a spring-and-piston actuator which positions the process valve. The process valve position is therefore proportional to temperature.

Pressure

Self-actuated pressure regulators are used to control gas streams, steam, and air. An example of a small regulator is that made by Fisher for air or inert gases (Fig. 71). The principle of operation is that the downstream (controlled) pressure must provide a force against the under side of a diaphragm which will balance the force on the top side due to compression of the main adjusting spring. If the downstream pressure drops, the diaphragm is moved downward, and the supply valve is opened to admit upstream gas or air. If the downstream pressure becomes too high, the

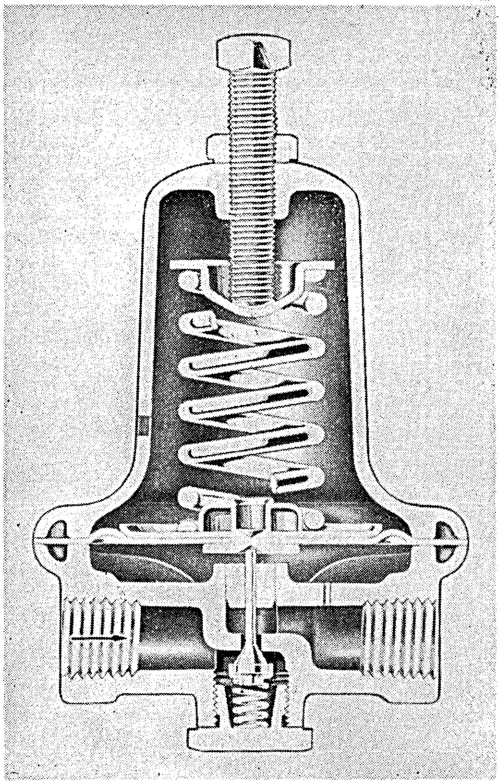


FIG. 71. Self-actuated pressure regulator. (Courtesy of Fisher Governor Company.)

diaphragm is moved upward, and thereby opens the exhaust valve which bleeds off the pressure. The diaphragm is isolated from the controlled fluid by a feedback chamber connected by a small port to downstream pressure. This provides stabilization and reduces the tendency of the regulator to buzz or chatter.

Flow Regulators

Small Flows. Regulators for small flows are widely used in laboratories, pilot plants, and for such applications as purge flows. They are available for gas or liquid service. A typical instrument in this category is the Moore Products Company Model 62 gas flow controller which will regulate gas flows in the range 0.2–2 scfh (see Fig. 72). It is designed to

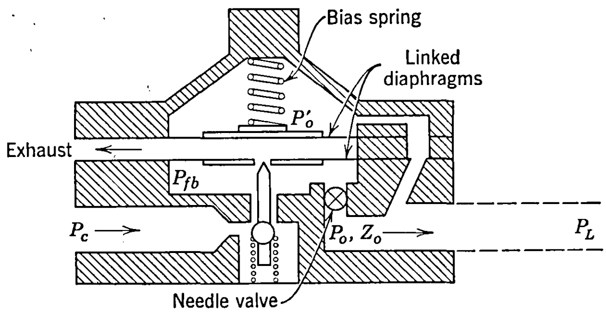


FIG. 72. Moore Model 62 gas flow controller. (Courtesy of Moore Products Company.)

hold flow constant in spite of fluctuations in either upstream or downstream pressure. The principle of operation is as follows.

The adjustable needle valve serves as a set-point adjustment. The regulator then attempts to maintain a constant differential pressure of 1.5 psi across this restriction, thereby maintaining flow constant. If, however, upstream pressure increases (or downstream pressure decreases), the differential pressure across the diaphragms increases, the diaphragms lift away from the exhaust valve, and the lower or inlet valve tends to close. The pressure drop across the inlet valve is therefore increased and the increased exhaust opening permits bleeding off the pressure under the lower diaphragm until the required 1.5 psi across the diaphragms is established again. A decrease in upstream pressure (or an increase in downstream pressure) causes the diaphragms to close down on the exhaust valve and to open the inlet valve wider, until the 1.5 psi differential is reestablished.

Rather than attempt to calibrate the needle valve precisely, most users insert a small rotameter or bubbler bottle in series with the regulator.

Large Flows. For the larger flows commonly encountered in chemical and petroleum plants, self-actuated regulators are sometimes used if control requirements are not too severe or if remote control and signal transmission are not required. An example of this kind of instrument is the Kates flow rate regulator for liquids which is available in ranges from 0.1–1.5 gpm up to 10–100 gpm. The principle of operation is as follows.

The flow enters at the bottom (see Fig. 73), passes into the inlet tube, then emerges through the control valve into the lower chamber. The control valve is a sleeve valve in which the inlet tube with its ports is fixed, while the sleeve with its matching ports is movable, thereby varying the effective valve port area. The flow is then split into two sections. The major portion goes through the adjustable orifice while the controlling flow goes through the fixed orifice. Both flows pass into the upper chamber and from there into the downstream piping. Any change in differential pressure between the upper and lower chambers is multiplied by the

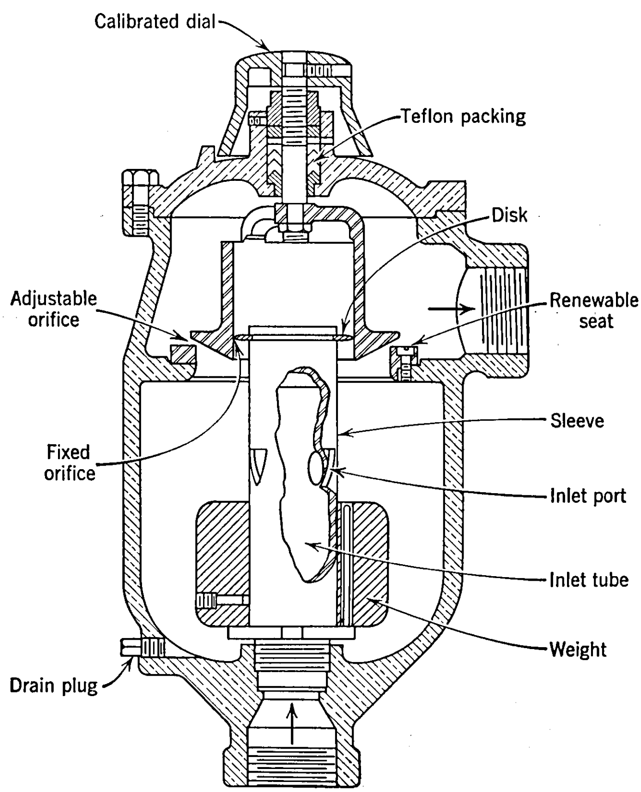


FIG. 73. Kates flow rate regulator for liquids. (Courtesy of W. A. Kates Company.)

area of the disk to produce a force which moves the sleeve until the differential pressure between the two chambers is restored to its proper valve. The desired flow is then obtained by adjusting the calibrated dial on top of the instrument that is connected to the adjustable orifice. The regulator shown is primarily suited for low flows; for higher flows the manufacturer furnishes an instrument which uses a spring rather than gravity for a driving force.

9. CONTROL PANELS

The following discussion will be limited to central control room practice where one or perhaps several central control rooms serve as nerve centers for an entire process. In large chemical and petroleum plants this is virtually standard practice, and should be distinguished from the custom in some industries of decentralizing instruments and controls and scattering them throughout the process. The central control room has the advantages of more convenient, unified process control and usually lower operating labor cost. Its chief disadvantage is somewhat higher investment cost due to the necessity of transmitting measurements from the process into the control room, and control signals back out again.

Conventional. Prior to the advent of miniature, unitized instruments, the major case recorder and recorder-controller were standard. Customarily they were arrayed on the panels in a geometrically regular fashion. Some users still prefer this kind of instrumentation and this arrangement (see Fig. 74). Since its chief disadvantage is its large space requirement (the major case instrument is typically 14 in. by 16 in. in contrast to a typical figure of $4\frac{9}{16}$ in. by $4\frac{9}{16}$ in. for miniature case instruments), some users retain the linear, geometric arrangement of instruments but use miniature case equipment. In either case, the panel is referred to as "conventional."

Graphic. Although there are a number of different kinds of graphic panels, the type which has been most popular consists of a large, picture flowsheet-control diagram on which miniature case recorders, indicators, and manual-automatic stations are appropriately located (see Fig. 75). By means of color coding and suitable symbols, the process streams and instrument functions are identified with a minimum of labeling. Although this kind of panel requires slightly more space than a conventional panel with miniature case instruments, it requires vastly less space than a conventional panel with major case instruments.

The big advantage of the graphic panel is the ease with which an operator may scan the board and tell how the process is doing. The possibility of the operator turning the wrong knob or switch in an emergency is greatly reduced. In addition, new operators are more rapidly trained on graphic panels than on conventional panels.

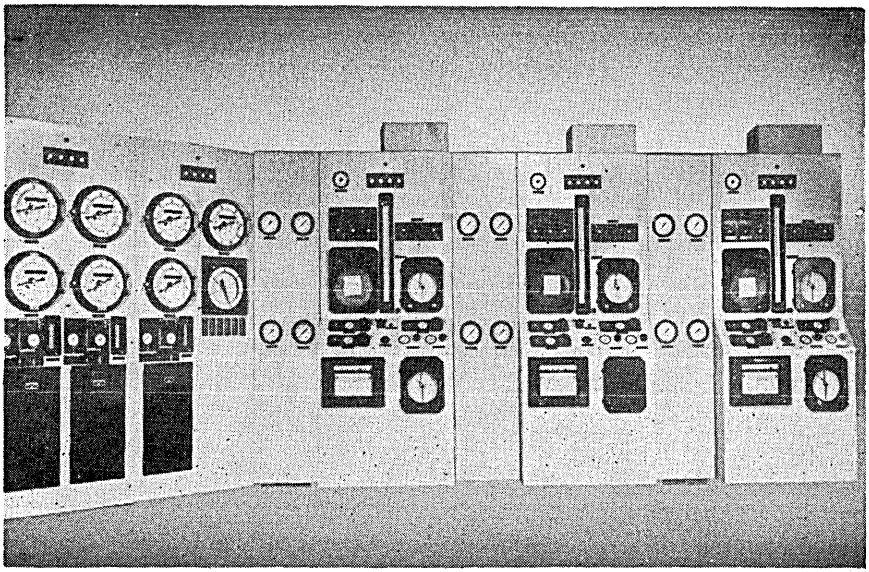


FIG. 74. Conventional control room panels using major case instruments. (Courtesy of E. I. duPont de Nemours & Company.)

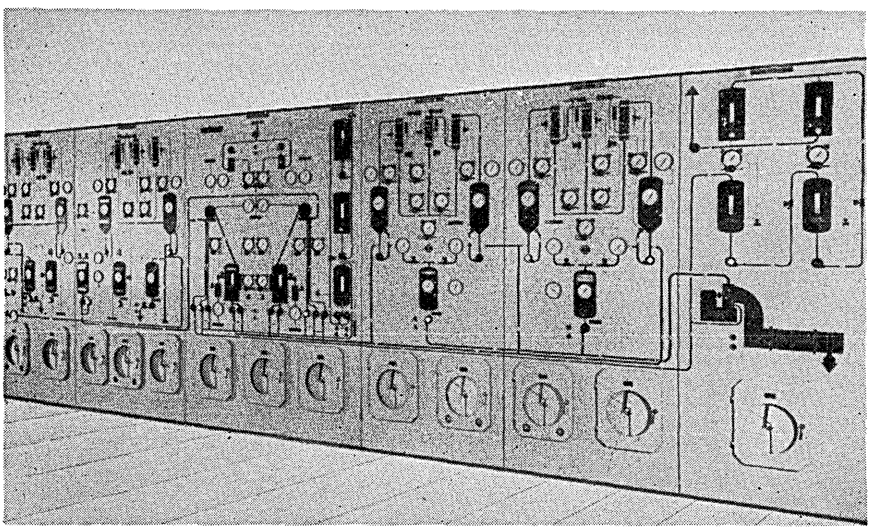


FIG. 75. Graphic panel using miniature case instruments. (Courtesy of E. I. duPont de Nemours & Company.)

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CHEMICAL PROCESS CONTROL SYSTEMS

D. CHEMICAL PROCESS CONTROL SYSTEMS

D. P. Eckman, Editor

Design Procedures

E. F. Holben

1. INTRODUCTION AND TERMINOLOGY

The purpose of this chapter is to outline the process control design procedures employed in the continuous processing industries, such as the chemical and petroleum industries. The emphasis in this treatment is on the control of the process rather than the instrumentation. Instrumentation systems are covered in Chap. 7, and a general discussion of principles and procedures for carrying out systems design is presented in Chap. 1, Systems Design.

This chapter serves as the introduction to Part D, Chemical Process Control Systems. The approach in this and later chapters of Part D is to outline the qualitative and quantitative theory now available for use in the design of process control systems. Chapters 9 to 14 deal with more specific aspects of process control system design, including test methods, single and multiple loop controls, nonlinearities, sampled data, computer control, and data processing. A detailed discussion of instruments, their construction, and their operation is covered in Chap. 7, Instrumentation Systems.

Terminology. The terminology used throughout Part D is that employed generally in the process control industries. In flow diagrams transmitters and controllers are shown as circles. The measuring element and the means for transmitting the measured value is called a transmitter, and is designated by T in a circle. Pressure, temperature, level, and flow transmitters are denoted by PT, FT, TT, and LT.

Controllers, denoted by C, are concerned with the same variables, pressure, flow, temperature, and level and are designated by PC, FC, TC, and LC in a circle. The controller has a set point which may or may not be shown as an input on the flow diagram. In some flow diagrams the transmitter is omitted, and a direct connection from the controller to the variable measured is shown.

A valve is shown as a restriction in the line with a stem attached to a bellows or a knob.

The block diagram symbols are those used in feedback control theory. Lower case letters stand for functions of time. Capital letters stand for Laplace transforms of time functions, and transfer functions are usually given in this form. An asterisk (*) indicates that the quantity is in sampled form. Constants may be either capitals or lower case letters.

2. SPECIFICATION OF QUALITY CONTROL

The general purpose of automatic process control is to provide a specified finished product under a wide variety of process conditions in the most efficient manner. The success of the automatic control system must always be evaluated in terms of the quality of the finished product. At the beginning of every instrument design study, the process design group must specify the desired quality of the finished product and the tolerances of deviation from this quality. The specification can be in the form of many physical quantities. A steam generator plant would define product quality in terms of degrees super heat and steam pressure; a plating plant would define it in terms of thickness of plating material; and most chemical plants would specify quality in terms of chemical analysis, acidity, viscosity, etc. The final determination will usually be a compromise and will, most likely, be arrived at after several design studies are made, cost estimates presented, and alternatives considered.

3. OPERATIONAL FACTORS

Process Variables. Many external and internal conditions affect the performance of a process. These conditions may be expressed in terms of process variables such as temperature, pressure, flow, liquid level, dimension, weight, volume, etc. The process may be controlled by measuring a variable representing the state of the product and automatically adjusting one of the other variables of the process. Ambient conditions must always be included in the list of process variables. The *controlled variable* of the process should be the one that most directly produces the desired form or state of the product. Direct control from product quality is most likely to insure proper performance of the process and to produce and maintain the desired quality of the product. Indirect control from a secondary variable of the process may be necessary when direct control is difficult to accomplish.

The *manipulated variable* of the process is the one selected for adjustment by the automatic controller to maintain the controlled variable at the desired value. The manipulated variable may be any one of the process variables that causes a fast response of the controlled variable and is relatively easy to manipulate.

The *load variables* of a process are all other independent variables except the controlled variable and manipulated variable. It is expected that the automatic controller will correct for fluctuation in load variables and maintain the controlled variable at the desired value.

Process Demand as a Load Variable. One of the most important load variables is the rate at which the finished product is used, called the process demand. It is necessary that the specification of process demand be clearly defined both in magnitude and time, for this will determine the maximum conditions for the control mechanism. Often a control system is handicapped by being called upon to operate under conditions of large variations in process demand for which it was not designed. The effect on the product quality may be such that a system operating satisfactorily with high process demand will produce undesirable results at low process demand.

Generally, a control system should be designed for one condition of process demand. Where variations in usage rate are expected, storage means can be provided to maintain a supply of final product and yet allow a constant feed from the process. In this manner, one load variable can be eliminated and a system design reduced to the control of the manipulated variables under relatively steady-state conditions of the controlled variable. The wise use of storage methods to reduce the magnitude of process demand variations will result in less complex control systems.

Supply Variations as a Load Variable. Although process demand variations probably have the greatest effect on the process quality, the supply of materials to the process may vary quite widely, thus adversely affecting the control system. For example, the gas-fired continuous heating furnace shown in Fig. 1 may have several sources of supply change. If the gas pressure changes, the flow of fuel will be altered; the heat content of the fuel may change appreciably and, therefore, affect the rate of

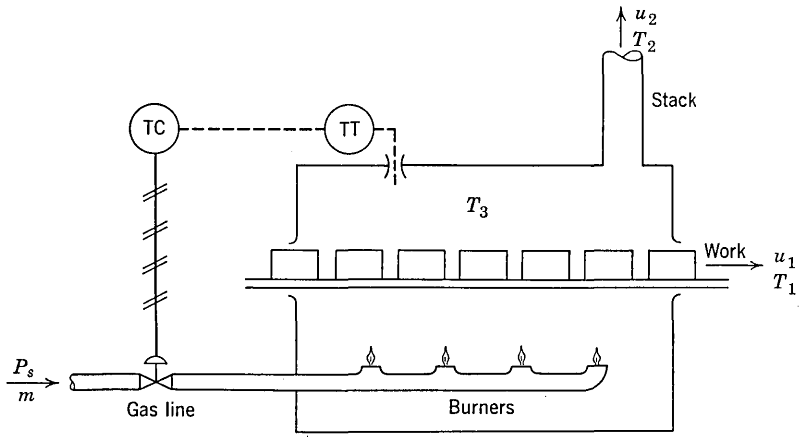

FIG. 1. Gas-fired continuous heating furnace; T = temperature, u = load variable.

heat application to the furnace. Another example is the clogging of the burner which will decrease the flow of fuel for any one valve setting. Therefore, it is necessary that these changes be specified with the same care as demand changes before the instrument design study is undertaken.

Degrees of Freedom of a Process. The complete specification of the load variables does not imply the need for the control of each. Every process can be defined by a set of equations, and there will be a certain number of degrees of freedom to control specific properties of the process. The number of *degrees of freedom* is derived from the following equation:

$$n = n_v - n_e, \tag{1}$$

where n equals number of degrees of freedom, n_v equals the number of variables of the system, and n_e equals the number of defining equations of the system. By definition, the number of independently acting controllers in a system or process may not exceed the number of degrees of freedom. However, there are quite often fewer automatic controllers than degrees of freedom and such systems are usually adequate. The number of automatic controllers to be employed will be determined by the allowable deviation of product quality.

Process Efficiency. Although product quality is probably the most important consideration in a design study, the efficiency with which the product is produced is also of great importance. The best economy is accomplished, if at all, by maintaining all process variables in a predetermined relation, such that the highest efficiency, least waste, and any other criteria are satisfied. To insure that the maximum efficiency of the process is maintained, it is necessary that this efficiency be calculated at various equilibrium conditions from maximum to minimum, considering all supply, load, and demand changes. Although an automatic controller might adequately maintain the state of the controlled variable under a wide range of load variables, this could be done at the sacrifice of process efficiency and, therefore, require a more complex control design in order to maintain the efficiency within practical limits.

Controllability of the Process. The remaining operational factor to be considered (preliminary to the actual system design) is the general controllability of a process. Many processes are comparatively simple and possess a certain degree of inherent stability. On the other hand, some processes possess a confusing array of capacities, lags, and load changes. In order to determine adequately the controller characteristics necessary to maintain the controlled variable at the desired value, it is imperative that the dynamic characteristics of the process be thoroughly understood. For example, to maintain the temperature of a heating oven at a fixed value is relatively simple and can be accomplished within a reasonable degree of temperature tolerances. In order to control the temperature of a process that contains exothermic reactions, however, the system design becomes more complex because the process tends to "run away with itself." In this case, adequate provisions in the control system must be made to compensate for or control the exothermic reactions.

Many flow control problems can be solved by the use of storage tanks. The proper use of storage tanks often results in a self-regulated system and, therefore, the automatic controller's job is greatly simplified. On the other hand, where storage tanks cannot be used and the flows are varying rapidly, the characteristics of the controller must be more complicated and include provisions for maintaining the desired value of the controlled variable under rapidly changing conditions.

4. SYSTEM DESIGN

Up to this point the system design has been mainly concerned with a definition of the control problem. This has consisted of determining the process characteristics under steady-state conditions and also under a wide range of load changes. This information must now be translated into a suitable control system.

Design Procedure and Flow Diagram Construction

A procedure for analyzing a process and applying instrumentation can be given as follows:

1. Divide the plant function into the smallest operation elements or operation units.

2. List all variables—temperature, pressure, flow rate, composition, etc. —that may affect each process operation element.

3. Add to this list ambient temperature, ambient humidity, barometric pressure, sun, and wind conditions.

4. Divide this list of variables into four groups: (*a*) variables to be controlled automatically; (*b*) variables to be measured continuously; (*c*) variables to be measured periodically; and (*d*) variables neither measured nor controlled.

5. For each variable of the automatic control group, select (*a*) a method of measurement that will provide data most indicative of the desired process performance; (*b*) a style of controller, nonindicating, indicating, or recording; (*c*) a mode of control that will provide the desired performance in view of the process dynamics.

6. For each variable of the continuous measurement group, decide (*a*) what method of measurement will provide data most indicative of the desired process performance; (*b*) whether signaling, indicating, or recording is most desirable; (*c*) whether a signal device, indicator, or recorder is to be located at the control center or at the unit.

7. For each variable of the periodic measurement group, decide on a method of performing the measurement and the frequency with which the measurement must be made.

8. Construct a flow diagram. The purpose of a flow diagram is to provide information quickly for use in the process analysis. It should be as simple as possible and yet pictorially describe the process unit. All information pertinent to the control problem should be indicated on the flow diagram, including the fluid or medium being controlled, size of vessels, length and size of piping, location of control equipment, pressures, temperatures, flows, liquid levels, and other process information. Although not necessarily indicated on the flow diagram, assignment of alphabetical letters or symbols to the system parameters should be made at this time. A typical flow diagram is shown in Fig. 2.

Development of Block Diagram from Flow Diagram

Having constructed a flow diagram, the block diagram of the control system should be constructed. From this block diagram will be developed the system equations and performance. The equations for each of the controlled variables will ultimately be combined to define completely the process control system.

The general block diagram is shown in Fig. 3. The diagram illustrates the behavior of the system by depicting the action of the variables of the system. The circle represents an algebraic function of addition; the rectangular box represents a dynamic function such that the output is a func-

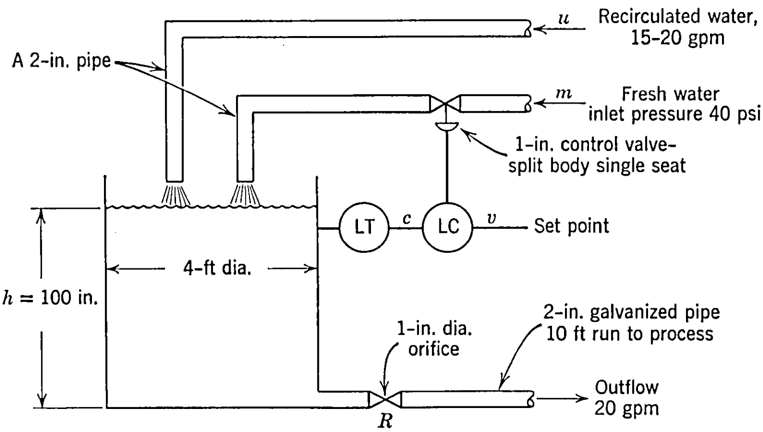

FIG. 2. Flow diagram; c = controlled variable, v = set point, u = load variable, m = manipulated variable, R = restriction.

tion of time and is also a function of the input. Notice that the terms input and output refer to signals and not necessarily to mass and energy flow. A block diagram is not unique, and its arrangement depends on the point of view of the analyst. The important point is that the block diagram illustrates the relation of the variables.

The operational equations for the system shown are of the following type:

$$C = \frac{G_1G_2G_3}{1 + G_1G_2G_3} V + \frac{N}{1 + G_1G_2G_3} U \tag{2}$$

where C = controlled variable,
V = set point,
U = load variable,
G = system transfer functions,
N = load transfer function.

For the moment, the transfer function H will be considered as equal to 1.

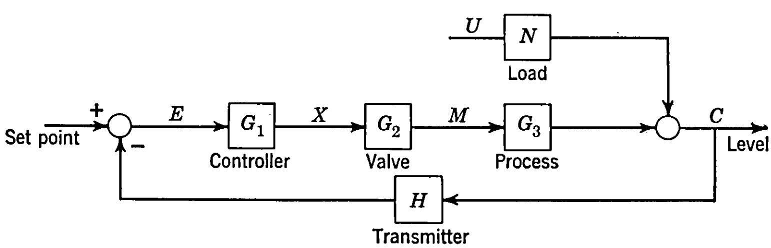

FIG. 3. Block diagram.

The effect of the function H on the process control system will be considered later in the chapter.

These equations give a complete description of the operation of the process unit and will be used to describe both the steady-state and dynamic performance of the process loop under a wide variety of process upsets. All the quantities are Laplace transforms.

Range of Operating Conditions and Transfer Functions

Before beginning the analysis of the process loop, it will be necessary to determine the operating ranges and transfer functions of each block of the diagram. The operating ranges of the controller will be dictated by the process variations and will be straightforward.

Controller Transfer Functions. The transfer function equations for the controllers of the continuous process control variety will be of the following general form:

Proportional Control

$$X = K_c E. \tag{3}$$

Integral Control

$$X = \frac{1}{T_i s} E. \tag{4}$$

Proportional plus Integral Control

$$X = K_c \left(\frac{1}{T_i s} + 1 \right) E. \tag{5}$$

Proportional plus Derivative Control

$$X = K_c (1 + T_d s) E. \tag{6}$$

Proportional plus Integral plus Derivative Control

$$X = K_c \left(\frac{1}{T_i s} + T_d s + 1 \right) E. \tag{7}$$

In these equations,

X = controller output,
E = error signal,
$K_c = \text{gain} = \dfrac{1}{\text{Proportional band}}$,
T_i = integral time, seconds,
s = Laplace operator, d/dt,
T_d = derivative time, seconds.

It is often necessary to simplify the method of analysis of systems by eliminating the mathematics and using graphical methods. The use of sinusoidal testing systems makes it possible to plot the transfer function. The method most often used to represent the transfer function graphically is the Bode plot of magnitude and phase versus frequency. The Bode plots of transfer functions for the various controllers are shown in Figs. 4 to 8. For more details of controllers see Chap. 7, Instrumentation Systems.

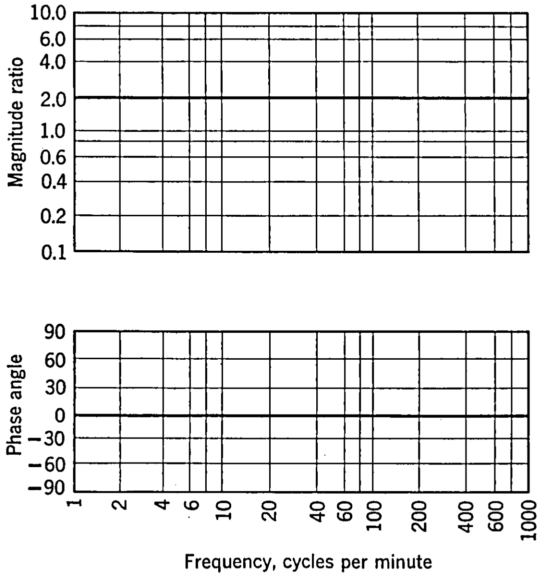

FIG. 4. Proportional control.

The final control element will have to be selected to accommodate the process variations, since this device will be doing the actual controlling of the manipulated variable. From the definition of the problem, the maximum and minimum conditions within the process loop have been determined. These must be translated into the operating range of the final control element.

In good system design the range is usually selected so that the normal maximum condition of the manipulated variable is taken as 70% of the full range of the final control element. This recommendation, of course, applies in general to all types of final control elements such as control valves, metering pumps, rheostats, autotransformers, and so forth (see Chap. 22, Actuators).

Control Valves. The most widely used final control element in the process industry is the throttling control valve. There are many types of

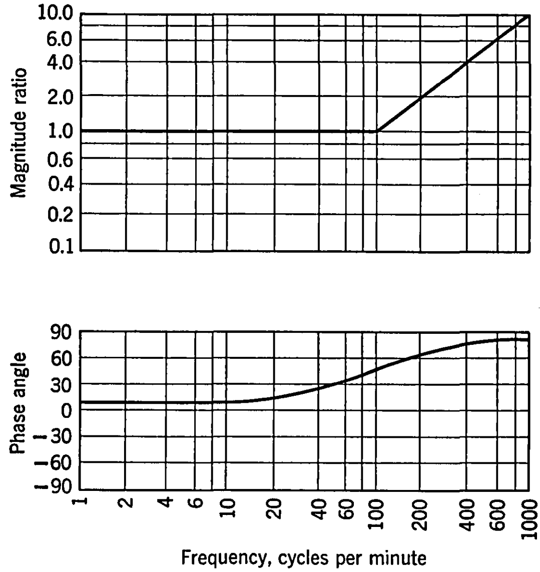

FIG. 5. Proportional-integral control.

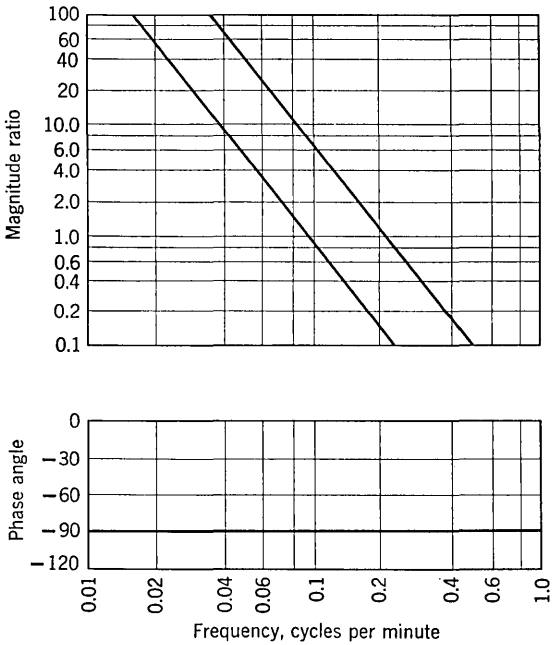

FIG. 6. Integral control.

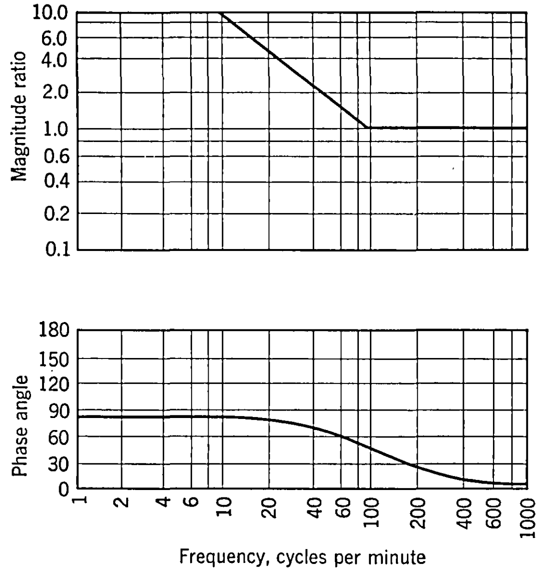

Fig. 7. Proportional-derivative control.

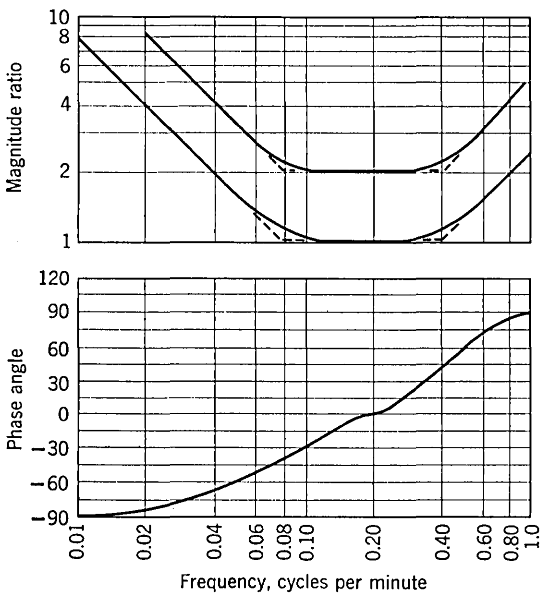

Fig. 8. Proportional-integral-derivative control.

throttling control valves available to meet various control problems. Those most generally used are single seat or double seat plug valves, gate valves, butterfly valves, plug cock valves, and slide valves. In addition to the design parameters already mentioned, the rangeability is quite important. Rangeability is defined as the ratio of the maximum flow of the valve to the minimum flow and generally falls between 20 and 70. For details of valve construction see Chap. 7, Instrumentation Systems.

Most manufacturers of control valves provide nomographs, slide rules, and formulas for determining valve sizes for various conditions and fluids. These are usually derived from the following general formulas:

Liquid

$$v = C_v \sqrt{\frac{\Delta p}{g}} \tag{8}$$

Gases

$$q = 1360 C_v \sqrt{\frac{\Delta p}{gT}} \sqrt{\frac{p_1 + p_2}{2}} \tag{9}$$

Vapor

$$w = K C_v \sqrt{\Delta p} \sqrt{\frac{p_1 + p_2}{2}} \tag{10}$$

where $K = 3$ for saturated steam,

$$K = \frac{3}{1 + 0.0007 T_s} \text{ for superheated steam.}$$

v = liquid flow, gallons per minute,
C_v = valve factor,
Δp = valve pressure drop, pounds per square inch,
g = specific gravity,
q = gas flow, standard cubic feet per minute,
p_1 = upstream pressure, pounds per square inch area,
p_2 = downstream pressure, pounds per square inch area,
T = temperature, degrees Rankine,
T_s = temperature, degrees superheat,
w = vapor flow, pounds per hour.

In the case of single seat or double seat plug valves, selection of the valve characteristic is quite important because of the effect on the automatic controller. Although many characteristics can be obtained, these generally fall into two categories, linear and equal percentage as shown in Fig. 9.

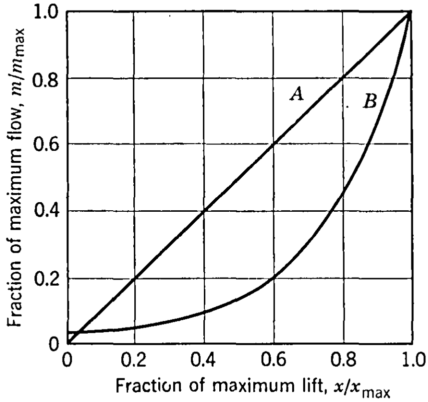

FIG. 9. Control valve characteristic; A = linear type, B = equal percentage type.

The transfer function of the final control element will be determined by the characteristics of the element itself and the response of the actuating mechanism. Final control element mechanisms are usually first or second order systems and can be defined by the following equations:

(11)
$$M = \frac{K_v}{T_v s + 1} X$$

(12)
$$M = \frac{K_v}{T_v^2 s^2 + 2\zeta T_v s + 1} X$$

where M = manipulated variable,
X = final element input,
K_v = final element gain,
s = Laplace operator, d/dt,
T_v = final element time constant, seconds,
ζ = damping factor.

The Bode plots of these functions are shown in Fig. 10.

The transfer function of the process itself is usually difficult to determine before a system is actually built and tested. In some cases, pilot plants or laboratory experiments can be used to approximate closely the actual characteristics. On the other hand, the equations of the process may enable the designer to calculate the expected response. The process functions can usually be broken down into simple single or double capacitance systems. Equations for typical systems are shown as follows:

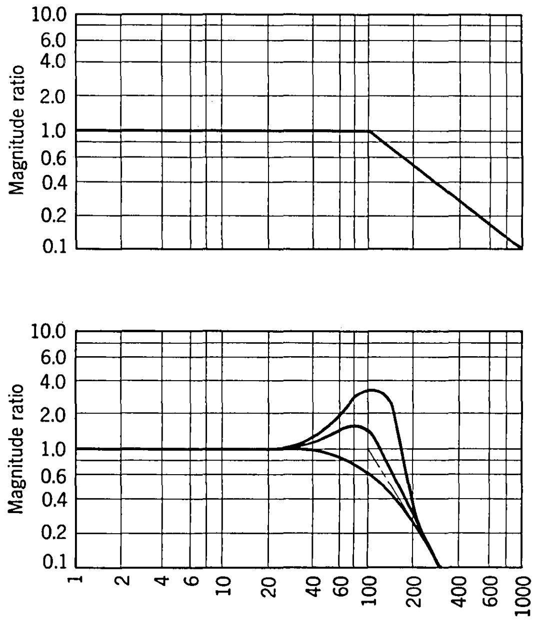

FIG. 10. Final control element response.

Single Capacitance

(13)
$$C = \frac{K_p}{T_p s + 1} M$$

Two Capacitance (general equation)

(14)
$$C = \frac{K_p}{T_p{}^2 s^2 + 2\zeta T_p s + 1} M$$

Two Capacitance ($\zeta > 1$)

(15)
$$C = \frac{K_p}{(T_{p1} s + 1)(T_{p2} s + 1)} M$$

where K_p = process gain,
T_p = process time constant.

System Analysis and Stability Criteria (See Vol. 1, Chap. 20, Fundamentals of System Analysis, and Chap. 21, Stability)

Analysis Procedure. After defining the transfer function of each element of the block diagram, the system can be analyzed and the proper controller settings of T_d, T_1, and K_c determined for best optimum control with stable operation. The insertion of the transfer functions of each element of the loop in the general equations and solving for the loop response is usually cumbersome, unless an analog computer is available for rapid manipulation of the equations. Also, the equations are based on linear theory by definition, and if a more accurate mathematical study of the nonlinear system is dictated, the use of a computer study is an absolute necessity. The graphical use of the frequency response plots of the elements is much simpler and gives an indication of the proper settings of the controller and the final response (see also Chap. 10, Single and Multiple Loop Controls).

The response plots are always made on logarithmic ordinates of magnitude ratio versus logarithmic plot of frequency. By definition, then, each plot can be graphically added together to give the final response.

Determining Optimum Stable Control. The optimum condition for the final response is determined by adjusting the parameters of the various elements. The optimum point is usually the fastest response just short of instability. The stability conditions are defined as follows:

1. The phase lag should not be more than 150° when the magnitude ratio is one or more. The 30° difference between the acceptable and unstable condition is called the phase margin.

2. At 180° phase lag the magnitude ratio should be equal to or less than 0.5. For a magnitude ratio of 0.5 it would be necessary to increase the ratio by a factor of two in order to make it unity and, hence, make the system unstable. The factor by which the magnitude ratio has to be increased to obtain instability is called the gain margin. A gain margin of two is, therefore, desirable for process control.

Effect of Load Changes. Graphical methods of determining system design are simple but do not show the effects of various upsets in the system. In order to analyze the system response, several mathematical checks can be made. The most important is the effect on the controlled variable of changes in the load or changes in the set point of the process. In eq. (2) for a load change, the set point V is taken as zero and the response due to load upset can be calculated. Also, for set point deviations, the load U is taken as zero and the response due to varying the set point can be calculated. As a general rule, process control analysis is concerned with the effect of a change in the load U variable.

The system equation and parameters determined from the graphical plot can now be converted to the differential form by taking the inverse transform of the equations. The purpose of this conversion is to test for response to a step change in the load variable. For example, consider the control of a two-capacitance system shown in Fig. 11. Manipulation

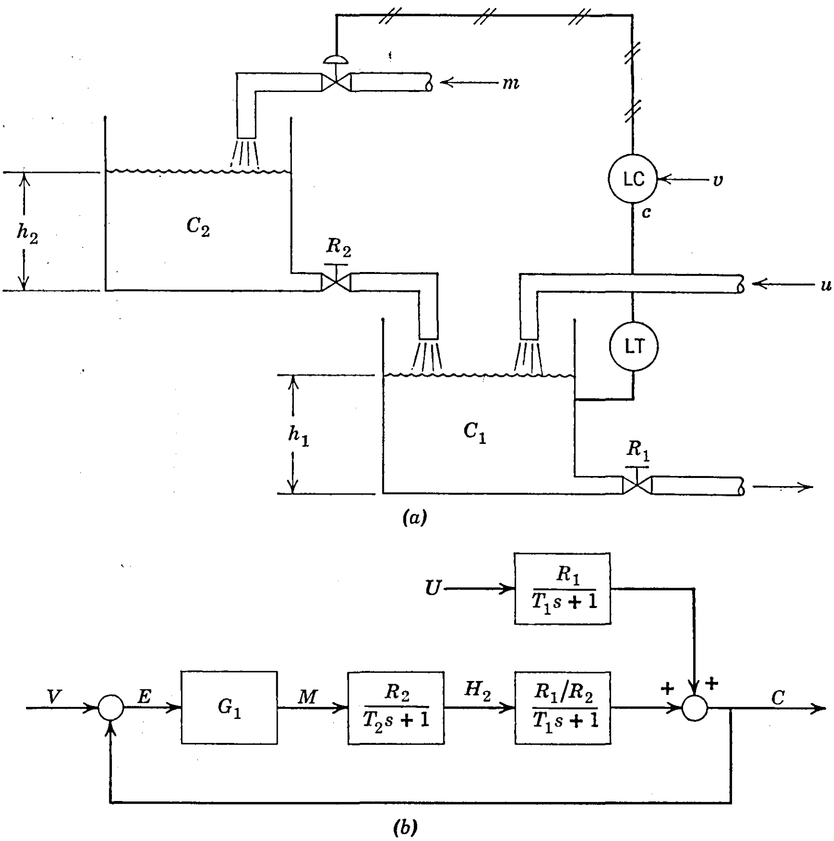

FIG. 11. Two-capacitance process: (a) flow diagram, (b) block diagram.

of the system equations to determine the effect on the deviation E of the process load changes gives the following:

$$(16) \quad E = \frac{(T_1s + 1)(T_2s + 1)}{(T_1s + 1)(T_2s + 1) + G_1R_1} V - \frac{(T_2s + 1)R_1}{(T_1s + 1)(T_2s + 1) + G_1R_1} U,$$

where E = error signal,
V = set point,
U = load variable,
T_1 = lower tank time constant,
T_2 = upper tank time constant,
R = valve resistance,
G = controller transfer function.

Figure 12 shows the deviation of the controlled variable from the original set point as a result of load changes for various modes of control.

1. Proportional-derivative control provides the smallest maximum error because the derivative part of the response allows the proportional sensitivity to be increased to a high value. The stabilization time is the smallest because of the derivative action. Offset is allowed, but is only one-half that experienced without derivative action.

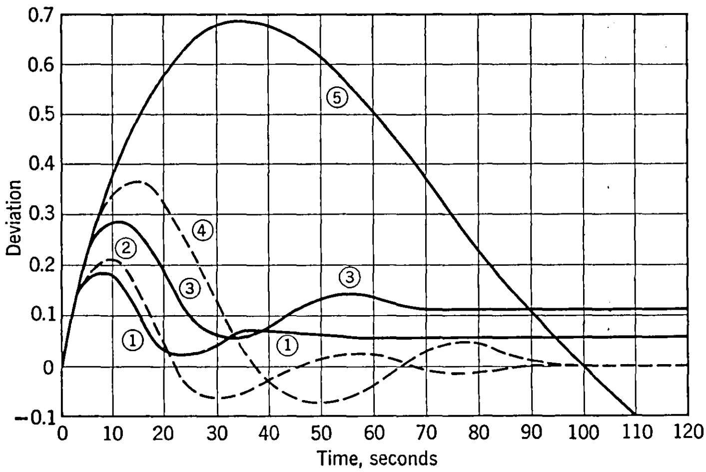

Fig. 12. Comparison of modes of control.

2. Proportional-integral-derivative control has the next smallest maximum deviation, and offset is eliminated because of the integral action. Stabilization time is increased, however.

3. Proportional control has a larger maximum deviation than controllers with derivative action because of the absence of this stabilizing influence. Offset is also larger.

4. Proportional-integral control has no offset because of the integral action. The unstabilizing influence of integral response is reflected in the large maximum deviation and the persisting deviation.

5. Integral control is best suited for the control of processes having little or no energy storage, and the results of the comparison are not representative of an integral control. Moreover, in this process, the results indicate a large maximum error and a long stabilization time.

The selection of the mode of control and the value of the controller function K_c, T_1, and T_d will, of course, determine the characteristics of the deviation. Final selection will depend on the amount of deviation that can be tolerated in view of the desired product quality. For economic reasons, the minimum number of controller functions should be used.

Instrument Location and Transmission Dynamics

Instrument Location. The general selection of the types of instruments to be used has been outlined (see also Chap. 7, Instrumentation Systems). Instruments and controllers are generally grouped at one location near the processing or manufacturing operation they serve. This grouping may be termed a control center. A central grouping has the advantage of coordination of all plant operations, and maintenance of instrumentation equipment is quickly and easily accomplished.

In small plants, all instruments can be grouped at one central control station. However, in larger plants, there may be several control groups for various major portions of the complete plant.

Individual units of instruments are either located in individual enclosures or, as is often the case when the plant is housed in a large building, mounted on open panel boards.

The arrangement of the instruments in the control center or on the control board depends on the type of process or operation being controlled. Very often the use of miniature recording and indicating instruments permits a graphic panel layout. This type of layout is a reproduction of the flow diagram, with the symbols for the controllers or indicating instruments replaced by the devices themselves.

Where larger recording or indicating instruments are desired, the graphic layout can become cumbersome. The layout will then depend upon a grouping of the instruments in an easily read group of similar measurements which will facilitate the rapid diagnosis of plant operation.

Transmission Lags. Although the centralization of recording and controlling instruments at one location is desirable, the distances involved in the transmission of signals between the controlling elements may vary from a few feet to thousands of feet. For pneumatic instruments, the pneumatic transmission is generally operated on pressures of 0 to 20 psig. Generally, the tubing is $\frac{1}{4}$ in. or $\frac{3}{8}$ in. o.d. The lag caused by the tubing results from the resistance and volume of the line. The lag of transmission is generally small, up to 500 feet, but can be tolerated up to 2000 feet providing the lag is reduced by the use of booster pilots. For a detailed discussion of transmission systems see Chap. 7, Instrumentation Systems.

The selection and location of the controllers or measuring means will affect the system dynamics, depending upon the measuring lags involved. The effect of a large measuring lag is almost always to cause large amplitude oscillations and slow return or stabilization. The introduction of a large measuring lag modifies the block diagram of Fig. 3 by the addition of a time element H in the feedback line as shown in Fig. 13.

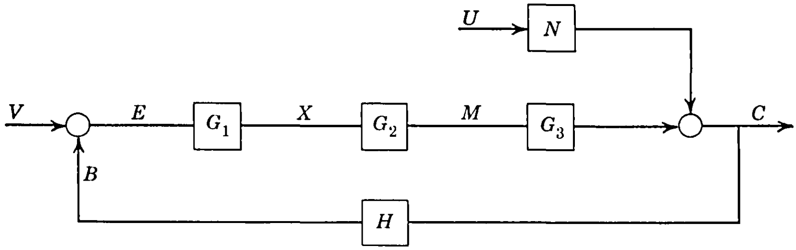

FIG. 13. Process with feedback lag.

The function H can be the transmission lags of the system or the characteristics of the transmitter if one is used in the control system.

The system of eq. (2) is similarly modified to include this element as follows:

$$C = \frac{G_1G_2G_3}{1 + G_1G_2G_3H} V + \frac{N}{1 + G_1G_2G_3H} U. \tag{17}$$

In addition, the equation for the feedback variable or measured variable can be shown to be

$$B = \frac{G_1G_2G_3H}{1 + G_1G_2G_3H} V + \frac{NH}{1 + G_1G_2G_3H} U. \tag{18}$$

The comparison of the response of the controlled variable C and the feedback variable B to a load upset for a given system for two measuring lags, 2 sec and 10 sec, is shown in Fig. 14. Both variables have the same general characteristics, but the feedback variable shows less change and is retarded from the actual controlled variable. Therefore, changes in the controlled variables are always larger in magnitude than those indi-

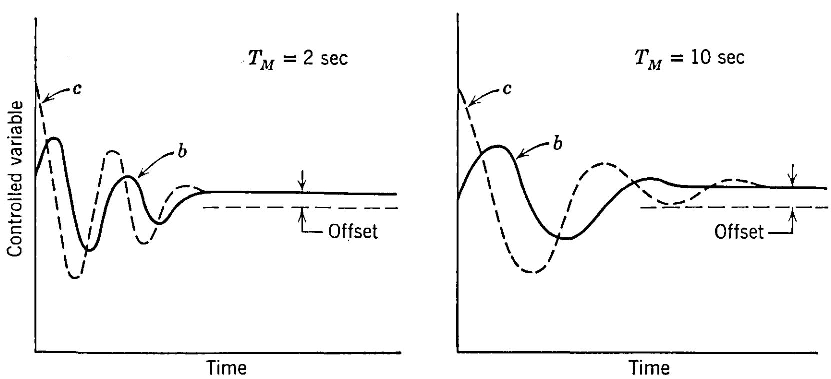

FIG. 14. Effect of measuring lag.

cated by the feedback. Longer measuring lags result in a larger difference between the controlled variable and the feedback variable and also result in a greater offset in the process from the set point. The recovery time is also slower. Although the effect of measuring lag will depend upon the particular system, as a general rule, the measuring lag should be at least one-tenth the largest lag in the process system.

The effect of transmission lags between the controller and the final control element, or the final control element and the process, is to add another time function in the block diagram. In the procedure for determining the controller settings, these lags will have a determining effect if they are relatively large compared to the other lags of the process. As a general rule, the transmission lags should be at least one-tenth the largest lag of the system.

The complete process analysis is determined by combining the individual loop analyses, just as the individual blocks of the block diagram were combined. The degree of success with which the control system maintains the desired product quality and efficiency is a direct function of the thoroughness of the system design. System analysis will many times prevent the embarrassing situation of a violently oscillating process, making evident the effect of a particular variable within the system.

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Process Test Methods

P. E. A. Cowley

1. INTRODUCTION AND TERMINOLOGY

Simple tests of a *controlled system,* that is, process and controller, are often used to obtain data from which optimum controller settings may be calculated. On the other hand, *tests of a process* are performed in order to measure the static and dynamic characteristics of the process. These characteristics may be required for a variety of reasons:

1. To investigate the cause of poor control in an existing control system.
2. To provide the basis for redesign of a control system.
3. To provide an experimental check on analytically determined characteristics.
4. To provide additional data to add to an accumulation of dynamic characteristics of processes of various types.
5. To provide the basis for plant modification or for new plant design.

The emphasis in this chapter will be on methods of testing to determine the process dynamics.

The particular method of testing the dynamics of a process will depend upon the object of the test and will be influenced by the following factors: (1) the special equipment needed for testing, (2) the permissible plant disturbance, (3) the time allowed for the tests, (4) the computational facilities available, (5) cost, and (6) accuracy. The factors are summarized in Table 1.

TABLE 1. FACTORS INFLUENCING THE CHOICE OF TESTING METHOD

Type of Test	Special Equipment	Time Required	Plant Disturbance	Essential Computational Facilities	Accuracy	Cost
Step function (simplest)	None	Short	Large	None	Low	Small
Step function	Recorder	Short	Medium	None	Medium	Small
Frequency response	Sine Wave Generator Recorder	Long	Medium to small	None	High	Medium
Statistical	Recorder	Very long	None	Large	Low to medium	Large

Terminology. The symbols used in this chapter are listed below.

a	a coefficient
$C(j\omega)$	frequency response of a controller
$f(t)$	an arbitrary input disturbance function
G	controller gain
$g(t)$	the process time response to $f(t)$
$H(j\omega)$	process frequency response
$H(s)$	process transfer function
$h_0(t)$	process step function response
$h_1(t)$	process impulse function response
K	constant coefficient
$P(j\omega)$	process frequency response
s	complex frequency variable of the Laplace transformation
T_1, T_2	time constants
T_d	dead time, or transportation delay
σ	a variable of integration
τ	delay, or shift of the correlation functions
$\Phi_{ii}(j\omega)$	power density spectrum at the process input
$\Phi_{io}(j\omega)$	cross power density spectrum between process input and process output
$\phi_{ii}(\tau)$	auto-correlation function of the process input
$\phi_{io}(\tau)$	cross-correlation function of the process input and the process output
ω	angular frequency

2. TUNING A CONTROL LOOP

The operation of *tuning* a controller in an automatic control loop is in effect a process test. This test is the simplest type of test and correspondingly yields the least information about the process. Nevertheless, the information is pertinent and in a form suitable to the task of adjusting the controller to give good control of the process.

There are many procedures for making controller adjustments in tuning a control system. Some of the procedures utilize the so-called "ultimate period" which is found by setting rate and reset action to minima and increasing the gain until the system oscillates. (Rate action expressed as "rate time" in minutes is set to the shortest time. Reset action may be expressed as "reset rate" in repeats per minute or as reset time in minutes. The longest reset time, or the smallest number of repeats per minute, gives the minimum reset action.) When the control system is in the state of just sustaining an oscillation, the product of the controller response and the process response is equal to unity:

$$C(j\omega)\,P(j\omega) = 1. \tag{1}$$

If at the frequency of oscillation (where G is the controller gain)

$$C = -G, \tag{2}$$

then

$$P = -1/G. \tag{3}$$

That is, the process has at the frequency of oscillation a phase shift of 180° and an amplitude ratio $1/G$. Many of the controller adjustment procedures (Ref. 1) then give rules of thumb whereby the controller knobs are set, using only the information gained thus far together with the desired degree of system damping.

To obtain further information about the process, reset or rate action may be introduced to shift the frequency of oscillation. The controller gain is adjusted so that the oscillation is just sustained. For each frequency the process response is obtained as the reciprocal of the controller response:

$$P(j\omega) = \frac{1}{C(j\omega)}.$$

Testing a process by this method is slow, since it may take several hours to determine the exact critical gain required to sustain oscillation.

Accuracy is poor since controller knob calibration cannot be relied upon to better than a factor of two. The only advantages of this method of testing are that no additional instruments are necessary and that the process is not taken "off control" for testing.

3. STEP FUNCTION TESTING

Procedure. The open loop step function test is extremely useful for determining process dynamics when the greatest accuracy is not required and when testing time is restricted. First the process is allowed to "line out" with the air pressure to the control valve at the normal operating value. Then the air pressure is suddenly altered by an increment Δp, and the transient response at the "controlled" variable is recorded. Thus far the test has been conducted without the need for special (test) instrumentation, the assumption being that the regular plant instrumentation has been utilized. The manual loading station can be used to apply the step function in the air pressure to the control valve, and the increment of pressure Δp can be obtained from successive readings of the air gauge at the manual station. This procedure is satisfactory where step amplitudes of 1 to 2 psi are permissible, and the best accuracy of the method is not sought. When circumstances are such that small step amplitudes must be employed, a special setup for testing such as that shown in Fig. 1 is almost essential.

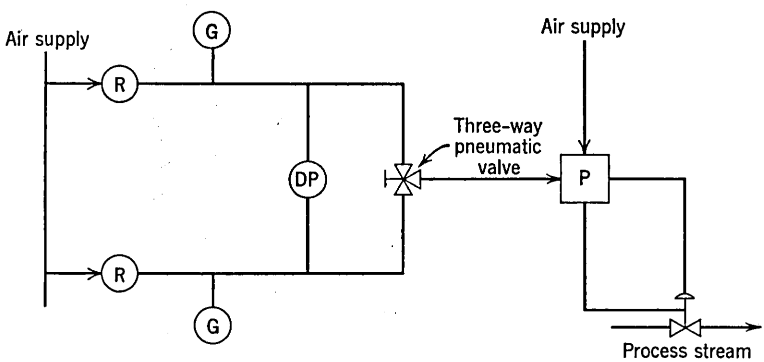
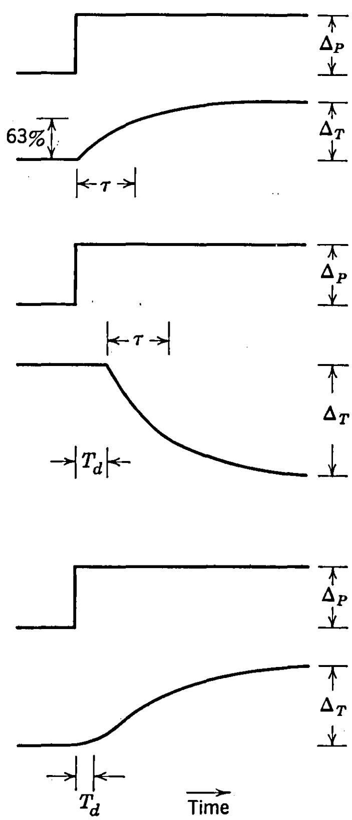


FIG. 1. Equipment setup for step function disturbance; DP = differential pressure, G = pressure gage, P = positioner, R = regulator.

Instrumentation. As recorded on the regular plant instruments, the transient response will in general yield only the crudest information about the process dynamics. The recording instrument that would be ideal for step function testing differs from the regular plant recorders in having (1) greater sensitivity, (2) narrower span (i.e., a large range suppression), (3) faster chart speed, (4) rectilinear recording, (5) less hysteresis, (6) greater linearity, and (7) faster response time. Although improvements in all these characteristics are not necessary for measurements on all processes, the first four are essential to the fullest exploitation of the step function test. For both the input disturbance to the process (i.e., the step function change in valve air pressure) and the process output response (i.e., the step function response of the process) to be recorded simultaneously, a two-channel recording system is desirable.

Graphical Analysis of Step Function Response

Oldenbourg and Sartorius Method. A great deal of information about the process may be gleaned from a quick visual inspection of the step function response. A number of step function responses together with the process characteristics usually determined from them are given in Fig. 2. When the step function response is other than a simple exponential curve, it is necessary to analyze the response to determine the time constants. A useful graphical method of analyzing the response of a system having two time constants is the Oldenbourg and Sartorius method (Ref. 2). The method depends upon finding the point of steepest slope and the slope itself. More convenient for analysis are the quantities T_A and T_C which may be obtained graphically as illus-



Process Characteristics		
Gain	Time Constants	Dead Time
$\frac{\Delta_T}{\Delta_P}$	τ	
$\frac{-\Delta_T}{\Delta_P}$	τ	T_d
$\frac{\Delta_T}{\Delta_P}$	Two or more	T_d

FIG. 2. Process step responses.

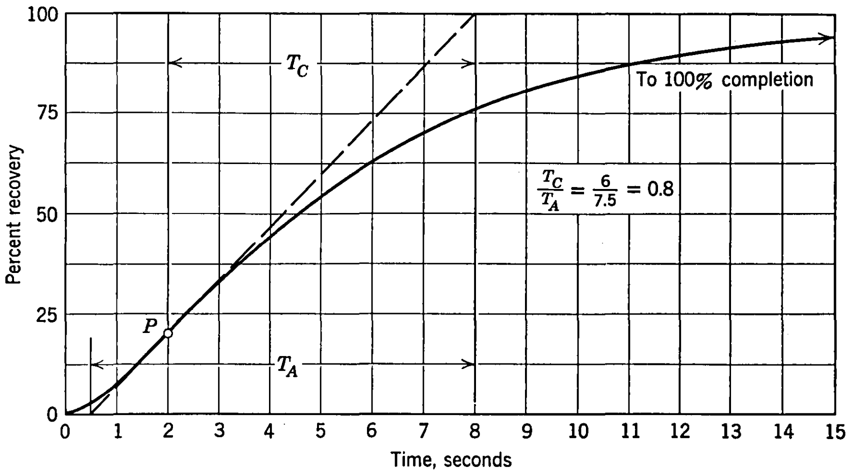


FIG. 3. Oldenbourg and Sartorius analysis of a two-time-constant system.

trated in Fig. 3. In the Oldenbourg and Sartorius diagram (Fig. 4) the ratio T_C/T_A is used as the intercept on each axis of the straight line. The straight line intersects the curve in two points, either of which gives the ratios T_1/T_A and T_2/T_A from which T_1 and T_2 may be found. The graph covers the whole range of possible ratios of T_1 to T_2 from infinity to unity.

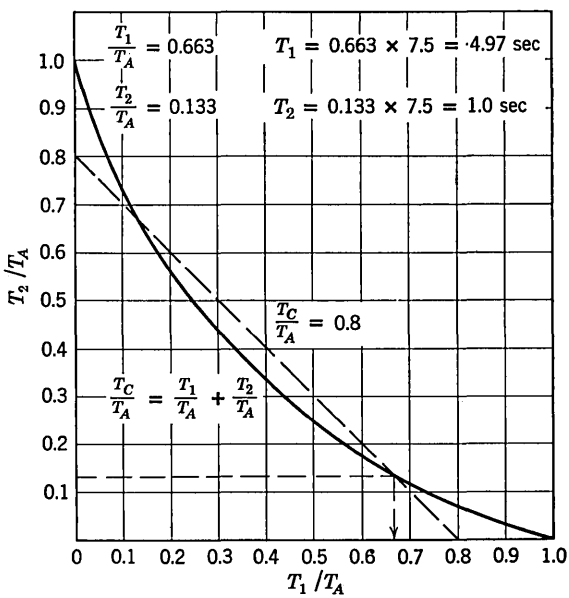


FIG. 4. Oldenbourg and Sartorius diagram.

The case of $T_1 = T_2$ results in the straight line being tangential to the curve.

The Oldenbourg and Sartorius method can be used for the analysis of the step function response of processes involving integration, provided the integral response can first be subtracted from the total response (Fig. 5). The method cannot be applied to analyze the step function response of processes having complex time constants, but such processes, although

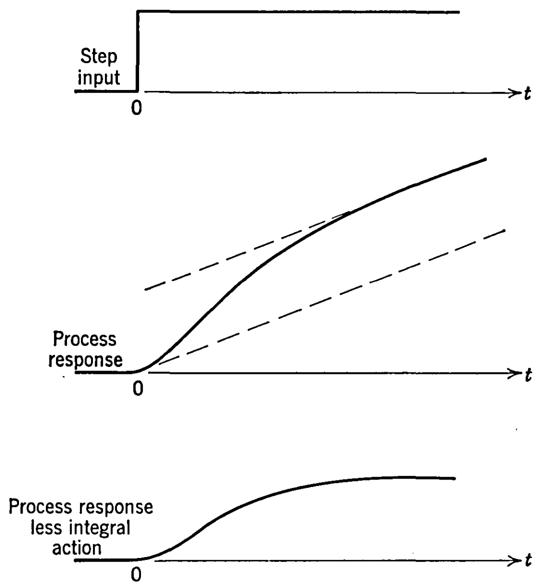


FIG. 5. Removal of the integral response.

common in servomechanisms, have not yet been reported in process control.

The slope intercept method is useful in obtaining a rapid analysis of step function responses when the response consists of the sum of two or three exponential responses. It is readily applied when the time constants are widely separated and becomes progressively more difficult and less accurate as any two of the time constants tend to become coincident. The percent of incomplete response (i.e., difference between the final value of the step function response and the response at any time) is plotted on semi-logarithmic paper (e.g., curve *A*, Fig. 6). One time constant T_1 is found as the time required for the asymptote to fall to 36.8% of the intercept. The numerical difference between the percent incomplete response and the asymptote is plotted in curve *B*, and the process is repeated to extract the second time constant T_2. If curve *B* deviates appreciably

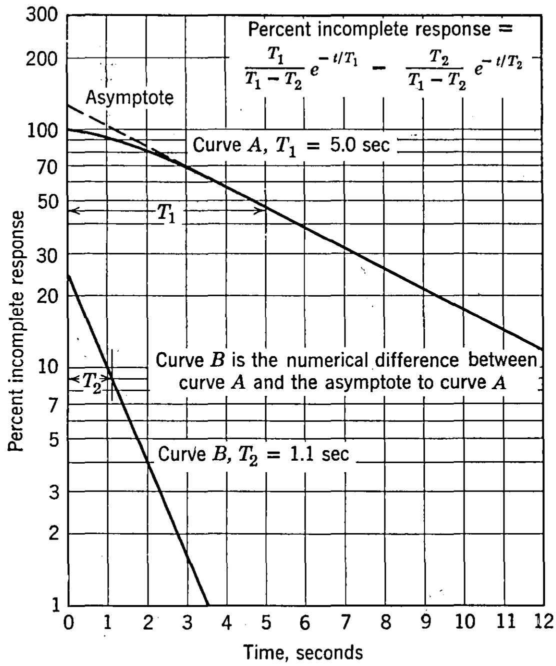


FIG. 6. Slope intercept method of determining time constants from step response.

from a straight line, it may be possible to determine yet a third time constant.

A method has been published (Ref. 3) which in certain cases will yield three time constants from the step response. A number of points are picked off from the step response, and by reference to a family of curves the three time constants are obtained.

Mathematical Analysis of Step Function Response

Where graphical methods of analysis are not sufficient, a mathematical analysis may be performed:

$$H(s) = s\int_0^\infty h_0(t)\, e^{-st}\, dt. \tag{4}$$

The step function response $h_0(t)$ of processes involving integration contains a linear term which it is convenient to remove (Fig. 5). The integral may then be evaluated in two parts. If

$$h_0(t) = h_0^{(1)}(t) + h_0^{(2)}(t), \tag{5}$$

where $h_0^{(1)}(t) = at$, then since multiplication by s is equivalent to differentiation in the time domain and since the derivative of the step function response is the impulse response $h_1(t)$, eq. (4) becomes

$$H(s) = \frac{a}{s} + \int_0^\infty h_1^{(2)}(t)\, e^{-st}\, dt. \tag{6}$$

Note. The step function response is measured and is differentiated to obtain the impulse response. The substitution $s = j\omega$ may now be made:

$$H(j\omega) = \frac{a}{j\omega} + \int_0^\infty h_1^{(2)}(t)\, e^{-j\omega t}\, dt, \tag{7a}$$

$$H(j\omega) = \frac{a}{j\omega} + \int_0^\infty h_1^{(2)}(t) \cos \omega t\, dt - j\int_0^\infty h_1^{(2)}(t) \sin \omega t\, dt. \tag{7b}$$

The integrals in eq. (7*b*) may be evaluated numerically for discrete values of ω. If the step function response $h_0\ (t)$ has been carefully measured with special instrumentation under conditions nearly free of noise and other disturbances, the numerical evaluation may give useful results over a wide range of frequency (e.g., 250 to 1). However, from step function tests conducted in process plants where measurement equipment and conditions are often less than ideal, the frequency range over which useful results may be obtained is not so wide (e.g., 16 to 1). The numerical evaluation may be performed manually but is somewhat tedious even for simple step function responses. A digital computer is desirable to obtain the best results.

The integrals in eqs. (7) may be evaluated graphically (Ref. 11) or by analog computation. Numerous devices and schemes have been devised for this purpose.

The frequency response $H(j\omega)$ is obtained from eq. (7) as a number of points which when plotted may be approximated by a smooth curve. The plot usually results in a great deal of scatter for the higher frequencies. This scatter may be due to (1) the effects of noise and disturbances which occurred during the step function test, (2) sampling errors and simplifying assumptions in the case of digital computation, (3) machine errors in the case of analog computation.

The plot of $H(j\omega)$ may be approximated by the type of transfer function desired.

4. IMPULSE FUNCTION TESTING

The impulse function test can be used as a variation of the step function test. The ideal impulse must be approximated by a pulse of finite amplitude and finite duration. A good choice of pulse duration cannot be made until the process dynamics are approximately known. Furthermore, the pulse amplitude must be large if a reasonable size of impulse response is to be recorded. Hence the test may extend beyond the linearized "small disturbance" range which is implicitly assumed in most discussions of process control theory and practice. Perhaps for these reasons impulse function testing has found little application in the process industry.

Analysis of Impulse Function Response. The impulse function response is the derivative of the step function response. Hence the graphical and mathematical methods of analysis of the step function response may be readily adapted to the analysis of the impulse function response.

Equation (4) becomes

$$H(s) = \int_0^\infty h_1(t)\, e^{-st}\, dt. \tag{8}$$

The impulse function response $h_1(t)$ of processes involving integration contains a constant term which it is convenient to remove. The integral in eq. (8) may then be evaluated in two parts. If

$$h_1(t) = h_1^{(1)}(t) + h_1^{(2)}(t), \tag{9}$$

where $h_1^{(1)}(t) = a$, then

$$H(s) = \frac{a}{s} + \int_0^\infty h_1^{(2)}(t)\, e^{-st}\, dt. \tag{10}$$

Equation (10) is identical to eq. (6) in form. The difference lies only in the method of obtaining $h_1^{(2)}(t)$. In eq. (10) it is obtained directly from the experimental measurement of the impulse function response whereas in eq. (6) it is derived from the step function response by differentiation. The evaluation of the integral may be carried out in a similar manner.

The frequency range over which $H(j\omega)$ may be obtained is greater with impulse function tests than with step function tests. However, to achieve this wider range, it may be necessary to make a number of impulse function tests (using impulses of various duration). Under laboratory conditions frequency ranges of 400 to 1 have been reported (Ref. 4), but under field test conditions in plants the useful frequency range obtainable may be only 32 to 1 or even less.

Arbitary Function Testing. If an arbitary input function $f(t)$ produces a process response $g(t)$, the process transfer function is given by

$$H(s) = \frac{\int_0^\infty g(t)\, e^{-st}\, dt}{\int_0^\infty f(t)\, e^{-st}\, dt}. \tag{11}$$

Testing with arbitary input functions has found little application in the process industry, although the method is used commonly in the aircraft industry, which substitutes triangular pulses for $f(t)$.

5. FREQUENCY RESPONSE TESTING (Ref. 5)

The frequency response test is the standard by which other test methods are judged. The frequency response test is well established in other industries, notably communications and servomechanisms, where it has contributed greatly to the understanding of system dynamics. To carry out a frequency response test the control loop is first put on manual control. The pneumatic sine wave generator is then set to give a high-frequency signal having a small amplitude and a mean pressure equal to that of the air pressure at the control valve head (Fig. 7). The block

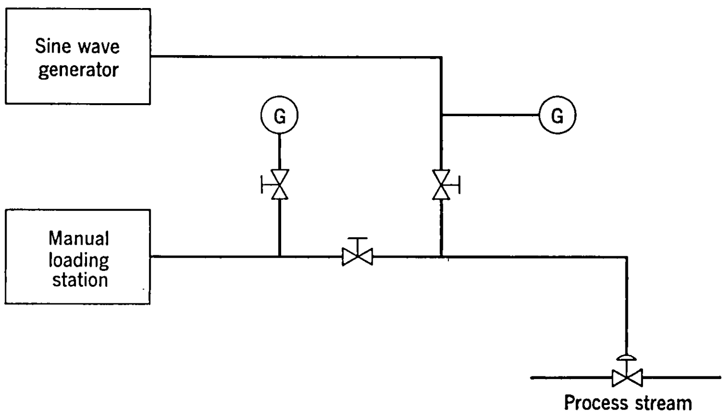


FIG. 7. Equipment setup for sine wave disturbance; G = pressure gage.

valve between the pneumatic sine wave generator and the control valve is then opened, and the block valve between the manual loading station and the control valve is closed. When the "switch" has been accomplished, the amplitude of the pneumatic sine wave may be increased to a relatively large value.

The recording system is arranged to record the controlled variable and also the air pressure on the control valve. Recordings are made over as wide a range of frequency as is necessary for a complete description of the process. The test is usually begun with a high frequency of oscillation, using the largest amplitude of oscillation consistent with obtaining a reasonably linearized measure of the process. The frequency is reduced in approximately octave intervals until the response of the controlled variable becomes apparent. The amplitude ratio and phase shift are then measured from the recording as shown in Fig. 8. Measurements are obtained at approximately octave intervals of frequency, and the amplitude of the sine wave disturbance is reduced if necessary in order to maintain reasonable limits of excursion of the controlled variable. The frequency range over which measurements are made (or attemtped) is that range necessary for the complete description of the process. Quite often this will be to the low-frequency limit necessary to properly locate the longest time constant of the process and to determine the gain (or integration rate) of the process.

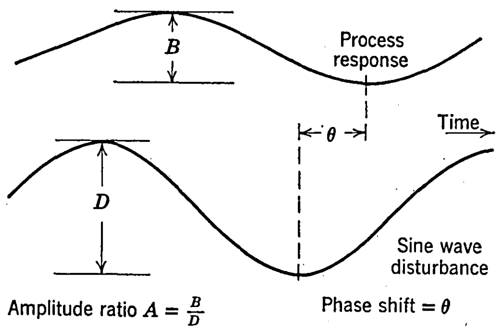


FIG. 8. Amplitude ratio and phase shift.

The measured frequency response is plotted as shown in Fig. 9. Some scatter in the measured points is expected, but usually the best curves drawn through the amplitude ratio points and the phase points are capable of approximation in terms of time constants and the dead time. The measured frequency response curves of Fig. 9 are, for example, representative of a lumped parameter system of transfer function:

$$H(s) = \frac{Ke^{-T_d s}}{(1 + T_1 s)(1 + T_2 s)}, \tag{12}$$

where T_d is the dead time, and T_1 and T_2 are the process time constants.

As with step function tests, frequency response tests are easier under laboratory conditions than under field test conditions. However, even under field test conditions it is usually possible to make measurements over a frequency range of two to three decades.

The upper frequency limit to which frequency response measurements may be made is often established by process noise rather than by instrument sensitivity. There is a low-frequency limit for processes involving an integration owing to the difficulty of introducing extremely low-ampli-

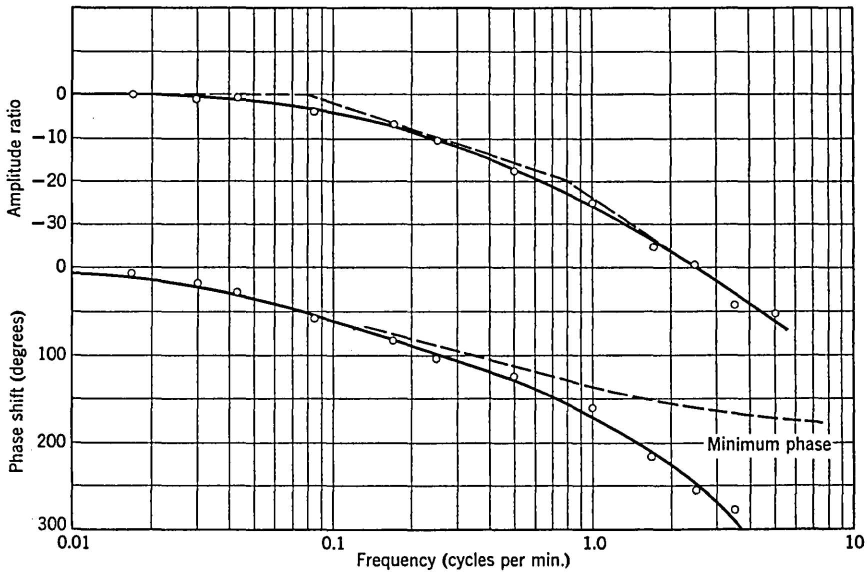


FIG. 9. Measured frequency response.

tude sinusoidal disturbances into the process and the difficulty of maintaining such a process without automatic control for long periods.

Where process noise limits the frequency range of frequency response measurements, special steps may be taken to overcome the difficulty (Ref. 10). Such steps are necessary for the frequency response measurement of processes, such as temperature control of exothermic reactions, which cannot be taken off control.

6. STATISTICAL METHODS FOR THE MEASUREMENT OF PROCESS DYNAMICS (Refs. 6 and 7)

Statistical methods for the determination of process dynamics have been reported (Refs. 8 and 9). The advantages claimed for these methods are that they require no plant disturbances and that normal operation and control need not be changed for testing.

The method involves a considerable amount of computation with data recording, reproduction, and storage. The computation and data handling may be in analog or digital form. General purpose digital computers may be utilized, but the equivalent calculations in analog form have been done by some special purpose computers.

It is required to find the transfer function $H(j\omega)$ of a linear system under excitation by a random stationary function having a power density spectrum $\Phi_{ii}(j\omega)$. If the cross power density spectrum between input and output is $\Phi_{io}(j\omega)$, the following relationship applies:

(13) $$H(j\omega) = \frac{\Phi_{io}(j\omega)}{\Phi_{ii}(j\omega)}.$$

Furthermore, if $\Phi_{ii}(j\omega)$ is essentially flat over the range of ω which is of interest in $\Phi_{io}(j\omega)$, then

(14) $$H(j\omega) = \Phi_{io}(j\omega).$$

Since the cross correlation functions are more readily computed than the cross power density spectrum, these equations may be replaced by

(15) $$H(j\omega) = \frac{\frac{1}{2\pi}\int_0^\infty \phi_{io}(\tau)\, e^{-j\omega\tau}\, d\tau}{\frac{1}{2\pi}\int_{-\infty}^\infty \phi_{ii}(\tau)\, e^{-j\omega\tau}\, d\tau}$$

and

(16) $$H(j\omega) = \frac{1}{2\pi}\int_{-\infty}^\infty \phi_{io}(\tau)\, e^{-j\omega\tau}\, d\tau.$$

Alternately, these equations may be written as

(17) $$\phi_{io}(\tau) = \int_{-\infty}^\infty h(\sigma)\, \phi_{ii}(\sigma - \tau)\, d\sigma.$$

In a novel method (Ref. 8) of "deconvolution," the system impulse response $h(t)$ of an analog is adjusted while being repetitively convolved with the auto-correlation function, which is generated as a time function. Adjustments are made until the result of the convolution has the shape of the cross-correlation function. Equation (17) is then satisfied, and the impulse response of the analog is therefore analogous to that of the process plant on which $\phi_{ii}(\tau)$ and $\phi_{io}(\tau)$ were measured.

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Single and Multiple Loop Controls

J. E. Rijnsdorp

1. INTRODUCTION AND LIST OF SYMBOLS

In principle, process controls are not different from servomechanisms. Their dynamic behavior can be described in the same way, and techniques common to the servomechanism field have frequently been applied to the design of process control systems.

However, there are some differences which mark the individuality of the process control field. In the first place it is difficult to obtain clear-cut specifications for the performance of the process control system, largely because most process controls do not directly control the quality of the plant products, only some easy-to-measure process variable such as temperature, pressure, level, and flow rate.

Another aspect is the lack of quantitative data on the plant disturbances. The latter are often not even random enough to apply correlation techniques such as Wiener's optimization (see Vol. 1, Chap. 17, Smoothing and Filtering).

In the third place, the dynamic behavior of the process varies with process conditions (plant load, etc.), which interferes with optimum adjustment of the controller.

All these considerations emphasize the need for a simple, qualitative theory and semiquantitative rules of thumb. This is the line of thought to be followed in this chapter.

List of Symbols.

A	area of response
a_1, a_2	constants
C	controller (transfer function)
C_1	primary controller } in cascade control systems
C_2	secondary controller } in cascade control systems
D	derivative action
e	2.718...
F	frequency of oscillation
FC	flow controller
FT	flow transmitter
G	coupling factor of two-variable systems $\left(= \dfrac{P_{12}P_{21}}{P_{11}P_{22}}\right)$
H_1, H_2	disturbances
h	height of step disturbance
I	integral action
j	$\sqrt{-1}$
K	gain factor
K_c	proportional gain of controller
K_p	steady-state gain of process
K_{cu}	ultimate proportional gain of controller (at limit of stability)
K_1, K_2, K_3	process gains
LC	level controller
M	transfer function of measuring device
N	number of time constants
n	order number
P	process (transfer function)
P	proportional action
P_a, P_b, etc.; P_1, P_2, etc.	process transfer functions
PC	pressure controller
PT	pressure transmitter
Q	flow rate
RC	ratio controller
S	integration coefficient (see Fig. 23b)
T	dead time

T_d	derivative action time
T_i	integral action time
T_1, T_2, etc.	process time constants
TC	temperature controller
v	slope of constant rate disturbances
X, X_1, X_2	controlled variables
Y, Y_1, Y_2	manipulating variables
α	argument of complex frequency
β	constant
γ	constant
ΔPT	pressure difference transmitter
δ	constant
ϵ	deviation
θ	temperature
μ	gain factor
$\prod$	product of
ρ	modulus of complex frequency
τ	time constant
ϕ	phase angle
ω	angular frequency
ω_{max}	frequency at the peak of the deviation ratio curve
$\bar{\omega}$	complex frequency

2. BLOCK DIAGRAM OF SINGLE LOOP CONTROL

Figure 1 gives an example of a single loop control, the control of the outlet temperature of an oil furnace. The block diagram is shown in Fig. 2.

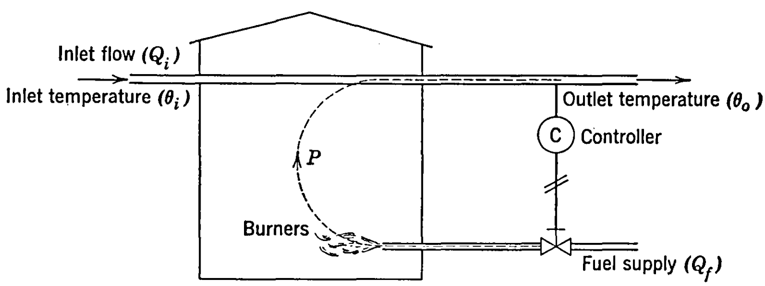


FIG. 1. Control of the outlet temperature of an oil furnace.

Contrary to what is normal in servomechanisms, the set point is generally at a fixed value, so that the process control system is usually a regulator. In other words, the main task of the control system is to reduce the effect of disturbances on the controlled variable. The quality of control can be judged by comparing the remaining deviations in the controlled condition to the requirements.

Sometimes noise has a strong influence on control, for instance in flow control systems where turbulence causes a noise signal in the measuring

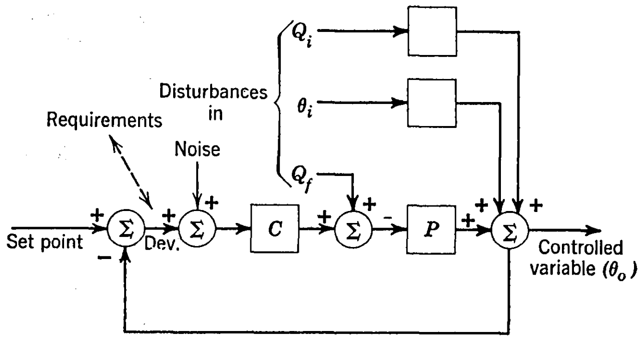


FIG. 2. Block diagram of the furnace control system of Fig. 1.

device. In Fig. 2 all noise sources have been combined into one source at the input of the controller.

3. REDUCTION OF SINUSOIDAL DEVIATIONS

The Deviation Ratio. The most powerful method of studying the dynamic behavior of closed loop systems is by frequency response (see also Refs. 1–7). In this chapter the frequency response of single loop controls are expressed in the so-called deviation ratio, as has been done by Ahrendt and Taplin (Ref. 6) and by Janssen (Ref. 7). The deviation ratio equals

$$\frac{\text{Deviation with control}}{\text{Deviation without control}} = \left| \frac{1}{1 + P(j\omega)C(j\omega)} \right| \tag{1}$$

where $P(j\omega)$ is the transfer function of the process and $C(j\omega)$ is the transfer function of the controller.

Formula (1) is based on the assumption that the system is linear. However, by using describing function methods (see Vol. 1, Chap. 25, Nonlinear Systems), the formula can be extended to nonlinear systems.

Figure 3 gives an example for a furnace control system similar to that shown in Fig. 1 (see also Ref. 13). The frequency has been plotted in dimensionless form; the yardstick is the frequency at which the deviation ratio has its maximum value.

Westcott (Ref. 8) has found an interesting property of the deviation ratio curve:

$$\int_0^\infty \log \left| \frac{1}{1 + PC} \right| d\omega = 0. \tag{2}$$

Expressed in words, this formula says that improvement of control in one frequency region deteriorates control in other frequency regions. Gen-

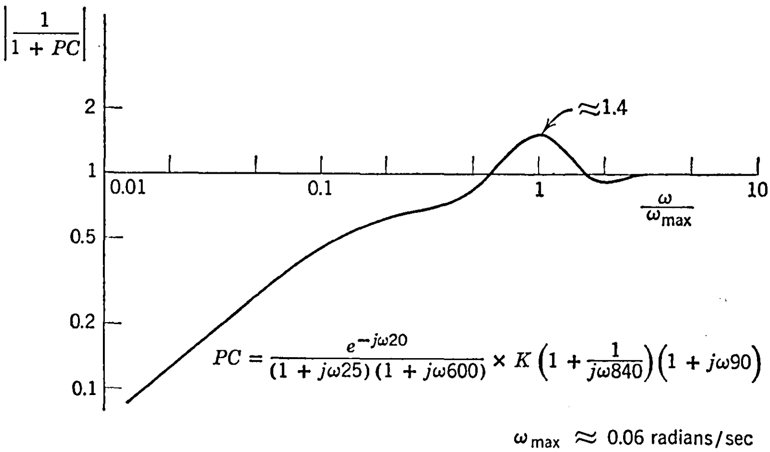


FIG. 3. Deviation ratio of the furnace control system of Fig. 1.

erally good control at low frequencies is desired, namely, a small value of the deviation ratio. The result is that a resonance zone has to be tolerated at higher frequencies (see Fig. 3).

At very high frequencies the loop gain is always small, so that control is ineffective and the deviation ratio is roughly unity.

The most important characteristics of the deviation ratio are the value of ω_{max} (the resonance frequency) and its values for low frequencies, where $|PC| \gg 1$.

As long as noise and saturation do not interfere, a higher value of ω_{max} is desirable because it means a larger bandwidth and a faster response.

The values for low frequencies are a measure of the control of slow disturbances which occur frequently in chemical and refinery processes. This aspect of process control corresponds to the consideration of error constants in servomechanisms (see Vol. 1, Chap. 20, Fundamentals of Systems Analysis).

The height of the resonance peak is not characteristic of the process control system because it can be adjusted to any convenient value by merely changing the gain of the controller. In practice the peak height is seldom larger than 2.

4. TRANSFER FUNCTION OF THE CONTROLLER

The following controller transfer function will be used for the examples of this section:

$$C(j\omega) = K_c\left(\frac{1}{j\omega T_i} + 1\right)\left[\frac{1 + j\omega(1+\beta)T_d}{1 + j\omega\beta T_d}\right], \tag{3}$$

where K_c is the proportional gain factor, T_i is the integral action time, T_d is the derivative action time, and β is a constant (here chosen equal to 0.1).

It is interesting to compare eq. (3) with the corresponding part of the transfer function of a servomechanism:

$$C(j\omega) = \frac{K_c}{j\omega T_i}(1 + j\omega T_i)\left[\frac{1 + j\omega(1 + \beta)T_d}{1 + j\omega\beta T_d}\right]. \tag{4}$$

The first factor represents the integration of the servomotor and the gain of the amplifier, whereas the second and the third factor stand for phase lead networks (see Vol. 1, Chap. 23, Feedback System Compensation).

Thus a process controller with three actions corresponds to a servo with double lead compensation. Many controllers have transfer functions which are different from eq. (3) (see also Refs. 9 and 10). For instance,

$$C(j\omega) = K_c \frac{T_i}{T_i - T_d}\left(1 + \frac{1}{j\omega T_i}\right)(1 + j\omega T_d),$$

$$C(j\omega) = K_c \frac{(1 + \beta)T_d/T_i + [1 + j\omega(1 + \beta)T_d](1 + 1/j\omega T_i)}{1 + j\omega\beta T_d}.$$

A transfer function often used in theoretical work is that of the pure three-term controller:

$$C(j\omega) = K_c\left(\frac{1}{j\omega T_i} + 1 + j\omega T_d\right). \tag{5}$$

However, only a few process controllers operate according to this formula.

Some controllers have only proportional action: $T_i = \infty$, $T_d = 0$ (P controllers). Others have proportional and integral action: $T_d = 0$ (PI controllers); or proportional and derivative action: $T_i = \infty$ (PD controllers).

When the plant is being designed, the type of controller that can be expected to do the job is chosen. In general, the controller actions are not adjusted before the plant is put into operation.

5. DYNAMIC BEHAVIOR FOR SOME TYPICAL PROCESSES

The line of thought followed in this section is the same as Janssen's in Ref. 10. For further information see this reference and the examples provided by Aikman (Ref. 12).

A Process without a Predominant Time Constant

First a process transfer function of the following kind will be considered:

$$P_a(j\omega) = K_p \prod_{n=1}^{N} \frac{1}{1 + j\omega T_n}, \tag{6}$$

where all time constants T_n are of the same order of magnitude and K_p is the steady-state gain of the process.

An *example* is a flow control where the flow is measured with a fast-responding device, such as a force balance pneumatic transmitter FT (see Fig. 4). The dynamic behavior of the transmitter, the pneumatic

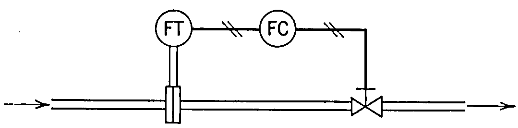


FIG. 4. Flow control system.

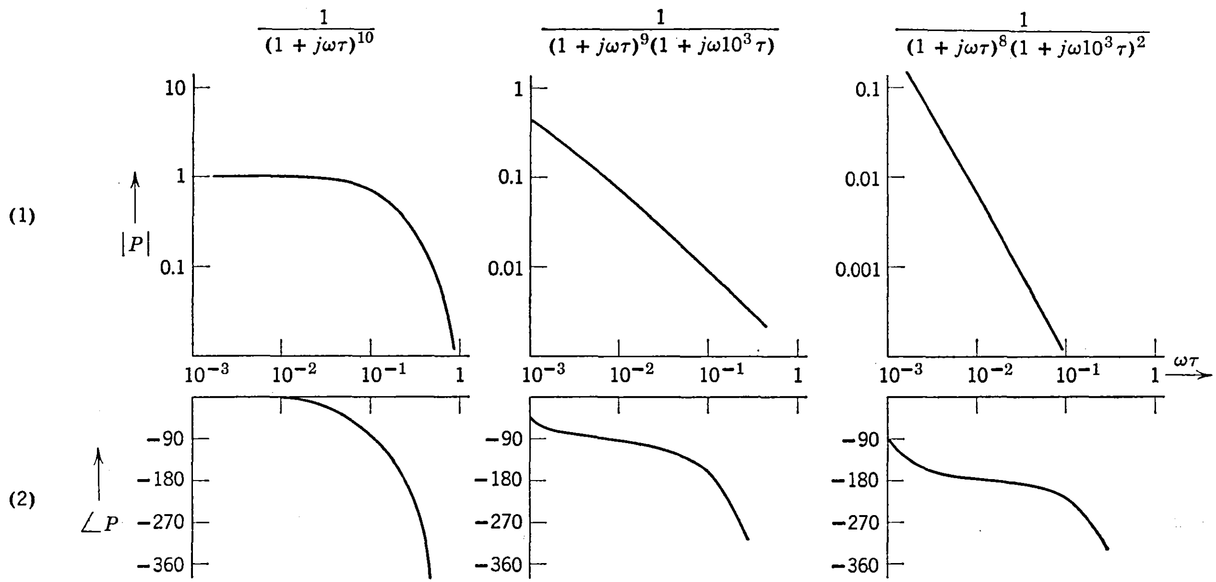
transmission lines, the control valve, and the flow line itself can be represented by a combination of many time constants of the order of a second or less. Then the process transfer function embraces everything in the closed loop except the pure controller transfer function [eq. (3)].

Frequency Response. Figure 5*a* shows the frequency response of the process and the deviation ratio for a P, a PD, an I, a PI, and a PID controller. P stands for proportional action, I for integral action, and D for derivative action. Figure 5*a* has been obtained from measurements on an analog computer. The controller has been adjusted according to the Offereins' method (see Sect. 7, Adjustment of the Controller Actions), with two modifications.

In the case of a PID controller, the integral action is adjusted as if the controller were a PI controller. Before this, the derivative action is adjusted in the normal way. The final adjustment of K_c (K_c/T_i in the case of I control) is such that the height of the peak of the deviation ratio curve equals 2.

Evaluation. It can be seen that proportional action as such is not very useful. The steady-state deviation is rather large, and the application of integral action is highly desirable. On the other hand, derivative action gives very little improvement.

Thus we can conclude that an integral controller or a proportional plus integral controller (which is easier to construct pneumatically) is the best



1 / (1 + jωτ)^10
1 / (1 + jωτ)^9(1 + jω10^3 τ)
1 / (1 + jωτ)^8(1 + jω10^3 τ)^2
(1)
|P|
10
1
0.1
0.01
0.001
10^-3
10^-2
10^-1
1
ωτ
(2)
∠P
−90
−180
−270
−360

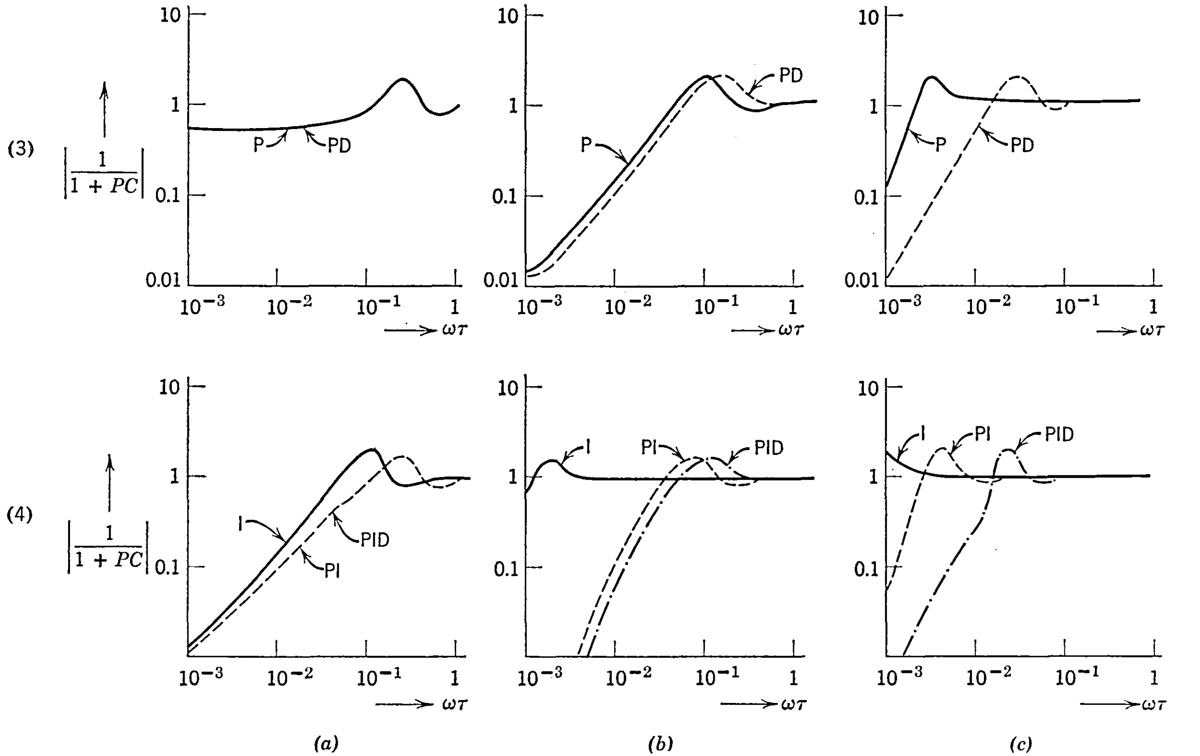


FIG. 5. Frequency response of process with (*a*) no, (*b*) one, or (*c*) two predominant time constants and the associated deviation ratios.

choice. In the latter case, a low proportional gain should be used. This is in line with the experience obtained for flow control systems with fast-responding transmitters (see Refs. 42 and 43).

A Process with One Predominant Time Constant

The transfer function of the process equals

$$P_b(j\omega) = \frac{K_p}{1 + j\omega T_1} \prod_{n=2}^{N} \frac{1}{1 + j\omega T_n} \tag{7}$$

where all time constants T_n are of the same order of magnitude and $T_1 \gg T_n$. An *example* is a flow control system for which the transmitter is a mercury U-tube type with a comparatively large time constant.

Another *example* is the pressure control on a buffer vessel of a refinery gas system (see Fig. 6). The transfer function between control valve and

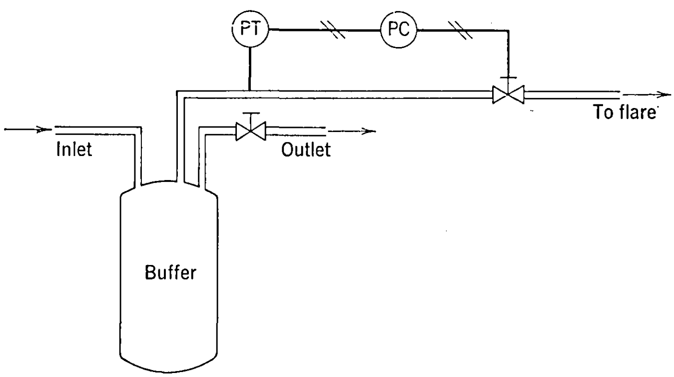


FIG. 6. Control of the pressure in a refinery gas system.

the pressure in the buffer is that of a large time constant, whereas the rest of the system is practically identical to the fast flow control shown in Fig. 4.

Frequency Response and Evaluation. Figure 5*b* shows the frequency response of the process and the deviation ratio for various types of controllers. It can be seen that proportional action is essential; without it control is very slow. On the other hand, derivative action does not give much improvement. Thus it can be concluded that a proportional plus integral controller is very suitable, whereas a proportional controller may be sufficient when it can be adjusted to a high proportional gain.

It is not advisable to use a P controller when the steady-state gain of the process is high. The effect of disturbances at the inlet of the process is then amplified by the high process gain, and the resulting offset can be considerable. Thus integral action is desirable and a PI controller should be used.

A Process with Two Predominant Time Constants

The transfer function of the process equals

$$P_c(j\omega) = \frac{K_p}{(1 + j\omega T_1)(1 + j\omega T_2)} \prod_{n=3}^{N} \frac{1}{1 + j\omega T_n} \tag{8}$$

where all time constants T_n are of the same order of magnitude and $T_1 \geq T_2 \gg T_n$.

An *example* is the heating furnace control discussed in Ref. 13 (see also Fig. 1). The large time constant is determined by the residence time of the oil in the furnace, the second time constant is that of the temperature-measuring element, and the small ones represent lags in the transmission lines, control valve, fuel supply, and the heat transfer from the burners to the oil in the furnace tubes. In this particular case heat transfer is fast because a great deal of it goes by direct radiation.

Frequency Response and Evaluation. Figure 5*c* shows the frequency response of a process according to eq. (8) and the deviation ratio for various types of controllers. It can be seen that derivative action is very effective because it increases the resonance frequency appreciably and makes possible a much better control for very low frequencies. The conclusion is that a controller with three actions is most suitable, whereas a proportional plus derivative controller can be used when it can be adjusted to a high proportional gain.

Limitations of Derivative Action

In practice it often happens that derivative action does not give as much improvement as might be expected from linear dynamic behavior. The reasons are

1. As has been reported by Farrington (Ref. 14), Boyd (Ref. 15), and Aikman (Ref. 41), the presence of dead zone effects ahead of the derivative action unit can have an unfavorable influence.
2. The control system should distinguish between signal and noise. When there is much high-frequency noise, the bandwidth of the system should be made rather narrow. Thus derivative action, which increases the bandwidth, can be disadvantageous.
3. Derivative action gives fast corrections. This can disturb the operation of other processes and control systems.
4. The adjustment of derivative action is rather critical and adds another degree of freedom to the controller. Therefore, in practice, controllers with derivative action are not always adjusted as they should be.

Summary of Control Systems for Processes with Zero, One, or Two Dominating Time Constants

Approximations. The process transfer functions, considered up until now, all contain a series of small time constants. A convenient approximation for such a series is a pure time delay or dead time:

$$e^{-j\omega T}, \tag{9}$$

where T is equal to the sum of the small time constants. Hence a good approximation of many process transfer functions is one by two time constants and a dead time, as long as second derivative action is not used. When first derivative action is also not used, process transfer functions can be approximated by a time constant and a dead time, or a reaction rate, S, and a dead time, T (see the transient response shown in Fig. 7). These approximations have been used rather often in theoretical work (see e.g. Refs. 19 and 23).

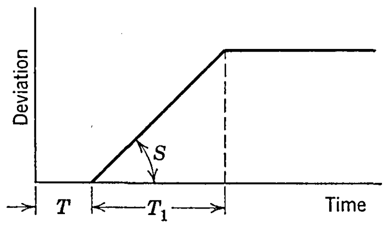


FIG. 7. Transient response showing dead time and reaction rate.

Suitable Types of Controller Transfer Functions. Figure 8 gives a survey of the most suitable types of controller transfer functions for control of processes with two time constants and a dead time. Its main value lies in showing where additional controller actions do not give much

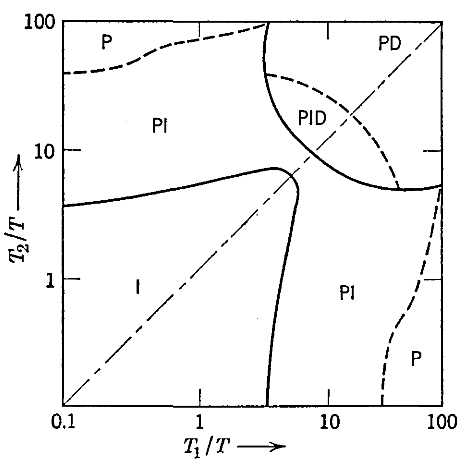


FIG. 8. Areas where various controller actions are must useful.

improvement. The cases shown in Figs. 5*a*–*c* correspond to the extreme points in Fig. 8, i.e., they correspond to the corner points. The diagram has been constructed from measurements on an analog computer. The adjustment of the controller is the same as that used for the construction of Fig. 5 (see earlier subsection on A Process without a Predominant Time Constant).

The boundary lines of Fig. 8 are the following.

1. The borderline between I and PI is the locus of the points where K_c/T_i [see eq. (4)] is two times as large for the PI controller as for the I controller. This factor 2 is considered to give enough improvement of control to justify the addition of proportional action.

2. The borderline between PI and PID is the locus of the points where K_c of the PD controller is two times larger than K_c of the P controller. The integral action has been omitted because the measurements indicated that it makes control here very slow.

3. The dotted lines represent the loci of the points where the steady-state loop gain with a P or a PD controller equals 30.

It must be emphasized that Fig. 8 does not provide a complete guide to the choice of controller types, which requires many additional data, such as the pattern and sources of disturbances, specifications, prices, amount of noise, etc. For instance, the application of derivative action can be disappointing (see earlier subsection Limitations of Derivative Action); and strong disturbances at the inlet of the process sometimes require the application of integral action (see earlier subsections on processes with one and two predominant time constants, and Using the Frequency Response of the Process in Sect. 7).

Processes with Different Behavior for Low Frequencies

When a process has one predominant time constant, the low-frequency part of its frequency response resembles Fig. 9, curve *a*. Multiplication of the steady-state gain K_p and of the predominant time constant T_1 by the same factor hardly changes the process transfer function except for very low frequencies (see curve *b* of Fig. 9). Thus the choice and adjustment of controller actions are hardly affected.

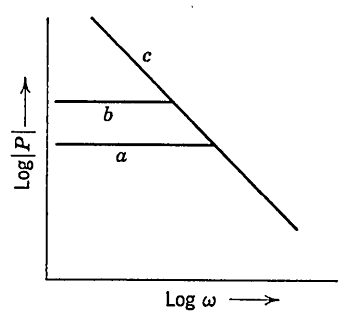


FIG. 9. Different low-frequency behavior of processes with identical high-frequency behavior.

It can be concluded that for processes with one predominant time constant the actual magnitude of this time constant is not very important. More important is the ratio K_p/T_1:

$$P_d(j\omega) = \frac{K_p}{1 + j\omega T_1} \prod_{n=2}^{N} \frac{1}{1 + j\omega T_n} \approx \frac{K_p}{T_1} \frac{1}{j\omega} \prod_{n=2}^{N} \frac{1}{1 + j\omega T_n}, \tag{10}$$

when $T_1 \gg T_n$. Formula 10 is exact when the process contains a pure integration (see Fig. 9, curve c). This is true for most liquid level processes in buffer tanks and accumulators.

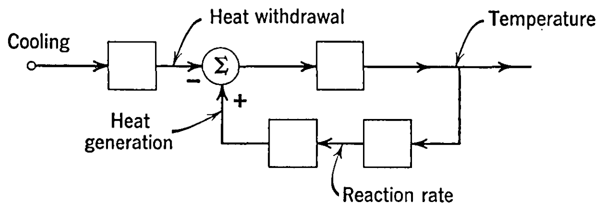


FIG. 10. Exothermic chemical reaction giving rise to positive feedback.

Unstable Process Transfer Functions

Instability often occurs for exothermic chemical reactions. There the process contains a positive feedback loop (see Fig. 10) because an increase in temperature increases the reaction rate, which in turn increases the production of heat by the reaction and thus also the temperature.

When the steady-state gain of the feedback loop exceeds 1, the process transfer function has a positive real pole, and the process is unstable. However, the combination of process and controller can be stable if the controller gain is neither too high nor too low. This can be seen from the Nyquist diagram (Fig. 11) where the point -1 must lie between A and B. (The polar plot must encircle the point -1 once in a counter-clockwise direction because of the positive real pole.) Such a control system is said to be conditionally stable.

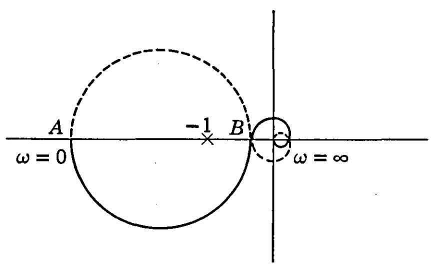


FIG. 11. Nyquist diagram for the process of Fig. 10.

Process Transfer Functions with Parallel Paths

Many processes contain parallel paths between input and output (see Fig. 12). The overall transfer function can be quite complicated, and approximation by two time constants and a dead time might fail.

Moreover, some peculiar phenomena might occur, such as one or more zeros with a positive real part. The transfer function is then nonminimum phase (see Ref. 16). An *example* is the response of steam drum level in a boiler to variations in heat supply. A sudden increase in heat supply increases the formation of steam in the tubes, which lifts the level in the drum (see Fig. 13). Simultaneously, the total amount of water in the boiler decreases at a constant rate because of the increased steam production. The total transfer function equals

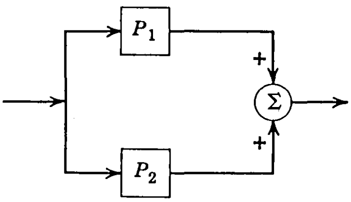


FIG. 12. Parallel paths between input and output of a process.

$$K - \frac{1}{j\omega T} = \frac{-1 + j\omega KT}{j\omega T}. \tag{11}$$

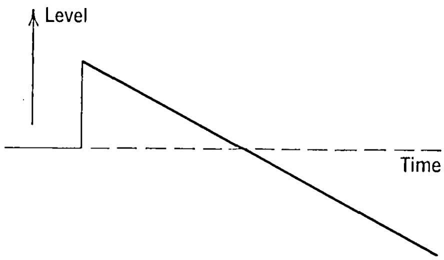


FIG. 13. Transient response of the drum level in a boiler.

With parallel paths it is also possible that the control system is conditionally stable (see Vol. 1, Chap. 21, Stability). An *example* is pressure control in a distillation column, where the accumulator level is controlled by the vaporous top product (see Fig. 14). A block diagram of the process is shown in Fig. 15. A change in cooling water supply to the condenser influences the rate of condensation. This has a direct effect on column pressure and an indirect effect via the accumulator level, the level controller LC, and the vaporous top product flow.

This parallel combination in the process gives a transfer function $1 + K_1K_2/j\omega$, which gives 90 degrees phase lag at low frequencies. When integral action is used in the pressure controller PC, the total phase lag at low frequencies becomes 180 degrees. Thus, the Nyquist diagram resembles Fig. 16; it is evident that the control system is conditionally stable.

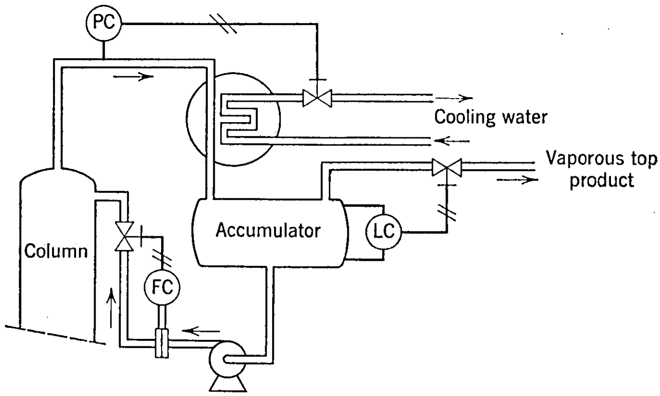


FIG. 14. Pressure control in a distillation column.

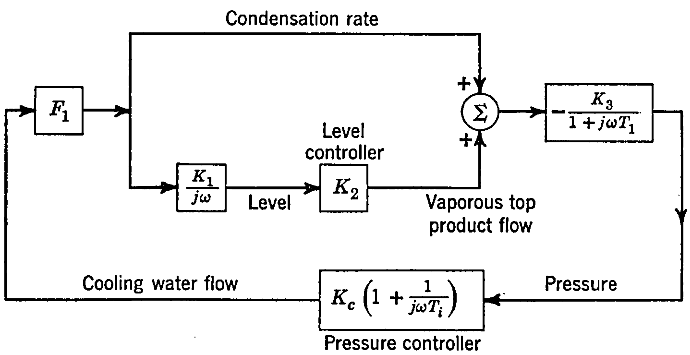


FIG. 15. Block diagram of the process of Fig. 14.

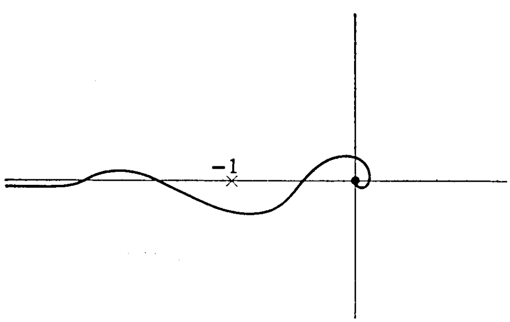


FIG. 16. Nyquist diagram for the process of Fig. 14.

6. RESPONSES TO STEP AND CONSTANT RATE DISTURBANCES

In this section no exact analysis of the transient responses of process control systems is given. Instead, rules of thumb are used, which can help to assess performance quickly.

Response to a Step Disturbance in the Controlled Variable (Output of the Process)

Figure 17*a* shows the general character of this response for a not too conservative adjustment of the controller. It is the "mirror image" of the response to a step change in the set point (see Fig. 17*b*). The following geometric data more or less determine the form of the response.

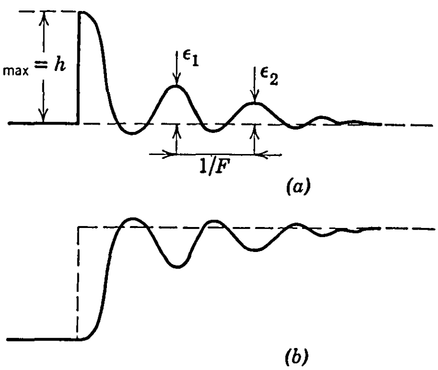


FIG. 17. Response to a step disturbance: (*a*) in the controlled variable, (*b*) in the set point.

1. Maximum Deviation (ϵ_{max}). This simply equals the height of the step change h.

2. Subsidence Ratio (S.R.). The degree of damping of the oscillation can be described by the ratio between two successive amplitudes in the same direction (ϵ_2/ϵ_1), the so-called subsidence ratio. Its value depends very much on the setting of controller actions and is often used as a criterion for the adjustment of the proportional gain factor (see Sect. 7, Adjustment of the Controller Actions).

3. Frequency of Oscillation (F). The frequency of oscillation is roughly equal to the frequency at the peak of the deviation ratio curve.

4. Total Area (A). The time integral of the response or total area equals

$$\int_0^{\infty} \epsilon\, dt = A = \frac{T_i}{K_p K_c} h \tag{12}$$

where T_i is the integral action time.

Response to a Step Disturbance in the Manipulated Variable (Input of the Process)

Figure 18 shows the general character of this response. The frequency of the oscillation is the same as in Fig. 17. However, the subsidence ratio

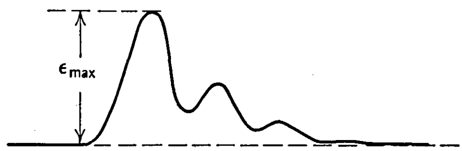


FIG. 18. Response to a step disturbance in the manipulating variable.

is not a good measure for the degree of damping of the oscillation because the nonoscillatory modes in the response are generally rather strong.

1. Maximum Deviation. Rutherford (Ref. 2) and Aikman (Ref. 3) give a simple formula for the value of the maximum deviation:

$$\frac{\epsilon_{max}\text{ (without control)}}{\epsilon_{max}\text{ (with control)}} \approx K_c K_p. \tag{13}$$

This ratio is called the deviation reduction factor. It is often used as an index of controllability, especially in the English literature.

The maximum deviation without control generally equals the steady-state gain of the process, multiplied by the step height:

$$\epsilon_{max}\text{ (without control)} = K_p h. \tag{14}$$

Thus eq. (13) can be written in the form

$$\epsilon_{max}\text{ (with control)} \approx h/K_c. \tag{15}$$

This formula is more convenient when the steady-state gain of the process is unknown.

2. Total Area. The total area of the response equals

$$A = T_i h/K_c. \tag{16}$$

The ratio of the total area and the maximum deviation gives some measure of the duration of the response.

Response to a Step Disturbance Entering at an Arbitrary Point (see Fig. 19)

1. Maximum Deviation. Janssen gives the following expression (Ref. 11):

$$\epsilon_{max} = \gamma h|P'(j2\pi F)|, \tag{17}$$

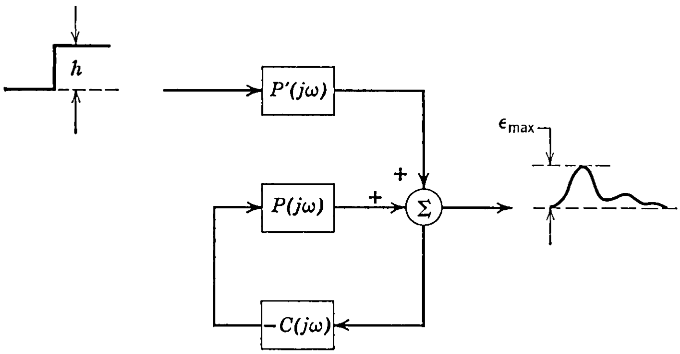


FIG. 19. Response to a step disturbance entering at an arbitrary point.

where $\gamma \approx 1.5$. It is based on the assumption that the oscillatory mode is dominant in the response. Thus it is inaccurate when P' has much attenuation for the resonance frequency, or

$$|P'(j2\pi F)| \ll |P'(0)|.$$

Another formula, based on Paynter's analog between pulse responses and probability distributions (see Refs. 17 and 18), is the following:

(18) $$\epsilon_{\max} = \delta \frac{h|P'(0)|}{F\sqrt{\Sigma T'^2_n}},$$

where the T'_n's are the time constants of the transfer function P'. This expression has been checked on an analog computer for a wide range of processes and practical controller adjustments (Ref. 44). The average value of δ was 0.14, and 92% of the results were in the range $0.07 < \delta < 0.21$.

When one of the time constants of response P' dominates, eq. (18) can be simplified to

(19) $$\epsilon_{\max} = \delta \frac{h}{F} \frac{|P'(0)|}{T'_1}$$

The last factor indicates that it is not necessary to know $P'(0)$ and T'_1 separately. Only their ratio must be determined. Equations (18) and (19) are not to be used when the control loop has a double integration.

2. Total Area. The total area is given by

(20) $$A = \frac{hT_i}{K_pK_c} P'(0).$$

Responses to Constant Rate Disturbances

These can be found from corresponding responses to step disturbances by means of integration. The steady-state deviation is typical. It can be found directly from the total area of the corresponding step response:

$$\epsilon_{\text{steady-state}} = A(\text{step}) \times v, \tag{21}$$

where v is the slope of the constant rate disturbance.

7. ADJUSTMENT OF THE CONTROLLER ACTIONS

The three principal approaches of setting the controller actions are

1. Using the limit of stability of the control system.
2. Using the transient response of the process.
3. Using the frequency response of the process.

However, practical considerations influence the choice of the final settings. They will be discussed first.

Practical Considerations

In general, it can be said that no adjustment procedure is foolproof. In most practical applications the final settings should be obtained by trial and error, taking into account the specific requirements of the case.

Discrepancies between Theory and Practice. There are several reasons for the discrepancy between the theoretical and the practical setting of the controller actions.

1. The pattern of disturbances has a strong influence on the choice of satisfactory controller settings, an influence not taken into account by most adjustment procedures. For instance, for a process with one predominant time constant, a proportional controller can give low values of the deviation ratio at low frequencies. However, when the steady-state gain of the process (K_p) is large, disturbances entering at the inlet of the process can still give large offsets (see also A Process with One Predominant Time Constant in Sect. 5). Hence a fair amount of integral action can be very desirable.

This can be generalized for all kinds of processes: low-frequency disturbances require more integral action, although the effect of high-frequency disturbances is better reduced with less integral action (see also Ref. 13). Another example is the double effect of derivative action: the effect of increased resonance frequency is more desirable when high-frequency disturbances are predominant, whereas the effect of increased proportional gain factor is more useful when there are many low-frequency disturbances.

2. Nonlinear effects lead to settings different from the adjustment procedures. An extreme example is the impossibility of using derivative

action, although the process dynamics seem suitable for its application (see Limitation of Derivative Action in Sect. 5).

3. In many cases the process dynamics change with process conditions, such as plant load. Hence a good setting for one set of process conditions can be unsatisfactory for another set and even lead to instability. The practical solution is to set the controller actions conservatively to ensure stability under all practical circumstances.

4. In many cases the controller can do the job easily, for instance in most flow control systems. Then a conservative setting is sufficient. Also when a strong mutual influence exists between control systems, it can be desirable to use conservative settings (see also Sect. 5, The Dynamic Behavior of Some Typical Processes).

In the next subsections the three principal approaches of setting the controller actions will be discussed.

Using the Stability Limit of the Control System

This approach is by far the most important. It is much less time consuming than the frequency or the transient response approach and is very suitable for all normal, not too critical, control systems.

Method of Ziegler and Nichols. One of the earliest recipes was developed by Ziegler and Nichols (see Ref. 23). First the integral and the derivative action of the controller is put out of operation. Then the proportional gain factor of the controller is increased until the control system is on the limit of stability and an oscillation is generated.

Because most control systems contain small signal nonlinearities, such as valve hysteresis, dead zone in the measuring device, etc., it can happen that no oscillation shows up, although the system is already in the unstable region. Therefore it is common practice to introduce small step or pulse variations while increasing the proportional gain factor in order to start any possible oscillations.

At the limit of stability, the period of oscillation P_u and the proportional gain factor K_{cu} are noted. Now Ziegler and Nichols give the following formulas:

(22) For P control $K_c = 0.5K_{cu}$;

(23) For PI control $K_c = 0.45K_{cu},\ T_i = 0.8P_u$;

(24) For PID control $K_c = 0.6K_{cu},\ T_i = 0.5P_u$;

$T_d = 0.125P_u$.

Offereins Method. Another method, proposed by Offereins (see Ref. 25), is particularly useful when the calibration of the controller knobs is not accurate. The proportional action is adjusted in the same way as with the Ziegler and Nichols method. Then the integral action is increased (for a PI controller) until again the limit of stability is reached. The integral action time is then increased by a factor 3, and the proportional gain factor is slightly decreased to ensure stable operation.

For a PID controller, first the proportional gain factor is increased until the limit of stability is reached. The derivative action time is then increased from zero until the limit of stability is reached again. After this, the derivative action time is decreased by a factor 3. Finally the integral action time is made about equal to the derivative action time. In this case, the proportional gain factor can generally be increased somewhat, as derivative action has a stabilizing influence.

Method of Rutherford, Aikman, and Ream. Rutherford (Ref. 2), Aikman (Ref. 3), and Ream (Ref. 24) give a method of obtaining the settings from the frequency response of the process. However, their method can also be used empirically.

1. The proportional gain is adjusted in such a way that the subsidence ratio (see Fig. 17*a*, ϵ_1/ϵ_2) of the response to a small set point change equals 3.

2. The integral action time is made equal to the period of oscillation of this response.

3. The derivative action is adjusted in such a way that the proportional gain factor of the controller has its maximum value, subject to 1 and 2.

Methods of Pessen and Clarridge. Pessen (see Ref. 27) has given a special adjustment recipe for obtaining good operation with automatic start-up of the process. The trouble with automatic start-up is the tendency to overshoot the set point (see Fig. 20*a*), which can be very unde-

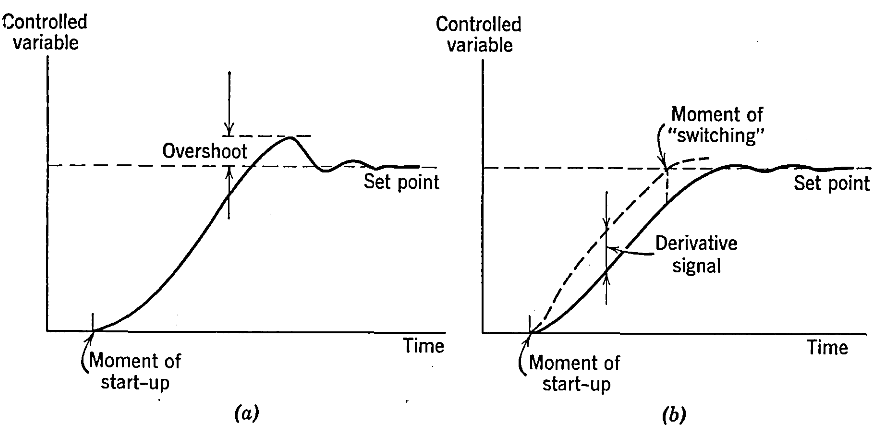


FIG. 20. Automatic start-up: (*a*) using conventional control, (*b*) using the arrangement of Fig. 21.

sirable, especially with chemical reactions. This overshooting is caused by the integral action which integrates the large initial error until the output of the controller reaches its saturation limit. When the controlled variable crosses the set point, this saturation does not disappear immediately and overshoot is unavoidable.

This problem has been solved (see Clarridge, Ref. 26) by arranging the controller in such a way that the derivative action element precedes the proportional-plus-integral element (see Fig. 21). Because of the positive

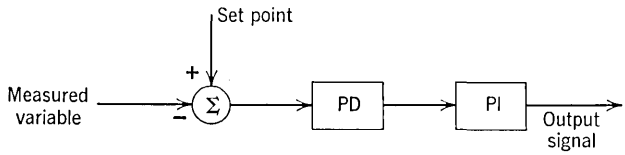


FIG. 21. Controller arrangement for avoiding overshoot.

rate of change of the controlled variable during the start-up period, the output signal of the derivative element crosses zero while the deviation is still negative (see Fig. 20*b*).

In this way the proportional-plus-integral element can recover from its saturation before the controlled condition crosses the set point, and overshooting can be avoided. This method of avoiding overshoot resembles the operation of so-called optimum relay servomechanisms, which also have a derivative action element preceding the servomotor (see Vol. I, Chap. 25, Nonlinear Systems).

Pessen gives the following formulas for setting the actions of the controller according to Fig. 21 when overshoot is to be avoided:

$$K_c = 0.25K_{cu}, \; T_i = 0.33P_u, \; T_d \approx 0.5P_u. \tag{25}$$

The adjustment of the derivative action time is very critical. Hence its final value should be obtained by trial and error.

Using the Transient Response of the Process

Many recipes obtain the settings of the controller from the unit step response curve of the process. To this end the latter is generally approximated by a dead time T and a time constant T_1 (see Fig. 22*a*), or by a dead time T and an integration S (see Fig. 22*b*), or by a dead time and a reaction rate (see Fig. 7). Finally, the controller settings are calculated from T, T_1, and/or S.

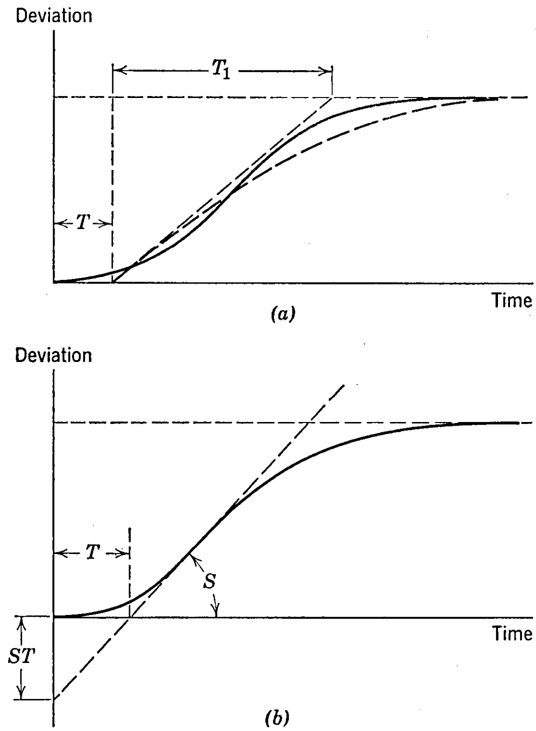


FIG. 22. Transient response approximated by (a) a dead time and a time constant, and (b) a dead time and an integration.

For example, Ziegler and Nichols (Ref. 23) give the following formulas:

(26) P action only $$K_c = \frac{1}{ST};$$

(27) PI action $$K_c = \frac{0.9}{ST}, \quad T_i = 3.3T;$$

(28) PID action $$K_c = \frac{1.2}{ST} \text{ to } \frac{2.0}{ST},$$

$$T_i = 2T, \quad T_d = 0.5T.$$

A disadvantage of these recipes is the rather crude approximation of the process response. As has been shown in Sect. 5, derivative action gives much improvement when the process contains two predominant time constants. Thus, in order to obtain good settings for controllers with derivative action, it is desirable to approximate the process response by at least a dead time and two time constants.

Moreover it is often difficult to interpret the step response of a process accurately because the plant disturbances easily spoil the result.

Using the Frequency Response of the Process

When the frequency response of the process is known, it is possible to apply servomechanism techniques for setting the controller actions, e.g., phase margin, gain margin, M contours, etc. (see Vol. 1, Chap. 19, Methodology of Feedback Control, and Chap. 21, Stability).

Method of Rutherford, Aikman, and Ream. However, there is also a technique which has been developed in the process control field. It is given by Rutherford (Ref. 2), Aikman (Ref. 3), and Ream (Ref. 24) and uses the following criteria:

1. The subsidence ratio (see Fig. 17*a*, ϵ_1/ϵ_2) of the oscillatory component in the transient response should equal e (Rutherford), 3 (Ream), or 4 (Coon, Refs. 20 and 21).

2. The integral action time should be made equal to the period of the oscillatory component.

3. The derivative action should be adjusted in such a way that the proportional gain factor of the controller has its maximum value, subject to 1 and 2.

Method Derived by Ream. In order to simplify the application of the first criterion, a method has been developed (derived by Ream in Ref. 24), based on a modification of the Bode diagram of the frequency response of the process (see Fig. 23). In this diagram curves are plotted to describe the response of the process to damped sine waves having the prescribed subsidence ratio. Ream gives the following approximate formulas for the distances between the original and the new curves:

$$\ln|P(j\bar{\omega})| - \ln|P(j\omega)| = -\alpha \frac{d[\arg P(j\omega)]}{d(\ln \omega)}, \tag{29}$$

$$\arg P(j\bar{\omega}) - \arg P(j\omega) = \alpha \frac{d[\ln|P(j\omega)|]}{d(\ln \omega)}, \tag{30}$$

where $j\bar{\omega} = -\sigma + j\omega = j\rho e^{j\alpha}$,

$\alpha = 0.175$ rad. (10°) for a subsidence ratio 3,

$\arg P(j\omega) =$ polar angle of $P(j\omega)$.

The right-hand side of eq. (29) equals the slope of the phase curve multiplied by $-\alpha$; the right-hand side of eq. (30) equals the slope of the gain curve times α. Because both slopes are generally negative (see Fig. 23), the response to damped sine waves has a higher gain and more phase lag than the frequency response.

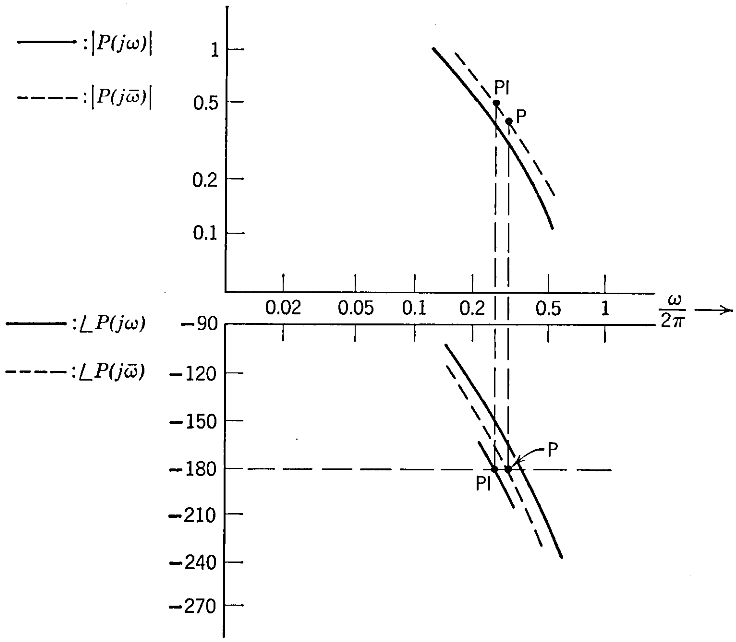


FIG. 23. Bode diagram of the process modified for finding the adjustment of the controller actions.

For a proportional controller the first criterion should be satisfied (subsidence ratio equal to 3), which corresponds to choosing the proportional gain factor in such a way that

$$1 + K_c P(j\bar{\omega}) = 0. \tag{31}$$

This is very easy to do in the modified Bode diagram (see Fig. 23). First the frequency is determined where the modified phase lag equals 180 degrees, which is the frequency of the oscillatory component in the transient response of the control system. The modified gain is then determined at this frequency. The reciprocal of this modified gain gives the setting of the proportional gain factor.

For a PI controller the second criterion fixes the modified gain and phase lag of the controller at the frequency of the oscillatory component. For a subsidence ratio equal to 3 they are 0.99 and 9 degrees, respectively. Adding this gain and this phase lag to the modified gain and phase lag of the process (see Fig. 23) easily leads to the setting of the proportional gain factor and the integral action time.

For a PID controller, the settings can be found by a trial-and-error procedure in the Bode diagram (see Coon, Refs. 20 and 21) or by a calculation in the log–gain–phase lag diagram (see Ream, Ref. 24).

Literature on Adjustment of Controller Actions

The literature on the adjustment of controller actions is very extensive. Therefore, only some surveys will be mentioned here.

Oppelt (Ref. 19) compares the various settings for a PI controller and a process transfer function consisting of a dead time and a time constant. His conclusion is that the various methods are not very much different for this particular case. The adjustments according to Ziegler and Nichols give a good average.

Izawa and Hayashibe (Ref. 22) also compare various methods on the base of the obtained phase and gain margins. Their conclusion is that the adjustment procedures for process control lead to less stable operation than those for servomechanisms.

Coon (Refs. 20 and 21) compares the two methods of Ziegler and Nichols (Ref. 23) and the one of Ream (Ref. 24). Her conclusion is that the last method gives the best control.

8. FEED-FORWARD CONTROL (See Refs. 28–30)

Figure 24 shows the block diagram of a feed-forward control system.

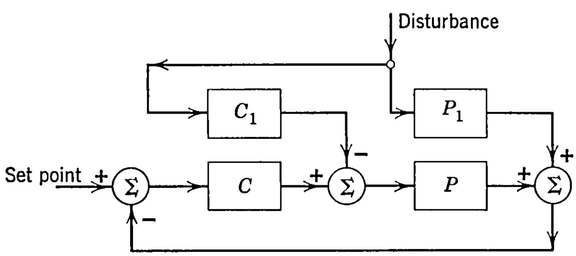


FIG. 24. Block diagram of a combined feedback and feed-forward system.

A source of disturbance influences the controlled variable via the transfer function P_1. An extra device, with transfer function C_1, compensates for the effect of the disturbance when

$$C_1P = +P_1 \qquad \text{or} \qquad C_1 = +P_1/P. \tag{32}$$

It is necessary that C_1 conform to eq. (32), both statically and dynamically. In practice this can be difficult when the process dynamics change much as a result of changes in the process conditions. The compensation is then nearly always imperfect, and the controller has to take the difference into account.

Feed-forward control can be used (see Refs. 28–30) when there is a strong source of disturbances whose effect cannot be sufficiently reduced by normal control, and also when the process dynamics do not vary too

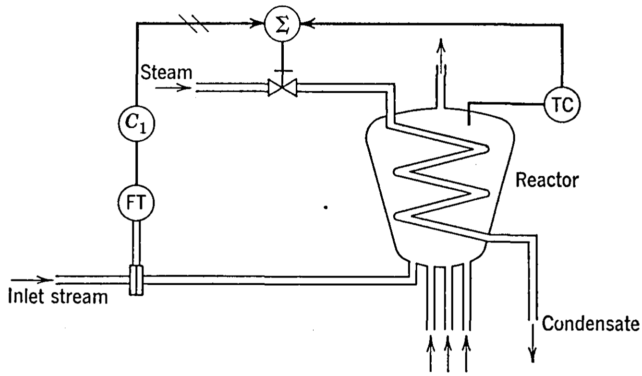


FIG. 25. Feed-forward control applied to the inlet stream of a chemical reactor.

greatly. An *example* is given in Fig. 25. An inlet stream to a chemical reactor shows large flow variations. The latter are measured by a flow transmitter FT and added to the output signal of the temperature controller TC via the compensating device C_1.

9. CASCADE CONTROL

A cascade control system can be defined as a system in which the output of one controller adjusts the set point of another controller (see Fig. 26). The characteristics are:

1. The complete system has one control valve and two closed loops.
2. Generally the gain K of the (pneumatic) set mechanism can be manually adjusted.
3. C_1 is called the primary or master controller, and C_2 the secondary or slave controller.

In the literature (see Refs. 31–35) the name cascade control system is also used for those systems in which the primary controller is merely a

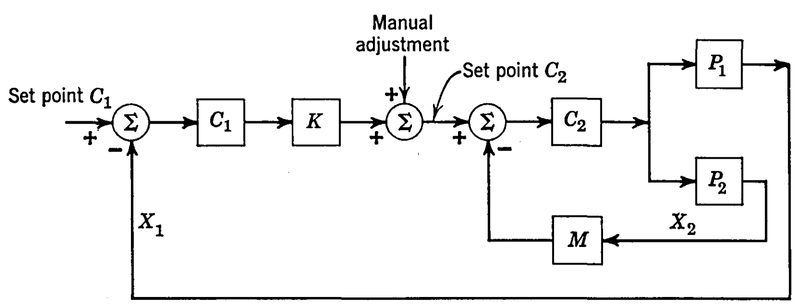


FIG. 26. Block diagram of cascade control.

measuring unit. However, such systems can also be considered as single loop systems with a controlled variable equal to

$$C_1KX_1 + X_2M \tag{33}$$

where C_1 is the transfer function of the primary controller, M the transfer function of the measuring unit of the secondary controller, K the gain of the set mechanism, X_1 the primary variable, and X_2 the secondary variable. Thus here the name cascade control system is somewhat misleading. Therefore in a later section the term *pseudo-cascade* control system will be used to describe systems in which the primary controller is merely a measuring unit.

The transfer function M has a strong influence on the dynamic behavior of the cascade control system and can thus be a valuable degree of freedom in the design.

True Cascade Control Systems

According to Ziegler (Ref. 34) the objects of cascade control are:

1. To maintain a desired relationship between variables.
2. To limit accurately the secondary variable.
3. To reduce the effects of disturbances and nonlinearities near their source.
4. To improve the quality of control of the primary variable.

1. Maintaining a desired relationship between variables. This object applies particularly to pseudo-cascade control systems, as will be shown later. The other three will be discussed separately, although they often occur in combination.

2. Limiting accurately the secondary variable. Figure 27 gives an *example.* A refinery furnace is heated by fuel oil. In order to guarantee good operation of the burners, the pressure at their fuel inlet should be within a certain range, say 150–450 psig.

By using a cascade control system where the temperature controller TC adjusts the set point of a secondary pressure controller PC, and by careful

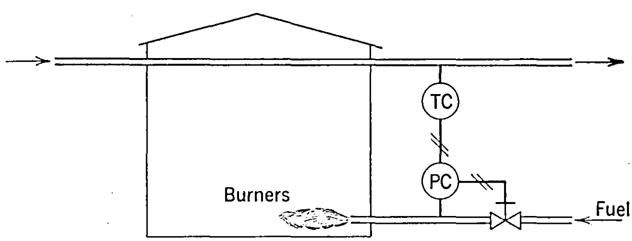


FIG. 27. Cascade control applied to a refinery furnace.

adjustment of the set mechanism, a full-range change of the output signal of the temperature controller can be made to give the desired range of set points for the pressure controller. Without a secondary controller accurate limitation is impossible, for, as in this case, disturbances in the fuel supply system change the pressure at the burners.

3. Reducing the effects of disturbances and nonlinearities near their source. A simple *example* is the use of a valve positioner to reduce the effect of control valve hysteresis (see Fig. 28). This system is so pop-

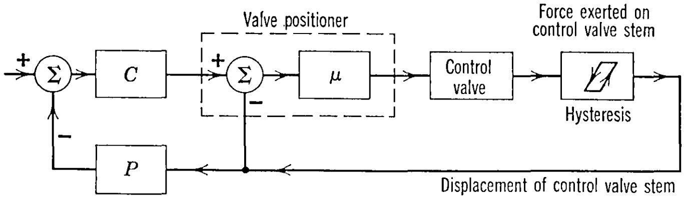


FIG. 28. Valve positioner for reducing the effect of control valve hysteresis.

ular in process control that it is generally not recognized as a type of cascade control system. The valve positioner loop reduces the effect of hysteresis by a factor 1 plus its loop gain, thereby improving the operation of the (primary) control system.

Another *example* is the system shown in Fig. 27. The effects of disturbances in the fuel supply are effectively reduced by the fast pressure control, and this lightens the task of the slower temperature control system.

A third *example* is the control of the liquid level in a buffer vessel (see Fig. 29). It must be remarked that the expression "liquid level control" is somewhat misleading here. The liquid level should not be kept constant but should change over the full range of the buffer vessel in order

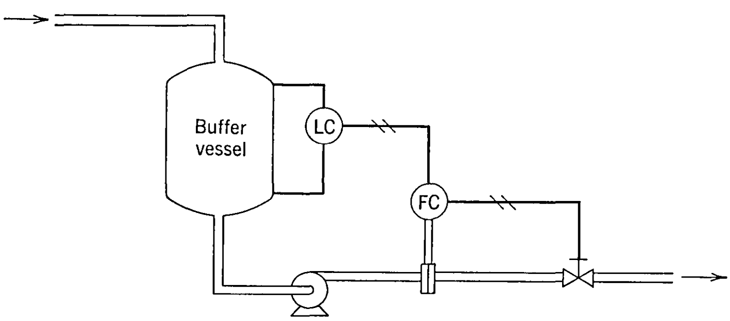


FIG. 29. Cascade control applied to a buffer vessel.

to keep the outgoing flow rate as constant as possible. Liquid level controllers are therefore often proportional ones, adjusted to a low gain. The secondary flow controller helps to keep the outgoing flow rate constant, irrespective of disturbances in the pressure drop over the line.

4. Improving the quality of control of the primary variable. This aspect of cascade control will be made plausible for the simple case shown in Fig. 30 (see also Franks and Worley, Ref. 33, and Gollin, Ref. 35).

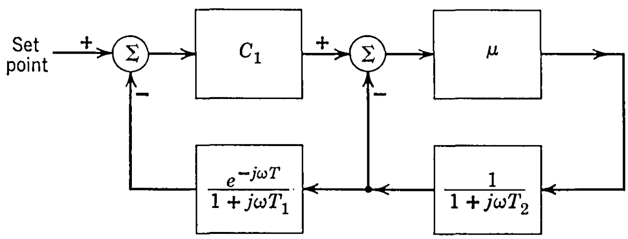


FIG. 30. Improving control quality by cascade control.

The process response pertaining to the secondary controller is a first order one. Hence the secondary controller can be of the proportional type adjusted to a high gain μ.

The loop gain of the primary control system equals

$$\frac{\mu}{1+\mu} C_1 \frac{e^{-j\omega T}}{(1+j\omega T_1)\left(1+j\omega \dfrac{T_2}{1+\mu}\right)}. \tag{34}$$

Without cascade control, the loop gain would have been

$$C_1 \frac{e^{-j\omega T}}{(1+j\omega T_1)(1+j\omega T_2)}. \tag{35}$$

This comparison shows that the secondary control effectively reduces the time constant T_2, thereby improving the controllability of the system. There is a similarity between this system and the so-called tachometric feedback in servos, where it is the purpose to reduce the time constant of the servomotor (see Vol. 1, Chap. 23, Feedback System Compensation).

Pseudo-cascade Control Systems

A good *example* of a pseudo-cascade control system is a flow ratio control system (see Fig. 31). The block diagram is a special case of Fig. 26 and is shown in Fig. 32. Because most flow-measuring devices have a

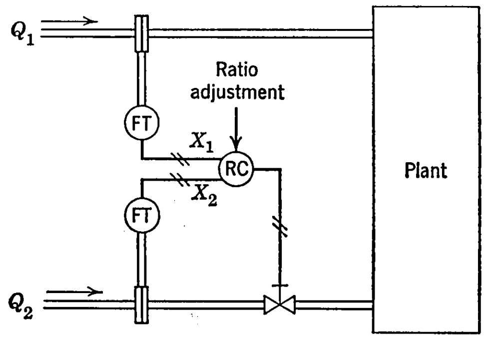


FIG. 31. Flow ratio control system.

quadratic characteristic, the steady-state relationship between flow and output signal can be expressed as

(36) $$X_1 = a_1 Q_1^2 \quad \text{and} \quad X_2 = a_2 Q_2^2,$$

where a_1 and a_2 are constants.

According to the block diagram the deviation, ϵ is given by

(37) $$\epsilon = +KX_1 - X_2.$$

Using eq. (36), the steady-state deviation becomes

(38) $$\epsilon_\infty = +a_1 K Q_1^2 - a_2 Q_2^2.$$

With integral action the steady-state deviation is zero; thus eq. (38) reduces to

(39) $$Q_1/Q_2 = \sqrt{a_2/a_1 K} = \text{constant}.$$

The ratio can be adjusted by changing K.

Flow ratio control systems are often applied to blending operations in which a product stream is made up from two or more component streams. They are also used for burners and other combustion processes, in order to maintain the desired ratio between fuel and air flow.

Another *example* of a pseudo-cascade control system is shown in Fig. 33. Here a bulb has been installed on a distillation column tray. A liquid mixture with the desired composition is in the bulb. By measuring the difference of the vapour pressures of the liquid mixture in the bulb and of the liquid mixture on the tray, a measure is obtained of the deviation from the desired composition (see Tivy, Ref. 36).

A simplified block diagram is shown in Fig. 34. All transfer functions have been linearized. P_1 and P_2 are process transfer functions, and M_2 represents the thermal lag of the bulb. The lag of the pressure measurement has been ignored.

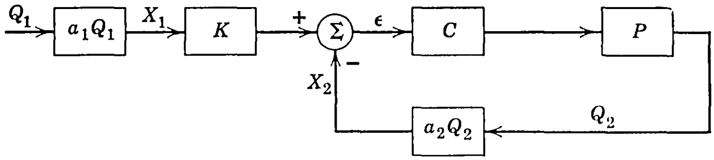


FIG. 32. Block diagram of flow ratio control system.

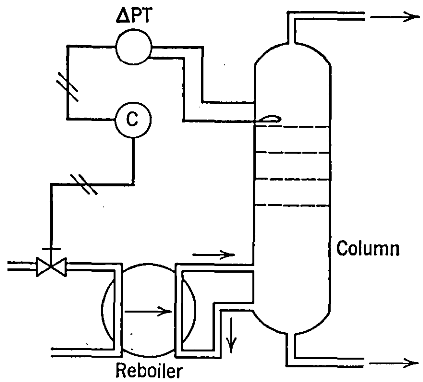
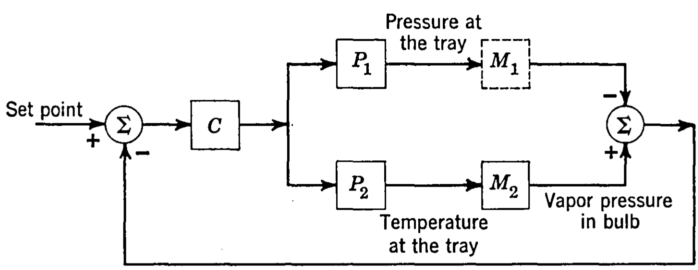


FIG. 33. Vapor pressure control applied to a distillation column.



$$M_2 = \frac{K_2}{1 + j\omega\tau},\ P_2 = \frac{1}{K_2} P_1 + P_3,\ M_1 = \frac{1}{1 + j\omega\tau}$$

FIG. 34. Block diagram of vapor pressure control system of Fig. 33.

The loop gain of the control system equals

$$(P_2M_2 - P_1)C. \tag{40}$$

Approximation of M_2 by a single time constant leads to

$$\left(\frac{P_2K_2}{1 + j\omega\tau} - P_1\right)C. \tag{41}$$

In general, the temperature transfer function P_2 consists of two parallel paths, one via the pressure with transfer function P_1/K_2 and the other via the composition with a slow transfer P_3.

Substitution into eq. (41) gives

$$\frac{(P_3K_2 - P_1j\omega\tau)C}{1 + j\omega\tau}. \tag{42}$$

The step response will first go the other way because the second term of this equation initially gives the strongest effect. Thus the response has a nonminimum phase character.

The nonminimum phase factor can be obviated by elimination of the time constant τ of the bulb, or by introduction of the same time constant into the pressure measurement at the tray (see dotted lines in Fig. 34).

In the latter case, the loop gain of the system equals

$$\frac{P_3K_2}{1 + j\omega\tau}C, \tag{43}$$

which is generally much more favorable for good control than eq. (42).

Summary. It can be said that pseudo-cascade control systems are used to maintain a specific relationship between two or more process variables. They can be interpreted as single loop systems, although their behavior is often more complicated than that of conventional single loop systems.

10. USE OF ANALYTICAL INSTRUMENTS FOR PROCESS CONTROL

In addition to the conventional instruments, analytical instruments are finding an increasing field of application in process control (see Chap. 24, Continuous Analyzers). In many cases they give a direct measurement of the composition of a product stream, for instance by means of infrared spectroscopy, mass spectroscopy, etc. In other cases they measure some important property of the product stream, such as pH, viscosity, density, refractive index, dielectric constant, etc.

A problem with most analytical instruments is the difficulty of obtaining a clean and representative sample from the product stream. Elaborate sampling systems are often necessary. This easily leads to long time constants and long dead times in the transfer function between product stream and instrument indication. Thus control by means of an analytical instrument is often slow and gives insufficient reduction of the effects of plant disturbances.

Another difficulty is that analytical instruments are less reliable than conventional instruments. This often makes it desirable to leave the control action to the human operator; the instrument is then used as a recorder or as an indicator. Even when the analytical instrument is connected to an automatic controller, we must take into account the fact that the instrument requires servicing and that the automatic control system will often be out of operation.

Both difficulties mentioned here can be partly overcome by using cascade control systems. The analytical control system thus adjusts the set point of a secondary control loop equipped with conventional instrumentation. The secondary loop is able to reduce sufficiently the effects of many disturbances and maintains automatic control when the analytical instrument is not in operation.

11. MULTIVARIABLE CONTROL SYSTEMS

Multivariable control systems contain, as the name implies, two or more controlled variables. However, contrary to cascade control systems, the set points of the controllers are adjusted independently of each other.

Figure 35 gives the block diagram of a two-variable system. X_1 and X_2 are the controlled variables, Y_1 and Y_2 are the manipulated variables, and H_1 and H_2 are the effects of plant disturbances on the controlled variables X_1 and X_2, respectively. An *example* is the control of distillation

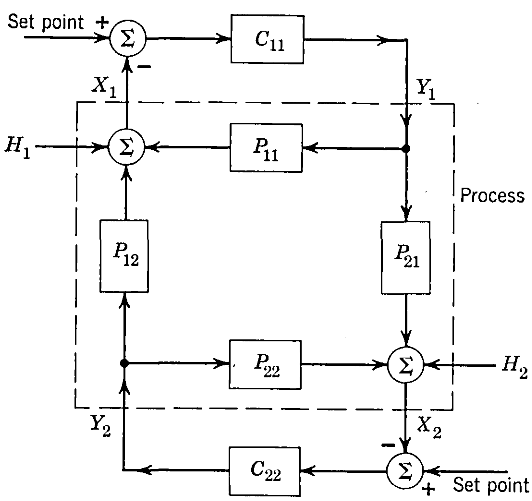


FIG. 35. Block diagram of a two-variable control system.

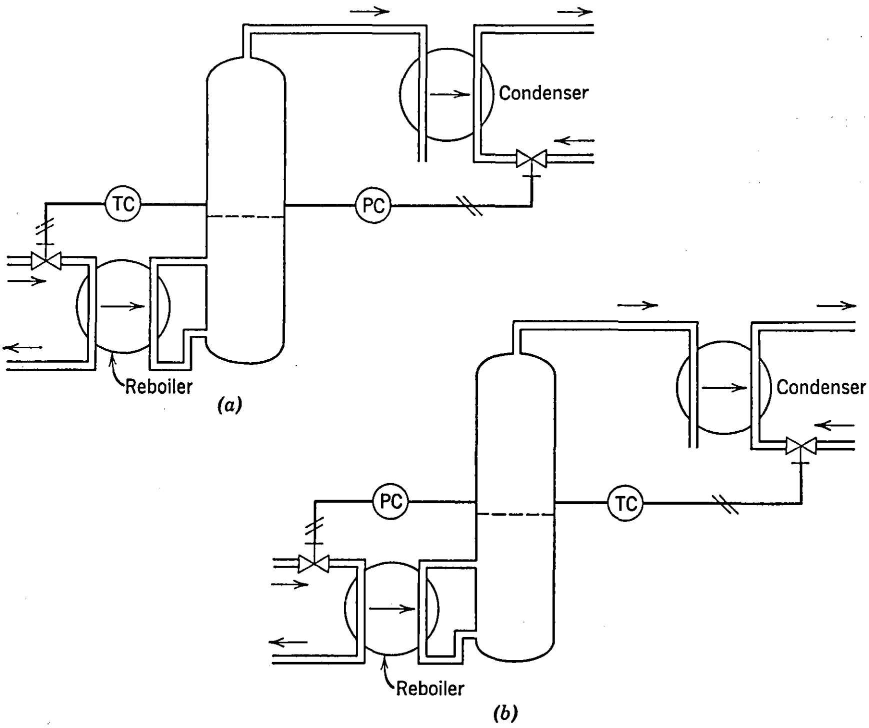


FIG. 36. Pressure and temperature control of a distillation column: (*a*) practical system, (*b*) impractical system.

columns, where pressure and temperature control often form a two-variable control system (see Fig. 36*a*).

Condition for Stability of Multivariable Control Systems

First a simple condition which is of direct practical significance will be derived.

In process control it is desirable to have stable control systems, irrespective of the fact that other control systems are in or out of operation. For instance, when a controller is in repair, or when it is saturated or inoperative for some other reason, the process together with the other controllers should form a stable system.

For the system of Fig. 35 this means that the loops $P_{11}C_{11}$ and $P_{22}C_{22}$ should both have negative feedback:

$$(P_{11}C_{11})_{\omega=0} > 0, \qquad (P_{22}C_{22})_{\omega=0} > 0. \tag{44}$$

But the control loop of C_{11} should also have negative feedback when controller C_{22} is in operation.

By using formula (44), this leads to the condition

$$\left(\frac{1}{P_{11}P_{22}}\begin{vmatrix} P_{11}P_{12} \\ P_{21}P_{22} \end{vmatrix}\right)_{\omega=0} > 0 \quad \text{or} \quad \left(\frac{P_{12}P_{21}}{P_{11}P_{22}}\right)_{\omega=0} < 1 \tag{45}$$

where the vertical lines stand for determinant.

The same result is obtained when the control loop of C_{22} is considered with C_{11} in operation.

Formula (45) only contains the steady-state values of the process responses. It indicates whether the two-variable system is practically realizable. Figure 36*b* gives an *example* of a control system that does not satisfy condition (45). When the pressure controller is not in operation, more cooling water through the condensor generally gives a lower temperature. When the pressure controller is in operation, temperature variations are directly related to composition variations. More cooling water then gives more top product and less bottom product, and consequently heavier compositions and higher temperatures.

It can be concluded that the sign of the temperature response is different, depending on whether C_{22} is operating or not, and the control system is impractical. It should be replaced by the scheme of Fig. 36*a*, whose value of $P_{12}P_{21}/P_{11}P_{22}$ is the reciprocal of that of the scheme shown in Fig. 36*b*.

Condition (45) can be generalized for application to n-variable systems. Figure 37 shows the block diagram of an n-variable control system. X_1, X_2, $\cdots$, X_n are the controlled variables; $[C]$ is the controller matrix, Y_1, Y_2, $\cdots$, Y_n are the manipulated variables; $[P]$ is the process matrix; and H_1, H_2, $\cdots$, H_n are the effects of the plant disturbances on the controlled variables.

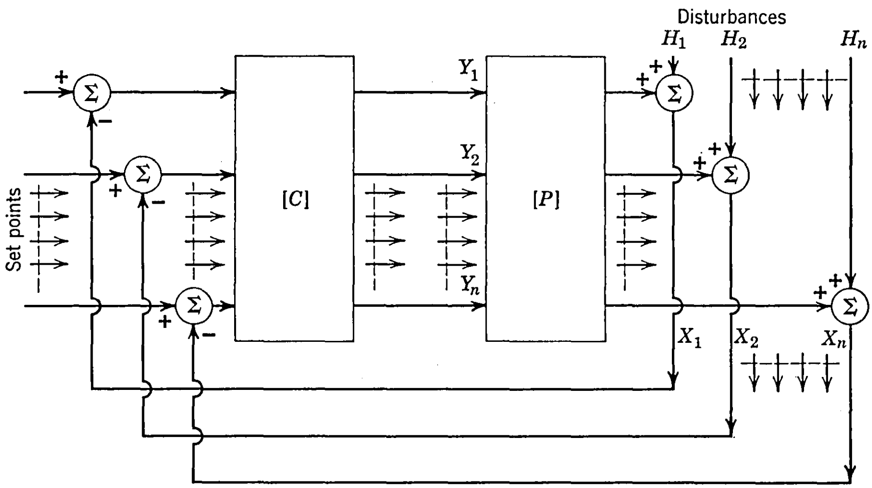


FIG. 37. Block diagram of a n-variable control system.

A necessary but inadequate condition for stability of the system is

$$(46) \qquad (|[P]\,[C]|)_{\omega=0} > 0$$

where the vertical lines stand for determinant.

This formula can be derived from the equality of the signs of the first and of the last term of the characteristic equation of the system. It is further based on the assumption that integral action is used in the controller transfer functions between X_1 and Y_1, X_2 and Y_2, $\cdots$, X_n and Y_n.

When all other controller transfer functions are without integral action, and

$$(47) \qquad (P_{11}C_{11})_{\omega=0} > 0, \quad (P_{22}C_{22})_{\omega=0} > 0, \quad \cdots, \quad (P_{nn}C_{nn})_{\omega=0} > 0,$$

then formula (46) can be simplified to

$$(48) \qquad \left(\frac{1}{P_{11}P_{22}\,\cdots\,P_{nn}}\,|[P]|\right)_{\omega=0} > 0$$

where $P_{11}P_{22}\,\cdots\,P_{nn}$ is the main diagonal of matrix $[P]$.

For an n-variable system condition (48) should also be applied to the $(n-1)$-variable control systems which can be formed from it, etc.

Multivariable systems in more general cases can be handled by computer control (see Chap. 13, Computer Control, and Chap. 14, Data Processing).

The Dynamic Behavior of Two-Variable Control Systems

The stability of two-variable systems can be investigated in the Nyquist diagram of the equation

$$(49) \qquad 1 - G\,\frac{P_{11}C_{11}}{1 + P_{11}C_{11}}\,\frac{P_{22}C_{22}}{1 + P_{22}C_{22}} = 0$$

where

$$(50) \qquad G = P_{12}P_{21}/P_{11}P_{22}.$$

The factors $\dfrac{P_{11}C_{11}}{1 + P_{11}C_{11}}$ and $\dfrac{P_{22}C_{22}}{1 + P_{22}C_{22}}$ are easily determined. Generally they have the character of a low-pass filter.

Before using eq. (49), first the stability of the loops $P_{11}C_{11}$ and $P_{22}C_{22}$ should be investigated. From the practical point of view it is desirable that both be stable in themselves.

Symmetrical Two-Variable Control System. A simple case is the symmetrical two-variable control system, where

$$(51) \qquad P_{11}C_{11} = P_{22}C_{22} = L.$$

An *example* is the combination of two identical units in a plant, each with a control system manipulating a common supply of steam, water, or fuel (see Fig. 38). The internal impedance of the utility supply causes the coupling and gives rise to the two-variable system.

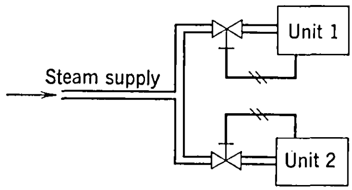


FIG. 38. Symmetrical two-variable control system.

In this particular case the characteristic equation of the system can best be written in the form

$$[1 + L(1 + \sqrt{G})]\,[1 + L(1 - \sqrt{G})] = 0$$

or

(52) $$\left(\frac{1}{1 + \sqrt{G}} + L\right)\left(\frac{1}{1 - \sqrt{G}} + L\right) = 0.$$

Formula (52) can be represented by two Nyquist diagrams, one with $-1/(1 + \sqrt{G})$ instead of the point -1, the other with $-1/(1 - \sqrt{G})$ instead of the point -1. Figure 39 gives the two Nyquist diagrams for two specific cases: G is real and positive, and G is real and negative. It can be seen that in both cases the limit of stability is reached earlier than for the corresponding single variable control systems.

This means that the stronger the coupling, the less sensitive the settings to which the controllers can be adjusted.

The reduction of the effect of very low-frequency disturbances is given by the formula

(53) $$\left(\frac{X_1}{H_1}\right)_{\omega \to 0} = \left[\frac{1}{2 + L(1 - G)}\right]_{\omega \to 0}.$$

When $G = 1$, the reduction is only a factor 2, which is very unsatisfactory.

Two-Variable Control System with One Fast Control Loop. Another simple case is the two-variable system in which one control loop is much faster than the other one. The dynamic behavior of the fast loop is hardly influenced by the slow loop. On the other hand, the influence of the fast loop on the slow loop can be appreciable. It is given by (see also Ref. 13)

(54) $$\frac{X_1}{H_1} = \frac{1}{1 + P_{11}C_{11}(1 - G)}$$

where $P_{22}C_{22}$ is fast compared to $P_{11}C_{11}$.

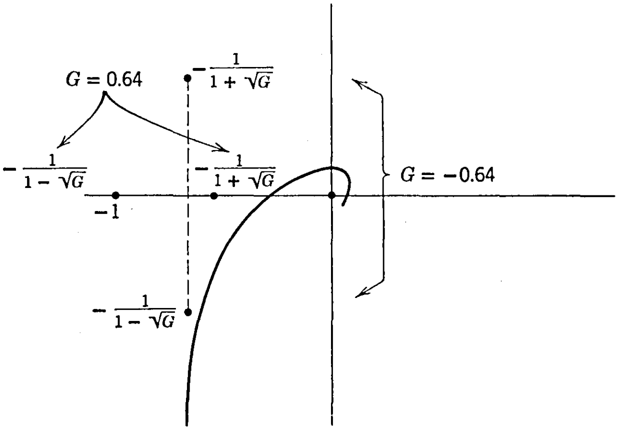


FIG. 39. Nyquist diagram for symmetrical two-variable control system of Fig. 38.

Thus the dynamic behavior of control loop $P_{11}C_{11}$ is modified by the factor $1 - G$, which can improve or deteriorate the control of X_1. An *example* is temperature and pressure control of a distillation column, where the pressure control loop is often much faster than the temperature control loop.

12. SPECIAL SUBJECTS

Reswick (see Ref. 37) has published the idea of disturbance response feedback. In theory disturbance response feedback gives improved control for high-order processes, but the practical application is difficult.

Draper and Li (see Refs. 38 and 39) give a series of methods of optimalizing control, whereby a special control mechanism searches for the maximum value of some process condition, for instance the process efficiency (see also Ref. 40).

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Nonlinearities

C. G. Laspe and T. M. Stout

1. INTRODUCTION

This section describes the types and sources of nonlinearities frequently encountered in process control systems and indicates how these might be coped with. Chapter 25, Nonlinear Systems, in Vol. 1 of this Handbook, presents a detailed treatment of the fundamental mathematical relationships as they apply to nonlinearities in general. In the paragraphs that follow the nonlinearities are classified in a somewhat different manner. This general treatment will give the systems engineer an insight into the mode of attacking any process control problem involving nonlinearities.

Sources of Nonlinearity. There are at least three sources of nonlinearity encountered in process control. These arise from the measurement instruments, from the process itself, and from the control equipment.

Each of these individual items will be discussed further in the paragraphs that follow.

2. NONLINEARITIES IN MEASUREMENT INSTRUMENTS

Flow Measurements. Perhaps the most popular and most frequently used flow measurement device in the process industries is the simple orifice meter. The orifice meter, the venturi tube, and the Pitot tube meters compose a group of fluid-measuring instruments that for the action depend upon the differential pressure or head produced across the primary element by the fluid that is flowing. According to Bernoulli's equation, this pressure differential is proportional to the square of the velocity of the fluid. In its simplest form the relationship between flow and differential pressure is as shown in eq. (1):

$$Q = C\sqrt{h} \tag{1}$$

where Q = flow rate,
C = discharge coefficient,
h = differential pressure.

By rearrangement of eq. (2) the head produced is directly proportional to the square of the quantity flowing:

$$h = AQ^2, \tag{2}$$

where A = proportionality constant.

With a differential head meter of this type the effective gain of the meter depends upon the absolute value of the flow. This is readily seen by differentiating the head produced with respect to the flow, giving

$$\frac{dh}{dQ} = 2AQ. \tag{3}$$

As a consequence, in flow control systems using an orifice meter as the primary measuring element, the control system stability depends upon the absolute flow level. In most flow transducers the differential head h is converted into a proportional and linear signal which is supplied as the measured variable signal to conventional controllers.

A further complication in the nonlinearity of flow measurement arises from the nonconstancy of the discharge coefficient C in eq. (1). As long as the flow is turbulent, C is relatively constant. However, as the flow is decreased, which, in turn, decreases the system's turbulence, the flow patterns become more and more streamlined. This results in a gradual increase of the discharge coefficient to a certain maximum value which is a function of the geometry of the system. In most commercial applications, however, the orifice meter is so designed as to operate well out into the turbulent region where the discharge coefficient C may be assumed relatively constant.

Temperature Measurements. Certain temperature-measuring instruments also exhibit a considerable degree of nonlinearity in converting the measured variables into the control signal. This is especially noticeable in instruments using the vapor pressure characteristics of liquids as the motive force in the temperature sensitive element. In process control jargon, these elements are known as Class II filled systems. The vapor pressure–temperature relationship for most liquids can be expressed by an equation of the form

$$\log P = a - \frac{b}{T + c} \tag{4}$$

where P = vapor pressure,
T = temperature,
a, b, c = constants for a particular compound.

Vapor pressure data for ethyl chloride, a common filling agent, is presented in Table 1. These data are also shown graphically in Fig. 1. Note that

TABLE 1. VAPOR PRESSURE OF ETHYL CHLORIDE

Temperature, °F	Vapor Pressure, psig
60	1.5
80	9.5
100	20.0
120	33.0
140	53.0
160	72.0

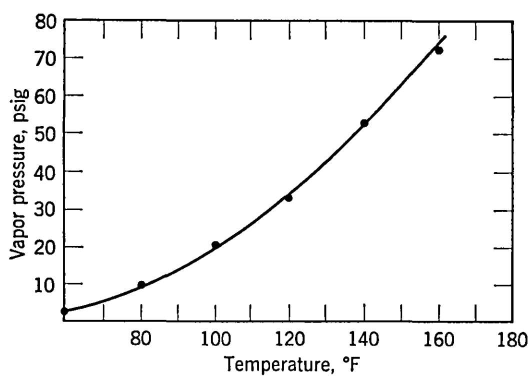

FIG. 1. Vapor pressure curve for ethyl chloride.

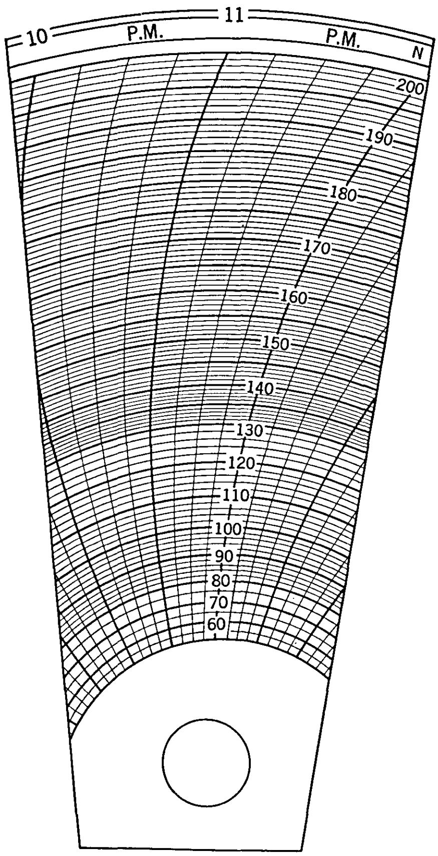

FIG. 2. Section of a recorder chart for a vapor responsive temperature-sensing element.

as the temperature increases, the vapor pressure increases at a much more rapid rate. The gain of such an element, of course, increases rapidly as the temperature is raised. A section of a typical temperature recorder chart utilizing the Class II vapor-filled system is shown in Fig. 2.

The main advantage of this type of element lies in its sensitivity in the control region while, at the same time, permitting the same instrument to be used to bring the process on stream when the temperatures are considerably below the normal operating point.

The Class II vapor-filled system exhibits a nonlinear gain as a function of the temperature level. For relatively short temperature ranges the vapor pressure–temperature relationship may be expressed by

$$\log P = a - b/T. \tag{5}$$

Since the transducer output signal is normally proportional to the vapor pressure exerted, differentiation of eq. (5) will allow an estimate to be made of the gain of the instrument:

$$\frac{dP}{dT} = \frac{a'b}{T^2}\, e^{-b/T}. \tag{6}$$

The exponential term in temperature by far outweighs the inverse square term so that the gain of the system increases as temperature is increased.

Thermocouples are a second type of primary temperature-measuring element which is commonly used in the processing industries. The thermocouple depends for its action upon the emf developed at the junction of two dissimilar metals when heated to an elevated temperature. A typical relationship between the emf developed and the junction temperature is

$$E = k_1(T - T_r) + k_2\,(T - T_r)^2, \tag{7}$$

where k_1, k_2 = constants,
T_r = reference temperature,
T = junction temperature,
E = emf developed.

For temperature spans of 200°F or less, the relationship is sufficiently linear that the quadratic term may be dropped without creating an error any greater than normal instrument error. In wide-range measuring instruments, however, this nonlinearity must be taken into account.

Speed Measurement. The rotary speed of large machines such as the four-cycle gas engine and turbine-driven compressor is frequently measured by devices containing a flyball mechanism as the primary sensing element. In one commercially available speed transducer the centrifugal force created by the rotation of the flyball mechanism is counterbalanced by a linear and proportional air pressure. According to the laws of dynamics, this force is directly proportional to the square of the rotating speed:

$$F = Ma = M\omega^2 r, \tag{8}$$

where F = force,
M = mass,
a = acceleration,
ω = rotary speed,
r = radius to center of mass.

Differentiation of this force with respect to rotary speed gives the effective gain of the device as a measuring element:

$$\frac{dF}{d\omega} = 2Mr\omega. \tag{9}$$

Note the similarity between eq. (9) and eq. (3) for an orifice meter. In fact, both systems are identical in that the measurement gain is directly proportional to the magnitude of the measured variable.

pH Measurement. In the electrometric method of determining hydrogen ion concentration in aqueous chemical solutions (commonly called pH measurement) there exists a logarithmic relation between the electrode potential and hydrogen ion concentration. Equation (10) defines this relationship:

$$\text{(10)} \qquad \mathrm{pH} = \log \frac{1}{C_\mathrm{H}},$$

where C_H = hydrogen ion concentration.

Values of emf as a function of pH are listed in Table 2 for the glass-calomel system of electrodes.

TABLE 2. pH–EMF RELATIONSHIP FOR THE GLASS-CALOMEL ELECTRODE SYSTEM AT 25°C

pH	Emf, μv
3	−211
7	+25
10	+202

3. NONLINEARITIES IN THE PROCESS

Almost without exception most processes today involve at least one or two chemical reaction stages. Since these chemical reactions are subject to the laws governing mass transfer, invariably the reactor itself will be violently nonlinear when considering the relationships between the input and output quantities.

Process Gain. Consider a simple reaction such as shown in eq. (11) where one molecule of A is transformed into a single molecule B, for example, as in butane isomerization:

$$\text{(11)} \qquad \mathrm{A} \underset{k_2}{\overset{k_1}{\rightleftarrows}} \mathrm{B}.$$

Assuming that this is a first-order reversible reaction and that isothermal conditions prevail, the rate equation governing this reaction is

$$\text{(12)} \qquad \frac{dX}{dt} = k_1(1 - X) - k_2 X,$$

where k_1 = forward velocity constant,
k_2 = reverse velocity constant,
X = mole fraction of A converted.

Upon integration this equation results in eq. (13) which relates the degree of conversion as a function of the contact time within the reactor:

$$\text{(13)} \qquad X = X_e[1 - e^{-k_2(1+K)t}],$$

where X_e = equilibrium value of X,
K = equilibrium constant = k_1/k_2.

In a continuous flow reactor the contact time is inversely proportional to feed rate to the reactor. Inspection of eq. (13), therefore, will reveal that the effect of feed rate upon yield is a function of the conversion level. This is a simple and frequently encountered example of nonlinearity in process gain. Most chemical reactors are even more complicated than this in that more reactants and products are involved. For infinite values of reactor contact time there is a definite fixed maximum yield which can be expected for any particular system. This maximum value is a function mainly of the thermodynamic factors involved in the process. Thus, returning to the butane isomerization example, the maximum yield under normal operating conditions is in the neighborhood of 50 to 60%.

Distillation. Separation of the product from unreacted feed materials is frequently carried out by means of fractionation by distillation. Although it is beyond the scope of this present chapter to go into the theory of distillation, suffice it to say that the fractionating column consists of a series of stages wherein the liquid and vapor at each stage are in thermal equilibrium. The separation of two components is possible by fractionation owing to differences in volatility of the two constituents. Equation (14) presents a relationship between the concentration of the lighter constituent in the vapor and the concentration of the lighter constituent in the liquid:

$$y = \frac{x\alpha}{1 + (\alpha - 1)x}, \tag{14}$$

where y = mole fraction of lighter component in vapor,
x = mole fraction of lighter component in liquid,
α = relative volatility.

This equation is applicable for a single plate or tray. When a fractionating column consists of N plates counted from the feed tray to the top of the column, this formula is repeated N times, assuming infinite reflux. The resulting recursion formula will then describe the top composition as a function of the composition of the feed to the column. When the reflux is not infinite, then, in addition, the material balance must be considered at each tray. For feed mixtures which contain more than two components, the problem becomes even more complex.

pH Process. Previously the nonlinearity in the pH-measuring element was considered. However, an even greater nonlinearity exists in the pH process itself. Figure 3 represents a typical titration curve which might be obtained when titrating a basic aqueous solution with an acid. The nonlinearity becomes even more apparent when it is recognized that the abscissa or pH values are in themselves logarithmic values of the hydrogen ion concentration. It immediately becomes apparent that the stability of

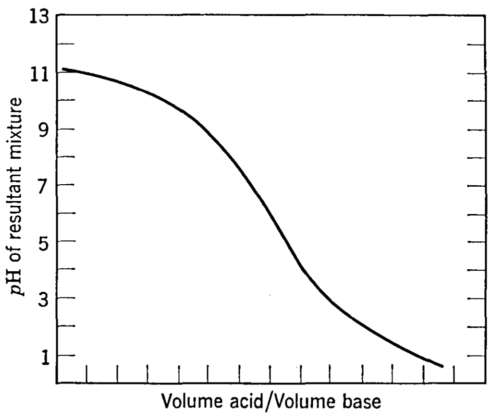

FIG. 3. Typical titration curve.

the pH control system will depend entirely upon the level of the pH value which is to be maintained. Further, the presence of salts which in themselves may contain ions common either to the acid or to the basic material in the system may cause a buffering effect which will effectively change the curvature of the titration curve presented in Fig. 3.

Liquid Level. Another frequently encountered nonlinearity exists in the measurement of liquid level in horizontal cylindrical tanks. Although quite obvious, this effect is frequently ignored when designing liquid level control systems. This particular type of nonlinearity is most annoying when applying control to maintain a level in the upper or lower portion of the horizontal vessel. In the central regions of the vessel the nonlinearity can normally be ignored. The problem is graphically shown in Fig. 4, and the capacity of the tank is shown in Table 3 as a function of the liquid depth.

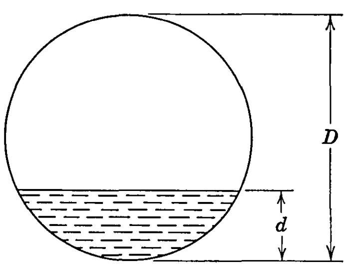

FIG. 4. Illustrating the variation of capacitance of horizontal cylindrical vessels as a function of liquid depth.

TABLE 3. CONTENTS OF HORIZONTAL CYLINDRICAL TANKS

Per Cent Depth	Per Cent Capacity
0	0
10	5.2
20	14.2
30	25.2
40	37.3
50	50.0
60	62.7
70	74.8
80	85.8
90	94.8
100	100.0

4. NONLINEARITIES IN CONTROL EQUIPMENT

Perhaps the most common piece of control equipment having nonlinear characteristics is the familiar diaphragm–motor control valve. Many excellent papers have been written which concern themselves with the practical aspects of valve characteristics in process control. The control valve or final control element is a device by which compensation for other nonlinearities may be made by the control engineer. For example, by proper selection of control valve characteristics it is possible to eliminate the nonlinear characteristics introduced by the primary transducing element, such as an orifice meter or the flyball type speed control mechanism.

Valve Characteristics. There are many practical valve characteristics in use today. However, the wide majority of these characteristics can be classified in one of three categories: (1) linear, (2) parabolic, and (3) exponential. Mathematically, these characteristics may be described by eqs. (15), (16) and (17) in which the fraction of maximum flow is given as a function of the actuating signal or valve stroke position:

$$\text{Linear, } \omega = \sigma, \tag{15}$$

$$\text{Parabolic, } \omega = \frac{1}{\rho} + \left(1 - \frac{1}{\rho}\right)\sigma^2, \tag{16}$$

$$\text{Exponential, } \omega = \rho^{\sigma - 1}, \tag{17}$$

where ω = fraction of maximum flow,
σ = fraction of maximum stroke,
ρ = rangeability of valve,
= maximum flow/minimum flow.

As noted above, the rangeability of a control valve is expressed as a ratio of the maximum to minimum controllable flow. For the parabolic characteristic value is approximately 25 to 1, whereas with the exponential valve a rangeability of 50 to 1 is common.

The basic law relating the flow of fluids through control valves is quite similar to that governing flow through an orifice. For liquids, this flow relationship may be simply expressed as

$$Q = C_v \sqrt{\frac{\Delta P}{G}}. \tag{18}$$

C_v, the valve discharge coefficient, is a function of the maximum valve capacity, the valve characteristic, and the percentage of valve closure. Equation (19), along with eqs. (15) through (17), will allow an estimation to be made of the discharge coefficient provided the maximum capacity of the valve is known:

$$C_v = \omega(C_v)_{\max}. \tag{19}$$

Equations (15) through (19) are based upon the assumption of constant pressure drop across the control valve. In practical control systems this is not generally the case. Since valve pressure drop represents wasted power, good engineering practice is to keep this value at a minimum consistent with good control. As a rule of thumb, approximately one-third to one-fourth of the total system pressure drop should be absorbed by the control valve. Assuming a constant pressure source, because the pressure drop in the system varies as the flow changes, a variable pressure drop is observed across the control valve. The valve and the system may be considered as two flow restrictions hooked in series. The equivalent discharge coefficient of the combination may be computed with the aid of eq. (20):

$$C_E = \sqrt{\frac{1}{\left(\dfrac{1}{C_v}\right)^2 + \left(\dfrac{1}{C_L}\right)^2}}, \tag{20}$$

where C_E = equivalent discharge coefficient,
C_v = valve discharge coefficient,
C_L = line (or system) discharge coefficient.

The effective characteristics of the valve are altered considerably by the presence of flow resistance in the remainder of the system. When it is necessary to design a control system which requires that only a small fraction of the total available pressure drop be taken across the control valve, the exponential characteristic is the logical choice. It can be shown that the overall control valve effectiveness is more nearly independent of load level in the exponential valve than in any other type.

Stiction and Unbalanced Stem Forces in Valves. Before leaving the subject of control valves, several other idiosyncrasies are worthy of mention. These include the effects of stiction and unbalanced stem forces. The term "stiction" applies to the Coulomb friction between the stem and the valve-packing gland. The effect of stiction is to require a rather large change in the activating signal in order to overcome the friction that is present. Once the valve stem has begun to move, however, the stroke is nearly proportional to the applied force. There are several services in which stiction is particularly annoying. These include (1) large valves; (2) valves with long stuffing boxes, such as those in steam service or those in refrigerated service; (3) valves under high pressure; and (4) valves in a service where slurry or other gum-forming materials in the process stream may find their way into the packing gland. Stiction in any of these instances can be remedied through the proper use of a valve position controller.

The question of unbalanced stem forces arises only in those applications in which there is a large pressure drop across the valve itself. In the diaphragm motor control valve, a force applied to the diaphragm motor is opposed by means of a compression spring. Assuming that Hooke's law is applicable, the position which the valve stem will take is directly proportional to the applied diaphragm pressure. However, due to the unequal areas on which the upstream and downstream pressures operate, unbalanced forces result which tend to alter the motion imparted by the diaphragm motor. The final valve stem position is not a direct or linear function of the diaphragm motor pressure. A valve stem position controller can often be used to compensate for the undesirable effects of unbalanced stem forces.

5. NONLINEAR CONTROL DEVICES

On-Off Controllers. A great majority of controllers currently used in process control systems are linear devices. However, occasionally we find the nonlinear on-off controller in use. The common home thermostat is a good example of this type of controller. In process control systems its use is normally limited to very simple systems in which the ratio of system capacity to supply is rather large. A very excellent discussion of the on-off control system is presented in Ref. 1.

In some systems, such as the speed control of reciprocating compressors, it is desirable to place limit stops upon the maximum possible excursion of the set point. This is essential where a master controller, such as a pressure controller, will reset the set point of the speed control loop. In still another instance it may be desirable to limit the minimum closure position of a valve because of safety reasons. In this case a low-limit stop would be placed in the signal line from the controller to the valve. Both of the foregoing examples are illustrative of linear systems which have imposed upon them an arbitrary nonlinearity when a certain value is reached. This is essentially equivalent to saturation of an element in the control system.

Pneumatic Transmission Lines. Perhaps one of the more interesting nonlinear elements in control systems are pneumatic transmission lines. Since air is a compressible medium, the control relationships are nonlinear. Therefore it is meaningless to speak of the time constant of a pneumatic transmission line. When dealing with small signal changes, however, it is permissible to linearize the flow equation about the operating point and to determine an effective time constant at that particular pressure level. If we restrict ourselves to small perturbations and, further, if we define the time constant as the time required for the system to reach 63% of its final value, then for a deflating system eq. (21) will hold:

$$e^{-\lambda T_k} = \frac{0.63 + 0.37r + \sqrt{(0.63 + 0.37r)^2 - 1}}{r + \sqrt{r^2 - 1}}, \tag{21}$$

where r = absolute pressure ratio of initial to final values of pressures,
T_k = 63% response time,
λ = geometric factor.

The apparent time constant is a function of the absolute pressure ratio of initial to final pressure, and also a function of the geometry of the system, including diameter and length of tubing as well as terminal volume, molecular weight, and temperature of the gas. A simpler form of this expression is presented as eq. (22) and plotted in Fig. 5:

$$\lambda T_k = \phi(r). \tag{22}$$

As the absolute pressure ratio approaches unity, the effective time constant will approach zero. If the effective time constant is known at any particular pressure ratio r, Fig. 5 will allow the estimation to be made of the effective time constant at a different pressure ratio.

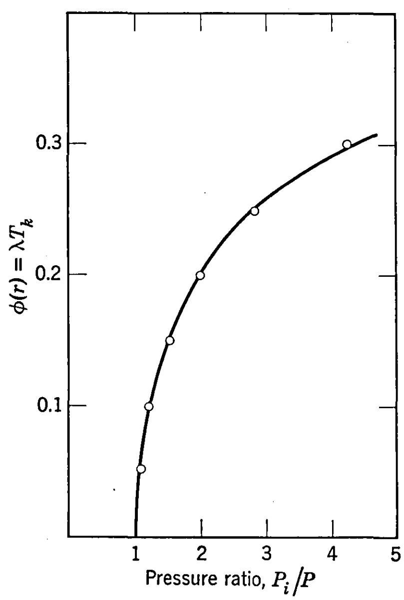

FIG. 5. Effect of pressure ratio upon effective time constant of deflating pneumatic systems.

6. CLASSIFICATION OF PROCESS NONLINEARITIES

Process nonlinearities may be classified either in accordance with their effect upon the static response or the dynamic response. Under the static response category the effects may be further divided into continuous and discontinuous nonlinearities.

Static, Continuous Nonlinearities. Static, continuous nonlinearities are those in which a smooth continuous relationship exists between the input and output quantities of a system and in which the derivative of this relationship with respect to the input variable is continuous within the region of interest. This particular type of nonlinearity can be characterized by a variable system gain which is a function of the signal level.

Static, Discontinuous Nonlinearities. Static, discontinuous nonlinearities can be conveniently classified on the basis of the signal size. Small signal discontinuities, as far as process control is concerned, are principally (1) dead zone, (2) backlash or hysteresis, and (3) stiction. Graphical representations of these effects are shown in Figs. 6, 7, and 8.

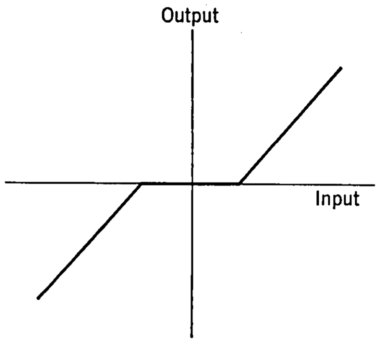

FIG. 6. Graphical representation of dead zone.

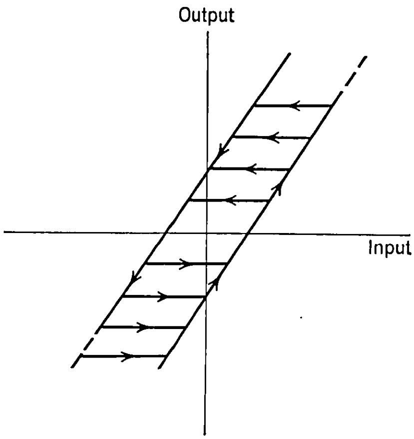

FIG. 7. Graphical representation of hysteresis.

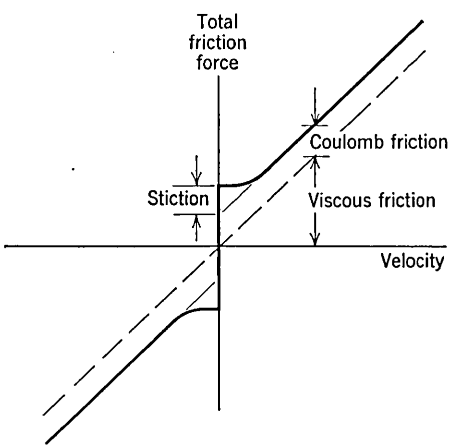

FIG. 8. Graphical representation of stiction.

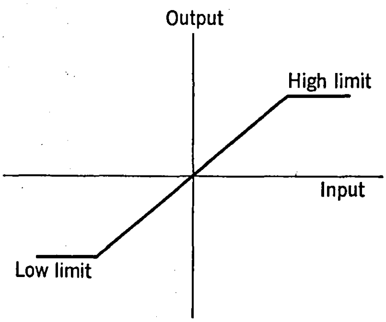

FIG. 9. Graphical representation of saturation.

The main static discontinuity caused by large signal changes is the effect known as saturation, such as complete opening or closing of a control valve. Saturation is graphically depicted in Fig. 9.

Dynamic Nonlinearities. Dynamic nonlinearities depending on signal amplitude and level were mentioned briefly in a prior discussion of pneumatic transmission lines. Another good example is the effect of flow rate upon the effective time constant of a temperature control system. The time constant of a thermal-measuring element may be estimated through the use of eq. (23) in which h represents the overall heat transfer coefficient:

$$T_k = \frac{C}{Ah}, \tag{23}$$

where C = thermal capacity = mass $\times$ specific heat,
A = area across which heat transfer is effective,
h = overall heat transfer coefficient.

Now according to eq. (24) the heat transfer coefficient for fluids flowing outside of a tube is a function of the 0.6 power of the mass velocity. The reason now becomes apparent from the following equation why some temperature control systems are sensitive to changes in flow rate:

$$\frac{hD_0}{k} = 0.33\left(\frac{C_p\mu}{k}\right)^{1/3}\left(\frac{D_0G}{\mu}\right)^{0.6}, \tag{24}$$

where D_0 = diameter of tube,
k = thermal conductivity of metal,
μ = viscosity of the fluid,
C_p = specific heat of the fluid,
G = mass velocity of the fluid.

There is still another dynamic effect which is peculiar to processes themselves. This is the result of degradation of process performance as a function of usage or time owing to catalyst activity decline, heat exchanger fouling, compressor valve wear, etc. The rates at which these changes occur are functions of process variables. The effects therefore constitute a kind of nonlinearity. In most cases, however, they can be regarded as long term time variations in process parameters which do not affect the short term linearity or nonlinearity of the process. These items in general are quite unpredictable, and compensation for them is best obtained by periodic adjustment of the control system.

7. EFFECTS AND TREATMENT OF NONLINEARITIES

Process systems are always nonlinear to some extent. For this reason it might appear that numerous examples could be found in the published literature to illustrate the performance characteristics of such systems, the various analytical tools, and the techniques for circumventing or using nonlinearity. This is not the case.

The primary explanation is economic. Process control instruments are sold in great numbers and at relatively low prices. Neither the manufacturer nor the user can expend much time or effort in an engineering analysis to discover the best way to fit a controller to a process. Ordinarily, therefore, standard, off-the-shelf controllers are installed, and their adjustments are determined by trial and error, guided by experience or possibly by more systematic procedures having some theoretical basis (Refs. 2 and 3). If the control system cannot be made to operate properly by these procedures and if the incentives are great enough, a more rigorous analysis may be undertaken. Advanced analytical techniques (see Vol. 1, Chap. 25, Nonlinear Systems) may be used, or the assistance of computers may be enlisted. A number of papers have been published describing such studies (Refs. 4 to 22).

The second explanation for the shortage of published material on process nonlinearities is technical. The analysis of nonlinear systems is not an easy matter. Many methods are available, but none of them is universally applicable or quickly mastered. Few of the methods are taught in undergraduate courses, with the result that most instrument men have not been exposed to them. Computing facilities adequate for realistic problems are somewhat expensive. Only comparatively large companies have had people or equipment capable of detailed investigation of process control problems. These deficiencies are being remedied by education, formal and informal, and by more widespread availability of computers. Subject to the economic constraints mentioned, increasing amounts of data can be expected on nonlinearities in process systems.

Because the variety of process nonlinearities is so great and the analytical techniques are so numerous, it is impossible to treat them all in an encyclopedic fashion. However, a few examples may serve to show some of the phenomena that can be expected, to illustrate some of the techniques presented in Vol. 1, Chap. 25, Nonlinear Systems, and to suggest some cures for undesirable effects of nonlinearities.

8. ADJUSTMENT OF CONTROLLER CONSTANTS

In a process that operates at widely different conditions from time to time, noticeable variations in its static and dynamic characteristics may be encountered. Since extreme changes might result in instability of the process and its control system, preventive measures are clearly necessary. The simplest measure that can be taken is an adjustment of the controller constants so that some degree of control is maintained under the worst possible conditions. This solution, by far the commonest method of attacking nonlinearities, will be illustrated by two examples.

Variable Static Gain

Control Valve. In nearly all control systems encountered in chemical processes a valve is manipulated at some point. The controlled variable may actually be the flow of an important raw material. In many cases a flow is varied in order to regulate a temperature, pressure, level, concentration, or some other significant process condition. By suitable interpretation of the diagram, Fig. 10 can therefore be applied to a great many process situations.

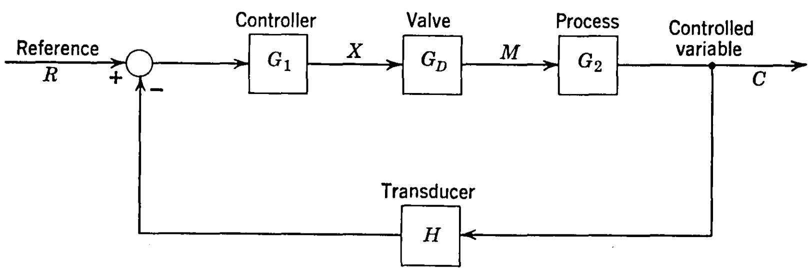

FIG. 10. Typical nonlinear process involving control of a valve.

In Fig. 10 the symbols G_1, G_2, and H represent the characteristics of the controller, process, and feedback transducer, respectively. These elements are assumed here to be described by linear differential equations or, as a kind of shorthand, by transfer functions in the Laplace transform variable s. The symbol G_D denotes the nonlinear relation between the valve position X and the flow M. This relationship is sketched in Fig. 11 for a typical valve.

For small variations about specified operating points, the valve characteristic can be linearized. For example, when $x = 0.25$, the curve can be approximated by a straight line having a slope

$$\left.\frac{dm}{dx}\right|_{x=0.25} \approx \frac{\Delta m}{\Delta x} = \frac{0.075}{0.25} = 0.3. \tag{25}$$

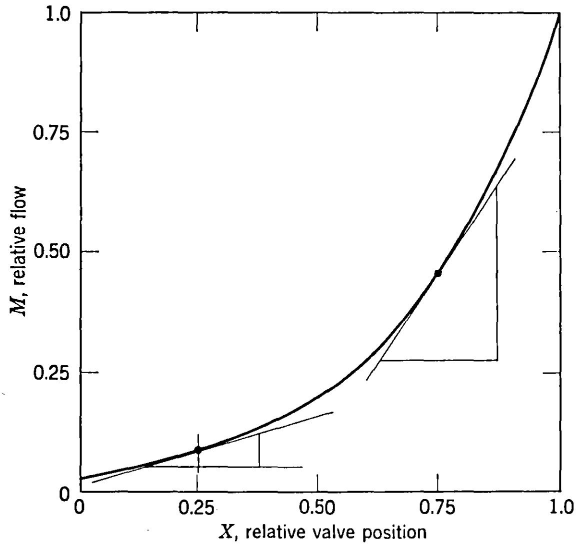

FIG. 11. Relation between valve position and valve flow (G_D in Fig. 10).

When $x = 0.75$, the slope is

$$\left.\frac{dm}{dx}\right|_{x=0.75} \approx \frac{\Delta m}{\Delta x} = \frac{0.375}{0.25} = 1.5. \tag{26}$$

The 5:1 variation in slope can have a significant effect on the performance of the control system. An example will illustrate this point. Take

$$\begin{aligned} G_1 &= K, \\ G_2 &= \frac{e^{-s}}{(s+1)(10s+1)}, \\ H &= 1. \end{aligned} \tag{27}$$

The expression for G_1 implies that a simple proportional controller is being used; G_2 describes a process having a pure time delay of 1 sec (not in itself a nonlinearity) and two first-order transform factors with time constants of 1 and 10 sec; $H = 1$ is assumed for convenience. The frequency response corresponding to G_2H is plotted in Fig. 12.

As can be seen from Fig. 12, the phase shift is $-180°$ at a frequency of 0.15 cps; the magnitude ratio at this frequency is 0.08. It follows that the controller constant K is limited by

$$KG_D = 1/0.08 = 12.5 \tag{28}$$

since, at this value for KG_D, the system is on the verge of instability. In practice, of course, a smaller value must be used to obtain a response

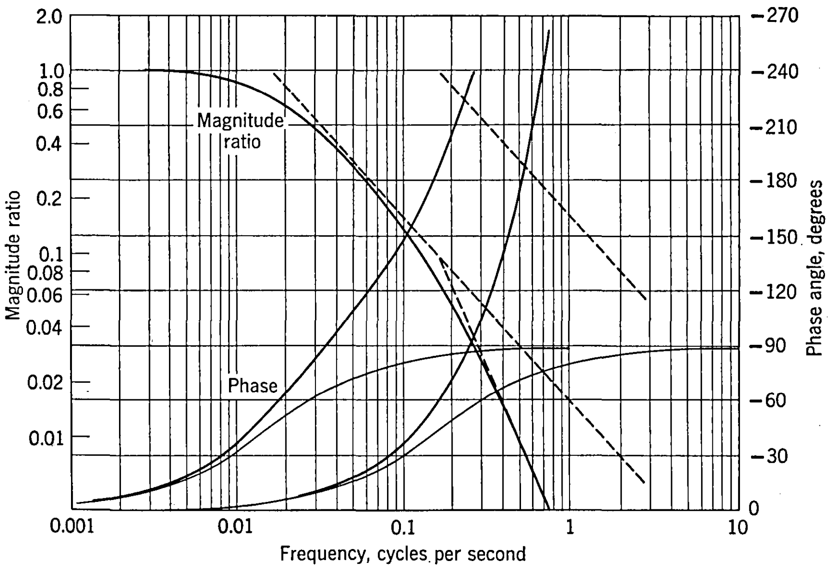

FIG. 12. Frequency response corresponding to G_2H (see Fig. 10, $H = 1$ for convenience).

with a satisfactory damping and to allow for variations in G_D. The value of K must be determined to accommodate the largest expected value of G_D. In this example, $K = 3$ might be satisfactory.

Closed loop frequency response curves are plotted in Fig. 13 for $G_D = 0.3$ and $G_D = 0.5$, using $K = 3$ as suggested above. While the system would be stable and operate fairly satisfactorily at both operating points, the dynamic behavior changes. At $x = 0.25$, the system would show a slow and nonoscillatory response; at $x = 0.75$, the system would have a faster and more oscillatory response. For the specified value of K, the system would be stable as the operating point approached $x = 1$ where the slope of the valve characteristic is greatest. With a larger value of K, however, say $K = 10$, the system would be stable for small values of x and unstable at large values of x.

Variable Dynamic Characteristics

Processes Involving Storage. In systems involving storage of materials, process dynamic characteristics can be expected to vary with the amount of material stored. A simple example is presented in Fig. 14. One stream from a previous processing step enters the mixing tank at a rate of F_1 gal/sec, and it contains a low and variable concentration a_1 of some important ingredient. The second stream is added at a controlled rate of F_2 gal/sec and contains the same ingredient at a high and fixed concentration a_2. Because of the thorough mixing provided by an agita-

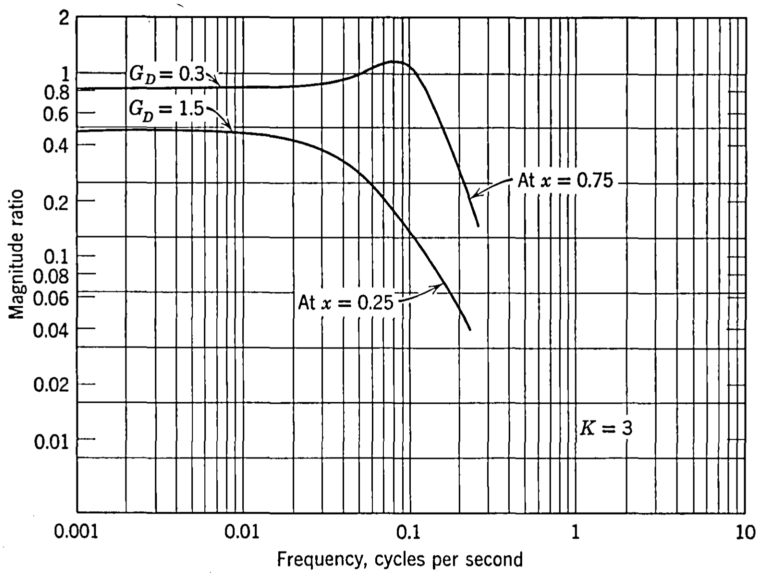

FIG. 13. Closed loop frequency response curves for $G_D = 0.3$ and $G_D = 1.5$, with $K = 3$.

tor, the contents of the tank are assumed to reach instantaneously a uniform concentration a_0. Material of concentration a_0 is removed from the tank at a rate of F_0 gal/sec. The quantity of material in the tank at any time is V gal.

The function of the control system is to maintain the outlet concentration at a fixed value a_{0d} despite changes in the inlet concentration a_1. The composition analyzer measures the actual outlet concentration. The measured concentration a_{0m} is compared with the desired concentration

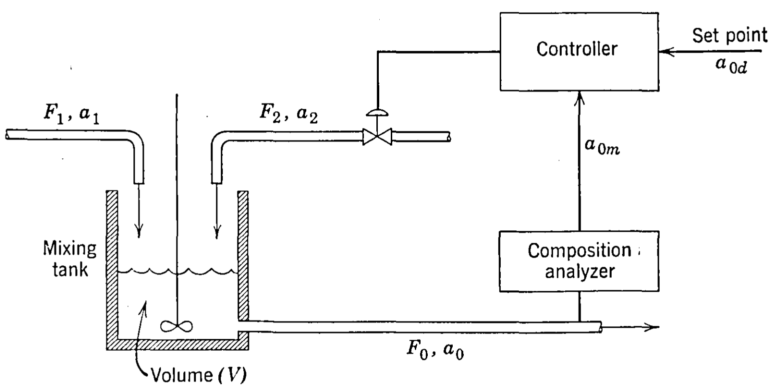

FIG. 14. Process involving storage of materials having nonlinear dynamic characteristics.

a_{0d} in the controller, generating an error signal which actuates the valve governing the flow rate F_2.

The total material balance equation for the mixing tank is

$$\frac{dV}{dt} = F_1 + F_2 - F_0. \tag{29}$$

The material balance equation for the ingredient of interest is

$$\frac{d(a_0 V)}{dt} = a_1 F_1 + a_2 F_2 - a_0 F_0. \tag{30}$$

If F_2 is small and F_0 is almost equal to F_1, the rate of change of V given by eq. (29) will be small. Hence, over short time intervals at least, the quantity V in eq. (30) can be regarded as a constant. With this simplification eq. (30) can be written

$$\frac{V}{F_0}\frac{da_0}{dt} + a_0 = a_1 \frac{F_1}{F_0} + a_2 \frac{F_2}{F_0}. \tag{31}$$

In transfer function notation the process can be represented as shown in the area bounded by the dashed lines in Fig. 15. For ease of analysis, the composition analyzer will be characterized by a simple constant K_a; the dynamic characteristics of the analyzer, as well as the time required for material to travel from the tank to the analyzer, will be ignored. The controller will be assumed to be a pure integral or "reset" device with the transfer function K_c/s.

For variations in a_1, the block diagram can be reduced to the form

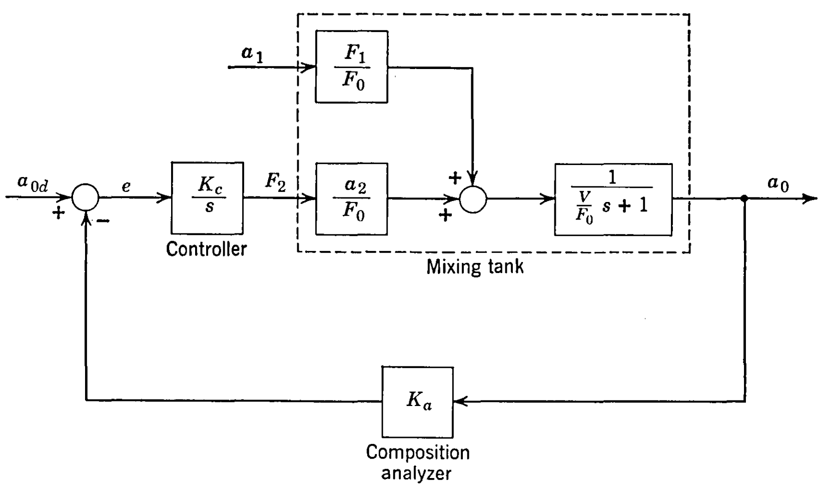

FIG. 15. Block diagram of process of Fig. 14. The process is bounded by the dashed lines.

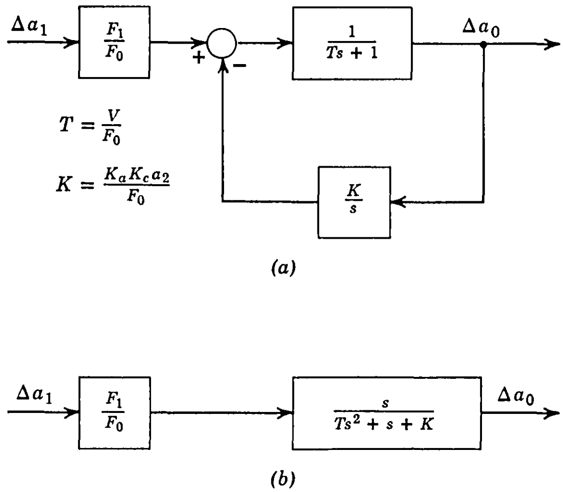

FIG. 16. Simplification of the block diagram of Fig. 15, assuming that the composition analyzer can be characterized by a simple constant K_a and the controller is a pure integral device represented by K_c/s: (a) reduction for variations in a_1, (b) further reduction.

shown in Fig. 16a. A further reduction gives Fig. 16b (see Vol. 1, Chap. 20, Fundamentals of System Analysis for theorems covering transformation and reduction of block diagrams), corresponding to the overall transfer relation

$$\Delta a_0(s) = \frac{s}{Ts^2 + s + K} \frac{F_1 \Delta a_1(s)}{F_0}, \tag{32}$$

$$\text{where} \qquad T = \frac{V}{F_0}, \tag{33}$$

$$\text{and} \qquad K = \frac{K_a K_c a_2}{F_0}. \tag{34}$$

The variation of T and K with V and F_0 is a manifestation of the nonlinear nature of the process. By virtue of the various assumptions, the complete system has been represented by a standard second order transfer function with the following parameters:

$$\text{Undamped natural frequency} = \omega_0 = \sqrt{K/T}, \tag{35}$$

$$\text{Damping ratio} = \zeta = \frac{1}{2\sqrt{KT}}, \tag{36}$$

$$\text{Damped natural frequency} = \omega_n = \omega_0\sqrt{1 - \zeta^2}. \tag{37}$$

Examples of Response Curves. By application of the final value theorem to eq. (32), it is found that there is no steady-state change in a_0 for step disturbances in a_1; this behavior is a direct result of the use of an integrating controller. There are transient changes in a_0, however, whose size and duration are dependent on the system parameters. To demonstrate the variation in dynamic behavior of the system with changes in V, three response curves will be calculated. In all three cases the following values will be assumed:

$$\begin{aligned} F_0 = F_1 &= 4 \text{ gal/sec}, \\ a_1 &= 0.05\ (t = 0^-), \\ &= 0.03\ (t = 0^+), \\ a_{0d} &= 0.08, \\ a_2 &= 1.00, \\ K &= 0.25. \end{aligned}$$

When steady-state conditions are reached in the linearized representation of the system, the flow F_2 will be

(38)
$$\begin{aligned} F_2\big|_{ss} &= \frac{(0.08)(4) - (0.03)(4)}{1.00} \\ &= 0.20, \end{aligned}$$

from eq. (30). The rate of change of V from eq. (29) is therefore

(39)
$$\begin{aligned} \left.\frac{dV}{dt}\right|_{ss} &= 4 + 0.20 - 4 \\ &= 0.20. \end{aligned}$$

The validity of ignoring this small rate of change can be judged better after the duration of the transients is established.

Case 1, V = 16. For $V = 16$ gal:

$$\begin{aligned} T &= \tfrac{16}{4} = 4 \text{ sec}, \\ \omega_0 &= \sqrt{0.25/4} = 0.25 \text{ radians/sec}, \\ \zeta &= \frac{1}{2\sqrt{(0.25)(4)}} = 0.50, \\ \omega_n &= 0.25\sqrt{1 - (0.5)^2} = 0.217 \text{ radians/sec}. \end{aligned}$$

For a step change in a_1 from 0.05 to 0.03, the change in a_0 is a damped sinusoid with a period of $2\pi/0.217 = 29$ sec and a damping ratio of 0.50 (one-half critical damping), shown in Fig. 17.

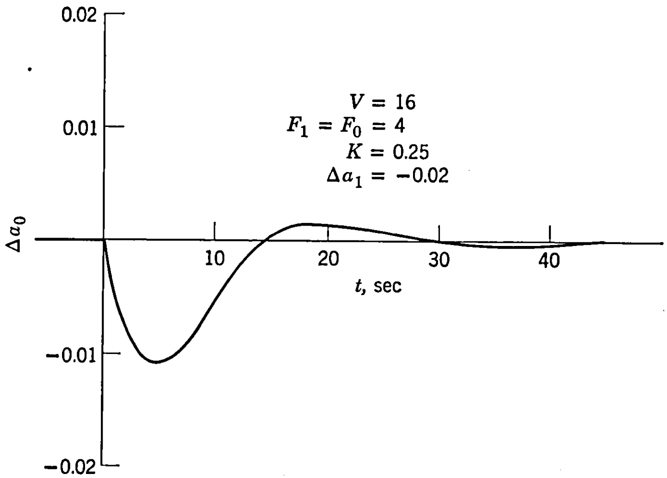

FIG. 17. Response curve, change in concentration Δa_0 with time, for case 1, $V = 16$.

Case 2, $V = 100$. If the same disturbance occurs when the tank contains 100 gal, we obtain:

$$T = \tfrac{100}{4} = 25 \text{ sec},$$

$$\omega_0 = \sqrt{0.25/25} = 0.10 \text{ radians/sec},$$

$$\zeta = \frac{1}{2\sqrt{(0.25)(25)}} = 0.20,$$

$$\omega_n = 0.10\sqrt{1 - (0.2)^2} = 0.098 \text{ radians/sec}.$$

The change in a_0 is now a damped sinusoid with a period of $2\pi/0.098 = 64$ sec and a damping ratio of 0.20 (two-tenths critical damping), shown in Fig. 18. Because of the greater dilution of the incoming flow in the material contained in the tank, two effects are observed: the maximum change in a_0 is smaller and the transient lasts longer.

Case 3, $V = 0$. For an empty tank the change in a_1 is felt immediately and completely by the composition analyzer, and the controller acts promptly to return a_0 to its desired value. Examination of eq. (32) shows that the process transfer function reduces to a first order form with a single time constant:

$$t = \frac{1}{K} = \frac{1}{0.25} = 4 \text{ sec}. \tag{40}$$

The response is therefore a simple exponential, shown in Fig. 19.

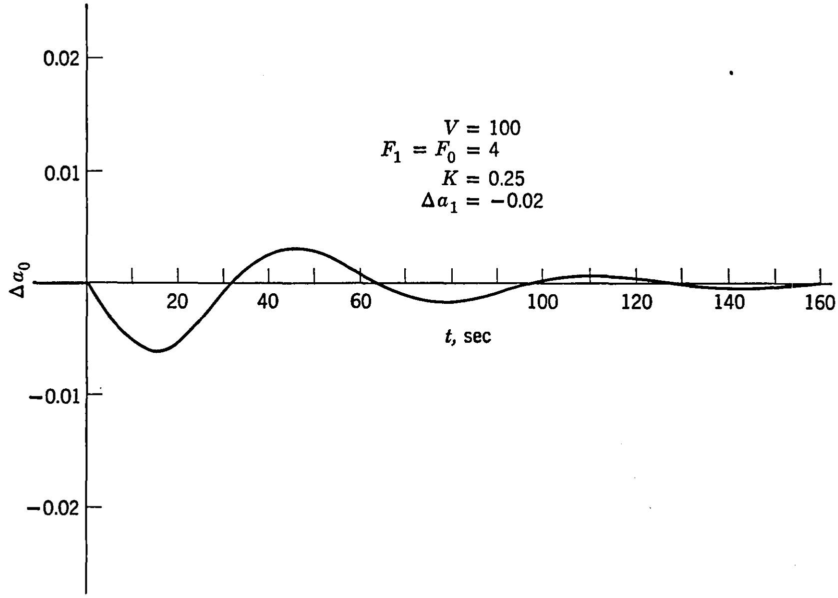

FIG. 18. Response curve, change in concentration Δa_0 with time, for case 2, $V = 100$.

The desirability of any of the response curves cannot be judged without a knowledge of the requirements of downstream processing steps receiving the flow F_0. It is clear, however, that the inventory V has a noticeable effect on the dynamic behavior of the system. Using more realistic

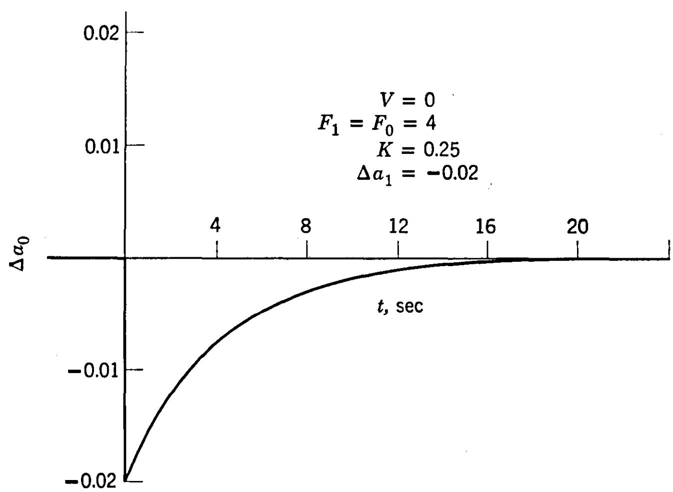

FIG. 19. Response curve, change in concentration Δa_0 with time, for case 3, $V = 0$.

characteristics for the composition analyzer and its associated sampling system, it is likely that the control system would be unstable for large values of V (or T). This difficulty might be alleviated if F_1 or F_0 could be varied to maintain a constant level in the tank; whether this step could be taken would depend on the requirements of the upstream or downstream processes. Varying F_0 would introduce further nonlinearities which, although they would complicate analysis of process behavior, would not necessarily be detrimental to system performance.

Although the example treated here was a mixing process, similar problems arise in heating and cooling processes. If hot and cold materials are combined to produce a mixture at a specified temperature, the temperatures appear in a manner analogous to concentration in this example. However, these problems can be complicated by the phenomena of condensation and vaporization, differences in specific heats, and heat losses. Tubular heat exchangers are further complicated by the necessity for considering space as well as time variations. For a discussion of some problems of this kind, the reader should see Ref. 3. There are, of course, many other situations in which the dynamic characteristics of the process vary with changes in the important process variables.

9. USE OF LOCAL FEEDBACK LOOPS

The transfer function for a feedback system is

$$\frac{C}{R} = \frac{G}{1 + GH} \tag{41}$$

where C is the transform of the controlled variable, R is the transform of the reference variable, and independent G and H are the transfer functions of the forward and feedback paths, respectively. If GH is made very large, the system transfer function becomes

$$\frac{C}{R} \approx \frac{1}{H}. \tag{42}$$

Under these conditions the system behavior is independent of G. Where G includes a nonlinearity, its importance is effectively minimized, and the system characteristics are determined primarily by the elements in the feedback path (represented by H). This principle is not easily applied to complex systems in which numerous time delays limit the attainable loop gain, but it can be used to advantage on a smaller scale by placing feedback paths around individual nonlinearities.

An example, already mentioned, is the valve positioner. This device is used to counteract the nonlinear friction forces acting on the valve stem, generally a combination of stiction and Coulomb friction as suggested in Fig. 20*a*. Without a valve positioner the valve stem does not move until sufficient force is developed to overcome the stiction force. With a valve positioner, as shown in Fig. 20*b*, the stem position is measured and com-

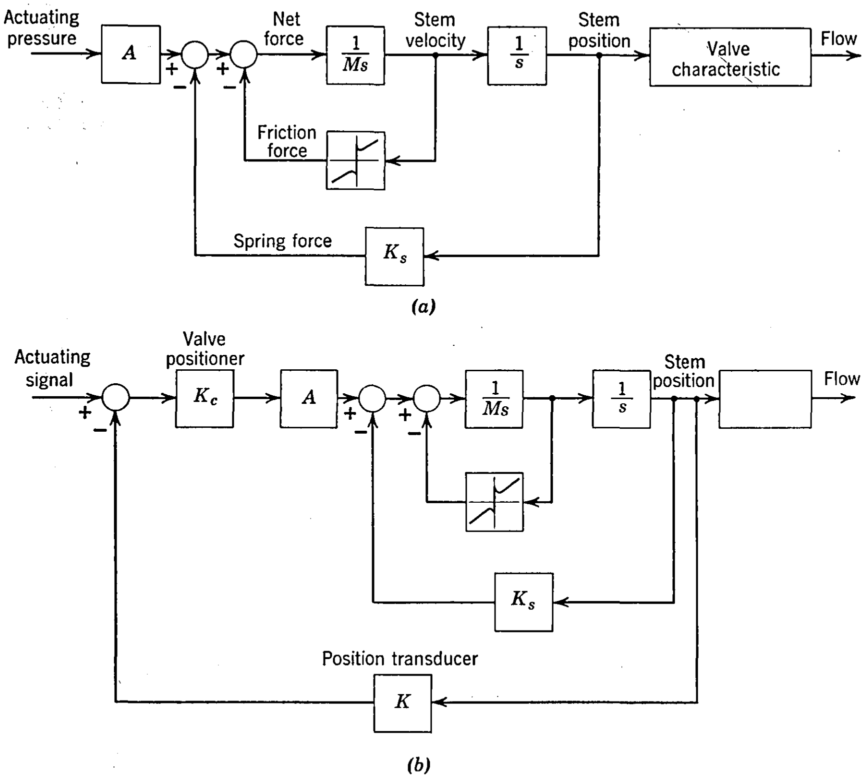

FIG. 20. Block diagrams of valve control illustrating (*a*) stiction and Coulomb friction and (*b*) valve positioner employing local feedback loop to control valve stem position in accordance with the actuating signal.

pared with the actuating signal. The difference signal is sent to a controller (acting as the valve positioner) and amplified. Since the controller gain can be relatively high, sufficient force is easily developed to move the valve stem in spite of the friction forces.

10. COMPENSATION FOR NONLINEARITIES

Another treatment for the harmful effects of a nonlinearity is the deliberate introduction of a second nonlinearity. In other cases nonlinearities are introduced into linear systems to (*a*) reduce response time or (*b*) minimize overshoots by better utilization of power elements. In process control, this approach is employed most frequently in connection with the square and square root relationships arising in the regulation of flow rates.

Three methods for compensating for the nonlinear characteristics of a valve are shown in Fig. 21 (Ref. 23). In the first method, Fig. 21*a*, a

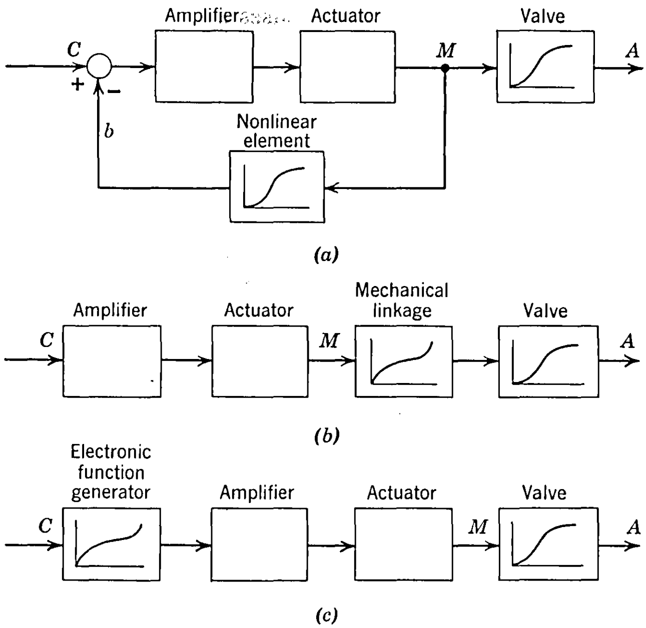

FIG. 21. Methods of compensating for the nonlinear characteristics of a valve: (*a*) use of a nonlinear element in a feedback loop around the valve, (*b*) nonlinear mechanical linkage in series with valve, and (*c*) nonlinear electronic function generator in series with the valve.

nonlinear element having a characteristic like that of the valve is placed in a feedback path around the valve actuator and its amplifier. Because of the well-known property of feedback systems, that is, to have a transfer characteristic which is approximately the reciprocal of the characteristic of the feedback element, the feedback loop preceding the valve will have a characteristic which roughly cancels the nonlinearity of the valve. This approach has the disadvantage that the dynamic properties of the feedback loop vary with the signal levels in the system.

The second and third methods use a nonlinear element, in series with the valve, which has a characteristic selected to compensate directly for the effects of the nonlinear valve characteristic. In the system of Fig. 21*b* the nonlinear device is a mechanical linkage, whereas the system of Fig. 21*c* can use an electronic function generator.

An example of the use of deliberate nonlinearity is found in a pressure control system for a jet engine test cell (see Fig. 22 and Ref. 24). In this system the pressure level may change over a 15:1 range, and the nonlinear flow position characteristic of the butterfly valves causes a further

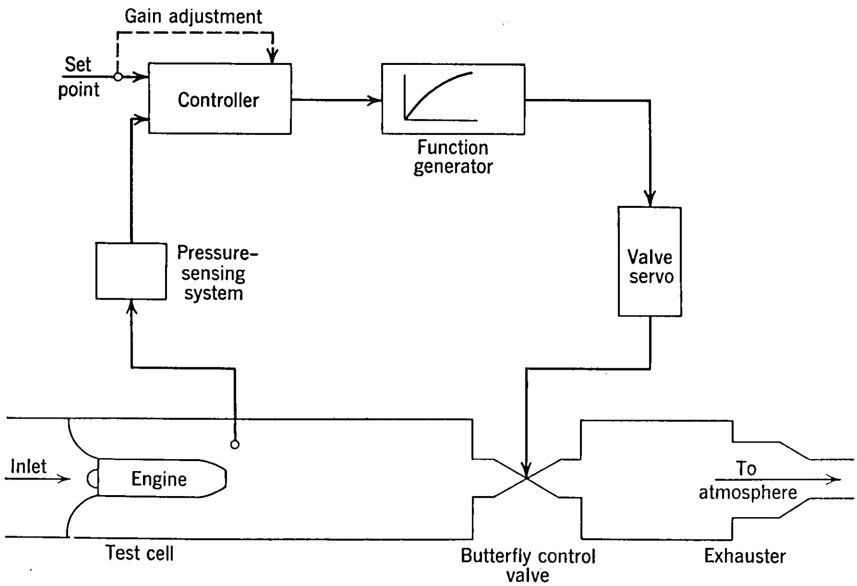

FIG. 22. Deliberate use of nonlinear function generator for pressure control of jet engine test cell.

variation in loop gain. To compensate for these variations, the pressure set point is mechanically coupled to a potentiometer which determines the controller gain, and a nonlinear function generator is used to furnish a signal to the valve servo.

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Sampled-Data Control

R. E. Kalman

1. INTRODUCTION

At present, there are but a few sampled-data systems in process control. Such systems have important advantages, however, over conventional controllers and may be expected to be used more frequently in the future.

The block diagram of a sampled-data system for the control of a single input, single output process is shown in Fig. 1 (Ref. 1). We use terminology of the z-transform theory discussed in Vol. 1, Chap. 26, Sampled Data Systems and Periodic Controllers. The error signal $e_1(t)$ is converted by the sampling switch into $e_1{}^*(t)$, which is a sequence of narrow pulses, spaced at intervals of T seconds; the area of the pulse at $t = kT$ is $e_1(kT)$. The sampled signal $e_1{}^*(t)$ is converted by the *sampling controller* into another pulse train $e_2{}^*(t)$, which, after passing through a hold circuit, becomes the input signal $m(t)$ to the process. The sampling switches shown in Fig. 1 are used only for mathematical convenience.

The sampling controller performs the function of converting a sequence of numbers $e_1(0)$, $e_1(T)$, $\cdots$, $e_1(kT)$, $\cdots$ into another sequence of numbers,

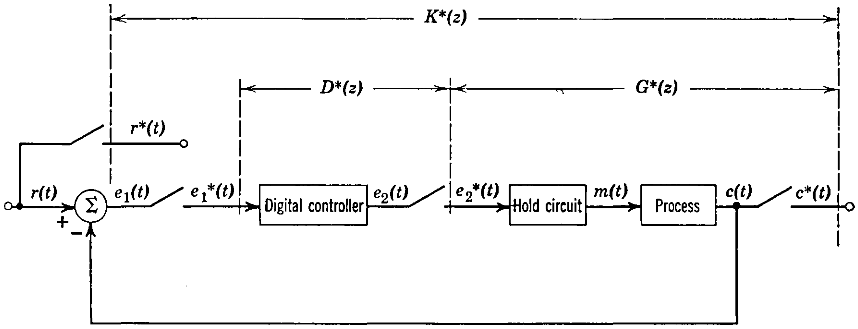

FIG. 1. Block diagram of a sampled-data system for the control of a single-input, single-output process.

$e_2(0),\ e_2(T),\ \cdots,\ e_2(kT),\ \cdots$. Any *linear* linear sampling controller is describable by the difference equation

$$(1)\quad e_2(kT) = a_0e_1(kT) + a_1e_1((k-1)T) + \cdots + a_me_1((k-m)T) - b_1e_2((k-1)T) - \cdots - b_ne_2((k-n)T)$$

or, equivalently, by the z-transform

$$(2)\qquad D^*(z) = \frac{a_0 + a_1z^{-1} + \cdots + a_mz^{-m}}{1 + b_1z^{-1} + \cdots + b_nz^{-n}}$$

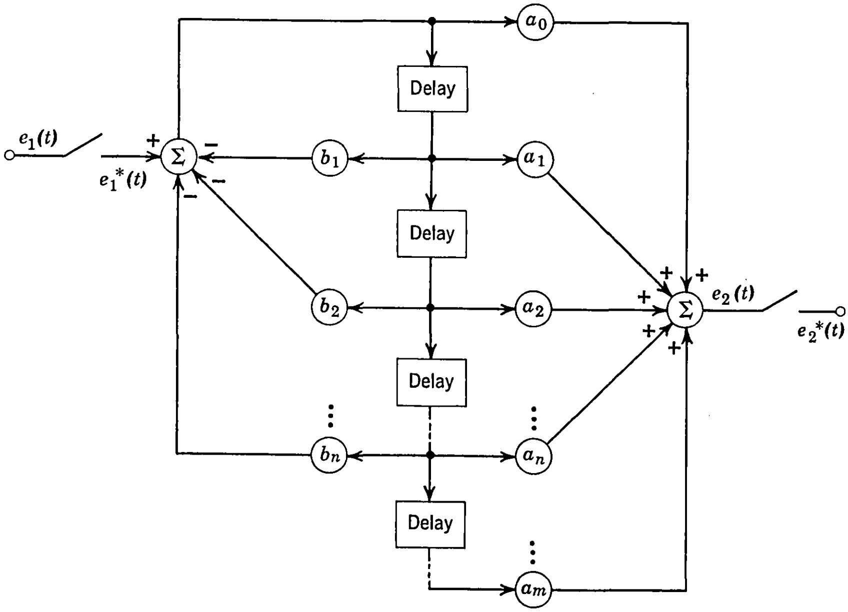

FIG. 2. Schematic diagram of a sampling controller.

where $z = e^{sT}$. A schematic diagram of the sampling controller is shown in Fig. 2.

It is generally assumed that the process is governed by a linear differential equation with constant coefficients, at least for small deviations about equilibrium. Then the closed loop z-transform of the control system relating the sampled values of the output to the sampled values of the input is given by

$$K^*(z) = \frac{C^*(z)}{R^*(z)} = \frac{D^*(z)G^*(z)}{1 + D^*(z)G^*(z)} \tag{3}$$

where $G^*(z)$ is the z-transform of the combined hold circuit and process.

2. APPLICATION CONSIDERATIONS

In process control, the principal advantages of sampled-data systems over conventional systems appear to be the following.

Flexibility. By proper adjustment of the coefficients $a_0, a_1, \cdots, b_n$ in eq. (1), the overall control system can be made to have any desired kind of dynamic behavior subject to the limitations imposed by available power at the control input to the process and to the assumption that the process is linear for small deviations about equilibrium.

Physical Realizability. The sampling controller is readily realizable by either a general purpose digital computer (which may be time-shared to simulate many sampling controllers) or by the special purpose unit described in Sect. 5.

Time Scale. By proper selection of the sampling period T, the sampling controller can be "matched" to processes having arbitrarily long time constants. (In conventional pneumatic control instrumentation an upper limit on the time constants of the controller is imposed by the size of the equipment; in electronic control instrumentation the upper limit is due to the cost of stabilizing operational amplifiers.) In the special type of sampling controller described in Sect. 5, the sampling period can be set by an electric clock.

Transportation Lags. Sampled-data systems can be used effectively in controlling processes with large transportation lags, without sacrificing steady-state accuracy. See Sect. 4.

Adaptability to Sampled Analysis. In many processes, measurement of basic process variables requires chemical or physical analysis of batch samples. This often results in time delays, which are analogous to the transportation lags in the process. By designing a sampled-data system to take into account the time delay involved in batch type chemical analyses, a high-performance system can be achieved in a simple way. If the sampling is relatively slow, eq. (1), which gives the new control setting

$e_2(kT)$ each time a batch has been analyzed, can be implemented by using just a slide rule or a desk calculator.

Several precautions must be observed in attempting to put a sampled-data system into operation under plant conditions.

Steady-State Accuracy. The hold circuit, relays, and other components in the system which affect steady-state accuracy must be of good quality, for otherwise it may be difficult to maintain required accuracy in the steady state.

Accurate Knowledge of Process Dynamics. The dynamics of the process to be controlled [i.e., the transfer function $G^*(z)$] must be known fairly accurately (1–5%) in order to obtain substantial benefits from sampled-data operation. If this is not possible, $G^*(z)$ must be computed more accurately from input-output data of the process observed during actual operation. Methods for accomplishing this are available (Ref. 6). Since the dynamic characteristics of the process may change in time (scale build-up in heat exchangers, changes in ambient temperature, etc.), it may be desirable to repeat this step frequently. Trial and error adjustment of the sampling controller is not recommended unless the correct values of $a_0, \cdots, b_n$ are known fairly accurately.

Disturbances. Measurements on the process should be performed sufficiently fast so that the accumulated effect of process disturbances between samples does not become too large.

3. DESIGN PROCEDURES

A variety of methods is available to determine how the coefficients of the sampling controller should be selected. Three of these methods are illustrated here; many other methods have been discussed in the literature; see the textbooks of Ragazzini and Franklin, Jury, and Tou (Refs. 1–3).

Method of Bergen-Ragazzini (Refs. 4 and 5). Suppose we wish to adjust the coefficients of the sampling controller in such a fashion that the process output is related to the system input by (see Fig. 1)

$$K^*(z) = \frac{C^*(z)}{R^*(z)} = \frac{D^*(z)G^*(z)}{1 + D^*(z)G^*(z)}.$$

Rearranging this equation leads to

$$D^*(z) = \frac{K^*(z)}{G^*(z)[1 - K^*(z)]}. \tag{4}$$

Thus if the desired response $K^*(z)$ and the process dynamics $G^*(z)$ are known, the coefficient settings for the sampling controller may be found directly from eqs. (4) and (2). This method has two major limitations.

1. For the z-transform $D^*(z)$ to be physically realizable, it is necessary that in the inverse z-transform of $K^*(z)$,

$$k_0 + k_1 z^{-1} + k_2 z^{-2} + \cdots$$

the terms $k_0 = k_1 = \cdots = k_q$ equal zero where $(q - 1)T < \tau < qT$, τ being the combined measurement and transportation lag of the process.

2. Poles of $G^*(z)$ which lie outside the unit circle in the z-plane must not be canceled by corresponding zeros of $D^*(z)$.

The chief disadvantage of this method is that it controls the response of the system only at the sampling points. It is desirable to check the behavior between sampling points to make sure that the design is satisfactory (see Table 1).

TABLE 1. RESPONSE OF PROCESS AT REST TO UNIT STEP INPUT

Time	Method of Bergen-Ragazzini		Method of Kalman-Bertram	
	$m(t)$	$c(t)$	$m(t)$	$c(t)$
*0	12.92	0	8.041	0
*0.5	−7.504	0	0.2058	0
*1.0	7.765	0	2.000	0
*1.5	−1.496	0	2.000	0
*2.0	4.121	0	2.000	0
2.25	4.121	0.3161	2.000	0.1968
*2.5	0.7142	1.000	2.000	0.6225
2.75	0.7142	1.298	2.000	0.9272
*3.0	2.780	1.000	2.000	1.000
3.25	2.780	0.8197	2.000	1.000
*3.5	1.527	1.000	2.000	1.000
3.75	1.527	1.109	2.000	1.000
*4.0	2.287	1.000	2.000	1.000
4.25	2.287	0.9340	2.000	1.000
*4.5	1.826	1.000	2.000	1.000
4.75	1.826	1.040	2.000	1.000
*5.0	2.106	1.000	2.000	1.000

* Denotes sampling points.

Method of Kalman-Bertram (Refs. 6 and 7). We wish to bring the process to equilibrium as soon as possible following the application of a prototype input signal, such as a unit step. If the input is a unit step, it can be shown (Ref. 3) that

$$D^*(z) = \frac{Q^*(z)}{1 - P^*(z)} \tag{5}$$

where $$P^*(z) = \frac{p_0 + p_1 z^{-1} + \cdots + p_m z^{-m}}{p_0 + p_1 + \cdots + p_m}$$

$$Q^*(z) = \frac{q_0 + q_1 z^{-1} + \cdots + q_n z^{-n}}{p_0 + p_1 + \cdots + p_m}$$

and $G^*(z) = P^*(z)/Q^*(z)$. Formula (5) is valid only if all poles of $G^*(z)$ are *inside* the unit circle. If one pole is located at $z = 1$, eq. (5) still applies but it is necessary to factor out the term $1 - z^{-1}$ from the numerator and denominator of $D^*(z)$. An extension of this method, which is not subject to the limitations just mentioned, is also available (Ref. 7).

The main advantage of the Kalman-Bertram method is that the steady-state error in response to the prototype input becomes identically zero at the sampling points as well as *between* the sampling points after at most n steps, where n is the number of poles of $G^*(z)$ plus q, q being defined earlier.

Method of Kalman-Koepcke (Ref. 8). More freedom of design is obtained by considering performance criteria of the type

$$\sum_{k=0}^{\infty} \alpha[e_1(kT)]^2 + \beta[\dot{e}_1(kT)]^2 + \gamma[m(kT)]^2$$

and adjusting the parameters of the controller so as to minimize the value of the particular performance index chosen.

This is the most general technique and many variations are possible. The calculations generally require a digital computer, and the use of the method is not recommended when knowledge of process dynamics is inaccurate.

Other Considerations. The choice of the sampling period depends on the speed of response desired. Any of the procedures described above may lead to unsatisfactory design if the sampling period is chosen too small in relation to the time constants of the process, because then the required control signal $m(t)$ may become excessively large, violating the fundamental assumption of linearity (small deviations from equilibrium). In practice, it is not recommended that the sampling period be chosen to be less than about 10% of the smallest measurable time constant of the process. Also, the smaller T is, the more accurately $G^*(z)$ must be measured in order to achieve theoretically possible performance. Methods are also available for designing sampled-data systems in which the basic sampling rate (batch analysis, etc.) is slower than the desired response of the process (Ref. 9).

4. EXAMPLES

Suppose that the process to be controlled has the transfer function

$$H(s) = \frac{e^{-2s}}{(s+1)(s+2)}.$$

The transportation lag $\tau = 2$ may be associated solely with the process or it may include also a lag due to the measurement of $e_1(kT)$. Let T equal 0.5. Then the z-transform of a zero-order hold circuit and process combination is

$$G^*(z) = \left[\frac{(1-z^{-1})e^{-2s}}{s(s+1)(s+2)}\right]^* = \frac{(1-z^{-1})z^{-4}}{2}\left(\frac{1}{s} - \frac{2}{s+1} + \frac{1}{s+2}\right)^*.$$

Using Table 1, Vol. 1, Chap. 26, rows *d*, *g*, Sampled-Data Systems and Periodic Controllers, leads to

$$G^*(z) = \frac{0.0774z^{-5} + 0.0470z^{-6}}{1 - 0.9744z^{-1} + 0.2231z^{-2}}.$$

Method 1. Suppose that $K(z) = z^{-5}$. This is the fastest possible response given the value of T and the requirement that $D^*(z)$ be physically realizable. From eq. (4),

$$D^*(z) = \frac{12.92 - 12.59z^{-1} + 2.882z^{-2}}{(1 + 0.6065z^{-1})(1 - z^{-5})}.$$

Method 2. From eq. (5),

$$D^*(z) = \frac{8.041 - 7.835z^{-1} + 1.794z^{-2}}{1 - 0.6225z^{-5} - 0.3775z^{-6}}.$$

The response of the control system for this process as designed above is shown in Table 1. [The responses between sampling points are computed by first finding $G^*(z)$ for $T = 0.25$ and then using the known values of $m(t)$ and the difference equation corresponding to $G^*(z)$.] The system designed by method 1 responds somewhat faster but requires more control signal power and energy and has more oscillatory transient behavior than the system designed by method 2.

5. SPECIAL PURPOSE COMPUTER

A special purpose computer designed to act as a single sampling controller is shown in Fig. 3. The values of $e_1(kT)$, $\cdots$, $e_1((k-m)T)$ and $e_2((k-1)T)$, $\cdots$, $e_2((k-n)T)$ are stored in the form of settings of shaft positions of potentiometers. These potentiometers are positioned by a common servomotor, a given potentiometer being selected by engaging a clutch. As soon as the measured signal $e_1(kT)$ is available, $e_2(kT)$ is com-

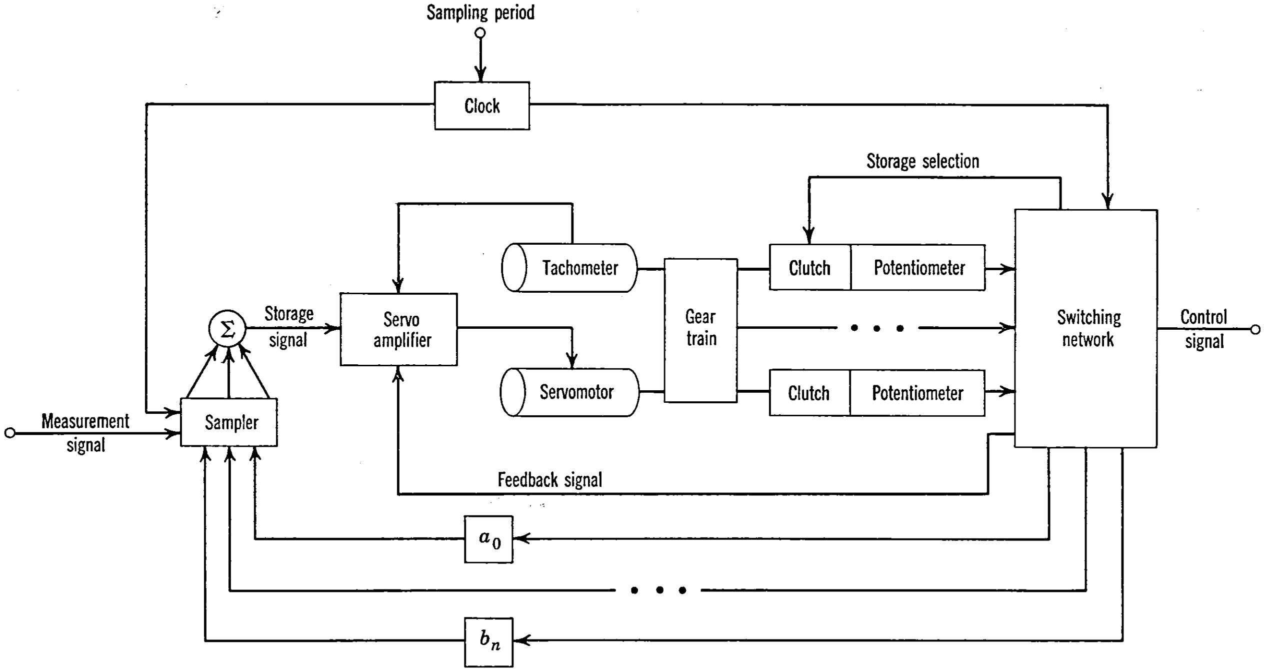

FIG. 3. A special purpose analog computer designed to act as a single sampling controller.

puted by an electronic summing circuit in accordance with eq. (1). Since a value $e_1(kT)$ stored on a potentiometer becomes $e_1[(k-1)T]$ at the next sampling point, the storage potentiometers must be switched around in cyclical order at each sampling point, new samples being stored by potentiometers which hold signals from eq. (1) that are no longer needed. The time interval between samples can be changed by adjusting a timing clock which triggers the sampling operations and the switching network.

6. FUTURE SYSTEMS

As mentioned, the chief problem in achieving high-performance operation of a sampled-data system is to obtain an accurate description of process dynamics. For this reason future systems are expected to incorporate automatic means for determining and monitoring the transfer function from operating data (Ref. 10). Moreover, future systems will be designed to abstract maximum usable information from data provided by the primary measuring elements.

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Computer Control

I. Lefkowitz

1. THE TREND TO COMPUTER CONTROL

As a result of ever-increasing demands upon the means of production for greater output, lower costs, and improved quality of products, industry has expanded in size and complexity and adopted faster and more intricate processes operating within tighter specifications. These technological advances have been possible only through the prior development and application of more advanced systems for measurement and control. For example, in such applications as the control of (1) critical processes characterized by high-speed interactions, (2) complex, multivariable processes under dynamic loading conditions, or (3) nonlinear processes under widely varying operating conditions and loads, the variables of the process must be maintained in proper dynamic relationships to one another in order to satisfy modern control objectives of optimum process performance and dynamic stability.

Conventional process control is limited, for the most part, to simple linear relationships involving a relatively small number of variables. Thus, relationships of the following form are readily handled by summing relays, ratio controllers, and the like:

$$q = \sum_{i=1}^{n} K_i m_i,$$

where q = output signal,
m_i = input signals, $i = 1, 2, \cdots$,
K_i = constant coefficients.

This equation may be further generalized to include dynamic functional relationships generated by process controllers with integral and derivative response modes:

$$Q(s) = \sum_{i=1}^{n} G_i(s)\, M_i(s),$$

where $G_i(s)$ = transfer function relating $Q(s)$ to $M_i(s)$,
$M_i(s), Q(s)$ = Laplace transforms of input and output signals respectively.

The $G_i(s)$ may be of the general form

$$G_i(s) = K_i\left(1 + \frac{1}{T_{I_i}s} + T_{D_i}s\right),$$

where K_i = proportional gain,
T_{I_i} = integral time constant,
T_{D_i} = derivative time constant.

There are, in addition, many examples of simple nonlinear functions such as the two-variable multiplication obtained by the remote set type ratio control and the square root extraction used in flow metering.

Design Procedures for Complex Conventional Systems. The more general requirements of complex functional interrelationships among process variables are handled in conventional systems by combinations of the following approaches: (1) design of a system so that essentially steady-state operation under a very narrow range of operating conditions is maintained. This is effected by means of storage tanks, stabilizing chambers, independent control of input variables, blending operations, etc.; (2) use of the operator (aided perhaps by operating graphs, computing devices, etc.) to manipulate the set points of the different variables in order to maintain an acceptable performance of the system.

Computer Control. As the scale of operations becomes faster and more complex, the human operator imposes an increasing limitation to proper maintenance of the necessary interrelationships. This limitation is eased by incorporating within the control loop modern computers capable of handling at high computation rates the complex mathematical relationships. Computer control may be designed to improve system controllability and performance over wide ranges of operating conditions in the following applications:

1. Feed-forward compensation for load and disturbance variations.
2. Feedback control of derived variables or performance indices computed from measurements of the process variables.
3. Adaptive control to compensate for large changes in system parameters.
4. Optimizing control which maximizes a specified performance criterion.
5. Model generation based on analysis of the process behavior.

2. CONTROL BASED ON COMPUTED FUNCTIONS

Data Logging. There is fairly extensive use of computing components in modern data logging and reduction systems. These include: square root extraction for orifice flowmeters, temperature and pressure compensation for fluid flow, calculation of yields and efficiency factors from measured data (see Chap. 14, Data Processing). Although most of these applications are for measurement and recording purposes, there are some extensions to control.

Composition Analyzers. Algebraic equation solvers are employed in connection with multicomponent composition analyzers. The primary example is the mass spectrometer which provides a set of output signals bearing a linear relationship to the concentrations of the various components of the mixture. The composition is determined by solving a matrix of n equations in the n unknown component concentrations. Other examples include analysis by infrared spectroscopy and chromatographic methods.

Analog Computer Control. There have been several recent publications describing analog computer application to on-line control of a derived quantity. Tolin and Fluegel (Ref. 1) describe a computer applied to the control of an exothermic chemical reactor. The control is based upon computation of production rate and reactor concentration by means of heat and material balances around the reactor. A constant production rate is maintained by manipulating one of the inputs. Lupfer and Berger (Ref. 2) describe a method of computer control of internal reflux in petroleum fractionation columns. The effective reflux is determined by a simple heat balance computation based on measurements of the external reflux flow rate and the temperature difference between the reflux stream and the reflux plate.

Digital Computer Control. An increasing number of digital computer control applications are being reported in the literature (Ref. 50). In general, the digital facility is more adaptable to the control of large scale processing systems; the on-line control of a derived quantity is usually a part of an overall control complex involving data logging and monitoring, multivariable control functions, system analysis, and optimization. Specific applications and references are presented in Sect. 7, Applications of Computer Control. Extensive bibliographies of computer control have been prepared by Grabbe (Ref. 50) and Freilich (Ref. 51).

Decoupling. The application of computer elements to effect noninteraction in multiple loop control systems has also been considered. This involves compensation for any internal coupling between control variables. Such coupling generally degrades system performance and may lead to instability because it tends to propagate disturbances from one control loop to another. The general method was introduced by Boksenbom and Hood (Ref. 3) in application for the control of a turbojet engine where the controlled variables, speed, turbine temperature, and tail pipe fuel flow, interact. Ergin and Ling (Ref. 4) apply similar principles to a boiler control system which compensates for the interaction between the header pressure and drum level control loops.

3. OPTIMIZING CONTROL

The major contribution of computer control lies in the area of optimizing system performance. The objective is to maximize (or minimize) a specified performance criterion for the controlled system subject to both disturbing and constraining influences.

Criteria for Optimization. The performance criterion is presumably specified by management. Most generally it will be an economic criterion based on production costs or profit and expressed as a linear combination of the costs of raw materials, energy, labor, maintenance, depreciation, off-standard product, etc. These cost components will in turn be functions of the operating conditions. The process objectives will often be expressed in more limited terms such as maximizing the yield or throughput rate or minimizing by-product formation.

It is assumed that the performance p can be expressed as a function of the system variables (see Fig. 1):

$$p = f(u_i, m_j, q_k), \tag{1}$$

where u_i = independent input variables (load or disturbance variables), $i = 1, 2, \cdots, I$,

m_j = dependent input variables (manipulated variables), $j = 1, 2, \cdots, J$,

q_k = system output or state variables, $k = 1, 2, \cdots, K$.

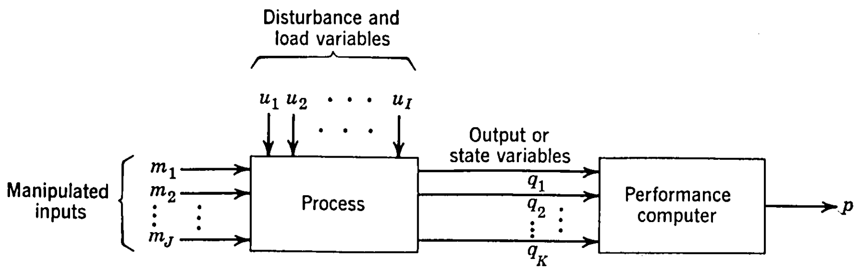

FIG. 1. Block diagram of process variables.

Since only a relative measure of the performance is required for optimizing purposes, elements of eq. (1) which are essentially constant over the range of operating conditions may be conveniently suppressed, e.g., some overhead and labor charges, research expenses, etc. Indeed, the equation may often be simplified by considering the variables only in terms of their deviations from appropriate reference values. The performance equation may be simplified further by dropping terms which are small in magnitude compared to the dominant terms. *Note.* In practice, both system equations and measured quantities are subject to some degree of uncertainty; consequently, there is little to be gained by refining the control criterion beyond a certain point. This is discussed further in the section on self-checking.

Constraint Equations. The variables of the system are assumed to be interrelated by a set of equations of the form

$$(2) \qquad g_l(u_i, m_j, q_k) = 0 \qquad l = 1, 2, \cdots, L.$$

These constraint equations may be derived analytically by applying principles of physics, chemical kinetics, thermodynamics, and other disciplines. In particular, the equations may be based on material and energy balances, heat and mass transfer relationships, laws of motion, chemical and thermodynamic equilibria, etc. Of necessity, some of the system equations will be determined empirically, based on experimental observation and past experience. Examples might include the effect of operating conditions on catalyst activity; the tendency of fouling a heat transfer surface as a function of fluid temperature and composition; fractionating column plate efficiencies as functions of the vapor and liquid flow rates and compositions. A more extensive discussion of the development of system equations and performance criteria may be found in Refs. 5 and 6.

4. ANALYTICAL METHODS OF SYSTEM OPTIMIZATION

The excess of variables over equations yields the number of degrees of freedom of the system. By referring to eqs. (1) and (2) and assuming that all the u_i are independently determined,

$$F = J + K - L, \tag{3}$$

where F = number of degrees of freedom.

The necessary conditions for the optimum are determined by first using K of the L constraint eqs. (2) to eliminate q_k from eqs. (1) and (2). Then the partial derivatives of p with respect to any F manipulated variables are set equal to zero.

$$\frac{\partial p}{\partial m_j} = 0 \qquad j = 1, 2, \cdots, F. \tag{4}$$

The F equations of (4) coupled with the remaining $L - K$ constraint eqs. (2) provide a total of J equations ($F + L - K = J$), which are sufficient to determine the J unknowns, $m_1, m_2, \cdots, m_j$.

Lagrangian Multipliers. The method of Lagrangian multipliers eliminates the need to solve eqs. (2) explicitly for the q_k variables. If we let

$$\phi = p(u_i, m_j, q_k) + \sum_{l=1}^{L} \lambda_l g_l(u_i, m_j, q_k), \tag{5}$$

where λ_l are the Lagrangian multipliers (arbitrary constants at this point), the necessary conditions for the optimum are expressed compactly as

$$\frac{\partial \phi}{\partial m_j} = 0 \qquad j = 1, 2, \cdots, J. \tag{6}$$

Note that the L Lagrangian multipliers may be eliminated from the final result by solving eqs. (6) simultaneously with the L constraint eqs. (2).

Inequality Constraints. The analytical approach just given must be modified if the optimizing control conditions call for violation of any inequality constraints imposed by the process or control elements. The most common inequality constraints are the upper and lower limits imposed on each of the manipulated variables by the rangeabilities of the respective control valves. Thus,

$$M_{L_j} \le m_j \le M_{U_j}, \tag{7}$$

where M_{L_j} = lower limit of m_j,
M_{U_j} = upper limit of m_j.

The constraint may also relate to one of the state variables of the process in terms of some function of other variables as, for example, conditions determining the flooding point in a distillation column. This inequality constraint may be expressed in the form

$$h(q_n, q_{n+1}, \cdots) \leq H. \tag{8}$$

The optimization of a process with a single manipulated variable is illustrated in Fig. 2. The performance curve is plotted as a function of m for three values of the load u. If m has the fixed upper limit M_U, then p_{max} is determined by the solution of eq. (4) for loads U_1 and U_2 (note, however, that p_{max} occurs at different peaks). As for load U_3, eq. (4) does not satisfy the condition for p_{max} which occurs at the limit M_U. Thus, in general, in-

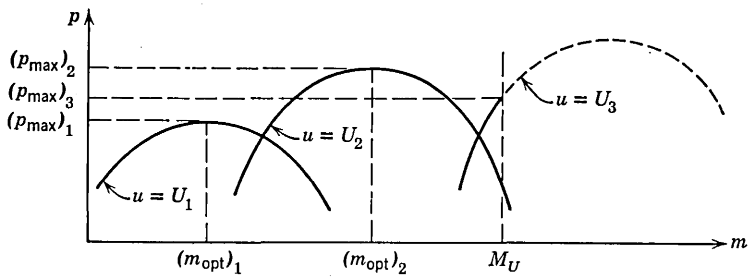

FIG. 2. Performance curves for different loads.

equality constraints require some trial and error operations superimposed on the analytical solution for the optimum.

Dynamic Processes. The foregoing is based on steady-state or quasi-static process operation and is applicable where the system inputs vary slowly in relation to the dominant response time of the system. If, however, the inputs change relatively rapidly, the constraint eqs. (2) must be expressed in the forms of differential or integral equations. The performance specified by eq. (1) then becomes a function of time and the objective of the optimizing control may be more properly stated in terms of maximizing (or minimizing) the time averaged performance $\bar{p}$. Thus,

$$\bar{p} = \frac{1}{T}\int_0^T f(u_i, m_j, q_k; t)\, dt, \tag{9}$$

where T represents the total duration of the batch process or the time between successive steady states in the continuous process.

Calculus of Variations

The calculus of variations may be applied to determine the necessary conditions for a maximum (or minimum) of the integral of eq. (9) (see Refs. 7, 8, and 9). Here again, equality constraints may be incorporated by use of Lagrangian multipliers. Inequality constraints of the forms of eq. (7) or eq. (8) impose restrictions on the analytical solution and require trial and error procedures to determine the optimum. One method of handling such constraints is based on the idea of parametric representation. This is used by Miele (Ref. 10) to derive an optimum trajectory for a rocket missile subject to upper and lower bounds on the fuel flow rate.

A simple formulation of the calculus of variations problem is the determination of a function $y(x)$ which satisfies prescribed boundary values $y(x_1)$ and $y(x_2)$ and yields an extremum for the integral (extremum refers to either a maximum or a minimum):

$$I = \int_{x_1}^{x_2} f(y, y', x)\, dx, \tag{10}$$

where $y' = \dfrac{dy}{dx}$. A necessary condition for the solution of this problem is provided by the Euler-Lagrange equation,

$$\frac{\partial f}{\partial y} - \frac{d}{dx}\frac{\partial f}{\partial y'} = 0 \tag{11}$$

provided the function $f(y, y', x)$ has continuous second partial derivatives with respect to x, y, and y' within the region of interest.

The integrand of eq. (10) may be much more general, including higher order derivatives and multiple dependent and independent variables. The necessary conditions for the extremum are then given by a set of equations similar in form to eq. (11). For example, the necessary conditions for a minimum of the integral,

$$I = \int_{x_1}^{x_2} f(y_1, y'_1, y_2, y'_2, \cdots, y_m, y'_m; x)\, dx, \tag{12}$$

are determined by the following:

$$\frac{\partial f}{\partial y_i} - \frac{d}{dx}\frac{\partial f}{\partial y'_i} = 0 \qquad i = 1, 2, \cdots, m \tag{13}$$

where $y_i(x_1)$ and $y_i(x_2)$ are prescribed for all i.

The boundary values $y_i(x_1)$ and $y_i(x_2)$ need not all be prescribed. However, for each value not specified, a "natural boundary condition" must be imposed. This requires that for the integrands of eqs. (10) and (12) the condition

$$\left.\frac{\partial f}{\partial y'_a}\right|_{x=x_1 \text{ or } x_2} = 0 \tag{14}$$

be satisfied for each value $y_a(x_1)$ or $y_a(x_2)$ not prescribed.

The variables $y_1, y_2, \cdots, y_m$ may be interrelated through equality constraints of the form

$$g_i(y_1, y'_1, y_2, y'_2, \cdots, y_m, y'_m, x) = 0 \tag{15}$$

or as definite integrals of the form

$$\int_{x_1}^{x_2} g_j(y_1, y'_1, y_2, y'_2, \cdots, y_m, y'_m, x)\, dx = J_j, \tag{16}$$

where J_j is a constant.

By defining

$$\phi = f + \sum_{k=1}^{n} \lambda_k(x)\, g_k \tag{17}$$

where $\lambda_k(x)$ = Lagrangian multipliers,
g_k = functions of the form indicated by eqs. (15) and (16),
f = integrand of eq. (12),

the necessary conditions for an extremum of the integral of eq. (12) are given by the set of equations

$$\frac{\partial \phi}{\partial y_i} - \frac{d}{dx}\frac{\partial \phi}{\partial y'_i} = 0 \qquad i = 1, 2, \cdots, m. \tag{18}$$

Equations (18) coupled with the constraint eqs. (15) and (16) provide $m + n$ equations, sufficient to solve for the m $y_i(x)$ unknowns and the n $\lambda_k(x)$ parameters.

Dynamic Programming

Dynamic programming provides an alternative procedure for system optimization. The problem is formulated as a sequence of discrete decision processes and as such is particularly well adapted to digital computation. This method has the advantage of being applicable to a very much broader class of problems than that treated by classical methods; in particular, it is not restricted by questions of continuity and inequality constraints. A major disadvantage, however, is that a solution of the optimizing conditions often requires excessively large computing time and storage capacity.

Dynamic programming is based on Bellman's principle of optimality, which states that whatever the initial state and initial decision may be, the remaining decisions must constitute an optimal policy with respect to the state resulting from the first decision (Ref. 11). The application of this principle is illustrated by means of the previous example: minimization of the integral of eq. (10) subject to prescribed boundary conditions,

$y(x_1) = y_1, y(x_N) = y_N$. First, the integral is replaced by a finite summation,

$$I_N = \sum_{n=1}^{n=N} f(y_n, y'_n, x_n)\,\Delta x, \tag{19}$$

where a constant interval Δx is assumed and where the subscript n denotes an appropriate value of the variable within the nth interval. By separating out the first term of eq. (19),

$$\begin{aligned} I_N &= f(y_1, y'_1, x_1)\,\Delta x + \sum_{n=2}^{n=N} f(y_n, y'_n, x_n)\,\Delta x \\ &= f(y_1, y'_1, x_1)\,\Delta x + I_{N-1}. \end{aligned} \tag{20}$$

Since y_1 is already specified, the only choice with regard to the first interval is the value of y'_1. Denoting $I_N{}^0$ and $I_{N-1}{}^0$ as the desired minimum values of the summations I_N and I_{N-1} respectively, the principle of optimality states that

$$I_N{}^0 = \min_{y'_1} [f(y_1, y'_1, x_1)\,\Delta x + I_{N-1}{}^0]. \tag{21}$$

Thus, y'_1 is determined such that I_N is a minimum, assuming that an optimal policy is provided for the remaining $N - 1$ intervals. With y_1 and y'_1 specified, y_2 can be determined and the formulation of eq. (21) repeated:

$$I_N{}^0 = \min_{y'_1, y'_2} [f(y_1, y'_1, x_1)\,\Delta x + f(y_2, y'_2, x_2)\,\Delta x + I_{N-2}{}^0].$$

This leads to an iterative procedure for the determination of the optimal policy, $y_1, y_2, \cdots, y_N$.

This approach may be generalized to handle either discrete or continuous functions, equality and inequality constraints, and multivariable systems. It has been considered for a number of process optimization applications: control of an exothermic chemical reactor, optimum catalyst replacement program, optimum multistage cross-current extraction, and optimum temperature gradients in a chemical reactor (Refs. 12 and 13).

There are two general approaches to the implementation of optimizing control, the direct approach and the model approach.

5. DIRECT METHODS OF OPTIMIZING CONTROL

General Approach. In the direct approach the system inputs are manipulated according to the observed effects of previous input changes on the system performance. This is illustrated by the block diagram of Fig. 3 in which the control computer makes decisions regarding the changes in manipulated variable m based on measurements of the performance p.

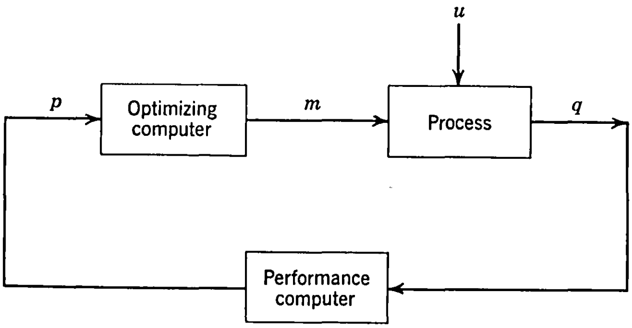

FIG. 3. Block diagram of direct optimizing controller.

The performance is measured continuously or sampled, depending on the specific control scheme employed. It may be measured directly, as in the maximizing of engine power output, or determined inferentially from measurements of the system variables, as in the minimizing of production cost rate according to a specified cost function.

Single Manipulated Input. The basic technique is illustrated in Fig. 4 representing a system with a single manipulated input and negligible dynamics. The decision rule for manipulation of input m is as follows. If the previous change in m caused an increase in p, change m again in the same direction; if the result is a decrease in p, reverse the direction for the next change in m. Thus, referring to Fig. 4, the direction of change of m is reversed at steps 4, 6, 10, $\cdots$, according to the polarity of the change of p in the previous step. At step 7 a load change is indicated which shifts both the value and location of the optimum. Since the controller is continually hunting, it detects the shift in performance and immediately seeks the new optimum.

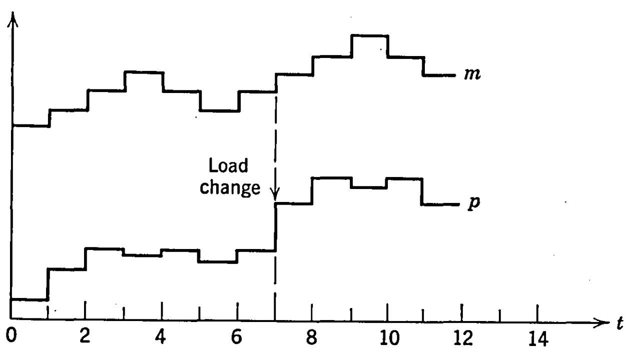

FIG. 4. Simple strategy for direct optimization.

The Quarie controller represents a commercial application of this technique (Ref. 14). The size of the input step is made proportional to the difference between the actual and desired output changes corresponding to the previous step. The system may be controlled to any specified slope of the performance curve; control to a maximum or a minimum is obtained by setting the controller to zero slope.

Method of Draper and Li. Several modifications of the basic method were introduced by Draper and Li (Ref. 15). Their peak holding method, for example, varies the input signal at a constant rate until the performance drops a preset amount from the peak value measured during the current cycle. The direction of change of the input signal is then reversed and the operation repeated as shown in Fig. 5.

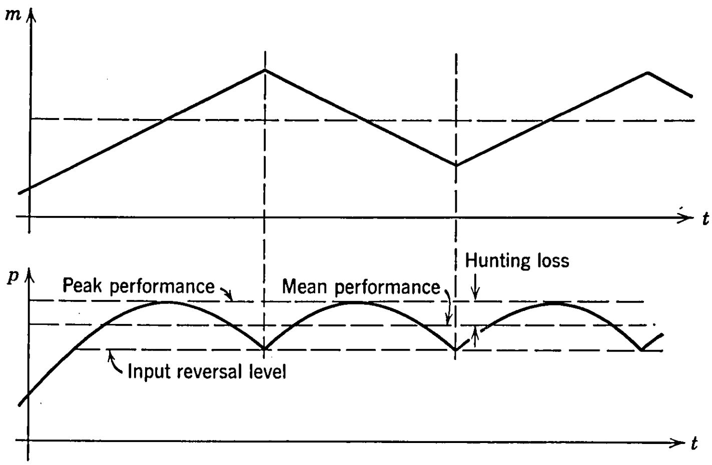

FIG. 5. Peak holding optimalizing control.

Sinusoidal Perturbation Method. It is apparent from eq. (4) that a necessary condition for a maximum (or minimum) performance is that the slope of the performance curve be zero (assuming the function $p(m)$ to be smooth and continuous in the operating region). Thus, another approach to direct optimization is the manipulation of the process input to seek the point at which the slope of the performance curve changes sign. The system shown in Fig. 6 represents an application of this approach to an aircraft engine for the purpose of minimizing the fuel consumption for given engine speed and load (Ref. 16).

A continuous perturbation signal $A \sin \omega t$ is added to the process input signal m. It is assumed that the performance curve may be approximated reasonably well in the vicinity of the optimum by the parabolic function

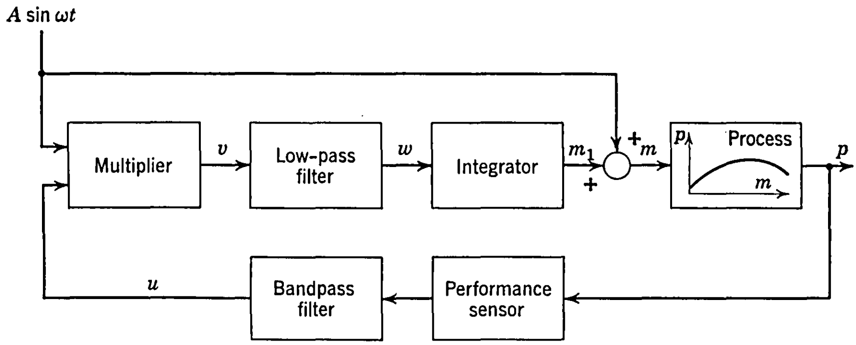

FIG. 6. Direct optimization by means of sinusoidal perturbation signal.

(22) $$p - P = K(m - M)^2,$$

where K is a constant and M and P are the optimum values of the manipulated variable and performance, respectively. Referring to Fig. 6,

$$m = m_1 + A \sin \omega t.$$

By inserting into eq. (22) and expanding,

$$p = P + K(m_1 - M)^2 + \frac{KA^2}{2}(1 - \cos 2\omega t) + 2KA(m_1 - M) \sin \omega t.$$

A bandpass filter in the feedback rejects all the components of p except the fundamental. Thus,

$$u = 2KA(m_1 - M) \sin \omega t.$$

The fundamental is then multiplied by the perturbation signal,

$$v = KA^2(m_1 - M)(1 - \cos 2\omega t).$$

A low-pass filter rejects the harmonic to yield

$$w = KA^2(m_1 - M).$$

This is integrated with respect to time to produce the main component of the input signal,

(23) $$m_1 = M + (m_{10} - M)\, e^{-KA^2Bt},$$

where m_{10} = initial value of m_1,
B = integrator constant.

Thus, m_1 approaches the optimum exponentially with an effective time constant of $1/KA^2B$.

Optimization without Perturbation. Direct optimization may proceed without employing a perturbation or test signal; one such method is shown in Fig. 7 (Ref. 17). The divider in effect generates the derivative of the performance with respect to the manipulated variable. This derivative

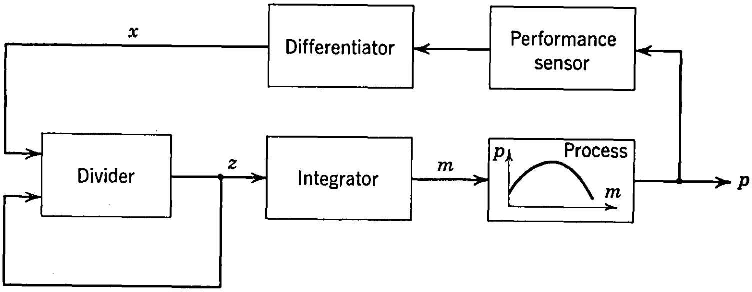

FIG. 7. Direct optimization by means of divider circuit.

is integrated with respect to time to yield the process input m. The relationships are readily established by reference to Fig. 7:

$$z = \frac{x}{z}$$

but

$$x = T_1 \frac{dp}{dt}$$

$$z = T_2 \frac{dm}{dt},$$

where T_1 and T_2 are the differentiator and integrator time constants, respectively. By combining these equations,

$$\frac{dm}{dt} = \frac{T_1}{T_2} \frac{dp}{dm}. \tag{24}$$

Thus, the rate of change of m is proportional to the slope of the performance curve, and m is forced in the direction to reduce the slope to zero.

This method has the great advantage of simplicity and easy realizability. It has been applied effectively to a pilot plant fractionating column. Analysis of the dynamics shows that the system is stable for a number of practical applications (Ref. 18).

Extensions to Multiple Manipulated Inputs. The various techniques outlined above may be extended to systems with multiple inputs. The inputs may be manipulated sequentially; that is, m_1 is first varied until p is maximized, then m_2 is varied, then m_3, etc., with the cycle repeated until the performance is maximized with respect to all the inputs. This procedure tends to be very slow and inefficient.

Method of Steepest Descent. Various strategies have been developed to improve the efficiency of the exploration procedure; one such strategy is the method of steepest descent (Refs. 19 and 29). In terms of N manipulated inputs, this is expressed as

$$\frac{dm_n}{dt} = k\left(\frac{\partial p}{\partial m_n}\right)\bigg|_{\text{operating point}} \qquad n = 1, 2, \cdots, N, \tag{25}$$

where k is a negative or positive constant depending upon whether the performance is to be a minimum or a maximum, respectively. The gradient $\partial p/\partial m_n$ may be determined either analytically if the function is known or experimentally by basing calculations on the measured changes in p resulting from the preceding manipulations of the m_n variables. For manipulation in discrete steps of fixed duration T, eq. (25) yields

$$\Delta m_n = kT\left(\frac{\partial p}{\partial m_n}\right)\bigg|_{\text{operating point}}. \tag{26}$$

The value of k yielding the fastest approach to the optimum depends on both the operating point and the nature of the performance contour.

Design of an optimizing controller based on the method of steepest descent is presented by Feld'baum (Ref. 20). A detailed experimental study of a two-channel optimizer based on similar methods is given by Stakhovskii (Ref. 21).

The Opcon controller represents a commercial application of the direct method to multiple input systems (Ref. 22). The controller employs a special strategy to determine the next move based on the results of previous moves. Applications include a catalytic process for the dehydrogenation of ethyl benzene and a distillation unit for the binary mixture isobutane–n-butane (Refs. 22 and 23).

Multiple Frequency Perturbations. Simultaneous manipulations of multiple inputs may also be achieved by employing multiple frequency perturbations. There are limitations, however, in the frequency range that can be employed and in the resolution of the various frequency components.

Evaluation of Direct Methods of Optimization. The efficiency of the direct method of optimizing control depends, in general, on the amount of phase lag in the system and the noise level in the output signals (Ref. 24). The phase lag limits the maximum rate or frequency of change of the input signal and hence limits the speed with which a disturbance from the optimum can be corrected. In particular, the transient following an input change must attenuate sufficiently to permit a meaningful interpretation of the response. Where a sinusoidal perturbation signal is used, the phase shift of the output relative to the reference sinusoid causes the useful component of the correction signal to decrease. Indeed, excessive phase lag causes a reversal in phase of the correction signal, resulting in instability. System dynamics may be compensated to some extent by use of phase-compensating networks or appropriate logic.

The controller must be able to distinguish between the indicated variations in performance caused by the manipulated inputs and those caused by spurious signals or noise. Thus, the noise level determines the lower limit on the hunting amplitude of the manipulated input. This introduces a hunting loss (see Figs. 4 and 5), which is the time-averaged deviation of the system from peak performance produced by the hunting of the input signal about its optimum value. The hunting loss is shown to be proportional to the square of the hunting amplitude for several systems (Ref. 24). The noise effect may be reduced by proper choice of the frequency of hunting or perturbation. Use of dynamic filters or correlation schemes are also effective; however, they tend to decrease response speed and increase system complexity.

6. OPTIMIZING BY COMPUTER CONTROL

System Design. The process consists of a fixed structure of elements and interconnections supplied by a limited number of energy and material flows. In general, only the energy and material balance relationships can be modified by the control computer. As a result, the effectiveness of the optimizing control is directly related to the initial plant design and operating practice. Ideally, the system should be designed optimally from the start; that is, the selection and design of equipment, instrumentation, and control computer should be included in the overall performance evaluation along with the determination of operating procedures and on-line optimizing control functions. This requires a broad systems design approach which, at the present level of know-how, can only be applied to a very limited extent.

The underlying theories of repetitive control, continuous process optimization, and self-checking are discussed in Sect. 7, Applications of Computer Control, in connection with specific applications.

Model Methods of Optimizing Control. Model methods provide an alternate approach to optimizing control. The model refers to the set of relationships and inequality constraints which describe the process behavior. The mathematical model is the most general form of expression; it consists of equations in the form of eqs. (2), (7), and (8). The model may, however, be alternatively expressed in terms of some appropriate physical simulation or analog of the process or be represented graphically as a multidimensional surface. A further discussion of models and the analytical and experimental methods for their determination is presented in Chap. 14 (see also Refs. 5 and 6).

The model provides the basis for determining the optimizing conditions for the system in terms of its present state, the desired final state, and the specified performance criterion. In general, information defining the present state is provided through measurements of appropriate system variables. The results of the optimizing procedure are transmitted back to the system through actuators of the system input (manipulated) variables.

The model may form an identifiable entity in the control loop or it may be only an implied basis for analytical derivation of the optimizing control equations. If the former, then the system optimum can be determined by applying to the model or analog a direct method of optimization such as described in the preceding section. The two-time scale control of Ziebolz and Paynter (Ref. 25) and some of the experimental methods of Box (Ref. 26) may perhaps be considered in this category.

Many of the limitations of the direct method can be eliminated by its application to the model rather than to the system itself. Two of the factors involved are

1. The model may be scaled to run very much faster than the system, so that the time required for a complete exploration of the performance contours is negligible compared to the dominant time constants of the system. Thus, problems of unfavorable dynamics, multiple inputs, and local maxima or minima are bypassed.
2. The system need not be perturbed by the optimum-seeking procedure. Consequently, losses due to system transients and hunting are minimized.

7. APPLICATIONS OF COMPUTER CONTROL

Computation of Operating Guides. The mathematical model permits rapid extrapolation of the process performance beyond its present state and provides the means for automatic computation of the necessary optimizing conditions. In a large percentage of existing applications, these conditions are transmitted to the process through the human operator. Thus, the computer serves essentially as an operating guide: it computes periodically (or on demand) the optimum set point values for each of the controlled variables and then displays the results in appropriate form for operator action. A diagrammatic representation of a digital computer application as an operating guide is shown in Fig. 8.

Closing the feedback loop through the operator has the obvious advantage of greatly simplifying the computer control facility. In particu-

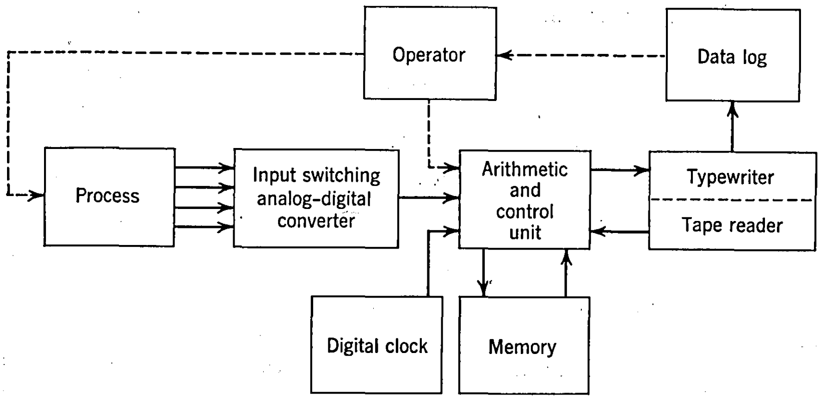

FIG. 8. Digital computer as an operating guide.

lar, the experience, judgment, and intuitive "feel" factors of the good operator bypass for the time being the problems of system response characteristics and stability as well as the problem of confidence in the reliability of the computed results.

A number of process applications of operating guide computers are presented in the literature. In some of these applications the computer is designed to close the loop as soon as sufficient operating data, know-how, and confidence are established.

Operation of a distillation column to achieve maximum operating economy with fixed-product specifications is discussed by Engel (Ref. 27). The computer determines optimum feed-tray, heat input, and reflux rate based on a mathematical model derived from mass and energy balances and vapor equilibrium relationships. Actual installations of computer-controlled columns include:

1. Special purpose analog computer applied to optimizing tower efficiency (Humble Oil Refinery) (Ref. 28).
2. Digital differential analyzer applied to an experimental unit for the separation of the two-component mixture of n-butane and isobutane (Sun Oil Refinery) (Ref. 28). This unit is referred to in Sect. 5, Direct Methods of Optimizing Control, in connection with experimental application of the Opcon controller.

An application of an operating guide computer for optimum operation of a catalytic cracking unit (Esso Baton Rouge Refinery) has also been described (Refs. 28 and 29). Here the computer is designed to provide information for more efficient operation, faster recovery from upsets, etc.

A computer system designed to increase open-hearth efficiency and yield is described (Ref. 30). The system scans, computes, and logs furnace data and provides the operator with scheduling guides for optimum performance.

Economic Power Distribution. The electric power utilities have made extensive use of the optimizing computer as an operating guide (Refs. 31, 32, and 33). Here the objective is to allocate a given power load among a number of generating units in such a way that the total cost of supplying the power is minimized. The result is to be consistent, of course, with the specified boundary conditions regarding frequency and voltage variations, upper and lower limits on each unit, and allowable rates of load change. On-line computer control of a power distribution system is described in Ref. 34.

It is readily shown that the optimum load distribution among several generating units is obtained when the incremental cost of received power is the same from each source. This is expressed as the following control condition,

$$\frac{dF_n}{dP_n} L_n = \lambda \qquad n = 1, 2, \cdots, N, \tag{27}$$

where F_n = cost of operating the nth power source,
P_n = power generated by the nth source,
L_n = penalty factor assigned to the nth source,
λ = incremental cost of received power,
N = total number of sources.

The penalty factor is given by the expression,

$$L_n = \left(1 - \frac{\partial P_L}{\partial P_n}\right)^{-1} \tag{28}$$

where P_L = power lost in transmission.

Equations (27) and (28) are derived by the method of Lagrangian multipliers [see eqs. (5) and (6)]. The total cost F is to be minimized subject to supplying a given amount of power P where

$$F = \sum_{n=1}^{N} F_n,$$

$$P = \sum_{n=1}^{N} P_n - P_L.$$

To define,

$$\phi = F - \lambda P = \sum_{n=1}^{N} (F_n - \lambda P_n) + \lambda P_L \tag{29}$$

where λ is the Lagrangian multiplier; the necessary conditions for a minimum are determined by differentiating with respect to P_n and setting the result equal to zero [refer to eq. (6)]:

$$\frac{dF_n}{dP_n} - \lambda\left(1 - \frac{\partial P_L}{\partial P_n}\right) = 0 \qquad n = 1, 2, \cdots, N. \tag{30}$$

Note that use is made of the assumption that F_n is a function only of P_n and is independent of the power outputs of the other sources. Equation (30) is equivalent to the desired result given by eqs. (27) and (28).

A graphical representation of the incremental cost approach is given in Fig. 9 for a simple two-source system with negligible transmission losses.

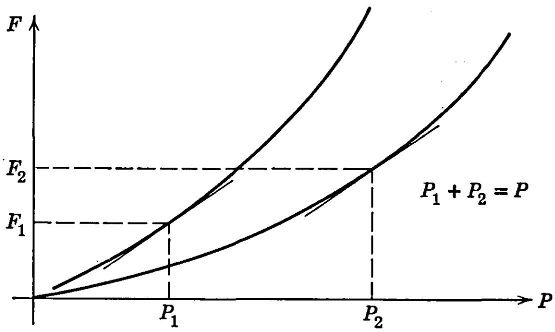

FIG. 9. Equal incremental slope method.

For every given load P the source outputs P_1 and P_2 must adjust themselves so that their sum equals P and the slopes of the curves at the indicated outputs are equal.

This result is applicable to a wide range of systems where there are a number of parallel producing or consuming units, e.g., economic distribution of fuel flow through a pipeline system or optimum feed allocation to a bank of catalytic reactors (Ref. 35).

On-Line Computing Control. The extension of the operating guide computer to a closed loop optimizing control application is, in concept, fairly straightforward. As mentioned previously, there are, however, practical difficulties with respect to the complex dynamics often encountered. These difficulties are circumvented by (1) incorporating the process dynamics into the control equations or (2) limiting the frequency and rate of change of control signals transmitted to the process, thereby restricting the system to a quasi-static state. Current on-line computer control applications are, for the most part, based on the second approach.

An increasing number of on-line computer control applications are being reported. The first is the optimizing control of a catalytic polymerization unit (Texaco Port Arthur Refinery) with the objectives of improving operating efficiency and reducing catalyst replacement costs (Ref. 36; references of related interest are 6 and 35). The computer determines optimum values for reactor pressures, catalyst temperatures, recycle flow rate, and reactor feed rates; these values are then transmitted to the set point inputs of process controllers in conventional feedback control loops. The system consists of ten parallel reactors, and an important aspect of the optimization control is the distribution of feed to the reactors according to their relative catalyst activities (Ref. 35). Similar approaches to digital computer control of chemical processes are presented in Refs. 37 and 38.

Another on-line control application of interest is the computer control of a continuous annealing furnace (Ref. 39). Here, the computer maintains closed loop control of the various zone temperatures and other functions, including data acquisition and processing.

There are many other areas of the steel mill considered for computer control. One of the more challenging and potentially offering, perhaps, the most significant returns is the optimization of the blast furnace. Some of the results of Soviet efforts in this direction are outlined in Ref. 40.

Predetermined Program Control. The conditions for controlling to a specified system performance may be predetermined when the model is reasonably complete and exact. Often these conditions can be stated explicitly; that is, the manipulated inputs may be specified in the following forms: (1) as algebraic functions of the measured system variables, (2) as functions of time or some other independent parameter, or (3) as a library of discrete operating procedures or practices.

It is apparent that, once computed, the optimizing conditions can be stored on punched tape, magnetic drum or even, for a simple two-dimensional model, on a mechanical cam. The system variables are then manipulated according to the playback of the appropriate stored program.

There are several recently announced computer control installations for electric power generating stations which are based to a large extent on predetermined program control. These include installations at the Southern California Edison Company, Huntington Beach Station, and the Louisiana Power and Light Company, Little Gypsy Station. The stored programs direct station start-up and shutdown operations under various normal and emergency conditions. The main objective is to assure correct operating procedures so as to prevent costly accidents. The computer provides, in addition, continuous logging and control of the system variables.

Automatic control of start-up and shutdown operations, sequencing of cyclic operations, etc., are also important in the process control field. The development of predetermined optimizing programs for such operations may prove fruitful in many computer control applications.

Repetitive Computer Control. The predetermined control scheme is essentially open loop with respect to the desired system performance; that is, there is no feedback of information to verify either that the process performs as specified or that the model accurately describes the system behavior. Accordingly, if there are any factors tending to cause the system to deviate from the model, such as disturbances, the system performance may be expected to deviate from the computed optimum.

The predetermined optimization concept is modified by repetitive feedback of information describing the state of the system. Thus, as shown in Fig. 10, the q_k variables are periodically sampled by the optimiz-

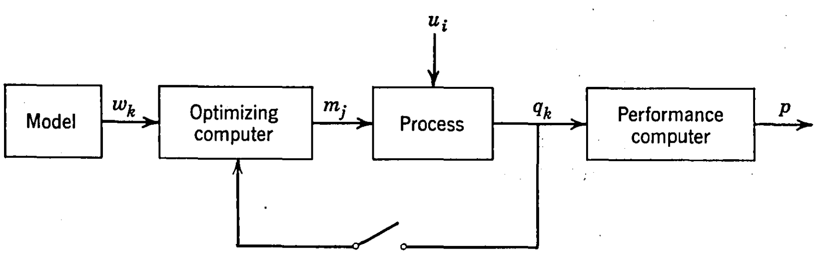

FIG. 10. Repetitive optimizing control scheme.

ing computer, providing the basis for repetitive computation of the optimizing conditions. In this way, each computation is based on the most recent information describing the state of the process. As a result, deviations of the system from the postulated model do not cause cumulative errors. The repetitive computer action tends to force the system to the desired performance, despite significant inadequacies of the model.

The repetitive control concept has been applied to the optimizing control of a pilot plant scale batch hydrogenation reactor (Refs. 41, 42, and 43). A brief description of this process is now presented.

The reaction mixture is made up of three chemical components identified as X, Y, and Z. Hydrogen under pressure and in the presence of a catalyst reacts with X and Y according to the following reaction scheme,

$$X + H_2 \underset{k_1}{\rightarrow} Y$$

$$Y + H_2 \underset{k_2}{\rightarrow} Z,$$

where k_1 and k_2 represent kinetic reaction coefficients. A reasonable approximation to the kinetic behavior of this process is given by the following equations:

$$\frac{dx}{dt} = -k_1 x \tag{31a}$$

$$\frac{dy}{dt} = k_1 x - k_2 y \tag{31b}$$

$$x + y + z = 1, \tag{31c}$$

where x, y, and z represent molar concentrations of components X, Y, and Z, respectively.

The kinetic coefficients are functions of the operating conditions—pressure, temperature, catalyst, agitation, etc. Assuming that only pressure is to be manipulated and that all other influencing factors are relatively constant, the coefficients may be expressed as

$$k_1 = A_1 p^{N_1} \tag{32a}$$

$$k_2 = A_2 p^{N_2}, \tag{32b}$$

where A_1, A_2, N_1, and N_2 are assumed constant and p equals the process pressure.

Based on a mathematical model consisting of eqs. (31) and (32), the necessary conditions for optimum process performance may be derived. In the particular process under study, control to a specified product composition consistent with minimum processing time is established as the performance criterion.

It is convenient to transform these equations to new variables, u, v, and k defined as follows:

$$u = \frac{y}{x} \tag{33a}$$

$$v = \log_e \frac{x_0}{x} \tag{33b}$$

$$k = \frac{k_2}{k_1} = \frac{A_2}{A_1} p^{N_2 - N_1}. \tag{33c}$$

Making the substitutions in eqs. (31a) and (31b), a single equation defining the processing path independent of the time variable is obtained:

$$\frac{du}{dv} = (1 - k)u + 1. \tag{34}$$

Since the kinetic coefficient ratio k may be varied during the course of the reaction by manipulating the pressure [eq. (33c)], there are an infinite number of operating paths which can satisfy the boundary conditions, x_0, y_0, z_0 representing the initial composition and x_f, y_f, z_f representing the desired final composition. (In terms of the u, v coordinates, these boundary conditions are expressed as u_0, 0 and u_f, v_f, respectively.) This degree of freedom permits the introduction of an optimizing condition. Assuming that the processing time is the predominant factor in the cost equation, the optimizing problem reduces to the determination of a control path which will minimize the time to go from the raw material state to the desired product state.

An expression for the processing time t_f is derived from eqs. (31), (32), and (33):

(35) $$t_f = A \int_0^{v_f} k^{1-B}\, dv,$$

where

$$A = \left[\frac{A_2^{N_1}}{A_1^{N_2}}\right]^{\frac{1}{N_2 - N_1}}$$

$$B = \frac{N_2}{N_2 - N_1}.$$

The necessary condition for minimizing this integral is derived by applying the Euler-Lagrange conditions [eq. (18)]. The following optimizing equation is obtained:

(36) $$\frac{dk}{dv} = -\frac{1}{B}\frac{k}{u}.$$

The optimizing computer is programmed to solve eqs. (34) and (36) simultaneously for the given boundary conditions u_0, 0 and u_f, v_f.

If eqs. (31) and (32) described the process behavior exactly, one computation based on eqs. (34) and (36) and the specified boundary conditions would suffice to define the optimum control path $p(t)$. Thus, the $p(t)$ schedule could be recorded on tape or other storage medium and played back through appropriate transducers and pressure controller to manipulate the process pressure according to the schedule. In the example under consideration, however, the model only approximates the process kinetics because such factors as variations in catalyst activity, other components in the reaction mixture, higher order terms in the kinetic equations, etc., are neglected. Open loop control of the process would lead, therefore, to very significant deviations from the desired end point.

When the repetitive control concept is used, eqs. (34) and (36) are solved for the control path leading from the current state of the process (based on the most recent composition measurement of the reaction mixture) to the specified final composition. Thus, each time a new composition measurement is made available to the computer, a new control path is computed. This technique has been shown to be very effective in forcing the process to the prescribed performance.

Optimizing Control of Continuous Processes with Significant Dynamics. As noted previously, the optimizing control of a continuous process may be treated statically; that is, the control conditions are determined on the basis of the process going from one steady-state configuration to another. In general, the process performance during the transition period is not considered, except perhaps to make sure that the transient response of the manipulated variables are reasonably stable.

There are two factors limiting the static approach for systems whose dynamics are significant in relation to the frequency of disturbances or input changes:

1. Off-optimum control during the transient period may significantly degrade overall performance.
2. Manipulation of the process inputs in seeking the optimum may in fact continually upset the process equilibrium.

The process may be considered in terms of a succession of steady states if the inputs and disturbance variables are normally relatively constant except for changes occurring at discrete intervals (Ref. 44). The transition from one steady-state level to another may then be treated as a batch operation with maximizing the performance during the transient phase, the objective. Thus, the calculus of variations (or other optimizing technique) is applied to a performance function of the form of eq. (9) to define a control path leading the process from its present state to a new steady-state optimum. The new steady state is determined by conventional static optimization procedures. One approach to this problem is presented by Sandelien (Ref. 44).

Note that, except for very special cases, determination of the optimum path requires a priori knowledge of the nature of the time variations of the (independent) inputs. This is feasible in certain applications; for example, a surge chamber can be employed to convert arbitrary flow variations into a sequence of discrete step changes in flow. In general, however, the inputs must be approximated by arbitrarily assumed time functions (corrected perhaps by a repetitive control scheme). Alternatively, the optimum path may be based on an appropriate mean input function arrived at statistically.

A generalized chemical processing model which embodies the basic limitations pointed out above has been formulated by Williams (Ref. 45). The problem, considered representative of the process industry, involves optimization of a continuous reactor that has unfavorable dynamics and is subject to large input variations.

Model Adaptation (Self-Checking). In practical applications of the model method of optimization, the postulated model will generally deviate significantly from the actual system behavior. There are several reasons for this:

1. Not enough is known about most industrial processes to derive a complete and accurate analytical representation. Indeed, plant design and operation are generally based on very approximate and empirical relationships.
2. The complexity of most systems precludes comparable detail in the formulation of the mathematical model because the resulting computer capacity would be prohibitive.
3. Many variables affecting system behavior cannot be satisfactorily measured with existing instrumentation (e.g., catalyst activity); hence, they cannot be employed directly in the computer control function.
4. The state of the system can generally be specified only within statistical limits because of nonhomogeneity, random fluctuations of measured quantities, etc.

The inadequacies of the model are compensated, in part, by the repetitive control technique described earlier. However, the effectiveness of such compensation depends on the nature and degree of the approximations in the model and is limited by the repetitive period, measurement lags, dead time, and bounds on the manipulated variables.

More effective compensation for the approximations in the mathematical model is possible through a self-checking or model adaptation technique (Refs. 46 and 47). This technique involves a periodic adjustment of the parameters of the model in order to force a "best" fit of the model to the observed system behavior in the vicinity of the operating point. In essence, the parameters are adjusted to minimize the deviations between the actual processing path, defined by appropriate system variables, and the path predicted by the model for the same operating conditions. The resulting parameter corrections are applied to the proper terms of the optimizing control equations.

The self-checking operation may be considered an application of optimizing control to the model, where the performance criterion is defined as some measure of the effectiveness of the model in describing the system behavior. Thus, any of the available optimizing techniques may be applied to the problem.

The self-checking concept is illustrated by the block diagram of Fig. 11. The deviations of the system variables from their respective model variables generate error signals (as functions of time) from which the performance criterion is determined. An optimizing computer manipu-

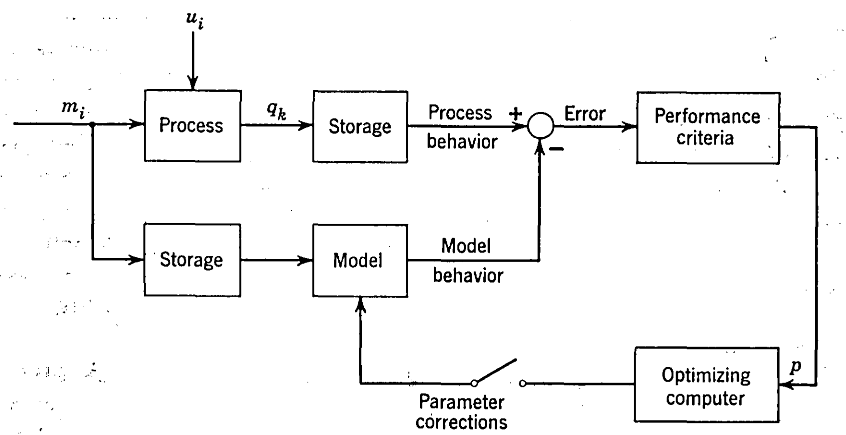

FIG. 11. Self-checking control scheme.

lates the parameter values to minimize this criterion. Storage elements are inserted for synchronizing in time the system and model time functions. Finally, the corrective action is sampled to provide an intermittent self-checking action compromised between the rate at which the parameter values are changing and the limitations imposed by stability considerations and the rate of information flow to the self-checking computer.

The formulation of the model depends on both the complexity of the system and on how much is known of the system equations. Several possibilities may be considered:

1. The system equations are known reasonably well, with the model based directly on these equations. Self-checking may be applied to correct for slow drifts of some of the parameter values.

2. The model represents only the dominant characteristics of the system equations, because either knowledge of the system is limited or the equations have been intentionally simplified. The parameter values are then functions of the state of the system and change with changes in operating conditions. The self-checking determines average values of the parameters for the region about the operating point.

3. The model does not include the influence of one or more system variables (because their effects are unknown or because they are not satisfactorily measured). Here again, the self-checking determines average parameter values for the region of the operating point.

4. The theoretical background for the system analysis is totally lacking, and a generalized expression, such as a power series, is used for the model. The self-checking determines effective values of the parameters in the same manner as for models 2 and 3.

The self-checking technique may be intentionally applied for the purpose of simplifying the equations or reducing the number of pertinent variables. In general, the optimizing computer requirements are reduced at the expense of the self-checking facility. Thus, as the equations become more generalized or approximate, either the number of parameters to be adjusted is increased or the parameters must be adjusted more frequently as operating conditions change. It is expected, however, that the overall computer demand can be minimized by a judicious compromise between the two systems; in particular, the process equations may advantageously describe only the dominant first order effects, which relate only the rapidly changing variables.

Computer-Controlled Pilot Plant. A promising extension of the model adaptation concept is the application of a computer to control a pilot plant for the purpose of automatically determining a satisfactory process model (Ref. 48). The computer manipulates the pilot plant through a programmed series of experiments. Measurements of the system response are fed back into the computer for subsequent analysis and correlation.

Adaptive Control. The adaptive concept has attracted a great deal of attention in the servo field, particularly for aircraft and missile control (Ref. 49). The major emphasis in this work has been the automatic modification of controller parameters to achieve a desired transient response of the system under widely varying operating conditions. Thus, the gain and perhaps the time constants of the controller function are periodically adjusted to satisfy a performance criterion based on some appropriate time function of the error signal. Effective implementation of this approach in the general complex system usually requires computer control.

The adaptive control approach may be applied to process systems in which the nonlinearities result in very different response characteristics under varying operating conditions. In particular, the adaptive control approach may be coupled with the optimizing control function to assure that the manipulated variables respond properly over the operating range.

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Data Processing

E. M. Grabbe

1. INTRODUCTION

Basic Principles. Emphasis on the data processing aspect of chemical and other industrial processes has been increasing steadily. With common usage of analog and digital computers, widespread application of automatic control theory, and increased knowledge of the behavior of processes, it has thus become easier to mechanize much of the computing, data manipulation, and logical decision processes associated with the control of industrial systems so that they may be carried out by machines rather than by human operators.

Data processing is the monitoring, recording, computing, and evaluating of important information concerning process variables in order to obtain improved operation of the process. The computations range from simple scaling to calculating operating guides which an operator can use to control the process or plant more effectively. In systems using a computer, the loop may be closed and the computer supplies signals which automatically make calculated changes in process variables. Other names applied to this type of data processing system are *data acquisition,* *data logging, data recording, computing control, and computer control.* The theory and methods of computer control are covered in Chap. 13, Computer Control.

Continuous monitoring means that all points are observed at all times. Each variable being monitored must have its own instrumentation and detection equipment. *Scanning* systems are programmed to observe or sample sequentially a large number of points. The recording and indicating equipment is time shared, thereby reducing cost. Both standard and special purpose data processing systems are in use. The emphasis in this chapter will be on standardized equipment and digital systems.

Benefits of Monitoring. The fundamental benefits of monitoring are protection of equipment and personnel. In complex systems computation may be required to determine whether dangerous conditions exist in a process, for example in nuclear power plants (see Ref. 10). Automatic monitoring removes the human bias and error introduced in manual logging or conversion of analog records into digital form. Hence, the output information is far more reliable and accurate. The plant operators are also freed from the tedious task of gathering large quantities of data, and they can concentrate on improving plant efficiency and in coping with unanticipated emergencies when proper and expeditious action is needed.

For plants which represent large capital investments and have high production rates, data processing has become a must. Proper data gathering systems will provide accurate accounting data, improve maintenance procedures, reduce down time, and increase the operating life of industrial plants. With the trend toward larger plants the need and value of data processing as a protective measure increase. *Examples.* Individual electrical power generating plants of the future may represent investments in the order of $25,000,000. Nuclear power plants represent large investments, and personnel must be protected against radiation hazards.

Benefits of Closed Loop Computer Control. When a computer is used in a closed loop system, substantial benefits may result from carrying out the control functions described in Chap. 13, Computer Control. In complex multivariable processes the computer can improve yield, increase throughput, improve quality, and decrease operating costs. Computer control is especially useful in systems in which process variables interact strongly, input materials vary in characteristics, output product specifications are changed frequently, environment variations are important, etc. These are situations in which man cannot cope with the speed and complexity of action required for optimum control. Computer control has been applied to many processes in which the payout (time to recover investment) is six months to two years.

Objectives. There are many different levels of on-line data processing in industrial activity. The objectives, listed in order of increasing complexity and sophistication, are as follows.

1. As a safety device to sound warnings, to provide visual indication of malfunction, and to shut down equipment automatically when unsafe or emergency conditions exist.

2. To supply accurate records for cost analysis, inventory, and other accounting functions.

3. To provide records for analyses of processes. *Examples* are determinations of static and dynamic behavior, recording of events during emergency shutdown, and collection of maintenance data.

4. To compute operating guides which may then be used for adjusting set points of controllers to obtain optimum plant operations.

5. To close the control loop and accomplish completely automatic control for optimum plant operation (feed-forward or feedback).

6. To provide for automatic adjustment of mathematical models of the process to account for slow changes in the process.

TABLE 1. MONITORING AND DATA PROCESSING FUNCTIONS

1. Alarms (off-normal conditions)
    - Audible annunciator, Visual annunciator } Continuous or scanned
    - Equipment shutdown
    - Analog indication (visual)
        - Meter
        - Oscilloscope
2. Analog Recording
    - Chart records
        - Continuous
        - Scanned
        - Off-normal indication
    - Programmed (relays or plugboard)
3. Digital Recording (logging and simple computation)
    - Analog computations (scaling, zero offset, square root, etc.)
    - Digital indication (visual digital display)
    - Digital records (typewriter, punched cards, punched tape, magnetic tape)
    - Programmed (plugboard)
4. Digital Control Computers (computation, recording, and control)
    - Process Optimization
        - Operating guides (manual control)
        - Closed loop control (feed-forward or feedback)
        - Automatic modification of mathematical equations (computer program)
    - Internally stored program

Items 4 and 5 are concerned with short-term process optimization, whereas item 6 is concerned with long-term changes which require modification of the process transfer functions as represented by a mathematical model (see Chap. 13, Computer Control).

Table 1 shows the types of data processing equipment which may be used as part of control systems. The equipment ranges from simple alarms to general purpose control computers. Figures 1, 2, and 3 illustrate three levels of data processing. Figure 1 is a block diagram of a monitoring

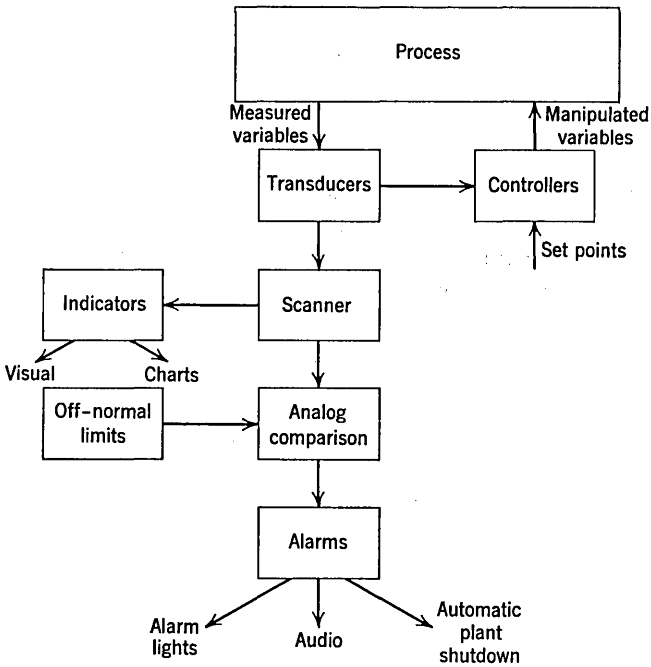

FIG. 1. Monitoring and recording system block diagram.

and analog recording system. Figure 2 shows the functions of a typical digital data logger, and Fig. 3 illustrates the use of a digital process control computer.

The digital data logger of Fig. 2 provides all the functions of the simpler monitoring and recording system of Fig. 1. In addition, it can provide some analog computation and digital records such as printed data, punched cards, punched tape, and magnetic tape. The process control computer of Fig. 3 adds the features of computation of operation guides, automatic closed loop control, and automatic long time optimization

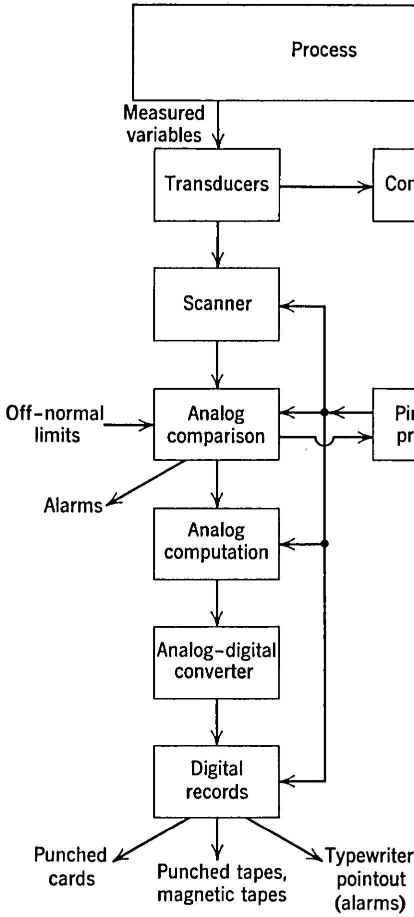

FIG. 2. Typical digital data logger system block diagram.

through adjustment of mathematical models (see Chap. 13, Computer Control, Sect. 7, Applications of Computer Control).

Loggers are employed for gathering process data for analysis at a later time. On-line computing systems are employed for large and complicated processes for which rapid computation and control involving many variables are required and are beyond the scope of a human operator.

Data Processing Inputs. The inputs to a data processing system originate for the most part in physical quantities, the variables of the process. Measuring elements (transducers) convert the signals into suitable pneumatic or electric form for monitoring and data processing. Chapter 20, Measuring Elements and Sensors, describes the basic principles of measuring elements and sensors with emphasis on devices that provide

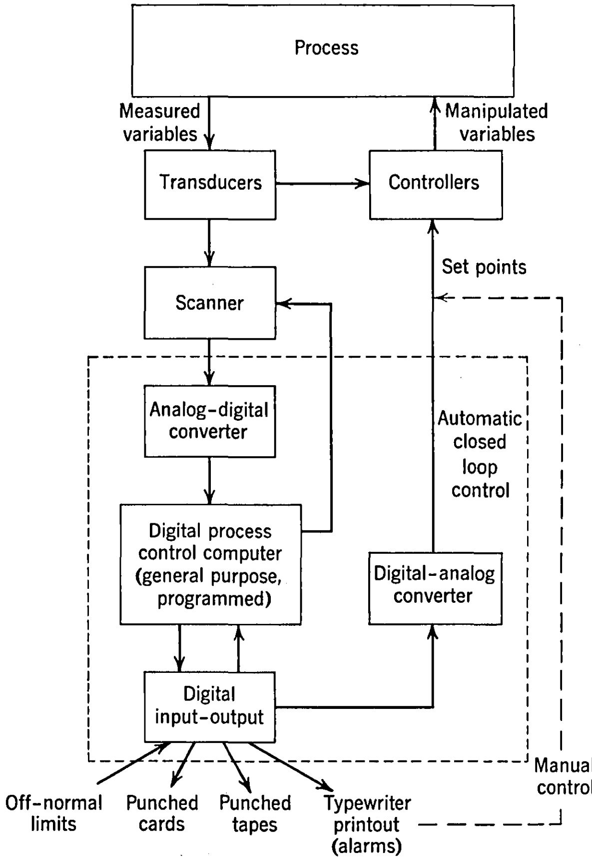

FIG. 3. Process computer control system block diagram.

electrical signals as their outputs. Chapter 7, Instrumentation Systems, covers the basic types of measuring instruments used in the process industries. The most common values for variables measured in industrial control systems are temperature, liquid level, pressure, and flow. Other important measurements come from strain gages, tachometers, *p*H cells, and continuous analyzers (see Chap. 24, Continuous Analyzers).

The off-normal limits must be entered into the system as constants. For analog systems these limits are set by potentiometers. For digital systems the information is recorded in the storage or can be read in coded form when needed, for example, by a punched card reader.

Programming the System. Some monitors operate continuously, and duplicate equipment is required for each point. When large numbers of points are to be monitored, analog systems and digital data loggers usually use plugboard programming for sequencing the equipment and for selecting the functions to be performed on each channel. Digital computers have internally stored programs. Such programs may include more sophisticated system checks as well as off-limit checks at the will of the programmer. For example, the computer program might include (1) tests for computer malfunction, (2) reasonableness checks on raw or computed data, (3) redundancy features, (4) detection of instrument failures, and (5) automatic optimization.

Data Processing Outputs. The system outputs are listed in Table 1 as the equipment functions. Alarms for emergency action or automatic shutdown are among the basic outputs of any data processing system. Analog and digital records provide the output under normal operating conditions. The form and format of the recorded output will depend on the particular requirements. If cumulative data is required in analog systems, the output production is recorded continuously and planimetering is necessary. On the other hand simple digital computing can provide cumulative records of production during prolonged periods. A digital computer can also extrapolate values, and in the event of instrument or transmission system failure it can compute good estimates for quantities such as cumulative production. When analog-to-digital converters are used, the output data appears in digital form. As such it may be displayed by digital indicators, printed out by electric typewriter, or recorded on punched or magnetic tapes or punched cards. This recorded information may then be used for other purposes as desired (see Vol. 2, Computers and Data Processing).

Installation. Installation of monitoring equipment is expensive. It includes installing transducers, fittings, conduits, wiring, alarms, etc. For simple systems the cost of installation may be three to four times the cost of monitoring equipment. The cost may be reduced by connecting closely located transducers to a central point in the area or zone. A stepping switch is used to select a desired variable, and a single pair of wires is needed to make the connection to the control room.

Time sharing of a transducer may also be accomplished, as in the Fischer and Porter multiple pressure readout system. This equipment uses one readout detector for sampling a number of pressures from processing units located reasonably near to one another.

Installation of data logging and computing equipment will require more detailed system planning. Control room air conditioning is usually desirable to prolong the life of electronic equipment. For computer control systems the installation may also require transducers for converting pneumatic signals to electric signals and continuous composition or quality analyzers. For stored program computer systems, program checkout is part of the installation procedure.

Maintenance. To be useful, data processing equipment must be highly reliable and easy to maintain. Complex equipment is subject to the rules laid down in Chap. 19, Reliability. The basic design of the system should be such that it is fail-safe. Test procedures and test equipment must be available to diagnose and correct any malfunctions.

Solid state electronic equipment designed for high reliability has demonstrated excellent performance. On-call maintenance personnel are provided for computers and data loggers. It is often better practice to avoid scheduled maintenance but to carry out preventive maintenance only when the process being monitored is down for other purposes.

2. MONITORS AND DATA LOGGING EQUIPMENT

Table 2 shows a survey of typical monitoring and logging equipment

TABLE 2. MONITORING AND DATA LOGGING EQUIPMENT CHARACTERISTICS

Characteristic	Typical Data
Number of points	4 to 10 (continuous monitors) 60 to 100 (digital loggers)
Expandability	Modular
Scanning rates	30 to 2000 points per minute
Logging rates	1 to 2 points per second
Logging cycle	1, 2, 5, 10, 15, 30, 60 minutes and off
Error	0.1 to 2% of range
Inputs (electrical)	0 to 100 mv, thermocouples
Functions	Audio alarms, alarm indicator light, analog or digital indication, analog or digital recording
Variables	Temperature, pressure, force, flow, strain, speed, *p*H, level, etc.
Transducers	Thermocouples, strain gauges, *p*H electrodes, other standard detectors (see Chap. 20, Measuring Elements and Sensors)

for production and processing operations (Ref. 1). Scanning rates become important for systems with large numbers of variables. Only the simplest systems monitor continuously.

Monitors. Typical alarm monitors come in modules of a small number of points, 4, 6, or 10, such as the temperature monitoring equipment shown in Fig. 4. When the temperature reaches an alarm setting adjustable for each channel, a sensitive relay detects the condition and closes a contact. The alarm action consists of actuating a light on the front of the unit, lighting a direct reading designation card, and actuating remote visual and audible anunciators. A meter may be plugged in to read the monitored variable. External wiring can automatically shut

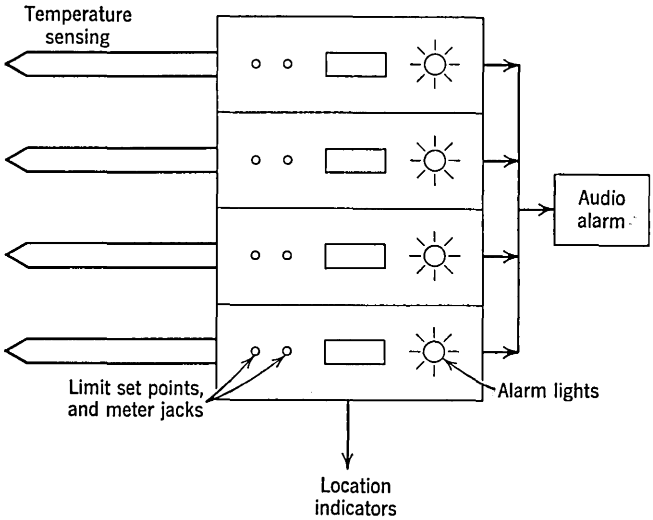

FIG. 4. Temperature monitoring system.

down the operating equipment if desired. Equipment of this type is adaptable to zone or centralized installation. Its modular construction permits panel or closure mounting.

Scanning and Recording Monitors. Scanning systems have been developed to fill the need for compact and economical methods of displaying and/or recording large quantities of information in multivariable systems (Ref. 2). Typical scanning systems come in modules that monitor 50 to 1000 or more variables. Scanning rates are in the range of 2 to 3 per second for mechanical switches or relays (Fig. 5). Electronic scan-

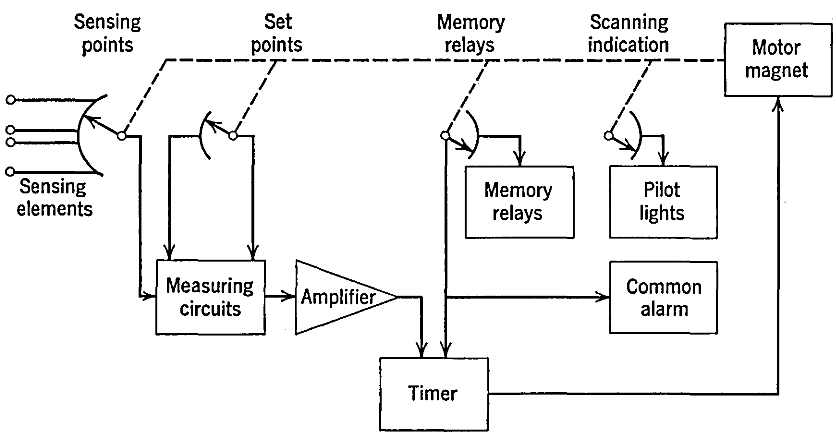

FIG. 5. Scanning monitor block diagram. (Courtesy Thermo Electric Co.)

ning rates are as high as 2000 per second. In some systems displays are on an oscillograph screen at the central receiving point. Heights of the pulses displayed on the screen are proportional to the measured function, and a number of variables may be displayed simultaneously as a train of pulses on the calibrated oscilloscope.

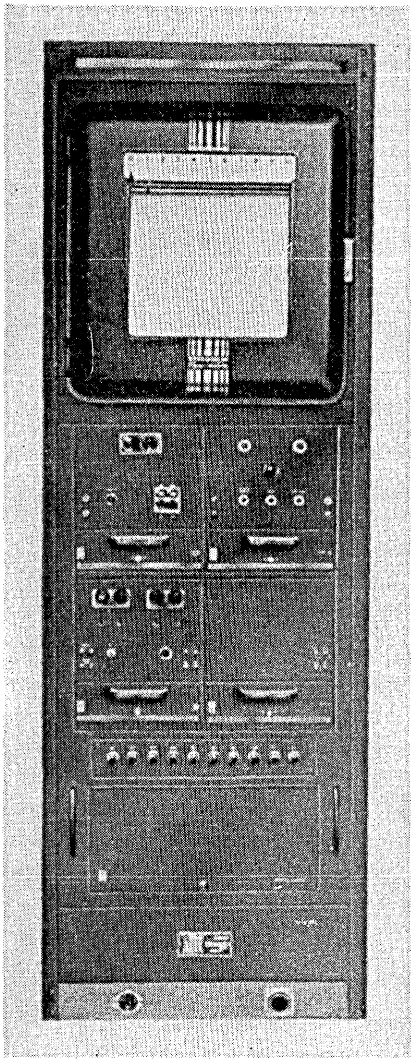

FIG. 6. 100-channel scanning and analog recording system. Each point is recorded with identifying code and time. (Courtesy Leeds & Northrup.)

Other scanning devices are single pen multipoint analog recorders with added functions. A typical setup such as that shown in Fig. 6 may cover 100 points. This device operates in similar fashion to alarm monitors and scanners but provides a chart record of the variables. The points are recorded sequentially on a strip chart and an identifying number and the time is printed by each point. The monitoring unit has stepping switches and controls for program selection.

Scanning monitors are programmed to record data, say, hourly but to scan for off-limit values at the maximum rate between recordings. The operator should be able to override the programming controls to select any desired value, to initiate recording, to omit any variable, and to reset to the beginning of the cycle.

Digital Data Loggers

Information Recording in the Chemical and Petroleum Industries. Large scale data loggers were first developed for the chemical processing industries (Refs. 2 and 3). Similar equipment, often called data processing systems or data reduction systems, has found broad applications in aircraft and missile testing, nuclear and conventional power generating plants, and a variety of other industries. Each new generation of data loggers has increased computing capabilities because of the demands of the users, and some data loggers have been discontinued by manufacturers in favor of digital control computers. It is difficult to justify a data logger when the output is a punched tape which is later fed into a scientific computer. The use of a process control computer eliminates duplication of equipment and intermediate steps. Data loggers will continue to be used when recording without too much computation or evaluation is required.

Characteristics of Data Loggers. Typical data logger diagrams are shown in Figs. 7 and 8. In most equipment computations such as scaling,

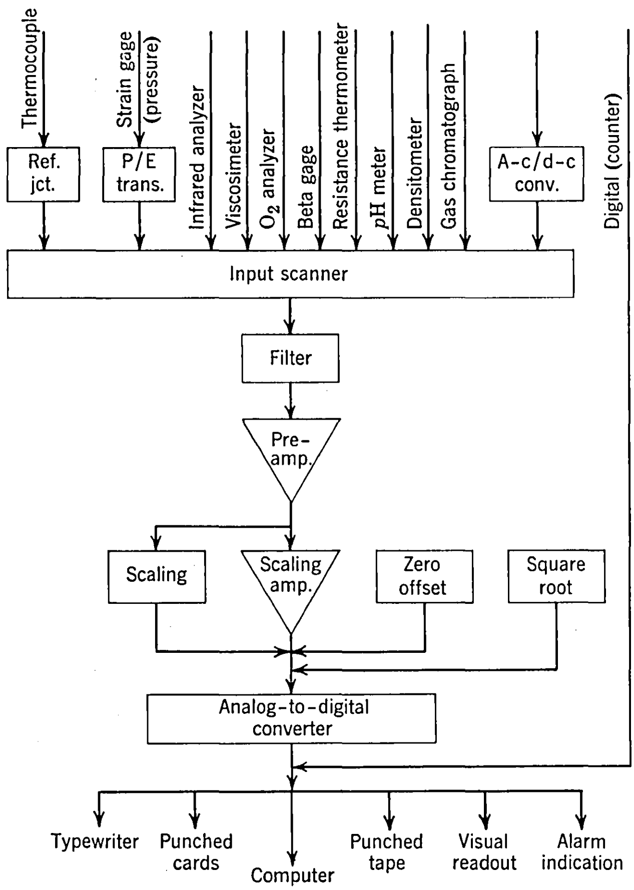

FIG. 7. Operating diagram of the 123 data logger system. (Courtesy Beckman Instruments.)

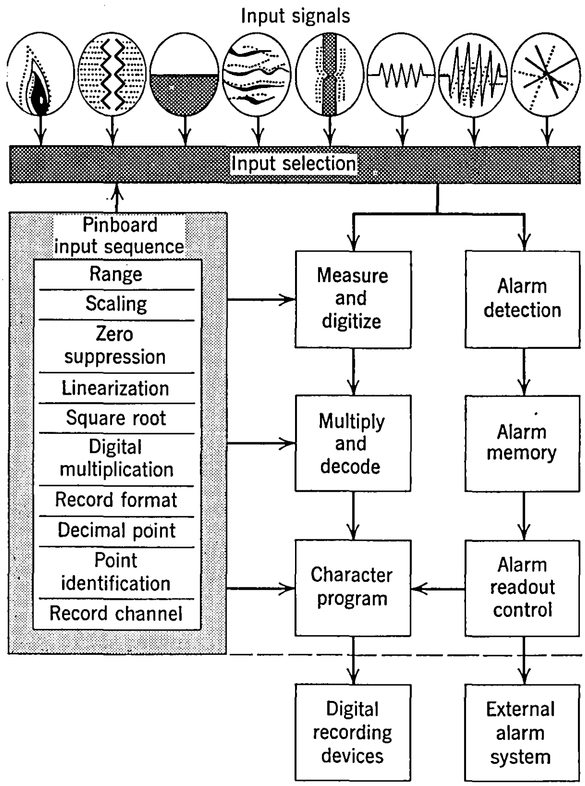

FIG. 8. Data logger block diagram. (Courtesy Fischer & Porter).

zero offset, and square root are done in analog form. Inputs to the system are usually in electrical form. The number of channels available ranges from 100 to 2000 and the equipment comes in modules so that theoretically there is no limit to the number of points that can be handled. Modern equipment is transistorized and designed for in-plant on-line operation. Some recent machines provide digital multiplication. This is useful for logging cumulative values but requires a storage register for each quantity which is repeatedly brought up to date by adding accumulated production at specified time intervals. The Kybernetes logger performs multiplication by including a multiplying register as part of the basic analog-to-digital converter. A variety of analog-to-digital converters are used in digital loggers (see Vol. 2, Chap. 20, Input-Output Equipment for Digital Computers).

In operation the logger is programmed to print out all variables at specified intervals of an hour or so. Between logging cycles the equipment scans all variables for off-limit values. When detected these values are printed out in red with the time of occurrence. No further printout is made until the variable returns to normal, when the time is again recorded. Programming of data loggers is by plugboard. In one typical system, shown in Fig. 9, each channel has a row of pinholes for program-

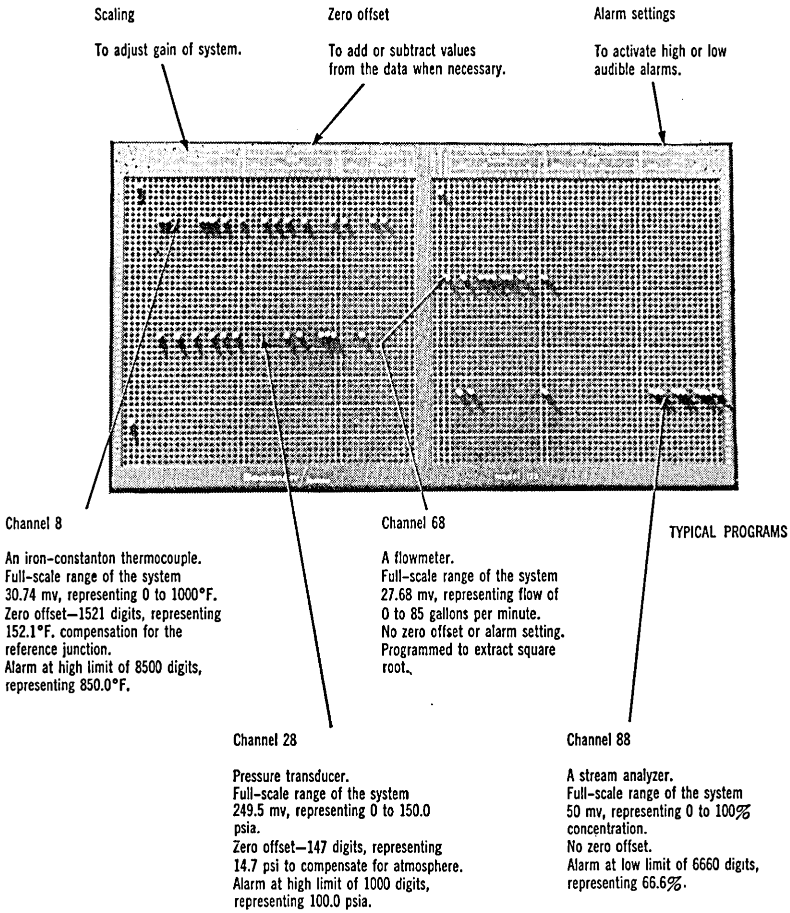

FIG. 9. The 123 data logger programming pinboard. (Courtesy Beckman Systems.)

ming functions. Pins are inserted to add up to the desired digital value associated with the function. This procedure is applied to scaling, zero offset, and off-normal limit.

3. PROCESS CONTROL COMPUTER EQUIPMENT

Computer Characteristics. A control system is a complex combination of equipment, operating and supervisory personnel, and operating procedures whose function is to guide or direct a process to attain some desired objective. If the control system employs a digital computer as a component for carrying out computations associated with the proper operation of the process being controlled, the system is often called a *digital computer control system.* Alternate names for process control by computers are *real time control, on-line control,* and *in-line control.* These names all indicate that the computations carried out by the computer are fast enough to control the process in a useful and stable manner. Analog computers are discussed in Chap. 13, Computer Control, and Vol. 2, Computers and Data Processing. Process controllers are available which manipulate one or two process variables to optimize some performance factor p, which is determined by a special purpose analog computer (see Chap. 13, Computer Control, Section 5, Direct Methods of Optimizing Control). Available equipment includes the Quarie Controller, Westinghouse's Opcon, and Daystrom's Flowcon. The analog computer parts of the systems are special purpose devices designed for the particular application.

General purpose process control computers differ from other general purpose digital computers in the following two respects: (1) multichannel analog inputs and outputs, and (2) high reliability.

Typical applications in the process control field will have 50 to 300 inputs and 10 to 50 outputs. The inputs come from variables measured by transducers located at various points in the process. These measurements are analog in nature, and an analog-to-digital converter is necessary. Alarm scanning is done by the input switching device, usually under the control of the computer. The computer detects off-normal conditions and controls alarm devices. The outputs resulting from computations are analog signals to change set points of controllers, and a digital-to-analog converter is required for closed loop control. Alternately, the computer could compute and print out operating guides such as yields, conversions, heat rates, etc., which the operator evaluates to determine controller set points. Computing operating guides is frequently a preliminary step leading to closed loop control.

Process control computers are designed for extremely high reliability. Manufacturers quote better than 99% reliability. This means that for less than 1% of the time of operation the computer will be down for maintenance or repair. Since computer control systems are designed to lock controllers at the last computed set point, during maintenance periods the plant will continue to operate at the last computed settings. This differs little from operator manual control, and the operator can adjust any of the controllers during the maintenance period. In a digital system a computer might be down once or twice during 1000 hours of

Table 3. Characteristics of Process Control Computers[a]

Company	Thompson Ramo Wooldridge	Daystrom Systems	General Precision Equipment Corp.	General Electric	Information Systems, Inc.	Minneapolis-Honeywell
Computer	RW-300	Daystrom	Libratrol-500	GE-312	ISI-609	Honeywell 290
Storage	drum	mag. core	drum	drum	mag. core	mag. drum and core
Capacity (words)	8,000–15,000	1,000–16,000	4,000	2,000–16,000	4,000	1,000–12,000
Word length (bits)	17 + sign	20 + sign	30 + sign	20 + sign	38 + sign + parity bit	17 + sign
Components						
Transistors	580	1,800	250	1,600	2,400	1,500
Diodes	4,000	5,000	1,850	2,000	1,000	10,000
Cores	–	–	–	–	800	72,000
Tubes	13	–	171	–	–	
Instruction						
Addresses	1 + 1	1	1	1, 1 + 1	1	1
Per word	0.5	1	1	1	1	1
Number	21	46	16	60	64	50
Speed						
clock (kc)	153.6	50	136	250	167	50
addition (ms)	0.78	0.44	0.25	0.096	0.720	0.14
multiplication (ms)	2.99	9.24	15.0	0.29–2.02	2.80	0.80
Maximum input switching speed (points/sec)	3,840	284	200	300	5–350	1,000
Input signals	0–10.23 v	0–50 mv	0–10 v std	0–10 mv	0–60 mv	0–10 v
Output signals	0–15 v or 0–5 ma	–	–	0–20 v	–	–
Weight (lb)	600	2,000	1,000	3,000	–	1,150
Power (kw)	0.5	<2	1.5	4	<2	1.4

[a] All machines are binary, serial general purpose computers, except the Honeywell machine which is parallel.

operation and require 2 to 5 hours of repair and maintenance. During this period the plant will be operating less close to the optimum than during computer control, but over 99% of the benefits of computer control are obtained. Design for reliability and the use of solid state circuitry have made possible the remarkable performance of digital computer systems for process control.

Table 3 lists typical commercial process control computers (Refs. 4 and 5). The computer control equipment can perform the functions of monitoring (scanning and alarm), storing and recording information, computing operating guides and set points for controllers, actuating controllers if closed loop control is used, and checking itself and the instrumentation equipment.

Process control computers are usually serial binary machines and operate at clock speeds of 50–250 kc. Word lengths are 18 to 40 binary digits, with shorter word lengths sufficient because of the low precision input data from process instruments.

Storage. Process control computers use magnetic drums or cores for storage. Magnetic drums and disks have been used in most equipment. Drums are reliable but expensive to manufacture. Access time may be a problem, and usually fast access circulating registers are provided on the drums.

Magnetic cores require more equipment for selecting reading and writing information, and the overall cost of core storage per bit of information is more than for drum storage. Access time is read time since cores have random access. The core storage has no moving parts but is subject to errors from regeneration of information. When a core storage is read the information is destroyed and must be regenerated. Most core systems have a parity bit to detect errors in reading and writing. Parity bits are not normally used in drum storage. In case of a power failure the words being read may be destroyed. If these are instructions or constants, they must be rewritten into the storage before operation is resumed.

Instruction Codes. Typical process computers have single or $1 + 1$ address systems. The $1 + 1$ address instruction contains the address of the quantity to be operated on and also the address of the next instruction. This system is particularly valuable in drum computers where access time can limit the speed of operation of a program. The number of instructions ranges from 16 to 64. Inputs and outputs are limited by switching speeds and speed of analog-to-digital conversion. Typical computers time-share the analog-to-digital conversion device, and rates of 200 to 1000 or more points per second are available. The ranges of input and output signals have not been standardized but are dependent on the system and method of tie-in with instrumentation.

Future Outlook. Typical applications of process control computers have been discussed in Chap. 13, Computer Control. Installations have been made in electric power industry largely for monitoring, in the chemical and petroleum industries for closed loop control, and in the steel industry for data processing and control. Users of computers are demanding larger storage capacity both to scan larger numbers of variables and to perform auxiliary functions. As programs become larger the need for higher operating speeds also increases.

4. PLANNING FOR COMPUTER CONTROL

Design Problems. Applications of computer control of processes indicate that the control of a process can be based on complex calculations and decisions that involve large numbers of measurements of process variables. Adjustment can be made on a large number of manipulated variables with the speed and freedom from error that is unattainable with the most capable and conscientious operators (Refs. 7 to 10).

The two fundamental types of computer control have been discussed in Chap. 13, Computer Control. These are systematic experimentation, with logical control of the experiments, so that the process moves in the direction of the optimum. In the second kind a mathematical representation or model of the process is used to predict the reaction of the process to disturbances, and control equations are used to calculate optimum conditions for control action. This section outlines the procedure for planning for computer control (Refs. 11 to 15) employing mathematical relationships.

In designing a computer control system for chemical and other industrial processes four types of mathematical relationships between process variables are important. The final model for control will contain all these relationships directly or indirectly. They are as follows:

1. A profit equation which expresses the economic objectives of process operation.

2. Constraints or restrictions imposed on process operations by factors such as material availability, product quality, specifications, equipment ratings, safety requirements, etc.

3. Relationships developed from fundamental physical and chemical principals including material and energy balance, reaction kinetics, equilibriums, and material properties. These relationships define the transfer functions for the process which relate the output behavior to the changes in input variables and enable the computer to control the process in an optimum manner.

4. Relationships which permit the model of 3 to adapt itself automatically to changes in the process itself.

The designers of a computer control system must have a thorough understanding of material flows, characteristics, equipment layout and limitations, product specifications, control objectives and procedures, and process economics. In system analysis they must carry out the following steps.

1. Check the adequacy of existing and available instrumentation.
2. Develop correlations of operating variables.
3. Express process objectives in quantitative fashion.
4. Study process dynamics to verify that the computer will provide stable and close control in accordance with the objectives.
5. Define system functions and specify the equipment and procedures by which they are carried out.
6. Program the computer for the control problem.
7. Take into account any safety consideration, fail-safe procedures, etc.

Mathematical Models. Optimization of processes by mathematical models and some examples of models are given in Chap. 13, Computer Control. An excellent compilation of typical models including material and heat balances, chemical reactions, catalyst deactivation, material transport, separation, and blending has been made by Stout (Ref. 16). The literature on the subject has been growing rapidly (Refs. 16–22). *Dynamic models* are in the form of differential equations of the process. *Static* or *steady-state* models do not depend on time and are expressed as algebraic equations.

Dynamic models have been determined by the following methods.

1. Step and frequency response (Refs. 23 and 24).
2. Analog computer simulation (Refs. 25 and 26).
3. Correlation techniques. These are in the early stages of development (Ref. 27).

A steady-state model is a simplification of an associated dynamic model and can be developed without reference to the dynamic characteristics of the process. Ordinary process calculations are made in this manner, and much information that can be adapted to the needs of the computer control system designer is available.

Development of a Model. The development of relationships among process variables, including quantitative expressions of the process objectives and statements of constraints of the process operation, is the most difficult and time-consuming part of the system design.

For most operating processes there is usually a wealth of plant data in company files. Periodicals and patents are another source of information. Even for new processes laboratory and pilot plant data are generally available. In almost all cases a theoretical treatment which will furnish the framework for interpretation of the available process data is possible.

A successful approach to establishing relationships suitable for computer control should have the following characteristics.

1. The analyst should have the attitude that the relationships exist.
2. He should appreciate that the relationships do not need to be perfect.
3. The mathematical model should be able to predict the results of changes in the major process variables.
4. The accuracy of prediction should be comparable to the accuracy of process instruments.
5. The computer itself can be used to refine a crude model when improved knowledge and better instruments become available.
6. The analyst must be aware of the tools available for analyzing process control systems. These include auto- and cross-correlation techniques, graphical analysis, and standard statistical procedures of many kinds.

Process Objectives. The fundamental objective of most industrial process operations is economic, i.e., to earn money for the operators. The quantitative expression for process objectives is therefore often called a profit equation and may be written as

$$\text{Profit} = \text{Income} - \text{Expense}.$$

Income generally arises from sale of products including valuable by-products or waste, whereas expenses arise from a variety of sources as shown in Table 4.

TABLE 4. EXPENSES ASSOCIATED WITH A PROCESS

Materials	*Overhead*
Raw materials	Employee benefits
Chemicals	Plant supervision
Catalysts	General administration
Packages	*Miscellaneous*
Utilities	Depreciation
Water	Taxes
Steam	Insurance
Fuel	Selling expenses
Electricity	Research expenses
Labor	
Operating	
Maintenance	
Laboratory	
Supervision	

Expenses will be of two types, fixed and variable. The system designer is mainly concerned with the income and expenses which vary with the way the process is operated. Costs that are truly fixed and independent of process conditions can be considered separately and need not appear in the calculations. The cost of raw materials or utilities can be represented by an equation of the form

$$C = V_F F + k_1, \tag{1}$$

where C is the total cost in dollars,
F is the quantity of material used in appropriate units, pounds, gallons, cubic feet, kilowatt-hours, etc.,
V_F is the cost per unit of the quantity F, and
k_1 is fixed cost in dollars.

The total income from sale of a product can be expressed in a similar fashion as

$$V = V_Q Q + k_2, \tag{2}$$

where Q is the quantity of product,
V is the total income,
V_Q is the value per unit quantity, and
k_2 is a fixed cost.

Figure 10 shows the concepts of fixed and variable costs used in these equations. In general, equations such as (1) and (2) should be written

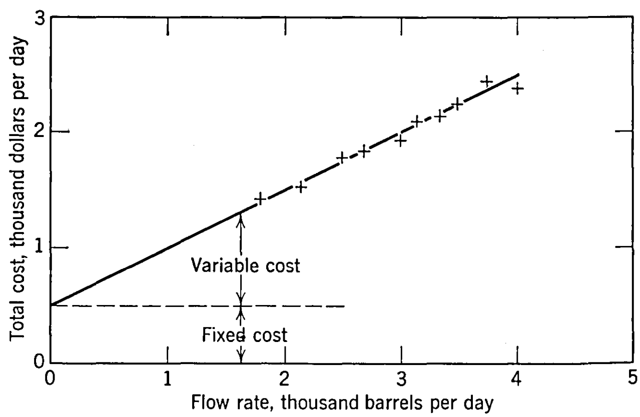

FIG. 10. Representation of material costs, total cost versus flow rate.

for all the significant streams to reach realistic conclusions on how to adjust the process. In particular it is advisable to allocate all operating costs to the product streams.

Where the process uses materials which are the outputs of previous processes and furnish a product which becomes the feed material for subsequent processes, the values of the various materials can be somewhat nebulous and difficult to determine. Even in relatively straightforward situations the systems engineer may need help from the accounting department in order to obtain suitable values of the slopes, intercepts, and equations.

System Boundaries. The boundaries for the process considered must be fixed, and all income-producing or cost-incurring materials which cross these boundaries must be included in the profit equation for the process. Hence, a multiterm equation results:

$$P = \sum_{1}^{M} V_{Qi}Q_i - \sum_{1}^{N} V_{Fj}F_j - P_0, \tag{3}$$

where P is the total profit,

Q_i is the amount of the ith product,

V_{Qi} is the unit value of the ith product,

F_j is the amount of the jth raw material or utility which feeds the process,

V_{Fj} is the unit value of the jth raw material or utility, and

P_0 is the fixed cost.

If the Q's and F's in these equations are given as rates rather than quantities, this equation gives the profit rate in dollars per hour or day. Continuous processes can be optimized by making adjustments so that the profit rate is always a maximum.

Batch and semicontinuous processes as well as the start-up and shutdown periods of continuous processes can be optimized only by considering the time variations of P that are due to programmed variations of process conditions, deactivation of catalysts, etc. Although the values of materials (V's) have been defined as independent of Q and F, they will vary as changes in the market occur and must frequently be brought up to date. In rare instances fluxations in market may be sufficient to require frequent adjustment.

Product Specifications and Production Quotas. Product specifications may vary from (1) the minimum which ensures that the desired product is made to (2) stringent requirements on product characteristics. Typical product specifications are

1. *Physical properties of material:* strength or hardness limits, viscosity, specific gravity, color, etc.

2. *Chemical properties:* composition, impurity limits, pH, etc.

Many of the quality specifications may appear as constraints on the control process conditions. An impurity limit, for example, may restrict reaction temperature. Poisoning of a catalyst may inhibit reactivity. Equipment rating will also constrain the production rate.

Constraints. *Constraints* or *restrictions* on process operations generally take the form of inequalities. Process variables may be restricted to have values (1) greater than some minimum value, (2) less than some maximum value, or (3) within both minimum and maximum limits. These constraints arise from product quality specifications or production quotas or from material availability, equipment ratings, safety requirements, etc.

Optimization. Optimizing is performed at two levels: (1) frequent adjustment of variables to maintain operation as close as possible to the desired limits set by the equations, (2) periodic adjustments of constants of the equations to bring the mathematical model into agreement with process operation. The latter function may be performed every ½ hour to 4 hours, whereas the short-term adjustments are made every 2 to 5 minutes depending on the process. Characteristics of a typical operating installation with both of these types of optimization is described in Refs. 9 and 28.

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INDUSTRIAL CONTROL SYSTEMS

Transmission Systems

L. M. Silva

1. INTRODUCTION

A measure of the effectiveness of a technological society is its ability to effectively transfer information from the source to the user. Information may originate from instrumentation, computers, or weapon systems. In each particular situation the effectiveness of information transfer or transmission is determined by the rate at which information transfer is accomplished and the degradation or loss of information in the process.

A transmission system, to perform within the limits set by the allowable degradation and at the specified rate, is designed by selecting a configuration which is appropriate for the distances involved and in which the total equivocation or ambiguity introduced by all the system components is within the specified limits.

Symbols. The symbols defined in the following list are used in a consistent manner throughout this chapter. Symbols not listed are defined where they are used.

A	Rms amplitude.
A_0	Peak carrier amplitude.
A_{0t}	Peak carrier amplitude, at the threshold point.
A_n	Rms noise amplitude.
A_p	In-phase carrier amplitude component.
A_q	Quadrature carrier amplitude component.
A_s	Peak amplitude of a signal voltage.
$A(\omega)$	Amplitude frequency characteristic of a system, filter, etc.
$A_0(\omega)$	An assumed or ideal amplitude characteristic for pulse transmission systems. Also, the amplitude characteristic of a reference system used as a standard of comparison.
a	Rms value of $\alpha(\omega)$ over the transmission bandwidth.
a_0	Ratio of rms carrier amplitude to rms noise amplitude.
B	Bandwidth in cycles per second. For high-order filters, B should be interpreted as the half-power bandwidth or 3-db amplitude bandwidth.
B_c	3-db bandwidth of carrier circuits in cycles per second (e.g., the bandwidth of the bandpass filter ahead of an FM discriminator).
B_g	Bandwidth of a Gaussian filter, cycles per second.
B_o	Output filter 3-db bandwidth, cycles per second.
B_p	3-db bandwidth required to track a signal in a phase lock loop.
B_s	Signal or source bandwidth, cycles per second; where signal refers to message or information which is being transmitted.
B_T	Transmission system bandwidth, cycles per second.
B_T/B_s	Ratio of $\dfrac{\text{Transmission system bandwidth}}{\text{Source bandwidth}}$.
$B_{1/2}$	½ power bandwidth.
$B_{1/2a}$	½ amplitude bandwidth, cycles per second.
b	Moments of a frequency distribution or spectrum around some reference frequency (e.g., carrier frequency) or point. The zeroth moment b_0 is equal to the mean noise power and is equal to N for the message bandwidth spectrum and N_T for the transmission system bandwidth spectrum.
$\underline{b}$	Rms phase deviation in radians over the transmission bandwidth, $B_{1/2a} = \omega_1/2\pi$.
C	Information rate measured, bits per second.
D	A distortion power factor.
$D_0(t)$	Time-varying angular modulation [$2\pi \times$ Time-varying frequency modulation]. For sine wave modulation, $D_0 \sin \omega_s t$. $D_0 = 2\pi \times$ Maximum frequency deviation.
$\overline{E}^2$	Time-averaged mean square error.
f	Frequency, cycles per second.
f_0	A reference frequency, or the center frequency of a filter.
f_c	Carrier frequency.
f_s	Signal frequency.
$f(t)$	Any function of time.
I	Information measured in bits.
K	Message dynamic range; sluggishness index; ratio of quadrature to in-phase carrier power; a constant; a peak factor.
k	Number of redundancy bits; also a constant.
$M(t)$	A modulated voltage waveform.

$m = n + k$ Sum of information bits and redundancy bits in a digital code representing a single sample of the signal. Control bits are not included. Also, modulation index of an amplitude modulated carrier.

N Mean noise power or the noise power in the signal or message bandwidth.

N_0 Noise power in output of AM detector when carrier is not present.

N_c Mean noise power in the carrier bandwidth or bandwidth of band-pass filter.

N_T Rms noise power in transmission circuit bandwidth.

n The number of bits in a digital code representing a single sample of the signal. Redundancy and control bits are not included in n. Also, the number of identical sections in a filter.

P_{dc} The power component at the output of a device which contains no information and is equivalent to a d-c bias, a fixed amplitude carrier component, etc.

P_s The maximum power component associated with the signal or information, $P_s = S$ if $P_{dc} = 0$.

P_T Maximum average transmission system power.

$\widehat{P_T}$ Maximum peak transmission system power.

$p(x)$ Probability density function. The probability that a particular measured value lies in an interval of length dx centered at x.

R_0 Saturation level of limiter.

$R(t)$ The modulated signal received at the receiving end of a transmission system.

S Rms signal power.

S/N Signal-to-noise power ratio. For a specific measurement it is the ratio of signal power to noise power. For a device or component it is the ratio of maximum available signal power to noise power.

$(S/N)_{\text{eff}}$ Signal-to-noise power ratio realizable at the output of a device, component, or system or the signal-to-noise power ratio required at the output of a device, component, or system.

$(S/N)_{\text{I}}$ S/N for narrow band FM.

$(S/N)_{\text{II}}$ S/N for wide band FM.

s Power efficiency factor; s in decibels is equal to the excess power required (s is negative) to realize a given $(S/N)_{\text{eff}}$.

s_A Power efficiency factor for a transmission system with an average power limitation.

$s_{\max}$ Power efficiency factor for maximum decibel loss or minimum efficiency.

$s_{\min}$ Power efficiency factor for minimum decibel loss or maximum efficiency.

s_p Power efficiency factor for a transmission system with a peak power limitation.

T Time constant, etc. Time over which an average is taken.

t Time.

U Intersymbol amplitude interference (see Pulse Modulation section).

$\underline{U}$ Rms intersymbol amplitude interference for pulse transmission.

$\overline{u}$ Peak intersymbol interference.

V_E Carrier envelope voltage or response as a function of time.

V_i Input voltage.

V_n Noise voltage.

V_o	Output voltage.
V_p	In-phase carrier voltage component.
V_q	Quadrature carrier voltage component.
V_s	Signal voltage.
$V(t)$	Time-varying voltage.
$W(f)$	Power spectrum as a function of frequency.
X_0	True value of an arbitrary quantity, a series of measurements, etc.
x	A measured value of an arbitrary quantity.
$Z(j\omega)$	Amplitude characteristic of a filter as a function of frequency.
$\alpha(\omega)$	Fractional deviation of an actual amplitude frequency characteristic from that of an ideal system or of a reference system. a is equal to the rms value of $\alpha(\omega)$ over the transmission bandwidth $B_{\frac{1}{2}a} = \omega_1/2\pi$.
$\beta(\omega)$	Fractional deviation of an actual phase characteristic from that of an ideal characteristic or reference system. b is equal to the rms phase deviation in radians over the transmission bandwidth $B_{\frac{1}{2}a} = \omega_1/2\pi$.
Γ	A ratio of rms modulation to a bandwidth factor that is a function of the spectral distribution of the noise.
ϵ	Maximum fractional error.
μ	Deviation rate (for sine wave modulation μ = deviation ratio).
ρ	Bandwidth factor equal to ratio: $\dfrac{\text{Mean squared value of noise power spectrum about } \omega_0}{\text{Mean noise power}}$
σ	The standard deviation, root mean square error.
σ_n	Rms phase jitter.
σ_2	Variance.
τ	Period or time interval.
τ_d	Transmission delay time.
ψ	Phase deviation in FM systems or phase characteristic versus frequency for pulse transmission systems.
$\psi_0(\omega)$	An assumed ideal linear phase characteristic for pulse transmission systems. Also the phase characteristic of a reference system used as a standard of comparison.
ω	Radian or angular frequency.
ω_0	A reference angular frequency, the center frequency of a filter.
ω_c	Angular frequency of carrier.
ω_d	Angular detuning.
ω_s	Angular signal frequency.

2. INFORMATION

Information typically originates from instrumentation and is in effect a measure of a physical quantity or quality. It is characterized by being continuous over a finite range and for a given measurement precision can have a number assigned to every possible state. Typical signal sources include voltages from transducers, pulse signals in which the information is contained as the time displacement between pairs of pulses, and sinusoidally varying voltages, the frequencies of which contain the information.

The process of measuring or assigning a number to a particular magnitude of the signal is limited by the presence of noise or uncertainty in the measuring system. For scientific measurements it is normal to assume a Gaussian or normal law of errors.

Gaussian Distribution. If X_0 is the true value of a quantity, the Guassian distribution states that the probability $p(x)$ of any particular value is

$$p(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-(x - X_0)^2/2\sigma^2\right].$$

The distribution of $p(x)$ as a function of $x - X_0$ is shown in Fig. 1*a*. The *standard deviation* or root-mean-square error σ, the significant parameter of this distribution, describes the magnitude of the errors associated with the measurement. In particular for the Gaussian distribution, 67% of the measurements will have an error less than σ. The quantity $1/\sigma$ is an index of precision which defines the number of identifiable amplitude levels for a given measurement system. The quantity σ^2 is called the *mean square error*. It is numerically equal to the average value of the sum of the squares of the individual deviations about the mean.

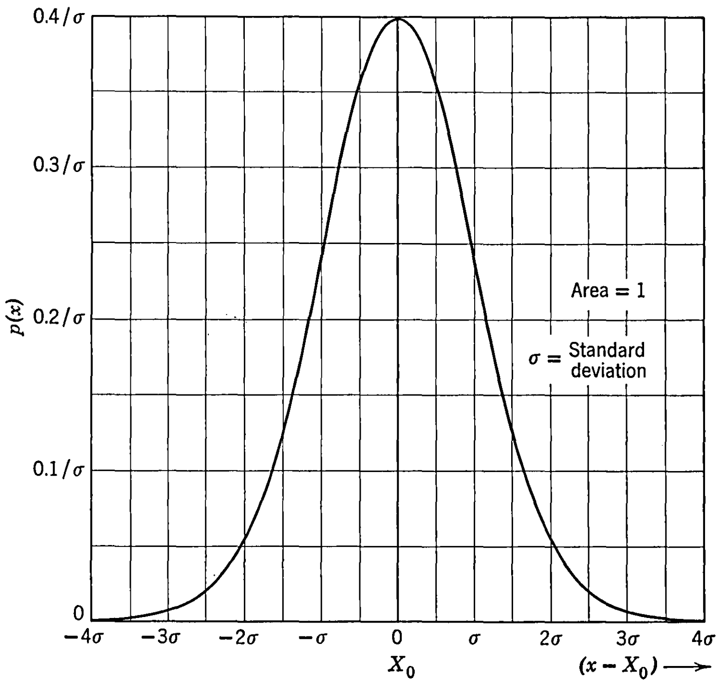
A characteristic of the Gaussian distribution is that it is possible to obtain large errors for some fraction of the measurements. Thus it is impossible to define a unique peak error or factor. Instead a qualified peak factor can be defined as the ratio of the peak exceeded only a specified percentage of the time to the rms error value. It is convenient to express peak factors in decibels above the rms value. A table of peak factors for Gaussian distribution is given in Fig. 1*b*.

Signal-to-Noise Ratio. In a system in which a random rms noise voltage V_n is superimposed on a steady signal voltage V_s, the quantity V_n is equal to σ and $(V_n)^2$ is equal to σ^2. The quantity $(X_0/\sigma)^2$ is thus equal to $(V_s/V_n)^2$. Since $V_s{}^2$ and $V_n{}^2$ are proportional to signal power S and noise power N respectively, the ratio of $(X_0/\sigma)^2$ is numerically equal to the signal-to-noise power ratio, S/N.

The rms deviation σ sets a lower limit to the precision with which a quantity can be described. This limit is by nature associated with a power sensitivity, $\sigma^2 = N$, due to thermal noise. The extremes of the range or saturation limits of a device imposes an upper limit on the magnitude of the information. The ratio of the maximum and minimum power levels is called the *signal-to-noise ratio:*

$$\frac{S}{N} = \frac{\text{Signal power at extreme of range}}{\text{Mean square noise power}}.$$

If the signal includes a steady d-c term which contains no information and



(a)

Percent of Time Peak is Exceeded	$\frac{\text{Peak}}{\text{Rms}}$	Peak Factor in db = 20 $\log_{10}$ (Peak/Rms)
10.0	1.615	4.32
1.0	2.576	8.22
0.1	3.291	10.35
0.01	3.890	11.80
0.001	4.417	12.90
0.0001	4.892	13.79

(b)

Fig. 1. (a) Gaussian distribution; (b) peak factors for Gaussian noise.

which limits the dynamic range of the signal, the signal-to-noise ratio becomes

$$\frac{S}{N} = \frac{P_s - P_{dc}}{N}$$

where P_s = maximum signal power,
P_{dc} = steady-signal power component,
N = noise power.

The concept of S/N, signal-to-noise ratio as a measure of *quality,* is applicable to most physical processes if S is interpreted as the square of the range of the physical variable and noise as the variance or mean square value of uncertainty.

The quantity N/S is a per unit or normalized variance and is independent of units or power levels. It is interesting to note that Fisher's (Ref. 3) measure of total information is equivalent to $1/(N/S) = S/N$.

The signal-to-noise ratio for an analog signal is a measure of the number of distinguishable power levels or the dynamic range of the signal.

Shannon-Hartley Law. In order to complete the specification of a signal, it is necessary to define the amount of information contained in the signal. If a signal can be measured to a precision of 1 part in 128 (2^7), then 128 *amplitude* levels or different values of the signal can be distinguished. *Information* is defined (Ref. 1) as the log to base 2 of the number of distinguishable amplitude levels. Thus for the case in point,

$$I = \log_2 (2)^7 = 7 \text{ binary digits.}$$

The unit of information is the *binary digit* or *bit.*

For the general case of a signal with added Gaussian noise, the number of amplitude levels is equal to the square root of the number of distinguishable power levels (Ref. 2):

$$\text{Distinguishable amplitude levels} = \sqrt{1 + \frac{S}{N}}.$$

The addition of 1 to S/N is required because zero power is an allowed level. Thus the information contained in a single measurement for a signal with noise power N and signal power S is

$$I = \log_2 \sqrt{1 + \frac{S}{N}} = \tfrac{1}{2} \log_2 \left(1 + \frac{S}{N}\right).$$

Next it is necessary to specify the number of measurements required per unit time to completely describe the signal. This is the subject of Shannon's sampling theorem (Ref. 2).

SHANNON'S SAMPLING THEOREM. *If a function $f(t)$ contains no frequencies higher than B cps, it is completely determined by giving its ordinates at a series of points spaced $1/2B$ seconds apart.*

Thus $2B$ samples per second are required to specify a continuous analog function, and hence the information content C of the analog signal having a frequency bandwidth B cps is

$$C = 2B\,\tfrac{1}{2}\log_2\left(1 + \frac{S}{N}\right) = \text{Information rate,}$$

$$C = B\log_2\left(1 + \frac{S}{N}\right) \text{ bits per second (Shannon-Hartley law).}$$

One of the qualifications in the formulation of the sampling theorem is that the signal of bandwidth B shall contain *no power* outside of this band of frequencies. This infinite attenuation characteristic is not realizable in practice, and as a result sampling rates in excess of the two samples per cycle of bandwidth are required (Refs. 11 and 12).

Effective Signal-to-Noise Ratio. Consider a system in which the signal has a bandwidth B, and transmission is required with an accuracy of 0.1%. To obtain the required accuracy 1000 amplitude levels must be distinguishable. Hence,

$$\sqrt{1 + \frac{S}{N}} = 1000,$$

$$\frac{S}{N} \cong 10^6 = 60 \text{ db.}$$

Thus a minimum signal-to-noise power ratio of 60 db is required for the received signal.

For an ideal transmission system a 60-db S/N at the input would appear at the output as a signal having a 60-db S/N. It is characteristic of practical transmission systems that the original signal-to-noise ratio is degraded by the transmission process. Thus if a specified S/N is required at the output of the system, the actual signal-to-noise power ratio required at the input will have to exceed the output S/N to allow for degradation. A convenient method for comparing systems is to express the *effective signal-to-noise ratio,* $(S/N)_{\text{eff}}$, obtained at the output of the system in terms of the actual signal-to-noise power ratio required at the input. For actual systems the information capacity, C, is meaningful only if the effective signal-to-noise ratio is used in the Shannon-Hartley expression. That is,

$$C = B\log_2\left[1 + \left(\frac{S}{N}\right)_{\text{eff}}\right] = \text{information capacity practical system,}$$

where B = message bandwidth.

Error Rate. For the ideal situation described by the Shannon-Hartley law, the magnitude of B and S/N defines a system with an arbitrarily small error rate or frequency of errors exceeding $\sqrt{N/S}$. In actual systems the effective signal-to-noise ratio is a function of the error rate in addition to the input power and noise levels. Thus the comparative evaluation of actual systems can only be meaningful if the error rate or the distribution of errors is specified in addition to the effective signal-to-noise ratio.

3. TRANSMISSION SYSTEMS

All methods of transmission of information can be classified as either direct or carrier. The *direct* systems transmit the information in its original form or in the form of code groups obtained by encoding or quantizing the information. *Carrier* type systems use some form of common carrier which is modulated by the signal information. The configurations of these systems are shown in Fig. 2. The transmission mediums are typically wire lines or RF links.

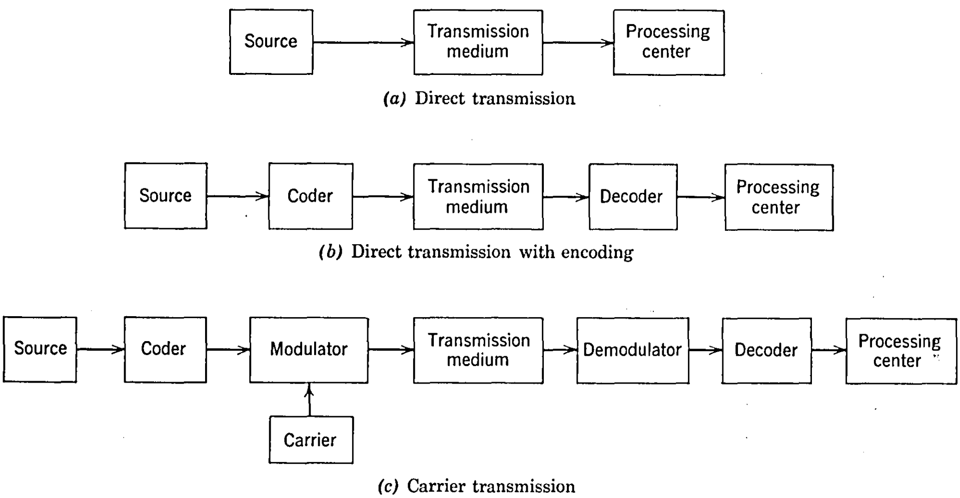


FIG. 2. Transmission systems.

Transmission System Specification. The basic parameters specifying an element of a transmission system are the power and bandwidth requirements in conjunction with the capacity of the system to transmit information. The evaluation of a particular physical component or system requires a means of quantitatively assessing whether the performance limitations of the component are fundamental or due to the design of the equipment. The quantities used to evaluate performance are: (*a*) input signal-to-noise ratio, (*b*) effective output signal-to-noise ratio, (*c*) error rate or error statistics, (*d*) information capacity, and (*e*) bandwidth.

The difference between input and output signal-to-noise ratio is a measure of the degradation or loss of information associated with either a component or the entire system. If an overall output signal-to-noise ratio is specified for a component or a system having a loss of information,

$$\left(\frac{S}{N}\right)_{\text{in}} - \left(\frac{S}{N}\right)_{\text{out}} = \text{db}_{\text{loss}},$$

the specified output S/N can only be realized in practice by increasing the power level or the signal-to-noise ratio at the input.

The error rate of the system or component can, in most instances, be quantitatively specified, and the design of a system to a prescribed error rate is equivalent to employing excess power to provide a margin over noise.

The formula for information capacity involved only the S/N and the bandwidth. Since the signal-to-noise ratio is specified at the output by the dynamic range of the message, the required signal-to-noise ratio at the input is equal to the output signal-to-noise ratio plus the margins required to account for loss of information through the system and for satisfying the prescribed error rate. The only other specification required to complete the description of the system is the bandwidth of the input or output information.

Power-Bandwidth Exchange. The Shannon-Hartley law, $C = B \log_2 (1 + S/N)$, is an exchange relationship between the variables C, B, and S/N. Thus, signal to noise can be bartered for bandwidth for a constant communication capacity. The idea that a smaller signal-to-noise ratio will suffice if greater bandwidth is used did not come as a surprise, for exchange in this direction existed in frequency modulation and various types of pulse modulation prior to its inception. A reason for the delay in discovering the possibility of exchange of bandwidth for power is that to approach the ideal limit set by the Shannon-Hartley law some means of encoding or quantizing a continuous signal into a finite number of elements is required. The development of pulse code modulation, PCM, in which information is transmitted as a series of binary pulses, was a significant accomplishment.

Given a particular task for a transmission system, it is important to have a grasp of the physical significance of the Shannon-Hartley law in terms of bandwidth and power. For this purpose it is convenient to derive an alternate form for the Shannon-Hartley law (Ref. 21) in which the variables are

$$\frac{P_T}{P_s} = \text{Power ratio,}$$

$$\frac{B_T}{B_s} = \text{Bandwidth ratio,}$$

$$\sqrt{1 + \frac{P_s}{N}} = \text{Message dynamic range,}$$

where P_T = transmission system power,
P_s = signal power or message power,
B_s = message or signal bandwidth,
B_T = transmission system bandwidth.

In order to derive the relationship in terms of these variables, consider the direct transmission of the information from a source having a bandwidth B_s and a dynamic range of $\sqrt{1 + P_s/N}$. Then the information rate of this source is

$$C = B_s \log_2 \left(1 + \frac{P_s}{N}\right) = \text{Information rate of source.}$$

By the Shannon-Hartley law, this same rate can be realized by exchanging bandwidth and power in a modulator or information transformer prior to transmission. Thus for the transmission system

$$B_T \log_2 \left(1 + \frac{P_T}{N}\right) = B_s \log_2 \left(1 + \frac{P_s}{N}\right).$$

This equation can be rearranged into the form

$$\frac{P_T}{P_s} = \frac{B_T/B_s}{P_s/N} \left[\left(1 + \frac{P_s}{N}\right)^{B_s/B_T} - 1\right].$$

By letting $K = \sqrt{1 + P_s/N}$ (message dynamic range),

$$K^2 = 1 + \frac{P_s}{N},$$

then

$$\frac{P_T}{P_s} = \frac{B_T/B_s}{(K^2 - 1)} (K^{2B_s/B_T} - 1).$$

This equation gives the power ratio P_T/P_s associated with the bandwidth ratio B_T/B_s for a given dynamic range or message signal-to-noise ratio. Figure 3 is a plot of this ratio for three values of dynamic range 2, 32, and 1000. Note that the power ratio is a function of the signal dynamic range, and that *when the dynamic range is low, the power saving by bandwidth*

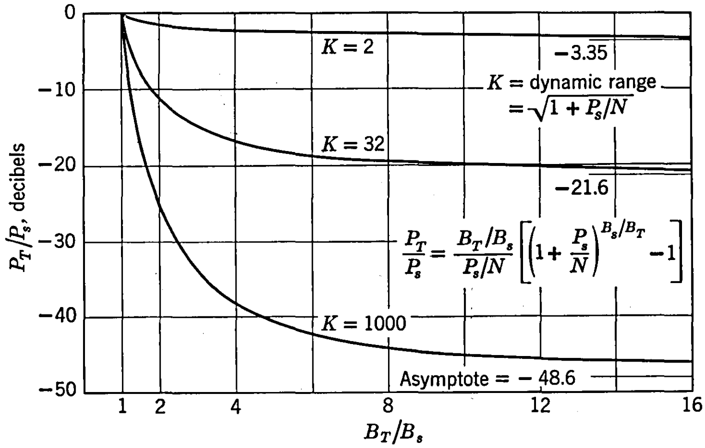


FIG. 3. Power ratio vs. bandwidth ratio for a specified dynamic range.

expansion is also low regardless of bandwidth ratio. For example, the power saving for a signal with a dynamic range of 2:1 is less than 3½ db for infinite bandwidth ratio.

In Fig. 4, the limiting value of the power ratio P_T/P_s versus dynamic range K is given. This limiting value of P_T/P_s is the maximum possible power gain for a given dynamic range.

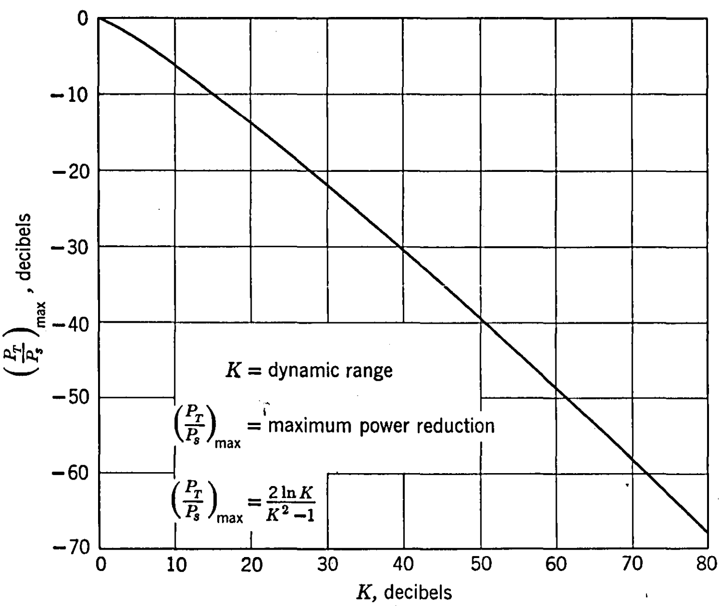


FIG. 4. Maximum power reduction by exchanging bandwidth for power.

It is important in most applications to compare only power and bandwidth. The Shannon-Hartley law, for a fixed information rate, actually is a relationship involving the signal-to-noise ratio and the bandwidth. For example, assume the message has a dynamic range of approximately 32:1 and the bandwidth is unity (see Table 1). Then, from the Shannon-

TABLE 1. EFFECT OF BANDWIDTH ON SIGNAL-TO-NOISE RATIOS

	S/N	$1 + S/N$	B	C
Case 1	1023	$1024 = 2^{10}$	1	10
Case 2	1	$2 = 2$	10	10

Hartley law for a bandwidth of 10, the signal-to-noise ratio will be unity.

Since the ratio of the two values of S/N is approximately 1000:1, the first reaction is to say that a power saving of 30 db was realized by increasing the bandwidth by a factor of 10. This is, of course, wrong, since it ignores the increase in power required owing to the increase in bandwidth. The net power saving is approximately 20 db, which is in agreement with the information obtained previously for a dynamic range of 32:1:

$$\frac{P_1}{P_2} = \frac{1023N_1B_1}{1N_1(10B_1)} = 102.3 \cong 20 \text{ db.}$$

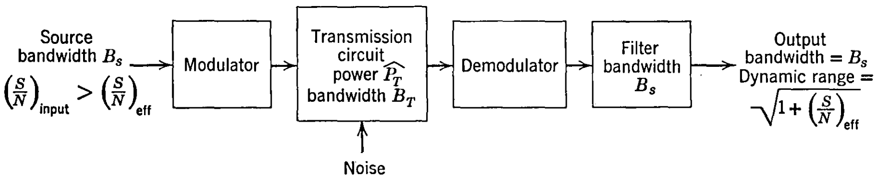
Modulation and Encoding. The modulation and encoding operations indicated in Fig. 2 represent the transformation of the signal into a form suitable for transmission by the system. These operations match the characteristics of the signal and transmission system by (1) shifting the signal spectrum to the spectrum occupied by the transmission medium, (2) exchanging bandwidth for power to permit operation within the power capabilities of the system, and (3) altering the redundancy of the signal to minimize transmission bandwidth or increase the reliability of transmission.

When the noise level of the transmission system is low, the proper modulation or encoding method is generally determined by convenience. This is also the case where the power available is relatively large for the information to be transmitted. The modulation or encoding method is sometimes determined by the practical considerations that one terminal of the system be simple and compact with greater complexity permissible at the other terminal. Many air-ground installations are coded with particular emphasis on this requirement.

Modulation Methods

It is characteristic of most transmission systems that the information received at the destination is usually reconstructed into the form in which it existed prior to transmission. Thus, two information transformations are usually required: at the source, a transformation from source bandwidth and dynamic range to transmission medium bandwidth and dynamic range, and the reverse process at the destination. These two conversions in the transmission system are identified as modulation and demodulation.

System Normalization. In order to provide a simple means for comparing different modulation and demodulation systems, it is convenient to describe the demodulator output signal-to-noise ratio in terms of the power and bandwidth requirements of the transmission circuits (Fig. 5.). This is equivalent to normalizing (Ref. 22) the system with respect to the output.



B_T = effective transmission circuit bandwidth
B_s = message bandwidth
$\hat{P}_T$ = transmission circuit peak power

FIG. 5. Transmission system.

This method of normalizing system performance with respect to the output permits describing the system performance in terms of the message bandwidth, B_s, and the effective signal-to-noise ratio, $(S/N)_{\text{eff}}$, required at the output for a specified dynamic range.

These two specifications, output bandwidth and message dynamic range, in conjunction with the input-output S/N characteristics for each of the elements, will determine the signal-to-noise ratio required at the source and the power and bandwidth requirements of the transmission circuits. The information rate of a system which is normalized in this manner is simply

$$C = B_s \log_2 (S/N)_{\text{eff}},$$

with $(S/N)_{\text{eff}}$ large compared to unity.

Classification of Modulation Methods. Four modulation methods are listed in Table 2. Each of these represents a unique class of modulating devices in the sense that information is represented in a different manner.

Direct Transmission–Amplitude Modulation. For the simplest transmission system, direct transmission over a pair of wires, the output signal-to-noise ratio is equal to the input signal-to-noise ratio providing

TABLE 2. MODULATION METHODS

Method	Information Representation
Amplitude modulation	In terms of an amplitude function
Frequency modulation	In terms of a frequency deviation from a reference frequency (carrier)
Pulse code	In a pulse group as a digital code
Pulse modulation	In terms of a time interval

the amplifier or transmitter at the source does not have a peak power limitation and only thermal noise exists in the trnasmission circuit. In practice, however, the power limitation is more likely to be a peak power limitation than an average power limitation. Shannon (Refs. 20 and 22) has shown that under these circumstances the effective signal-to-noise ratio will lie within a range

$$\frac{2}{\pi e^3}\frac{\widehat{P_T}}{N} \leqq \left(\frac{S}{N}\right)_{\text{eff}} \leqq \frac{2}{\pi e}\frac{\widehat{P_T}}{N},$$

for S/N large,

$\widehat{P_T}$ = peak power,
N = noise power in message bandwidth.

Thus if a 60-db signal-to-noise ratio is required at the output of the system, the input signal-to-noise ratio will correspond to

$$75\text{ db} \geqq \frac{\widehat{P_T}}{N} \geqq 66.3\text{ db},$$

where $(S/N)_{\text{eff}} = 60$ db. For this example, direct transmission with a peak power limitation, the effective signal-to-noise ratio is simply a constant times the required signal-to-noise ratio at the input to the transmission circuits. The only constraints are those of a peak power limitation and the fact that the effective signal-to-noise ratio is substantially greater than unity.

For the generalized amplitude modulation process, the expression for the effective signal-to-noise ratio can be written

$$s_{\max}\frac{\widehat{P_T}}{N} \leqq \left(\frac{S}{N}\right)_{\text{eff}} s_{\min} \leqq \frac{\widehat{P_T}}{N},$$

where the s's are called power efficiency factors.

A measure of the excess power required for a particular system is the ratio $\widehat{P_T}/P_s$ of peak power $\widehat{P_T}$ to the amount of power P_s required for an ideal system. In this instance, this ratio is simply the reciprocal of the power efficiency factors:

$$\frac{1}{s_{\max}} \geqq \frac{\widehat{P_T}}{P_s} \geqq \frac{1}{s_{\min}}.$$

P_s is the message power or the transmission circuit power for direct transmission without a peak power limitation and without losses.

These results are applicable to all transmission systems in which information is contained in the form of an amplitude. The only differences between individual systems are the magnitudes of the power efficiency factors.

Frequency Modulation. In the case of FM systems, Goldman (Ref. 22) has shown that the effective signal-to-noise ratio for wide deviation FM and for large $\widehat{P_T}/N$ is

$$\frac{2}{\pi e}\frac{\widehat{P_T}}{N}\left(\frac{B_T}{B_s}\right)^2 \geq \left(\frac{S}{N}\right)_{\text{eff}} \geq \frac{2e}{\pi}\frac{\widehat{P_T}}{N}\left(\frac{B_T}{B_s}\right)^2,$$

where N = thermal noise power in message bandwidth B_s,
B_T = effective transmission circuit bandwidth
= maximum frequency deviation (note transmission circuit bandwidth $= \beta B_T > 2B_T$).

In this instance the effective signal-to-noise ratio varies directly as the product of peak power and the square of the deviation ratio B_T/B_s. Thus, for a given $(S/N)_{\text{eff}}$ and bandwidth B_s, power is exchanged as the square of the bandwidth of the transmission medium.

It is characteristic of all systems that exchange bandwidth for power that a threshold value of $\widehat{P_T}/N_T$ exists below which performance deteriorates rapidly. For the case of FM, the threshold occurs when the amplitude of the carrier signal equals approximately four times the rms noise voltage in the carrier passband. An expression from Nichols and Rauch (Ref. 7), for the threshold power level is

$$\left(\frac{\widehat{P_T}}{N_T}\right)_{\text{threshold}} \cong 16\beta,$$

where $\beta \cong 3$ for deviation ratios in excess of 4,
βB_T = RF bandwidth.

The quantity β is a function of the deviation ratio, dynamic range of the message, and first derivative of the message waveform.

For the general case of a system in which power is exchanged on the basis of the bandwidth expansion ratio squared, $(S/N)_{\text{eff}}$ becomes

$$s_{\max} \frac{\widehat{P_T}}{N} \left(\frac{B_T}{B_s}\right)^2 \leq \left(\frac{S}{N}\right)_{\text{eff}} \leq s_{\min} \frac{\widehat{P_T}}{N} \left(\frac{B_T}{B_s}\right)^2.$$

The ratio $\widehat{P_T}/P_s$ is given by

$$\frac{1}{s_{\max}} \left(\frac{B_s}{B_T}\right)^2 \geq \frac{\widehat{P_T}}{P_s} \geq \frac{1}{s_{\min}} \left(\frac{B_s}{B_T}\right)^2.$$

Binary PCM–Direct Transmission. In pulse code modulation (PCM) the signal is sampled periodically, and an n-bit binary code is generated which corresponds to the amplitude level of the signal at the instant of sampling. The device which performs the sampling is called an analog-to-digital converter. For each sample, its output is a serial group of n pulses which are transmitted over the transmission circuits. In addition to the n pulses which carry the information, one or more additional pulses are used to synchronize the transmitter and receiver.

If there are n binary pulses per sample, the resolution of the measurement is limited to $1/2^n$. This quantity is called the quantizing step and determines the accuracy of the conversion. Since the resolution is finite, the quantized value will, on the average, be in error by plus or minus one-half a quantizing step or $1/2^{n+1}$. This error is called quantizing noise.

If B_s is the bandwidth of the message, then $2B_s$ samples per second are required by the sampling theorem. Each sample will include a group of n pulses, where $n = \log_2 K$ with K the dynamic range.

The transmission system will transmit $2nB_s$ pulses per second. Since a transmission circuit of bandwidth B_T can transmit up to $2B_T$ binary pulses per second, then

$$B_T = nB_s = B_s \log_2 K = \text{Transmission bandwidth.}$$

If the error rate of the transmission circuit is small, the received message signal-to-noise ratio is *independent of the transmission circuits.* Error rates of less than 10^{-6} (Ref. 5) can be realized with values of $\widehat{P_T}/N_T$ of 20 db.

The power required for PCM transmission is proportional to the number of bits or the dynamic range. The power ratio $\widehat{P_T}/P_s$ can be obtained from

$$\frac{\widehat{P_T}}{N_T} = \frac{1}{s} \qquad \text{where } \frac{1}{s} = 20 \text{ db for } 10^{-6} \text{ error rate.}$$

Then

$$\frac{\widehat{P_T}}{P_s} = \frac{1}{s}\frac{1}{P_s/N_T} = \frac{1}{s(P_s/N)(B_s/nB_s)} = \frac{n}{s(P_s/N)} = \frac{1}{s}\frac{\log_2 K}{K^2 - 1}.$$

Thus for PCM the bandwidth varies as the $\log_2$ of the dynamic range, and transmission power ratio $\widehat{P_T}/P_s$ varies inversely as the square of the dynamic range.

If it is assumed that transmission will be effected at a negligible error rate, the only source of noise in the output of the system will be that resulting from quantizing noise (Ref. 22), and the effective signal-to-noise ratio is

$$(S/N)_{\text{eff}} = [2(2^n - 1)]^2 = [2(2^{\log_2 K} - 1)]^2.$$

Pulse Modulation. Pulse modulation is the method used by another class of modulators which have found widespread use. Their characteristics are intermediate to the extremes represented by amplitude modulation and frequency modulation. This can be established by considering that information is contained in the interval between two pulses or the duration of a pulse. If the transmission circuit has a peak power limitation, then the signal to noise ratio for the detection of a single pulse or pulse edge will be of the form, $s(\widehat{P_T}/N_T)$. The time resolution of the system is proportional to the ratio B_T/B_s, of transmission circuit bandwidth (pulse rise time) to message bandwidth (pulse repetition rate). The ratio B_T/B_s, is an amplitude function; hence it will appear in the effective S/N as $(B_T/B_s)^2$ since the power ratio must vary as the square of the amplitude variations. Thus, the effective S/N will be of the form

$$\left(\frac{S}{N}\right)_{\text{eff}} = s\frac{\widehat{P_T}}{N_T}\left(\frac{B_T}{B_s}\right)^2 = s_P\frac{\widehat{P_T}}{N}\frac{B_T}{B_s},$$

with s the power efficiency factor. Thus, for a peak power limitation, pulse modulation (PPM or PDM) systems exchange bandwidth directly for power for a given effective signal-to-noise ratio.

The question of power is fundamental in this form of modulation as in amplitude type systems. If it is assumed that an average power limitation exists rather than a peak limitation, the exchange relationship between power and bandwidth is more attractive. If the average power remains constant as the bandwidth is varied and the peak power varies directly as bandwidth, then

$$(P_T)_{\text{av}} = \widehat{P_T}\frac{B_s}{B_T} = \text{Constant},$$

$$\widehat{P_T} = P_T \frac{B_T}{B_s},$$

and $(S/N)_{\text{eff}}$ becomes

$$\left(\frac{S}{N}\right)_{\text{eff}} = s \frac{P_T}{N} \left(\frac{B_T}{B_s}\right)^2.$$

Thus, for the case of pulse modulation with an average power limitation, power is exchanged directly as the bandwidth squared as in FM. This example serves to illustrate the importance of a realistic appraisal of the physical characteristics and limitations of a system.

Comparison of Modulation Methods. The basic power and bandwidth characteristics of the various forms of modulation provide a basis for comparison and evaluation. A summary of these characteristics is given in Table 3. In order to obtain a quantitative grasp of the power efficiency of the various methods in terms of the dynamic range of the transmission, the quantity $\widehat{P_T}/N$ is used since it is directly proportional to the transmission power required for a particular system:

$$\frac{\widehat{P_T}}{N} = \frac{\widehat{P_T}}{P_s} \frac{P_s}{N}$$

but

$$\frac{P_s}{N} = \left(\frac{S}{N}\right)_{\text{eff}} = K^2 - 1.$$

Hence,

$$\frac{\widehat{P_T}}{N} = \frac{\widehat{P_T}}{P_s} (K^2 - 1).$$

This quantity $\widehat{P_T}/N$ also appears in Table 3.

In the derivation of expressions for the effective signal to noise of the various systems, the following assumptions or constraints were imposed:

1. There is linear superposition of signal and noise.
2. A filter at the output of the demodulator limits the noise bandwidth of the overall transmission system to the message bandwidth B_s.
3. Excess noise contributed by system elements (modulators, receivers, etc.) is negligible.
4. All systems receive the same noise power per unit bandwidth.
5. Noise has a Gaussian amplitude distribution.
6. No allowance for losses associated with the transmission circuits or other system elements.
7. $\widehat{P_T}/N_T \gg 1$.

The assumption of negligible losses throughout the system is not restrictive, providing the individual elements do not contribute noise in excess

TABLE 3. POWER AND BANDWIDTH CHARACTERISTICS FOR BASIC METHODS OF MODULATION

Information Coordinates (modulation)	Power Ratio $\frac{\widehat{P_T}}{P_s}$	Power Ratio $\frac{\widehat{P_T}}{N}$	Actual Transmission Circuit Bandwidth	Remarks
Amplitude	$\frac{1}{s}$	$\frac{1}{s}(K^2 - 1)$	$\beta[B_s]$	A. $B_T = B_s$ B. $s < 1$ C. $1 < \beta < 2$
Frequency	$\frac{1}{s}\left(\frac{B_s}{B_T}\right)^2$	$\frac{1}{s}\left(\frac{B_s}{B_T}\right)^2(K^2 - 1)$	βB_T	A. Threshold limitation B. $\beta \geq 3$
Time	$\frac{1}{s_P}\left(\frac{B_s}{B_T}\right)$	$\frac{1}{s_P}\left(\frac{B_s}{B_T}\right)(K^2 - 1)$	βB_T	A. Peak power limitation B. $1 \leq \beta \leq 2$
	$\frac{1}{s_A}\left(\frac{B_s}{B_T}\right)^2$	$\frac{1}{s_A}\left(\frac{B_s}{B_T}\right)^2(K^2 - 1)$	βB_T	A. Average power limitation B. $P_T = \widehat{P_T}\left(\frac{B_s}{B_T}\right) = \text{constant}$
Pulse code	$\frac{1}{s}\frac{\log_2 K}{(K^2 - 1)}$	$\frac{1}{s}\log_2 K$	$B_s \log_2 K$	

of that attributed to the thermal noise associated with the bandwidth of the element. The fact that losses do exist in a system merely implies the need for providing excess power at the transmitter or power amplification at the receiver. The assumption of Gaussian or White noise is restrictive in the sense that the results do not apply for impulsive or Rayleigh noise sources which are encountered in RF transmission systems.

The inclusion of the filter at the output is typically not restrictive, since by definition no information is contained in the message for frequencies greater than B_s, the message bandwidth.

The assumption of linear superposition of noise is required for simplicity of analysis. With the exception of binary pulse transmission, the assumption that $\widehat{P_T}/N_T$ is greater than unity is not restrictive since the information content of the message will normally require that this condition be satisfied. In the case of FM or pulse systems, the threshold condition places a limitation on the minimum value of this quantity.

Table 4 presents the expressions obtained for the effective signal-to-noise ratio for all the systems and in addition indicates the range over which the expressions are meaningful.

In the case of amplitude-modulated carrier type systems, no distinction is made for synchronous systems in which the phase of the carrier is used in the demodulator to effect a S/N improvement. The use of synchronous detection will provide a maximum of 3-db improvement in the effective signal-to-noise ratio of the double sideband AM systems in which a single modulator is used at the transmission end. With more sophisticated modulation methods (two modulators or two signals per channel), it is possible to obtain up to an additional 3-db improvement. Thus in the limit double sideband (DSB) modulation can be as efficient as single sideband (SSB) modulation while avoiding the difficulties of SSB transmission.

Example. To illustrate exchange of bandwidth and power and the performance of the various methods of modulation, assume that a transmission system is required to transmit 0.1% data. This corresponds to a dynamic range of 1000:1 or 10 binary bits. If the results are plotted in terms of P_T/N versus bandwidth ratio B_T/B_s, the curves (Fig. 6) will indicate the actual power required for transmission over a lossless system. This statement is not true for pulse position modulation systems in which the actual pulse width is a fraction of the repetition period. In this instance, the actual power will be equal to the peak power multiplied by the duty ratio.

For this particular application, if the conservation of spectrum or bandwidth is the primary consideration, then SSB-AM is the best choice. If bandwidth is available, either PCM-DSB-AM or PCM-FM provides the greatest power saving. Note that if the duty ratio is small for the pulse

TABLE 4. EFFECTIVE SIGNAL-TO-NOISE RATIOS FOR BASIC MODULATION METHODS

Symbol	Description	$\left(\frac{S}{N}\right)_{\text{eff}}$	Threshold	Remarks
AM	Direct transmission with average power limitation	$\frac{P_T}{N}$		
AM	Direct transmission with peak power limitation	$\frac{2}{\pi e^3}\frac{\widehat{P_T}}{N} \leq \left(\frac{S}{N}\right)_{\text{eff}} \leq \frac{2}{\pi e}\frac{\widehat{P_T}}{N}$		
DSB-AM	Double sideband amplitude-modulated carrier	$\frac{1}{2\pi e^3}\frac{\widehat{P_T}}{N} \leq \left(\frac{S}{N}\right)_{\text{eff}} \leq \frac{1}{2\pi e}\frac{\widehat{P_T}}{N}$		$2B_s$ = actual transmission bandwidth, upper bound uncertain (Ref. 20)
SSB-AM	Single sideband amplitude-modulated carrier	$\frac{1}{2\pi e^3}\left(\frac{4}{1}\right)\frac{\widehat{P_T}}{N} \leq \left(\frac{S}{N}\right)_{\text{eff}} \leq \frac{1}{2\pi e}\left(\frac{4}{1}\right)\frac{\widehat{P_T}}{N}$		
FM	Wide band frequency modulation	$\frac{2}{\pi e}\frac{\widehat{P_T}}{N}\left(\frac{B_T}{B_s}\right)^2 \leq \left(\frac{S}{N}\right)_{\text{eff}} \leq \frac{2e}{\pi}\frac{\widehat{P_T}}{N}\left(\frac{B_T}{B_s}\right)^2$	$\frac{\widehat{P_T}}{N} = 16\beta$	βB_T = actual RF bandwidth B_T = max. frequency deviation $\beta \cong 3$ for deviation ratios $\frac{B_T}{B_s} = 5$ (Ref. 7, p. 51)
Binary PCM	Direct transmission of binary pulse code modulation	$\left(\frac{S}{N}\right)_{\text{eff}} = [2(2^K - 1)]^2$ $\frac{\widehat{P_T}}{N} = \frac{1}{s}\log_2 K$	$\frac{\widehat{P_T}}{N} = \frac{1}{s}$	n = no. of bits = $\log_2 K$ K = dynamic range $\frac{1}{s}$ = 20-db, error rate $\leq 10^{-6}$
PPM	Direct transmission pulse position modulation	$S_p\frac{\widehat{P_T}}{N}\frac{B_T}{B_s}$	$\frac{\widehat{P_T}}{N} = 16\frac{B_T}{B_s}$	Peak power limitation: Values of s ranging from 1 to 0.01 reported in literature
		$S_A\frac{P_T}{N}\left(\frac{B_T}{B_s}\right)^2$	$\frac{\widehat{P_T}}{N} = 16\frac{B_T}{B_s}$	Average power limitation $P_T = \widehat{P_T}(B_s/B_T)$
PPM-DSB-AM	Pulse position modulation of an amplitude-modulated carrier	$S_p\frac{\widehat{P_T}}{N}\frac{1}{4}\frac{B_T}{B_s}$	$\frac{\widehat{P_T}}{N} = 16\beta\frac{B_T}{B_s}$	Peak power limitation

$\left(\frac{S}{N}\right)_{\text{eff}}$ = effective signal to noise power ratio of system referred to output = $\frac{\text{Max. signal output power}}{\text{Average noise power}}$

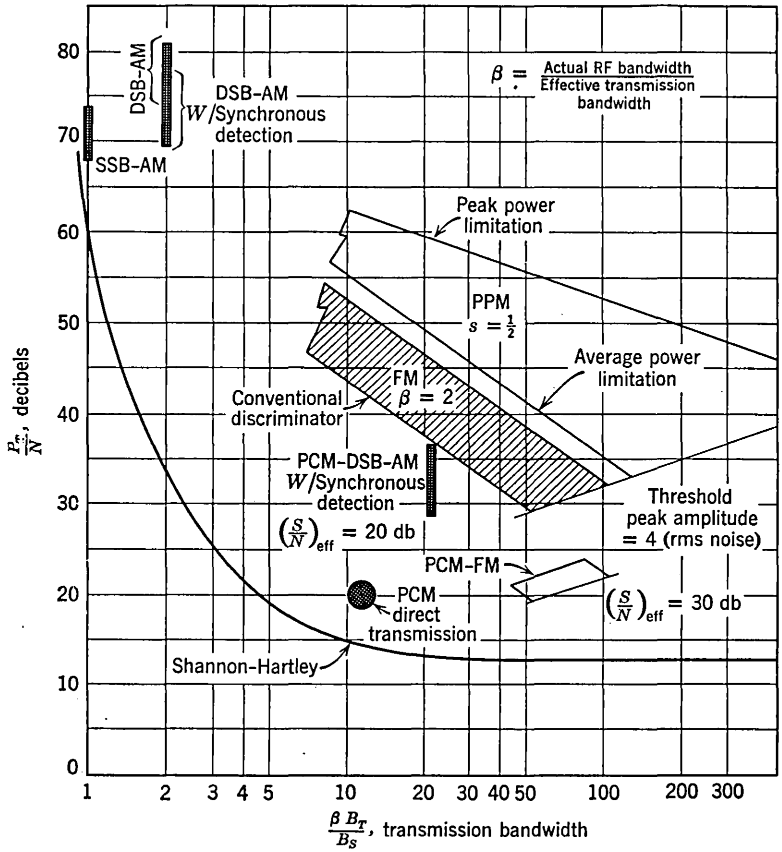


FIG. 6. Power and bandwidth exchange for transmission of signal having a dynamic range of 1000:1 and bandwidth B_S.

position modulation system, the total power required will be comparable to that required for the PCM systems. For PCM-FM system, a signal-to-noise ratio of 30 db was used in order to obtain a deviation ratio of 2.5 to 5. The 30 db was necessary in order to maintain the power level above the threshold. Note that for PCM-FM the threshold is obtained relative to $\beta(B_T/B_s) = 10$ since the output frequency of the PCM modulator is ten times the input frequency. If phase lock demodulators are used for the FM systems, the threshold can be reduced by 10 to 20 db. (See Sect. 4, FM Demodulation and System Errors.)

An alternate method of comparing transmission systems has been proposed by Halina (Ref. 39) which relates the actual bandwidth and power characteristics of systems to the information capacity.

Digital Codes

The use of digital codes other than straight binary in transmission systems can usually be identified with one of the following requirements:

1. Codes required to correct for apparatus limitations.
2. Codes which compress information into less time or bandwidth.
3. Codes which reduce errors due to random system disturbances.
4. Codes which are basically decimal for operator convenience.

Operators are accustomed to information in the decimal notation. The trend in modern communication systems is to employ binary pulse transmission. If this is carried to the extreme, information will be coded as a binary number, and the final user must either memorize the binary number system or have a binary-to-decimal converter at his disposal.

For a message with a dynamic range of 1000:1, a 10-bit binary code must be used to transmit the information. If the number were transmitted in decimal notation, then 30 bits (3×10) would be required and the channel capacity would be increased by a factor of 3. A compromise between these two extremes would be to code individual decimal digits in binary notation and to send 12 binary bits (3 groups of 4 bits) for a system having a dynamic range of 1000:1. The user is now required to memorize only the ten binary numbers 0–9. This code, called *binary-coded decimal* (BCD), is shown in Fig. 7. It requires a 20% increase over straight binary in channel capacity (10 to 12 bits per sample).

Codes Designed to Compensate for Apparatus Limitations. A characteristic of the BCD code is that in going from number to number (e.g., 3 to 4), the values of two or more of the binary bits may change. In devices such as shaft position digitizers, a group of four brushes per decimal digit is used to read the code corresponding to the position of the shaft. It is impossible to align the brushes so that in going from one number to the next the binary bits all change at exactly the same time. Thus, if the code is changing from 3 (0011) to a 4 (0100) and the "1's" bit goes to 0 before the "4's" bit changes to a 1, a reading of 2 (0010) will be obtained. This problem can be solved by using codes in which the value of only one bit changes in going from one number to another. Two of these codes, the *reflected binary* (Gray code) and *cyclic binary decimal,* are illustrated in Fig. 7. Note that the reflected binary code cannot be used as a reflected binary-coded decimal code because of errors that can occur during change between decimal digits 9 and 10 (00001101 and 00010000).

Thus, the reflected binary code solves the ambiguity problem, but the decoding and visual interpretation problem are just as bad as for the straight binary. A solution to this problem suggested by Glixon (Ref. 16)

Decimal Number	Binary	BCD 8421	Reflected Binary	Cyclic Binary Decimal
0	0000	0000	0000	0000
1	0001	0001	0001	0001
2	0010	0010	0011	0011
3	0011	0011	0010	0010
4	0100	0100	0110	0110
5	0101	0101	0111	0111
6	0110	0110	0101	0101
7	0111	0111	0100	0100
8	1000	1000	1100	1100
9	1001	1001	1101	1000
10	1010	1010	1111	11000
11	1011	1011	1110	11001
12	1100	1100	1010	11011
13	1101		1011	11010
14	1110		1001	11110
15	1111		1000	11111
16	10000		11000	11101
17	10001		11001	11100
18	10010		11011	10100
19	10011		11010	10000
20	10100		11110	110000
100	1100100		1010110	11000000
1023	1111111111		1000000000	1100000110010

Fig. 7. Digital decimal codes.

is to combine the reflected and binary decimal code into a cyclic binary decimal code that is easy both to decode and to interpret.

Error-Detecting and Correcting Codes. The impetus for the development of codes which would detect errors was provided initially by the need for a reliable method of teletype radio communication (Ref. 23) in the presence of fading. Initial efforts were directed toward methods that would detect one or two errors per code group. The development of the digital computer and attempts to improve the reliability of communications systems led to more sophisticated codes that would *detect* and *correct* errors.

The construction of noise- or error-reducing codes is typically approached by utilizing redundant or excess binary bits to represent a single symbol or measurement; e.g., more than 4 bits for the decimal numbers 0–9.

These codes are called systematic codes and are based on using k out of a total of m binary bits for error detection or correction. Thus n bits represent the information, k bits represent redundancy (for checking), and

$$\text{Total} = m = n + k \text{ binary bits.}$$

The ratio $m/n = R$ is the redundancy and is a measure of the efficiency of the code (Ref. 17).

Parity Check–Single Error Detection. Single error detection codes use an additional bit, called the parity bit, to detect *single errors*. Parity check is defined as odd or even in the sense that the sum of one's in a symbol code is odd or even. Thus, for example, the decimal number 7 in BCD with even parity check would require a parity bit, but it would not if an odd parity scheme were employed. Table 5 illustrates the use of an odd parity check bit.

TABLE 5. ODD BIT PARITY CHECK

Decimal Number	8 4 2 1	Parity Bit	Sum
0	0 0 0 0	1	1
1	0 0 0 1	0	1
5	0 1 0 1	1	3
6	0 1 1 0	1	3

Hamming Codes. The construction of single error correcting codes has been described by Hamming (Ref. 17). The number of check bits k required for a given number of message bits is shown in Table 6. From the table, for $m = 4$, $k = 3$, and thus a total of 7 bits per BCD symbol are required to correct single errors. Hence, a total of $7 \times 4 = 28$ bits are required for 0.1% data samples as compared to 12 bits for BCD and 10 for straight binary. Note that k increases from $j - 1$ to j at the number $2j$ and remains constant until the number 2^{j+1} is reached. An elegant description of the single error correcting code is given in Ref. 18.

TABLE 6. HAMMING'S SINGLE ERROR CORRECTING CODES

m, Total No. Bits	n, Message Bits	k, Redundancy Bit
1	0	1
2	0	2
3	1	2
4	1	3
5	2	3
6	3	3
7	4	3
8	4	4
9	5	4
10	6	4
11	7	4
12	8	4
13	9	4
14	10	4
15	11	4
16	11	5
	Etc.	

$m = n + k =$ total bits per symbol

Power Efficiency of Redundant Codes. Voelcker (Ref. 19) has evaluated the efficiency of error-correcting codes on the basis of power gain. The conventional 5-bit (32 character) Baudot code used in Teletype communications was used as a standard of comparison. Two distinct approaches were considered: (1) systems in which one way transmission is used and coding is used for error detection and correction, and (2) systems in which coding is used only for error detecting and retransmission is used for error correction.

In the latter system, called "Decision Feedback Systems," each character or symbol is checked for errors. If no error is detected, the character is accepted. If an error is detected, the error detector in the receiver automatically signals the transmitter to repeat the character. Two types of error-detecting codes were considered for the "Decision Feedback Systems": (*a*) a 6-bit parity check code (5 message bits, 1 check bit), and (*b*) a 7-bit *constant ratio code.* This code, the Moore code, is termed constant ratio because 3 of the 7 bits are constrained to be a 1 (the remaining 4 bits are zeros). This 7-bit code contains 35 useful characters and will detect one, three, or five errors but not two or four errors.

For the one-way transmission systems, a 9-bit Hamming code was selected in which 5 bits were used for the message and 4 bits for checking and error correcting.

The results for the various systems are shown in Fig. 8 and include the evaluation of each of the systems for fading signal with additive noise. The fading signals were assumed to exhibit Rayleigh characteristics. Two limiting fading rates are considered: fast fading such that independent signal levels are obtained for adjacent binary bits, and slow fading in which the fading is slow enough to result in essentially constant signal level during the duration of a single character (7 bits). Experience indicates that slow fading is a more realistic assumption. It is further assumed that linear synchronous detection is used and the transmitter and receiver are synchronized.

The results of Fig. 8 confirm the conclusions set forth by Laemmel (Refs. 13 and 14) that error-correcting codes are uneconomical in view of the small increase in power required to obtain the same improvement. This is particularly true when the cost of the elaborate equipment required to detect and correct errors is considered.

4. FM DEMODULATION AND SYSTEM ERRORS

The equipment required for demodulation of FM signals typically consists of a bandpass filter, limiter, discriminator, and a low-pass filter as indicated in Fig. 9. In telemetry or carrier frequency application, the

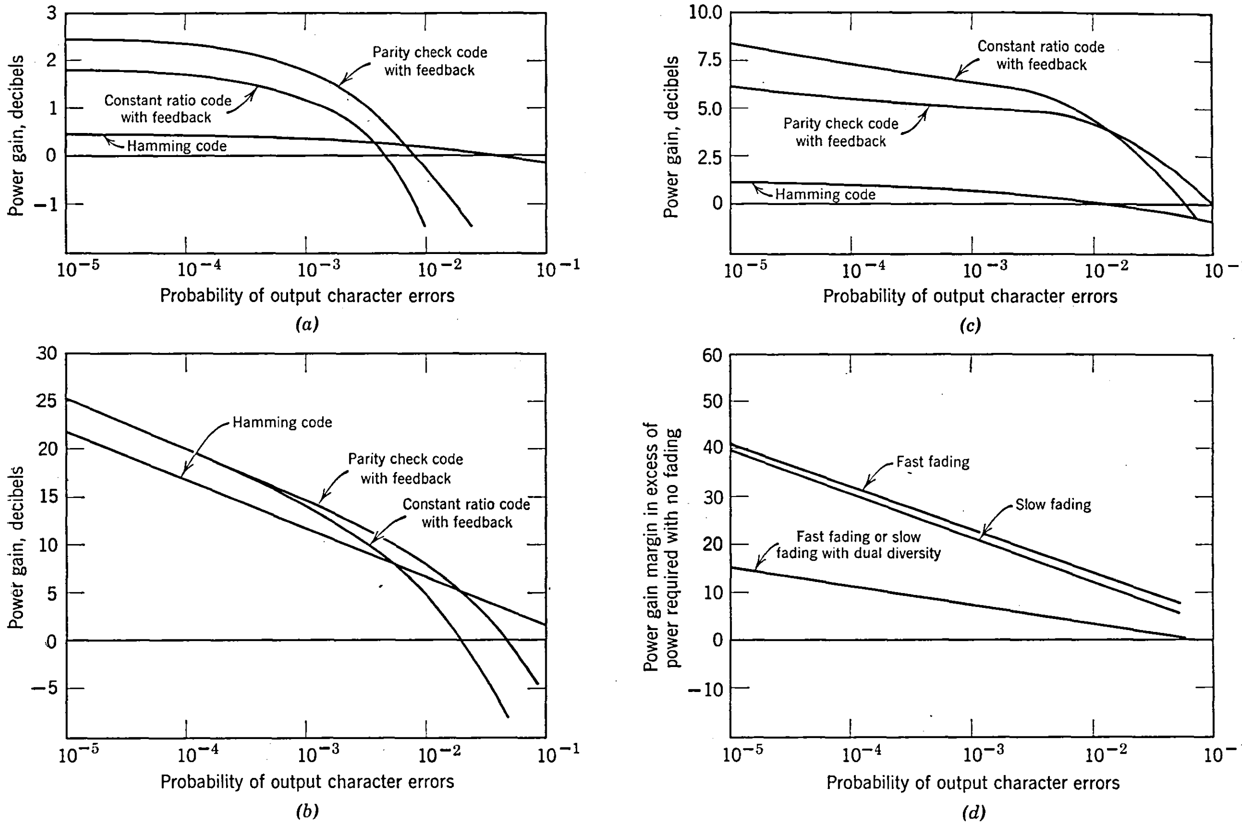


FIG. 8. Power efficiency of redundant codes and decision feedback transmission systems: (*a*) nonfading signal; (*b*) fast-fading signal; (*c*) slow-fading signal; (*d*) standard Teletype system.

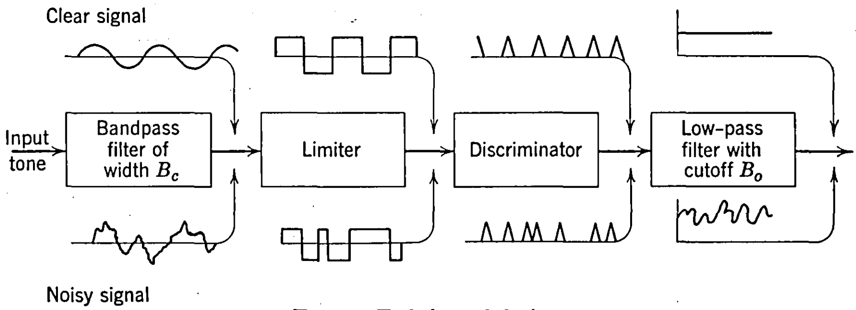


FIG. 9. FM demodulation.

bandpass filter is used to separate the desired channel from adjacent channels in a frequency-multiplexed carrier. In radio reception, the bandpass filter is the IF amplifier which provides the selectivity for separating adjacent channels. The signal from the output of the bandpass filter is clipped or squared off to remove amplitude modulation, and the resulting signal is sent to the discriminator which performs the function of FM-to-AM conversion. The ideal discriminator provides an instantaneous output voltage which is proportional to the deviation of the instantaneous frequency from the carrier or reference frequency. The output of the ideal discriminator is then a perfect reproduction of the signal modulating the transmitter or FM oscillator.

FM Discrimination Based on Zero Axis Crossing. In the system of Fig. 9 the discriminator sends out equal strength impulses for every crossing of the zero axis. The low-pass output filter smooths these impulses into a relatively slowly varying voltage, the reconstructed signal. For this example, frequency is measured as the *average rate of zero axis crossings,* and the averaging is accomplished by the low-pass filter having a time constant $T_0 = 1/2\pi B_0$. If the axis crossings of the limiter output were counted by a counter for a period of time T_c, the output of the counter could be the average rate of zero axis crossings during the interval T_c. Note that these two averages are not the same; the counter perfectly retains the information for the period T_c and discards it entirely and instantaneously before starting a new count. The low-pass filter, on the other hand, provides a weighted average. Physically this means that at any time t the output of the filter is equal to the sum of the contributions from the input at all past times. Each input pulse appears in the output multiplied by the weighting function $Ke^{-t/T}$ where t is the time interval between the present time t and the time of occurrence of the particular input pulse; T is the time constant of the filter. $Ke^{-t/T}$ thus specifies the weight with which any input at any past moment contributes to the present output.

Discriminator with Dual Resonant Circuits. Another common form of discriminator, Fig. 10, measures frequency by impressing the FM signal on two resonant circuits which are tuned to different frequencies. The voltage developed across the two tuned circuits is rectified and the difference of the rectified voltages is a measure of the frequency (Refs. 24 and 25).

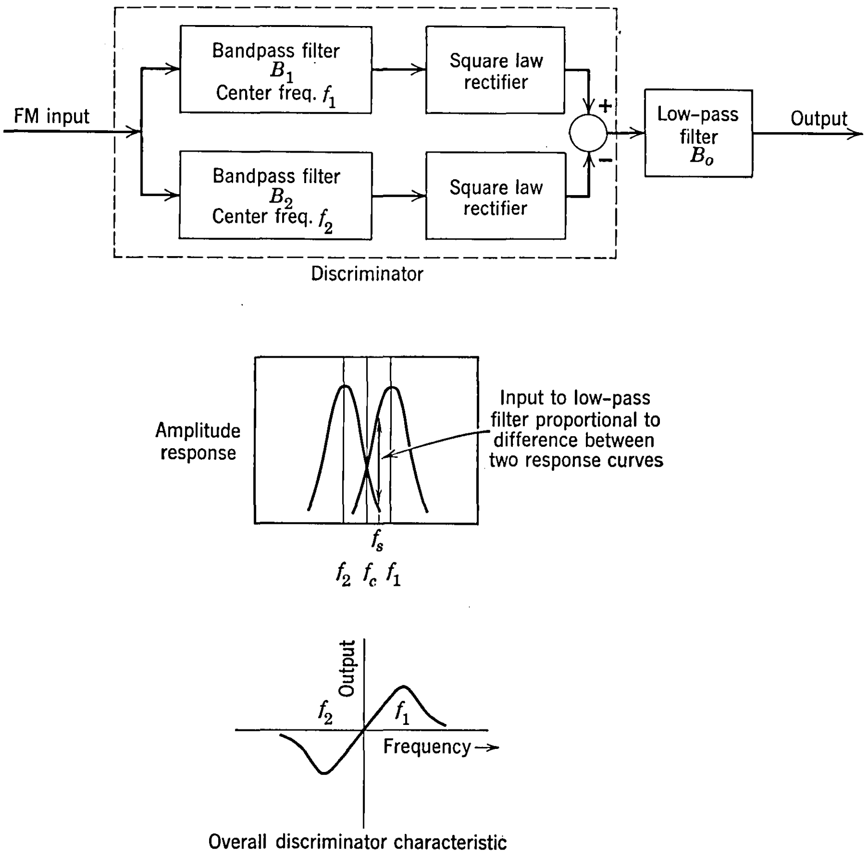


FIG. 10. Conventional FM discriminator.

FM—Threshold Phenomena. In the example of Fig. 9, when the carrier amplitude is of the same order of magnitude as the noise voltage in the circuit, some of the axis crossings do not occur and, in addition, new axis crossings appear which are independent of the signal. The discriminator output now contains appreciable noise components, some of which will appear in the output of the low-pass filter. It is intuitively obvious that the transition from normal operation to noisy operation occurs when the carrier S/N is small and limiting action fails. This is the familiar threshold phenomena of FM systems and for the type of demodulators described above is a function of both S/N and the limiter characteristics.

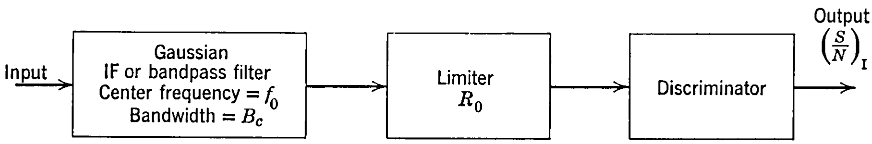
Middleton (Refs. 26, 27, 28, and 29) has developed the relations for signal-to-noise ratios at the output of an FM system as a function of the input signal-to-noise ratio, clipping level, and bandpass and audio filter characteristics. The problem is solved on a general basis for both narrow band and wide band FM where:

Narrow band FM: Maximum carrier frequency deviation is of same order of magnitude as highest modulating frequency.

Wide band FM: Maximum carrier frequency deviation is considerably greater than highest modulating frequency.

Narrow Band FM

For the narrow band FM case (Fig. 11) consider the system consisting of a band pass or IF filter having a half-power bandwidth $B_{1/2}$, a limiter, and a discriminator. For carrier-to-noise power ratios in excess of 10,



$$\left(\frac{S}{N}\right)_{\text{input}} = \alpha_0 = \frac{A_0}{\sqrt{2b_0}}$$

A_0 = carrier peak amplitude
b_0 = mean noise power in bandwidth = B_c

Limiter

Wide band frequency response—no distortion of clipped wave
R_0 = saturation level

$$r_0 = \frac{R_0}{\sqrt{2b_0}}$$

Discriminator

Linear response vs. frequency for all frequencies appearing at output of limiter

FIG. 11. Narrow-band FM demodulation system.

Middleton derives the expressions for output $(S/N)_I$ for the two extremes of no limiting and heavy limiting.

No Limiting. Narrow Band FM.

$$(S/N)_I \cong 2a_0^2\Gamma^2,$$

where $a_0^2 > 10$,

$\Gamma^2 \leq \frac{1}{2}$,

$$a_0 = \frac{A_0}{\sqrt{2b_0}},$$

A_0 = peak carrier amplitude,

b_0 = mean noise power.

Heavy Limiting. Narrow Band FM.

$$\left(\frac{S}{N}\right)_{\mathrm{I}} \cong \frac{2a_0{}^2\Gamma^2}{1 + \Gamma^2}.$$

The quantity a_0 is the ratio of rms carrier to rms noise and Γ is a ratio of the rms modulation to a bandwidth factor that is a function of the spectral distribution of the noise. The definition of $(S/N)_{\mathrm{I}}$ is

$$\left(\frac{S}{N}\right)_{\mathrm{I}} = \frac{P_s - P_{dc}}{N_c},$$

where P_s = measure of signal power associated with the modulation,
P_{dc} = d-c power component appearing in the output,
N_c = mean power associated with noise alone and including modulation cross product components generated by the nonlinear discriminator-limiter operating on signal and noise but excluding dc.

The noise at the output of the IF or bandpass filter has a mean noise power b_0 and a spectral distribution $W(f)$. A bandwidth factor ρ is used to characterize the noise:

$$\rho^2 = \frac{b_2}{b_0} = \frac{\text{Mean squared value of noise power spectrum about } \omega_0}{\text{Mean noise power}},$$

where $b_n = \int_0^\infty (\omega - \omega_0)^n W(f)\, df$,
$W(f)$ = spectral noise power distribution at output of bandpass or IF filter.

The b's are called the moments of the distribution about the frequency ω_0. The zeroth moment b_0 is equal to the mean noise power of the noise signal described by $W(f)$, and b_1 equals the mean value of noise power spectrum about the frequency ω_0. If the distribution is symmetrical about ω_0, the mean b_1 will be zero. The second moment b_2 is equivalent to the radius of gyration of the noise power spectrum about the frequency ω_0. If the distribution is symmetrical about ω_0, the mean is zero and b_2 is simply the variance or square of the standard deviation of the noise power spectrum referred to the frequency ω_0. This follows since the variance σ^2 is defined as the mean-squared value about the mean (Ref. 59):

$$\sigma^2 = b_2 - (b_1)^2 = \text{Variance of power spectrum about mean.}$$

The quantity Γ in the expressions for the signal-to-noise ratio is the ratio of the rms modulation to the noise factor ρ defined above:

$$\Gamma = \frac{\text{Rms modulation}}{\rho}.$$

Thus Γ is a ratio of the rms modulation to a bandwidth factor that is a function of the spectral distribution of the noise.

Although the above expressions apply only for a_0 greater than 10, the original work (Ref. 26) covers the case of $a_0 \leqq 1$ where the noise power equals or exceeds the carrier power. Figure 12 is reproduced from the original work and indicates the signal-to-noise ratio for narrow band FM when the modulation is sinusoidal and the system is as indicated in Fig. 11.

For the example given in the figures, the signal is given by Refs. 6 and 26:

$$V(t) = A_0 \cos (\omega_c t + \psi) + V_n(t),$$
$$= [V_c + A_0 \cos (\omega_d t + \psi)] \cos \omega_0 t - [V_s + A_0(\omega_d t + \psi)] \sin \omega_c t,$$

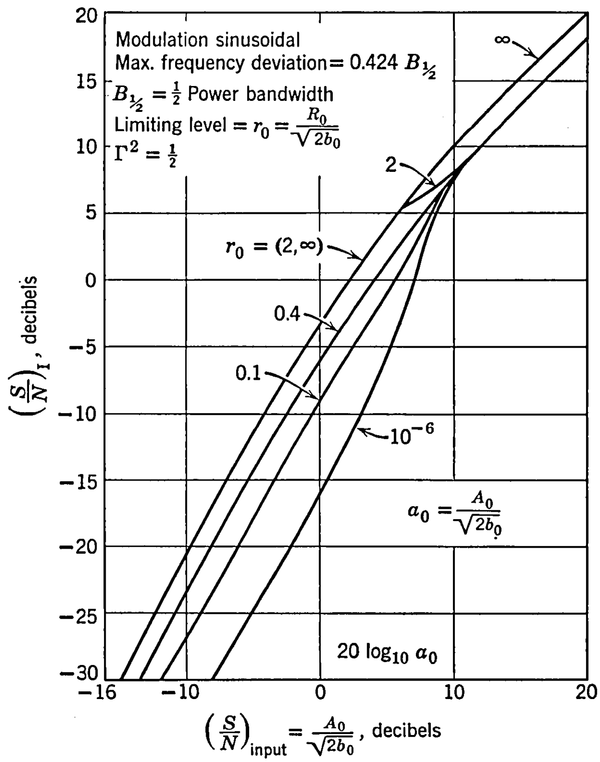


FIG. 12. Narrow-band FM output signal-to-noise ratio.

where A_0 = peak carrier amplitude,

$\frac{\omega_d}{2\pi}$ = detuning in cycles per second $= f_0 - f_c$,

f_0 = center frequency of bandpass filter,

f_c = carrier frequency,

$\psi(t) = \int_0^t D_0(t)\,dt$ = time integral of modulation,

$D_0(t)$ = angular modulation,

$V_n(t)$ is a Gaussian noise source limited in bandwidth to the IF or bandpass filter bandwidth.

The total modulation ϕ is the sum of the detuning effect ω_d and the signal modulation:

$$\phi = \omega_d t + \psi,$$

$$\dot{\phi} = \omega_d + \dot{\psi} = \omega_d + D_0(t).$$

The quantity Γ is defined in terms of $\dot{\phi}$:

$$\Gamma = \sqrt{\frac{[\dot{\phi}(t)^2]_{\mathrm{av}}}{\rho^2}},$$

where $[\dot{\phi}(t)^2]_{\mathrm{av}}$ represents the average mean square value of the total modulation.

For the specific case illustrated in Fig. 12, the modulation is sinusoidal and it is assumed that $\omega_d = 0$. The frequency modulation consists of a single sine wave of maximum amplitude D_0:

$$\dot{\psi} = D_0 \cos \omega_s t.$$

Further, a Gaussian filter of bandwidth B_g (in cycles per second) is assumed for the bandpass filter. For a Gaussian filter $W(f)$ the power spectrum is

$$W(f) = \omega_0 \exp\,(-f^2/B_g^2).$$

The half-power bandwidth $B_{1/2}$ for a Gaussian filter is related to the B_g by

$$B_{1/2} = 2\,(\ln 2)^{1/2} B_g.$$

For the Gaussian filter

$$\rho = \frac{2\pi B_g}{\sqrt{2}} = \frac{\pi(B_{1/2})}{\sqrt{(\ln 4)}}.$$

Hence, substituting an expression for

$$\Gamma = \frac{D_0/\sqrt{2}}{\rho},$$

$$D_0 = \rho\Gamma\sqrt{2} = \frac{\pi(B_{1/2})}{\sqrt{\ln 4}}\sqrt{2}\,\Gamma.$$

The curves in Fig. 12 are drawn for $\Gamma = \frac{1}{\sqrt{2}}$. Thus

$$D_0 = \frac{\pi(B_{1/2})}{\ln (4)}.$$

Note that D_0 is the maximum angular deviation. For the case of sine wave modulation, the maximum frequency deviation is $D_0/2\pi$. Thus

$$\text{Maximum frequency deviation} = \frac{D_0}{2\pi} = 0.424B_{1/2}.$$

Wide Band FM

For wide band FM, the system of Fig. 11 includes, in addition, a low-pass output filter having a bandwidth B_o. Two new variables are introduced to characterize the system; these are

$$\Omega_A = B_o/B_s,$$

$$\mu = D_o/\omega_s.$$

where ω_s = frequency of sine wave modulation ($\omega_s = 2\pi f_s$),
μ = deviation rate.

Note that the definition of μ is equivalent for sine wave modulation to the frequency deviation ratio, since for sine wave modulation the frequency deviation is $D_0/2\pi$:

$$\mu = D_0/2\pi f_s = \text{Frequency deviation}/f_s.$$

For the case when the carrier-to-noise ratio is large and Γ is again less than or equal to one-half the signal-to-noise ratio $(S/N)_{\text{II}}$ is given by Ref. 26:

$$\left(\frac{S}{N}\right)_{\text{II}} = \frac{\pi^{1/2}\,\Gamma^2\,a_0^2}{\sum}$$

where $a_0^2 \gg 1$,
$\Gamma^2 \leqq \frac{1}{2}$.

The modulation is sine wave and the limiting is heavy ($R_0 \ll A_0$),

R_0 = limiter saturation level,

A_0 = peak carrier amplitude,

$$a_0 = \frac{A_0}{\sqrt{2b_0}},$$

$$\Sigma = \sum_{n=0}^{\infty} a_n J_n(\mu)^2 \left(\frac{n-1}{2} \Omega_A \exp\left[-(n+1)^2 \Omega_A{}^2\right] \right.$$
$$- \frac{n+1}{2} \Omega_A \exp\left[-(n-1)^2 \Omega_A{}^2\right]$$
$$\left. + \frac{\pi^{1/2}}{2} (n^2 \Omega_A{}^2 + \tfrac{1}{2}) \{\Theta[(n+1)\Omega_A] - \Theta[(n-1)\Omega_A]\} \right),$$

and

$$a_n = \begin{cases} a_0 = 1, \\ a_{n \geq 1} = 2, \end{cases}$$

$$J = \text{Bessel functions,}$$

$$\Theta = \text{error function,}$$

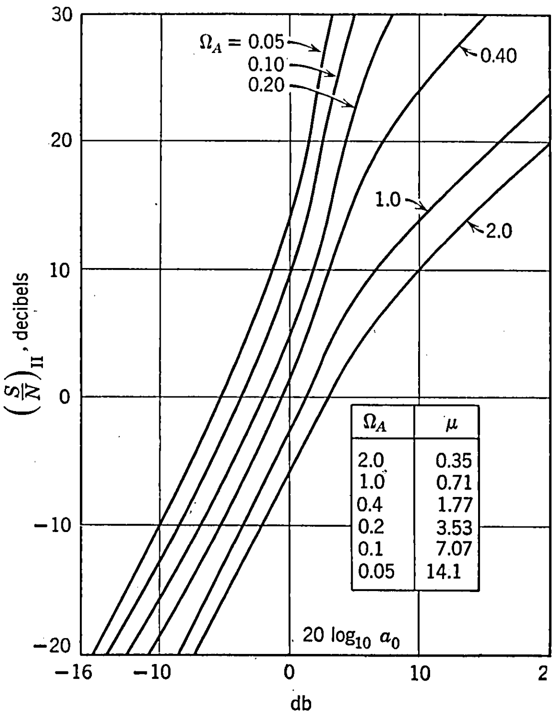
$$\Omega_A = \frac{B_0}{B_{1/2}} [2(\ln 2)^{1/2}]$$

With the exception of Ω_A, μ, and $(S/N)_{II}$, all the quantities are defined as in the narrow band case. The signal-to-noise ratio, $(S/N)_{II}$ in the case of wide band FM, is referred to the output of the low-pass output filter. Figure 13 illustrates the variation of output signal-to-noise ratio $(S/N)_{II}$ as a function of carrier input level. It is assumed that the IF bandwidth remains constant, that the modulation is sinusoidal, that $\Gamma^2 = 1/2$, and that the deviation ratio μ is varied so that the frequency deviation remains constant.

The curves of Fig. 13 indicate the FM threshold effect which occurs at about a 10-db input signal-to-noise power ratio; below this point the output signal-to-noise falls at a rate proportional to the square of the deviation ratio. At some value of the input signal-to-noise less than 0 db, the individual curves become tangent to lines having a slope of 2 [output S/N varies as input $(S/N)^2$].

From the viewpoint of signal-to-noise degradation, a comparison of Figs. 12 and 13 indicates that (*a*) narrow band FM is inferior to broad band FM for all input carrier levels; (*b*) limiting is detrimental in narrow band FM; (*c*) broad band FM with heavy limiting provides the best performance (Ref. 26).

These remarks apply for ideal discriminators and wide band limiting action. Departure from these conditions will typically increase the interference effects of noise versus signal, since filter characteristics that are narrow compared to the spectrum of the amplitude-limited noise at output of limiter will tend to restore randomness to the clipped wave and destroy the limiter's action.



$$\alpha_0 = \frac{A_0}{\sqrt{2b_0}} = \frac{\text{Rms carrier}}{\text{Rms noise}}$$

A_0 = peak carrier amplitude

b_0 = mean noise power

$\left(\frac{S}{N}\right)_{\text{II}}$ = signal-to-noise power ratio at output of low-pass filter

$\Gamma^2 = \frac{1}{2}$ (freq. deviation $= 0.424\ B\frac{1}{2}$)

$B_{1/2}$ = half-power bandwidth of IF or bandpass filter

$$\Omega_A = \frac{(B_o)_2}{B_{1/2}}(\ln 2)^{1/2} = \frac{B_o}{B_g}$$

B_o = low-pass output filter bandwidth

B_g = Gaussian IF or bandpass filter bandwidth

$$\mu = \frac{\text{Frequency deviation}}{\text{Modulating frequency}} = \frac{\text{Frequency deviation}}{f_s}$$

f_s = sine wave modulating frequency

FIG. 13. Wide-band FM output S/N vs. input S/N for sine wave modulation.

Since the superiority of wide band FM depends heavily on limiting, care should be exercised to provide limiter and discriminator characteristics that are wide compared to the IF bandwidth. The IF or bandpass filter characteristics should have negligible delay or amplitude distortion over the range of signals. If this condition is not realized, the bandpass filter will act like a nonlinear discriminator, causing serious mixing of signal and noise. For a discussion of errors introduced by the frequency characteristics of discriminators, see Baghday (Ref. 38).

Threshold Effects for Systems Exchanging Power for Bandwidth. The threshold effect observed for wide band FM is typical for transmission systems that exchange power for bandwidth. For this general type of system, the output signal-to-noise ratio will vary with the input signal-to-noise ratio as indicated in Fig. 14. The S/N characteristic for these systems has three distinct regions:

Region 1: Output S/N varies directly as input S/N.
Region 2: Rapid loss of improvement gained by exchanging bandwidth for power.
Region 3: Output S/N varies as the square of input S/N.

The square law characteristic of Region 3 is not peculiar to FM systems but is also found for AM and other forms of modulation. The boundary between Regions 1 and 2 is the threshold point of operation. For broad band FM systems, the threshold occurs at an input S/N of approximately 10 db.

Phase Lock Discriminator–Synchronous FM Detection

The threshold value of 10 db is not unique to broad band FM and does not represent an inherent lower bound for acceptable performance. Rather, it is the result of the methods used to demodulate the FM wave. For purposes of illustration, assume that a bandpass filter has a bandwidth of 1000 cps and that data or message bandwidth is 10 cps. For the normal discriminator, the threshold occurs when the signal-to-noise ratio of the carrier is 10 db. The limitation is the noise power of the carrier bandwidth and not the message bandwidth. It is obvious that a substantial improvement could be realized if the threshold were limited by the noise power in the message bandwidth rather than in the carrier bandwidth. The *phase lock* or *correlation* discriminator approaches this end result (Ref. 30).

Figure 15 illustrates the basic principle of the phase lock discriminator. In this discriminator the output of the bandpass filter is applied to a phase

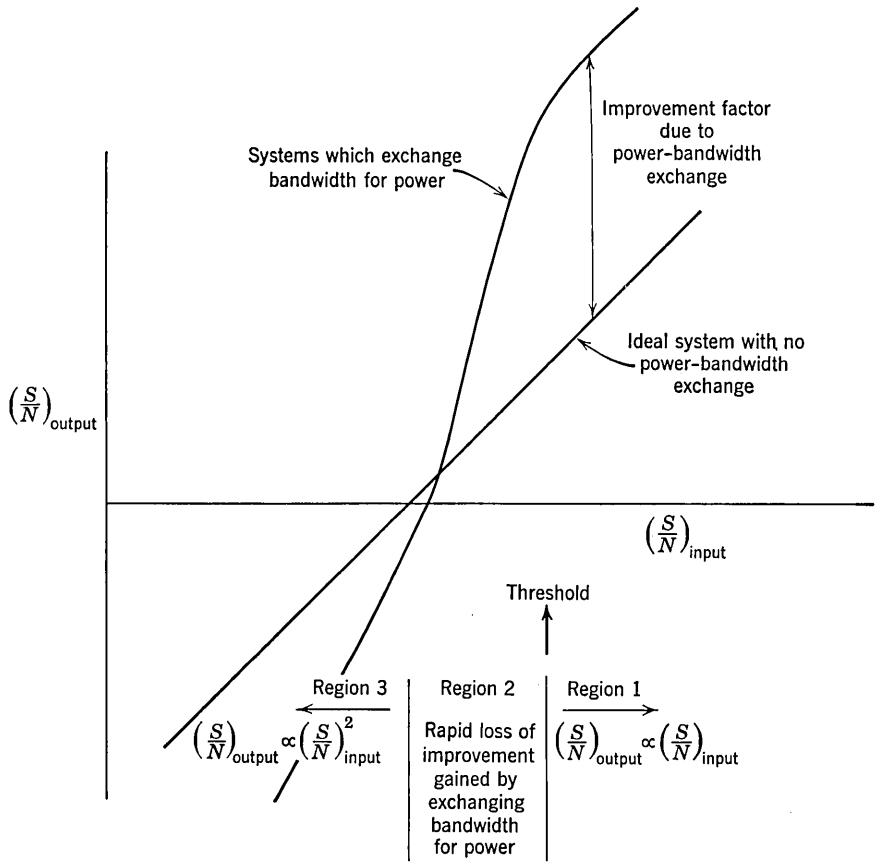


FIG. 14. Signal-to-noise variations in vicinity of threshold region.

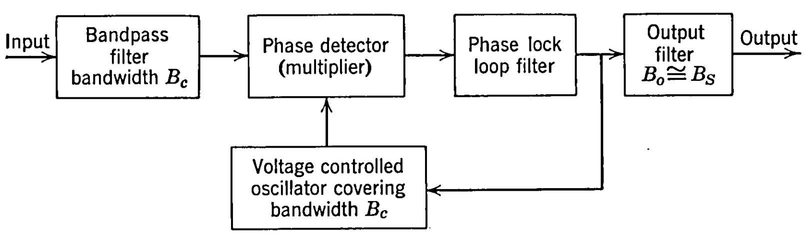


FIG. 15. Phase lock discriminator.

detector where it is literally multiplied by the signal from the voltage-controlled oscillator (VCO) whose output is a linear function of its input. The output of the phase detector is filtered by a loop filter, the output of which is applied to the VCO to close the loop. The effect of the feedback loop is to lock the oscillator at the frequency of the incoming signal and 90° out of phase with it. The design of the loop filter is critical and is optimized to minimize the phase error between the signal and VCO output over the range of specified input signals. The response of the phase lock servo loop is typically limited to the information or message bandwidth B_o (where $B_o \cong B_s$). The voltage input to the VCO is a measure of the frequency of the input signal.

The threshold for this system occurs when the servo loop loses synchronization. This normally occurs when the rms phase jitter between the input signal and the VCO signal has a magnitude of approximately 60°. For a carefully designed servo loop, a rms phase error of 60° will occur when

$$\left(\frac{\text{Rms carrier}}{\text{Rms noise}}\right)^2_{\text{threshold phase lock}} = \frac{A_0^2}{2b_0} = \frac{3B_p}{4\sqrt{2}\,(B_{1/2})}$$

where B_p = the largest of the quantities, $\begin{cases}\sqrt{12\dot{f}_{\max}} \\ 4\sqrt{f},\end{cases}$

f = frequency of signal being tracked by phase lock loop,

$\dot{f}_{\max}$ = maximum rate of change of frequency f in cycles per second^2.

For sine wave modulation, the frequency modulation consists of a single sine wave of amplitude D_0:

$$D_0(t) = D_0 \cos \omega_s t \text{ radians/sec}$$

or

$$= (D_0/2\pi)(\cos \omega_s t) \text{ cycles/sec}^2.$$

The frequency deviation is $D_0/2\pi$ cps. The maximum rate of change of frequency

$$\dot{f} = \frac{d}{dt}\left(\frac{D_0(t)}{2\pi}\right)_{\max} = \frac{\omega_s D_0}{2\pi} \text{ cycles/sec}^2.$$

Hence

$$B_p = \sqrt{\frac{12\omega_s D_0}{2\pi}}.$$

Assuming $\Gamma^2 = \frac{1}{2}$, as in the case of the linear discriminator, the one-half power bandwidth of the IF or bandpass filter is related to $D_0/2\pi$, the maximum frequency deviation, by

$$D_0/2\pi = 0.424 B_{\frac{1}{2}}.$$

By substituting the expressions for $B_{\frac{1}{2}}$ and B_p in the threshold equation

$$\left(\frac{\text{Rms carrier}}{\text{Rms noise}}\right)^2_{\text{threshold phase lock}} \begin{array}{l} \cong 2\sqrt{\omega_s/D_0} \\ \cong 2\sqrt{\dfrac{\text{Modulation frequency}}{\text{Frequency deviation}}}. \end{array}$$

Thus, for sine wave modulation, the threshold signal-to-noise power ratio varies inversely as the square root of the frequency deviation.

For a frequency deviation of 100:1, the threshold will occur at -14 db for the phase lock loop as compared to $+10$ db for the linear discriminator. The phase lock discriminator thus provides a means to extend the exchange of bandwidth for power and permit a greater power reduction factor or, conversely, for a given dynamic range, the phase lock discriminator will permit operation at thresholds 10 to 30 db below that possible for conventional discriminators.

The rms phase jitter σ_n for the phase lock loop (Ref. 30) in radians per second is

$$\sigma_n = \sqrt{\frac{b_0 B_p}{2\sqrt{2}\, B_{\frac{1}{2}}}\left(2\frac{A_0}{A_{0t}} + 1\right)},$$

where $A_{0t} = \sqrt{\dfrac{3b_0 B_p}{2\sqrt{2}\, B_{\frac{1}{2}}}}$ = peak carrier level at threshold,

b_0 = mean noise power in bandwidth $B_{\frac{1}{2}}$,

σ_n = rms phase jitter in radians.

For sine wave modulation, the maximum angular deviation in radians is

$$\psi = \int D_0 \cos \omega_s t = (D_0/\omega_a) \sin \omega_s t,$$

$$\psi_{\max} = D_0/\omega_s \text{ radians maximum},$$

$$= D_0/(\sqrt{2}\,\omega_s) \text{ radians rms}.$$

The effective output signal-to-noise ratio will be

$$\left(\frac{S}{N}\right)_{\text{eff}} = \left(\frac{D_0/\sqrt{2}\,\omega_s}{\sigma_n}\right)^2 \quad \text{phase lock loop at threshold } \sigma_n = 1\ (60°).$$

Hence:

$$\left(\frac{S}{N}\right)_{\text{eff threshold}} = \left(\frac{D_0}{\sqrt{2}\,\omega_s}\right)^2 = \tfrac{1}{2}\ (\text{frequency deviation ratio})^2.$$

FM Transmission System Transient Errors

The analysis of the transmission of variable frequency signals through filters and other circuit elements is difficult. In general, the principle of superposition is not valid and the rise time, overshoot, and nature of the transient response depend upon the size of transient and the position of the signal with respect to the center frequency of the filter or element (Refs. 31, 32, 33, and 34).

Two general methods are available for approaching this problem, the *spectral approach* and the *dynamic approach*. In the spectral approach, transmission of the signal is analyzed in terms of its individual components. The usefulness of this method is limited by the complexity of the computations. Papers by Carson and Fry (Refs. 35, 36, 37, and 40) appear to be the earliest publications discussing the dynamic response of a linear system with variable frequency signals.

The basic viewpoint in the dynamic method is that the dynamic responses can be broken up into two components—the quasi-stationary component and the distortion component. The quasi-stationary component represents the part of the response that is obtained from conventional sinusoidal steady-state circuit theory by *substituting the variable instantaneous frequency for the assumed constant frequency.*

In practical systems, the *sluggishness* of a filter or element will set a limit on the speed with which the output can faithfully follow the input frequency excursions. Thus the instantaneous output frequency will deviate from the assumed steady-state or quasi-stationary value by an amount equal to the distortion or error component.

Baghday (Ref. 38) has extended the work of Carson and Fry to arrive at an explicit statement for the maximum magnitude of the transient error as a function of the rate of change of input frequency and the characteristics of the filter or system.

Consider a signal or excitation having an instantaneous frequency ω_s:

$$\omega_s(t) = \omega_c + d\psi/dt,$$

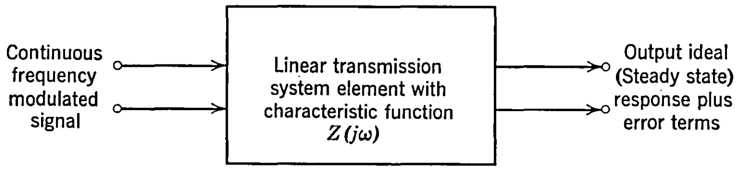
$$\omega_c = \text{Carrier frequency},$$

$$d\psi/dt = D_0(t) = \text{Angular frequency modulation},$$

$$\psi = \int^t D_0(t)\,dt = \text{Phase deviation},$$

$$D_0(t)/2\pi = \text{Frequency modulation in cycles per second}.$$

The system or element is assumed to have a linear characteristic and the input signal is assumed to have a finite spectrum or the error in assum-



$$Z(j\omega)\begin{cases}\text{Linear with no poles on } j\omega \\ \text{axis or in the right half-plane}\end{cases}$$

FIG. 16. Transmission system errors.

ing a finite spectrum is negligible (Fig. 16). The output dynamic response of the system should closely follow the steady-state response as predicted by a-c steady-state circuit theory.

The maximum fractional instantaneous error, the difference between the ideal or quasi-static response and the actual response, will be equal to or less than

$$\text{Error}_{\max} \leqq \tfrac{1}{2}(D'_0(t))_{\max}\cdot\left(\frac{Z''(j\omega)}{Z(j\omega)}\right)_{\max} \ll 1,$$

where $D'_0(t) = \dfrac{dD_0(t)}{dt}$,

$D_0(t)$ = angular modulation,

$\dfrac{D_0(t)}{2\pi}$ = frequency modulation in cycles per second,

$Z''(j\omega) = \dfrac{d^2Z(j\omega)}{d(j\omega)^2}$,

$$\text{Error}_{\max} = \left(\frac{\text{Actual response} - \text{Ideal response}}{\text{Ideal response}}\right)_{\max}.$$

The above bound for the error is the most restrictive and applies for all types of modulation of the FM carrier. If the modulation, variation of frequency, is sinusoidal, a less restrictive bound for the maximum error can be used:

$$\text{Error}_{\text{max periodic}} \leqq \tfrac{1}{2}\left(|D'_0(t)|\cdot\left|\frac{Z''(j\omega)}{Z(j\omega)}\right|\right)_{\text{max}} \ll 1.$$

The restrictive form of the error limits in which the individual maximums are multiplied specifies the magnitude of the error that will not be exceeded, regardless of which part of the filter or element response characteristic is being swept by the variable instantaneous frequency.

It is thus applicable to the general problem of spectrum analysis in which the element is scanned with a variable frequency source. As a simple example assume

$$\psi = \frac{D_0}{\omega_s}\sin(\omega_s t + \phi_m),$$

$$\frac{d\psi}{dt} = D_0(t) = D_0 \cos \omega_s t,$$

$$\frac{D_0}{\omega_s} = \text{Frequency deviation ratio},$$

$$D'_0(t) = \frac{D_0}{\omega_s}(\omega_s)^2 \sin \omega_s t$$

$$= D_0\omega_s \sin \omega_s t,$$

$$D_0 = \text{Maximum angular frequency deviation.}$$

Thus

$$(D'_0)_{\text{max}} = D_0\omega_s.$$

If the filter consists of a single-tuned parallel high-Q circuit, then

$$Z(j\omega) = \frac{V_o(j\omega)}{V_i(j\omega)} = \frac{1}{1 + j(\omega - \omega_0)/\pi B},$$

ω_0 = Center frequency,

πB = One-half the overall bandwidth in radians per second between half-power points,

$$= \frac{2\pi B}{2} = \frac{2\pi B_{1/2}}{2}.$$

The maximum value of $Z''(j\omega)/Z(j\omega)$ occurs at $\omega - \omega_0 = 0$ and has a magnitude of $1/(\pi B)^2$. Hence

$$\text{Error}_{\max} \leqq \frac{\omega_s}{\pi B} \cdot \frac{D_0}{\pi B} \ll 1,$$

$$\leqq \frac{f_s}{B_{1/2}/2} \cdot \frac{f_{\text{deviation}}}{B_{1/2}/2},$$

where $f_{\text{deviation}}$ is the frequency deviation in cycles per second.

This condition states that the product of the modulation frequency and the maximum frequency deviation, when each is measured in units of one-half the filter bandwidth, is equal to the maximum transient error provided this error is small ($\leqq 10\%$). The same result has been obtained by Clavier (Ref. 34).

The appearance of the $Z''(j\omega)$ term in the expression for the errors emphasizes the importance of considering the phase characteristics of filters in addition to the amplitude characteristics. The specification, of a maximum error, is in effect a restriction on the variation of the time delay characteristics of the filter over the swept frequency range.

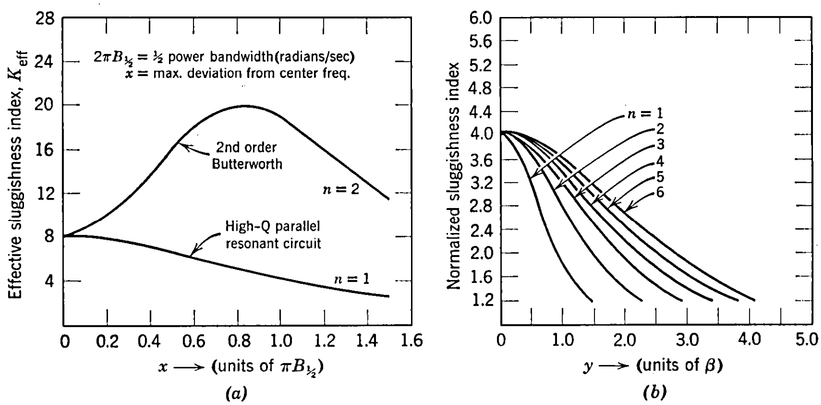
Filter Characteristics. For common types of filters, the quantity $Z''(j\omega)/Z(j\omega)$ is always expressible in the form

$$\left| \frac{Z''(j\omega)}{Z(j\omega)} \right|_{\max} = \frac{K_{\max}}{(2\pi B)^2},$$

where $K_{\max}$ = maximum sluggishness index,
$2\pi B$ = bandwidth in radians per second between half-power points.

The *sluggishness index* K is a single parameter which relates the time delay characteristics of filters of widely varying form. For a given bandwidth, it is directly proportional to the quantity $Z''(j\omega)/Z(j\omega)$. $K_{\max}$ refers to the maximum value of the quantity Z''/Z. In practice it is often convenient to operate over a fraction of the filter bandwidth. In this instance the effective sluggishness index K_{eff} should be used, where K_{eff} is the maximum value of K over the range of swept frequencies.

Baghday (Ref. 38) has plotted the characteristics of conventional filters as a function of the deviation from center frequency (Figs. 17 and 18). The results are normalized, and for the high-Q parallel resonant circuit and the nth-order Butterworth filters the ordinate is the effective sluggishness index K_{eff}. With the exception of the high-Q parallel resonant circuit, the effective sluggishness index is the quantity that applies for a particular filter providing the signal frequency does not sweep beyond the frequency at which K_{eff} is determined. By using only a fraction (0.4 to 0.7) of the half-power bandwidth, the advantage of the high attenuation character-



Normalized sluggishness index $= \dfrac{K_{eff}}{4[n(n+1)(2^{1/n}-1)]}$ each stage a high-Q single-tuned amplifier

$$Z(j\omega) = \frac{1}{\left(1 + \dfrac{f - f_0}{B_1/2}\right)^n}$$

$$\beta = \frac{2\pi B_n}{2} \text{ radians/sec}$$

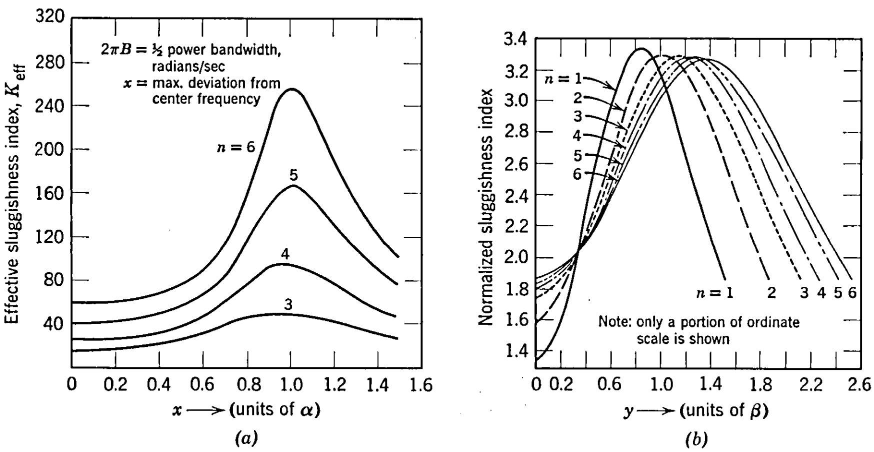
B_1 = bandwidth of a single section, cycles per second

B_n = overall bandwidth, cycles per second

y = max. deviation from center frequency ω_0

FIG. 17. n identical cascaded single-tuned amplifiers: (a) sluggishness index as a function of filter bandwidth utilization, (b) normalized sluggishness index as a function of overall bandwidth utilization.

istic of a 6th-order Butterworth can be realized with a sluggishness index of a 4th-order filter. In the case of the high-Q parallel resonant circuit, the maximum value of K occurs at the center frequency, and this value should be used. Figure 18 presents the effective sluggishness index for n identical cascaded amplifiers. Note that β is the effective half-power bandwidth of the n stages. $2\pi B$ is still considered as the half-power bandwidth of a single stage. In order to obtain the effective sluggishness index, it is necessary to multiply the ordinate by the bracketed term which is a function of n, the number of stages.



Normalized sluggishness index $= \dfrac{K_{eff}}{4[2n(2n+1)\sqrt{2^{1/n}-1}]}$ each stage a second order Butterworth

$$Z(j\omega) = \frac{1}{\left[1 - \left(\dfrac{f - f_0}{B_1/2}\right)^2 - j\sqrt{2}\left(\dfrac{f - f_0}{B_1/2}\right)\right]^n}$$

$$\beta = \frac{2\pi B_n}{2} \text{ radians/sec}$$

B_1 = bandwidth of a single section, cylces per second

B_n = overall bandwidth, cycles per second

y = max. deviation from center frequency ω_0

FIG. 18. n identical cascaded Butterworth tuned amplifiers: (a) nth-order Butterworth filter sluggishness index as a function of filter bandwidth utilization, (b) normalized sluggishness index as a function of overall bandwidth utilization.

Table 7 lists the pertinent characteristics of the filters. The Butterworth filter is usually characterized by a pole pattern in which the poles fall on a semicircle whose center lies on the $j\omega$ axis and whose radius equals one-half the overall bandwidth of the filter between half-power points. The exact position of the poles of an nth-order Butterworth filter are at the locations of the $2n$th roots of $(-1)^{n+1}$ that lie in the left half-plane.

TABLE 7. FILTER CHARACTERISTICS

Butterworth Bandpass Filters

$\Omega = f - f_0$ = frequency deviation from center frequency of the filter

$\alpha = B/2 = \frac{1}{2}$ overall bandwidth (in cycles per second) between half-power points

Filter	$Z(j\omega)$
High-Q parallel resonant circuit	$\dfrac{1}{1 + j\left(\dfrac{\Omega}{\alpha}\right)}$
Second order Butterworth (critically coupled, or flat-staggered high-Q pair)	$\dfrac{1}{1 - \left(\dfrac{\Omega}{\alpha}\right)^2 + j\sqrt{2}\left(\dfrac{\Omega}{\alpha}\right)}$
Third order Butterworth	$\dfrac{1}{1 - 2\left(\dfrac{\Omega}{\alpha}\right)^2 + j\left[2\left(\dfrac{\Omega}{\alpha}\right) - \left(\dfrac{\Omega}{\alpha}\right)^3\right]}$

Cascaded tuned amplifiers

n identical stages, α = half-bandwidth of each stage

Single Stage Filter	Overall $Z(j\omega)$	β = half overall bandwidth
Single-tuned, high-Q, parallel resonant circuit	$\dfrac{1}{\left(1 + j\dfrac{\Omega}{\alpha}\right)^n}$	$\alpha\sqrt{2^{1/n} - 1}$
Second order Butterworth	$\dfrac{1}{\left[1 - \left(\dfrac{\Omega}{\alpha}\right)^2 + j\sqrt{2}\left(\dfrac{\Omega}{\alpha}\right)\right]^n}$	$\alpha\sqrt[4]{2^{1/n} - 1}$

Specification of Filter Bandwidths. The dynamic and static responses of transmission filters approach each other as the error becomes small. Instead of expressing the error as a function of the rate of change of frequency and the filter sluggishness, it is convenient to express the bandwidth required for a given error and input signal:

$$2\pi B = K\sqrt{|D''_0(t)|_{\max}}$$

where B = half-power bandwidth in cycles per second,

$K = \sqrt{\dfrac{K_{\text{eff}}}{2\epsilon}}$,

K_{eff} = maximum value of sluggishness index over range of frequencies swept by input signal $D_0(t)$,

ϵ = maximum fractional error.

5. AM DETECTION AND SYSTEM ERRORS

Amplitude modulation describes a class of systems in which the basic principle is the multiplication of the signal by a periodic function of time. The frequency of the periodic function is called the carrier frequency f_c. For each frequency f_s present in the signal, the amplitude modulation or multiplying process gives rise to a pair of frequencies displaced from the carrier frequency by f_s as indicated in Fig. 19.

The effect of the modulation is thus to shift the signal spectrum into two sidebands symmetrically displaced about the carrier frequency. These sidebands contain all the information in the original signal. The amplitude of each sideband component is equal to one-half the corresponding amplitude in the original signal.

If the periodic multiplying function is a sine wave, all the sideband components are centered about the carrier fundamental frequency. For

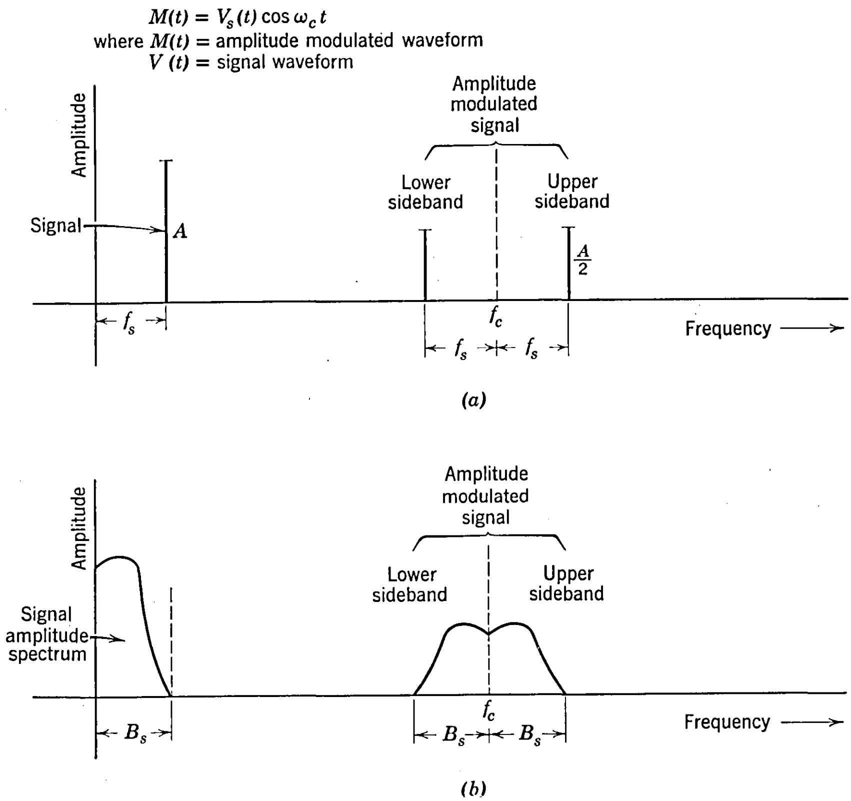


FIG. 19. Frequency spectrum of an amplitude-modulated signal modulated by a sine wave carrier: (a) single frequency signal; (b) signal complex with bandwidth B_s.

the general case of multiplication by an arbitrary periodic function, sideband components are produced at harmonic multiples of the carrier frequency in addition to the pair of sidebands centered on the fundamental. If the complex modulated wave containing harmonic sidebands is passed through a filter to eliminate the harmonic sidebands, the result is equivalent to modulation by sine wave carrier.

Other common forms of periodic multiplying functions are the square wave and the impulse or sampling modulator. Full wave mechanical choppers and solid state switching modulators are examples of modulators using a square wave multiplying function. For this type of modulator, the modulated signal includes the dominant sidebands centered at the carrier frequency f_c and, in addition, sideband pairs centered at all odd harmonics, nf_c (n odd), of the carrier frequency. The amplitude of the sidebands centered on the nth harmonic varies inversely as the order of the harmonic, $1/n$.

Classification of AM Systems. Amplitude modulated systems can be grouped into three classifications depending on whether a carrier is added to the sideband information. These are:

1. DSB-EC-AM: Double sideband–emitted carrier–amplitude modulation.
2. DSB-SC-AM: Double sideband–suppressed carrier–amplitude modulation.
3. SSB-SC-AM: Single sideband–suppressed carrier–amplitude modulation.

In DSB-EC-AM the carrier carries no information except the phase and amplitude of the unmodulated wave. It is added to permit demodulation of the modulated signal by means of nonlinear circuits that do not require explicit use of the carrier signal. Because the addition of the carrier is equivalent to the addition of a constant to the message signal, emitted carrier systems are not useful for the accurate transmission of instrumentation or wide dynamic range data.

When a carrier is not added to the signal, suppressed carried modulation results. Two types of systems are in common use: double sideband, in which both sidebands are transmitted, and single sideband, in which only one sideband is transmitted to conserve bandwidth. For both SSB and DSB systems, the demodulation of the modulated signal requires the synchronization of the demodulator with the carrier.

DSB-EC Demodulation

Detectors or demodulators in common use in emitted carrier amplitude-modulated systems give an output which depends on the amplitude, but not on the phase, of the input carrier. The "square law" and "linear" detector are examples of this nonsynchronous method of demodulation. For the square law detector, the output signal is proportional to the square of the amplitude of the input signal, whereas a linear relationship exists between input and output amplitudes for the "linear detector." The chief advantage of these nonsynchronous detectors is the simplicity of circuits required to effect demodulation. This advantage, however, is offset by the requirement that a steady carrier signal be transmitted with the message signal. In comparison with other types of demodulators, these nonsynchronous methods characteristically have poor noise rejection and increased distortion.

The calculation of the transmission of signal and noise voltage in amplitude-modulated systems is a familiar and simple process for linear elements in the system. In these linear elements, superposition holds and the output of an element supplied with signal and noise is the sum of the outputs that would be obtained if each were applied separately. However, this principle of superposition cannot be applied when the signal and noise pass through a nonlinear device such as a rectifier or limiter.

Rectification of Signal and Noise. Signal Suppression. Understanding the rectification of noise and of a mixture of noise and signal is important in relation to (1) its effect on the signal-to-noise ratio following demodulation, (2) the measurement of a continuous signal in the presence of noise, and (3) the detection of pulsed signal superimposed on noise. A characteristic of nonsynchronous demodulators is the existence of a threshold below which output S/N decreases faster than input S/N. This effect is often referred to as suppression of the carrier or signal by noise and is the result of the noise interacting with the weak carrier signal to form additional noise components.

The interaction of noise and a weak carrier is due to the generation of beat frequency components produced by noise frequencies beating with the carrier. These beat frequencies will have an amplitude proportional to the carrier amplitude A_0. Since these new terms are random with respect to each other, the total noise power in the output will be the sum of the noise plus a term proportional to the square of the carrier amplitude:

$$N = N_0 + kA_0^2 = \text{Total noise power in demodulator output}$$

where N_0 = noise power in output in absence of carrier,
k = constant of proportionality,
N = noise power in output of demodulator,
$kA_0^2 \ll N_0$.

This expression is valid only for very small input signal-to-noise ratios. For large values of signal-to-noise ratio, Middleton has shown that the increase in noise power due to intermodulation of carrier and noise components tends to a limit which is primarily a function of the spectral distribution of the noise (Refs. 41 and 42). The results are given in Fig. 20 for a rectangular or uniform spectral distribution and for an optical spectral distribution. The optical spectrum is a term used to represent a spectral distribution equal to the selectivity curve of a single-tuned circuit. The results apply to systems in which the carrier bandwidth is equal to or greater than three times the output bandwidth.

From Fig. 20 it is evident that the output noise increases by 4 to 7 db when a carrier is present. This characteristic is important in applications in which the detection of weak-pulsed signals is required. Note that in Fig. 20, the ordinate is the ratio of output noise power with carrier to output noise power without carrier. An alternate representation is to plot the ratio of rms output noise to rms input noise for the demodulator. This is given in Fig. 21 which was taken from a paper by Burgess (Ref. 43).

Characteristics of Nonsynchronous Demodulators. For small input signal-to-noise ratios, it can be assumed that the output signal amplitude is proportional to the difference between noise voltages when the carrier is present and those when the carrier is not present:

$$
\begin{aligned}
V_o &= h(\sqrt{N_0 + kA_0^2} - \sqrt{N_0}), \\
&= h[\sqrt{N_0}\,(1 + kA_0^2/2N_0) - \sqrt{N_0}], \\
&= hkA_0^2/(2\sqrt{N_0})
\end{aligned}
$$

where h and k are constants of proportionality. The output signal-to-noise ratio is

$$\frac{V_o^2}{N_0} = \left(\frac{h}{2}\frac{kA_0^2}{N_0}\right)^2,$$

$$(S/N)_{\text{output}} = C(S/N)^2_{\text{input}}.$$

NOTE: N is the output noise power but $N \cong N_0$ since $kA_0^2 \ll N_0$.

Thus, when the signal is small, the output signal-to-noise ratio varies as the square of the input signal-to-noise ratio, and signal suppression by the demodulator exceeds the noise suppression indicated in Fig. 21.

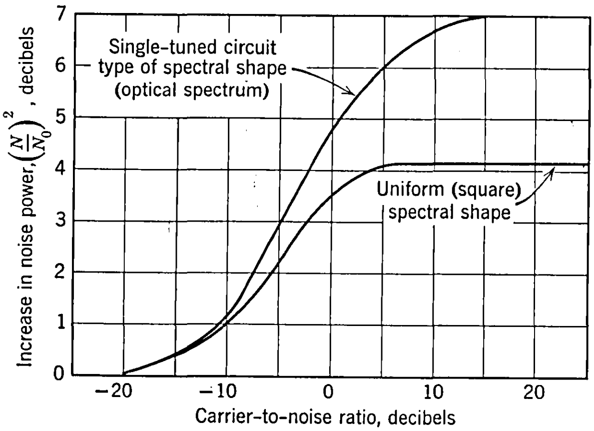


FIG. 20. Increase of noise power at output of linear detector; N = output noise power with carrier, N_0 = output noise power without carrier.

For large signals the noise output remains constant as the input signal-to-noise ratio is varied (Fig. 21). This means that the output S/N must vary linearly with input S/N. Therefore, if input S/N in decibels is plotted versus output S/N in decibels, a curve should be obtained which is asymptotic to a line with unity slope for large input S/N, and to a line with a slope of 2 for small signals. The work of Middleton and associates confirms this characteristic for sine wave modulation and for all types of nonsynchronous demodulators and noise characteristics.

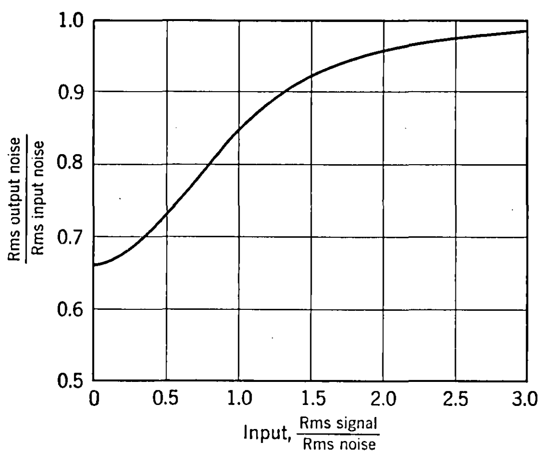
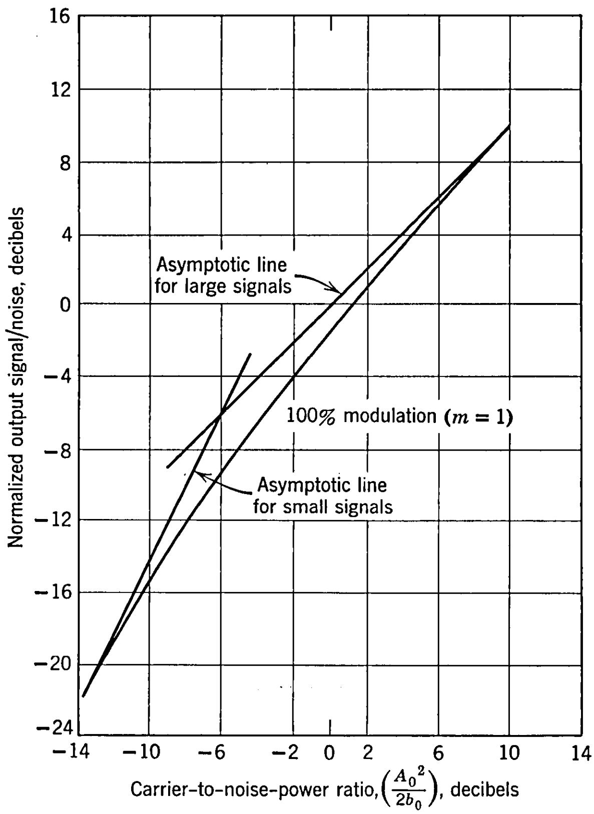


FIG. 21. Linear detector input-output noise characteristic.

Universal Input-Output S/N Curve for Nonsynchronous AM Demodulator. The universal relationship between input and output S/N ratios is given in Fig. 22. The results apply for carrier bandwidth in excess of three times demodulator output filter bandwidth, for sine wave modulation, and for an optical noise spectrum. The curve of Fig. 22 is a



1. To obtain ordinate scale for a given $\dfrac{\text{Carrier bandwidth}}{\text{Output bandwidth}}$ ratio, add $10 \log \dfrac{\text{Carrier bandwidth}}{2 \times \text{Output bandwidth}}$ to ordinate scale.

2. This method is valid only for sinusoidal modulation with carrier bandwidth $\geqq 3$ times output bandwidth.

3. For $m \neq 1.0$ decrease ordinate scale by $20 \log \dfrac{1}{m}$.

m = modulation index
A_0 = carrier peak amplitude
b_0 = rms noise power in carrier bandwidth

FIG. 22. Universal curve for output signal-to-noise ratio for nonsynchronous demodulators.

universal curve that is valid for any carrier bandwidth and any output filter bandwidth as long as the ratio is in excess of three as just indicated.

The curve is normalized to take into account variations in bandwidth ratio and modulation index m. The actual output S/N is obtained by adding a correction term to the normalized value obtained from the curves:

$$\left(\frac{S}{N}\right)_{\text{output}} = \left(\frac{S}{N}\right)_{\text{norm}} + 10 \log \frac{\text{Carrier bandwidth}}{2 \times \text{Output filter bandwidth}}.$$

For example, if the input S/N ratio is -2 db, the carrier bandwidth is 1 kc and the output filter bandwidth 50 cps. Then

$$\left(\frac{S}{N}\right)_{\text{output}} = -4 \text{ db} + 10 \log_{10} \frac{1000}{2 \times 50} = +6 \text{ db}.$$

DSB-SC-AM and SSB-AM Demodulation

Amplitude modulation is descriptive of a general class of systems in which the amplitude of a carrier wave is a function of the message or modulating signal. To gain a physical insight into the difference between the various forms of amplitude-modulated systems, consider a signal waveform $V(t)$ which modulates a carrier having a radian frequency ω_c. Then

$$M(t) = A_c[1 + kV(t)] \cos \omega_c t,$$

where $kV(t) \leq 1$ = modulation index,
$[1 + kV(t)]$ = modulating function,
ω_c = carrier frequency.

The modulated wave $M(t)$ is the product of the modulation function, $[1 + kV(t)]$, and the sinusoidal carrier, $\cos \omega_c t$. Instead of a cosine multiplying function of the carrier frequency, a square wave or sampling or impulse type function of the carrier frequency could be used.

The modulated signal $M(t)$ is the sum of a fixed carrier component, $A_c \cos \omega_c t$, and a varying amplitude component, $A_c kV(t) \cos \omega_c t$. For suppressed carrier systems the unity term in the modulation function is omitted and the modulated wave becomes

$M(t) = A_o kV(t) \cos \omega_c t$, double sideband suppressed carrier modulation.

Since the amplitude of the message-bearing signal is twice as large for the suppressed carrier waveform as compared to the general case in which a steady carrier is added, the signal or message power is four times greater (amplitude doubled). Hence, a 6-db improvement in S/N results for a given peak power limitation.

If $V(t)$ is assumed to be a cosine wave,

$$V(t) = A_s \cos \omega_s t,$$

then $M(t)$ can be written in terms of the sum and difference frequencies:

$$M(t) = A_o \cos C + \tfrac{1}{2}kA_0A_s[\cos (C + V) + \cos (C - V)],$$

where $C = \omega_c t$,
$V = \omega_s t$;

or, letting

$$A = \frac{kA_0A_s}{2} = \text{Upper and lower sideband amplitudes,}$$

$$M(t) = A_0 \cos C + A \cos (C + V) + A \cos (C - V).$$

The general modulated wave consists of a fixed carrier term A_0 and two sideband terms of equal amplitude at the sum and difference frequencies.

For ideal SSB transmission, only one of the two sidebands is transmitted:

$$M(t) = \begin{cases} A_{\text{SSB}} \cos (C + V) \\ \qquad \text{or} \\ A_{\text{SSB}} \cos (C - V). \end{cases}$$

For DSB-SC, both sidebands are transmitted:

$$M(t) = A_{\text{DSB-SC}}[\cos (C + V) + \cos (C - V)].$$

For the general amplitude-modulated case, DSB-EC-AM, both sidebands and the carrier are transmitted. Figure 23 indicates the type of waveforms typical for each of the systems.

NOTE: For both DSB-SC and SSB, the amplitude of the modulated wave does not have a close resemblance to the original modulating signal. In both of these systems it is necessary to utilize both the carrier phase and amplitude in the demodulator to reconstruct the original signal.

If the peak power for both SSB and DSB-SC are the same, then SSB will ideally have a 3-db (factor of 2) power advantage over DSB-SC since the latter requires twice the bandwidth. If SSB and DSB-SC are compared with conventional DSB-EC-AM, then SSB transmission has a 9-db power advantage and DSB-SC has a 6-db power advantage.

Quadrature Component Generation in DSB-SC Systems. To appreciate the practical problems involved in the transmission of AM signals, it is convenient to identify the message signal components in terms of real

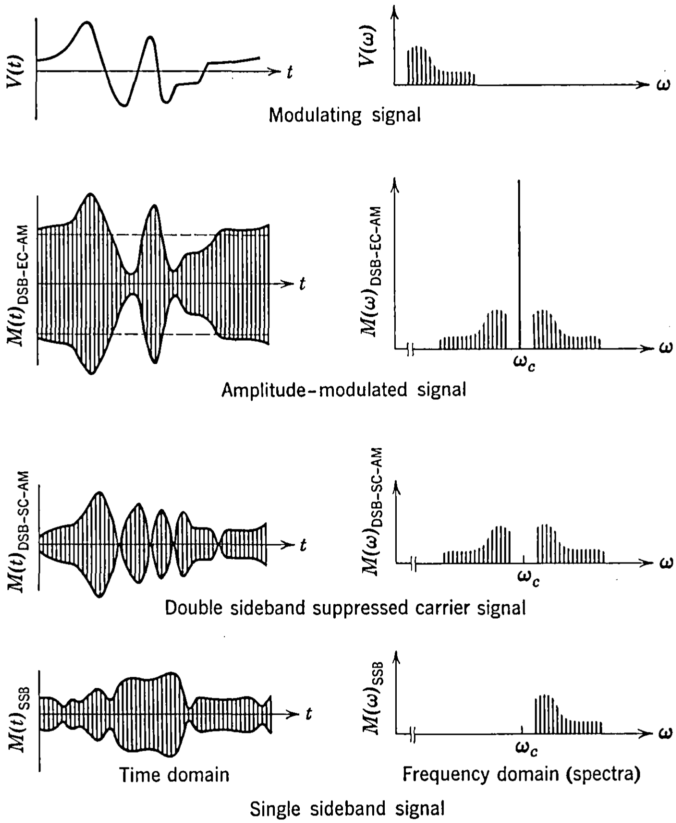


FIG. 23. Amplitude-modulated waveforms.

frequencies and to evaluate the effects of amplitude and phase distortion by the transmission system. For a message consisting of a single sine wave, the modulated wave consists of two sinusoidal sideband frequencies. These sidebands can be represented as a sum of an in-phase and quadrature carrier frequency waveform. Where the in-phase component will be a cos $\omega_c t$ time function, since a cos $\omega_c t$ modulating function is assumed, the quadrature component will be a sin $\omega_c t$ time function. As before, let the suppressed carrier modulated signal be represented as

$$M(t) = A \cos (C + V) + A \cos (C - V).$$

After transmission and amplification, the received modulated waveform $R(t)$ will be distorted owing to the amplitude and phase characteristics of the transmission system:

$$R(t) = A_1 \cos (C + V + \theta_1) + A_2 \cos (C - V + \theta_2)$$

where A_1, A_2 = amplitude of upper and lower sidebands after transmission,
θ_1, θ_2 = phase shift of upper and lower sidebands after transmission,
$C = \omega_c t$,
$V = \omega_s t$.

$$R(t) = [A_1 \cos (V + \theta_1) + A_2 \cos (V - \theta_2)] \cos C + [-A_1 \sin (V + \theta_1) + A_2 \sin (V - \theta_2)] \sin C. \tag{1}$$

The cos C term is the in-phase, A_p, carrier component and the sin C term the quadrature component, A_q. Note that if the amplitude and phase distortion is negligible, the amplitude of the quadrature component is zero.

DSB-SC Demodulator. For DSB-SC systems, both the quadrature and in-phase components are received. The intelligence, however, is ideally contained only in the in-phase component. As a result, a simple method for obtaining an in-phase carrier at the receiver consists of varying the phase of a locally generated carrier so that the output of a quadrature demodulator is zero. The output of an in-phase demodulator will then contain all the message information. This type of DSB-SC demodulator is illustrated in Fig. 24 (Ref. 44). In the demodulator of Fig. 24 the outputs of both the in-phase and quadrature demodulators are combined to effect cancellation of correlated noise or disturbances present in their carriers. The synchronous demodulators in Fig. 24 are typically phase sensitive rectifiers.

The block diagram of Fig. 24 is typical for DSB-SC systems used in wire or RF transmission. For RF transmission the RF signal is the input, and

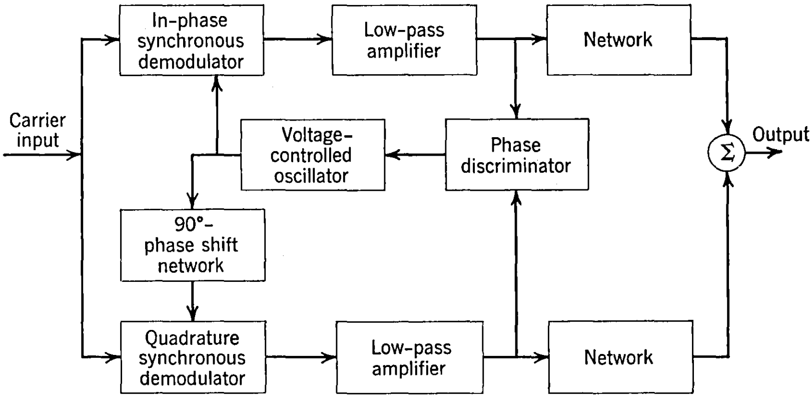


FIG. 24. Synchronous demodulator for DSB-SC-AM.

no IF amplifiers are used. As a result no problem exists with image responses, and the selectivity of the system is determined by the output low-pass filter characteristics.

SSB-SC Demodulation. In the case of SSB transmission, one of the carriers is not transmitted, and either A_1 or A_2 equals zero in Eq. (1). By letting A_2 equal zero, then for SSB

$$R(t) = [A_1 \cos (V + \theta_1)] \cos C + [-A_1 \sin (V + \theta_1)] \sin C.$$

Thus, for SSB the message information is transmitted on both the in-phase and quadrature components of the carrier. As a result no simple method exists for obtaining a local carrier which is locked in phase with the transmitted carrier (45). To circumvent this problem, a small carrier frequency signal is transmitted continuously or in bursts along with the modulated waveform. The VCO in the receiver must be synchronized to this pilot carrier signal in order to effect demodulation. To provide reliable operation, the amplitude of the pilot carrier is typically 10 to 30 db below that of the normal operating level of the system.

Synchronous Demodulators

In the previous section an amplitude-modulated wave was represented in terms of an in-phase carrier component and a quadrature carrier component. For DSB-SC transmission the information was ideally contained in the in-phase component only.

Synchronous demodulation is the recovery of the message signal by means of a time-varying circuit, the parameters of which are varied periodically at the carrier frequency.

Most practical synchronous demodulators can be represented as two cascaded operations: (*a*) multiplication of the received signal by a periodic function of the carrier frequency and (*b*) filtering. The multiplication operation shifts the signal spectrum to its original location prior to modulation and, in addition, creates new frequency components at multiples of the carrier frequency. The resulting signal must then be filtered in order to recover the original messages.

The design of a practical demodulator involves the selection of the multiplying function and the design of a suitable filter based on the relative amounts of in-phase and quadrature signals present in the input.

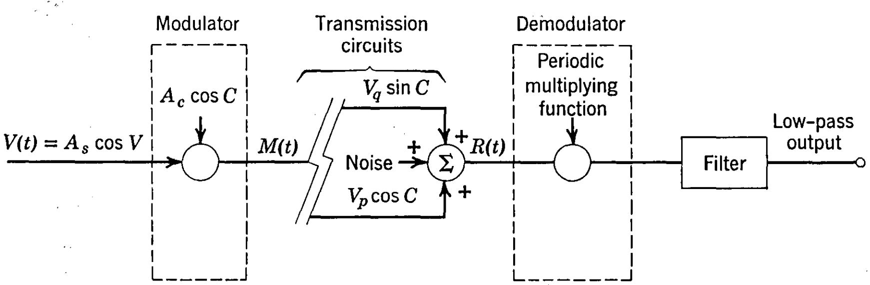
Demodulator Multiplying Functions. Three multiplying functions are sufficient to describe most practical synchronous demodulators: the square wave, the periodic impulse train, and the sinusoid.

If it is assumed that the signal to be demodulated is the product of a sinusoidal carrier and the message, the ideal demodulator, if no disturbances are present, will be a division of the demodulator input by a sinusoid. Even if the practical difficulties of such a division are neglected, the periodically infinite values of the multiplying function lead to a great sensitivity to quadrature signals.

The multiplying function used in most synchronous demodulators is the square wave. Bridge, ring, shunt series, and switching demodulators in which tubes, rectifiers, transistors, choppers, or relays are synchronously switched with the carrier are examples of square wave demodulation (Refs. 46 and 47).

A demodulator which periodically samples the input waveform and performs a holding operation (sample and hold) between samples has a periodic impulse train for its multiplying function. In this instance, the filter and the demodulator are not physically separated since the holding operation is a pseudo equivalent of a filter.

DSB-SC Synchronous Demodulators. Figure 25 is a block diagram of DSB-SC synchronous transmission system. It is assumed that no quadrature signal exists at the modulator output and that the quadrature signal is the result of transmission circuit distortion. Any quadrature component generated by the modulator should be added to that resulting from the transmission process.



$V(t) = A_s \cos V$
$R(t)$ = received modulated signal $= V_p \cos C + V_q \sin C + N(t)$
$M(t)$ = modulated signal $= 2A \cos v \cos C$
$A = \dfrac{KA_0A_s}{2}$
K = modulation index
$C = \omega_c t$
$V = \omega_s t$
V_p = received in-phase carrier component $= [A_1 \cos (V + \theta_1) + A_2 \cos (V - \theta_2)]$
V_q = received quadrature carrier component $= [-A_1 \sin (V + \theta_1) + A_2 \sin (V - \theta_2)]$
θ_1, θ_2 = transmission circuit phase shift upper and lower sideband frequencies
A_1, A_2 = received upper and lower sideband amplitudes

FIG. 25. Synchronous transmission system.

Demodulator Output Amplitude Spectrum. The demodulation process, in addition to shifting the message spectrum to its original location, introduces harmonic spectrums centered about multiples of the carrier frequency. These harmonics are error signals in the sense that they did not exist in the original message. The amplitude spectrum resulting from the demodulation of a modulated carrier are indicated in Fig. 26 for the square wave, sinusoidal, and impulse train multiplying functions. The components of the demodulated signal spectrum around even multiples

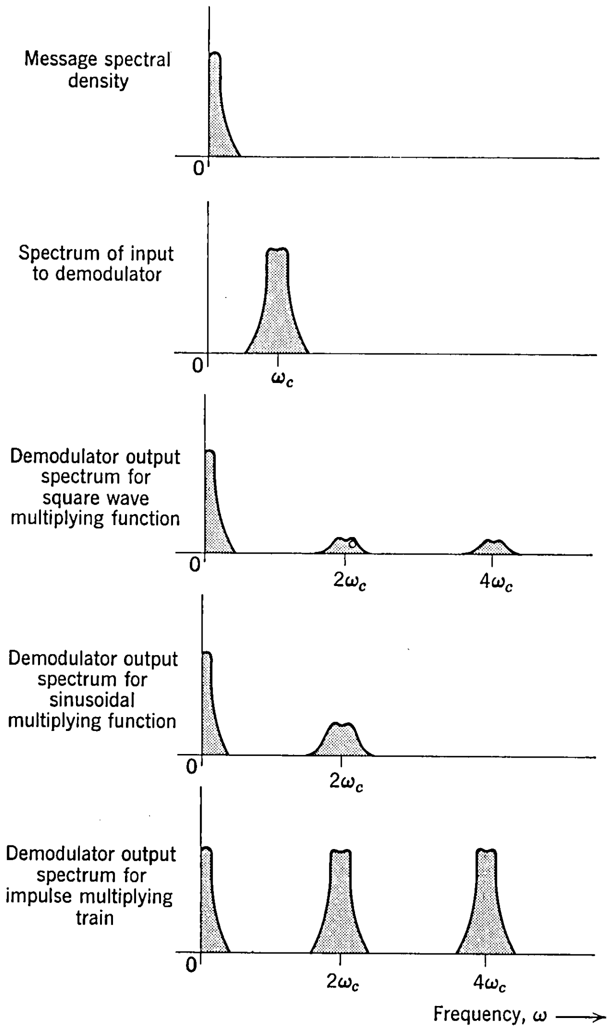


FIG. 26. Demodulator output amplitude spectrum for various demodulators

of the carrier frequency are shifted replicas of the message or signal spectrum with amplitudes determined by the Fourier coefficients of the multiplying function.

Output Filtering. The output filter must pass the message spectrum without distortion and must reject all other components. The other components have spectra centered around the various harmonics of the carrier and are called harmonic error. It is obvious that the design of a satisfactory filter becomes progressively more difficult as the highest significant message frequency approaches the carrier frequency. When the highest significant frequency exceeds the carrier frequency, the message spectrum and the second harmonic error spectrum overlap and cannot be separated by a filter. This limitation is related to the Shannon Sampling Theorem and is called *frequency folding error.*

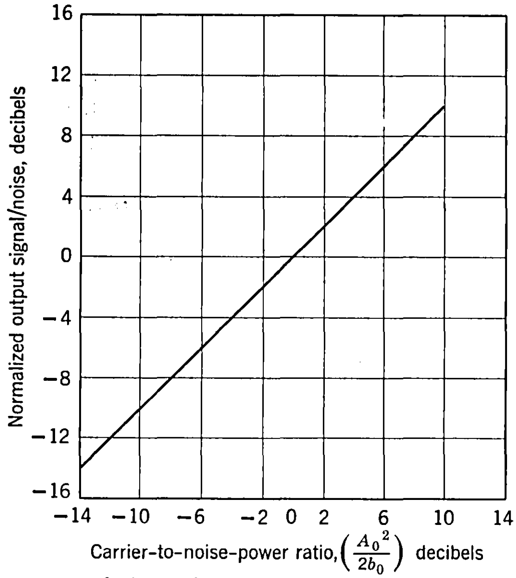
Synchronous Demodulator Input—Output S/N. The performance of synchronous demodulators has been described in the literature (Refs. 48, 49, 50, and 51). As is true of the phase lock or synchronous FM discriminator, the output S/N of the synchronous demodulator does not have the threshold phenomena characteristic of nonsynchronous demodulators. For the linear envelope demodulator signal suppression below a zero-db input S/N limited the usefulness of this device for low S/N. The synchronous demodulator operates effectively for high and low signal-to-noise ratios. The relationship between input S/N and output S/N is given in Fig. 27 for synchronous demodulators used in DSB-SC-AM systems. The results apply for carrier frequencies much larger than message bandwidth and for negligible phase error between the demodulator and modulator carriers.

In conclusion, for large signal-to-noise ratio, the synchronous demodulator is no better than a linear rectifier demodulator. However, for small signal-to-noise ratios, the synchronous demodulator is superior, being free from the additional overmodulation noise or carrier suppression effect.

DSB-SC-AM System Errors

For an ideal DSB-SC system, the information is all contained in the in-phase carrier component and the quadrature component is absent. If the transmission system does not have an even amplitude symmetry and odd phase symmetry about the carrier frequency, the received signal $R(t)$, eq. 1, will contain a quadrature component.

The received signal will, in addition to the in-phase and quadrature components, contain a noise component (see Fig. 25). This noise component will have a bandpass spectrum equal to the carrier signal bandpass and can be represented as a sum of in-phase and quadrature carrier com-



1. To obtain ordinate scale for a given

$$\frac{\text{Carrier bandwidth}}{\text{Output bandwidth}} \text{ ratio, add } 10 \log \frac{\text{Carrier bandwidth}}{2 \times \text{Output bandwidth}} \text{ to ordinate scale.}$$

2. This method is valid only for sinusoidal modulation with carrier bandwidth ≥ 3 times output bandwidth.

A_0 = carrier peak amplitude
b_0 = mean noise power in carrier bandwidth

FIG. 27. Universal curve for output signal-to-noise ratio for synchronous demodulators in DSB-SC-AM.

ponents. Then the signal at the demodulator input can be completely represented in terms of the in-phase and quadrature carriers.

The objective of the demodulation process is the recovery of the signal that modulates the in-phase carrier and the rejection of the quadrature carrier. The final separation of the message from the noise after demodulation of the in-phase carrier is not a part of the demodulation process but rather a separate filtering problem. The discussion of DSB-SC system errors is thus the evaluation of the effects of quadrature carrier signals on the information contained in the in-phase carrier demodulator output.

The total quadrature signal at the demodulator input is, in the general case, the sum of three components: (1) quadrature signals representing one component of the input noise, (2) quadrature signals resulting from amplitude and phase errors in the transmission system, and (3) in two-channel suppressed carrier systems using carriers in quadrature, each channel representing a quadrature signal to the other channel.

Error Criterion. In order to evaluate demodulator performance, an error criterion or a measure of performance is required. Mathematical simplicity leads to the use of the mean square value of the total error, in which the error is defined as the difference between the message and the output of the demodulator filter. However, this criterion is restrictive in that time delay introduced by the filter and system has a pronounced effect on the error. For most applications time delay is not of importance, and the fidelity of the output signal amplitude and phase is the prime concern. For these applications the time average of the mean square error will provide a more realistic measure of system performance and will be assumed in the following discussion.

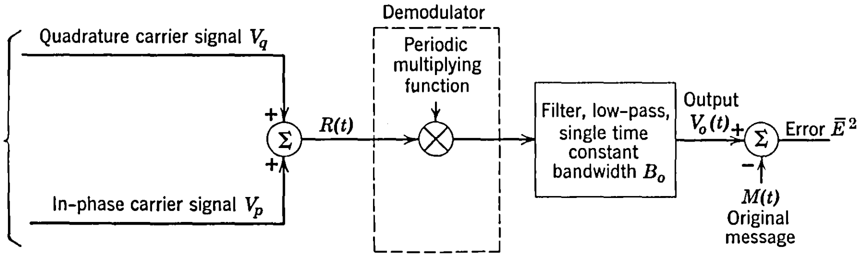
DSB-SC Errors. Since the original message $m(t)$ is assumed to contain information, it cannot be considered a known function of time. The message, the future values of which are unknown and uncorrelated, is thus a random function of time. In the error discussion that follows it is assumed that the message has the characteristics of random noise with a bandwidth B_s. This method of representation is realistic for actual systems, providing the actual message bandwidth is considered rather than the bandwidth of the transmission circuits used to transmit the message. In instrumentation information transmission systems, the transmission circuit bandwidth typically exceeds the actual message bandwidth by a factor of 2 to several hundredfold, depending on the constants imposed by transient performance or sampling specifications, etc.

Figure 28 is a representation of the synchronous demodulation process under consideration. The signal at the demodulator input is represented by an in-phase component and a quadrature component. It is assumed that both components are modulated by a noise-like signal of bandwidth B_s, and that the original message signal is contained in the in-phase carrier component. The ratio of quadrature to in-phase carrier power will be denoted by K:

$$K = \frac{\text{Quadrature carrier power}}{\text{In-phase carrier power}}.$$

The output filter has a bandwidth B_o and is assumed to be a single time constant RC filter (6 db/octave). The output of the filter is the recovered input signal and will differ from the original input, $M(t)$, by the error, $\bar{E}^2$, where $\bar{E}^2$ is the time averaged mean square error.

Neglecting noise in the output due to the noise component in the in-phase carrier, the demodulator output will include three error components in addition to the desired message signal: (*a*) harmonic error, (*b*) quadrature error, and (*c*) filter error.



$B_o \geq B_s$

$M(t)$ = noise-like message signal

B_s = message bandwidth

$R(t)$ = amplitude-modulated signal at demodulator input

$V_o(t)$ = demodulator output

$\bar{E}^2$ = time-averaged mean square error

$$K = \frac{\text{Quadrature carrier power}}{\text{In-phase carrier power}} = \left(\frac{V_q}{V_p}\right)^2$$

FIG. 28. Synchronous demodulator.

The harmonic error is the result of the new spectral components created by the multiplying operation in the demodulator. These error components are centered at the harmonics of the carrier frequency. The amplitude of these harmonic components is proportional to the magnitude of the signal. Hence, in order to obtain a given dynamic range at the filter output, it is necessary to provide sufficient filtering to reduce the magnitude of the harmonic components to less than 1/dynamic range.

The quadrature error represents the shifted spectrum of the quadrature carrier signal after the demodulation process. The amplitude of the quadrature signals at the demodulator output is of the same order of magnitude as that existing at the input and hence will depend on the ratio of quadrature to in-phase components.

The filter error represents the loss of information in a system in which the output filter bandwidth is less than the message bandwidth.

In Figs. 29*a*, *b*, and *c* the time average mean square error $\bar{E}^2$ is plotted as a function of ω_c/B_o and B_s/ω_c, where B_s/ω_c is the normalized message bandwidth. The curves are reproduced from a paper by Booton and Goldstein (Ref. 52) and apply for the case of no quadrature component, Fig. 29*a*, and for a quadrature-to-carrier ratio of 1 and 9, Figs. 29*b* and *c*. Note that for a given value of normalized message bandwidth there exists an optimum filter bandwidth which minimizes the error.

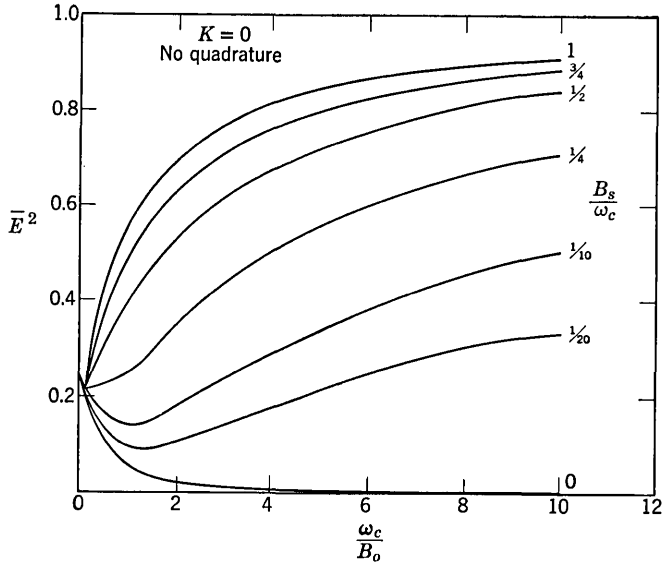


FIG. 29a. Mean square error for square wave multiplying function.

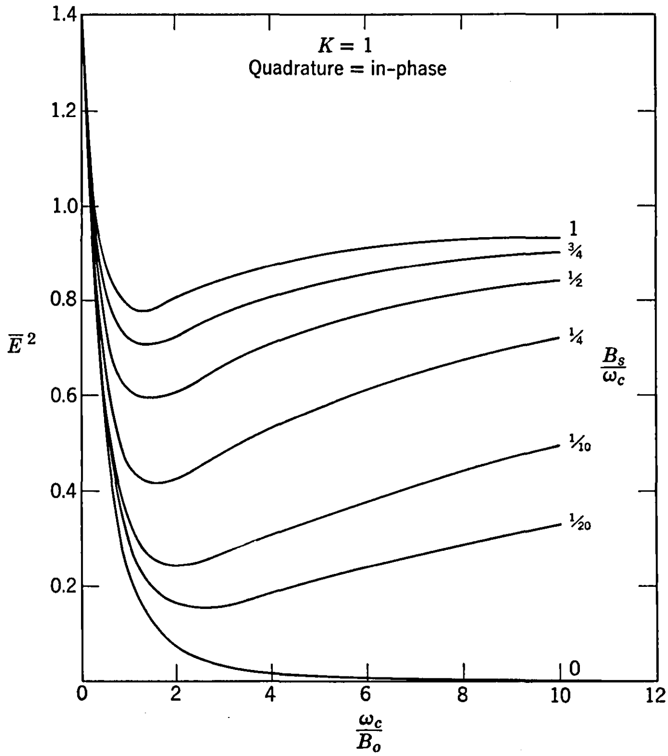


FIG. 29b. Mean square error for square wave multiplying function.

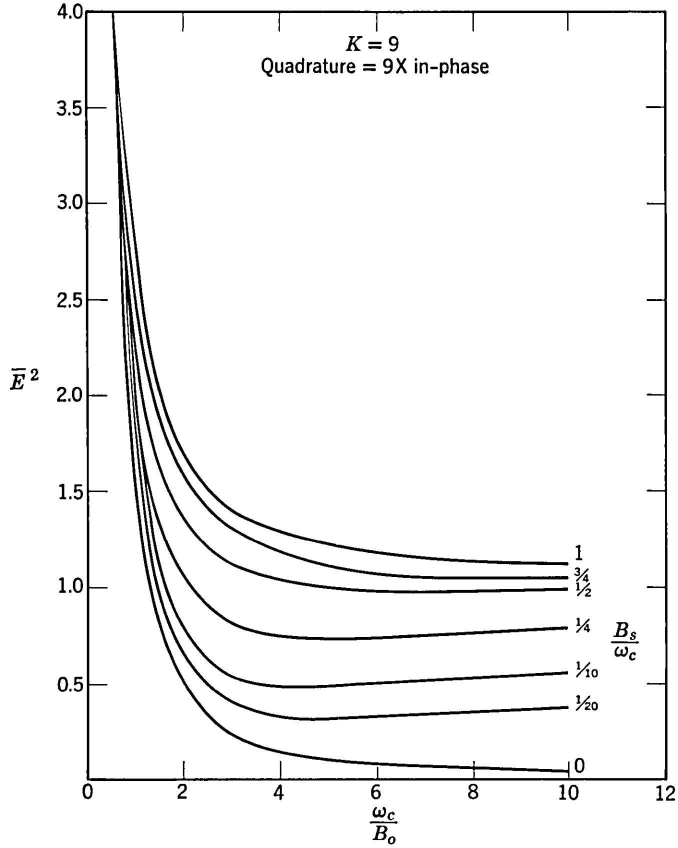


FIG. 29c. Mean square error for square wave multiplying function.

Figure 30 is a plot of error as normalized message bandwidth for various values of K and for the optimum value of output filter bandwidth. Although the optimum filter bandwidth varies with K and the message bandwidth, it is of the same order of magnitude as the message bandwidth. From the figures it is obvious that the quadrature carrier component has a pronounced effect on the error existing at the filter output.

Since the amplitude of the quadrature carrier component is zero at the time the in-phase carrier amplitude is maximum, a sampling demodulator which samples the in-phase carrier at this instant will not be influenced by the magnitude of quadrature component. In practice a substantial improvement in demodulator performance can be realized by using a sampling demodulator in this manner.

At the extreme of no quadrature component, the square wave multiplying function is found to be optimum for DSB-SC synchronous demodula-

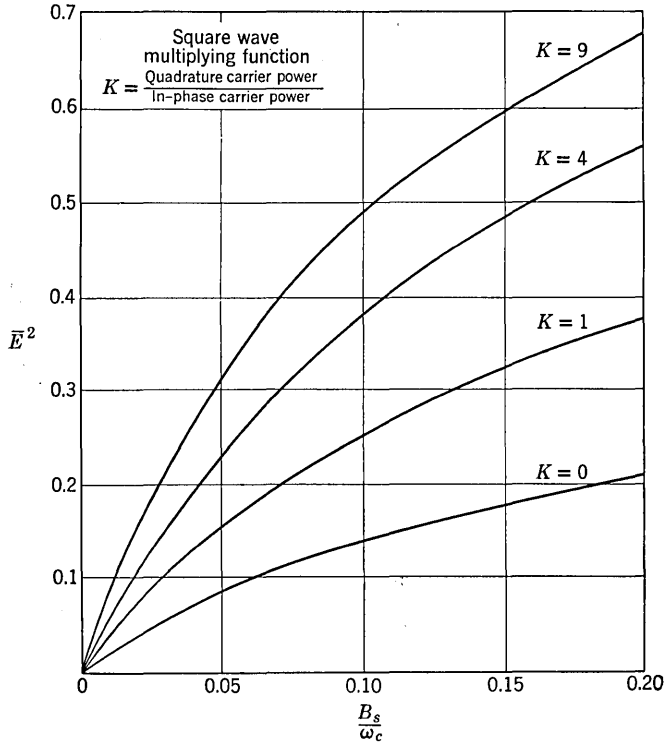
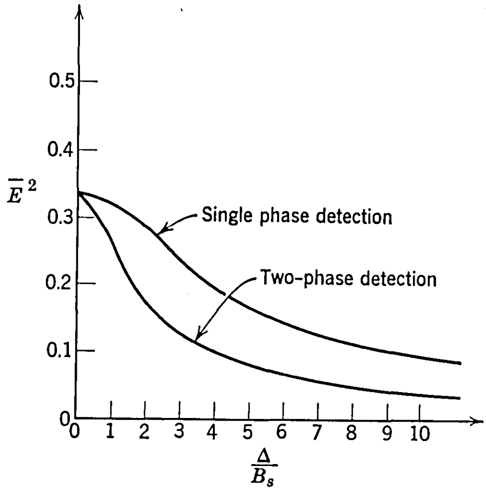


FIG. 30. Mean square error as a function of quadrature component and message bandwidth.

tors. At the other extreme, when the quadrature component is appreciably larger than the in-phase carrier, the sampling demodulator (impulse train multiplying function) is optimum. For intermediate values of K, a sinusoidal multiplying function will prove most effective.

DSB-SC-AM Interference Characteristics. Since the quadrature and in-phase components are independent in an ideal system, it is possible to use the quadrature carrier to transmit a second message and to add a pilot carrier to obtain a phase-locked carrier at the receiver. Under these circumstances the DSB system performance in terms of power and bandwidth utilization is equivalent to that obtained with SSB systems.

In many practical applications the system limitation is not the noise associated with the circuits and their corresponding bandwidth but rather adjacent channel interference. Under these circumstances it is found that the two-phase demodulator scheme of Fig. 24 will give a two-to-one advantage over demodulation systems in which only a single in-phase demodulator is used. The improvement in interference rejection is indi-



$$\frac{\text{Average power desired signal}}{\text{Average power undesired signal}} = 1$$

Δ = separation of interfering carrier and desired carrier, cycles per second
B_s = message bandwidth, cycles per second
$\overline{E}^2$ = mean square error

FIG. 31. Adjacent channel interference DSB-SC-AM.

cated in Fig. 31 for the situation in which both signals have DSB-SC modulation and both signals have the same power and spectral distribution. Note that for a given mean square error, single phase detection requires about twice the center frequency separation Δ of two-phase detection. For example, if $\overline{E}^2 = 0.2$, $\Delta = 1.6B_s$ for two-phase detection, where as a separation of $\Delta = 4B_s$ is required if only cosine detection is used.

The advantage of two-phase detection stems from the fact that the output of both demodulators is correlated, and thus it is possible, by combining the outputs of the two demodulators, to cancel some of the effects of the interference. If DSB systems with two-phase demodulation are compared with SSB systems, it is found (Ref. 53) that in almost every case DSB operation is superior to SSB.

6. PULSE TRANSMISSION

With the exception of direct wire facilities, transmission systems require the use of a carrier to effect transmission. In FM and AM systems, a parameter of the carrier waveform contains the information and the transmission process is an analog operation in which the variation of the carrier waveform parameter is continuous over the dynamic range. For these systems, the transmission of nonsteady data at accuracies exceeding 1% requires gain and phase tolerances which are difficult to realize in practice.

If the information is digitally coded to permit representation in terms of a train of binary bits, only two levels or magnitudes of the carrier waveform parameter are required.

With respect to the transmission facility, a digital system is in reality an analog operation in which only two levels are required to effect transmission. This exchange of carrier dynamic range for bandwidth in digital systems permits the *efficient utilization of nonideal facilities* and an overall performance which approaches the ideal set by the Shannon Hartley law.

The transmission of information as binary pulses has the further advantages that the pulses can be regenerated to avoid the accumulation of distortion from noise and other system imperfections, the speed of transmission can be varied to suit the application, and finally pulse transmission systems permit multiplexing data channels on a time division basis rather than a frequency division basis.

Methods of Representing System Characteristics. The performance of a pulse transmission system in the absence of noise can be predicted by either the *pulse transmission characteristic* or the *transmission-frequency characteristic.* The pulse transmission characteristic describes the shape of the received pulse for a given applied pulse shape and for a given transmission channel. It is limited, however, in that the characteristic for two transmission systems in tandem cannot be easily predicted from the pulse transmission characteristics of the individual systems.

This is not true for representation of a system in terms of its transmission frequency characteristic, and as a result this method of representation has gained universal acceptance. The transmission frequency characteristic for a system is simply its steady-state gain and phase characteristics expressed as a function of frequency. For this method, the design of a pulse transmission system reduces to the specification of the system transmission frequency characteristic which will provide a given transmission performance, or alternately the specification of the transmission performance given the transmission frequency characteristic.

The fundamental problem in pulse systems is the distortion of pulses by system imperfections in the form of gain and phase deviations or by the low-frequency cutoff characteristics of the transmission system. The distortion resulting from low-frequency cutoff is referred to as *characteristic distortion.* The effect of pulse distortion is to cause adjacent pulses to overlap or to interfere. In the limit, interference between pulses will result in the loss of information since individual pulses cannot be reliably distinguished or identified.

Ideal System with Sharp Cutoff

In pulse transmission theory (Refs. 54, 55, and 56), the capacity of an ideal system is usually described in terms of a fictitious low-pass system having contant amplitude and delay in the passband, zero-amplitude response beyond the cutoff frequency ω_1, and *linear phase variation in the passband.*

$$\begin{aligned} A(\omega) &= 1 \qquad 0 \leq \omega \leq \omega_1, \\ &= 0 \qquad \omega > \omega_1, \\ \psi(\omega) &= \omega\tau_d = \text{Linear phase variation}, \\ \tau_d &= \text{Transmission delay (constant over bandwidth—}B_{½a} = \omega_1/2\pi). \end{aligned}$$

The impulse transmission characteristic for this system is given in Fig. 32.

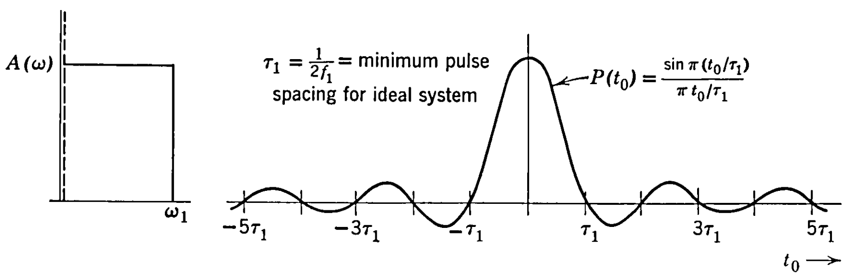


FIG. 32. Impulse response of ideal low-pass system with sharp cutoff.

The impulse response is zero at times $n\tau_1$ where

$$\tau_1 = 1/2f_1 = \text{minimum pulse spacing for an ideal system.}$$

Impulses can thus be transmitted at intervals of τ_1 seconds without mutual interference between peaks of the received pulses. This is a basic theorem relating the physical limitation on pulse transmission rate and bandwidth for an ideal system.

Ideal Systems with Gradual Cutoff and Linear Phase Characteristic

The ideal sharp cutoff system is impractical in the sense that the characteristic is difficult to realize in practice, and further that the oscillatory nature of the response would result in appreciable interference between pulses in a practical system. The oscillations in the impulse characteristic can be reduced with a gradual rather than a sharp cutoff characteristic as illustrated in Figs. 33*a* and *b*. The example in Fig. 33*a* also assumes that

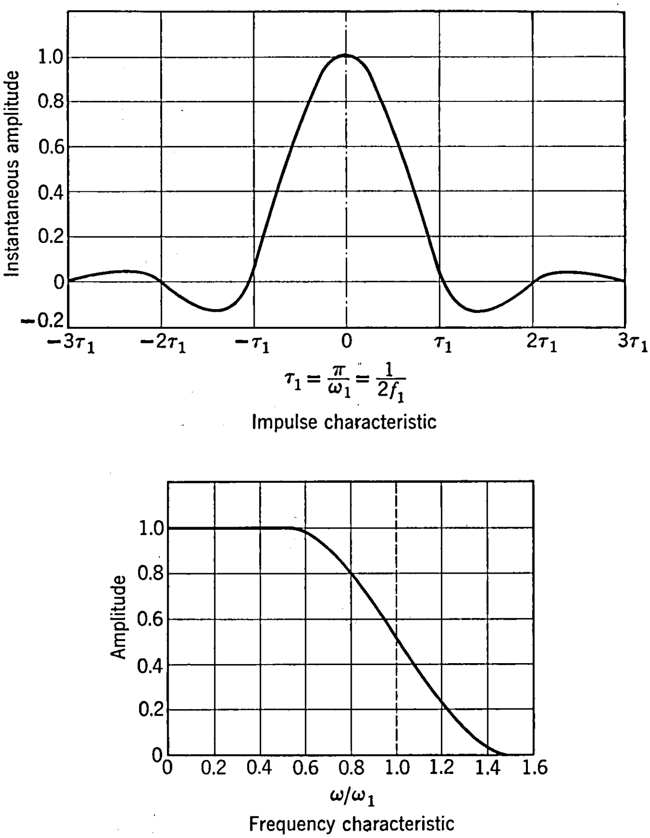


FIG. 33*a*. Low-pass system with gradual cutoff and linear phase characteristic.

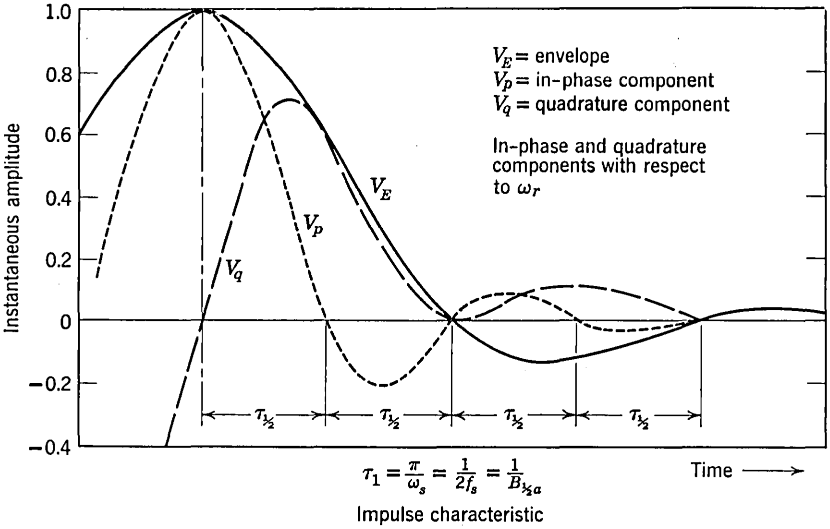
the phase variation is linear. In comparison with the sharp cutoff case, the introduction of a gradual rolloff and linear phase variation effects a considerable reduction in the length and amplitude of the oscillatory tail. Note that *the bandwidth* $B_{½a}$ *is the half-amplitude bandwidth.*

For the example used in Fig. 33, the rolloff characteristic is assumed to have odd symmetry about the half-amplitude cutoff frequency ω_1,

$$A(\omega) = 1 \qquad 0 \leq \omega \leq \omega_1/2,$$

$$A(\omega) = 1 - \tfrac{1}{2}(1 - \sin \pi\omega/\omega_1) \qquad \omega_1/2 \leq \omega \leq 3\omega_1/2.$$

The impulse characteristic is also given for a symmetrical bandpass frequency characteristic. In the figure the in-phase and quadrature components are in terms of a reference frequency ω_r which is displaced from the midband frequency ω_c (Ref. 54).



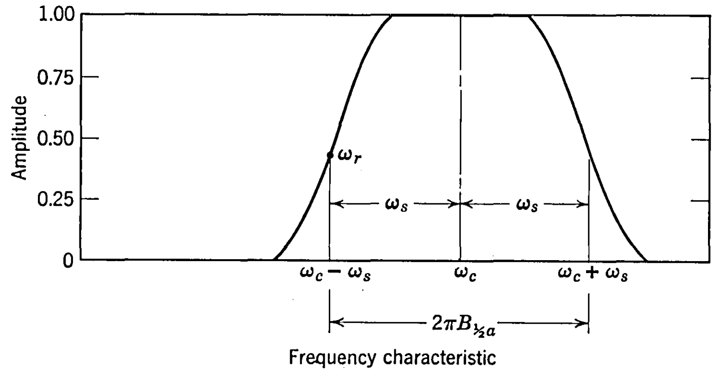


FIG. 33b. Symmetrical bandpass system with gradual cutoff and linear phase characteristics.

Note that the assumed gradual rolloff characteristic does not have a natural linear phase variation. As a result, a practical system with this rolloff characteristic will require phase equalization to obtain the linear phase characteristic. This system is, however, typical of the gradual rolloff characteristics found in many bandpass or low-pass transmission systems.

Ideal System with Linear Phase Characteristic

In the previous example, a typical rolloff characteristic was assumed, and phase equalization was required to obtain the indicated impulse characteristic. It is possible to obtain an almost perfect linear phase shift characteristic by modifying the rolloff characteristic such that rolloff is gradual across the entire passband. The impulse characteristic for a transmission system with natural linear phase shift is indicated in Fig. 34. The linear phase shift characteristic is the result of a rolloff characteristic of the form

$$A(\omega) = 1 - \tfrac{1}{2}(1 - \sin \pi\omega/2\omega_1) \qquad 0 \leq \omega \leq 2\omega_1.$$

Note that the magnitude of the oscillations is reduced over the previous example in which the rolloff characteristic did not extend over the entire

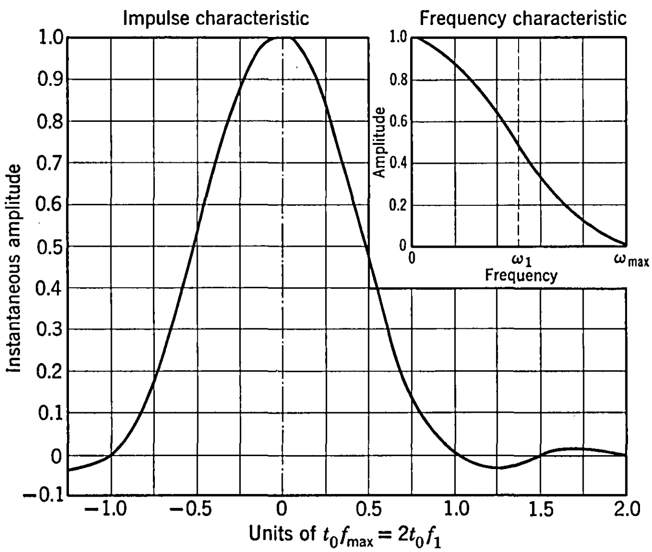


FIG. 34. Impulse characteristic for low-pass system with natural linear phase shift variation.

passband. Further, note that the phase variations must be controlled over a passband of $2\omega_1$, and hence the transmission bandwidth is, in effect, twice the theoretical bandwidth $B_{\frac{1}{2}a} = \omega_1/2\pi$ required to transmit pulses at a rate of $2f_1$.

Practical Considerations

In dealing with pulse transmission systems, it is convenient to consider the response of the system or the pulse transmission characteristic for impulse signals. Since the impulse signal has uniform spectral distribution for all frequencies, the result of applying impulses to a network or system is to pass only those frequencies within the passband of the system. *If the actual pulses applied to a system have an approximately uniform spectral distribution over the passband of the system, the response of the system to the actual pulses will be adequately described by the response to impulse signals.*

The principal advantage of using impulse signals is the simplicity of analysis and interpretation of system characteristic. In order to obtain a physical feel for the difference in system characteristics for impulse signals as compared with rectangular pulses, consider that pulses of finite duration are applied to a system and that the rolloff characteristic is to be modified to give the same output response as that obtained with impulse signals. Figure 35 indicates the minor modification required in the amplitude frequency characteristic to satisfy the requirement of identical output responses.

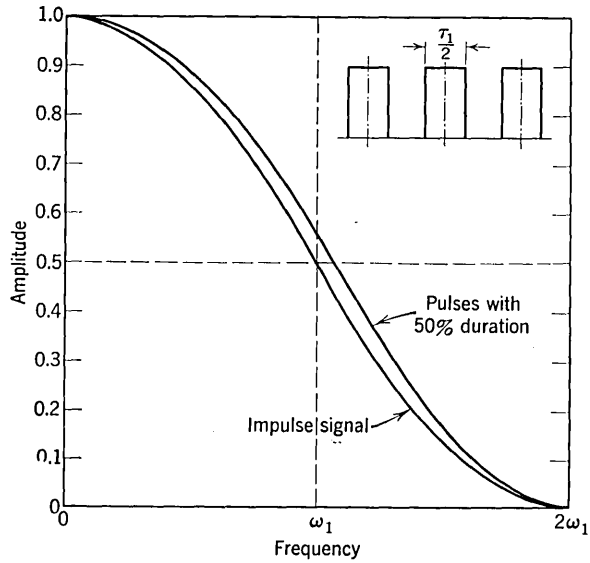
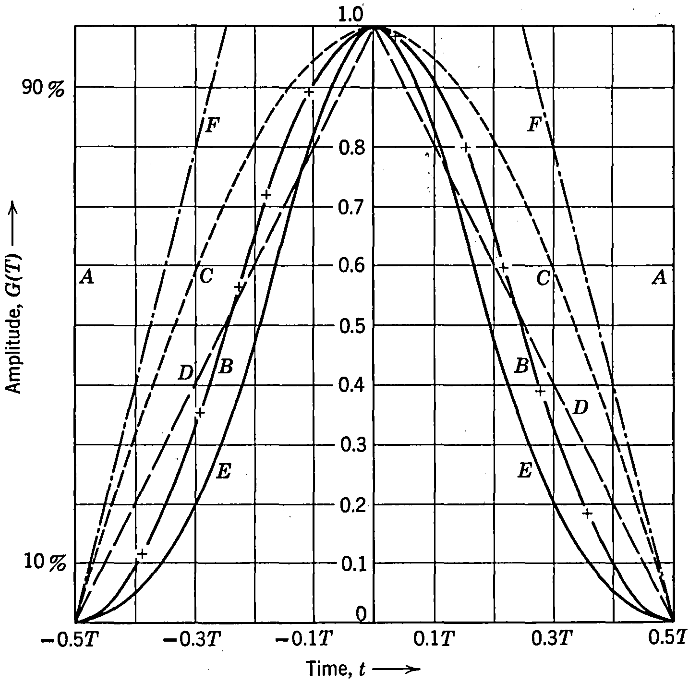


FIG. 35. Modification of frequency characteristic to obtain identical output response for pulses of finite duration.

The impulse and the rectangular pulse of finite duration have a frequency spectrum which extends far beyond the passband used for transmission. If the system has pronounced phase or amplitude variations at the edges of the passband owing to a sharp cutoff filter, the frequencies in the input signal should be confined to a band in which distortion is low in order to minimize oscillation and pulse interference. Figure 36 gives



A, rectangular
B, cosine squared
C, half-cosine
D, triangular
E, Gaussian
F, trapezium

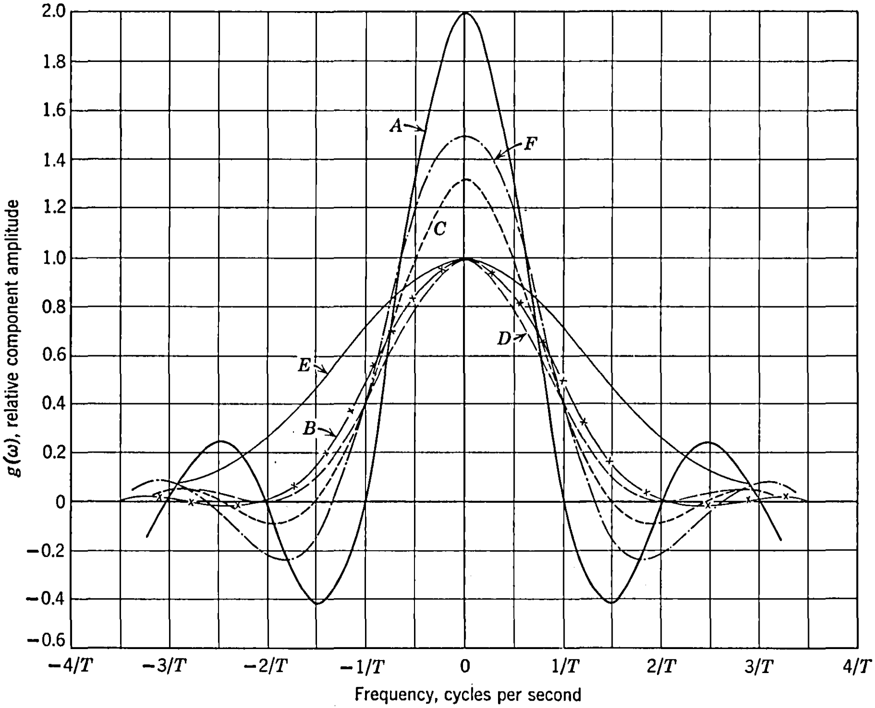
FIG. 36*a*. Pulse waveforms.

the pulse waveform and the corresponding spectral distribution for a variety of pulse shapes.

The diagrams indicate the effect of modifying the shape of the pulse and permit a considerable amount of mental interpolation. The peak amplitudes in the frequency spectra diagrams are not significant, the intention being to display the relative amplitude versus frequency.

The Gaussian pulse has the characteristic that the waveform and spectrum both have a Gaussian shape. A single section low-pass filter with a nominal cutoff frequency equal to the reciprocal of twice the width of a rectangular pulse has a response which closely approximates the Gaussian curve. The actual response is compared with the Gaussian waveform in Fig. 37.

If a transmission system has a Gaussian amplitude characteristic and linear phase shift, the impulse response is damped (no overshoots or oscillations). However, the bandwidth required for a given pulse rate is slightly larger than that required in previous examples.



A, rectangular
B, cosine squared
C, half-cosine
D, triangular
E, Gaussian
F, trapezium

Fig. 36b. Pulse spectra.

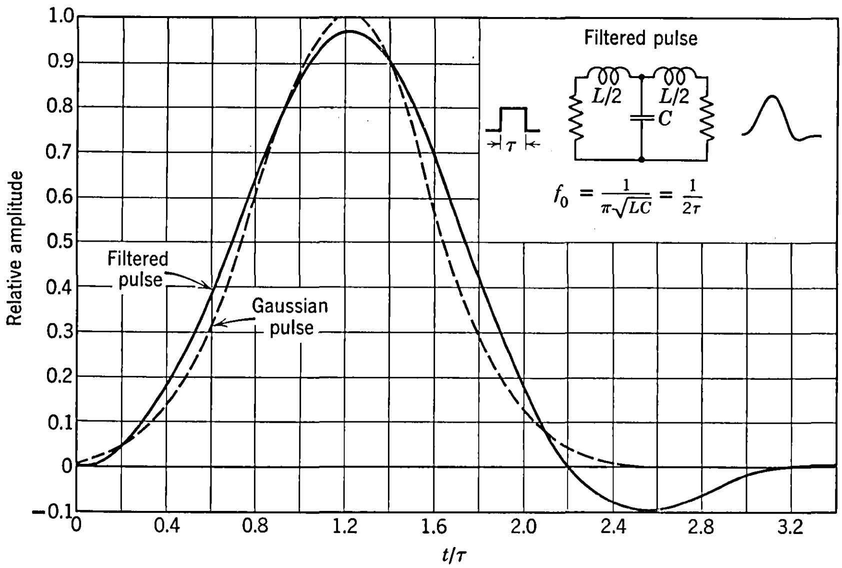


Fig. 37. Filtered rectangular pulse and Gaussian pulse.

Correlation between Bandwidth and Pulse Transmission Characteristic. Figure 38 gives the response of an ideal system having a sharp cutoff characteristic with a bandwidth $B_{\frac{1}{2}a}$ to a pair of test pulses with finite duration. From the figure, it is evident that a bandwidth

$$B_{\frac{1}{2}a} = f_1 = 1/2\tau_1$$

is required to resolve the two pulses. Note that the pulse spacing is equal to the pulse width. If the bandwidth is decreased below the above value,

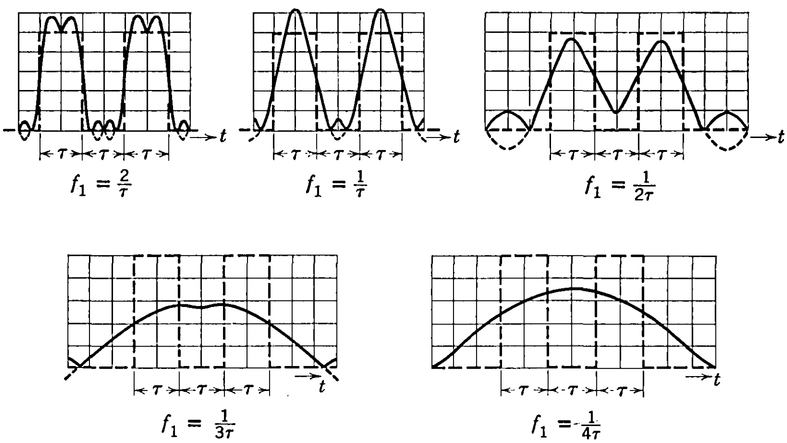


FIG. 38. Effect of bandwidth on the transmission of detail (low-pass filter with cutoff at f_1).

the detail or individual pulses are "washed out." The effect of increasing the bandwidth beyond this value is principally to sharpen the sides of the edges. To illustrate details of the figures, assume that the pulse width is 1 μsec and that a separation of 1 μsec exists between pulses (pulse interval = 2 μsec). Then, for a 250-kc bandwidth, no sign of two pulses exists; for 500 kc two pulses are clearly evident, and for 2 Mc details of the individual pulses become discernible.

Performance Specification

The derivation of the pulse transmission characteristic for an actual system in which the amplitude and phase variations deviate from the previous examples is an involved procedure. In practice, a satisfactory evaluation of performance can be obtained by assuming that the pulse transmission characteristic is approximated by one of the previous examples, namely, (*a*) sharp cutoff linear phase shift, (*b*) gradual cutoff linear phase by equalization, or (*c*) gradual cutoff natural linear phase shift, and that deviations of the actual system from the ideal will result in an rms intersymbol interference $\underline{U}$, which is evaluated in terms of the phase and amplitude deviations from the selected ideal system. The intersymbol interference U is a measure of the amplitude of distortion components resulting from nonideal phase and amplitude characteristics.

For the ideal system, the received pulse amplitude is uniquely determined by the transmitted pulse pattern. In a system with distortion, echoes and pulse overlaps add to or subtract from the assumed ideal response, and in the limit the identity of pulses may be lost. The intersymbol interference U is a ratio of the distortion component amplitude to the peak received pulse amplitude in the absence of distortion:

$$U = \frac{\text{Amplitude of distortion component}}{\text{Peak pulse amplitude with no distortion}}.$$

For $U = 1$, the distortion component is equal to the ideal received pulse amplitude.

Since the transmitted pulse train will vary in a random manner, the distortion component will also exhibit random variations. As a result, the intersymbol interference U, will vary as a function of time and will have an rms magnitude $\underline{U}$. Experience has indicated that for small values of U a peak factor of 3 to 4 provides a realistic measure of the peak intersymbol interference. For example, if rms interference $\underline{U}$ is 10%, the peak interference will usually be less than 30 to 40%.

The performance of an actual transmission system thus requires, in addition to the specification of transmission bandwidth and pulse rate, the specification of the rms intersymbol interference. Note that the intersymbol interference sets a limit to the transmission rate which is independent of the power level. If transmission power is limited, then it is necessary to specify, in addition, the error rate as a function of S/N.

Nonideal Transmission Systems

A convenient method of representing nonideal systems is to specify the amplitude and phase deviations from one of the ideal linear phase systems.

Let $A_0(\omega)$ = Ideal amplitude characteristic,

$\psi_0(\omega) = \omega\tau_d$ = Linear phase characteristic of ideal system.

Then, most systems can be described by an amplitude characteristic:

$$A(\omega) = A_0(\omega)(1 + a_1 \cos \omega T + a_2 \cos 2\omega T + \cdots),$$

$$A(\omega)/A_0(\omega) = 1 + \alpha(\omega),$$

where $\alpha(\omega)$ is the fractional deviation of the amplitude frequency characteristic from that of the ideal, $A_0(\omega)$. By letting $\underline{a}$ equal the rms value of $\alpha(\omega)$ over the transmission bandwidth $2\omega_1 = \omega_{\max}$ where $\omega_1/2\pi = B_{\frac{1}{2}a}$, then

$$\underline{U} = \eta(\pi/\omega_1\tau)^{\frac{1}{2}}\underline{a}$$

where $\eta = 1$ sharp cutoff linear phase system,
$\eta = 0.866$ gradual cutoff natural linear phase,
$2f = 1/\tau$,
$\omega_1 = \frac{1}{2}$ amplitude low-pass bandwidth,
$\tau =$ pulse separation.

If $2\pi f_1 = \omega_1$, the rms intersymbol interference $\underline{U}$, owing to amplitude deviations from the ideal amplitude characteristic, is simply equal to the rms value of the amplitude deviations over the transmission bandwidth $2\omega_1 = 4\pi B_{\frac{1}{2}a}$.

By applying the same procedure to phase deviations from the ideal linear phase characteristic $\psi_0(\omega)$,

$$\psi(\omega) = \psi_0(\omega) + \beta(\omega),$$
$$\beta(\omega) = b_1 \sin \omega T + b_2 \sin 2\omega T + \cdots.$$

Letting $\underline{b} =$ rms phase deviation over the transmission bandwidth $2\omega_1$, Sunde (Ref. 54) obtains the result

$$\underline{U} = \eta(1/\omega_1\tau)^{\frac{1}{2}}\underline{b}$$

where $\underline{b}$ is the rms phase deviation in radians. The total rms intersymbol interference due to both amplitude and delay distortion is

$$\underline{U} = \eta(\pi/\omega_1\tau)^{\frac{1}{2}}(\underline{a}^2 + \underline{b}^2)^{\frac{1}{2}}.$$

In the above expressions, note that T is not the delay τ_d of the transmission medium. *T in both the amplitude and phase characteristics is the period of the Fourier approximation of the deviations. It is numerically equal to the separation of echoes produced by the nonideal transmission system* (see Goldman, Ref. 55, page 106).

Note further that the transmission phase and amplitude characteristic must be controlled to $\omega_{\max}$, not just $\omega_1(\omega_1 = 2\pi f_1)$, the ideal bandwidth for a transmission rate of $2f_1$ pulses per second. $\omega_{\max}$ will be at least a factor of 2 greater than ω_1 for optimum transmission methods [values of 3 to 4 are common in conservative system design (Ref. 57)].

Sinusoidal Phase Deviation. In many practical applications, phase deviations from the ideal linear phase relation set the performance limit. In many of these instances, the phase deviation can be approximated by a single sine term which has odd symmetry about the center of the transmission bandwidth. The resulting amplitude characteristic will exhibit a cosine deviation and is indicated in Fig. 39.

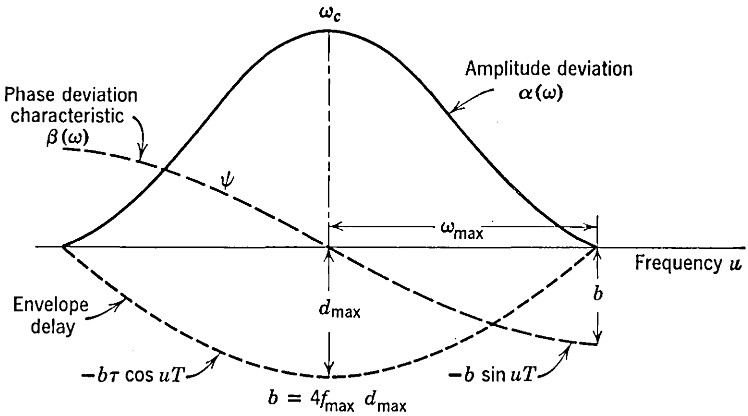


FIG. 39. Sinusoidal phase deviation; ω_{max} is the frequency limit for which amplitude and phase characteristics are controlled to permit operating with an effective bandwidth

$$B_{\frac{1}{2}a} = f_1 \leq \frac{f_{max}}{\sqrt{2}}$$

The rms phase deviation is simply $b/\sqrt{2}$ (Ref. 54) where

$$b/\sqrt{2} = (4/\sqrt{2}) f_{\max} d_{\max},$$

$$f_{\max} = \omega_{\max}/2\pi,$$

$$d_{\max} = \text{Maximum variation of envelope delay over the transmission bandwidth } \omega_{\max}.$$

Hence

$$\underline{U} = \eta(\pi/\omega_1\tau)^{\frac{1}{2}}(4/\sqrt{2}) f_{\max} d_{\max},$$

$$\omega_1 = \tfrac{1}{2} \text{ amplitude low-pass bandwidth} = 2\pi B_{\frac{1}{2}a},$$

$$\tau = \text{Pulse separation},$$

$$\eta \cong 1.$$

Intersymbol Interference Resulting from Low-Frequency Cutoff

Owing to use of transformers or a-c coupled components in transmission systems, the low-pass characteristic exhibits a low-frequency cutoff. The effect of a low-frequency cutoff can be avoided by employing a symmetrical bandpass characteristic in conjunction with a DSB-SC transmission system. In order to select the appropriate transmission method, direct or modulated carrier, it is necessary to evaluate the pulse transmission rates and accompanying intersymbol interference for each method.

For a DSB-SC system in which the pulse rate is appreciably less than the carrier frequency, the intersymbol interference can be determined by the methods of the previous section. As the pulse rate approaches the carrier frequency, increased distortion or intersymbol interference results from the modulation-demodulation operations. This distortion component can be evaluated from the rms error curves given in the section describing DSB-SC operation.

As the pulse rate approaches the carrier frequency, the rms error due to the modulation-demodulation process becomes large, and it is necessary to minimize intersymbol interference due to phase variations by careful phase equalization (Ref. 57).

For systems with low-frequency cutoff in which direct transmission is used, intersymbol interference results from the displacement of the zero or base line as the transmission rate is increased. This effect is called *zero wander* and is most pronounced for a long train of unipolar pulses. The number of pulses of one polarity, or of nearly all the same polarity, which can be transmitted before serious distortion occurs, depends on the extent of the low-frequency cutoff. If the low-frequency cutoff is unappreciable, this number may be sufficiently large that the probability of encountering such a sequence in a random pulse train is small. Hence the resultant error rate due to low-frequency cutoff may be disregarded.

If it is assumed that positive and negative impulses are applied at random to the transmission system at intervals of $\tau = 1/2f$, and that the transmission system has a sharp cutoff at ω_0 and ω_1, then the rms intersymbol interference is simply (Ref. 54):

$$\underline{U} = (\pi\omega_0/\omega_1{}^2\tau)^{1/2},$$

$$A(\omega) = 1 \qquad \omega_0 \leqq \omega \leqq \omega_1,$$

$$= 0 \qquad \begin{cases} \omega \leqq \omega_0 \\ \omega \geqq \omega_1 \\ \text{phase shift linear.} \end{cases}$$

In actual systems, the low-frequency cutoff will be gradual between $\omega = 0$ and ω_0, rather than abrupt as assumed above. With a linear variation in the amplitude characteristic between 0 and ω_0, $\underline{U}$ becomes

$\underline{U} = (\pi\omega_0/3\omega_1{}^2\tau)^{1/2}$ = Rms intersymbol interference resulting from low-frequency cutoff at ω_0,

τ = Pulse separation,

$\omega_1 = 2\pi f_1$ = High-frequency cutoff,

$\omega_0 = 2\pi f_0$ = Low-frequency cutoff.

For transmission at a rate $2f_1 = 1/\tau_1$ with an rms interference of $\underline{U} = 0.25$, the frequency ratio from the equation above, ω_0/ω_1, would have to be less than 0.188. Actually a substantially smaller ratio would be required since the phase distortion introduced by the low-frequency cutoff was neglected in the above evaluation. Low-frequency cutoff in a system in which direct transmission is attempted thus places severe restrictions on the performance of the system.

From either of the expressions for $\underline{U}$, it is evident that the effect of low-frequency cutoff can be reduced by transmitting narrow pulses at longer intervals than $\tau_1 = 1/2f_1$, the theoretical minimum. If acceptable performance requires increasing the interval by a factor of more than 2 to 4, the modulated carrier transmission may offer a decided advantage.

Dipulse Transmission. The zero wander of a system with a low-frequency cutoff characteristic can be minimized by transmitting a pair of opposite polarity pulses for each pulse in the original pulse train. This is called *dipulse transmission* and is accomplished by letting a positive pulse followed by a negative pulse indicate a true signal, and a negative followed by a positive indicate a false signal. For dipulse transmission, the d-c level taken over a pulse pair is zero, and improved performance can be obtained at the expense of doubling the bandwidth for a given transmission rate or halving the transmission rate for a given bandwidth. For dipulse transmission, the peak interference due to low-frequency cutoff is about equal to (Ref. 54)

$$\hat{U} \cong f_0/f_1 = \text{Peak interference.}$$

Note that to compare this with previous rms interference values it is necessary to multiply the rms quantity by a peak factor of 3 to 4.

Intersymbol Interference Resulting from Band Edge Phase Deviations

In pulse transmission systems in which phase equalization is employed, it may be impractical or unnecessary to equalize over the entire transmission band. As a result, a residual phase distortion will exist near the band edges. This type of distortion can give rise to pulse distortion extending over appreciable time intervals if the band edge phase deviations are large. This results from the fact that frequency components outside the linear phase range are transmitted with increasing delay.

Consider a low-pass system (or symmetrical bandpass) having a sharp cutoff, no amplitude variations, and a parabolic deviation from a linear phase characteristic between ω' and $\omega_{max} = \omega_1$ (see Fig. 40).

As a result of the assumption of a parabolic phase deviation, the delay distortion varies linearly in the band between ω' and ω_1. For this example,

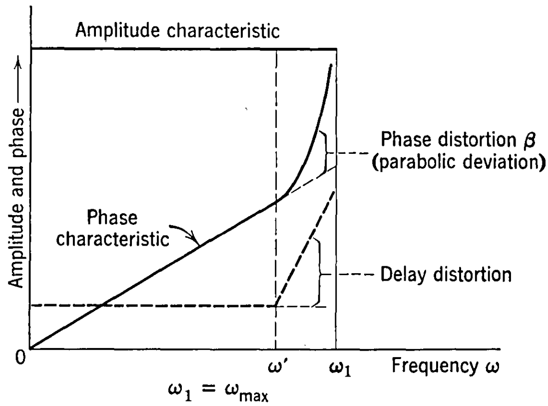


FIG. 40. Constant amplitude characteristic with band edge phase distortion.

the phase deviation β is given by

$$\beta = \beta_1[(\omega - \omega')/(\omega_1 - \omega')]^2,$$

β_1 = Phase deviation from linear phase characteristic at frequency ω_1.

For this example, Sunde gives the following rms intersymbol interference for impulse type signals:

$$\underline{U} = (\pi/\omega_1\tau)^{1/2}[(\omega_1 - \omega')/\omega_1]^{1/2}F(\beta_1),$$

ω_1 = Low-pass sharp cutoff bandwidth,

τ = Pulse separation,

β_1 = Phase deviation at band edge ω_1,

$F(\beta_1)$ = the table below.

β_1	0	0.25	1	4	∞
$F(\beta_1)$	0	0.14	0.43	1.24	1.42

If, for example, phase distortion were confined to the upper 10% of the transmission band ω_1, then $[(\omega_1 - \omega')/\omega_1] = 0.1$. For a maximum phase deviation of 1 radian at the edge of the transmission band, F equals 0.43 and $\underline{U}$ equals 0.135 if $\tau = 1/2f_1$. By allowing a peak factor of 4, the peak intersymbol interference becomes 4 (0.135) = 0.540 or 54%.

The severe tolerances on band edge phase deviation for a sharp cutoff low-pass system can be overcome by employing a gradual cutoff natural linear phase shift characteristic as indicated in Fig. 41. Note that the transmission bandwidth $\omega_{max} = 2\omega_1$ is doubled. If the phase characteristic is linear between $\omega = 0$ and ω_1, and if the phase deviation is parabolic be-

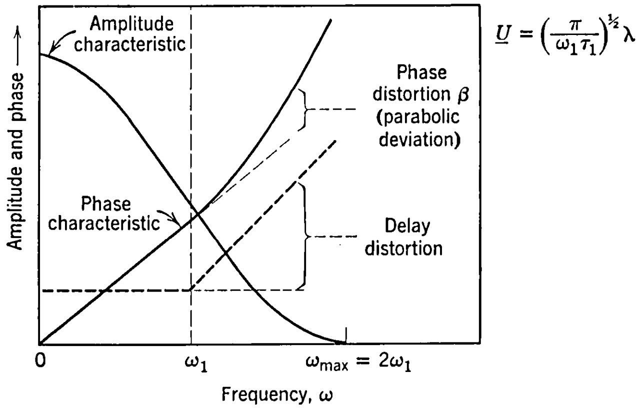


FIG. 41. Typical transmission frequency characteristic with phase equalization over 50% of transmission band.

tween ω_1 and $\omega_{\max} = 2\omega_1$, the phase deviation β becomes

$$\beta = \beta_1[(\omega - \omega_1)/(2\omega_1 - \omega_1)]^2 = \beta_1(1 - \omega/\omega_1)^2$$

where β_1 is phase deviation from a linear phase characteristic at $\omega = 2\omega_1 = \omega_{\max}$.

Then

$$\underline{U} = (\pi/\omega_1\tau)^{\frac{1}{2}}\lambda = \text{Rms intersymbol interference}$$

where

β_1	π	2π	4π	∞
$d_{\max}f_{\max}$	2	4	8	∞
λ	0.070	0.120	0.185	0.330

Note that if the pulse separation τ is the maximum given by $\tau = 1/2f_1$, then $\underline{U} = \lambda$. For the assumed parabolic phase deviation, the delay distortion varies linearly and is a maximum $d_{\max}$ at the band edge:

$$d_{\max} = 2\beta_1/\omega_1.$$

The product of this delay distortion with the maximum frequency $f_{\max}$ is

$$d_{\max}f_{\max} = 2\beta_1/\pi.$$

The quantity $d_{\max}f_{\max}$ is also tabulated above.

Symmetrical Systems—Amplitude and Phase Tolerances vs. Dynamic Range

In a symmetrical system, the amplitude characteristic has even symmetry and the phase characteristic odd symmetry with respect to a properly chosen frequency. A low-pass transmission system is thus symmetrical with respect to zero frequency, if negative frequencies are included. A double sideband system is symmetrical if the amplitude characteristic has even symmetry and the phase characteristic odd symmetry with respect to the midband frequency.

The previous discussion has been concerned with the transmission of bipolar pulses having a fixed amplitude. Sunde has extended the evaluation of transmission systems to include the transmission of pulses of varying amplitude (Ref. 54).

If a continuous analog signal is considered to have q identifiable amplitudes and if a time resolution of τ seconds is required to resolve the detail in the signal, then the transmission of the analog signal is equivalent to the transmission of impulse type signals having q amplitudes and a separation of τ seconds (see Ref. 55), page 85, and Ref. 58, page 33). If the amplitude increments are equal, the successful recovery of the signal will require the ability to distinguish the smallest amplitude increment A_{max}/q. Further, the distortion or intersymbol interference introduced by the system must be less than or at most equal to A_{max}/q, the smallest increment. Letting K represent a peak factor relating the rms intersymbol interference, Sunde (Ref. 54) has derived the following expression relating dynamic range and permissible distortion or intersymbol interference:

$1/(q-1) = L\underline{U}(\underline{A}/A_{max})$ limiting relation between intersymbol interference and message dynamic range,

q = Dynamic range,

= Number of identifiable amplitude increments,

L = Peak factor applicable to $\underline{U}$,

$\underline{U}$ = Rms intersymbol interference,

$\underline{A}/A_{max}$ = Ratio rms signal amplitude to maximum or peak amplitude.

If all negative and positive amplitudes have equal probability, then there are $q/2$ negative and $q/2$ positive amplitudes with equal steps $\dfrac{2A_{max}}{q-1}$ between pulse amplitudes. For the case of equal probability of amplitudes

$$\underline{A}/A_{max} = [(q+1)/3(q-1)]^{1/2}.$$

For this case, the limiting relationship becomes

$$1 = L\underline{U}[(q-1)/3]^{1/2}.$$

For large dynamic range $q \gg 1$:

$$1 \cong \frac{L}{\sqrt{3}} \underline{U} q,$$

$$1 \cong 2\underline{U}q,$$

$$L = 3.46 = \text{Peak factor.}$$

Thus, in order to transmit a bipolar message signal having a bandwidth f_1 or a time resolution $\tau = 1/2f_1$ with an accuracy of 1% ($q = 200$), the rms interference $\underline{U}$ must be less than or equal to ¼% over the bandwidth $0 - f_1$. Tolerances of this nature are difficult to realize and in practice can only be approached if transmission bandwidths 10 to 100 times greater than message bandwidth are utilized. The above result is a basic consideration in choosing between an analog or digital transmission system for non-steady-state data.

The rms interference can be evaluated from the expression for $\underline{U}$ in terms of $\underline{a}$ and $\underline{b}$, the rms amplitude and phase variations:

$\underline{U} = (\underline{a}^2 + \underline{b}^2)^{1/2}$ = Normalized rms interference,

$\underline{a}$ = Normalized rms amplitude variation over the message bandwidth ω_1,

$\underline{b}$ = Rms phase deviation (radians) over the message bandwidth.

Since the message is assumed to contain no frequencies above ω_1, then the quantities $\underline{a}$ and $\underline{b}$ need only be evaluated over the message bandwidth ω_1. If the message contains noise which includes spectral components beyond ω_1, it is necessary to control the amplitude and phase characteristic for frequencies beyond ω_1.

Limitation of Information Capacity by Distortion

For an ideal system with a bandwidth f_1 and a sharp cutoff characteristic with linear phase, the transmission capacity in bits per second is given by the Shannon-Hartley law:

$$C = f_1 \log_2 (1 + S/N).$$

This expression gives the limitation on channel capacity imposed by random noise. From the previous discussion, it is evident that a limitation also exists if distortion is present in the absence of noise. In idealized communication theory, distortion is disregarded in determining channel capacity on the premise that it is predictable and can therefore be corrected, at least in principle. In actual systems, however, complete elimination, although possible in principle, cannot be accomplished.

Consider a system in which $A_{\max}$ is the maximum amplitude and $\underline{A}_n$ is an rms noise amplitude. For convenience, let A refer to the amplitude of a small rectangular pulse which has a width equal to the time resolution required (Ref. 58). Then the number of identifiable amplitudes q is related to $A_{\max}$ and $\underline{A}_n$ by

$$A_{\max}/(q - 1) = L\underline{A}_n,$$

$$L = \text{Peak factor},$$

$$1/(q - 1) = L(\underline{A}_n/\underline{A})(\underline{A}/A_{\max}),$$

where $\underline{A}$ equals the rms amplitude of A. But from the previous section,

$$1/(q - 1) = L\underline{U}(\underline{A}/A_{\max}).$$

Hence

$$\underline{U} = \underline{A}_n/\underline{A};$$

by letting $D = \underline{U}^2$, then

$$\underline{U}^2 = D = \text{Noise power/Signal power}$$

This means that random characteristic distortion has the same effect as a random noise power DS, where D is a distortion factor.

In view of the above equivalence, the channel capacity in the presence of random distortion but without noise is

$$C = f_1 \log_2 (1 + 1/D) \qquad f_1 = B_{\frac{1}{2}a}.$$

With random interference from both distortion and noise, the interfering powers add directly so that (Ref. 54)

$$= f_1 \log_2 [1 + S/(DS + N)].$$

Since

$$D = \underline{U}^2 = \underline{a}^2 + \underline{b}^2,$$

it is necessary that $D \ll N/S$ for faithful reproduction of a transmitted signal as in a data transmission system. In the above, $\underline{a}^2$ is the normalized mean square amplitude variations and $\underline{b}^2$ the mean square phase variations, both taken over the message bandwidth.

Note that, unlike random noise, the transmission capacity of a system with distortion cannot be increased by increasing signal power. For a given system with D and the noise power N specified, the effective signal-to-noise ratio $S/(DS + N)$ is limited to a maximum of $1/D$ for large signal power and varies as S/N for small signal power.

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Nuclear Reactor Control

W. E. Shoupp and M. A. Schultz

1. INTRODUCTION

General Requirements—Nuclear Reactor Control. There are two reasons why the requirements of a control system for a nuclear reactor or power plant are different from those of any other system. First, safety requirements of a nuclear power plant are to some extent actually stipulated by law. Second, a nuclear reactor is a device which, if its control system fails, can create not only local damage but also damage on a statewide basis. This is a new concept in liability, and consequently, the primary requirement of a reactor control system is one of extreme safety. Secondary requirements of performance, efficiency, and economics are completely overshadowed by the safety requirements in the design of the control system.

Description of a Nuclear Reactor (Ref. 1). There are many types of nuclear reactors and to examine the control problem, a *heterogeneous thermal power reactor* may be used as an example. A heterogeneous reactor is one whose fuel is in discrete-sized pieces placed in definite relationship to other reactor components. Figure 1 indicates the elementary

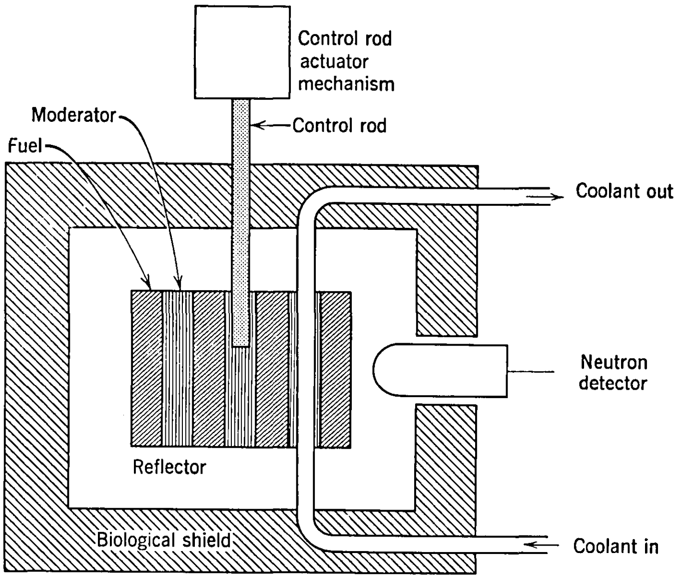

FIG. 1. Basic components of a power reactor (Ref. 3).

components of such a reactor. The fuel is located in the center section of the reactor which is sometimes called the *core*. This fuel is in solid form for a heterogeneous reactor and consists principally of uranium, either U^{235} or U^{238}, singly or in some combination. Other fuels, such as plutonium or thorium, may also be used.

Homogeneous reactors are those containing fuels in liquid or slurry form. In either case the fuel is generally intermingled with a moderator which is some suitable light element such as hydrogen, beryllium, or carbon. The *moderating material* is used to slow down neutrons created in the fission process to provide the desired energy spectrum. Passing through the core, or in close contact with it, is a heat transfer material or coolant. Gases, water, or liquid metals may all be used as coolants. Outside the core proper is some *reflecting material* which is used to conserve neutrons and reflect them back into the core.

Surrounding the reflector is a *biological shield* which attenuates the nuclear radiations emanating from the core. Inside the core or reflector is equipment for regulating the power level of the core by controlling the number of neutrons in it. The principal devices used for regulation are generally called *control rods*. Many other devices are available to change the number of neutrons in a reactor, but these devices have the same general effect as control rods.

Descriptive Terms and Definitions. These definitions are not completely accurate from an analytical point of view. The definitions have been abstracted with permission from Ref. 3.

Fission Process. When a neutron at a given energy is absorbed by a uranium nucleus, there is a finite probability this nucleus will split into two or more fragments. This process is called *fission,* and in addition to releasing a large amount of heat, two or three more neutrons each having an approximate energy of 2 Mev are released from the fragments. These neutrons are then available to split more uranium nuclei.

Multiplication Factor (k). The ratio of the number of neutrons in any one generation to the number of corresponding neutrons of the immediately preceding generation is known as the *multiplication factor.* If k is equal to or greater than unity, a *chain reaction* can take place. If k is less than unity the reaction will ultimately die down.

Reactivity (δk). *Reactivity* is proportional to the amount the multiplication factor differs from unity. $\delta k = (k - 1)/k$.

Neutron Lifetime (l). The average time between successive neutron generations in a reactor is defined as the neutron lifetime. l^* is used for the mean effective lifetime of a neutron in a finite reactor containing U^{235} ($l^* = l/k$).

Thermal Reactor. A reactor in which the bulk of the fissions are caused by neutrons having kinetic energies close to thermal levels (approximately 0.025 to 0.1 ev) is called a *thermal reactor.* Thermal reactors have neutron lifetimes in the 10^{-3} to 10^{-5} sec range.

Fast Reactor. A reactor in which the fission produced neutrons are not slowed down appreciably before fission again occurs is called a *fast reactor.* These reactors have neutron lifetimes in the 10^{-6} to 10^{-8} sec range.

Intermediate Reactor. A reactor using primarily neutrons in the region between fission and thermal energies is called an intermediate reactor. Neutron lifetimes are correspondingly between 10^{-5} to 10^{-6} sec.

Neutron Level (n). The number of neutrons in the core is proportional to the number of fissions occurring. For 3×10^{10} fissions per second one watt of power is produced.

Reactor Period (P). The period of a reactor can arbitrarily be defined as

$$P = \frac{1}{\dfrac{1}{n}\dfrac{dn}{dt}}$$

where n is the neutron level and dn/dt is the rate of change of neutron level.

Delayed Neutrons (β, β_i). Not all the neutrons that are created in the fission process are given off instantly. A small fraction of the neutrons are given off at discrete times after the fission process occurs. For a reactor fueled with U^{235} approximately 0.75% of the total neutrons procured are *delayed neutrons.* At least six distinct groups of neutrons are given off at different times and in different quantities. The symbol β is used to denote the total fraction of the delayed neutrons, with β_i being the fraction of the delayed neutrons in the ith group. The symbol λ_i is used to represent the decay constant of the ith group of delayed neutrons and is an inverse time constant.

Reactor State. This is defined by the value of the multiplication factor.

Subcritical Reactor. $k < 1$.

Critical Reactor. $k = 1$.

Supercritical Reactor. $k > 1$.

Prompt Critical Reactor. When a reactor is capable of sustaining a chain reaction without the use of the delayed neutrons, the reactor is said to be *prompt critical.* This corresponds to a multiplication factor $k = 1.0075$ for a thermal reactor containing U^{235} fuel.

Temperature Coefficient. The amount that the reactivity of a reactor changes per unit change in temperature is defined as the *temperature coefficient.* If, as a reactor heats up, its reactivity increases as a function of temperature, the reactor has a positive temperature coefficient. If as the reactor heats up its reactivity decreases, the reactor has a negative coefficient. Most present-day reactors are designed to have negative temperature coefficients.

Fission Product Poisoning. After a thermal reactor has been operated for a while, certain unwanted fission products, called *poisons,* are formed which have large probabilities of absorbing neutrons and removing them from the chain reaction. Two of the most common are $xenon^{135}$ and $samarium^{149}$. If these nuclear poisons are produced in appreciable amounts, they can affect the overall multiplication factor of the reactor.

Safety Rods. Control rods made of neutron-absorbing material, whose functions are primarily to shut down a reactor quickly, usually by rapid insertion, are called *safety* or *scramming rods.*

Shim Rods. Control rods which change large amounts of reactivity but which are moved slowly, are sometimes called *shim rods.*

Regulator Rod. A control rod which is used in an automatic control system to maintain a given power level or to change this power level is called a *regulator rod.* Regulator rods can, if necessary, be moved quickly but can change only small amounts of reactivity (usually limited to less than the difference in reactivity between critical and prompt critical).

2. REACTOR CONTROL SYSTEM REQUIREMENTS

Basic Functions of the Overall Reactor Control System

The basic functions are to start up, shut down, and to operate the reactor. These control functions are usually performed by three distinct interconnected systems. Figure 2 illustrates an elementary block diagram of a reactor control system.

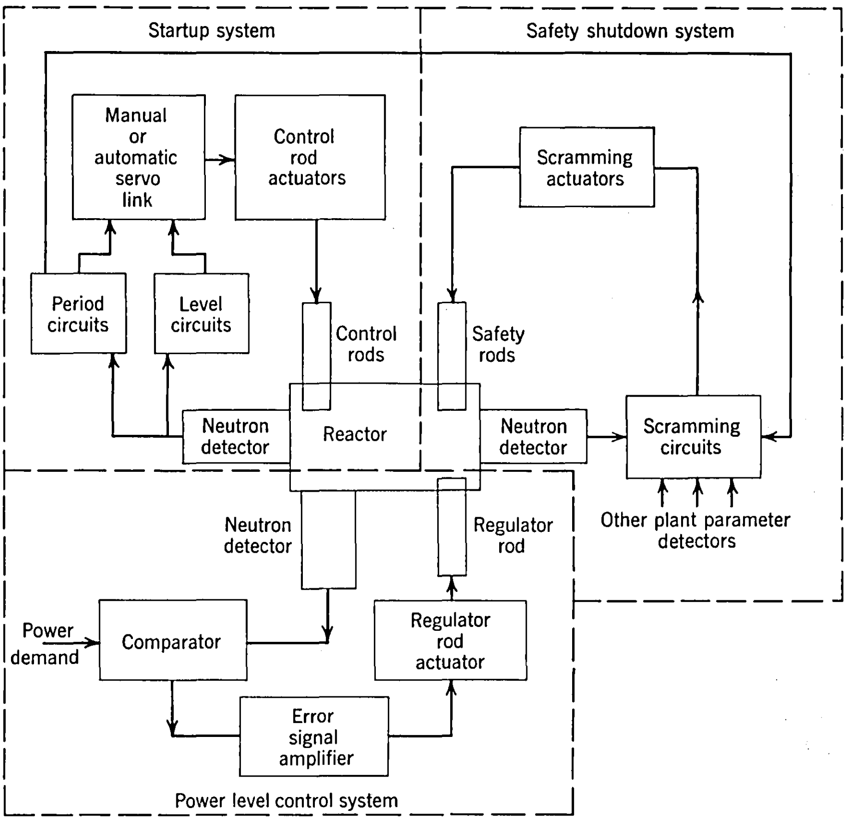

FIG. 2. Elementary block diagram—reactor control system.

The startup system contains a *neutron detector* or detectors which are capable of measuring neutron level over a wide range—up to ten decades. The range from 0 power to full power in a reactor covers nuclear fissions resulting in a few neutrons per second to many billions per second. No one detecting element is capable of covering this range, so a multiplicity of instrumentation is used. Neutron level, n, and period, P, are both measured and indicated. If the rate of change of power level as indicated by the period circuits is too high, a connection is made to the scramming circuits of the safety shutdown system whereby the reactor is quickly shut off or scrammed. Startup periods between 10 and 60 seconds are presently employed. The level in a startup is usually controlled by withdrawing control rods from the reactor. After the reactor is made critical, the control rods change the multiplication factor to slightly above one creating an excess of neutrons, and the power level is made to rise slowly. The neutron level may be observed by an operator or automatically and a measure of the level is used to control an actuator mechanism driving the control rod or rods. This type of startup system is used to bring the reactor into its normal power operating range.

The power level control system which takes over at this point starts from two inputs, first a neutron detector whose output is proportional to the power level, and a power demand reference signal representing the desired reactor output. These two signals are compared and any error signal amplified to control the motion of the regulator rod through an actuator mechanism. The regulator rod position then continuously changes the multiplication factor of the reactor to keep the power output at the demanded level.

The safety shutdown system is superimposed on the other controls for supersafety. Here many detectors are used, including a neutron detector and sometimes input signals from many other plant parameters. Variables such as temperature, pressure, coolant flow, and interlocks can be used to determine that the plant is not behaving properly and the reactor can be shut down immediately via the scramming circuits, scramming actuators, and safety rods.

Performance Requirements of the Control System

The startup requirements of the reactor control system are generally specified in terms of time rates. Variables involved are the neutron level, the startup level, the range of the startup operation, and the periods that will be permitted. In addition, a drastic limitation on startup time rates may be placed upon the reactor by thermal requirements of some of the components or auxiliaries. For example, a pressure vessel made of thick walls might have a limitation in temperature stress such that the temperature of the material inside the vessel would be permitted to change only by a degree or so per minute. Consequently, overall reactor startup times ranging from 15 minutes to several hours may be involved.

Power operation requirements are given in terms of stability, accuracy of power level demanded, time needed to change power level, and peak transient level permitted from either an internal or external cause.

The reactor shutdown requirements stipulate that no damage be caused to the reactor and no ancillary problems be caused in the plant or the neighborhood. A reasonable peak power level and energy limit in a transient burst must be specified. In addition, a listing of auxiliary failures which might cause reactor damage, such as loss of coolant flow, must be completely specified by a peak reactor temperature limit and a given amount of tolerable output energy. It is the function of the control system to see that all of these requirements are met.

3. THE REACTOR AS A SERVOMECHANISM COMPONENT

The Reactor as a Control Component without Temperature Coefficient. In the block diagram of Fig. 2, the reactor is a component in each of the three functional control systems. Its performance as a control element is therefore needed and this performance is most readily obtained by determining its *transfer function.*

The basic kinetic equations that determine the time behavior of an elementary reactor *not possessing a temperature coefficient* are (Ref. 2):

$$\text{(1)} \qquad \frac{dn}{dt} = \frac{\delta k - \beta}{l^*} n + \sum_{i=1}^{6} \lambda_i C_i \quad \textit{(Neutron level equation)}$$

and

$$\text{(2)} \qquad \frac{dC_i}{dt} = \frac{\beta_i}{l^*} n - \lambda_i C_i \quad \textit{(Delayed neutron equation)},$$

where n = the neutron level,
δk = the reactivity,
β = the fraction of the delayed neutrons,
β_i = the fraction of the delayed neutrons in the ith group,
l^* = the mean neutron lifetime,
λ_i = the decay constant of the ith group of delayed neutrons,
C_i = the concentration of the ith group of delayed neutrons.

Transfer Function. The solution of these equations for a sine wave input change in reactivity $\delta k(s)$ involves a linearization process in which the reactor is assumed to be operating at a steady state level n_0 and has a small perturbation $\delta n(s)$ superimposed upon it. The transfer function has been derived in this manner by several authors (Refs. 3–6), and is of the form:

$$\text{(3)} \qquad K_R G_R(s) = \frac{\delta n(s)}{\delta k(s)} = \frac{n_0}{l^*} \frac{1}{s\left[1 + \sum_{i=1}^{6} \frac{\beta_i}{l^*(s + \lambda_i)}\right]},$$

where $K_R G_R(s)$ is the reactor transfer function and $s = j\omega$ is the Laplace transform operator. For a U^{235} fueled reactor with a lifetime $l^* = 1.25 \times 10^{-3}$ sec and five principal groups of delayed neutrons, the reactor transfer function has been given as (Ref. 5)

$$\frac{\delta n(s)}{\delta k(s)} = \frac{n_0(s+14)(s+1.61)(s+0.456)(s+0.154)(s+0.0315)}{l^* s(s+14.4)(s+5.41)(s+1.41)(s+0.32)(s+0.08)}. \tag{4}$$

Reference 3 indicates the transfer function for a similar reactor with an $l^* = 10^{-4}$ sec and six groups of delayed neutrons as

$$\frac{\delta n(s)}{\delta k(s)} = \frac{\begin{Bmatrix} n_0(s+14)(s+1.61)(s+0.456) \times \\ (s+0.151)(s+0.0315)(s+0.0124) \end{Bmatrix}}{\begin{Bmatrix} l^* s(s+77)(s+13.38)(s+1.43) \times \\ (s+0.336)(s+0.0805)(s+0.0147) \end{Bmatrix}}. \tag{5}$$

Bode Diagram. Figure 3 indicates the transfer function gain and phase shift for reactors having different lifetimes but all possessing zero temperature coefficients. In this figure the gain is normalized to equal zero db at one cycle per second for $l^* = 10^{-4}$ sec.

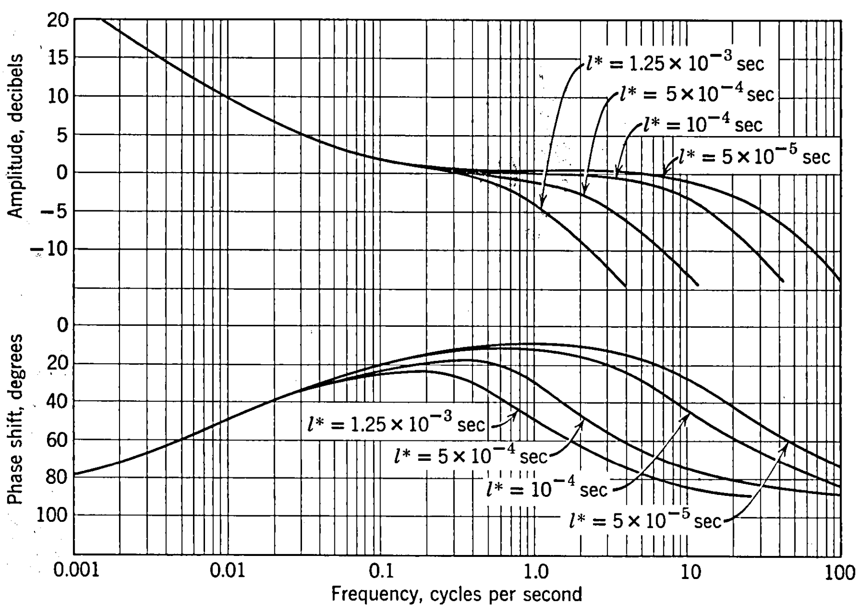

FIG. 3. Transfer functions of a U^{235}-fueled reactor with no temperature coefficient (Ref. 3).

Control features to be noted from eqs. (3–5) are: (*a*) the nonlinearity of the transfer function, i.e., the gain of the reactor as a circuit element depends upon n_0 the level at which the reactor is operating; (*b*) at zero frequency the gain is infinite connoting a catastrophic type of instability; and (*c*) the break point of highest frequency in the reactor transfer function is determined in eqs. (4) and (5) by the largest root, which in turn depends upon the value of β/l^*. It is the value of this root which specifies the time behavior that distinguishes controlwise a *thermal* reactor from an *intermediate* or *fast reactor.* In a fast reactor the value of l^* is so small that the last break in the Bode diagram occurs at a very high frequency well above 100 cps. Difficult component design problems arise in control systems operating in this frequency range. From the physical point of view, control out to this last break represents control on prompt neutrons. This procedure is generally unnecessary and slower control systems are more desirable.

The Reactor as a Control Component with Simple Temperature Coefficient Feedback (Refs. 7, 8)

The temperature coefficient in a reactor can be handled in two ways.

The *first method* is to modify the basic reactor kinetic eqs. (1) and (2) to include the effects of temperature. Solutions of these equations can be obtained with some approximation being needed. This method has been used successfully by Weinberg and Ergen (Ref. 9) for homogenerous reactors and by Lipkin (Ref. 10) for heterogeneous reactors. A complete example of this method and solution is presented in Rodgers and Thompson (Ref. 1).

The *second method* consists of obtaining the transfer function of a simple reactor as previously derived and modifying this transfer function by feedback networks which contain temperature effects. The feedback path in general consists of many branches which provide components of both negative and positive feedback. The combined feedback paths around the reactor usually results in an overall degenerative or negative feedback effect.

Transfer Function. This complex feedback in simplified form can be thought of as being in two basic parts. First, the power output of the reactor is transferred as heat from the reactor fuel to the moderator, and then a change in reactivity is caused by the change in temperature and density of the moderator and the fuel. Figure 4*a* indicates in block form these transfer functions with the temperature coefficient effect having the transfer function notation $K_{TC}G_{TC}(s)$. The two transfer functions can be combined as indicated in Fig. 4*b* into a new overall combined reactor and temperature coefficient effect $K_{RTC}G_{RTC}(s)$. This combined *reactor trans-*

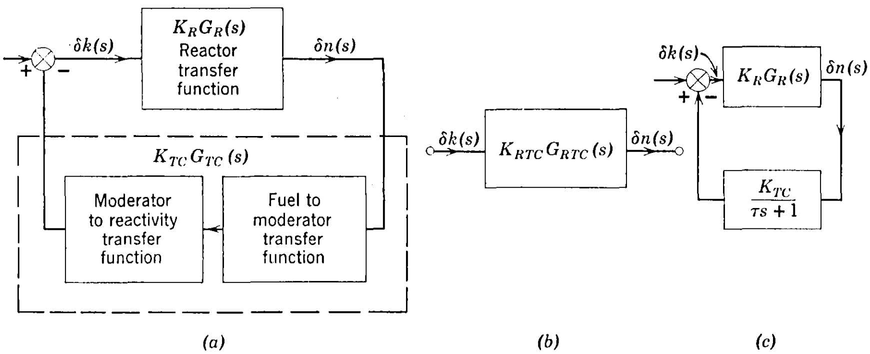

FIG. 4. Reactor transfer function with simple temperature coefficient feedback: (*a*) transfer of heat from fuel to moderator and moderator temperature affecting reactivity; (*b*) combined transfer function, reactor, and feedback; (*c*) temperature coefficient feedback in terms of single time lag.

fer function with temperature coefficient is of course derived as

$$K_{RTC}G_{RTC}(s) = \frac{K_R G_R(s)}{1 + K_R G_R(s) K_{TC} G_{TC}(s)}, \tag{6}$$

where $K_R G_R(s)$ is the transfer function of the elementary reactor as given in Fig. 3. A simple form of $K_{TC}G_{TC}(s)$ can be analyzed with fair accuracy by considering the two part process to consist of a lumped time lag in transferring the heat from the reactor to the moderator, and a gain term related to the power level and the value of the negative temperature coefficient. This type of approximation is quite accurate for homogeneous reactors of the type treated by Weinberg and Ergen (Ref. 9) and for some heterogeneous reactors. It is assumed that one simple time lag exists between the neutron power output and the ultimate reactivity change caused by the temperature coefficient. The transfer function of this lag is given by

$$K_{TC}G_{TC}(s) = \frac{K_{TC}}{\tau s + 1}, \tag{7}$$

where K_{TC} = constant multiplied by the temperature coefficient,
τ = the overall time constant of the thermal lag,
s = the Laplace transform operator.

The block diagram of this system is indicated in Fig. 4*c*.

Bode Diagram. An example of this combined reactor-temperature coefficient transfer function is presented in the Bode diagram of Fig. 5 for several values of K_{TC}, $\tau = 0.159$, and $l^* = 10^{-4}$ sec.

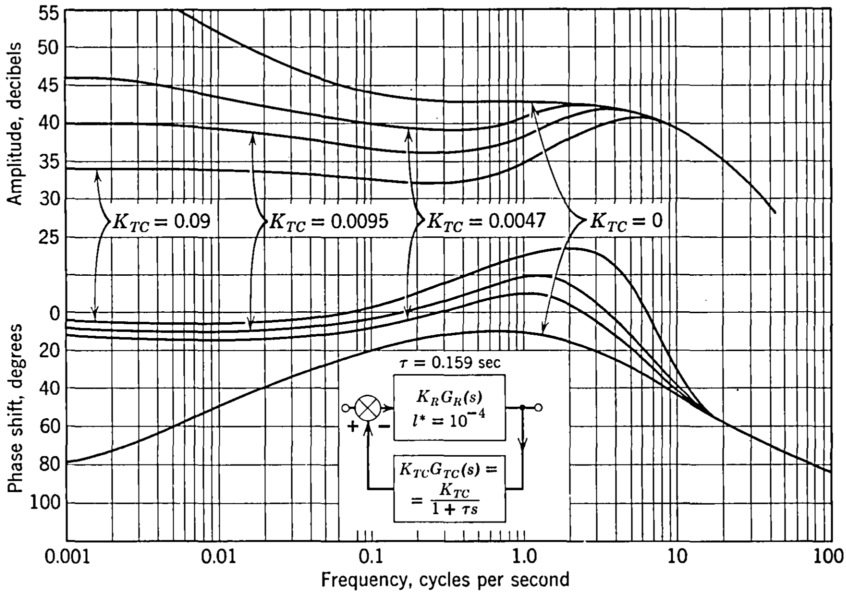

FIG. 5. Reactor transfer function with single time lag temperature coefficient feedback (Ref. 3).

Control Features

1. The amplitude curves of Fig. 5 indicate that at very low frequencies the overall combined transfer function is determined solely by the temperature coefficient effect feedback.

2. The gain at zero frequency is no longer infinite but a finite value equal to $1/K_{TC}$.

3. At very high frequencies the combined transfer function gain curves take on the shape of the elementary reactor transfer function whose high-frequency response depends principally upon l^*.

4. If the break frequency caused by the time delay τ is low compared with the highest natural break frequency of the reactor set by β/l^*, the high-frequency response of $K_{RTC}G_{RTC}(s)$ is not affected by the temperature coefficient.

5. Figure 5 also indicates that at low frequencies the phase shift for reactors with simple negative temperature coefficients approaches zero instead of 90° as in the case of elementary reactors.

6. At high frequency again the phase shift of the combined transfer function approximates the phase shift of the elementary reactor. Some phase lead may result from the combination of reactor and temperature coefficient. The position and magnitude of this lead would depend upon the value for τ.

Combined Transfer Function. The elementary representation of the combination of these two simple transfer functions (Eqs. 6, 7) is quite stable. Harrer and DeShong (Ref. 11) have derived and measured the transfer function for the CP-3 reactor, and from their work it is apparent that this type of transfer function form for the negative temperature coefficient feedback is a good approximation for their reactor.

The *behavior of circulating fuel reactors* has been formulated as a function of temperature coefficient under conditions of constant temperature. Reference 12 indicates that circulating fuel reactors are also very stable and that they can be made as stable as desired by making the parameter $\delta k t_1/l^*$ sufficiently small. In this expression δk is the reactivity, t_1 is the fuel transit time through the core volume, and l^* the mean neutron lifetime.

The Reactor as a Control Component with Two-Path Temperature Coefficient Feedback

A somewhat better reactor temperature coefficient feedback representation is obtained for heterogeneous reactors if the power out of the reactor is first presumed to heat up the fuel and change its temperature and then transfer heat to the moderator and then also change its temperature. If the fuel possesses a local temperature coefficient and the moderator possesses a local temperature coefficient, a block diagram similar to that of Fig. 6 results. From this figure, after the temperatures of both fuel and moderator are obtained, they are multiplied by local temperature coefficients α_F and α_M, and the resulting local reactivities can be added to produce the overall reactivity feedback. The time-dependent descriptions associated with this two-path feedback are given by (Refs. 10, 13):

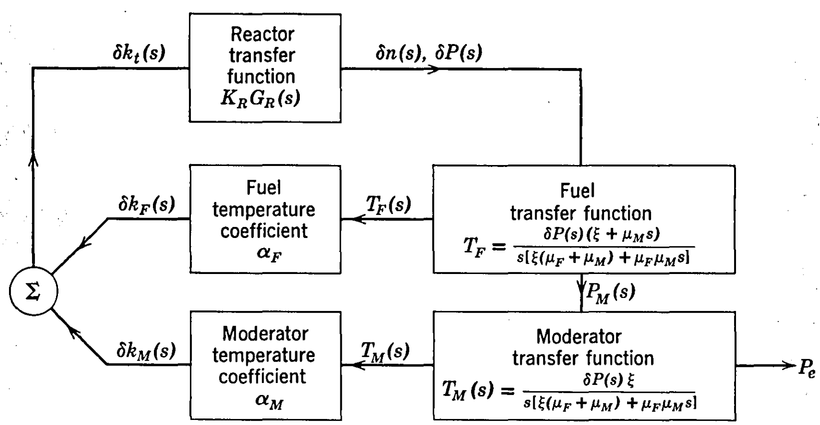

FIG. 6. Block diagram two-path temperature coefficient feedback.

(8) $$P = \mu_F \frac{dT_F}{dt} + P_M,$$

(9) $$P_M = \mu_M \frac{dT_M}{dt} + P_e,$$

(10) $$P_M = \zeta(T_F - T_M),$$

where T_F = fuel temperature;
T_M = moderator temperature;
$\mu_F = W_F C_F$ = the weight of the fuel times the specific heat of the fuel;
$\mu_M = W_M C_M$ = the weight of the moderator times the specific heat of the moderator;
ζ = a heat transfer coefficient between fuel and moderator; ζ is generally a function of power level but for small sinusoidal changes will be a constant;
P_e = the power output taken from the coolant which is also assumed to be constant at any convenient reactor output such as full power;
P_M = the heat transferred from the fuel to the moderator.

After the above equations have been solved simultaneously for the temperatures in the frequency domain and the temperatures multiplied by the temperature coefficients to give reactivity, the transfer function of the combined reactivity feedback becomes

(11) $$K_F G_F(s) = \frac{\delta k_t(s)}{\delta P(s)} = \frac{\zeta(\alpha_M + \alpha_F) + \alpha_F \mu_M s}{s[\zeta(\mu_M + \mu_F) + \mu_F \mu_M s]}.$$

Stability

1. Examination of the open loop product $K_R G_R(s)\ K_F G_F(s)$ by the Nyquist criterion indicates that there are many combinations of the local temperature coefficient values and geometric reactor parameters that must be examined before stability can be determined. Figure 7 shows the various regions of stability that are obtained by plotting α_F against α_M.

2. From an elementary physical point of view, if both α_F and α_M are positive, i.e., there is an overall gross positive temperature coefficient, then the loop is obviously an unstable one.

3. If the sum of a positive temperature coefficient and a negative temperature coefficient in either combination of α_F and α_M is positive, then the loop again is unstable. These situations are indicated by the area to the right of the 45° line in Fig. 7.

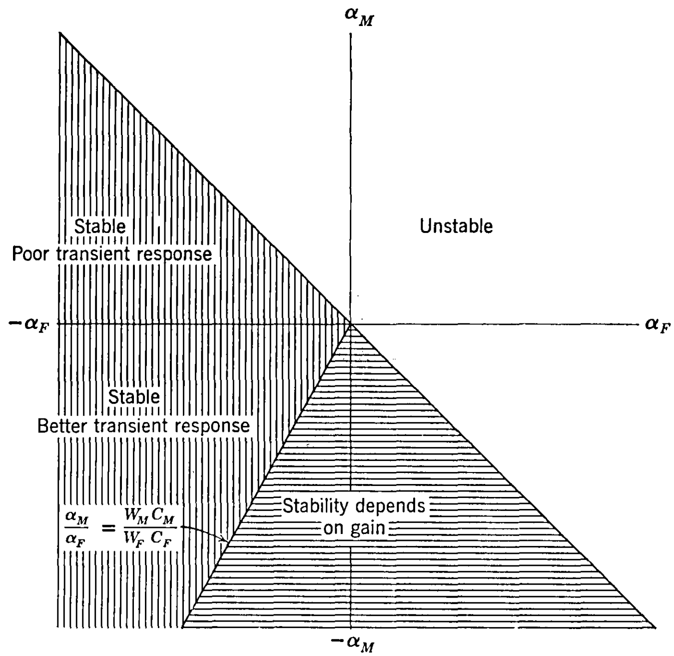

FIG. 7. Plot of α_F versus α_M indicating stable and unstable regions.

4. α_F is the dominant temperature coefficient. Feedback through the fuel occurs quicker than feedback through the moderator because of the series time delay in transferring the heat from fuel to moderator. Consequently, if α_F is negative and greater in magnitude than a positive α_M, the loop will always be stable.

5. On the other hand, the completely stable situation does not exist if α_M is negative and greater in magnitude than a positive α_F. In this case the loop may or may not be stable, depending upon the gain of the system.

6. Reference 13 indicates that a desirable design objective for complete stability and good transient response is achieved if

$$\frac{\alpha_M}{\alpha_F} < \left|\frac{\text{Moderator}}{\text{Fuel}}\right|_{\text{Weight}} \frac{C_M}{C_F}. \tag{12}$$

Bode Diagram. Figure 8 indicates the type of Bode diagram which results from this type of two-path feedback. This figure is an example of a reactor system in which the overall combined feedback of eq. (11) is simplified into:

$$K_F G_F(s) = \frac{A(1 + \tau_1 s)}{s(1 + \tau_2 s)}, \tag{13}$$

where

$$A = \frac{\alpha_M + \alpha_F}{\mu_M + \mu_F}; \quad \tau_1 = \frac{\alpha_F \mu_M}{\zeta(\alpha_F + \alpha_M)}; \quad \tau_2 = \frac{\mu_F \mu_M}{\zeta(\mu_F + \mu_M)}. \tag{14}$$

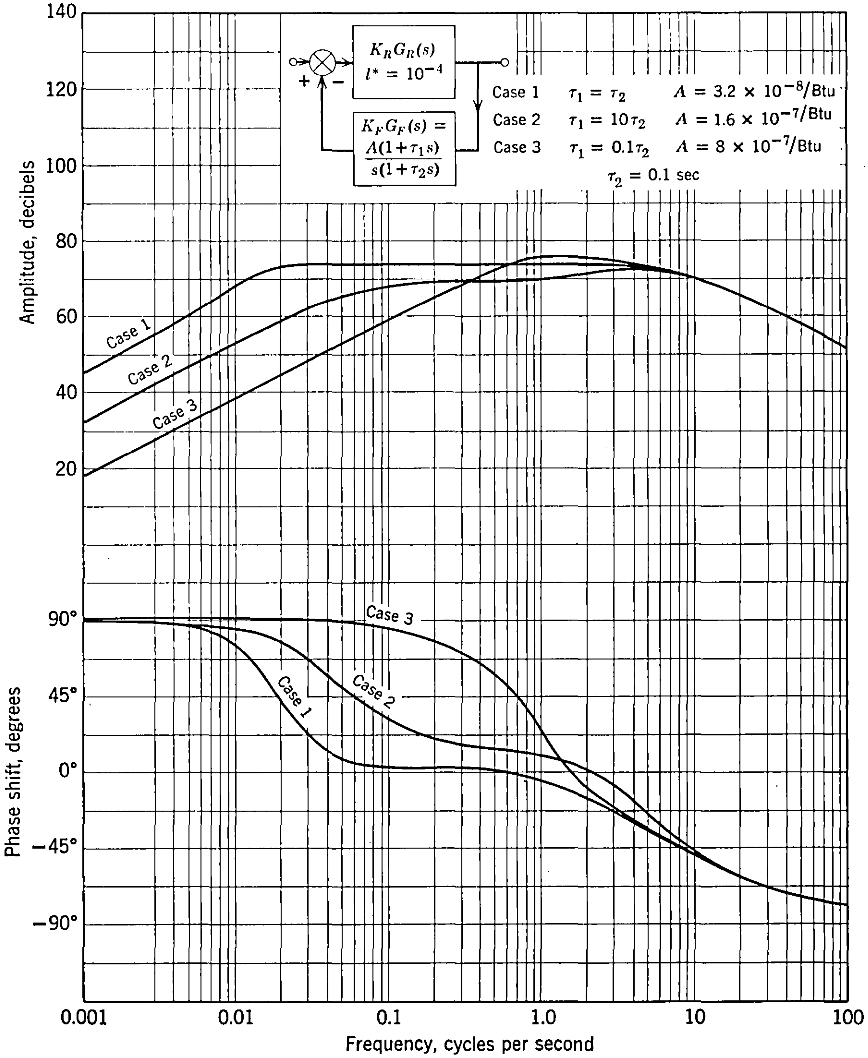

FIG. 8. Example of reactor transfer function with two-path temperature coefficient feedback (Ref. 3).

From Fig. 8 it can be seen that some of the dominant features of the single-time lag feedback given in Fig. 5 are still retained, i.e., infinite gain does not occur at zero frequency, and the high-frequency portions of the diagram are completely determined by the simple reactor neutron lifetime. The phase shift portion of the diagram also indicates that this configuration should be quite stable.

The Reactor as a Control Component with Multiple-Path Temperature Coefficient Feedback

The process indicated in Fig. 6 can be refined still further and multiple temperature coefficient feedbacks defined. Figure 9 indicates the block diagram of the temperature coefficient feedback paths for a testing reactor similar to the materials testing reactor (Ref. 14).

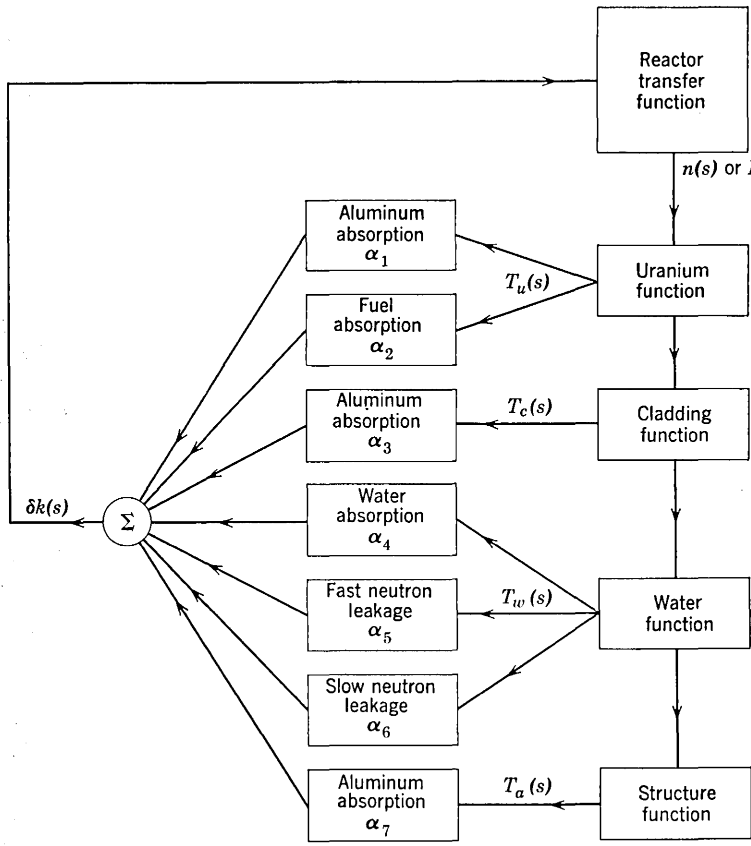

FIG. 9. Block diagram multiple path temperature coefficient feedback for a testing reactor.

Requirements

1. Here changes in the fuel temperature cause changes in the neutron absorption cross sections of uranium and aluminum.
2. Changes in the water temperature cause water cross-section changes and also affect the fast neutron and slow neutron leakage.
3. Finally, changes in the core structure can cause changes in neutron absorption and dimensional changes in the reactor.

To each one of these changes can be assigned a local temperature coefficient α which may be positive or negative. The net reactivity feedback effect is obtained by summing up all the individual reactivity changes as a function of frequency. The block diagram of Fig. 9 appears to be quite complex, but an overall reactor transfer function may be obtained graphically, or with the aid of an analog computer, providing the various temperatures, coefficients, and time constants can be obtained. The solution to this type of problem is usually a further refinement on the two-path case previously treated in this Sect. 3.

The Reactor as a Control Component with Poisoning Feedback

In a thermal reactor, fission product poisons can build up which affect reactivity. The isotopes causing the most reactivity change are Xe^{135} and Sm^{149}. The behavior of these poisons can also be represented as a feedback path around the reactor in a similar manner to the way that the temperature coefficient was represented. The Sm^{149} isotope behaves as a steadily increasing negative feedback depending upon the neutron flux level in the reactor and the duration of the reactor operation. This isotope is formed as the stable end product of the chain (Refs. 3, 15):

$$\text{(15)}\qquad \mathrm{Nd}^{149} \xrightarrow{1.7\ \mathrm{hr}} \mathrm{Pm}^{149} \xrightarrow{47\ \mathrm{hr}} \mathrm{Sm}^{149} \quad \text{(stable).}$$

This reaction occurs in approximately 1.5% of all fissions and Sm^{149} has a cross section for thermal neutron capture of approximately 5.3×10^4 barns. Kinetically the fission product feedback from Sm^{149} follows the equations

$$\text{(16)}\qquad \frac{dP}{dt} = \gamma_P\phi - \lambda_P P$$

and

$$\text{(17)}\qquad \frac{dS}{dt} = \lambda_P P - \sigma_S\phi,$$

where P = number of Pm atoms present per cubic centimeter at any time t;
S = number of Sm atoms present per cubic centimeter at any time t;

γ_P = fractional yield of Pm considering it to be the direct fission product. The Nd is ignored because the half-life of Nd is small compared with that of Pm; consequently, Pm may be mathematically considered to be the direct fission product;
λ_P = decay constant of Pm^{149};
ϕ = thermal neutron flux used interchangeably with previous definition of n;
σ_S = microscopic thermal neutron cross section of Sm^{149}.

The Xe^{135} reaction has a much higher probability and a more complex decay scheme (Refs. 15, 16).

$$(18) \quad Te^{135} \xrightarrow{1\ \text{min}} I^{135} \xrightarrow{6.7\ \text{hr}} Xe^{135} \xrightarrow{9.2\ \text{hr}} Cs^{135} \xrightarrow{2.1 \times 10^6\ \text{yr}} Ba^{135}.$$

This reaction occurs in approximately 5% of the fission products and as before, the Te^{135} decay to I^{135} can be ignored as occurring quickly compared with the other time constants involved. The buildup of the Xe^{135} poison behaves kinetically as

$$(19) \qquad \frac{dX}{dt} = \lambda_I I + (\gamma_x - \sigma_x X)\phi - \lambda_x X,$$

$$(20) \qquad \frac{dI}{dt} = \lambda_I I + \gamma_I \phi,$$

where X = number of atoms of Xe^{135} present per cubic centimeter at any time t,
I = number of atoms of I^{135} present per cubic centimeter at any time t,
γ_x = fractional yield of xenon as direct fission product,
σ_x = microscopic thermal-neutron absorption cross section of Xe^{135} $\sim (3.5 \times 10^6$ barns),
ϕ = thermal-neutron flux,
λ_I = decay constant of I^{135},
λ_x = decay constant of Xe^{135}.

By using a linearization technique whereby the variables are split into a steady-state term and an incremental variation about this steady state such that

$$(21) \quad X = X_0 + \delta X, \qquad \phi = \phi_0 + \delta\phi, \qquad \text{and} \qquad I = I_0 + \delta I,$$

a transfer function for small sinusoidal signals can be derived. This poisoning feedback transfer function has been found to be (Ref. 22)

$$(22) \qquad \frac{\delta X}{\delta \phi} = \frac{(\gamma_x - \sigma_x X_0)\{s + [\lambda_I \gamma_I / (\gamma_x - \sigma_x X_0)] + \lambda_I\}}{(s + \lambda_I)(s + \sigma_x \phi_0 + \lambda_x)}.$$

Before this expression may be used, a generalized relationship between X_0 and ϕ_0 must be obtained as

$$X_0 = \frac{(\gamma_x + \gamma_I)\phi_0}{\lambda_x + \sigma_x \phi_0}. \tag{23}$$

Characteristics. This expression and the transfer function can be examined numerically by using the following constants from Stephenson (Ref. 16):

$$\lambda_I = 2.9 \times 10^{-5}$$

$$\lambda_x = 2.1 \times 10^{-5}$$

$$\gamma_I = 0.056$$

$$\gamma_x = 0.003$$

$$\sigma_x = 3.5 \times 10^{-18}$$

1. From eq. (23) the steady-state poisoning rises linearly with flux until a flux level of approximately 10^{12} neutrons/cm^2-sec is reached.

2. At higher flux levels the Xe concentration rises more slowly until, at flux levels of approximately 10^{14} and higher, there is no further increase in poison concentration.

3. From a transfer function point of view, one flux level is of great interest. From eq. (22), when $(\gamma_x - \sigma_x X_0 = 0)$ corresponding to a flux level, with the above numerical values of $\phi_0 = 3 \times 10^{11}$, the phase of the transfer function shifts its ultimate end point with frequency. That is, at high frequencies for ϕ_0 less than 3×10^{11} there is a total phase shift in the Xe feedback path of $-90°$. When ϕ_0 is greater than 3×10^{11}, there is an ultimate $-270°$ phase shift in the transfer function. The transfer function gain at zero frequency when $\gamma_x - \sigma_x X_0 = 0$ is $\gamma_I/(\sigma_x\phi_0 + \lambda_x)$, and there is no discontinuity in gain as a function of ϕ_0.

Bode Diagram. Figure 10 shows the Xe^{135} concentration for small oscillations in flux normalized in gain about the $\phi_0 = 10^{14}$ case. The absolute level for the $\phi_0 = 10^{14}$ curve is $+19.1$ db. The gain at zero frequency is a constant for fluxes roughly below $\phi_0 = 10^{10}$. At higher levels the gain steadily decreases with flux.

To use this feedback transfer function in examining reactor stability means that another loop must be tied back around the reactor (Ref. 17). The poisoning loop to be considered can be around a simple reactor or one containing complex negative temperature coefficient feedbacks. As all reactors have a temperature coefficient of some sort, the representation of the reactor transfer function as $K_{RTC}G_{RTC}(s)$ rather than $K_RG_R(s)$ is reasonable. It will be recalled in the case of simple negative temperature

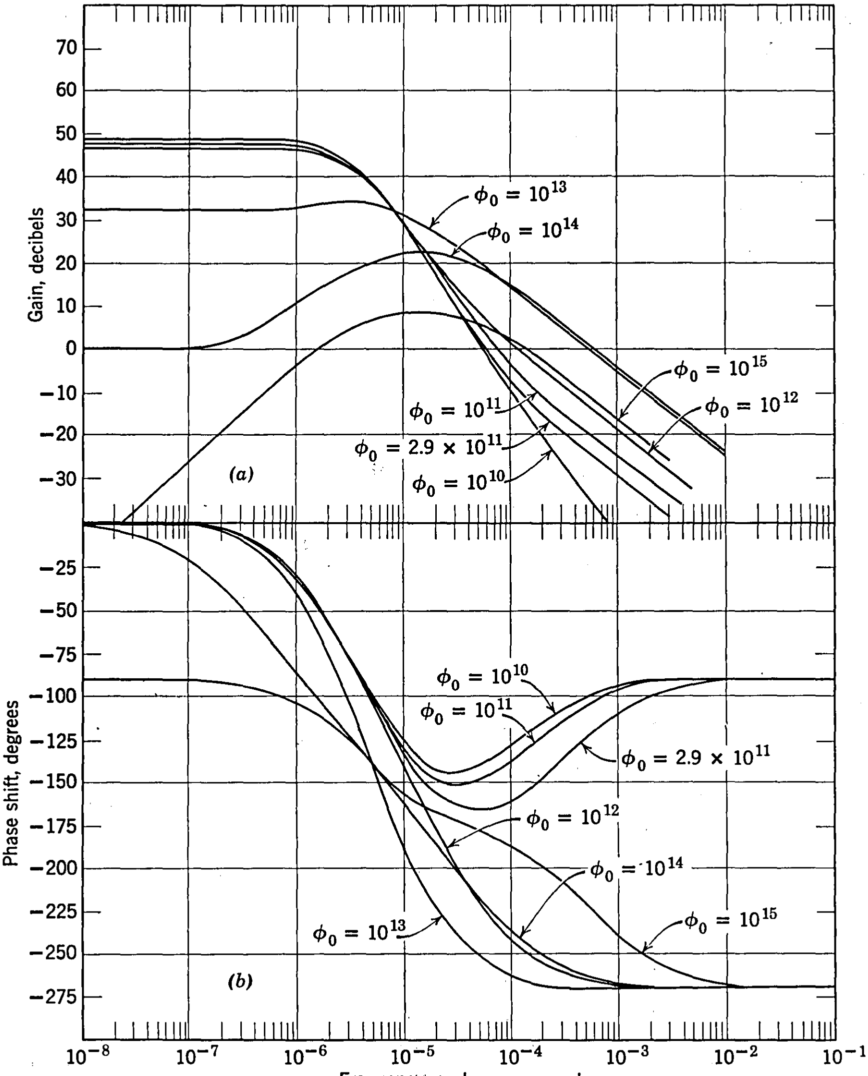

FIG. 10. Transfer function xenon poisoning feedback: (*a*) amplitude response, (*b*) phase shift (Ref. 3).

coefficient feedback that the reactor transfer function depends only upon the amount of feedback at zero frequency, and very low frequency responses could be approximated as $K_{RTC}G_{RTC}(s) = 1/K_{TC}$. As the poisoning feedback occurs only at extremely low frequencies, the use of this relationship for the reactor transfer function in poisoning feedback problems is a good approximation. The feedback loops involved are shown in Fig. 11. In this figure, before the complete poisoning loop is available, $\delta X(s)/\delta\phi(s)$ must be multiplied by a gain term which depends upon the fission cross section of the fuel used. The complete poison feedback from neutrons to reactivity is indicated by $K_xG_x(s)$.

For U^{235} fueled reactors (Ref. 22) indicates that thermal reactors operating up to flux levels of 3×10^{11} are inherently stable without any negative temperature coefficient in the reactor. At flux levels above 3×10^{11} a small negative temperature coefficient is required in order that

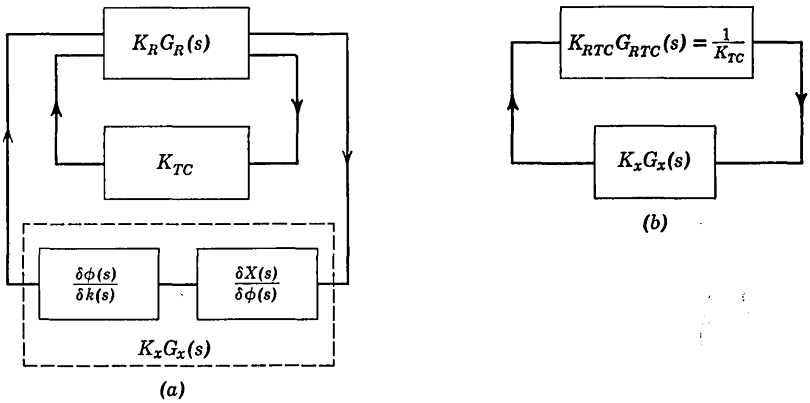

FIG. 11. Elementary block diagram of xenon poisoning feedback: (*a*) temperature coefficient loop and poisoning loop, (*b*) combined reactor and temperature coefficient.

the combined loops be stable. Although this reference shows that instability is possible if insufficient negative temperature coefficient is present, this is not a serious situation. An unstable Xe^{135} feedback loop will oscillate at frequencies of approximately 1 to 2 cycles per day. Normal control rod motions for other purposes usually tend to mask out these oscillations.

4. POWER LEVEL AUTOMATIC CONTROL

Description of Control Loop. Figure 2 indicates that the complete control system for a reactor consists of three operational loops. The startup loop is usually manual and open. The shutdown loop is either in an on or off state. The power level loop is the only one which exhibits regulatory action, and consequently it will be used as an example of how automatic reactor control is accomplished.

Control Modes. An automatic control system in the power level range consists of a control loop around a regulator rod or a shim rod group and can be operated either as a proportional regulating system or as a discontinuous regulating system. In this application, a *proportional regulating system* is one in which the position of the control rod or rods is changed in proportion to and in phase opposition with any error created either by an external power demand change or an internal system transient. Similarly, a *discontinuous regulating system* is one in which no control is exercised until the error is some fixed percentage away from a preset group of conditions set up in the control loop. When sufficient deviation occurs from these demanded conditions, a control rod position is changed usually at a fixed velocity. Discontinuous regulating systems have been built to hold reactor power level to within 0.5% of the demanded level. Higher accuracy if needed can be obtained with the proportional type of control system. The discontinuous system is used when there is a noise problem, in that it is less sensitive to random noises which may originate anywhere in the loop. In high power reactors noise is usually not a serious problem and either system may be used depending upon the accuracy requirements.

Figure 12 shows a *block diagram of the control system* in the power

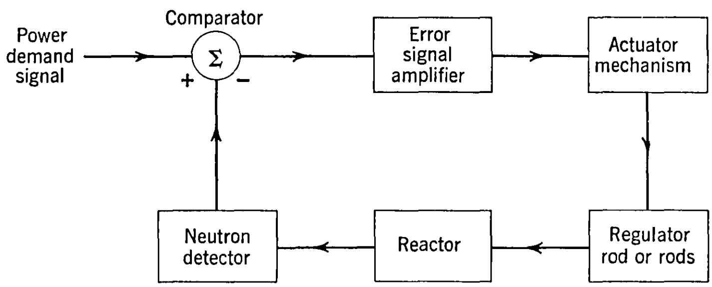

FIG. 12. Power level control block diagram.

level range. The reactor multiplication is changed by direct movement of the control rods. The output of the reactor is measured by a neutron detector, generally an *ionization chamber*. Where slow response can be tolerated, a *neutron thermopile* may be used. The reactor output is then compared with the desired power demanded in the comparator and the error between the output and the demanded output is amplified in the error signal amplifier. The output of the amplifier is used to control an actuator which moves the rods the proper amount and in the proper direction to eliminate the error. The comparator, error signal amplifier, and actuator may be of any suitable type. Pneumatic, hydraulic, electrical, and mechanical devices have all been used (Refs. 18–20). A brief description of these major components in the loop, from a control system point of view, follows.

The Reactor. The reactor transfer function for several reactor approximations has been given in Sect. 3. The principal feature to be noted is the *nonlinearity* of the transfer function. The reactor gain is proportional to the level at which the reactor is operating. This is usually an intolerable situation as the control loop must operate in a stable manner over a wide range in gain. Some means to eliminate the gain dependence upon level is generally added to the control loop.

Comparator (Refs. 21, 22). The comparator in an automatic control loop serves two purposes. First, it provides an error signal which is essentially a subtraction between the neutron detector signal and the power demand signal. Secondly, it is generally used to compensate also for the reactor nonlinearity. The preferred form of the comparator output signal is error/level. In this manner, the reactor nonlinearity is cancelled by the complementary nonlinearity of the comparator. Figure 13 shows two elementary forms of comparator circuits.

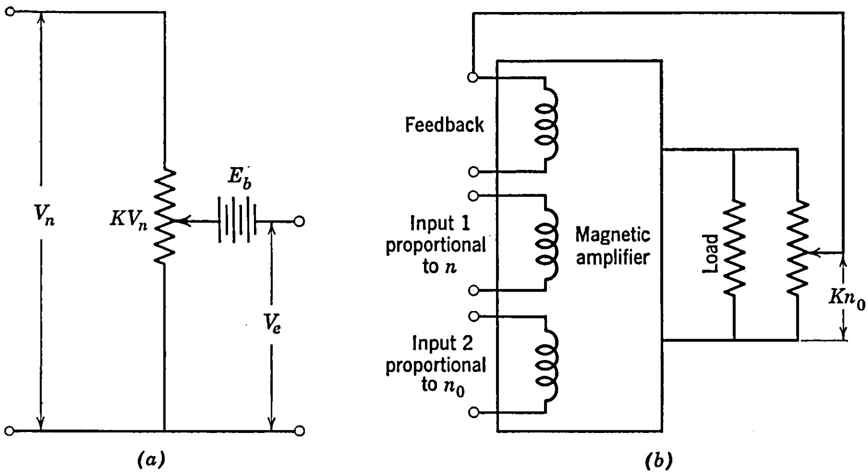

FIG. 13. Elementary forms of comparator circuits: (*a*) battery circuit, (*b*) magnetic amplifier circuit (Ref. 3).

Battery-Operated Circuit. Figure 13*a* is the simplest form, whereby the signal from the neutron detector is V_n, which is proportional to the neutron level of the reactor. If a reference voltage V_0 is arbitrarily defined as $V_0 = E_b/K$, then $V_e = KV_n - E_b = (E_bV_n/V_0) - E_b \sim E_b(n - n_0)/n_0$ where V_e is the output error signal, n is the actual reactor operating level, and n_0 is the demanded steady-state level. This circuit when connected to an output signal from the reactor does not quite cancel out the reactor gain dependence upon level. Actually a signal inversely proportional to n rather than n_0 is desirable, but the circuitry is usually more complex. The difference between the actual level n and the demanded steady-state level n_0 is usually quite small as the control system acts to make the two quantities the same. In a practical circuit, to ensure that no current is taken from V_e, analog computing techniques may be used and an operational amplifier input connected to V_e to prevent loading.

Magnetic Amplifier. The intense reliability required of reactor control circuits sometimes precludes the use of vacuum tubes in the control system, and consequently magnetic amplifiers have been used. Figure 13*b* indicates an elementary amplifier circuit which operates on the principle that, if sufficient negative feedback is used in an amplifier circuit, the gain of the amplifier depends inversely upon the amount of feedback. In the circuit of Fig. 13*b*, the load current $I_L = A(n - n_0)/(1 + AB)$ where A is the gain of the amplifier and B is the feedback factor. If AB is now made large compared with 1, $I_L = (n - n_0)/B$. It now only becomes necessary to make B proportional to either n or n_0 in order that the comparator output have the proper form. Again, in practical circuits, it is usually easier to obtain n_0 than n.

The Error Signal Amplifier. The error signal amplifier may be a conventional vacuum tube, magnetic, or hydraulic amplifier. It amplifies the error signal level from a few milliwatts to a few watts in order to control the actuating device. Because of the flexibility and high state of refinement of the control art, the frequency response of the amplifier presents no problem in comparison with the response of the rest of the system. The amplifier can be regarded and designed as pure gain in a servo concept.

Actuators. The actuator mechanism is complicated because it may be called upon to perform dual functions, i.e., in the case of shutting down a nuclear reactor a rapid action may be required, whereas for startup and power level control only a comparatively slow motion is needed. In the control loop of Fig. 12, the frequency response of the actuator may range from a few cycles per minute to a few cycles per second. The actuator may or may not contain its own power amplifier, and the output of the actuator is coupled directly to a control rod. In present-day reactors, the control rod may weigh between 25 and several hundred pounds.

5. EXAMPLE OF THE DESIGN OF A REACTOR AUTOMATIC CONTROL LOOP

Specifications and Limits. All the previously described components can now be put together in a loop to illustrate a design problem. To keep the example simple, the control rod speed will be assumed to be set by some operational feature, such as the desire to override Xe^{135} within a given time. Transient response of the loop will also be assumed to be of no consequence, as the speed of rod motions and that of other pertinent perimeters will be such that no large transients could occur. Consequently, the basic requirements for the design boil down into, "Given a reactor with a maximum rod speed, design a control loop for absolute stability." An auxiliary question is, "To what variations in power level is the resulting loop capable of maintaining the reactor."

Control Loop. For illustration, one can assume a discontinuous type control system as indicated in Fig. 14. The reactor of this block diagram

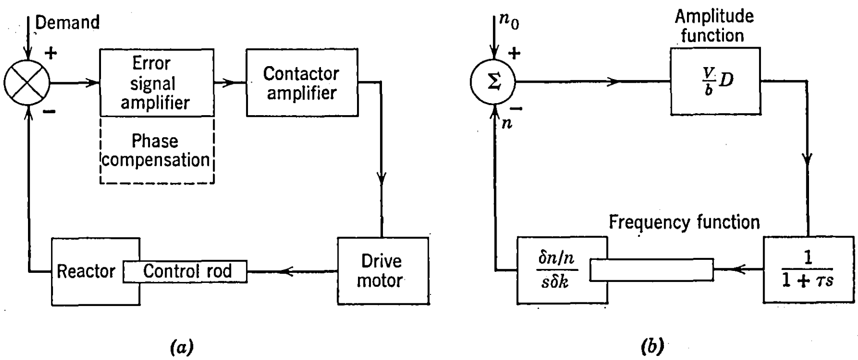

FIG. 14. Block diagrams of discontinuous type control loop: (*a*) component diagram, (*b*) elementary servo representation.

could have any one of the reactor transfer function representations previously indicated, but for a specific example the reactor chosen will have the transfer function given for the $K_{TC} = 0.0047$ case of Fig. 5. Additional phase compensation may or may not be needed in the loop of Fig. 14, and this will be determined at the conclusion of the analysis. The contactor amplifier indicated in the block diagram may be considered as a simple relay which closes a set of contacts when the error signal level reaches a fixed amount and opens these contacts when the error signal level drops below another fixed value. A corresponding set of contacts is used when the sign of the error signal reverses. This relay causes a motor to rotate in one direction or the other and to be stopped when the error signal level is too small to keep either set of contacts closed. Figure 15 illustrates the terminology that will be used. The error signal into the relays is proportional to $(n - n_0)/n_0$. As the error signal increases, it reaches the point b which closes the relay contacts that start the drive motor and creates a reactivity rate change signal V. Once the control rods are started moving in the direction to reduce the error signal, the

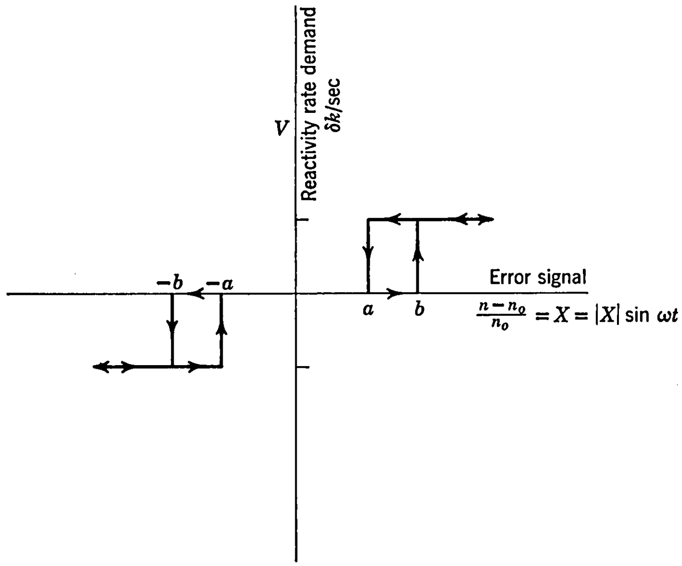

FIG. 15. Relay notation indicating start and stop points.

hysteresis of the relay causes the contacts in the relay system to remain closed until the point a is reached and the drive motor is turned off. The contactor amplifier is presumed to be symmetrical for negative signals.

The drive motor will be assumed as running at constant velocity after an initial time lag $\tau = 0.5$ sec in getting started. This is a reasonable value which many motors used in this service can exceed (Ref. 23). A control rod is geared to the motor and the control rod motion is characterized by being able to change reactivity at a fixed rate in δk/sec.

Analysis Procedure. The type of system indicated in Fig. 14 is a nonlinear system, the principal nonlinearity being caused by the contactor. The reactor nonlinearity is presumed to be wiped out by the action of the comparator. As is well known in control practice, the gain of a closed loop consisting of only linear components is a function of frequency. Any nonlinear component in a loop such as the on-off contactor in this reactor control loop causes the loop gain to be a function of both frequency and amplitude. When the amplitude-dependent functions in the system can be separated from those which depend upon frequency, the loop gain can be expressed as the product $BKAG$, where A is a complex function of amplitude, independent of frequency, G is a complex function of frequency independent of amplitude, K is a constant gain factor, and B is the feedback factor which equals -1 in this reactor control loop. The overall transfer function of the closed loop is the familiar expression (Refs. 24–26):

$$\frac{\text{Output}}{\text{Input}} = \frac{KAG}{1 + KAG} = \frac{KA}{G^{-1} + KA}. \tag{24}$$

Expressing the overall transfer function in this form indicates that the stability of the system can be examined by comparing G^{-1} and $-KA$. This comparison can be made simply on a polar plot of G^{-1} and $-KA$. The value of G^{-1} is plotted for all values of frequency, and $-KA$ is plotted for all values of amplitude. If the two loci intersect, that is, if $G^{-1} = -KA$, the system is capable of sustaining an oscillation.

Stability Plot. The loop components in the block diagram of Fig. 14 can be split up into the frequency function G and the amplitude function A as just described. The function G is the product of the transfer functions of the reactor, the error signal amplifier, and the drive motor. The function A comes from the contactor amplifier alone and describes the effects of the relays. The method of determining the response of the relays is usually based upon the development of Kochenburger (Ref. 27) and depends upon the assumption that only the fundamental component of the square wave signal of rod velocity coming out of the relay is significant. Higher harmonics are attenuated by the rest of the system, particularly by the motor, and consequently may be ignored. From Kochenburger, the terms which are of consequence in the analysis are the ratios b/a, V/b, and x/b. Briefly, when $b/a = 1$, the output V is in phase with the input x. When $b/a > 1$, phase shift occurs between x and V with the output phase lagging the input phase. The ratio V/b appears directly as a gain factor in the contactor amplifier and the complete amplitude function equals $(V/b)D$ where D is a complex function of b/a and x/b. This complex amplitude function is plotted in Fig. 16 for the case where $b/a = 2$; $V = 10^{-3}\ \delta k/\text{sec}$; and $b = 0.1$. This is a reasonable set of numbers which can be attained easily by the relay and by a typical control rod drive mechanism. Current practice in reactor operations limits V to a range of between 10^{-3} to $10^{-6}\ \delta k/\text{sec}$. The start limit b depends upon the accuracy to which it is desired to hold the power level, as b represents the dead zone.

Phase Compensation. The transfer function of the reactor now must be modified in that the input of the reactor is not a change in reactivity, but a change in rate of change of reactivity. Consequently, the output will be a change in level as a function of the change in reactivity input rate, that is, the transfer function is of the form $\delta n/n/s\delta k$. We can now combine all the frequency-dependent portions of the transfer function $G(s)$, including the $1/s$ term, invert this function, and plot it on the polar diagram as shown in Fig. 16. In this particular example the reactor

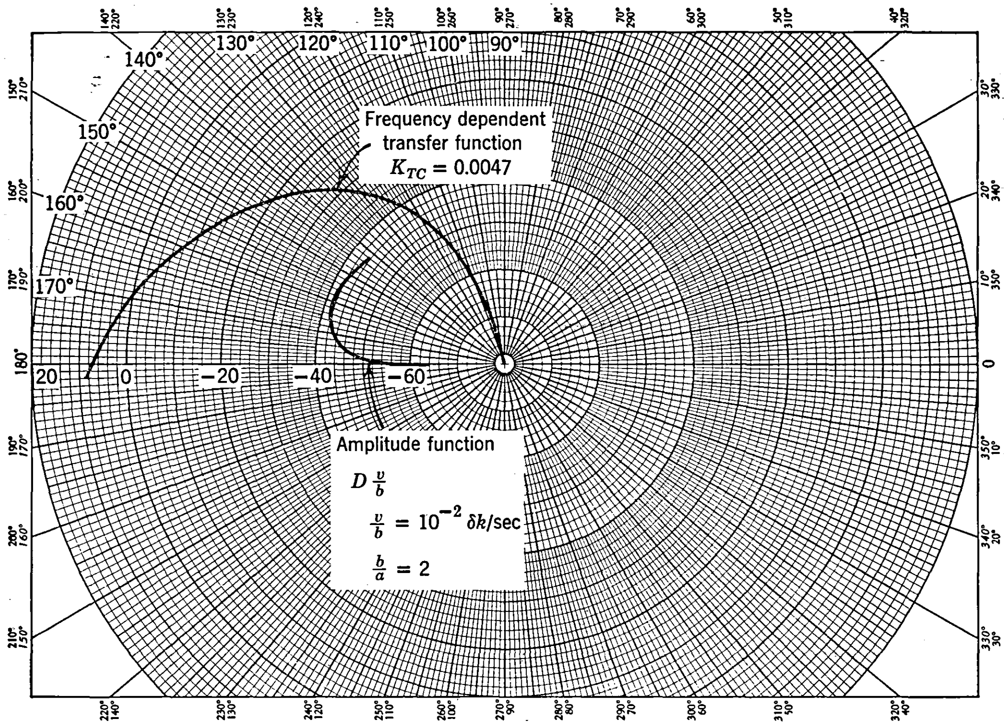

FIG. 16. Stability plot for discontinuous control system example.

control loop is stable in that the two curves do not intersect. The main margin is approximately 17 db, and the phase margin is approximately 21°.

The same stability criteria hold here as would hold for other systems and the phase margin appears to be somewhat low. Consequently, a phase correction network would probably be inserted in the block diagram of Fig. 14 to increase the phase margin to 35° or better. The above presented example for the design of a reactor control loop for stability is only one of the many types of control calculations that must be made before the complete reactor control system is available. However, the above techniques suggest that reactor control problems can be solved with simple presently available methods.

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Control of Interconnected Power Systems

Nathan Cohn

1. INTRODUCTION AND SCOPE

Automatic Control in the Power Industry. The important contributions which instruments and automatic controls can make to improved operating economy and reliability of electric power systems have long been recognized. Modern power stations make extensive use of automatic control equipment to regulate many of the parameters related to energy conversion and utilization. A modern coal-burning steam plant would typically be equipped for automatic control of combustion, feedwater flow, superheat and reheat temperatures, air heater and mill temperatures, feedwater pump recirculation, tank and heater levels, station voltage, and many other related variables. The objective that justifies such extensive use of automatic controls is the safe, reliable operation of the station at minimum cost. Such local station control loops, although of vital importance to plant operation, are not included in the discussion of this chapter.

The Area-Wide Concept. Another step forward in the use of automatic control in the power industry has been to relate automatically the operation of individual stations with one another so that the objectives of continuity of service and high economy are achieved for a far-flung network as a whole. There is the need and opportunity to coordinate the operation of the many generating stations of a network so that prevailing customer demand is fulfilled, power interchanges with neighboring networks are established and maintained, and the outputs of available alternative sources are maintained at such levels as will provide optimum overall economy.

This area-wide generation problem, which is more fully discussed in Refs. 1 and 2, is considered in this chapter. Basic concepts related to the control of generation and power flow on interconnected systems are outlined and analyzed, and steps leading to system optimization are defined and appraised.

In practice, there are many variations in the control systems which may be synthesized to solve the generation control problems of a given utility or a group of utilities. Each operating group will define its own objectives, which in turn will influence the control arrangements that are to be used. There are, however, common denominators in the concepts that define control objectives. It is these fundamental concepts and the basic nature of solutions that are discussed in this chapter.

Terminology and Definitions. Terminology in this chapter adheres generally to the prevailing day-to-day use by power systems engineering and operating personnel. No effort is made to translate this practical terminology into the language or symbology of feedback control specialists. There are in general no approved definitions or definitive terminology for many of the parameters, concepts, and philosophies encountered and utilized in this field. Therefore, to avoid ambiguity and misunderstanding, pertinent terms are defined where first used and all later usage adheres to these definitions.

Assumptions and Simplifications. Where graphical representations or performance equations are shown in the text, they generally apply to steady-state conditions that follow illustrative step function changes. This, and assumptions that are in each case stated, permits simplification of the analyses without destroying the basic validity of the resulting conclusions which illustrate and define the nature of automatic control responses on interconnected electric power systems.

2. INTERCONNECTED POWER SYSTEMS

Growth of Interconnections. Interconnections throughout the United States continue to grow and expand. Six major interconnections embrace most of the nation's central station facilities. The geographical extent of each of them and its approximate load peak in millions of kilowatts are shown in Fig. 1. Some of the adjacent interconnections shown in Fig.

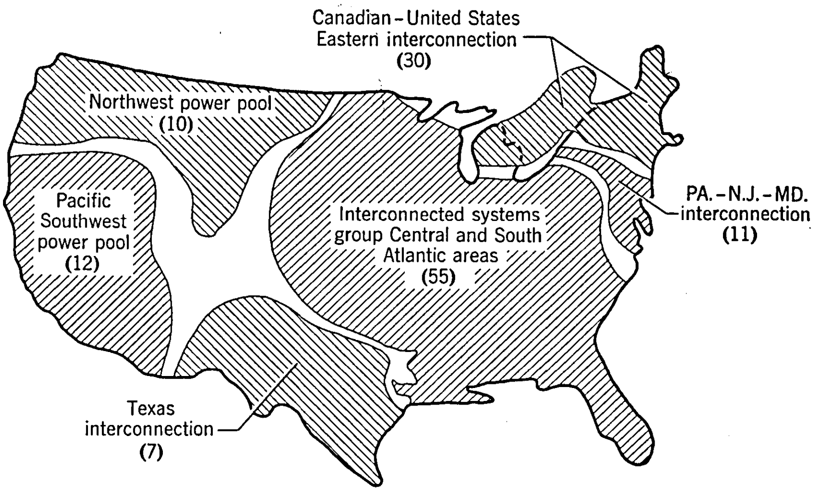


FIG. 1. Principal interconnections in the United States and eastern Canada. Figures in parentheses are the approximate interconnection peak load in millions of kilowatts.

1 have at times operated in parallel with each other, and continuous parallel operation in future years, reducing still further the total number of interconnections in the country, is not unlikely.

Advantages of Interconnections. Interconnection contributes to the two cardinal objectives of power systems operations: (1) continuity of service and (2) economy of power production. During normal operating periods, generation is shared. Interchanges between adjacent utilities are scheduled to take advantage of load diversity or available lower cost capacity, permitting lower overall operating costs and possible deferment of capital investment for new stations. Scheduled outages for maintenance can be staggered.

During emergencies, spinning reserve capacity is shared, thereby contributing to continuity of service.

Systems and Areas. The terms system and area are not always uniquely defined in power systems control discussions. Each is used at times to identify a part, or all, of an interconnection. For purposes of this chapter each will be given a specific meaning. This will permit ready identification of an interconnection as being of the *single area* or *multiple area* type, depending on its basic operating philosophy as related to absorption of customer load changes.

The term *interconnected system* identifies the complete interconnection. It embraces all the utilities or groups of utilities (all the generating sources and loads) which are linked together in the network. When the context makes its use clear, the word *system* is used alone, without the qualifying *interconnected.*

The term *area* identifies that part of an interconnected system which is to absorb its own load changes. It may be a single company, responding to its own load changes; it may be part of a company, operating to respond to load changes that occur in only a given part of the company's network; it may be a whole group of companies pooled together to absorb the load changes that occur anywhere within their collective boundaries.

A *single area interconnected system* is one in which load changes are absorbed by the system as a whole, regardless of where on the system they occur. Load changes that occur in any part of the system may be absorbed elsewhere within the system, in accordance with the allocation practices prevailing at that particular time. No one part of the system is expected to adjust its own generation to absorb its own load changes. Tie line power flows are, therefore, neither scheduled nor controlled. Synonymous terms are *single area system,* and *single area interconnection.* The Pennsylvania–New Jersey–Maryland interconnection, shown in Fig. 1, operates as a single area system.

A *multiple area interconnected system* is one that consists of a number of operating areas, each of which is expected to adjust its own generation to absorb its own load changes. Tie line power flows between areas are scheduled and maintained. Synonymous terms are *multiple area interconnection* and *multiple area system.* Five of the six interconnections shown in Fig. 1 are of the multiple area type.

Representation of an Interconnected System

Simplified representations of an interconnected system are shown in Figs. 2 and 3. Figure 2 shows several operating companies linked together in an interconnection. The intercompany ties are shown in simplified form. There would usually be additional links between the companies, but the simplified schematic will serve the purposes of this discussion. Figure 3 is a further representation of the interconnection. Each company has its own load, labeled L with a corresponding subscript, representing the aggregate of all loads within the area. Each company has

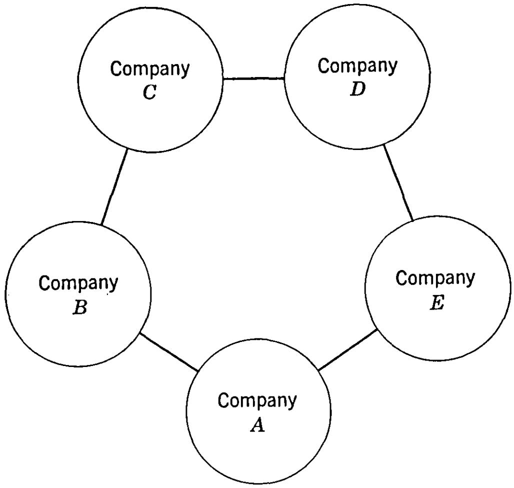


FIG. 2. Simplified diagram of several operating companies linked together to form an interconnection.

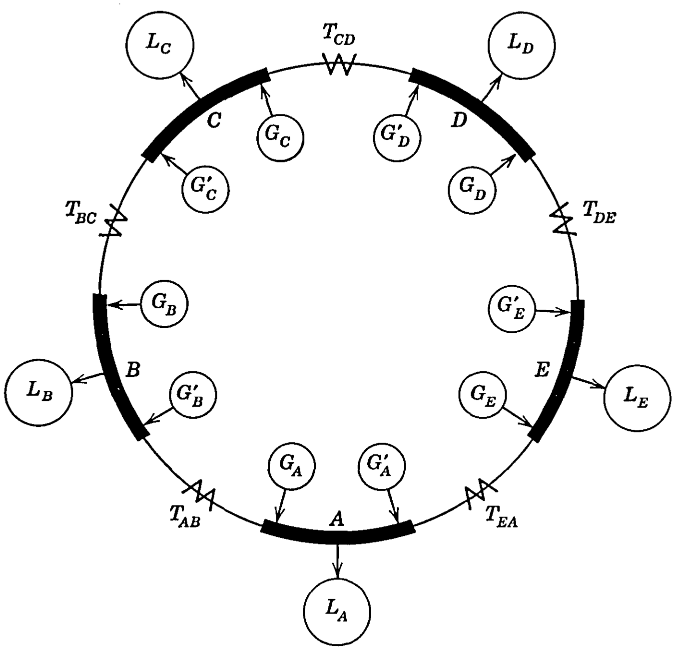


FIG. 3. Schematic representation of the interconnected system. Each company has its own load L, with a corresponding subscript, and its own alternative generating sources G and G', with corresponding subscripts. Each company has a tie line T with each of its two neighbors.

its own alternative generating sources, labeled G and G' with corresponding subscripts, representing all the generating sources within the company area. Each company has a tie line with each of its two neighbors, each tie representing all its links with that neighbor.

Figure 4 shows all the companies (the complete interconnection) oper-

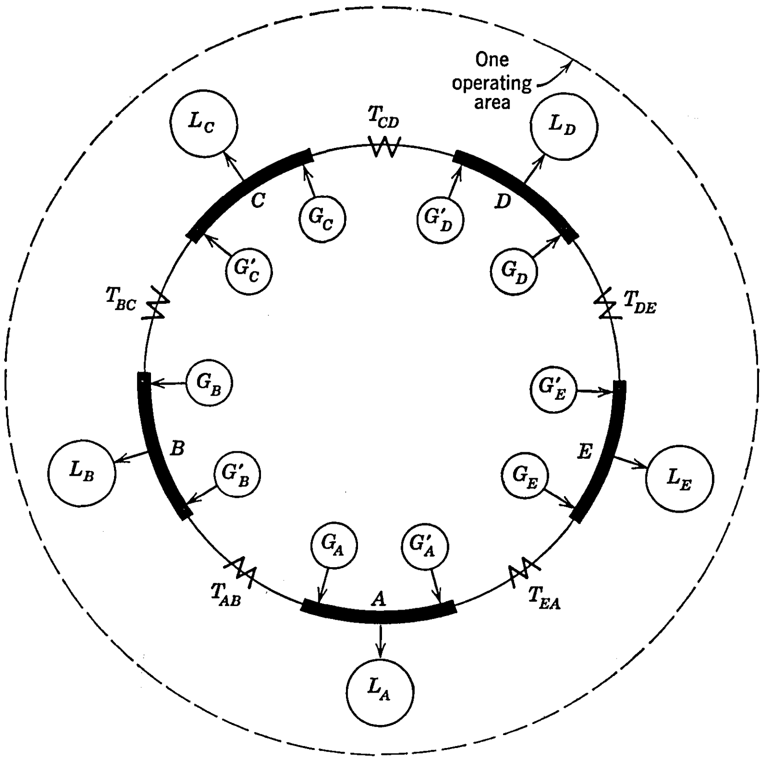


FIG. 4. The complete interconnection operating as a single area system. See Fig. 3 for definition of symbols.

ating as a single area, as indicated by the dash line circle. Intercompany tie line loadings are not of consequence, provided of course they are within the capabilities of the ties. Load changes that occur in the system are assigned, regardless of where they occur, in accordance with prevailing system-wide allocation programs. On this basis, any one of the companies is likely to absorb the load changes of another company. The five interconnected companies are at once a system and an area.

Figure 5 illustrates multiple area operation with each of the five companies of the interconnection operating independently as a separate area.

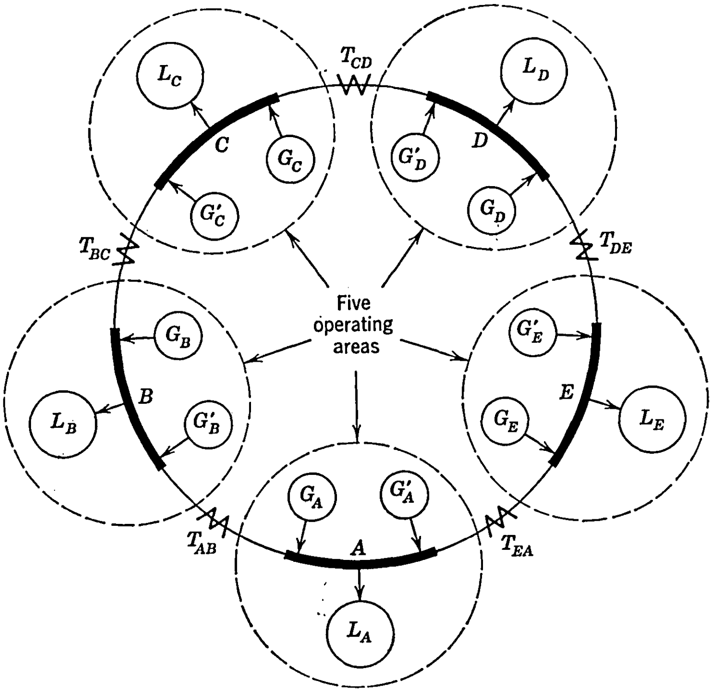


FIG. 5. The interconnection arranged to operate as a multiple area system, with five operating areas.

This is indicated by the five separate dash line circles. When a load change occurs in a given area, it is the generation of that area that is to be varied to accommodate that load change. Interchanges over the five interarea ties, labeled T with suitable subscripts, are now important and are scheduled to specific levels.

Figure 6 shows another multiple area operating arrangement for the interconnection. Companies A, B, C, and E are joined in a pool to operate as one area. Company D operates independently as another area. The complete interconnection now has two operating areas, indicated by the two closed dash lines. Power flow on the two interarea ties, T_{CD} and T_{DE}, is now important and is scheduled and regulated.

Responsibilities of Interconnections. While sharing in the benefits of interconnected operation, each participant is expected to share comparably in its responsibilites. This involves cooperative participation in system regulation in concordance with the established philosophies of the interconnection, so that smooth, neighborly, and mutually beneficial operation is achieved.

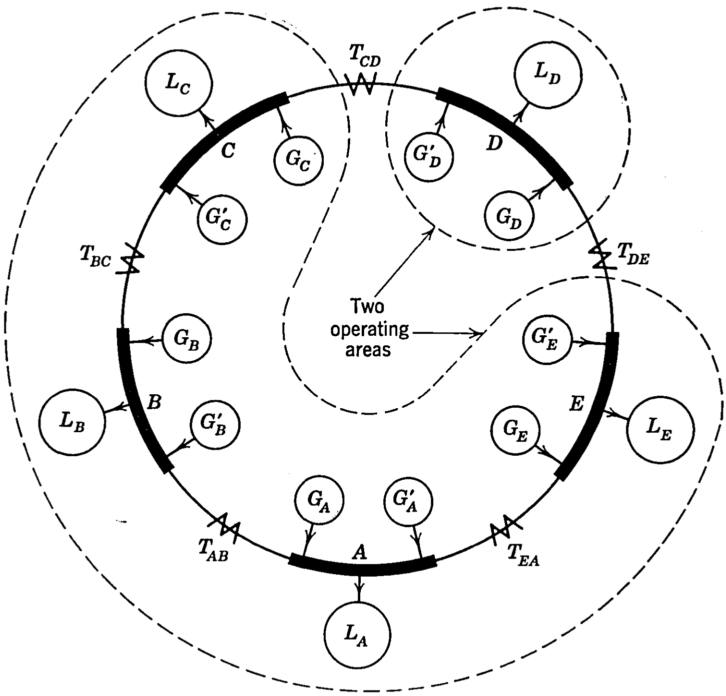


FIG. 6. The interconnection arranged to operate as a multiple area system with two operating areas instead of five as in Fig. 5.

3. THE GENERATION CONTROL PROBLEM

The Basic Problem

Fundamental Operating Objectives. A first objective in the operation of an electric power system is continuity of service to customers. This means that generation must be adjusted, in real time, to match prevailing demand. A second objective, to be achieved as long as it is consistent with continuity of service and dependable operation, is to generate the required total output at minimum overall cost.

Watt's Problem. The problem of matching output to demand is a fundamental one in energy conversion systems. James Watt faced this problem with his first steam engine. He solved it with his well-known flyball speed governor, itself a pioneer achievement in automatic feedback control, which automatically adjusted input until output satisfied demand. Speed governors are still part of today's problem, and of its solution. Watt, however, had only one energy source in his problem, and a single governor matched output to demand. On modern interconnected systems literally dozens or hundreds of alternative energy sources will be operating in parallel, each carrying a part of the total load and each speed-governed to change output in response to demand. Determining how to allocate demand changes among them, and achieving such allocations, adds a complex dimension to the generation control problem.

The Problem of Multiple Sources. The many generating sources of an interconnected system will be spread out over a large area, hundreds or thousands of square miles or more in extent. Important factors to be considered are:

1. The generating units will differ in size, type and age, and will have differing efficiencies, differing fuel and operating costs, varying load-carrying capabilities, and varying response characteristics.
2. The generating units will be at varying distances from load centers, and transmission losses will be influenced by the generation allocations that are utilized.
3. There will be buy and sell power interchange agreements between adjacent areas.
4. There will be limits to the power that can be carried over certain transmission lines.
5. Spinning reserves must be appropriately maintained in various areas.
6. Where hydroelectric power is involved, there will be problems of storage and stream flow.

All these factors, and related ones, will influence the allocation of generation to each of the sources. The dual objectives will be to secure the correct total generation to match prevailing total demand and to allocate this total among alternative sources for optimum economy consistent with continuity of service.

The Integrated Problem

For a given operating area of an interconnection, the control of total generation and its allocation among alternative sources may be regarded, despite its complexity and the number of variables encompassed, as a single integrated control problem. The typical steps of *perception, evaluation,* and *correction,* inherent in the solution of any control problem, may occur at widely separated points, and the information channels that link together the component parts of the measurement and control loops may be hundreds of miles in length. Telemetering problems, as discussed in Ref. 3 should be considered. There will be multiple objectives to be achieved and many significant parameters to be considered. Automatic computation will be utilized, and a number of controllers will be required, operating either in parallel or in cascade. The design objective is to synthesize means for fulfilling all the regulating requirements into a single coordinated solution.

General Steps in Planning a Solution. In considering the integrated generation control problem of an operating area and in planning for its solution, three general planning steps will be helpful. These are as follows:

1. *Define Operating Criteria.* Define the operating criteria which in turn will define the interrelations and set points for significant parameters. Some of the set points will be fixed; others will change with varying area conditions.

2. *Establish Set Points.* Provide means for establishing the set points, either by utilizing manually preset programs or by real time computation or control.

3. *Plan Control Execution.* Provide means for executing the control steps so that the set points established for the various parameters are achieved.

Points to be Considered. In carrying out these general steps, consideration should be given to a number of points whose proper treatment can contribute greatly to the overall effectiveness of the synthesized control system. These points are as follows:

1. *Coordination with Governors.* Control executions should be fully coordinated with governing responses.

2. *Coordination of Controllers.* Where multiple controls are used, they should be fully coordinated with one another to avoid interaction and hunting.

3. *Stable Controls.* Each control should be stable and should act to correct errors and not create them.

4. *Data Display.* Adequate display of pertinent parameters should be provided for dispatchers and operators.

5. *Channels.* Although the number of information channels should be kept at a minimum, their use must be carefully considered if they can serve vital purposes, such as providing appropriate feedback for control or pertinent information for display.

6. *Operating Ease.* Since supplementary controls are tools to help dispatchers and operators do a better job, they should be designed to ease operating problems and not complicate them. Controls should be easy to comprehend and easy to cut in or out of service.

7. *Safety.* Controls should be inherently safe and should be self-disabling, with appropriate alarms, if faults occur in their performance or in the channels that connect them to sources of information and locations of regulation.

Specific Steps for Achieving a Solution. Steps in the solution of the generation control problem on both single area and multiple area systems are summarized in Table 1. There are two steps on a single area system, and three on a multiple area system.

TABLE 1. THE GENERATION CONTROL PROBLEM FOR AN INTERCONNECTED SYSTEM

	For a Single Area System		For a Multiple Area System
	Step 1		Step 1
Objective:	Achieve correct total system generation, i.e., match total system generation to total system load.	Objective:	Achieve correct total system generation, i.e., match total system generation to total system load.
Defined as:	System governing.	Defined as:	System governing.
Criterion:	An unchanging system frequency.	Criterion:	An unchanging system frequency.
Achieved by:	Natural regulation.	Achieved by:	Natural regulation.
			Step 2
		Objective:	Allocate total system generation among the areas so that each follows its own load changes and does its share of frequency regulation, i.e., so that total area generation equals total area load ±scheduled area net interchange.
		Defined as:	Area regulation.
		Criterion:	Area net interchange is on schedule, i.e., area requirement is reduced to zero.
		Achieved by:	Net interchange tie line bias control.
	Step 2		Step 3
Objective:	Allocate total *system* generation among alternative *system* sources for optimum economy.	Objective:	Allocate total *area* generation among alternative *area* sources for optimum economy.
Defined as:	Economy dispatch.	Defined as:	Economy dispatch.
Criterion:	Sources loaded to equal incremental costs of power delivered.	Criterion:	Sources loaded to equal incremental costs of power delivered.
Achieved by:	Computation and control systems. See Sects. 8 and 9.	Achieved by:	Computation and control systems. See Sects. 8 and 9.

Specific Steps for a Single Area System. There are two specific steps in the solution of the generation control problem in a single area interconnection. The first is to achieve the correct total system generation; the second is to allocate this total among alternative system sources for optimum economy.

Step 1. Satisfying Total System Demand. This step is achieved when total system generation matches total system load. The criterion for determining when total demand has been satisfied is an *unchanging* system frequency. It is important to note that this does not mean a *unique* frequency, such as 60 cycles, but rather an *unvarying* frequency, at whatever level then prevails. It means an absence of acceleration or deceleration of the system.

Step 2. Allocating Total System Generation among Alternative Sources. Applicable criteria for this step, and various computation and control techniques for satisfying these criteria are discussed in Sects. 8 and 9.

Specific Steps for a Multiple Area Interconnection. There are three steps in the solution of the generation control problem on a multiple area interconnection. The first is to achieve the correct total system generation, the second is to allocate the total system generation appropriately among the areas of the interconnection, and the third is to allocate each area's generation among its alternative sources for optimum economy.

Step 1. Satisfying Total System Demand. This first step is the same on the multiple area system as it is on a single area system. An unchanging system frequency, zero acceleration, is the criterion that confirms a match between total system generation and total system load.

Step 2. Allocating Total System Generation among the Areas. This step for a multiple area system has no counterpart on a single area system. Its objective is to assign to each area the load changes that occur within its boundaries. Total area generation will accordingly vary in the manner that total area load varies. The two may not be equal, however, but may be displaced by a fixed amount corresponding to the prevailing interchange schedule between the area and the rest of the system. Maintenance of such interchange at its scheduled value is the criterion that the area generation is being adjusted to match changes in its own area load.

Step 3. Allocating Total Area Generation among Alternative Sources. This step for the multiple area system is comparable to step 2 of the single area system, except that here it is the total *area* generation that is allocated among alternative *area* sources for optimum economy. The criteria that define such allocation, and computation and control techniques utilized to achieve them, are discussed in Sects. 8 and 9.

Regulation

The term *regulation* applies generically to the matching of generation and load, to the transfer of generation among sources, and to the adjustment of stored spinning energy of a system to achieve a desired frequency. It is achieved on present-day power systems by direct speed governing action or by supplementary adjustment of speed governors. The following subsections define more specifically the types of regulating effects encountered and utilized in generation control systems.

Governing Regulation. The term *governing regulation* defines two effects. It applies to the adjustment of generator output by the action of turbine governors responding to changes in system speed; this adjustment is identified as *generation governing.* It also applies to the variation of connected load with changes in system frequency, identified as *load governing.* Both of these effects are frequently referred to as *natural governing* or *natural regulation.*

Supplementary Regulation. Turbine speed governors are normally equipped with a motor-driven adjustment that shifts the speed-output relationship of the generator. Operation of this device either manually or through automatic means varies the generation or speed level of the unit. Such operation is defined as *supplementary regulation.* It is also referred to as *imposed regulation.*

System Governing. The term *system governing* applies on an interconnected system to the matching of total generation to total load by governing action. It is *governing regulation* or *natural regulation* for the system as a whole. It is the aggregate effect of all the generator speed governors of the system plus any coefficient of system load as a function of system frequency. It is step 1 in the regulation of single area and multiple area interconnections (see Table 1).

Area Regulation. The term *area regulation* defines supplementary regulation applied, manually or automatically, to area generator speed governors, to cause each area to follow its own load changes and do its share of frequency regulation. It represents step 2 of the multiple area control problem.

Economy Dispatch. The term *economy dispatch* defines supplementary regulation applied, manually or automatically, to generator speed governors on either single area or multiple area systems, to allocate generation changes among alternative sources for optimum economy. Synonymous terms are *economic loading, incremental loading* and *sustained assignment.* In general, economy dispatch is achieved when alternative sources of an area are loaded to equal incremental costs of power delivered, as defined and discussed in Sect. 8. Economy dispatch represents step 2 in the single area control problem and step 3 in the multiple area control problem.

Load-Frequency Control. The term *load-frequency control* or *frequency-load control* has been used since the earliest days of the generation control art to identify generically supplementary regulation by automatic means responsive to frequency, time error, source loading, tie line power flow, area generation, and combinations of these or related parameters.

Area Control. The term *area control* is a contraction of *area-wide generation control.* On a single area system it would be synonymous with automatic economy dispatch. On a multiple area system, to which the term is more generally applied, it infers that the automatic control equipment is performing both area regulation and economy dispatch, thus fulfilling both steps 2 and 3 of the multiple area problem. It may do so in a single step or in successive steps. In the single step execution, all area generation changes are allocated in accordance with the economy dispatch program. In the aggregate such allocations provide the necessary area regulation. Such single step execution minimizes generation changes within the area. Each change is allocated once to the area source where it is to remain in accordance with the economy dispatch program. When the sources involved in such allocation cannot be responsive enough to provide the desired area regulation, generation changes are initially assigned elsewhere in the area for more rapid response, and then they are assigned in a subsequent step to the sources that are to retain them on the economy dispatch basis.

Area Assist. The term *area assist* or *area assist action* defines the component of area control that involves assignment of generation changes temporarily within the area for more rapid or extensive area regulation, before making the assignment to the sources of the area that are to retain the changes on the economy dispatch basis. A synonymous term is *initial assignment,* as distinguished from the economy dispatch sustained assignment. It is also identified as *fringe control*, *swing control*, and *proportional action.* On a single area system, the equivalent of area assist action is achieved by system governing, and no supplementary regulation is required for this purpose. On a multiple area system, area assist if desired would be part of the supplementary control.

Economy Interchange. The term *economy interchange* applies to the intentional supply of excess lower cost energy from a company which has it available to a company which can use it to displace its own higher cost energy. On a single area interconnection, economy interchange occurs when one company adjusts its generation on an economy dispatch schedule, in accordance with step 2 of the single area problem, to accommodate another company's load change. On a multiple area interconnection, economy interchange between areas is achieved by setting and holding interarea tie line schedules in accordance with step 2 of the multiple area problem.

Summary of the Generation Control Problem

Steps in the solution of the generation control problem for both single area and multiple area systems have been summarized in Table 1. Figure 7 summarizes the factors to be considered in allocating total generation

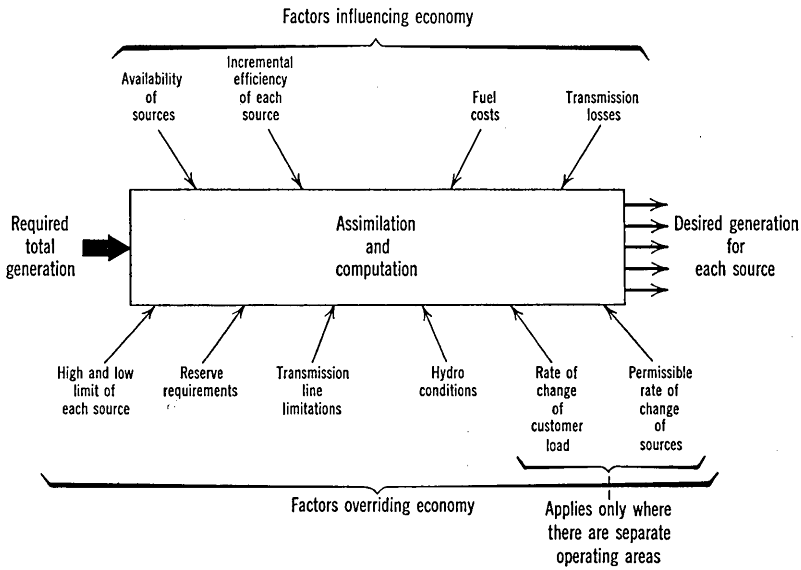


FIG. 7. Factors to be considered in determining how the total generation required of an area is to be allocated among available area sources.

for economy dispatch, step 2 of the single area problem, and step 3 of the multiple area problem.

Classification of Control Systems

Control systems may be broadly classified by the portions of the integrated control problem which they undertake to solve and by the nature of the programming technique used when multiple source loading is involved. They may be additionally subdivided by the nature of the common reference used for establishing source loadings. Also, in some of the subdivisions, optional arrangements, as will be individually specified, may be included.

Data on classifications and subdivisions, based on the categories suggested in Refs. 4 and 5, are included in the following paragraphs; their use makes for considerable convenience in describing the general nature of alternative approaches to the solution of generation control problems. The categories described apply fully to multiple area systems and partially to single area systems. It should be noted that in practice some control installations incorporate combinations of various classes and types.

Summary of Classifications. The three general classifications of control systems and the three subdivisions in the first class are as follows:

Class I. Area regulation only
- Type 1. Single source regulation.
- Type 2. Multiple source regulation, single source output reference.
- Type 3. Multiple source regulation, area requirement reference.

Class II. Economy dispatch, flexible programming.

Class III. Economy dispatch, fixed programming.

The subsections that follow discuss the area regulation techniques of Class I controls. For illustrative diagrams and a discussion of Class II and Class III economy dispatch systems, and of the three types of controls in each of these classes, see Sect. 8 of this chapter.

Class I Control, Area Regulation Only

This is a control system which undertakes to provide area regulation only. Such control is area assist control. Economy dispatch within the area, if practiced at all, is achieved by direct manual adjustment of the generation levels of area sources.

Subdivisions in this class are defined by whether control is on a single unit or multiple unit basis, and by the nature of the reference for multiple unit loading.

Class I, Type 1, Single Source Regulation. In this execution a Class I control is applied to a single generating unit. A block diagram would be Fig. 44 in Sect. 8, with control applied to source G_A only. Such control is usually undesirable, since it would generally impose too large a regulating burden on the unit. Very few installations of this type remain currently in operation. Control is preferably applied to several units, thereby spreading the regulation.

Class I, Type 2, Multiple Source Regulation, Single Source Output Reference. Here control is applied simultaneously to two or more units. Participating sources are automatically loaded with respect to the load level on one of them which is designated as the master. A schematic for such a control would be similar to Fig. 44 in Sect. 8, but with control applied to a limited number of units in the area. A few early installations of this type remain in operation.

Class I, Type 3, Multiple Source Regulation, Area Requirement Reference. In this execution a Class I control is applied to two or more units. Participating sources are automatically loaded with respect to a reference derived from area requirement, which is defined in Sect. 6. See Fig. 64 in Sect. 9 for a block diagram of such a control. Very few installations use this arrangement by itself. It is frequently used, however, in combination with Class II and Class III economy dispatch systems to achieve area assist control as discussed in Sect. 9.

4. SYSTEM GOVERNING

System regulation, matching total generation output to total load demand—step 1 of Table 1—is achieved by governing action. There are two components to such governing action. One is *generation governing,* the other *load governing.*

Generation Governing

General Nature of a Speed Governor. Speed governors are arranged to vary prime mover input automatically in response to changes in system speed. A speed sensitive element, usually a flyball assembly, is responsive to changes in speed and operates through suitable amplifying servos to adjust prime mover input until system acceleration or deceleration is arrested. A simplified schematic of a speed governing system is shown in Fig. 8. To permit parallel operation of prime movers, the speed-output governing characteristic of each is of a *drooping* nature. The extent of the droop determines how much speed change is required to induce a given change in output.

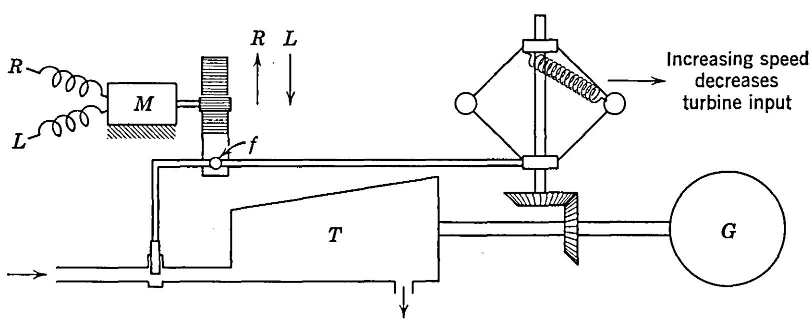


FIG. 8. Simplified schematic of flyball governor which regulates speed and output of generator G by controlling input to turbine T. Motor M permits application of supplementary control by adjustment of pivot point f. Upward adjustment of f raises output, downward adjustment lowers output.

Speed Changer. Speed governing systems are equipped with a supplementary means—identified as a *speed changer*—for shifting the speed-output relationship. The manner in which these supplementary means are utilized to allocate generation changes to various sources is discussed in Sect. 5.

The Speed-Output Characteristic. Figure 9 shows a hypothetical

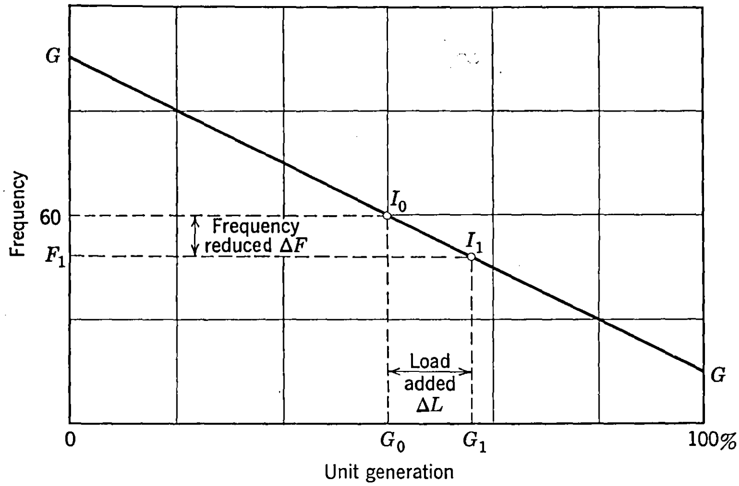


FIG. 9. Hypothetical generator governing characteristic.

speed–power output governing characteristic. It is shown as a straight line. Assuming a single unit system, and starting with output G_0 at 60 cycles, the figure shows how conditions move from point I_0 to I_1 as a new load ΔL is added and frequency is reduced by ΔF.

In practice, speed-output characteristics are not straight lines over the full range of generator operation as shown in Fig. 9. Instead, the drooping characteristic has an irregular pattern, as shown typically in Fig. 10. On steam units, such irregularities result from the characteristics of steam inlet valves.

Steady-State Speed Regulation. For a single turbine, *steady-state speed regulation* is defined (Ref. 6) as the change in steady-state speed, expressed in percent of rated speed, when output is gradually reduced from rated to zero power. For purposes of the present discussion, it may be regarded as the percent of nominal frequency which will cause generator output to change from no load to full load. A synonymous term is *percent droop*. It is graphically represented in Fig. 10 by the slope of the dash line.

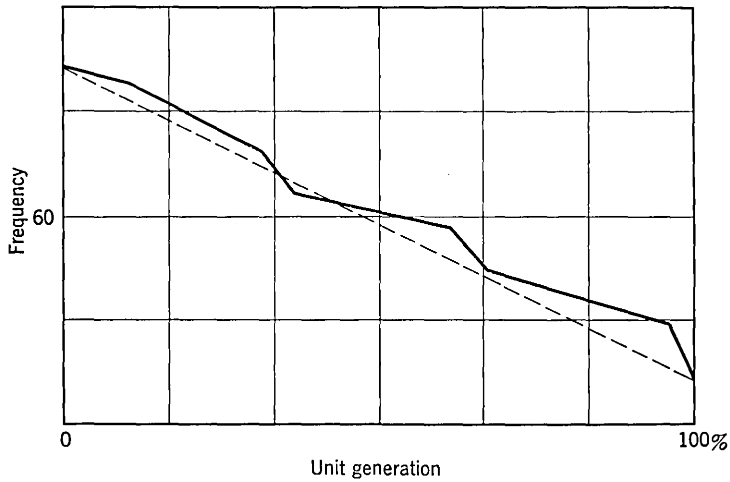


FIG. 10. Typical generator-governing characteristic. [Source: E. E. George, *Elec. World,* **23,** 85 (1945).]

Steady-State Incremental Speed Regulation. For a single unit, the *steady-state incremental speed regulation* is defined, at a given steady-state speed and power output, as the rate of change of steady-state speed with respect to power output, expressed in percent of rated speed (Ref. 6). A synonymous term is *percent incremental droop.* For a characteristic as shown in Fig. 10, it is a variable from no load to full load. It is graphically represented in this figure by the slope of each of the segments that make up the overall characteristic.

Prevailing Natural Generation Governing Characteristic. In analyzing generation control problems, an important term is the *prevailing natural generation governing characteristic.* It may apply to a unit, an area, or a system. For each of them, it has the same basic meaning that steady-state incremental speed regulation has for a unit, but it may be expressed in different terms.

For a Single Unit. The *prevailing natural generation governing characteristic for a single unit* is identical to the steady-state incremental speed regulation of the unit. It may be expressed as the same percentage droop. It may also be expressed in percent capacity per one-tenth cycle or megawatts per one-tenth cycle, in which cases for a 60-cycle system the following relations (see Ref. 7) apply:

$$N = \frac{100}{6D}, \tag{1}$$

$$N' = \frac{M}{6D}, \tag{2}$$

where N is the prevailing natural generation governing characteristic in percent of unit capacity per one-tenth cycle,
D is the steady-state incremental speed regulation in percent,
N' is the prevailing natural generation governing characteristic in megawatts per one-tenth cycle,
M is the unit capacity.

Thus, for example, applying eq. (1), a steady-state incremental speed regulation of 8% corresponds to a prevailing natural generation governing characteristic of approximately 2% of capacity per one-tenth cycle.

For an Area. An area will include a large number of generators of different types and sizes, with governors of correspondingly different sensitivities, dead bands, response times, and incremental speed regulation characteristics. At any given time, and for a given set of prevailing conditions, the area taken as a whole will have a characteristic generation response to speed and load changes. The *prevailing natural generation governing characteristic for the area* is defined by this aggregate response. Since it is made up of a number of nonlinear components, it is itself not likely to be linear over an appreciable range. However, over the small ranges of speed change and load change usually considered, it may, for a given combination of generators and loading, be regarded as linear. It is usually expressed in percent per one-tenth cycle or in megawatts per one-tenth cycle. For the former, the base used for the percentage computation must be identified. It may be area spinning capacity, prevailing area generation, or area peak load. The characteristic may also be expressed as a percent droop by using the relations of eqs. (1) and (2), where N, N', D, and M would each apply to the *area* instead of to a *unit*. Typical area characteristics may run from 1 to 3% of spinning capacity per one-tenth cycle.

For an Interconnected System. The *prevailing natural generation governing characteristic for a system* defines the generation response of the complete interconnection to changes in system speed or load. The characteristic is usually expressed in percent per one-tenth cycle or in megawatts per one-tenth cycle. For the former, the base for the percentage computation may be system spinning capacity, prevailing system generation, or system peak load, as specified. The characteristic may also be expressed as a percent droop by using the relations of eqs. (1) and (2), where N, N', D, and M would each apply to the *system* instead of to a *unit*. Typical system characteristics may run from 1 to 3% of spinning capacity per one-tenth cycle.

Load Governing

On most power systems total connected load has a frequency coefficient, load increasing with increasing speed. This adds a component of self-governing to the system. For example, when new load is added causing system speed to decelerate, the lowered system speed results in a lower effective rating of the already connected load, thereby making some of the prevailing generation available for the new load and decreasing the need for increased generation.

Natural Load Governing Characteristic. The term *natural load governing characteristic* defines the frequency coefficient of connected load of an area or system. It is a measure of the change in the rating of connected load with frequency. It may be expressed in percent per one-tenth cycle or in megawatts per one-tenth cycle. For the former, the base for the percentage computation should be the 60-cycle rating of the connected load. Frequently, for convenience in computation and analysis, the reference base is made the same as the reference base for the prevailing natural generation governing characteristic, namely, spinning capacity, prevailing generation, or peak load. Typical values for this characteristic lie in the range of 0.3 to 0.5% per one-tenth cycle.

Combined Governing

In practice, system regulation is achieved by the combined effects of generation governing and load governing.

Prevailing Natural Combined Governing Characteristic. The term *prevailing natural combined governing characteristic* defines the overall response of an area or system to changes in speed or load. It is a measure of the net generation response to such changes, taking into account the load governing effect as well as the generation governing effect. It may be expressed in percent per one-tenth cycle or in megawatts per one-tenth cycle. For the former, the base for the percentage computation may be spinning capacity, prevailing generation, or peak load.

The generation governing characteristic is a negative quantity, denoting the increase in generation with decrease in frequency. The load governing characteristic is positive, reflecting the decrease of rated load with decrease in frequency. The two may be combined as an algebraic difference or an arithmetic sum to obtain the combined governing characteristic. The latter, like the generation characteristic, is negative, reflecting an aggregate increase in generation for a decrease in frequency.

When both characteristics are expressed in megawatts per one-tenth cycle, they can be combined directly. When each is expressed as a percentage, they can be combined provided the same base reference has been used for each percentage value.

Combined governing characteristics encountered in practice may run for individual areas from 1 to 4% of spinning capacity per one-tenth cycle, corresponding [see eq. (1)] to system droops of approximately 16% and 4% respectively.

Graphical Representation

Figures 11 and 12 illustrate the responses of generation governing, load governing, and combined governing on an isolated area, for a step func-

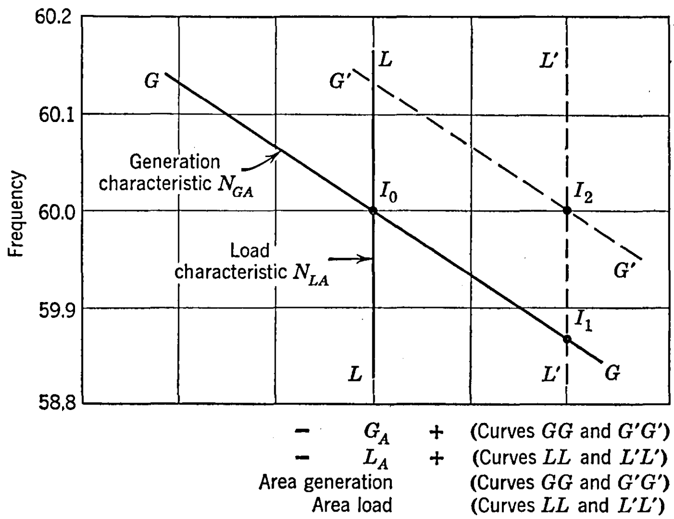


FIG. 11. Generation governing and load governing responses on an isolated area A for a step function load increase and zero load characteristic. G_A and L_A are initial generation and load respectively.

tion load increase. In each case, initial conditions are at I_0. GG is the generation governing characteristic, LL the load governing characteristic, and CC, where shown, the combined governing characteristic. It is assumed that for the small changes being considered, the characteristics may be shown as straight lines. Load is increased from LL to $L'L'$. These figures illustrate how system governing regulation achieves a match between total generation and total load of the area. For comments on supplementary regulation which shifts the generation characteristic from GG to $G'G'$, see Sect. 5.

Zero Load Characteristic. This is the case illustrated in Fig. 11. The load has zero frequency coefficient, as shown by LL being a vertical line. Initial conditions are defined by the intersection of the GG–LL characteristics, point I_0. After the load increase, the new conditions are defined by

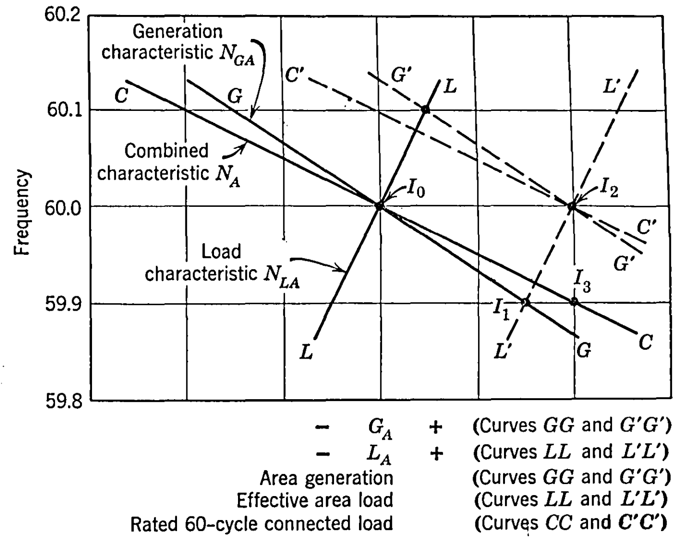


FIG. 12. Generation governing, load governing, and combined governing responses on an isolated area for a step function load increase and a positive load characteristic.

the intersection of the *GG–L'L'* characteristics, point I_1. Speed is decreased, and generation governing has increased generator output to balance the new load at I_1.

Positive Load Characteristic. This is the case illustrated in Fig. 12. Here the connected load has a positive coefficient of change with frequency, shown by the sloping *LL* characteristic. The *GG* and *LL* characteristics may be combined into the *CC* characteristic, with proper consideration to the parameter plotted on the abscissa. Initial conditions of frequency, generation, and load are defined by the intersection of the *GG–LL* characteristics, point I_0. For 60 cycles this is also the intersection of the *CC–LL* characteristics. After the load increase, the new conditions are defined by the intersection of the *GG–L'L'* characteristics, point I_1. Conditions of frequency and rated 60-cycle connected load are defined by point I_3 on the *CC* characteristic. For a given load increment, the frequency drop at balance and the generation pickup are less in Fig. 12 than in Fig. 11.

Determining the Combined Governing Characteristic

In area regulation, as discussed in Sects. 6 and 7, it is desirable, for proper coordination of supplementary regulation, to know the combined natural governing characteristic of the area. Two ways in which the combined characteristic can be determined are discussed in the following paragraphs.

On Interconnected Operation. When a disturbance occurs in one area of an interconnection, of sufficient magnitude to cause a discernible change in steady-state frequency and during a period when local load can be assumed to be relatively steady, computations of the prevailing natural combined governing characteristic in the nondisturbance areas of the interconnection may be made. In each case, this is done by noting from the net interchange recorder the magnitude of the coincident change in net tie line power flow for the observed frequency change and making the appropriate ratio computation.

On Isolated Operation. During a relatively constant load period, a disturbance, such as tripping a significantly sized and loaded generator, can be introduced. Knowing the magnitude of the induced disturbance and observing the coincident frequency departure permits a computation of the prevailing combined system characteristic.

5. SUPPLEMENTARY REGULATION

Supplementary regulation may have two purposes. The first is to correct frequency departures from normal that result from the drooping nature of the governing characteristic. The second, and by far the more important, is to shift generation between alternative sources, after initial governing responses have resulted in loadings which it is not desired to maintain.

Shifting the Governing Characteristic

Supplementary regulation is achieved by shifting the generation governing characteristic. An important consideration, frequently overlooked, is that *for prevailing load conditions every step of supplementary regulation is followed by governing action.* This follows because for the present state of the art governing action is more rapid than supplementary regulation and for a given load change will already have established a steady-state match between total generation and total load before supplementary regulation is applied. When supplementary regulation is then applied to one or more generators, say in the increase direction, there will be an excess of generation causing system acceleration. There will then be corresponding governing responses to reject the excess generation.

It is appropriate to temper this generalization by recognizing that small load or generation changes may be accommodated for an appreciable time from stored spinning energy of the system before there is a discernible change in system acceleration or deceleration. It remains of practical importance, however, for individual changes of significant size or for an accumulation of small changes which are of significant size in the aggregate.

One method of shifting the governing characteristic of a generator is shown in simplified schematic form in Fig. 8. A motorized mechanism shifts upward or downward the fulcrum point f of the lever linking the flyball unit to the turbine inlet valves. This in turn shifts the governing characteristic upward or downward with respect to the frequency axis, and to the right or the left with respect to the output axis, the new characteristic remaining essentially parallel to the initial characteristic. In Fig. 8 upward positioning of the fulcrum point f will result in greater input, and hence in greater output, for the same frequency, or higher frequency for the same output.

The Nature of Frequency Correction

System load changes are initially accommodated by changes in the stored spinning energy of the system. *Frequency correction* is the restoration of stored rotating energy to the level that corresponds to normal frequency, after initial accommodation of a system load change has altered the level of stored energy. In this sense, frequency correction involves a *temporary* change in total generation. It may also involve a *sustained* change to a new level to accommodate any frequency coefficient in system load. In both cases, frequency correction is achieved by shifting the governing characteristic by operation of the synchronizing motor or equivalent, either manually or through automatic control.

With Zero Load Characteristic. Consider a single generating unit with a governing characteristic as shown by aa in Fig. 13. Assume that the load connected to the unit, which is operating isolated, has zero frequency coefficient. Starting conditions are defined by point I_0. A step function load increase is added to the unit, and resulting steady-state conditions, after governing action, are defined by point I_1. Frequency is low, at F_1, and generation has been increased from G_0 to G_1 to accommodate the new load. Supplementary regulation is now applied to restore frequency to 60 cycles.

Moving the fulcrum point f of Fig. 8 upward by operation of synchronizing motor M increases input to the turbine and hence its output. Returning to Fig. 13, output is now greater than G_1, and since it exceeds connected load, system speed is accelerated. This is detected by the flyballs which operate to reduce output to its previous value G_1, where prevailing load will again be matched. During this cycle there has been an increase in the spinning energy of the system, the governing characteristic has been shifted from aa to bb, and steady-state conditions are defined by the intersection of bb and the G_1 coordinate. Similar successive steps of supplementary regulation are followed by steps of governing action until steady-state conditions defined by point I_2 are reached. The gov-

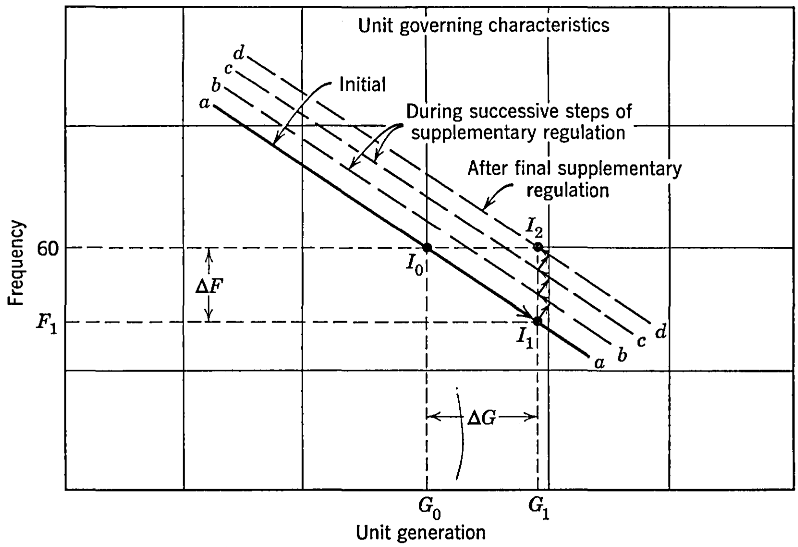


FIG. 13. Unit governing characteristics, before, during, and after supplementary regulation.

erning characteristic has now been shifted to *dd*, output remains at G_1, but spinning energy of the system has been increased in the aggregate sufficiently to restore speed to 60 cycles.

Referring to the area characteristics shown in Fig. 11, steady-state conditions after generation governing response to a load increase are defined by point I_1. Frequency correction is achieved by supplementary regulation which in the aggregate shifts the area generation characteristic from GG to $G'G'$. In the process, spinning energy of the system is restored to its 60-cycle level, and final steady-state conditions are defined by point I_2, the intersection of $G'G'$ and $L'L'$.

With Positive Load Characteristic. When connected load varies as a function of frequency, frequency correction has two components. First, generation must be temporarily increased in excess of connected load to restore spinning energy of the system to its 60-cycle level. Second, generation must be increased on a sustained basis by an amount equal to the increase in connected load rating with frequency. This is illustrated in Fig. 12. Point I_1 defines the steady-state conditions following the governing response. Point I_2 defines the steady-state conditions following supplementary regulation. Frequency has been restored to 60 cycles. There has been a sustained increase in generation, over and above the temporary increase needed to restore system spinning energy to its 60-

cycle value in order to accommodate the increase that occurred in connected load with the increase in frequency.

Reallocating Generator Loadings

The usual function of supplementary regulation is to reallocate generation among sources to achieve area regulation and economy dispatch.

Figures 14 and 15 illustrate the mechanics of supplementary regulation

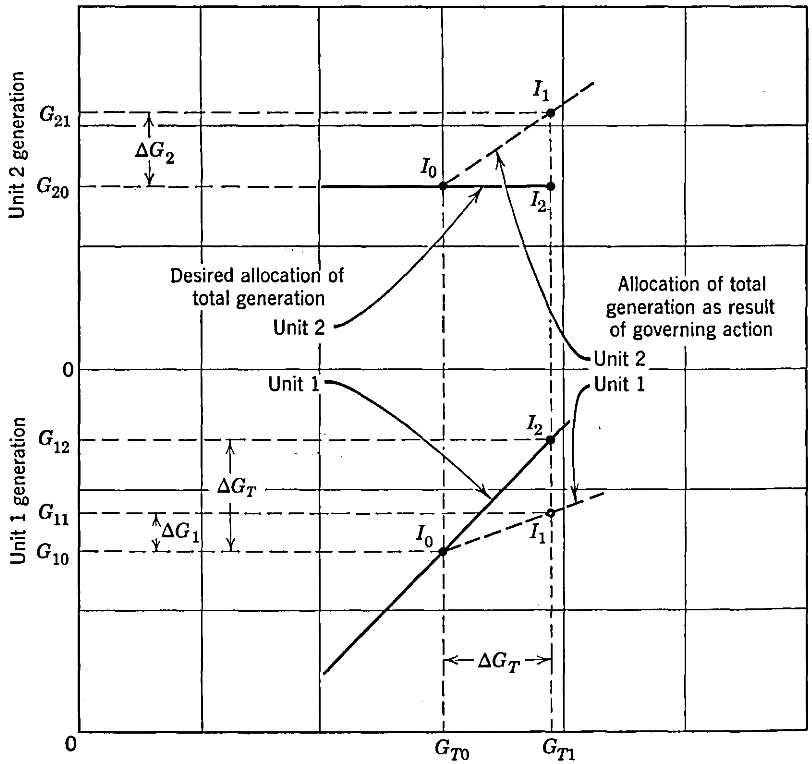


FIG. 14. Graphical representation of how governing responses following a step function load increase result in generation distributions which do not match scheduled allocations.

for reallocating generation loadings after an initial allocation by governing action. Figure 14 assumes a two-unit system. Starting conditions are defined by the I_0 points. As total generation for the system is increased from G_{T0} to G_{T1}, it is desired to load the two units along the solid lines. For a total generation increase of ΔG_T, it is desired to load unit 1 from G_{10} to G_{12}, whereas unit 2 is to stay loaded at G_{20}. The desired unit loadings for this total increment are defined by the points I_2.

Assume that the two units have governing characteristics as shown in Fig. 15 and assume that connected load has zero frequency coefficient.

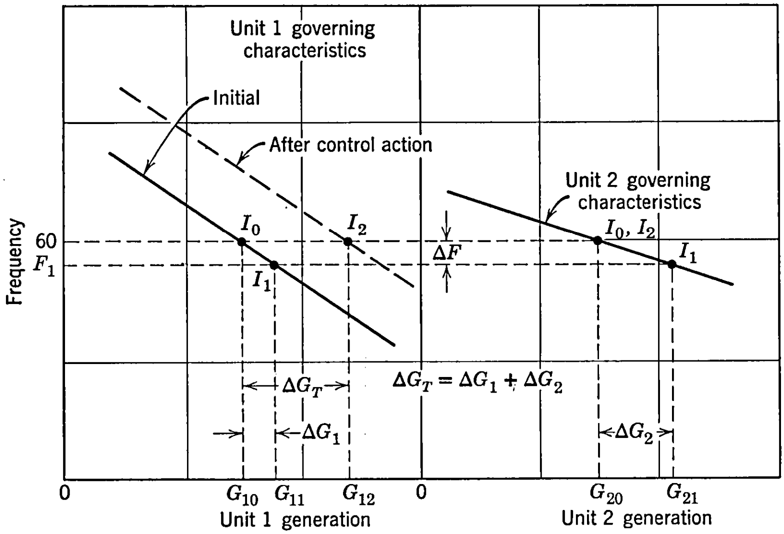


FIG. 15. Matching generation distribution to desired allocations by supplementary regulation, after initial governing responses to a step function load increase.

Application of a step function load change equal to ΔG_T will cause frequency to drop by ΔF, and governing action will cause the two units to pick up increments of generation ΔG_1 and ΔG_2 respectively. These loadings are defined by the I_1 points in Figs. 14 and 15. These loadings do not match the desired allocations. To achieve the desired loadings, reallocation is made by supplementary regulation, which shifts the unit 1 characteristic to the new position shown by the dash line in Fig. 15. Conditions are now as defined by points I_2 in Figs. 14 and 15. Unit 1 has absorbed the complete system load increment. Unit 2 to which no supplementary regulation was applied is back, as a result of its own governing action and as desired, at its original loading G_{20}. Generation has been allocated in accordance with the loading schedules.

This is an idealized example used to illustrate a principle. Governor vagaries, such as lack of sensitivity, dead band, or drift, would probably result in unit 2 returning to some value other than G_{20} in response to natural governing action alone, and unit 2 would then require its own imposed supplementary regulation for exact return to G_{20}.

6. AREA REGULATION

Governing action on an interconnected system cannot identify the source of a load change. Although governing action matches total system generation to total system load, it is not able, by itself, to accommodate the requirement of multiple area systems that each area absorb its own load changes. Each area must be equipped with a supplementary automatic control which will achieve such area regulation. Control used in each area must operate properly in parallel with controls in other areas, without inducing interarea hunting or instability.

Tie Line Interchanges

Since it is impracticable to make a direct comparison of total generation and total load of an area operating as part of a multiple area interconnection, indirect means must be used to determine whether a match between the two exists. Fortunately, such an indirect index is readily available. It is the tie line power that flows between an area and its adjacent areas. Such flow is the algebraic difference between area generation and area load.

A schedule is established for such tie line power flow. As long as an area adjusts its generation to maintain interchange flow on schedule, it follows that it is adjusting its generation to absorb its own load changes.

When an area has many ties with its neighbors, the question arises as to which of them can or should be scheduled and controlled. Also, there is the question of which of two adjacent areas should adjust its generation when the power flow between them is off schedule. These and related questions are discussed in later paragraphs of this section.

Area Net Interchange

When an area has more than one tie with its neighbor or neighbors, it is the algebraic sum of the power flow on all of them that should be scheduled and controlled. Stated another way, it is the *net interchange of the area with respect to the rest of the interconnection as a whole* that should be used as the guide to whether or not the area is adjusting its generation to follow its own load variations. This is true even though it means that the tie line flow from an area to one of its neighbors will not be at the level called for by a schedule between them. If all areas of the interconnection schedule their net interchanges properly, interchange schedules between each pair of adjacent areas will be achieved, even though flows may not be through the most direct routes.

Consider for example the five-area interconnection of Fig. 16. Assume that each pair of adjacent areas establishes a schedule between them as

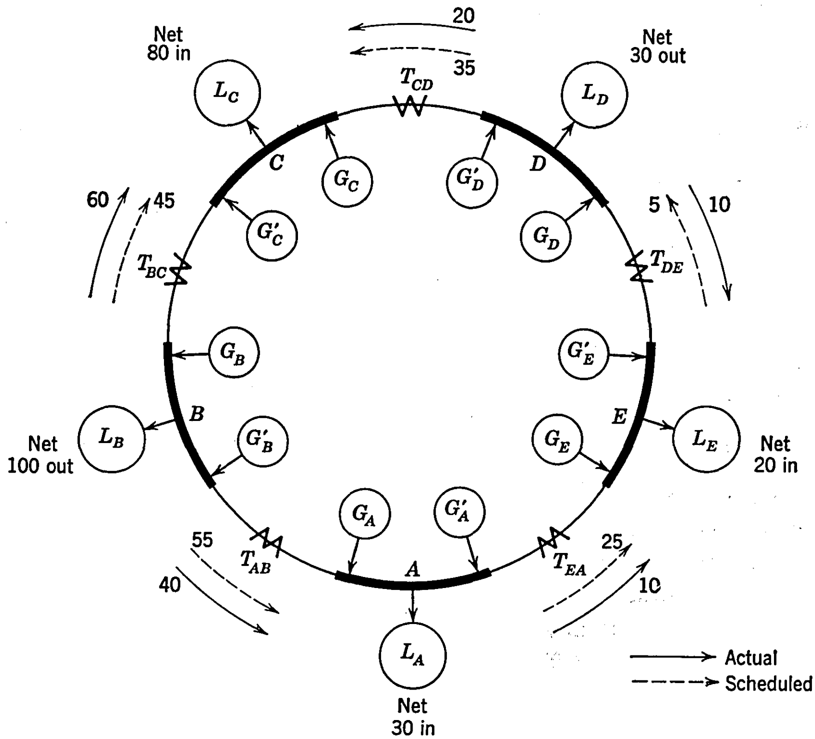


FIG. 16. Scheduled and actual area net interchange power flows for a five area interconnection. See Tables 2 and 3 for summary tabulations.

TABLE 2. SCHEDULED AND ACTUAL TIE LINE POWER FLOWS FOR THE EXAMPLE OF FIG. 16

Areas	Tie	Scheduled	Actual
A and *B*	*AB*	55 to *A*	40 to *A*
B and *C*	*BC*	45 to *C*	60 to *C*
C and *D*	*CD*	35 to *C*	20 to *C*
D and *E*	*DE*	5 to *D*	10 to *E*
E and *A*	*EA*	25 to *E*	10 to *E*

shown in column 3 of Table 2. These schedules are shown as dash arrows in Fig. 16. Assume finally that total system generation matches total system load, and that frequency is in balance at 60 cycles.

Inspection of power flows on each of the ties may show values typically as tabulated in the fourth column of Table 2 and as indicated by the solid arrows in Fig. 16. Individual tie line power flows are not on schedule, but, *for each area, area net interchange with the system as a whole is on schedule.* This is further illustrated in Table 3.

TABLE 3. ALGEBRAIC SUMS OF AREA NET INTERCHANGES FOR EXAMPLE OF FIG. 16

Power Flow		Tie	Scheduled	Actual
Between area A and	Area B	AB	55 to A	40 to A
	Area E	EA	25 to E	10 to E
	The interconnection as a whole (area net interchange, area A)		30 in	30 in
Between area B and	Area A	AB	55 to A	40 to A
	Area C	BC	45 to C	60 to C
	The interconnection as a whole (area net interchange, area B)		100 out	100 out
Between area C and	Area B	BC	45 to C	60 to C
	Area D	CD	35 to C	20 to C
	The interconnection as a whole (area net interchange, area C)		80 in	80 in
Between area D and	Area C	CD	35 to C	20 to C
	Area E	DE	5 to D	10 to E
	The interconnection as a whole (area net interchange, area D)		30 out	30 out
Between area E and	Area D	DE	5 to D	10 to E
	Area A	EA	25 to E	10 to E
	The interconnection as a whole (area net interchange, area E)		20 in	20 in
Algebraic sum of area net interchanges			0	0

Proper Scheduling. Balanced scheduling for the interconnection as a whole is achieved when for each pair of adjacent areas, and as a component of respective area net interchange, one of the areas schedules a power flow "in" while the other area schedules the same power flow "out." Each area algebraically adds all such components to establish its area net interchange. This results in the algebraic sum of all area net interchange schedules for the interconnection as a whole being equal to zero, a necessary criterion for proper system operation.

Multiple Area System Responses

On a multiple area interconnection, the sequence of responses to a step function load change (say an increase) in one of the areas is:

1. Accommodation of the load change at the expense of system stored energy, with corresponding speed deceleration.
2. Recognition of the deceleration with resultant governing action throughout the system to accommodate the load change and arrest deceleration, such accommodation upsetting scheduled tie line flows.
3. Supplementary regulation in the area in which the load change occurred to allocate the change to that area and return tie line schedules and frequency to normal.

Such responses are illustrated in simplified form in Figs. 17 and 18.

Areas A and B are of comparable size but are relatively small compared to the remainder of the interconnection illustrated by area C. For this specific example, it is assumed that area C is 92% of the interconnection, and areas A and B are each 4% of the interconnection. It is assumed also that all tie line schedules and flows are initially zero, that the load has no frequency coefficient, that line losses may be ignored, and that each area has the same prevailing generation governing characteristic, expressed in percent of its size per one-tenth cycle.

Figure 17 shows the responses for a load increase at A. Note that most of the initial accommodation comes from the large area C and note the upsets in tie line power flow. Supplementary regulation is needed at A, the area of load change, to lodge all the generation change there and return the ties to schedule. In Fig. 18 the load increase occurs at area B. The initial accommodation is mostly from the large area C through the CA and AB ties. Supplementary regulation is needed at B, the area of load change, to lodge all the generation change there and return the ties to schedule.

Shift of Schedule with Frequency

Figures 17 and 18 illustrate the need for supplementary regulating action in the area where the load change occurred. Supplementary action in other areas should be avoided, even though ties are off schedule, since any shift in governing characteristic in such areas would only have to be reshifted to original positions when the load change area finally adjusted its generation to absorb its change. This requirement may appear to be contradictory to an earlier statement that each area should maintain a

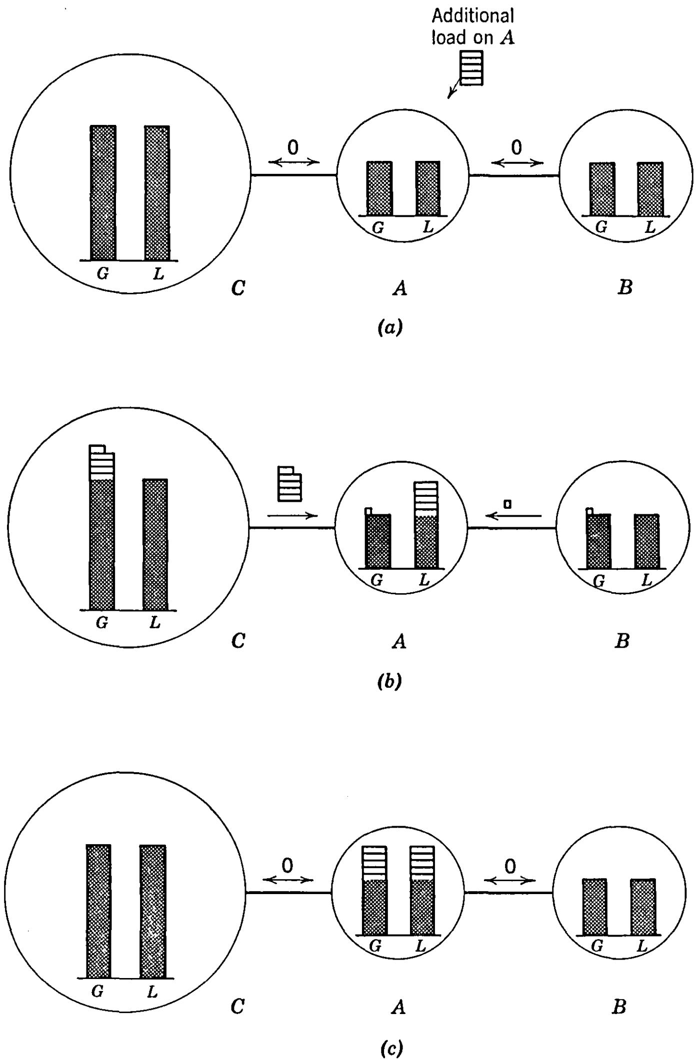


FIG. 17. Variation in area generation and interarea power flow following a load change in one area of an interconnection: (*a*) balanced conditions prior to load change at *A*, (*b*) conditions following governing action of the interconnection in response to load increase at *A*, (*c*) conditions following supplementary regulator action at *A*. The change is accommodated initially by spinning energy and governing action of the interconnection as a whole, as shown at (*b*). Supplementary regulator action then causes the load to be fully absorbed in the area *A* where the change occurred, as shown at (*c*). *G* and *L* are generation and load respectively in each of the areas. (Solid portions of *G* and *L* are not to scale in area *C*.)

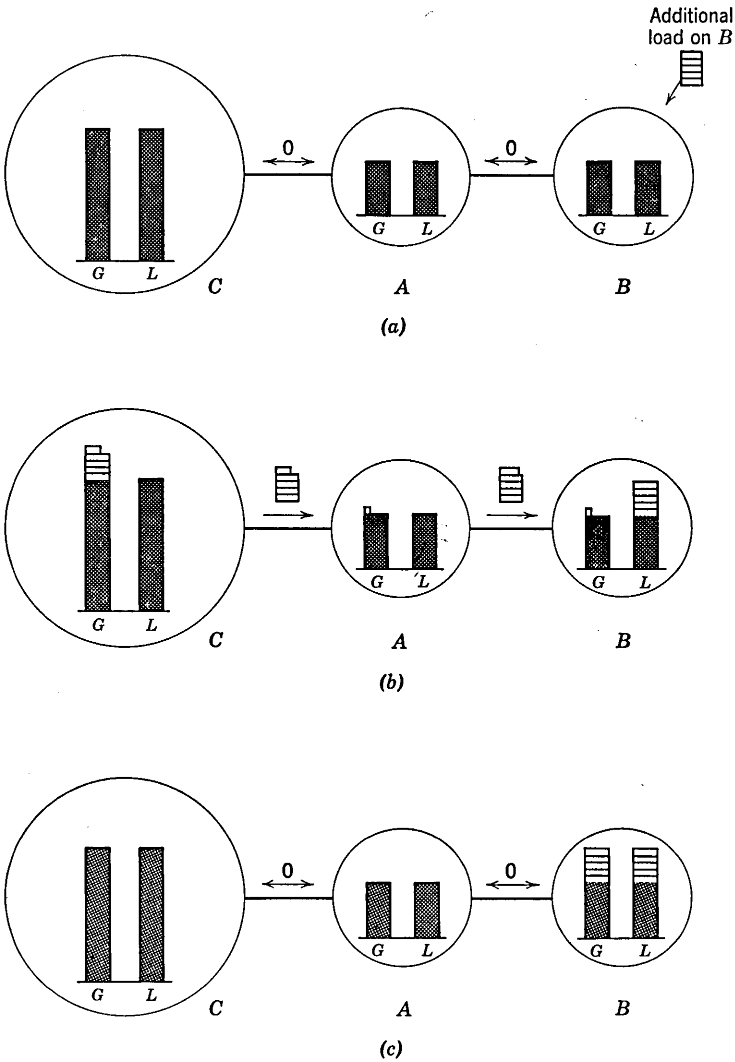


FIG. 18. Variations in area generation and interarea power flow following a load change in B: (a) balanced conditions prior to load change at B, (b) conditions following governing action of the interconnection in response to load increase at B, (c) conditions following supplementary regulator action at B. Sequence is similar to Fig. 17, but note the large fluctuation in power flow between areas C and A, although neither has had a load change. Proper supplementary regulation is required at area B, where the load change occurred, to restore power flow between areas C and A to normal.

scheduled net interchange. This apparent contradiction is resolved and stable parallel operation of area controls is achieved by assigning to each area a net interchange schedule that *shifts with frequency,* rather than one that is held rigidly constant independently of frequency variations.

Net Interchange Tie Line Bias Control

Bias Regulation. An area regulator which operates with a *shift* or *relaxation* of net interchange schedule with system frequency is identified as a *tie line bias controller.* The term *bias* identifies the extent of the schedule shift or relaxation with frequency toward the area of need. Bias itself is sometimes identified as *frequency bias* or *tie line bias.* The former is the preferred term. The literal designation of the controller is *frequency-biased tie line controller.*

Bias Regulation Characteristic. An area operating on tie line bias control has an imposed regulating characteristic as shown by the sloping solid line in Fig. 19. At normal frequency F_0, usually 60 cycles, the scheduled area net interchange is T_0. When frequency is *less* than normal, area net interchange is *higher* than T_0 in the "out" direction. When frequency is *higher* than normal, the area net interchange schedule is *less* than T_0 in the "out" direction.

When a point plotted for prevailing frequency and prevailing net interchange, as I_0 or I_1 in Fig. 19, falls on the bias characteristic, proper area regulation has been achieved and the area bias regulator is at balance. When such a point falls *below* the characteristic, as I'_1, in Fig. 19, area generation is too *low* and must be *increased.* When a point falls *above* the characteristic, area generation is too *high* and must be *decreased.*

Bias Setting. The extent of area net interchange schedule shift with frequency is determined by the slope of the bias regulating characteristic of Fig. 19. The reciprocal of the slope is defined as the *bias.* It is usually expressed in megawatts per one-tenth cycle or in percent per one-tenth cycle. When expressed in percent, the reference base may be prevailing area generation, area spinning capacity, or area peak load, depending on the practice of the interconnection. Decreasing the bias makes the slope of the bias regulating characteristic *steeper,* resulting in a *smaller* shift of net interchange schedule with frequency. Increasing the bias makes the slope characteristic *shallower,* resulting in a *greater* shift of net interchange schedule with frequency.

Zero Bias. When the bias is reduced to zero, the characteristic is the vertical dash line of Fig. 19. This characteristic provides for a rigid tie line schedule, unaffected by frequency. It is known as *flat tie line control.*

Infinite Bias. When the bias is infinite, the regulating characteristic would be as illustrated by the horizontal dash line of Fig. 19. Here there

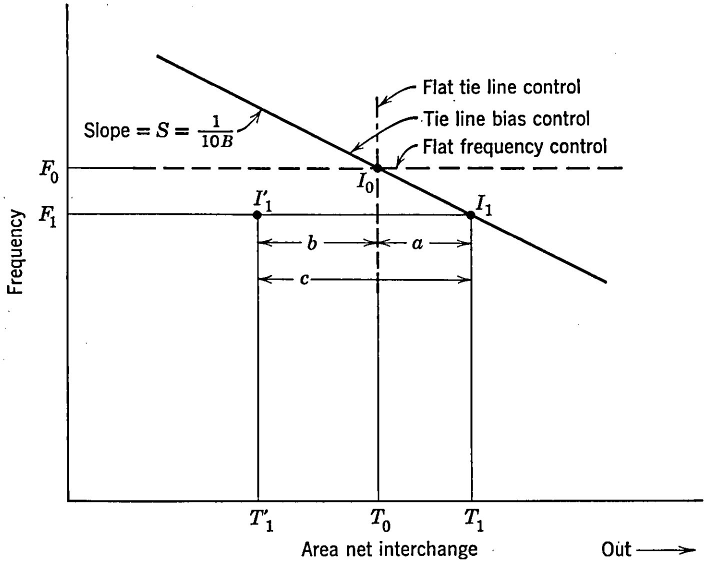


FIG. 19. Tie line bias regulating characteristic, showing how "area requirement" is computed from prevailing frequency and area net interchange. For points that fall on the bias curve, area requirement is zero. For a point not on the curve, such as I'_1, the quantitative magnitude of area requirement—the regulating need of the area—is represented by c, and is the sum of b, which is the deviation of the net interchange from the 60-cycle schedule, and a, which is the automatic shift of net interchange schedule with frequency.

is complete relaxation of the tie line schedule, in that there is no tie line schedule at all. The regulator would operate simply to hold a constant frequency, regardless of fluctuations in tie line power flow. This is identified as *flat frequency control.*

Parallel Operation of Area Regulators

Tie Line Bias Control. Net interchange bias regulators, with appropriate bias settings, will operate in parallel on a multiple area interconnection to provide stable and neighborly area regulation. Parallel operation of the area bias regulators, each equipped with a drooping regulating characteristic, is comparable on an interconnection to parallel operation on an isolated area of individual generators, each equipped with a drooping generation governing characteristic.

Flat Tie Line Control. This is an area net interchange regulator with zero bias, not generally used in present-day practice. In seeking to maintain a constant net interchange independent of frequency, it rejects its own governing responses to remote load changes, causing local overregulation as the remote area adjusts its own generation to absorb its load changes. For a detailed analysis of flat tie line control operation, see Ref. 7.

Flat Frequency Control. A practice sometimes followed, although decreasingly on present-day systems, is to designate one area to operate on flat frequency control, with remaining areas operating on area net interchange bias control. Such an arrangement places a large regulating burden on the flat frequency control area, since it will tend to provide supplementary regulation for all load changes, wherever they occur on the interconnection. For a detailed discussion of flat frequency control operating in parallel with flat tie line control or net interchange bias control, see Ref. 7.

Bias Regulators in Parallel

A simplified graphical representation of net interchange bias regulators operating in parallel on two equally sized areas of a two-area interconnection is shown in Fig. 20. Loads in each area are assumed to have zero frequency coefficient, and the two areas are assumed to have equal generation governing characteristics. The slope of each bias regulator is set to match the slope of its prevailing area governing characteristic. Step function load increases, first in area B and then in area A, are considered, and the resulting frequency, interchange, and generation changes are illustrated. Sequence of regulation is assumed to be simultaneous natural governing responses, followed by supplementary tie line bias regulation, first in one area and then the other.

In all four cases illustrated in Fig. 20, smooth cooperative regulation is obtained. Governing responses from the nondisturbance area persist until regulation in the disturbance area allocates the total generation change to its own area. With each bias characteristic set to match its prevailing area natural governing characteristic, no supplementary regulation is imposed on the nondisturbance area. For a fuller analysis of this graphical representation, see Ref. 7. See Sect. 7 for the effect in the nondisturbance area of bias settings which do not match the prevailing governing characteristic.

Area Requirement

Area requirement is a measure, in megawatts, of the prevailing area generation error. It is the amount by which the area is off its biased net interchange schedule. It is the amount by which area generation must be changed by supplementary area regulation in order that the area correct its net interchange and do its share of system frequency regulation.

Graphical Representation. Referring to Fig. 19, I'_1 is a point defined by prevailing system frequency F_1 and area net interchange T'_1; and the

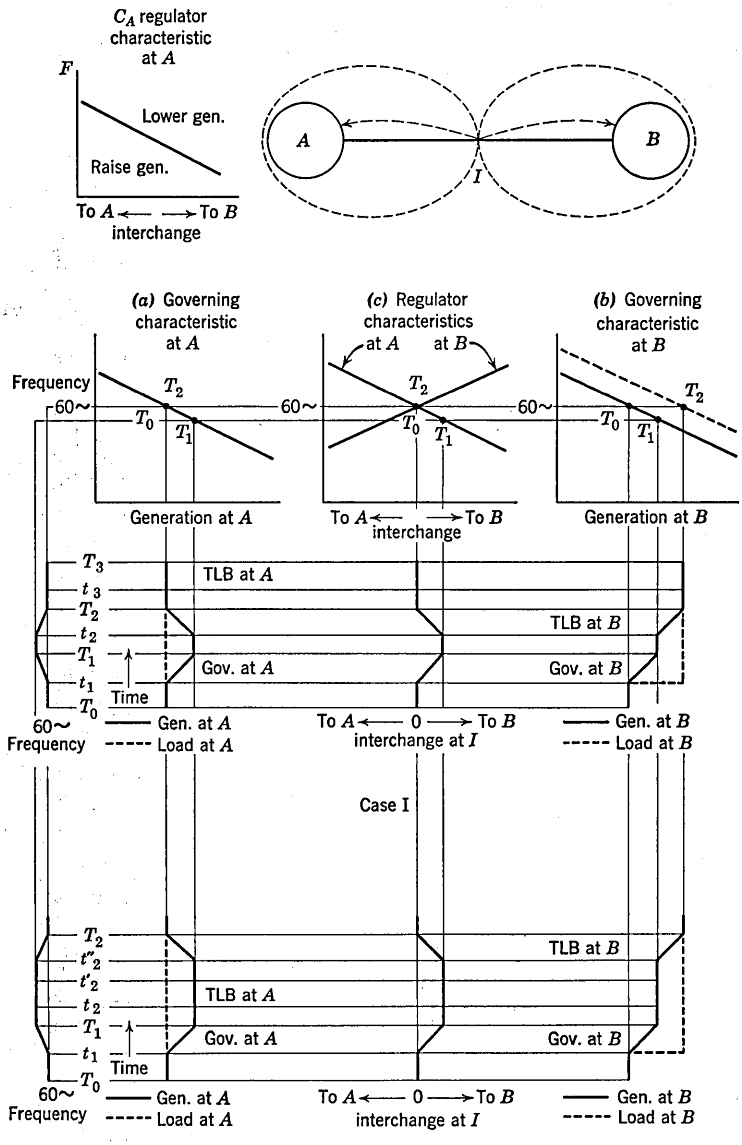
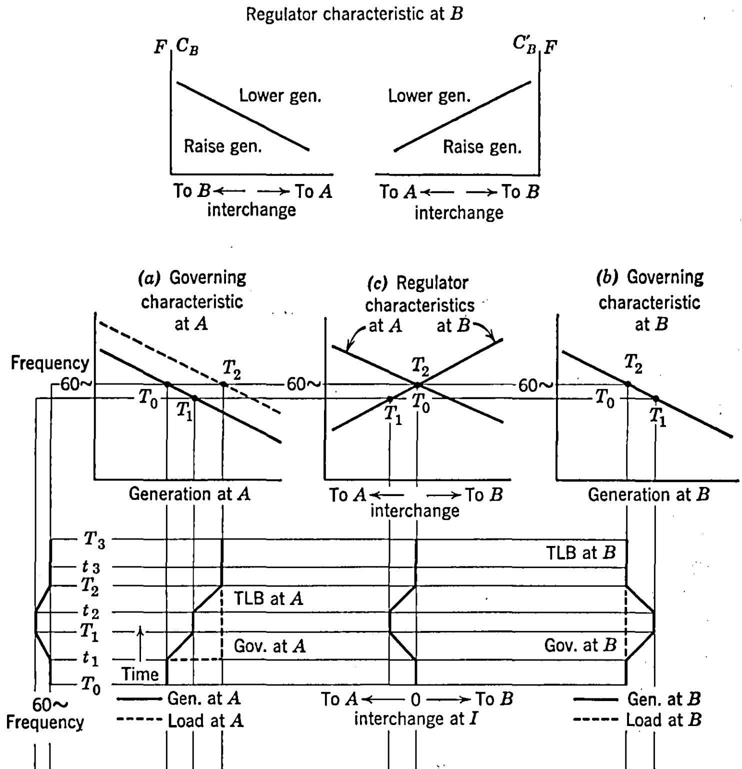
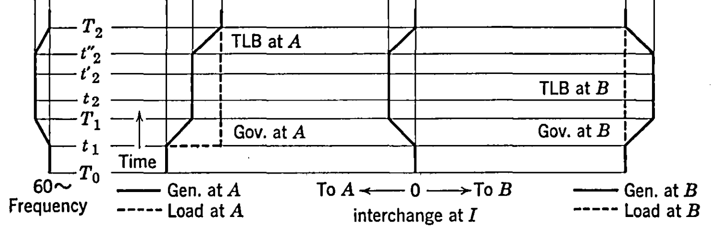


FIG. 20. Regulation responses with supplementary tie line bias control in both areas and no frequency regulator as such on the system. First one bias regulator and then the other is permitted to function after the initial governor responses that follow the designated load increase in one of the areas. In all four cases shown, whether the load change is local or remote, smooth cooperative regulation is obtained. The load change is fully absorbed by the area in which it occurred. System conditions are fully restored to normal, with no undue fluctuations, after the initial excursions due to governor action.



Case II



Case IV

Case I. Load increased at B. Regulation sequence: governors at A and B, tie line bias control at B, tie line bias control at A.

Case II. Load increased at A. Regulation sequence: governors at A and B, tie line bias control at A, tie line bias control at B.

Case III. Load increased at B. Regulation sequence: governors at A and B, tie line bias control at A, tie line bias control at B.

Case IV. Load increased at A. Regulation sequence: governors at A and B, tie line bias control at B, tie line bias control at A.

solid sloping line represents the net interchange bias regulating characteristic on which the area is operating. Area requirement, the prevailing deviation of area generation from biased schedule, is represented by c, which includes two components. The first component b represents the deviation of the net interchange from the 60-cycle schedule T_0. The second component a represents the automatic shift of net interchange schedule with frequency, from T_0 to T_1. When a and b are of equal magnitude and of opposite algebraic sign (see Ref. 24), the deviation from a 60-cycle schedule is that called for by the schedule shift with frequency, and area requirement is zero. This, for example, is true for point I_1, as for all other points that fall on the solid line curve of Fig. 19.

Equations. Equations for area requirement E may be written as follows:

$$E = c = T_1 - T'_1, \tag{3}$$

$$E = (T_0 - T'_1) - 10B(F_0 - F_1), \tag{4}$$

$$E = \Delta T - 10B(\Delta F), \tag{5}$$

where ΔT is the difference between the 60-cycle net interchange schedule T_0 and the prevailing net interchange T'_1,
B is the bias setting in megawatts per one-tenth cycle,
ΔF is the difference between normal frequency F_0 and prevailing frequency F_1.

Algebraic Conventions. Power flow out of an area is considered as plus. Power flow into an area is minus. Bias B is considered as minus, reflecting the negative slope of the bias characteristic. A plus area requirement means that there is a requirement for more generation in the area. A minus area requirement means that area generation must be decreased.

Computation. A functional and block diagram for the computation of area requirement in accordance with eq. (5) is shown in Fig. 21. Area A may be regarded as one of the five areas of the interconnection of Fig. 5. All area boundary tie lines are metered and summated. Telemetering over appreciable distances will probably be required, utilizing carrier or microwave channels. Care should be exercised to insure a simultaneous measurement of all ties so that on summation a true net interchange results. Similarly, there should be simultaneity of the frequency measurement with respect to the tie line measurements so that the computed area requirement reflects coincident area conditions.

Reduction to Zero. In accordance with step 2 of Table 1, supplementary regulation for an area of a multiple area system requires that area net interchange be *on schedule*. This will now be understood to mean the *biased schedule*, and the operating criterion may be stated as *zero area requirement.* Area requirement is reduced to zero by applying any existing area requirement signal through suitable controllers to adjust generation of area sources, as shown schematically in Fig. 21. When generation has been sufficiently adjusted, feedback into the computing loop from the tie line readings and the frequency measurement will have reduced computed area requirement to zero. Area net interchange will be on schedule, the schedule having been suitably and automatically shifted

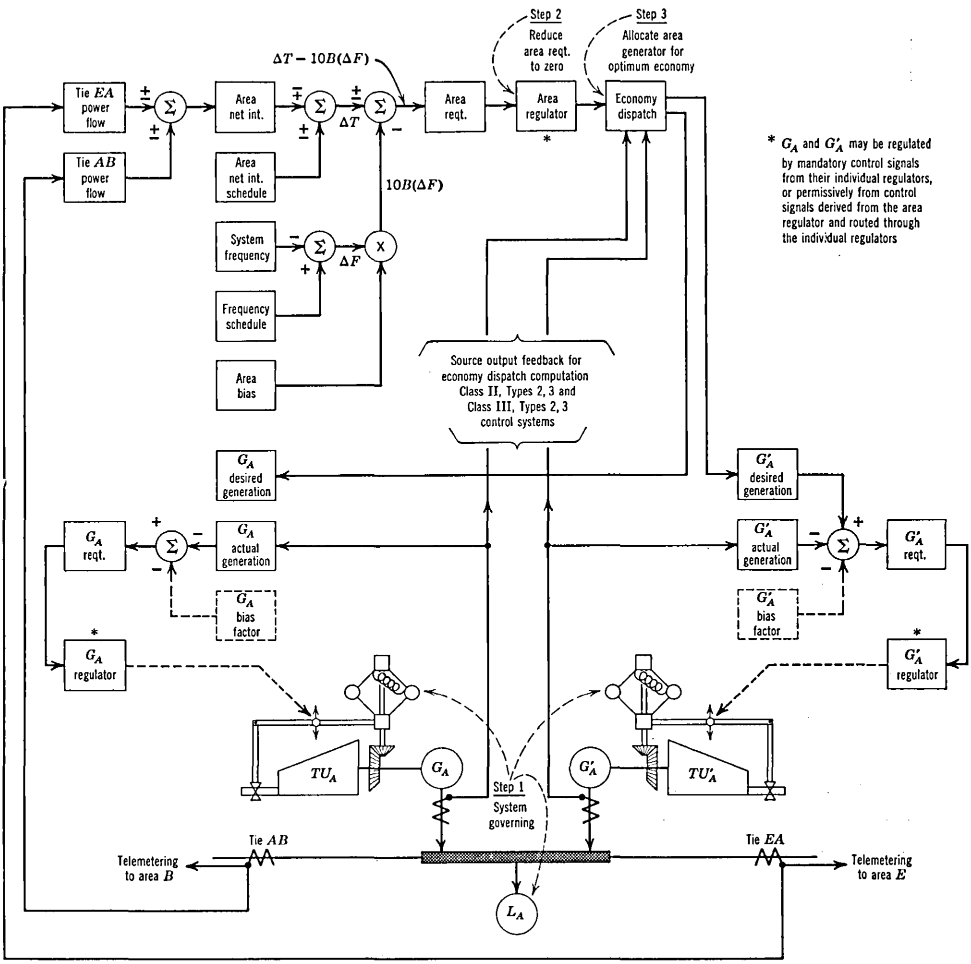


FIG. 21. Functional and block diagram of an automatic generation and power flow control system as applied to area A of Fig. 5. For discussion of source bias factors, shown in broken line blocks, see Section 9.

by the bias component of the computation if there is any prevailing frequency departure from normal.

Nature of Signals. In examining the nature of area requirement signals that will develop from the computing loop of Fig. 21 on occurrence of load changes, consideration must be given to the natural responses of the governors shown in Fig. 21 and of any frequency coefficient of connected load L_A.

Response to Local Load Change. When an area is small compared to the complete interconnected system, initial accommodation of a local load change is over its boundary ties as shown by the (*b*) sketches in Figs. 17 and 18. After occurrence of a local load increase, for example, computed area requirement will be defined by the position of point I'_1 with respect to the bias regulating characteristic in Fig. 19 and will be quantitatively equal to *c*. If the local load change has not influenced system frequency appreciably, component *a* will be relatively small and area requirement will largely be equal to *b*. In any case, control action within the area will cause local generation to increase to match the local load change. Area requirement will thereby be reduced to zero, and frequency and area net interchange will be returned to normal.

Response to Remote Load Change. When a remote load change sufficient to change system frequency occurs, governing action will occur in the local area in response to the frequency change. Thus, on a remote load increase, power will be sent toward the remote area over the local area boundary ties owing to (*a*) an increase of its generation by governing action and (*b*) generation releases that result from the frequency coefficient of the local load. The amount of power sent over the ties will depend on the extent of the frequency error and the combined natural governing characteristic of the local area.

If the bias regulating characteristic of the local area *matches* its natural combined regulating characteristic, the point defined by prevailing frequency and area net interchange will fall on the bias regulating characteristic, and area requirement will remain zero. No imposed regulation will occur in the local area. When the remote area has absorbed its own load change, frequency will have been restored to normal, and the local area net interchange will be back on 60-cycle schedule. Area requirement in the local area will have remained zero all during this operation.

For the nature of local responses to remote load changes when the bias regulating characteristic *does not match* the combined natural governing characteristic of the area, see Sect. 7.

Functions of Bias Control. Tie line bias control can be regarded as having three functions, reflecting the responsibilities of an area to the rest of the system when it becomes part of an interconnection. Terms frequently encountered in discussions of bias functions are *obligation* and *contribution.* Two of the three bias functions are classified as *obligation* functions, reflecting the need of each area to discharge its assigned regulating responsibilities. The third bias function is classified as a *contribution* function, reflecting the unscheduled assistance an area gives its neighbors when they need it. If each area fulfills its obligation functions, demands on other areas for contributions are minimized. An obligation for one area becomes, if performed by another area, a contribution from that area. The three functions are identified in the paragraphs that follow.

Absorbing Local Load Changes. Bias control will cause each operating area to absorb its own local load changes. This is an obligation function.

Sharing Frequency Control. In the generally accepted operating mode of all areas on net interchange bias control, with no central frequency regulating area, tie line bias control causes each area to do its share of system frequency control. This also is an obligation function.

Coordinating with Governing Responses to Remote Load Changes. Tie line bias control determines the steady-state response of an area to a remote load change, after the initial governing response and pending absorption of the remote load change by the remote area. Depending on its setting, the bias control may permit the natural governing response to persist, may add to it, or may diminish it. This is the contribution function.

System-Wide Use of Bias Regulators

Area Circle Diagrams. Each operating area of an interconnection is defined by the closed "circle" drawn through the boundary ties that are to be metered for computation of area net interchange. Generally each tie-metering point serves two adjacent areas, power "in" for one being "out" for the other. Using a common metering point for each tie between two adjacent areas avoids inaccuracies owing to unmetered tie line losses. Closed dash lines define the operating areas of Figs. 4, 5, 6, and 20. Application of net interchange tie line bias regulators to an interconnection having more complex tie line arrangements than was illustrated in those figures is shown in Fig. 22. Dash line circles drawn through the boundary tie points define the individual operating areas.

Area A illustrates a case of one utility A providing the regulation for two nonregulating utilities A_1 and A_2, contained within its regulating area. Area C is also of interest, illustrating how one operating area can encompass another. Area C expects the utility of C_1 to provide its own regulation, but if it fails to do so, area C—rather than the system as a whole—will undertake to make up the deficiency. From the viewpoint of the interconnected system, C and C_1 are regarded as a single area.

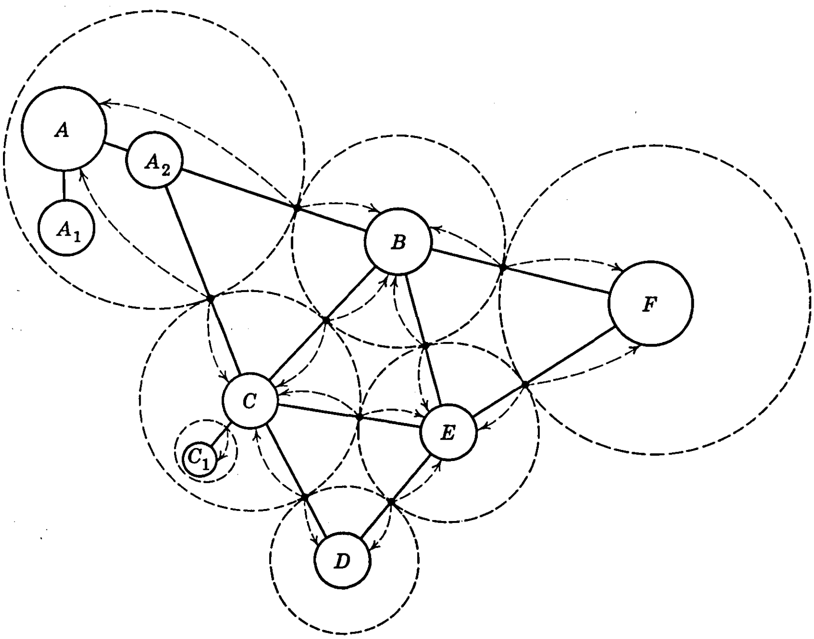


FIG. 22. Area circle diagrams. Solution of the regulating problem of a multiple area interconnection, using net interchange bias control in each regulating area. Area circle diagrams are drawn to define the several regulating areas. All boundary ties are metered and netted for each area.

Area Regulator Curves. Area net interchange bias regulator curves for the six operating areas of Fig. 22 are illustrated in Fig. 23. The abscissa of the figure represents, for each area, net interchange to or from the interconnection. Note that all scheduled "ins" must be equal to all scheduled "outs," illustrated by the top equation in the figure. In this example, all areas use a common bias setting, expressed as a percent of area capacity. The differing slopes of the curves therefore reflect the different sizes of the areas. With all areas thus operating on bias regulation, each area will absorb its own load changes and contribute to maintenance of system frequency, the two obligation functions of bias.

To illustrate the contribution function, assume that with all conditions balanced at the a points, Fig. 23, area D is suddenly disconnected from the system at both of its boundary tie points. Immediately before the emergency, area D was furnishing net interchange I_D to the system. Following loss of area D, there is therefore a deficiency of generation in the remainder of the system equal to I_D. Assuming zero frequency coefficient for connected loads, bias regulators in the remaining areas will operate to maintain or alter the area governing responses until all regulators come

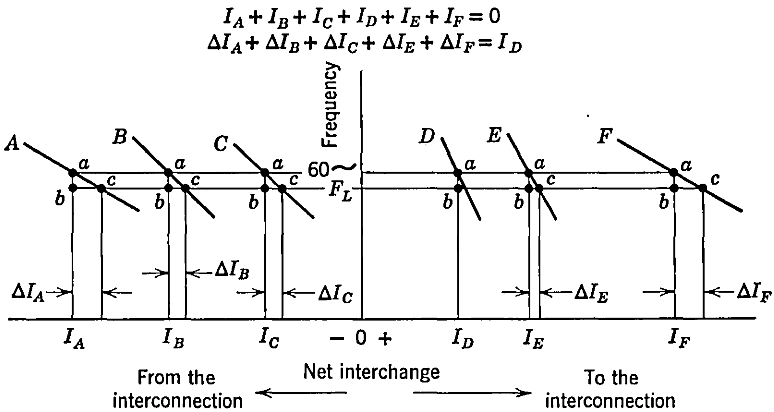


FIG. 23. Area regulator curves. Net interchange bias controllers in each area have the regulating characteristics as shown in curves A to F inclusive. Slopes of these curves are respectively matched as closely as practicable to the composite governing slope in each area. Algebraic sum of schedule settings, I_A to I_F inclusive, equal zero, yielding 60-cycle system frequency without a frequency controller as such on the system. Curves also illustrate the participation in regulation by each area when the whole system must be accelerated, and in addition they demonstrate how each area contributes proportionately to a general system generation deficiency, as on loss of area D.

to balance at the c points, with steady-state system frequency of F_L. The needed generation I_D is made up by the ΔI contribution increments in each of the areas, as shown by the second equation in Fig. 23.

System frequency can now be returned to 60 cycles only if area D returns to the interconnection and maintains its scheduled net interchange at I_D, or if the remaining five areas reset their interchange schedules to be algebraically equal to zero without I_D.

Constant Frequency as a By-product

When all areas of an interconnected system operate on a net interchange bias basis and no area operates on a flat frequency basis, constant frequency for the system is obtained as a by-product of area regulation provided the algebraic sum of all interchange schedules is zero.

For a Two-Area System. This may be illustrated for a two-area system by examining the bias regulator characteristics shown at either of the c sketches in Fig. 20. The characteristics are drawn with a common abscissa representing interchange between areas A and B. If each area has the same schedule with respect to the other area, their regulator characteristics intersect at a point T_0, defined by the prevailing interchange schedule and 60 cycles. If each area regulator operates to reduce its area requirement to zero, there is only one common balance point for the two regulators, namely, 60 cycles. At any other frequency, one regulator or the other, or both, will adjust generation to reduce the corresponding area requirement to zero and thereby establish the scheduled interchange and 60 cycles.

For a Multiple Area System. Frequency control as a by-product of area bias regulation on a multiple area system can be seen from Fig. 23. The only frequency at which net interchange schedules will algebraically add up to zero and area requirement of each area will be equal to zero is 60 cycles. For example, at the *b* points shown in Fig. 23, all area net interchanges are on 60-cycle schedule and the algebraic sum of the prevailing schedules is zero. But frequency is below normal and each area has a prevailing area requirement, as shown by the *b* points falling below their respective regulating characteristics. For this hypothetical case, all area regulators would increase generation, providing an excess over total connected load, until enough stored energy had been added to the system to restore frequency to 60 cycles. All regulators would now be at the *a* points. All area net interchanges would be on schedule, all area requirement regulators would be in balance, and frequency would be at the desired 60 cycles.

Effect of Errors

Errors may exist in the computation of area requirement owing to inaccuracies in frequency or tie line measurements or to inaccuracies in the computing network. Also, an area schedule may be improperly set so that the algebraic sum of all net interchange schedules around the system is not zero. The effect of such errors will be to yield a frequency other than 60 cycles and proportionate steady-state deviations in net interchange power flows.

On a Two-Area System. For a two-area system as shown in Fig. 20, these effects are illustrated in Fig. 24. Assume that area A is without metering or computation errors, that its bias regulator curve is as shown by curve A, and that its 60-cycle interchange schedule has been correctly set to I_0. Assume that area B, owing either to meter errors or incorrect schedule setting, has a regulating characteristic as shown by B instead of the theoretically correct curve B_0. This means that area B, in effect, has a 60-cycle interchange schedule of I_B instead of I_0. Area requirement in each area will be reduced to zero, and the regulators will be in balance at the intersection of the A and B characteristics. This will yield a frequency F_L instead of 60 cycles and an interchange I_1, intermediate between the true schedule I_0 and the incorrect schedule I_B.

To achieve 60 cycles on the two-area interconnection, both regulating characteristics can be offset with respect to the frequency base as shown

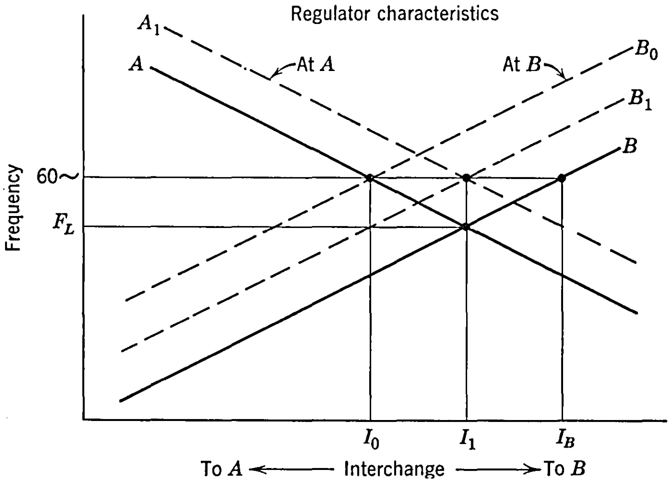


FIG. 24. Effect of incorrect schedule settings, unmetered ties, and metering errors on results obtained with tie line bias control in both areas of the two-area interconnection and no frequency regulator as such on the system. Any of these conditions is demonstrated by area B having regulating characteristic B instead of theoretically correct B_0. Net effect is to cause system frequency to come to balance at some point other than 60 cycles, with corresponding error in scheduled interchange. For the case shown, system balance is defined by frequency F_L and interchange I_1. Correct frequency can be obtained by properly offsetting the frequency base of the two bias regulators, as shown by curves A_1 and B_1.

by characteristics A_1 and B_1. This is the basis for automatic time error correction on an interconnected system as covered in a following paragraph. Such frequency offset yields 60 cycles but still results in interchange I_1 which is off schedule. Each of the areas will accordingly have unscheduled interchange, above schedule in the one and below schedule in the other.

On a Multiple Area System. The effects of metering or computation errors or improperly set schedules on a multiple area interconnection will be comparable to the effects given for the two-area interconnection and are shown by the c points in Fig. 23. Assume that area D is not part of the interconnection and that errors of metering, computation, or schedule setting result in effective 60-cycle schedules I_A, I_B, I_C, I_E, and I_F, which do not algebraically add up to zero. The system will then be at balance at the c points, at some frequency F_L other than 60 cycles and with interchanges correspondingly offset, some higher, some lower.

Inadvertent Interchange. Deviations from a 60-cycle schedule that result from bias regulator contribution responses to deviations of system frequency from 60 cycles or other assigned frequency base are generally regarded as scheduled deviations and are accepted as an inherent part of tie line bias regulation. Interchange deviations that result from metering or computation errors, or from improperly set schedules, are regarded as *inadvertent interchange* or *unscheduled interchange.*

Time Error Correction

Power systems endeavor to maintain integrated frequency within close limits so that synchronous electrical clocks will be accurate. Metering, computation, schedule setting, and regulating errors will contribute to inaccuracies in integrated frequency. In addition, normal excursions of frequency to either side of the reference value will not necessarily be symmetrical, contributing additionally to errors in integrated frequency. Reducing to zero any such prior errors in integrated frequency is defined as *time error correction.* The value of system frequency that should be maintained at any given time is defined as the *frequency schedule.* Synonymous terms are *frequency base* and *frequency reference.*

Corrections to time error are made by intentionally offsetting the frequency schedule of the area bias regulators. When this is done, the operating criteria of (1) the algebraic sum of area interchange schedules being equal to zero, and (2) all area regulators being in balance at zero area requirement (see discussion of Fig. 23), still apply, but they occur at a frequency other than 60 cycles.

Frequency Offset. The extent by which the frequency schedule is shifted from normal in order to correct for previously accumulated time errors is defined as the *frequency offset.* Instructions for frequency offset usually originate from a designated area of the interconnection where the master time reference standard for the system is maintained and where periodic comparisons of system synchronous time and standard reference time are made. If a common time error signal can be transmitted to, or generated at, each area, the frequency offset can be automatically achieved.

Whether the offset is manual or automatic, it is essential that it be achieved simultaneously and with equal magnitude in all areas. If this is done, time error correction in itself will not contribute to deviations from tie line interchange schedules or to inadvertent interchange.

Period of Correction. The time required to correct a prevailing time error with a given frequency offset is given by the following general relation:

$$m = \frac{F}{60}\left(\frac{t}{f}\right), \tag{6}$$

where m is the time in minutes required to achieve the correction,
F is the normal system frequency,
t is the prevailing time error in seconds,
f is the frequency offset, in cycles per second.

On a 60-cycle system, this relation becomes

$$m = t/f. \tag{7}$$

On a 60-cycle system, with a 0.05-cycle frequency offset, it would thus take 20 minutes to correct a 1-second error. With a 0.02-cycle offset, it would take 50 minutes to correct a 1-second error.

Summary Criteria for Area Regulation

The following steps in the synthesis and application of area regulators will provide effective cooperative control among the areas of a multiple area interconnection.

1. Define Area Boundaries. Each area will be that contiguous portion of the system which is normally to provide its own regulation. The several areas, taken together, should account for all the interconnection.

2. Meter Boundary Ties. Telemeter each boundary tie to the two areas that it interconnects. Each "out" will have a corresponding "in."

3. Compute Area Net Interchange. For each area, compute the algebraic net of its boundary tie power flows yielding area net interchange with the system as a whole.

4. Set Schedules on Net Interchange Basis. Assign a net interchange schedule to each area, representing its desired interchange over all its boundary ties with the remainder of the interconnection.

5. Set for Algebraic Sum Zero. The algebraic sum of all the area net interchange schedules for the interconnected system as a whole must be zero. This condition must be fulfilled even during periods of schedule change.

6. Assign a Common Frequency Base. Assign a common frequency base to the system. This will normally be 60 cycles. There will be prearranged common offsets from this value when time error correction is to be achieved.

7. Assign Area Bias Factor. Assign a bias factor to the area. If it matches the natural governing characteristic of the area, there will be full coordination with governing responses to remote load changes. Other settings will be used if other imposed regulation responses are desired.

8. Compute Area Requirement. Make this computation in accordance with eq. (5) and Fig. 21. Endeavor to have simultaneity for all metered variables that enter into the computation.

9. Execute Bias Regulation. Control the generation of the area with a net interchange bias regulator, which will act to reduce area requirement to zero.

10. Use Adequate Regulating Capacity. Back up the bias regulator with adequate and responsive capacity so that the regulating demands on the area can be met within the limits of area requirement deemed permissible by the interconnection.

7. REGULATION AS A FUNCTION OF BIAS SETTING

General Effects

A major matter in the operation of area net interchange tie line bias control is the question of how much bias should be used. As interconnected systems have grown and been strengthened, it has been possible for large blocks of load or generation to be suddenly lost, causing significantly large changes in frequency, but with the system holding together. Automatic bias controls stay in operation, and the action they impose on their respective areas is a function of the system frequency departure and the area bias setting. Bias settings are of greater importance in the contribution function of bias operation than in its two obligation functions. See Sect. 6 for definitions of these terms.

Obligation Functions. For the obligation function of absorbing local load changes, even a flat tie line controller with zero bias will operate satisfactorily. For the obligation function of participating in frequency control, any bias setting that is high enough to coordinate with the sensitivity of the bias controller, and not so high as to cause hunting between areas, is satisfactory.

Contribution Function. For the contribution function of responding to remote load changes the bias factor is critical. It determines the nature of the imposed responses of the area and the effectiveness and overall coordination of system regulation. It determines whether, during periods of system disturbance, nondisturbance areas will alleviate or enlarge the effects of the disturbance. This section analyzes the effect of bias settings on frequency, tie line power flow, and local generation in response to remote load changes.

Analysis Approach

Consider a two-area interconnection as shown in Fig. 25. This may be regarded as a simplification of the interconnection of Fig. 6, with area D representing the area in which a step function load change is to occur, whereas all the remainder of that interconnection is now represented by area A. Area A is appreciably larger than disturbance area D. After application of the step function change at area D, there will be natural

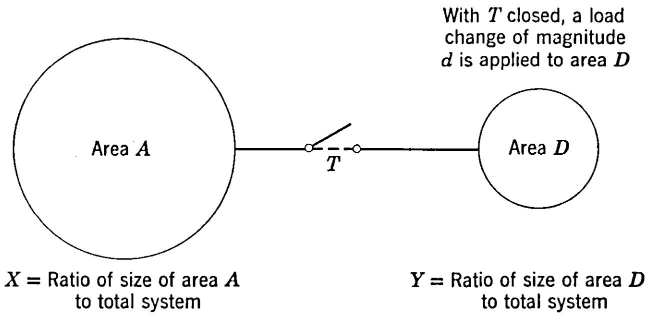


FIG. 25. Representation of an interconnected system. Responses at area A to a remote load or generation change at area D are analyzed.

governing responses by the entire system. Since bias responses to remote load changes are to be analyzed, it is assumed that system governing responses are followed by bias regulation at area A only, until area requirement for area A is reduced to zero. To permit full analysis of responses at area A, it is assumed that no supplementary regulation occurs at area D. The many operating entities that comprise area A would share in the aggregate action of this area in proportion to their sizes and operating characteristics.

Natural Governing Responses

Isolated Operation. With breaker T of Fig. 25 open, area A is operating isolated, and its natural governing characteristics with and without a frequency coefficient of load are shown in Figs. 11 and 12. In both cases, balance is achieved, after a step function load change, by natural governing action and occurs at the intersection of the generation and load characteristics.

Interconnection Operation. With breaker T of Fig. 25 closed, the natural generation, load, and combined governing characteristics of area A are shown in Fig. 26. Assuming zero initial interchange and initially balanced system conditions, the generation in area A will be defined by the intersection I_0 of its natural generation characteristic GG and its natural load characteristic LL. After a step function load increase in area D, assumed sufficiently large to reduce system frequency to 59.9 cycles, generation in area A will be increased to the I_2 point whereas its load will be decreased to the I_1 point. The difference between generation and load ΔT represents excess area generation and will flow out over the tie to the disturbance area, area D. This excess generation ΔT is also shown on the natural combined governing characteristic CC as the I_3 point.

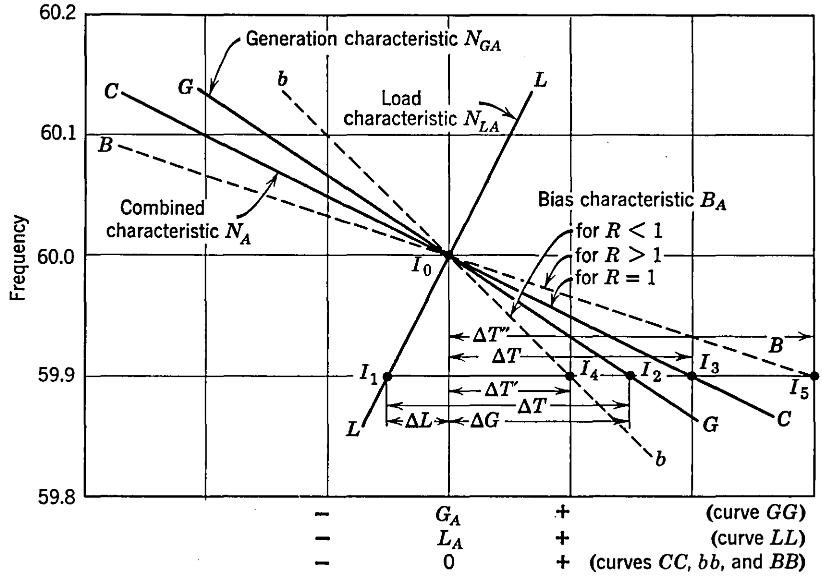


FIG. 26. Governing and regulating characteristics of area A, showing response to a step function load change at area D, Fig. 25. Curve GG is the area generation characteristic. Curve LL is area load characteristic. Curve CC is the combined area characteristic plotted as the difference between generation and effective load versus frequency. It is also the area bias characteristic when $R = 1$ and is then a plot of tie line power flow versus frequency. Curves bb and BB are area bias characteristics for $R < 1$ and $R > 1$, plotted as tie line power flow versus frequency.

Imposed Regulation Responses

The action of the bias regulator at area A will depend on the ratio R of the slope of the bias characteristic to the prevailing slope of the combined governing characteristic. With a matching slope, $R = 1$, there will be no imposed supplementary regulation. With a *steeper* slope, $R < 1$, reflecting a bias *lower* than the natural combined characteristic and illustrated by curve bb, there will be regulating action in the *decrease* direction *withdrawing* some of the natural governing response. With a *shallower* slope, $R > 1$, reflecting a bias *higher* than the natural combined characteristic and illustrated by curve BB, there will be regulating action in the *increase* direction *augmenting* the natural governing response. These supplementary responses will influence system frequency, tie line power flow, and effective load in area A as well as generation in area A. The nature of these effects are shown graphically in Fig. 27.

From T_1 to T_2 in Fig. 27, initial steady-state conditions prevail. At time T_2 a step function load increase occurs at area D. After transient responses to the disturbance, a new steady state is achieved at T_2 as a

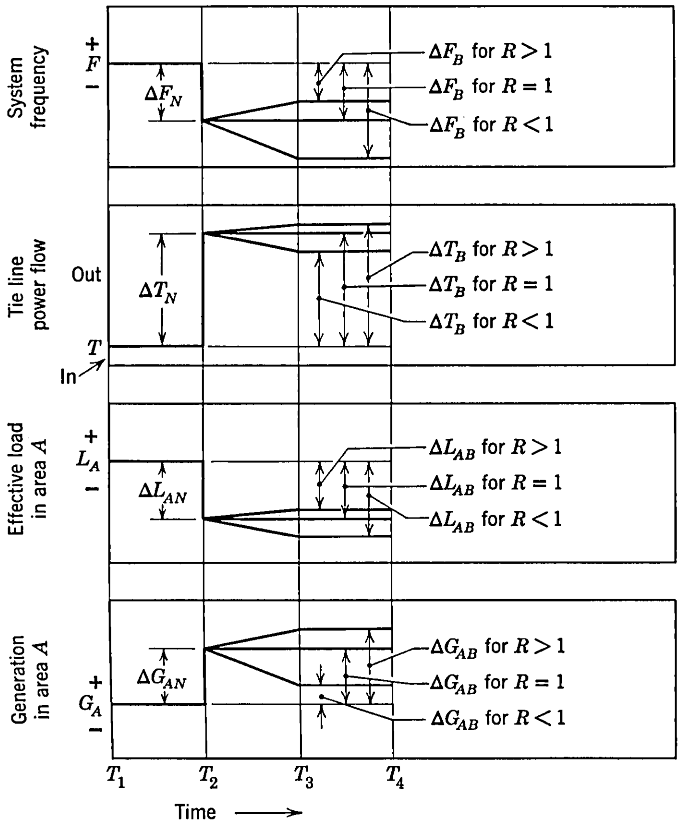


FIG. 27. Steady-state changes in frequency, tie line power flow, and load and generation in area A, after a load or generation disturbance in area D of Fig. 25.

result of the natural governing responses of the system. From T_2 to T_3, imposed bias regulation occurs in area A. For each of the four parameters considered, three curves are drawn from T_2 to T_3 corresponding to bias regulation effects when bias *matches,* is *less than,* or is *greater than* the natural combined governing characteristic of area A. Steady-state conditions for all curves are achieved at time T_3.

Criteria for Defining Bias Responses

Referring to Fig. 27, convenient criteria for defining the effect of different bias settings on frequency, tie line power flow, and generation in the nondisturbance area are the *imposed changes* after bias regulation as a *percentage of,* or *ratio to,* the *initial changes* caused by natural governing action.

Frequency Criterion. The criterion for the frequency effect is defined as the imposed change in frequency deviation as a percentage of the initial deviation. It is designated at $\%\Delta F$ and is given by the following equation:

$$\%\Delta F = \frac{\Delta F_B - \Delta F_N}{\Delta F_N}(100), \tag{8}$$

where ΔF_N is the steady-state change in frequency due to natural governing response to a remote step function load change,
ΔF_B is the net steady-state change in frequency from its initial value, after completion of local bias regulation.

Tie Line Criterion. The criterion for the tie line effect is defined as the imposed change in tie line contribution as a percentage of initial contribution. It is designated as $\%\Delta T$ and is given by the following equation:

$$\%\Delta T = \frac{\Delta T_B - \Delta T_N}{\Delta T_N}(100), \tag{9}$$

where ΔT_N is the steady-state change in tie line flow due to governing response to a remote step function load change,
ΔT_B is the net steady-state change in tie line flow from its initial value, after completion of local bias regulation.

Generation Criteria. Criteria for the generation effects may be established in three ways, differing by the reference base that is used. To define these criteria fully, assume that the lower set of curves in Fig. 27 applied to *regulated* rather than *total* generation in area A, adding an R to the corresponding subscripts.

Criterion I, designated $\%\Delta G_{AR}$, is defined as the imposed change in loading of the regulated generation of area A, as a percentage of its initial loading, and is given by the following equation:

$$\%\Delta G_{AR} = \frac{\Delta G_{ARB} - \Delta G_{ARN}}{G_{AR}}(100), \tag{10}$$

where ΔG_{ARN} is the steady-state change in regulated generation at area A owing to governing response to a remote step function load change,
ΔG_{ARB} is the net steady-state change in area A regulated generation from its initial value, after completion of local bias regulation,
G_{AR} is the initial regulated generation at area A.

Criterion II, designated $\%\Delta G'_{AR}$, is defined as the imposed change in loading of the regulated generation as a percentage of the initial total generation at area A and is given by the following equation:

$$\%\Delta G'_{AR} = \frac{\Delta G_{ARB} - \Delta G_{ARN}}{G_A}(100), \tag{11}$$

where G_A is the initial total regulated generation at area A.

Criterion III, designated $\%\Delta G''_{AR}$, is defined as the imposed change in loading of the regulated generation as a ratio to the initial generation response and is given by the following equation:

$$\Delta G''_{AR} = \frac{\Delta G_{ARB} - \Delta G_{ARN}}{\Delta G_{ARN}}. \tag{12}$$

Variables Affecting Bias Responses

Equations have been derived which quantitatively define the effect of bias settings and other variables on the frequency, tie line, and generation responses shown graphically in Fig. 27. Significant variables which appear in some of or all the equations are defined in the paragraphs that follow. For derivations and interpretations of the equations, and discussion of the assumptions that permit their reduction to the simplified forms shown in subsequent subsections, see Refs. 8 and 9.

Bias Ratio. The ratio R at the *nondisturbance area* of the bias setting to the combined natural governing characteristic of the area is defined as the *bias ratio.*

Size Ratio. The ratio Y of the size of the *disturbance area* to the size of the total system, based on respective intial generation magnitudes, is defined as the *size ratio.*

Governing Ratio. The ratio P at the *nondisturbance area* of the natural generation governing characteristic to the combined natural generation and load governing characteristic for the area is defined as the *governing ratio.*

Regulating Ratio. The ratio Q at the *nondisturbance area* of the amount of generation subject to regulation by the bias controller to the total generation within the area is defined as the *regulating ratio.*

Magnitude of Disturbance. The frequency, tie line, and one of the generation response equations are written to be independent of the *magnitude* of the step function disturbance. Two of the generation change equations include this variable, which is designated as d and is defined as the size of the disturbance in percent of initial total system generation.

Frequency Change Equation

The equation defining the imposed change in the frequency deviation may be written

$$\%\Delta F = \frac{(Y - 1)(R - 1)}{R - Y(R - 1)} (100). \tag{13}$$

Curves of $\%\Delta F$ versus Y for different values of R are shown in Fig. 28.

Appraisal. $\%\Delta F$ is not influenced by the magnitude of the disturbance, nor by the magnitude of P or Q. It is affected only by the R and Y ratios.

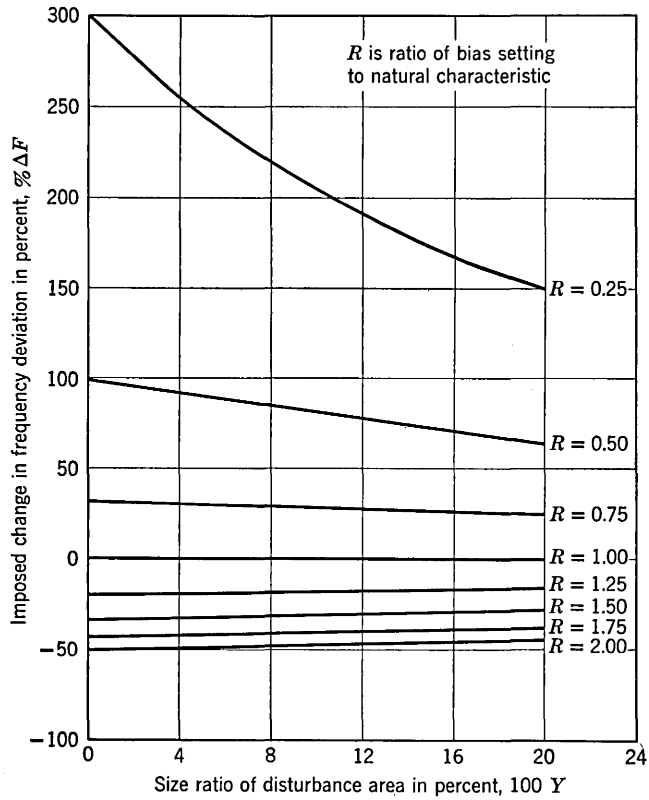


FIG. 28. Imposed change in system frequency as a function of the bias ratio and the size of the disturbance area.

When $R = 1$, $\%\Delta F$ is zero and there is no imposed regulating effect on frequency. When the disturbance area is 10% of the system, $R = 0.5$ imposes a change in frequency deviation of approximately 80% in the direction that further upsets system frequency. For the same size disturbance area, $R = 1.50$ imposes a change in frequency deviation of 30% in the direction that helps restore system frequency. A bias ratio a given percentage *below* the natural characteristic has a relatively greater effect on frequency in the *upsetting* direction than a bias ratio the same percentage *above* the natural characteristic has in the *corrective* direction.

Tie Line Change Equation

The equation for defining the imposed change in the tie line deviation may be written

$$\%\Delta T = \frac{Y(R-1)}{R - Y(R-1)}(100). \tag{14}$$

Curves of $\%\Delta T$ versus Y for different values of R are shown in Fig. 29.

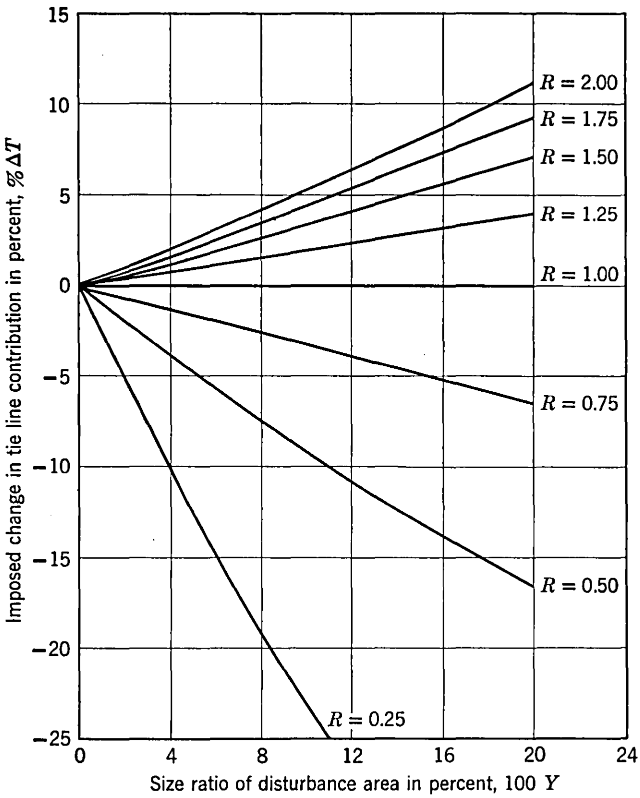


FIG. 29. Imposed change in tie line power flow as a function of the bias ratio and the size of the disturbance area.

Appraisal. As in the case of the frequency change equation, $\%\Delta T$ is independent of d, P, and Q.

Where $R = 1$, there is no imposed change in tie line contribution. When R is less than 1, part of the initial governing contribution is backed off. For a 10% disturbance area and for $R = 0.5$, this amounts to only 9%. When R is greater than 1, there is an additional tie line contribution but it does not add significantly to the initial contribution. With the 10% disturbance area and $R = 1.5$, the addition to the contribution is only about 3%.

Generation Change Equations

Three equations define respectively the three generation change criteria. All three are functions of P and Q as well as of Y and R, and two of

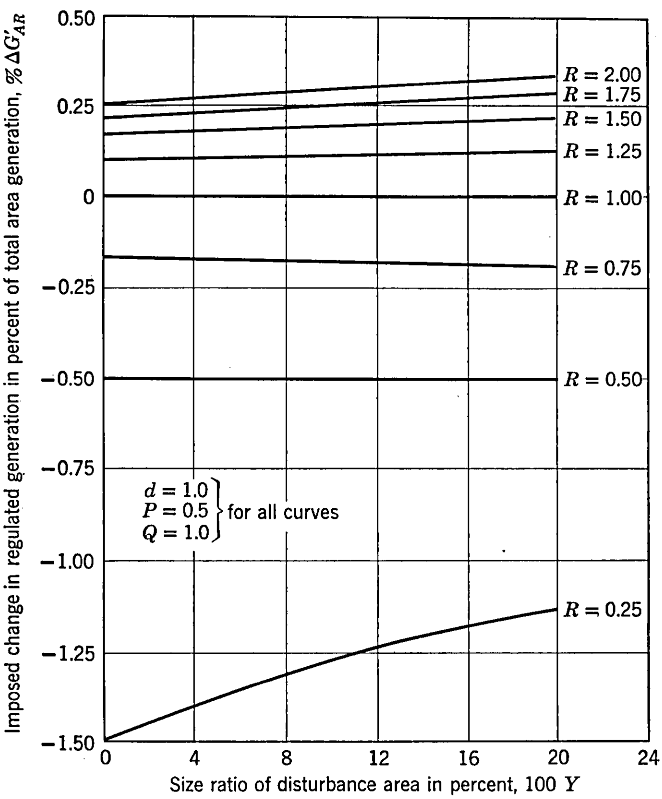


Fig. 30. Imposed regulation for a remote 1% of system disturbance, in percent of total generation. P and Q are constant.

them also include the variable d. The equations are as follows:

$$\%\Delta G_{AR} = \frac{d}{Q}\,\frac{[1 + PQ(Y - 1)](R - 1)}{R - Y(R - 1)}, \tag{15}$$

$$\%\Delta G'_{AR} = d\,\frac{[1 + PQ(Y - 1)](R - 1)}{R - Y(R - 1)}, \tag{16}$$

$$\%\Delta G''_{AR} = \frac{1}{PQ}\,\frac{[1 + PQ(Y - 1)](R - 1)}{R - Y(R - 1)}. \tag{17}$$

Curves based on these equations are shown in Figs. 30 and 31 for eq. (16); Fig. 32 for eq. (15); and Fig. 33 for eq. (17).

Appraisal. In appraising the generation changes at area A which are caused by imposed bias action, it should be remembered that any action

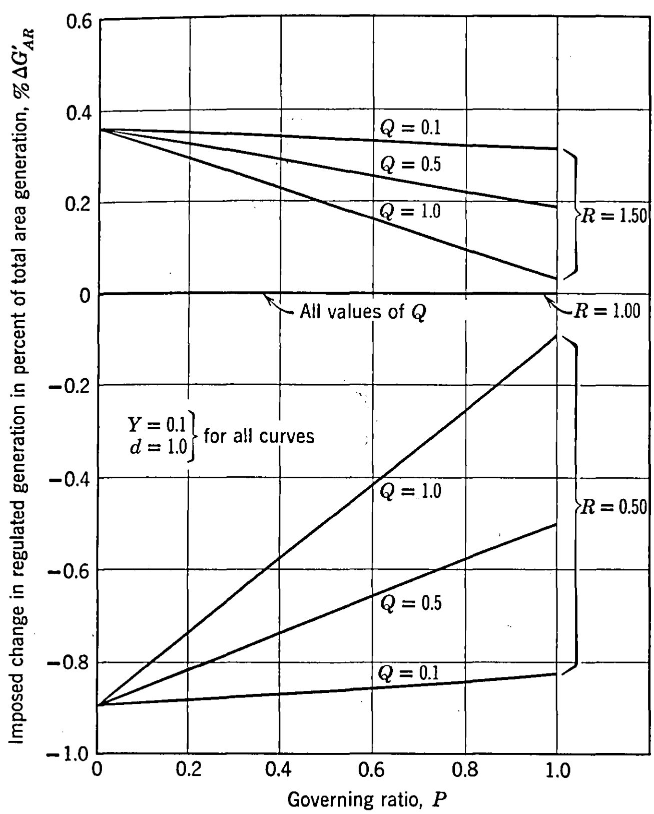


FIG. 31. Imposed regulation for a remote 1% of system disturbance, in percent of total generation, for selected bias ratios and for varying P and Q.

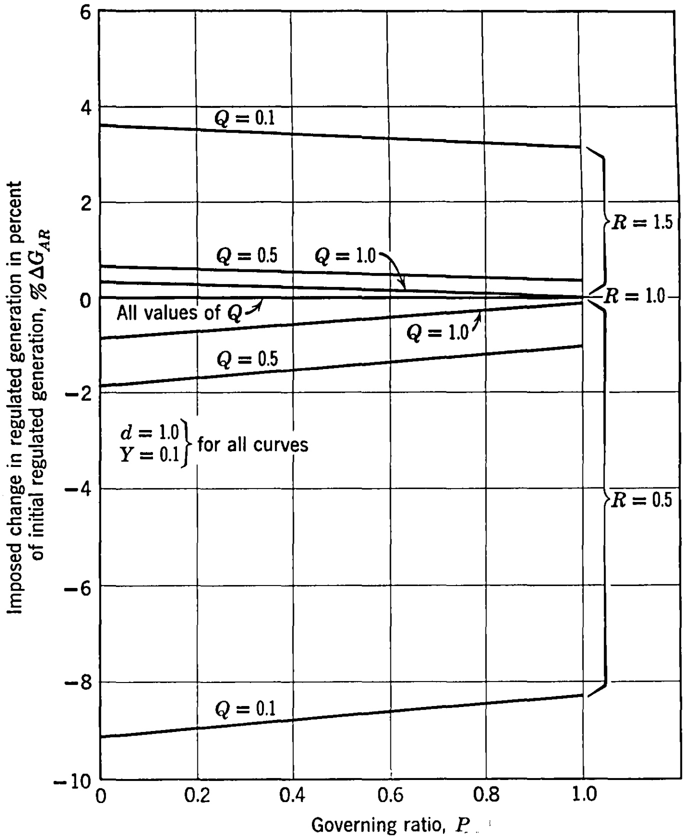


FIG. 32. Imposed regulation for a remote 1% of system disturbance, in percent of the regulated generation, for selected bias ratios and for varying P and Q.

taken by the bias regulator for a remote load change, whether in a direction to add to or subtract from the natural generation response, must ultimately be undone by subsequent bias regulator action in the opposite direction.

When $R = 1$, bias control imposes no additional regulation on the nondisturbance area.

When R is less than 1, the imposed generation change is in the direction opposite to the initial governing contribution. The effect is magnified as P increases. It is also magnified as Q decreases, meaning that the burden on regulating units is increased as less of the generation is placed on control, to an extent even greater than would be expected from the value of Q itself.

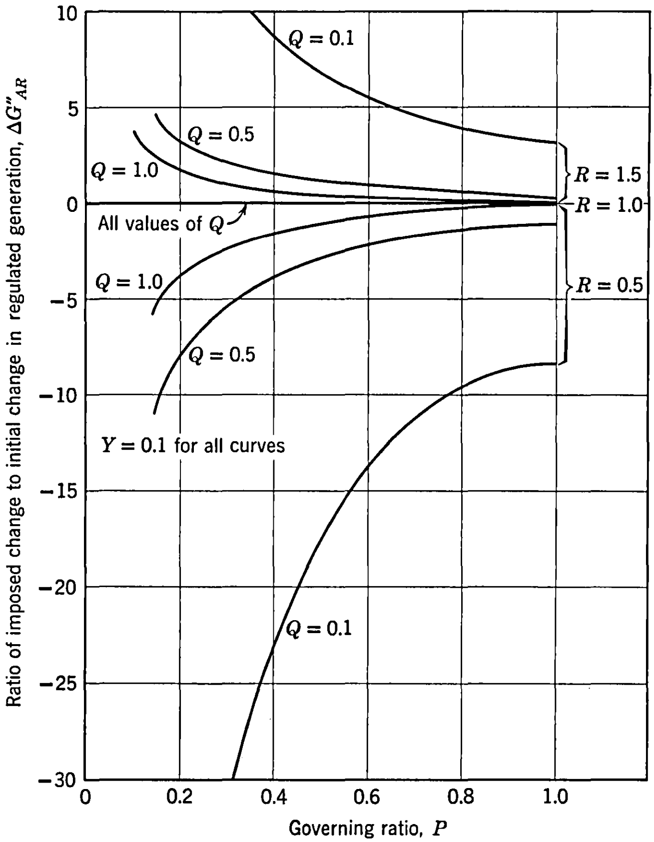


FIG. 33. Imposed regulation as a ratio to the initial governing response of the regulated generation, for selected bias ratios and for varying P and Q.

When R is greater than 1, bias control action adds to the initial generation increase, although in amounts relatively smaller than the decreases imposed when R is proportionately less than 1.

Example of Imposed Bias Responses

For Fig. 25, assume that area A is 36,000 Mw and area D is 4000 Mw. Assume that each area has a 1% combined natural characteristic and that the step function load increase d at area D is 400 Mw. The corresponding natural and imposed changes in system frequency, tie line power flow, effective load in area A, total generation in area A, regulated generation in area A, and unregulated generation in area A are shown for different values for P, Q, and R in Fig. 34. Table 4 shows

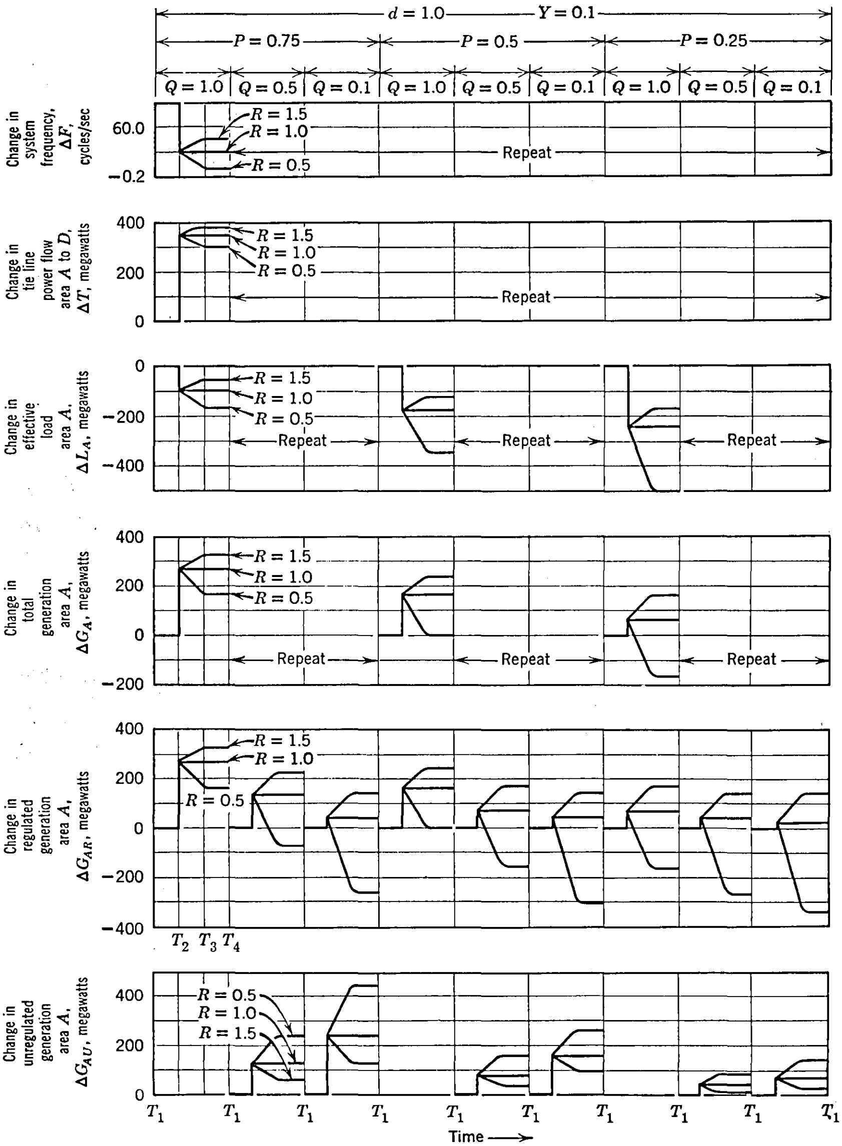


FIG. 34. Curves illustrating a numerical example of natural and bias-imposed responses to a remote load change, for various values of P, Q, and R. Data related to the example are contained in Tables 4 and 5.

TABLE 4. NATURAL AND IMPOSED RESPONSES
Q AT AREA A EQUAL TO UNITY

	After Natural Responses	After Bias Regulator Responses		
		$R = 1.0$	$R = 0.5$	$R = 1.5$
1. Change in frequency	−0.10	−0.10	−0.18	−0.07
2. Change in effective load, area A	−180	−180	−328	−124
3. Change in total generation, area A	180	180	0	248
4. Change in tie line power flow	360	360	328	372
5. Change in effective load, area D, exclusive of 400-Mw addition	−20	−20	−36	−14
6. Change in total generation, area D	20	20	36	14

These are natural and imposed responses after a 400-Mw load change in area D, for bias ratios equal to, 50% less than, or 50% greater than the combined natural characteristic, i.e., $R = 1$, $R = 0.5$, and $R = 1.5$ respectively.

Each area has a 1.0% per 0.1 cycle combined natural characteristic, made up of a generation characteristic of 0.5% per 0.1 cycle and a load characteristic of 0.5% per 0.1 cycle ($P = 0.5$).

It is assumed that all generation in area A is subject to bias regulator action ($Q = 1$), and that no bias regulator action occurs in area D.

All figures are net change from initial condition.

the magnitude of significant responses when all generation at area A is subject to bias control ($Q = 1$). Table 5 shows the significant additional effects on area A regulated and unregulated generation when all the generation in area A is not subject to regulation, for values of $Q = 0.5$ and $Q = 0.1$.

TABLE 5. NATURAL AND IMPOSED RESPONSES
Q AT AREA A NOT EQUAL TO UNITY

	After Natural Responses		After Bias Responses					
			$R = 1.0$		$R = 0.5$		$R = 1.5$	
	$Q = 0.5$	$Q = 0.1$	$Q = 0.5$	$Q = 0.1$	$Q = 0.5$	$Q = 0.1$	$Q = 0.5$	$Q = 0.1$
3. Change in total generation, area A	180	180	180	180	0	0	248	248
3*a*. Change in unregulated generation, area A	90	162	90	162	164	295	62	112
3*b*. Change in regulated generation, area A	90	18	90	18	−164	−295	186	136

These are natural and imposed responses in regulated and unregulated generation at area A, for the example of Table 4, but with first one-half of the total generation at area A subject to bias regulator action ($Q = 0.5$) and then one-tenth of its total generation so controlled ($Q = 0.1$).

Summary of Bias Effects

A summary of conclusions related to bias effects is tabulated below.

1. Frequency Coefficient of Load. A frequency coefficient of load increases the natural governing characteristics of an area. Where an area endeavors to match its bias characteristic to the combined governing characteristic, a correspondingly higher bias setting is required.

2. Effect on Obligation Functions. The magnitude of bias settings does not in general critically affect the obligation functions of responding to local load changes and participating in system frequency control.

3. Effect on Contribution Function. The magnitude of the bias setting is of considerable significance in the contribution function of a bias controller, establishing whether natural responses to remote load changes will be unaltered, added to, or opposed.

4. Bias Performance Equations. Bias performance equations for remote load changes show that

a. Bias settings which match the prevailing natural characteristic impose no further change on frequency, tie line power flow, or local generation.

b. Bias settings which are a given percentage lower than the natural characteristic impose a relatively greater change on frequency, tie line, and local generation than bias settings which are the same percentage higher than the natural characteristic.

c. Bias settings that differ from the natural characteristic impose relatively large changes on system frequency.

d. Bias settings that differ from the natural characteristic, in the range of 50% below to 100% above the characteristic, cause relatively little change in tie line contribution.

e. The greater the load component of the natural characteristic, the greater the changes in local generation caused by bias settings that differ from the natural characteristic.

f. The larger the amount of local capacity not subject to bias control action, the greater the changes in local generation caused by bias settings that differ from the natural characteristic.

8. ECONOMY DISPATCH

General Objective

As discussed in Sect. 3, *economy dispatch* is that step in the solution of the generation control problem which allocates generation changes of an area among alternative area sources to achieve optimum area economy.

Opportunity for Savings. Steady advances in recent years in the technology of energy conversion, characterized by increased sizes, pressures, and temperatures of steam generating units, have resulted in continued improvement of unit operating efficiencies. There has, correspondingly, been an increasing disparity between the heat rates of the alternative generating sources which might, at a given time, be used to satisfy consumer demand. Such disparity, coupled with varying fuel costs and varying transmission loss factors, makes possible very signifiicant fuel savings as a result of automatic economic loading of available sources.

In 1940 (Ref. 10) the average heat rate for electric energy produced from fossil fuels was 16,400 Btu/kwhr. By 1950 it had improved to 14,000 Btu/kwhr. By 1960 it was of the order of 11,000 Btu/kwhr with individual heat rates of 9000 Btu/kwhr or better.

A typical-sized power company may have a coal bill of 20 to 30 millions of dollars or more per year. It clearly does not require a large percentage saving to justify an appreciable investment in an automatic economy dispatch system.

Factors to be Considered. In achieving economy dispatch of an area, consideration must be given to the operating efficiencies of available sources, to fuel costs, and to transmission losses. Where opportunities exist to purchase economy interchange power from an adjacent utility, consideration will also be given to the relative cost of such purchased power at the tie point and the losses that would be involved in transmitting such power to the area load.

A simplified representation of the economy dispatch problem, applied, for example, to area A of Fig. 5 is shown in Fig. 35.

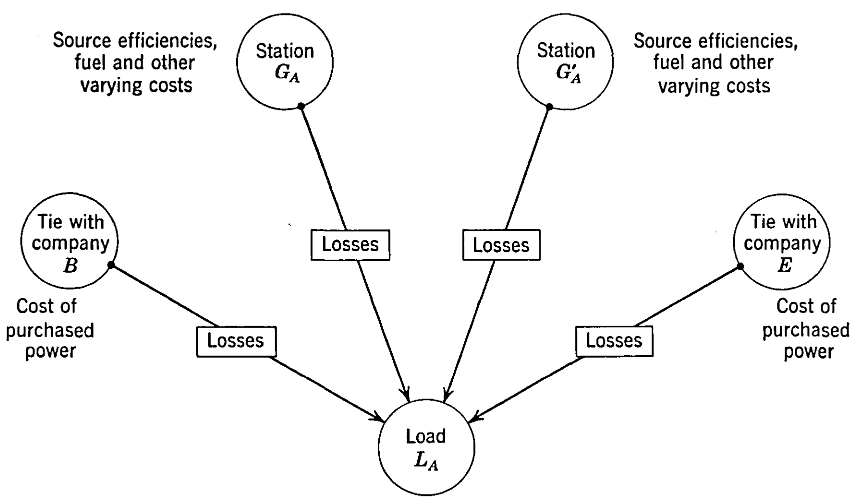


FIG. 35. Diagram showing factors to be considered in determining how to accommodate area load changes for optimum economy dispatch, considering fuel and other varying costs, source efficiencies, and line losses.

To illustrate, plant G_A may be able to generate the next needed increment of power with a better heat rate than could be achieved at plant G'_A. However, prevailing fuel cost at G'_A may be sufficiently lower than that at G_A for the actual cost of producing this next needed increment of power to be cheaper at G'_A than at the more efficient plant G_A. Further analysis may show, however, that there is a sufficiently greater transmission loss in getting this next increment of power from plant G'_A to the load than there would be in getting it from plant G_A to the load. Thus, even though it costs less to generate the next increment of power at G'_A, the power increment that would actually be delivered to the load from G_A may have a lower net cost than the same amount of additional power delivered to the load from G'_A. Under such conditions economy dispatch would call for assigning this next increment of generation to G_A.

Still further analysis may show, however, that it would be even cheaper to secure the next increment of power from neighboring utility B, which may be in a different time zone and may have some extra power for sale at a cost, even allowing for transmission losses, lower than any of the available sources of company A could deliver the same increment of power to load L_A at that time.

A rigorous treatment of comparative costs at alternative sources for the next needed increment of power requires that consideration be given to such additional varying cost factors as incremental maintenance, incremental supplies, ash handling, and labor, all frequently referred to as *O.T.F. costs* (costs other than fuel). In the discussions of this section it is assumed that O.T.F. costs can be expressed in terms of fuel and have been added to the fuel costs.

References. This section summarizes fundamental relationships between the several factors that influence economy dispatch. It then describes the basic concepts that have been utilized to achieve such economic loading with automatic equipment. For more detailed discussions of the derivation and utilization of transmission loss and coordination equations, see Refs. 11, 12, 13, and 14. References for detailed descriptions of typical actual automatic control systems that use the concepts described in this section are included in the discussion of each of the control type classifications.

Source Efficiencies and Generation Costs

The effectiveness with which a thermal plant fulfills its energy conversion task is defined by its input-output curve. In the interest of simplicity, this section considers input-output and related relationships for single boiler-turbine-generator units. For a discussion of common header boilers serving several turbines, see Ref. 15.

Input-Output Curve. A typical input-output curve is shown in Fig. 36. Input H is plotted in millions of British thermal units per hour, output P as megawatts.

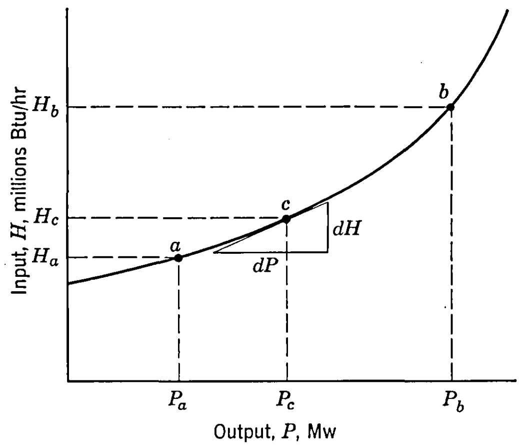


FIG. 36. Typical input-output curve.

Total or Average Heat Rate. For any given output the ratio of total input to total output represents the prevailing *total heat rate.* Thus, at points a and b in Fig. 36, the ratios H_a/P_a and H_b/P_b represent respectively the prevailing total heat rate at those two points. A curve of all such H/P ratios plotted against output is shown in Fig. 37. Total heat rate at any output is also referred to as *average heat rate* at that output. Its dimensions are British thermal units per kilowatthour.

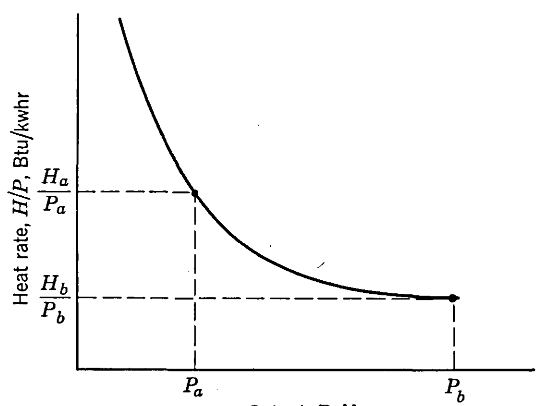


FIG. 37. Total heat rate curve, based on input-output curve of Fig. 36.

Average Efficiency. Source efficiency is based on the ratio of output to input. Hence, *average efficiency* at a given output is proportional to the reciprocal of average heat rate.

Incremental Heat Rate. The term *incremental heat rate* is defined, for any given output, as the ratio of a small change in input to the resultant change in output. It is a measure of how efficiently the next increment of output can be produced. It is the slope, at various outputs, of the input-output curve of Fig. 36. This is illustrated by the slope $\dfrac{dH}{dP}$ of the tangent at point c in Fig. 36. The slope of all such tangents, in British thermal units per kilowatt-hour is plotted against output in megawatts in Fig. 38, which represents a hypothetical incremental heat rate curve.

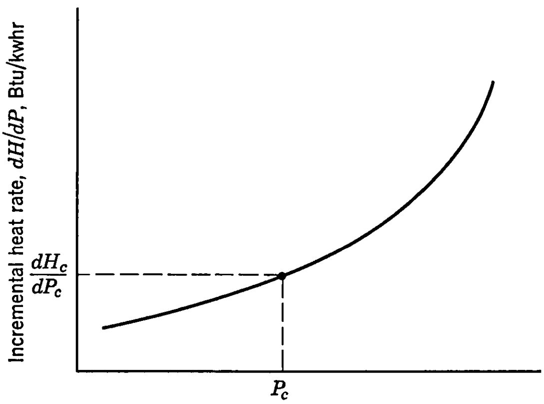


FIG. 38. Incremental heat rate curve, based on the input-output curve of Fig. 36.

Some of the economy dispatch techniques described in this section utilize simulations of incremental heat rate curves. Such use assumes that each heat rate curve can be represented by a succession of straight line segments and has no slope reversals. In practice, discontinuities and slope reversals are sometimes encountered. When that happens, approximations of the curves are used, so drawn that the criteria of continuity and no slope reversals are fulfilled. Errors resulting from such approximations are usually within the limits of the overall accuracy of the control method.

Incremental Efficiency. At any given output, *incremental efficiency* is proportional to the reciprocal of incremental heat rate.

Cost of Incremental Fuel. Where several fuels, or fuels of different costs, are being burned, the cost of the fuel that is then being adjusted in response to load demand is the fuel cost used in the economy dispatch computation for that source. This cost is defined as the *cost of incremental fuel.* As already noted in this section, varying costs other than fuel, such as labor, maintenance, and ash handling, are not treated separately in this discussion but are considered to have been appropriately included in the cost of incremental fuel.

Total Cost of Power Generated. The product of the total heat rate and the average cost of fuel is defined as the *total cost of power generated.*

Incremental Cost of Power Generated. The product of the incremental heat rate and the cost of incremental fuel is defined as the *incremental cost of power generated.* Synonymous terms are *incremental cost of power produced* and *incremental bus cost.* A curve of incremental cost of power generated versus output is shown in Fig. 39, where the following relation applies:

$$\frac{dF}{dP} = \frac{dH}{dP}(f), \tag{18}$$

where $\frac{dF}{dP}$ is the incremental cost of power generated,

$\frac{dH}{dP}$ is the incremental heat rate, and

f is the cost of incremental fuel, adjusted to include other varying station costs.

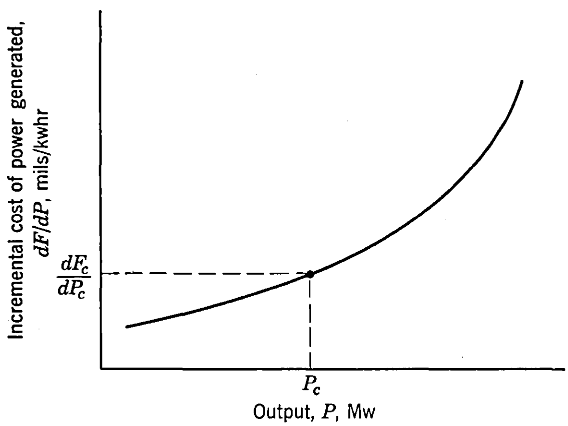


Fig. 39. Incremental cost of generation curve, based on Fig. 38 and Eq. 18.

Loading to Equal Incremental Costs within Station. In a station containing several units, minimum *total* cost of power generated is achieved for any given station output when participating units are loaded to equal *incremental* costs of power generated. This is demonstrated graphically for a two-unit station in Fig. 40.

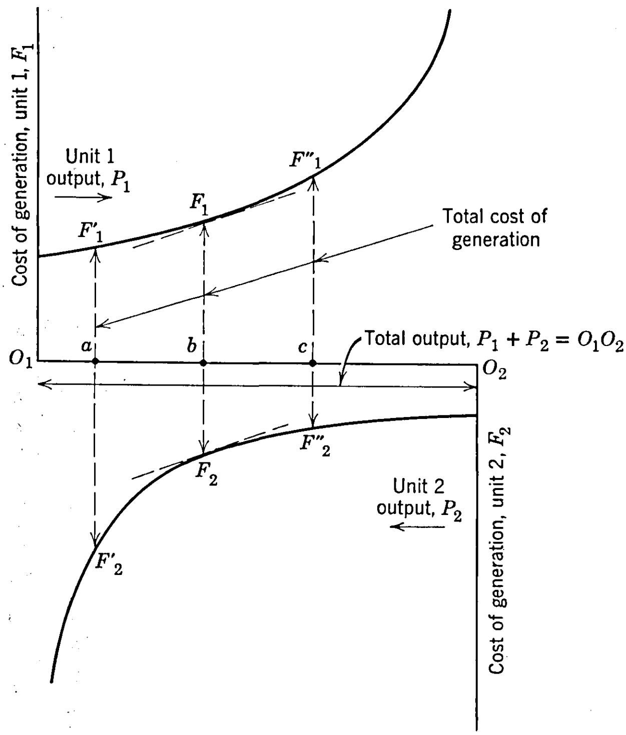


Fig. 40. Graphical demonstration of loading to equal incremental costs of power generated to yield minimum incremental total cost of power generated for a two-unit station.

In this figure a separate cost of generation versus output curve is drawn for each of the two units. Each curve would be similar to the curve of Fig. 36, except that the vertical ordinate is now total cost of power generated, F. The curve for unit 1 is drawn conventionally, with its zero coordinates, marked 0_1, at the left. Increasing cost for unit 1 is plotted upward, increasing output to the right. For unit 2 the curve is drawn "upside down." Its zero coordinates, marked 0_2, are on the right and spaced with relation to 0_1 so that the distance 0_10_2 represents the total output desired at that time from the station. Increasing cost for unit 2 is plotted downward, increasing output to the left.

The total desired output 0_10_2 may be divided in any way between the two units, as by the division points a, b, c. The figure permits a graphical determination of the load division that will result in minimum total cost of generating the desired output. In Table 6 the costs at unit 1 and unit 2 and the total are shown for load division points a, b, and c.

Note that minimum total cost of generation is represented by the shortest vertical line that can be drawn between the two curves. The shortest such line is the one that joins points of equal slope. In Fig. 40 this is the vertical line that joins the two points F_1 and F_2, which have the equally sloped tangents. This is the line through division point b. Since both of the cost-output curves turn away from the parallel slopes on both sides of the tangent points, any vertical line joining the two curves on either side of point b will necessarily be longer than the line through point b and hence will represent a larger total generating cost.

TABLE 6. COSTS FOR SEVERAL LOAD DIVISIONS BETWEEN TWO GENERATION UNITS

Division Point	Cost of Generation Unit 1	Cost of Generation Unit 2	Total Cost of Generation Units 1 and 2	Total Output Units 1 and 2
a	F'_1	F'_2	$F'_1 + F'_2$	O_1O_2
b	F_1	F_2	$F_1 + F_2$	O_1O_2
c	F''_1	F''_2	$F''_1 + F''_2$	O_1O_2

Since the slope of each cost-output curve represents incremental cost of power generated, loading the two units to *equal slopes* to achieve minimum total cost, as in Fig. 40, means that minimum total cost is achieved when load division is based on *equal incremental costs of power generated.*

The analysis can be extended to stations having more than two units. For a station containing n operating units, minimum total cost of generating any required output is achieved when the n units are loaded to equal incremental costs of power generated.

Units out of Range. In applying the postulate of loading the units of a station to equal incremental costs of generation, it is understood that some of the operating units in the station may be *out of range* of the incremental cost then being used for the loading allocations. Units with lower incremental costs over their entire operating range will have been fully loaded by earlier incremental cost allocations. Also, units having higher incremental costs over their entire operating range may be on the line for purposes of reserve and will be minimum loaded at higher incremental costs.

Transmission Losses

The potential savings to be achieved by attention to transmission losses in economy dispatch depend on the size and nature of the power system. A closely knit metropolitan network may find that the potential savings are only modest. On the other hand, a far-flung network covering a large geographical area may find potential savings from this source to be of considerable magnitude. One large power system which serves an area of about 100,000 square miles estimates that the fuel cost of power lost in transmission is approximately five million dollars per year (see Ref. 16). It has reported savings of approximately 4% of that figure, or about $200,000 per year, by the use of an economy dispatch system that gives appropriate consideration to transmission loss factors.

Total Transmission Losses. The total transmission losses of an area do not in themselves enter directly into the computation of area economy dispatch. Equations representing total losses are used, however, for the derivation and evaluation of incremental loss equations, the nature and significance of which will be discussed later in this section. A traditional total transmission loss equation, as developed and discussed in Refs. 11, 12, 13, and 14, will be reviewed and an augmentation thereof, as introduced in Ref. 17, will then be discussed. In this latter reference the authors identify the earlier equation as the simplified transmission loss equation and their augmented variation as the general transmission loss equation.

Traditional Loss Equation. Based on certain assumptions concerning area conditions, the *traditional* or *simplified total transmission loss equation* defines total transmission losses of an area, as follows:

$$P_L = \sum_m \sum_n P_m B_{mn} P_n, \tag{19}$$

where P_L is the total transmission loss,
P_m is the power output of source m,
P_n is the power output of source n,
B_{mn} are constants related to the nature and characteristics of the area, and
$B_{mn} = B_{nm}$.

Note that eq. (19) is a double summation which involves only the power outputs P_m and P_n of area sources and the corresponding constants B_{mn}. If an area, for example, has 15 sources, the m index in eq. (19) assumes all values of whole numbers from 1 to 15, inclusive, and the n index likewise assumes all values of whole numbers from 1 to 15, inclusive.

The assumptions on which eq. (19) are based are as follows:

1. The equivalent load current at any bus, defined as the sum of the line-charging, synchronous condenser, and load currents at that bus, remains a constant complex fraction of the total equivalent load current.
2. The generator bus voltage magnitudes remain constant.
3. The ratio of reactive power to real power of any source remains a fixed value.

4. The generator bus angles remain constant.

Where an area, for example, has three power sources, eq. (19) becomes

$$(20)\quad P_L = B_{11}P_1^2 + B_{22}P_2^2 + B_{33}P_3^2 + 2B_{12}P_1P_2 + 2B_{13}P_1P_3 + 2B_{23}P_2P_3.$$

For the general case of N power sources in the area, eq. (19) becomes

$$(21)\quad P_L = B_{11}P_1^2 + B_{22}P_2^2 + \cdots + B_{NN}P_N^2 + 2B_{12}P_1P_2 + 2B_{13}P_1P_3 + \cdots + 2B_{(N-1)(N)}P_{(N-1)}P_N.$$

Augmented Loss Equation. The traditional eq. (19) is said to be limited in its practical applications since the assumptions on which it is based are substantially invalid on typical power systems (Ref. 17). It has been pointed out that with changes in system load and reactive requirements, the equation becomes quite inaccurate. In some instances improved accuracy utilizing the traditional approach would require the impractical complication of separate sets of constants for various load levels and seasons of the year.

To achieve less dependence on the original assumptions, and introducing other assumptions which are more realistic, an *augmented* or *general total transmission loss equation* has been developed (Ref. 17) which is as follows:

$$(22)\qquad P_L = \sum_m \sum_n P_m B_{mn} P_n + \sum_n B_{n0} P_n + K_{L0},$$

where B_{n0} is a constant related to source n,

K_{L0} is a constant that may be regarded as representing total system losses under the imaginary condition of zero system power supply, and

B_{mn} is not necessarily equal to B_{nm}.

The conditions and assumptions on which eq. (22) are based are as follows.

1. That the variation in each substation load is a fixed percentage of the variation in system total load. This percentage is not related to the size of the load.

2. That the actual source bus voltages are established for both system peak load and minimum load conditions, and each bus voltage varies linearly with total system load from its value at peak load to its value at minimum load.

3. That the reactive power generation is that required to maintain the peak period and valley period bus voltage levels established in the cases studied on a network analyzer or digital computer to obtain the data used

in setting up the simultaneous equations for determining the loss equation constants.

4. That the power factor of each load varies from its value at system peak load to its value at minimum load linearly with total system load.

5. No assumptions are made with regard to source voltage phase angles.

Incremental Transmission Losses. It is the incremental transmission losses rather than the total transmission losses that are of significance in economy dispatch computations. *Incremental transmission loss* for a specific source is the change in transmission loss related to that source when the source output is changed by a small amount. It represents the fraction of the incremental power of that source which is lost in transmission. It is defined for each area source by the corresponding partial derivative of the total transmission loss equation.

The incremental transmission loss for a source may rise fairly rapidly with increased power output. Although the total transmission losses for a given level of power output may be only 5% or 6%, the incremental transmission loss for that given level may be many times these percentages.

Based on Traditional Loss Equation. The partial derivative of eq. (19) with respect to a given source represents the incremental transmission loss for that source, in accordance with the following equation:

$$\frac{\partial P_L}{\partial P_n} = \sum_m 2B_{nm}P_m, \tag{23}$$

where $\dfrac{\partial P_L}{\partial P_n}$ is the incremental transmission loss for source n.

For an area having three power sources, the incremental transmission loss for each of the sources is given by the following equations:

$$\frac{\partial P_L}{\partial P_1} = 2B_{11}P_1 + 2B_{12}P_2 + 2B_{13}P_3, \tag{24}$$

$$\frac{\partial P_L}{\partial P_2} = 2B_{21}P_1 + 2B_{22}P_2 + 2B_{23}P_3. \tag{25}$$

$$\frac{\partial P_L}{\partial P_3} = 2B_{31}P_1 + 2B_{32}P_2 + 2B_{33}P_3, \tag{26}$$

where $B_{mn} = B_{nm}$.

Based on Augmented Loss Equation. To achieve a higher degree of accuracy under a wide variety of area conditions, the use of the partial derivatives of the general loss equation, eq. (22), in preference to the partial derivatives of the simplified loss equation, eq. (19), for incremental transmission losses in economy dispatch computations has been proposed (see Ref. 17). The partial derivatives of eq. (22) are given by the following relation:

$$\frac{\partial P_L}{\partial P_n} = \sum_m 2B_{nm}P_m + B_{n0}. \tag{27}$$

For a three-source area, the incremental transmission loss equation for each area becomes

$$\frac{\partial P_L}{\partial P_1} = 2B_{11}P_1 + 2B_{12}P_2 + 2B_{13}P_3 + B_{10}, \tag{28}$$

$$\frac{\partial P_L}{\partial P_2} = 2B_{21}P_1 + 2B_{22}P_2 + 2B_{13}P_3 + \mathrm{B}_{20}, \tag{29}$$

$$\frac{\partial P_L}{\partial P_3} = 2B_{31}P_1 + 2B_{32}P_2 + 2B_{33}P_3 + B_{30}. \tag{30}$$

In eqs. (27), (28), (29), and (30), B_{mn} is not necessarily equal to B_{nm}.

The B Constants. The preferred equation for incremental transmission loss, eq. (27), includes three types of B constants, for source n, as follows:

B_{nm}, the *self constants*, which are always positive;
B_{mn}, the *mutual constants*, which may be positive or negative;
B_{n0}, the *added constants*, which may be positive or negative.

Methods of computing B constants suitable for a wide variety of system conditions are outlined in Ref. 17 and include use of the voltage phase angle concepts discussed in Ref. 18.

Typical values of B constants for a three-source study included in Ref. 17 are shown in Table 7.

TABLE 7. TYPICAL VALUES OF B CONSTANTS FOR EQUATION (27)

Source 1	Source 2	Source 3
$B_{11} = 0.016576$	$B_{21} = -0.010204$	$B_{31} = -0.013611$
$B_{12} = -0.010204$	$B_{22} = 0.021871$	$B_{32} = 0.002477$
$B_{13} = -0.013611$	$B_{23} = 0.002477$	$B_{33} = 0.016792$
$B_{10} = -0.026961$	$B_{20} = 0.027215$	$B_{30} = 0.025327$

Comparison of Results. The comparative effectiveness of eqs. (23) and (27) in determining incremental losses under a wide variety of area conditions is illustrated in Tables 8 and 9. Note that the checks between computed and actual incremental losses are not as good in Table 8, which is based on the traditional equation, eq. (23), as they are in

TABLE 8. COMPARISON OF ACTUAL AND CALCULATED VALUES OF INCREMENTAL AND TOTAL LOSS VALUES GIVEN BY SIMPLIFIED LOSS EQUATION

Case	Solution	Incremental Loss, percent Source 1	Source 2	Source 3	Total Loss, Mw
		Peak Period Loads			
A-1	actual	−25.76	37.41	16.36	157.93
	computed	−20.00	40.81	17.32	174.24
A-2	actual	2.29	1.37	0.72	11.33
	computed	1.43	2.57	2.63	12.27
A-3	actual	−25.76	21.74	23.71	110.73
	computed	−21.75	23.73	27.93	124.96
A-4	actual	5.69	9.08	−6.77	29.18
	computed	4.06	10.19	−4.91	28.24
A-5	actual	−15.57	24.48	10.49	79.01
	computed	−13.87	28.75	13.47	90.97
A-6	actual	−10.35	6.02	13.09	34.64
	computed	−10.20	8.74	17.33	43.33
		Median Loads			
B-1	actual	−13.86	17.55	10.82	48.44
	computed	−11.86	20.12	12.94	50.92
B-2[a]	actual	−1.56	4.66	2.74	8.83
	computed	−1.30	5.49	3.85	8.83
B-3	actual	−4.62	14.05	1.84	27.39
	computed	−4.28	16.13	3.32	28.57
		Valley Period Loads			
C-1	actual	7.33	3.83	−7.93	17.17
	computed	7.60	2.08	−9.39	17.93
C-2	actual	−20.66	14.25	19.32	69.51
	computed	−16.75	14.97	21.42	65.25
C-3	actual	−7.37	−0.26	10.89	21.03
	computed	−5.96	0.25	11.96	19.22
C-4	actual	−11.96	14.51	9.20	33.86
	computed	−9.53	15.80	9.93	30.55
C-5	actual	0.54	5.90	−1.09	7.68
	computed	1.42	5.46	−1.74	6.65
C-6	actual	−11.34	18.25	6.73	43.00
	computed	−9.02	20.04	7.17	40.70

$B_{11} = 0.013832 \qquad B_{12} = B_{21} = -0.009853$

$B_{22} = 0.025740 \qquad B_{13} = B_{31} = -0.013646$

$B_{33} = 0.020685 \qquad B_{23} = B_{32} = 0.004981$

[a] Base case used in the determination of B constants.

TABLE 9. COMPARISON OF ACTUAL AND CALCULATED VALUES OF INCREMENTAL AND TOTAL LOSS VALUES GIVEN BY GENERAL LOSS EQUATION

Power Supply, Mw					Incremental Loss, percent			
Source 1	Source 2	Source 3			Source 1	Source 2	Source 3	Total Loss, Mw
				Peak Period Loads				
200.00	800.00	357.93	A-1	actual [a]	−25.76	37.41	16.36	157.93
				computed	−22.14	35.41	13.07	157.75
600.00	200.00	411.33	A-2	actual [a]	2.29	1.37	0.72	11.33
				computed	1.92	1.26	1.00	11.05
200.00	400.00	710.73	A-3	actual [a]	−25.76	21.74	23.71	110.73
				computed	−23.58	19.66	22.94	111.14
629.18	400.00	200.00	A-4	actual	5.69	9.08	−6.77	29.18
				computed	4.55	8.37	−5.90	26.78
300.00	600.00	379.01	A-5	actual	−15.57	24.48	10.49	79.01
				computed	−15.31	24.72	10.07	81.33
400.00	200.00	634.64	A-6	actual	−10.35	6.02	13.09	34.64
				computed	−10.79	6.45	13.95	36.64
				Median Loads				
200.00	400.00	348.44	B-1	actual	−13.86	17.55	10.82	48.44
				computed	−13.71	17.86	10.77	50.07
400.00	200.00	308.83	B-2	actual	−1.56	4.66	2.74	8.83
				computed	−1.92	4.84	3.01	9.03
327.39	400.00	200.00	B-3	actual	−4.62	14.05	1.84	27.39
				computed	−5.45	14.53	2.32	28.16
				Valley Period Loads				
417.17	200.00	0.00	C-1	actual [a]	7.33	3.83	−7.93	17.17
				computed	7.05	2.96	−7.83	16.91
0.00	200.00	469.51	C-2	actual [a]	−20.66	14.25	19.32	69.51
				computed	−19.56	13.80	19.29	69.90
200.00	0.00	421.03	C-3	actual [a]	−7.37	−0.26	10.89	21.03
				computed	−7.53	0.73	11.23	20.89
100.00	300.00	233.86	C-4	actual	−11.96	14.51	9.20	33.86
				computed	−11.87	14.96	9.15	35.04
300.00	200.00	107.68	C-5	actual	0.54	5.90	−1.09	7.68
				computed	0.24	5.88	−1.03	7.87
100.00	400.00	143.00	C-6	actual	−11.34	18.25	6.73	43.00
				computed	−11.44	18.89	6.59	44.82

[a] Cases used for determination of loss equation constants.

Table 9 which is based on the augmented equation, eq. (27). These tables also show comparisons for total losses and again the augmented equation, eq. (22), yields better results than the traditional equation, eq. (19).

It will be apparent from these tables why eq. (27), derived from eq. (22), is the one that is preferred in the application of automatic incremental loss computations to economy dispatch systems. For a fuller discussion of the techniques used in preparing the data contained in these tables, see Ref. 17.

Practical Application. The prevailing incremental transmission loss for each source must be computed in automatic economy dispatch systems which are to make provision for the effect of prevailing transmission losses on allocation of area generation to available sources. By using eq. (27), such computations can be automatically made at a central dispatching location. Telemetered data on the prevailing output of each area source and preset B constants are fed to a suitable computing network, in the manner developed by Early (Ref. 19).

A block diagram illustrating this automatic computation for a three-source area, using eqs. (28), (29), and (30), is shown in Fig. 41. Appropriate combination and manipulation of the source outputs and the B constants yield the quantities $\dfrac{\partial P_L}{\partial P_1}$, $\dfrac{\partial P_L}{\partial P_2}$, and $\dfrac{\partial P_L}{\partial P_3}$, which are the prevailing incremental losses for sources 1, 2, and 3 respectively, for the prevailing magnitude and distribution of area generation.

Another factor which is computed for each source in Fig. 41 is $1 - \dfrac{\partial P_L}{dP}$, the significance of which in economy dispatch is discussed later in this section.

Coordination Equation

It is necessary to relate appropriately the incremental cost of power generated at each source with the incremental transmission loss for that source in order to determine the incremental cost of delivering power from that source to the load. A coordination equation and terms related to it are discussed in the paragraphs that follow (Ref. 20).

Incremental Fraction of Power Delivered. It has already been stated that the incremental transmission loss, $\dfrac{\partial P_L}{\partial P_n}$, represents the *incremental fraction of power lost.* The fraction not *lost* is *delivered.* Thus, the incremental fraction of power lost, subtracted from unity, yields the *incremental fraction of power delivered,* which is to say

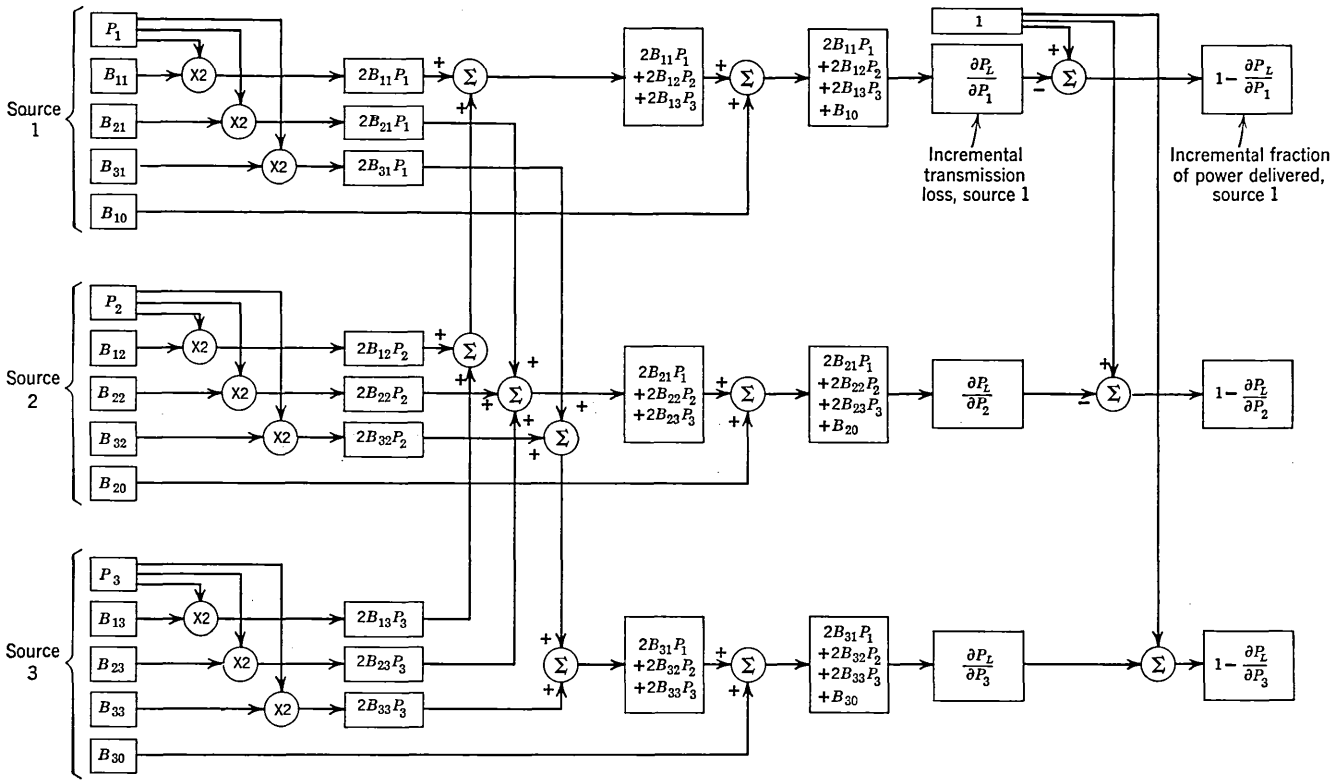


FIG. 41. Block diagram for automatic computation of incremental transmission losses for a three-source area, based on eqs. (28), (29), and (30). Computation of incremental fraction of power delivered per eq. (31) is also shown.

(31) $$\text{Incremental fraction of power delivered} = 1 - \frac{\partial P_L}{\partial P_n}.$$

It has already been noted that the automatic computation of $1 - \dfrac{\partial P_L}{\partial P_n}$ is included for sources 1, 2, and 3 in Fig. 41.

The *incremental percent of power delivered* is the incremental fraction of power delivered times 100, or $\left(1 - \dfrac{\partial P_L}{\partial P_n}\right) 100$.

Penalty Factor. The *penalty factor* is defined as follows:

(32) $$\text{Penalty factor} = \frac{1}{\text{Incremental fraction of power delivered}}$$

(33) $$= \frac{1}{1 - \dfrac{\partial P_L}{\partial P_n}}.$$

Incremental Cost of Power Delivered. The *incremental cost of power delivered* for a given source is defined as its incremental cost of power generated divided by its incremental fraction of power delivered, or as its incremental cost of power generated multiplied by its penalty factor. Thus

(34) $$\lambda_n = \frac{\dfrac{dF_n}{dP_n}}{1 - \dfrac{\partial P_L}{\partial P_n}}$$

and

(35) $$\lambda_n = \frac{\dfrac{dH_n}{dP_n} f_n}{1 - \dfrac{\partial P_L}{\partial P_n}},$$

where λ_n is the incremental cost of power delivered for source n,

$\dfrac{dF_n}{dP_n}$ is the incremental cost of power generated at source n,

$\dfrac{\partial P_L}{\partial P_n}$ is the incremental transmission loss for source n,

$\dfrac{dH_n}{dP_n}$ is the incremental heat rate for source n, and

f_n is the cost of incremental fuel for source n, adjusted to include other varying costs at source n.

Equation (35) provides a means of coordinating the incremental heat rate, the fuel cost, and the incremental transmission loss of each source as a step toward optimizing the performance of an area in the manner suggested by Fig. 35.

Other Types of Sources. The discussions in this section leading to eqs. (34) and (35) have all related to fuel-burning, steam-generating plants. Other types of sources which need to be considered on some installations are hydro plants, tie points to adjacent areas, and large nonconforming area loads.

Hydro Plants. Hydro plants may be handled with eq. (34) by assigning an equivalent incremental cost of generation to such sources (see Ref. 21).

Tie Points with Adjacent Areas. Tie line interchange points with adjacent areas may be considered as generating sources with the numerator of eq. (34) representing the incremental cost of purchased power at the tie point.

Large Nonconforming Loads. When determining incremental transmission losses with eq. (27), *a nonconforming load* is one that does not vary linearly with total area load. Special consideration of such a load is considered justified when its magnitude is large compared to the generation at or near the substation which supplies the load. Then the load is telemetered to the computing network and is treated as a negative power supply. This permits appropriate factors for such a load and its B constants to be introduced into the computations of incremental transmission loss for each of the actual sources (see Ref. 19).

Optimizing Area Performance

Loading to Equal Incremental Costs of Power Delivered. Comparable to the way in which a station's performance is optimized by loading its units to *equal incremental costs of power generated,* a complete area is optimized when its several operating sources are loaded to *equal incremental costs of power delivered.* For an area with n participating sources, optimum area performance from the point of view of economy dispatch is achieved when

$$\lambda_A = \lambda_1 = \lambda_2 = \cdots = \lambda_n, \tag{36}$$

where λ_A is the incremental cost of power delivered for the area as a whole, and

$\lambda_1, \lambda_2, \cdots, \lambda_n$ are the incremental costs of power delivered for sources $1, 2, \cdots, n$ respectively.

With all operating sources of the area loaded to equal λ, no change of level of generation at any of the sources (for the prevailing customer load and the prevailing combination of sources) will produce a lower total area cost of delivered power. One mathematical proof of eq. (36) is contained in Ref. 22.

The λ Versus Area Output Relationship. For a prevailing set of area load, generation, and power flow conditions, assuming that area sources are loaded to equal incremental costs of power delivered, in accordance with eq. (36), there is a specific value of λ_A for the then prevailing total area generation. An increase in area generation could be achieved with higher λ_A. A decrease in total area generation would result in a lower λ_A. For purposes of discussion, this general relationship between area incremental cost of power delivered, λ_A, and total area generation, P_A, may be shown as in Fig. 42.

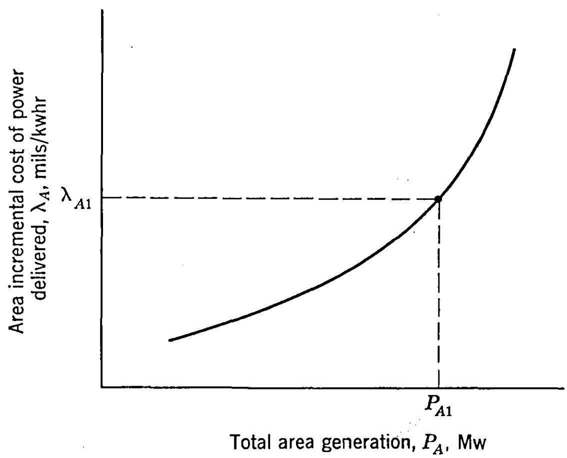


FIG. 42. General relationship between area optimum incremental cost of power delivered and total area generation.

The relationship between these two parameters is not a fixed one, and in actual practice the prevailing curve is not a smooth one, as drawn in Fig. 42. An actual curve is likely to have discontinuities and slope irregularities. Also, some sources may be *out of range*; some may have previously been fully loaded at lower λ values, and others may be minimum-loaded for reserve purposes at higher λ values. Note, however, that for a prevailing set of area conditions, the relationship between λ_A and total generation slopes upward as shown in the figure, and for a given set of prevailing conditions the total generation required of the area defines a unique λ value which will yield economy dispatch of the area.

Thus, in Fig. 42, λ_{A1} corresponds to a total area generation of P_{A1}. A typical range of λ values for a representative area is from 2 to 10 mils/kwhr.

The Area Daily Generation Curve. A typical daily generation curve for an operating area is shown in Fig. 43. It shows the customary morning pickup, the noonday drop, and the late afternoon peak.

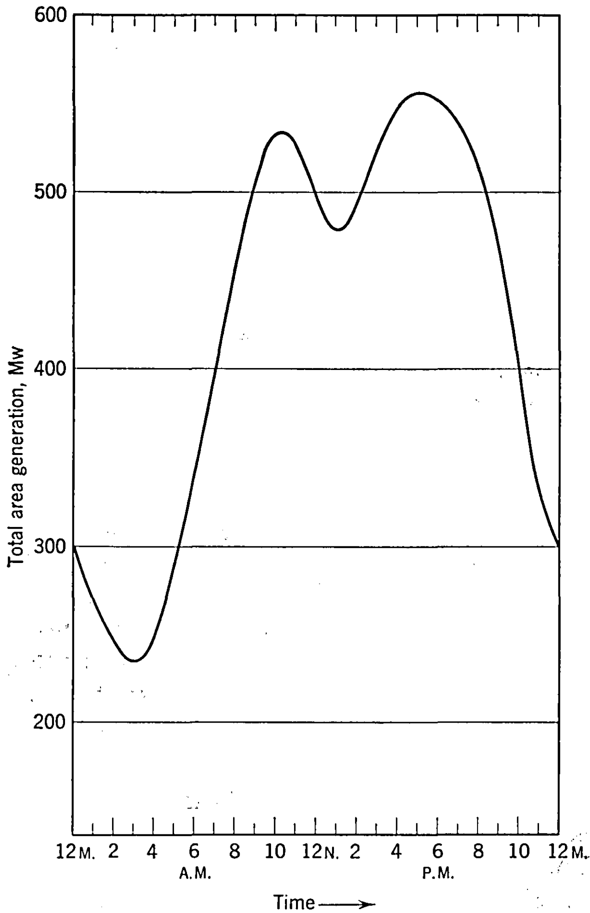


FIG. 43. A daily generation curve for the area.

The Problem Restated. The overall objective of economy dispatch is to adjust total area generation so that it matches the varying daily demand shown in Fig. 43 and at the same time loads the operating sources of the area to the λ_A value which, in Fig. 42, corresponds to the prevailing demand for total area generation.

Since, for each prevailing area condition, there is a unique value of λ_A that corresponds to the total required area generation, it can be postulated that to establish the appropriate λ_A there must be knowledge of the total generation then required of the area. This knowledge can be implied, as in some economy dispatch executions; or it can be direct, as in others. In Fig. 21, source outputs are shown as feedbacks to the economy dispatch block, corresponding to the use made of such feedback in executions based on direct rather than implied knowledge of required area generation.

In achieving economy dispatch in accordance with eqs. (35) and (36), consideration must be given to possible constraints or overriding factors which may mitigate against the achievement of theoretical economy dispatch. This is summarized in Fig. 7.

Desired Generation. The *desired generation* or *required generation* of a source is the loading level dictated for it by the prevailing allocation program.

Source Requirement. The *change* in generation required of a source to match its output to its programmed allocation is called the *source requirement.* It is the difference between the desired generation and the actual generation of the source.

Computation and Control Techniques

There are a number of differences in the arrangements that have been used to achieve automatic economy dispatch. Some of these differences reflect normal evolution and innovation in the state of the art; others reflect the competitive technologies of a competitive economy. Section 3 classifies automatic control systems on the basis of the portions of the integrated control problem which the system undertakes to solve, and on the nature of the programming technique utilized. Section 3 includes a discussion of Class I controls.

Classes II and III, and the three control types in each, refer specifically to control systems that see to achieve economy dispatch:

Class II. Economy dispatch, flexible programming
- Type 1. Single source output reference.
- Type 2. Total area output reference.
- Type 3. Total area required output reference.

Class III. Economy dispatch, fixed programming
- Type 1. Lambda reference adjustment through area control loop.
- Type 2. Lambda reference adjustment using area output feedback.
- Type 3. Lambda reference adjustment using area output feedback and area requirement feed-forward.

Class II Control, Economy Dispatch, Flexible Programming

A Class II control is an area control system which is applied to most of or all the generators of the area, and which, in achieving automatic economy dispatch, simultaneously achieves the objectives of area regulation. A common reference is used for source loading. The distinguishing characteristic of this class is that the loading programs are manually set and hence are flexible. Programs may be set to achieve true economic dispatch or to recognize the need for overriding economy dispatch with factors shown in Fig. 7.

In this class the program for a source with respect to the common reference depends upon the characteristics and conditions of other participating sources as well as on its own characteristics and conditions. This may be regarded as a limitation, for it generally requires that programs be reset for various combinations of participating sources.

Subdivisions in this class are identified by the nature of the common reference used for programming source loading.

Class II, Type 1, Single Source Output Reference. This designation applies to a control where the reference for source programming is the output of one of the sources which is designated as master. Manual settings for *base offset* and *ratio* are provided for each source, permitting preset programming in accordance with precalculated curves. Such programming tends to be complex, depending on which source is the master and which combination of sources is in operation.

In a Class II, Type 1 control, the equation for programming the output of a source n in terms of the output of the master source m is as follows:

$$D_n = \frac{b_m - b_n + r_m P_m}{r_n}, \tag{37}$$

where D_n is the desired generation of a followup source n,
P_m is the output of the master source m,
b_m and b_n are the base offset settings of sources m and n respectively, and
r_m and r_n are the ratio settings for sources m and n respectively.

For such a control the requirement of a source n is given by the following relation:

$$S_n = D_n - P_n = \frac{b_m - b_n + r_m P_m}{r_n} - P_n, \tag{38}$$

where S_n is the source requirement for source n, and
P_n is the actual output of source n.

A simplified block diagram of such a control, as applied to area A of Figs. 5 and 21, is shown in Fig. 44. One of the country's pioneer area-wide installations, still in successful operation, was of this type (see Ref. 23).

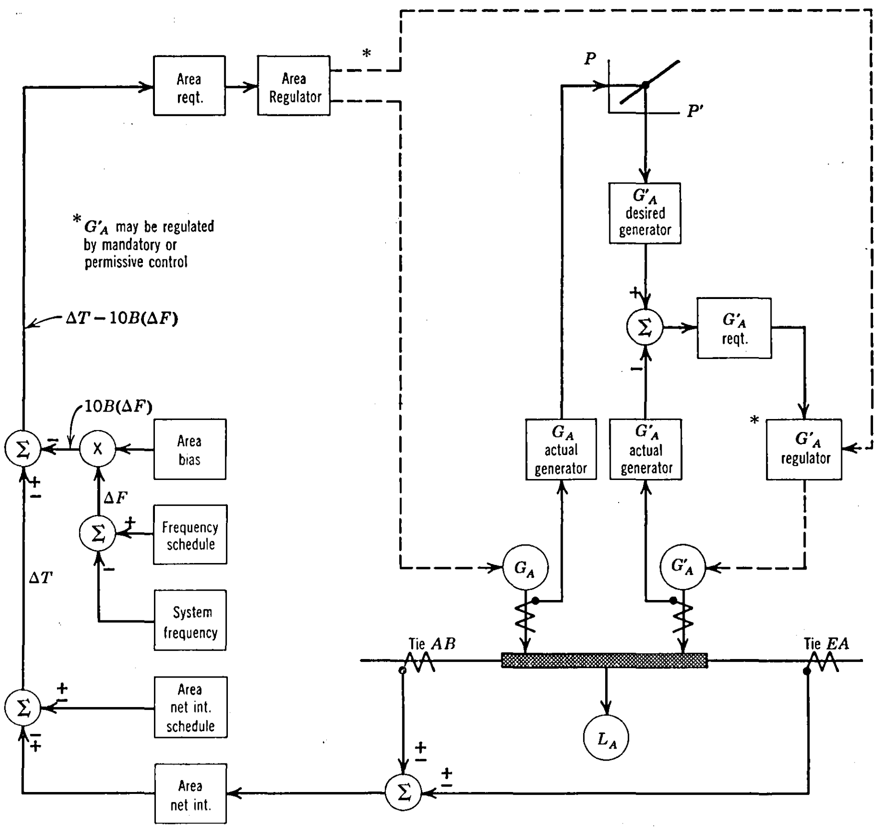


FIG. 44. Class II, Type 1 flexible program control. The common reference is the output of one of the sources. This figure also illustrates Class I, Type 2 control.

Class II, Type 2, Total Area Output Reference. This designation applies where the reference for source programming is the total prevailing output of participating sources. Data for the reference would be obtained from an intermediate feedback related to prevailing output of participating sources.

A limitation to this approach, a block diagram of which is shown in Fig. 45, is that computed generation desired from each source is based on the total generation *already existent* in the area, and not on the total

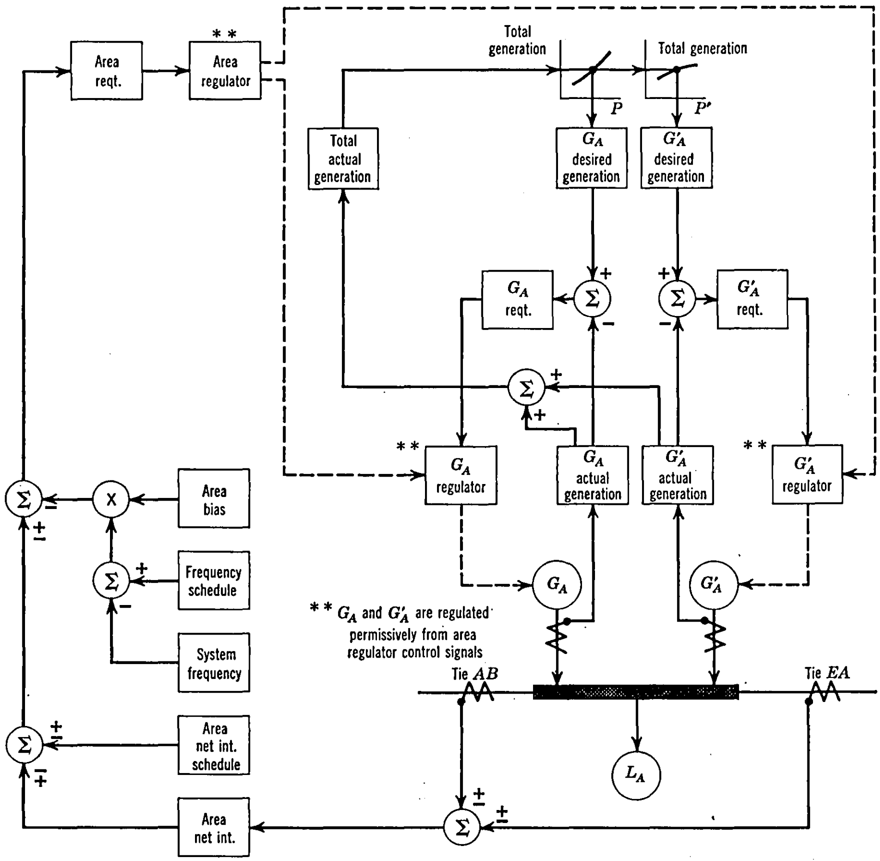


FIG. 45. Class II, Type 2 flexible program control. The common reference is a feedback from source outputs.

generation then *required* of the area. This type is listed as an illustrative step in the evolution to Class II, Type 3 control.

Class II, Type 3, Total Area Required Output Reference. In this type of control the reference for source programming is the total generation required of the participating sources to satisfy area load and reduce any prevailing area requirement to zero. The common reference is derived by using an intermediate feedback related to prevailing generation of the participating sources, as in Class II, Type 2, but combining with it a feed-forward related to prevailing area requirement. Thus, desired generation computed for each source represents the source output which is to prevail *after* area requirement is reduced to zero.

Block Diagram. The application of a Class II, Type 3 control to a two-area system is illustrated in Fig. 46.

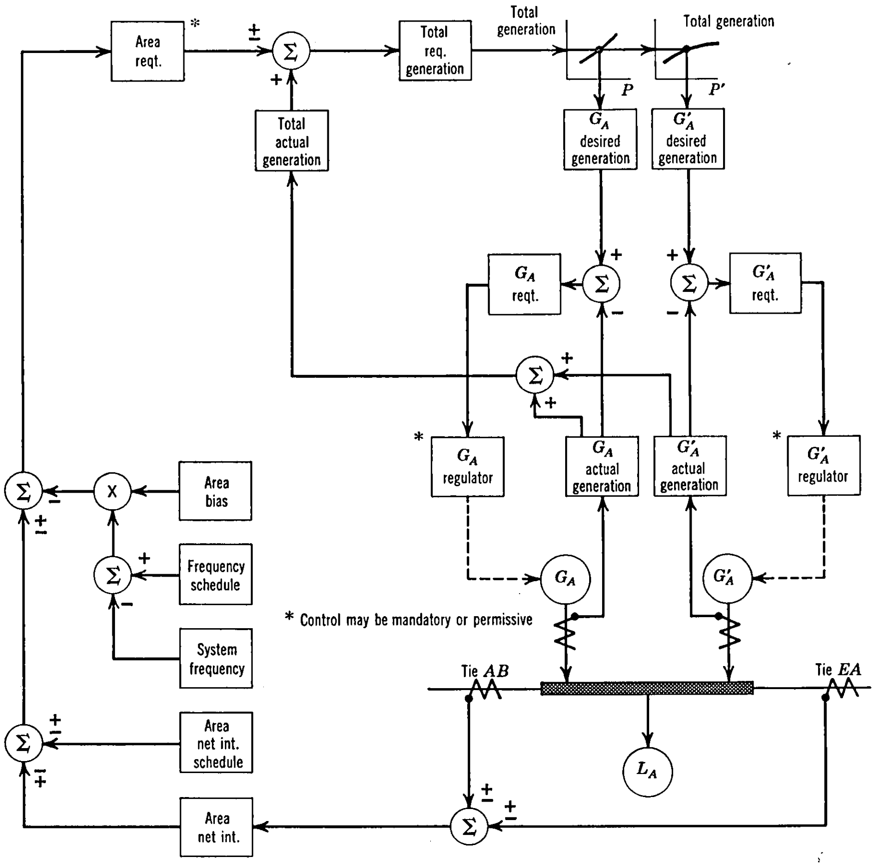


FIG. 46. Class II, Type 3 flexible program control. The common reference is a feedback from source outputs combined with a feed-forward from area requirement.

Loading Schedules. The output desired for each source is programmed by means of adjustable *base point* and *participation* schedule setters, which relate the output assignment for that source to total area generation. Schedules thereby take the form of a series of segmented straight lines, the base point settings defining successive points of inflection and participation settings defining the slope of each segment. Incorporated into the schedules are appropriate data previously derived from the coordination equation, eq. (35), and any other factors that need then be considered in loading area sources.

Illustrative loading schedules for a four-station area are shown in Fig. 47. A summary of base point and participation settings for the schedules of Fig. 47, taking two segments at a time, is shown in Table 10.

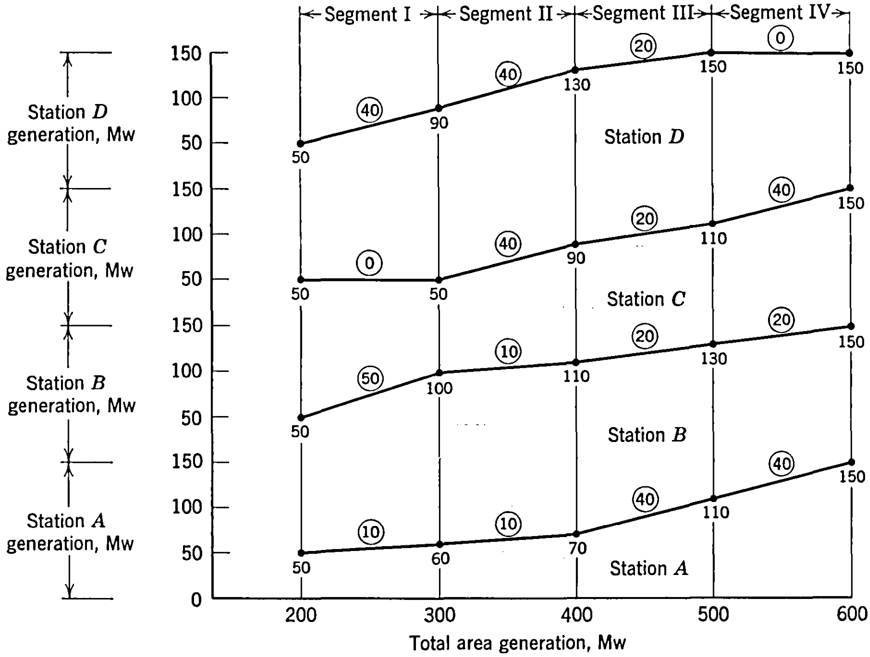
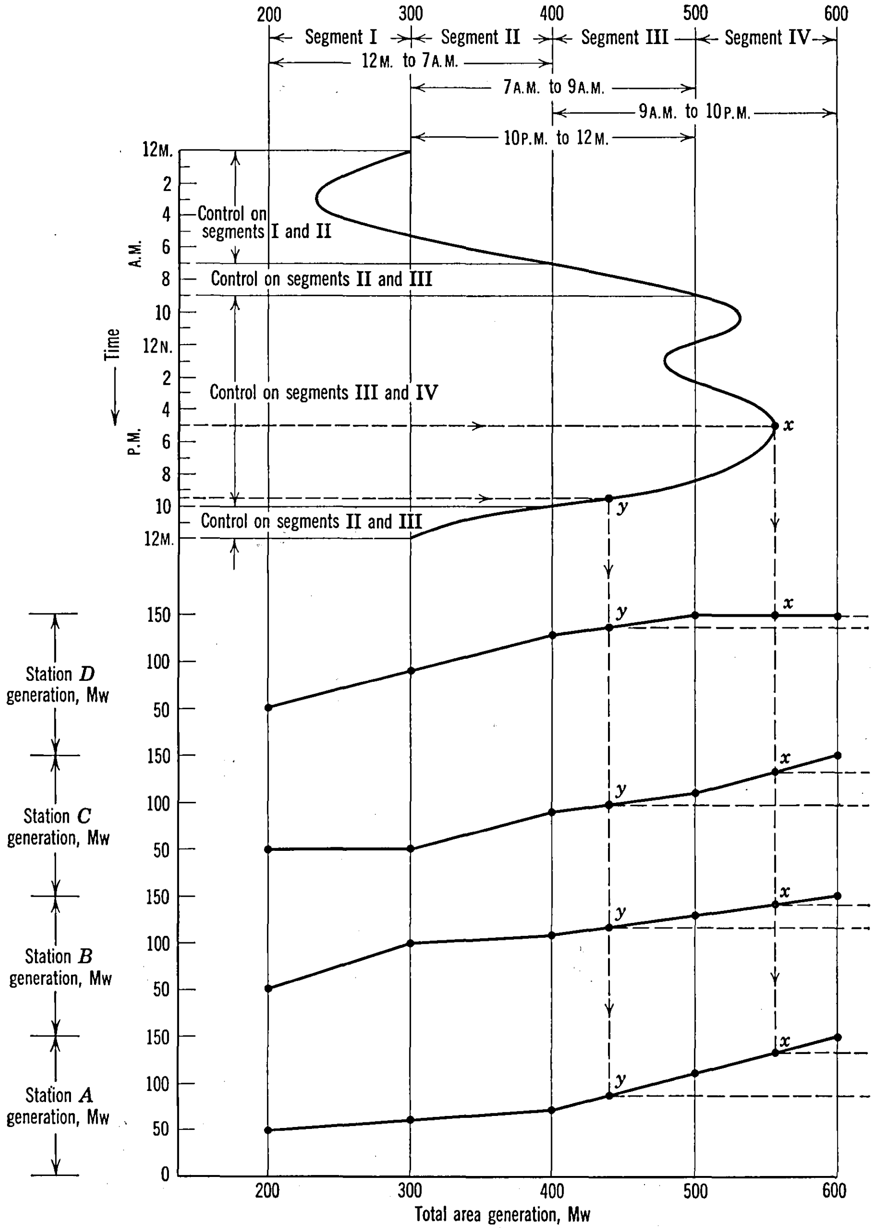


FIG. 47. Typical loading schedules for a four-station area for use with a Class II, Type 3 control.

TABLE 10. STATION BASE AND PARTICIPATION SETTINGS, AND REGULATING RANGES FOR TWO-SEGMENT CONTROL BASED ON STATION LOADING SCHEDULES OF FIG. 47

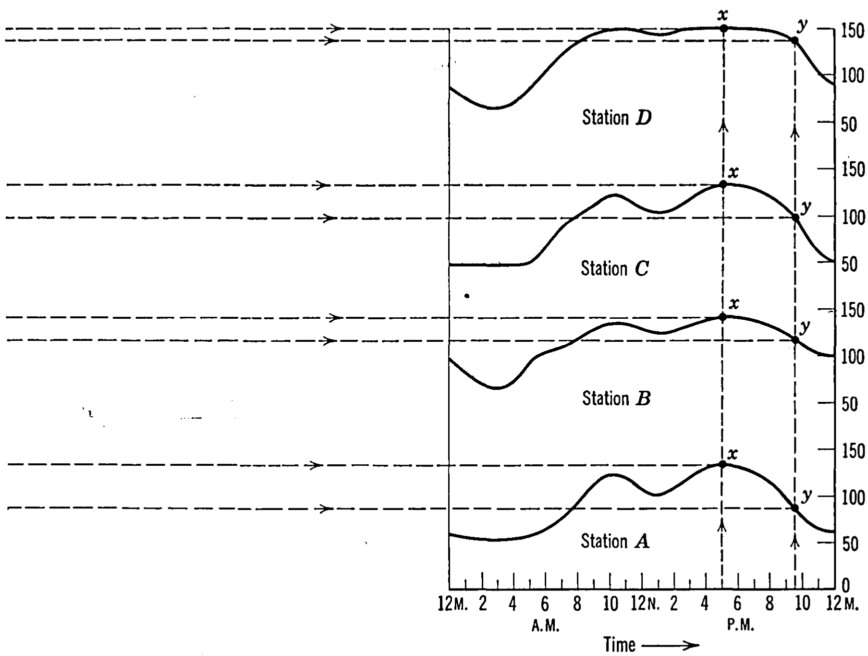
	Segments I and II Regulating Range, 200 to 400 Mw			Segments II and III Regulating Range, 300 to 500 Mw			Segments III and IV Regulating Range, 400 to 600 Mw		
	Participation below Base, Mw/100 Mw	Base Setting, Mw	Participation above Base, Mw/100 Mw	Participation below Base, Mw/100 Mw	Base Setting, Mw	Participation above Base, Mw/100 Mw	Participation below Base, Mw/100 Mw	Base Setting, Mw	Participation above Base, Mw/100 Mw
Station *A*	10	60	10	10	70	40	40	110	40
Station *B*	50	100	10	10	110	20	20	130	20
Station *C*	0	50	40	40	90	20	20	110	40
Station *D*	40	90	40	40	130	20	20	150	0
Total	100	300	100	100	400	100	100	500	100

Allocation of Total Area Generation. The net effect of the control scheme shown in Fig. 46 is to project the daily generation requirement curve for the area, Fig. 43, against the source loading schedules, Fig. 47, and produce corresponding individual source loadings in the manner illustrated graphically in Fig. 48. A significant point to note is that the daily generation curve from which the projection is made in Fig. 48 is assumed to include, and in actual practice does include, not only the generation already prevailing in the area but any prevailing area require-



Segment I
Segment II
Segment III
Segment IV
12 M. to 7 A.M.
7 A.M. to 9 A.M.
9 A.M. to 10 P.M.
10 P.M. to 12 M.
Control on segments I and II
Control on segments II and III
Control on segments III and IV
Control on segments II and III
Time
A.M.
P.M.
12 M.
12 N.
Station D generation, Mw
Station C generation, Mw
Station B generation, Mw
Station A generation, Mw
Total area generation, Mw

Fig. 48. Projection of the area desired generation curve of Fig. 43 on the loading schedules of Fig. 47, and the resulting desired generation curve for each of the participating stations. This projection is automatically performed in the Class II, Type 3 control of Fig. 46, the "total area generation" also including any prevailing area requirement.



ment as well. Thus, the assignment to each source represents the assignment it should carry when area requirement is reduced to zero.

Equations. The equation for establishing the loading of each source in terms of the common reference is given by the following relation:

$$S_n = t_n(R + E) - (P_n - b_n), \tag{39}$$

where S_n is the requirement for source n;
- t_n is the prevailing participation setting for source n;
- R is the *area regulation*, a parameter that is related to total area generation and is specifically the departure of total area generation above or below the sum of the prevailing station base point settings; it is defined by the expression $\Sigma P_n - \Sigma b_n$;
- E is the area requirement;
- P_n is the prevailing output of source n; and
- b_n is the base point setting for source n.

A summation of eq. (39) for all participating sources of the area yields the following relation:

$$E = \Sigma S_n. \tag{40}$$

Independence of Source Response Rates. The real significance of eq. (40) is that the generation change allocated to each source is not influenced by the rate at which other sources respond to their allocation changes. Control, in other words, is noninteracting. This is particularly significant when a large number of sources are responding to the automatic control. Each source then has a specific destination and approaches it independently of the rate at which other sources are approaching their respective destinations. This independence of response rates

TABLE 11. MAGNITUDE OF SIGNIFICANT PARAMETERS DURING THE REGULATING PROCESS AFTER A 5-MEGAWATT LOAD INCREASE IN AREA A [a]

	Net Interchange	Area Requirement	Station Requirement				Change in Station Generation				Area Regulation
			A	B	C	D	A	B	C	D	
Conditions following governing action of the interconnection.	4.80	5	2	1	1	1	0.080	0.040	0.040	0.040	0.20
Station A has regulated to reduce its station requirement to zero.	2.88	3	0	1	1	1	2.048	0.024	0.024	0.024	2.12
Station B has regulated to reduce its station requirement to zero.	1.92	2	0	0	1	1	2.032	1.016	0.016	0.016	3.08
Station C has regulated to reduce its station requirement to zero.	0.96	1	0	0	0	1	2.016	1.008	1.008	0.008	4.04
Station D has regulated to reduce its station requirement to zero.	0	0	0	0	0	0	2	1	1	1	5.00

[a] Area A is assumed to have four stations participating in the regulations, with applicable loading schedules per segment III, Fig. 47. See Ref. 24 for a detailed discussion of this table.

derives from the use of the intermediate feedback combined with feedforward to achieve the common reference, as shown in Fig. 46.

Table 11 illustrates how for a given load change the requirement computed for each source is independent of the response rates of other sources.

Approximately fifty Class II, Type 3 installations were in operation in the United States in 1960 (see Refs. 24–29).

Class III Control, Economy Dispatch, Fixed Programming

A Class III control system, like those of Class II, applies automatic control to most of or all the generators of the area. Like Class II, it fulfills area regulation objectives while achieving automatic economy dispatch and uses a common reference for source loading. The distinguishing characteristic of this class is that the common reference is incremental cost.

Control execution for economy dispatch is based on loading participating sources to equal incremental cost. Programming of a source is based on a preset relation between incremental cost and output for that source. In that sense the economy dispatch program is fixed and has the advantage that it need not be changed for various combinations of participating sources. Also, this approach coordinates well with automatic computation and application of transmission loss factors.

This approach has been used increasingly in recent years. The common reference may be incremental cost of power delivered, or (in instances where transmission losses are not considered to be significant) the common reference may be incremental cost of power generated.

Grouped in this class, but subdivided as different types, are control systems where the common reference either continuously represents incremental cost or represents incremental cost only during certain conditions of balance.

Class III, Type 1, λ Reference Adjustment through Area Control Loop. This designation applies to a control where the common reference is not independently computed as incremental cost but is established by a floating search initiated from prevailing area requirement. In this method of λ_A determination there is not a direct use of the total generation required of the area; λ is adjusted until area requirement is reduced to zero. By that inferential means, required λ_A is related to required area generation. Control is a series cascade, with feedback through the power network and the boundary tie lines determining when the reference λ has been properly established.

Block Diagram, without Transmission Loss or Fuel Factors. A simplified block diagram showing the application of this type control to a two-source area is shown in Fig. 49. For simplicity, no transmission loss

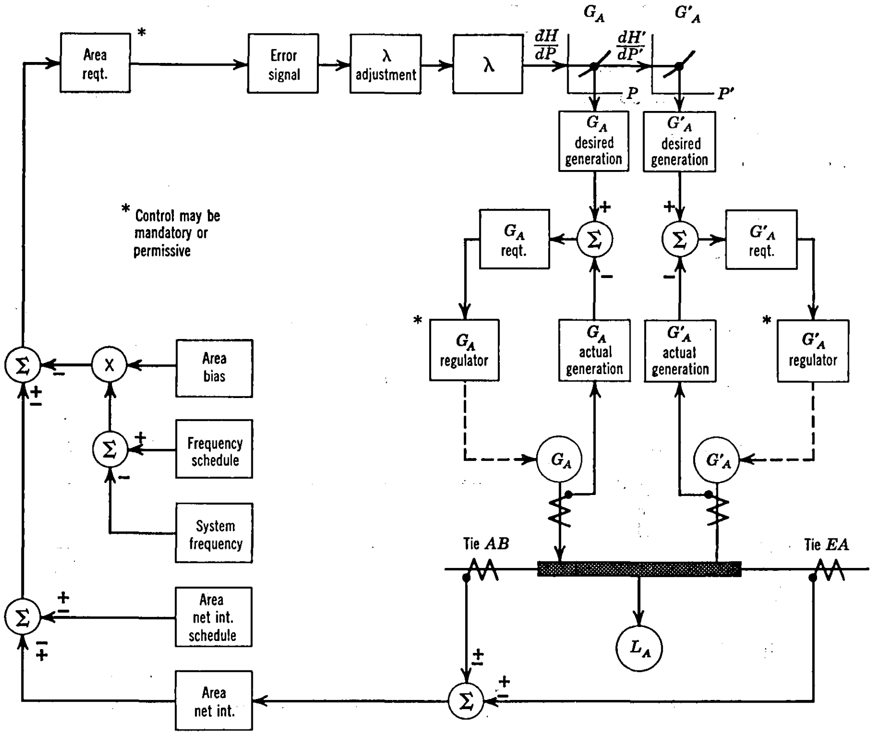


FIG. 49. Class III, Type 1 economy dispatch. The common λ reference adjustment is achieved by feedback through the area control loop.

or fuel factors are shown or considered in this schematic, and it is assumed that the λ_A reference may be regarded as a common incremental heat rate reference for the sources. The incremental transmission loss penalty factors, and fuel cost settings, could be injected, manually or automatically, between the common λ reference and the source incremental heat rate curves.

Block Diagram, with Transmission Loss and Fuel Factors. An expanded block diagram for a Class III, Type 1 control in which the incremental transmission loss for each source is automatically computed, using the technique illustrated in Fig. 41, is shown in Fig. 50. Fuel cost settings are also included in this figure, which utilizes coordination eq. (35) to make the economy dispatch allocations.

Equations for λ_A Balance. Note from Figs. 49 and 50 that the λ adjustment continues to shift the λ_A reference until area requirement is reduced to zero. The common reference therefore accurately represents the λ_A

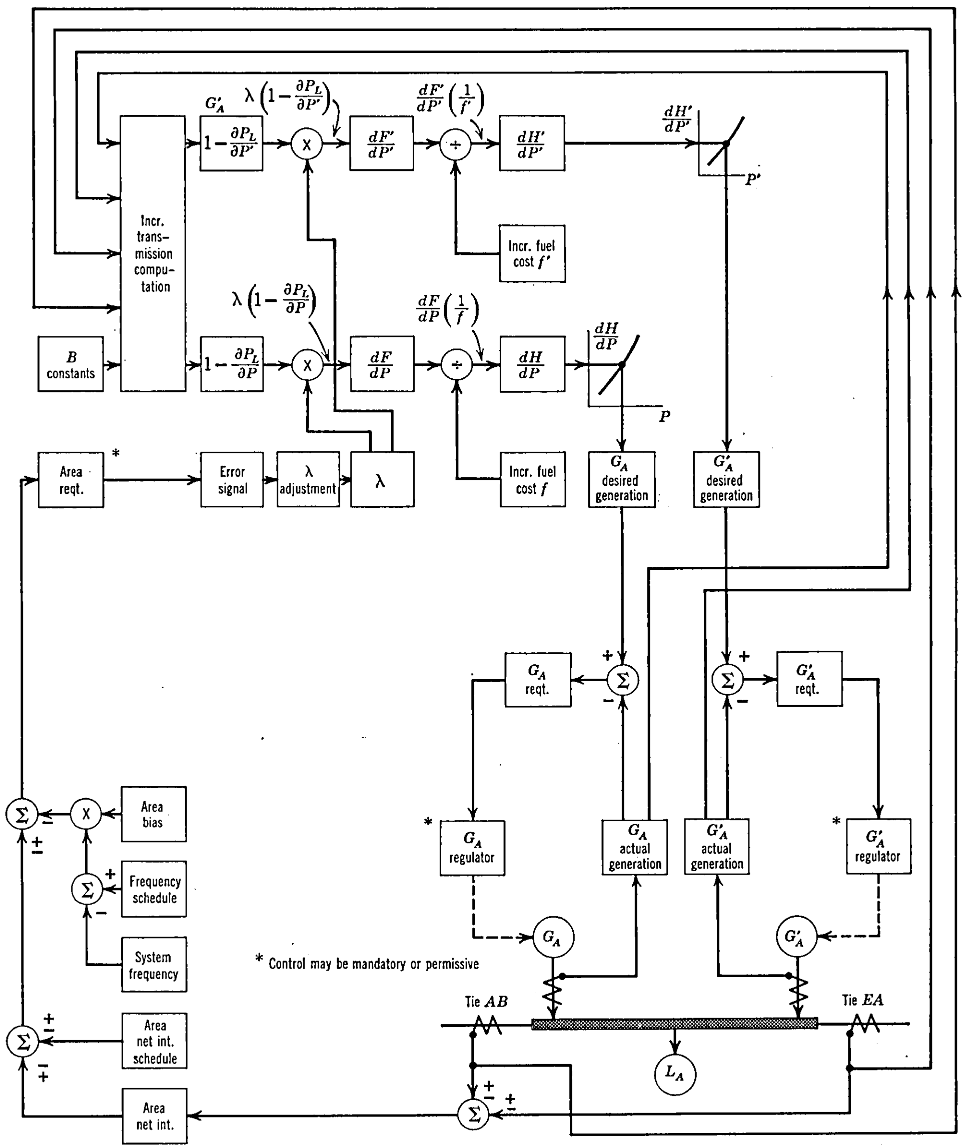


FIG. 50. Class III, Type 1 economy dispatch of Fig. 49 expanded to include incremental transmission loss computation and fuel cost adjustments.

required for prevailing conditions only when the two following conditions are fulfilled:

$$E = 0 \tag{41}$$

$$S_n = 0, \tag{42}$$

where E is area requirement, and
S_n is the requirement of each participating source n.

For a detailed description of installations utilizing this type of automatic economy dispatch, see Refs. 30 and 31.

Class III, Type 2, λ Reference Adjustment Using Area Output Feedback. In this type of control the common λ reference is established independently of the area control loop; λ adjustment is based on a comparison between summated desired generation of the participating sources and total prevailing area generation.

Block Diagram, without Transmission Loss or Fuel Factors. A basic block diagram for this type of control, which does not include incre-

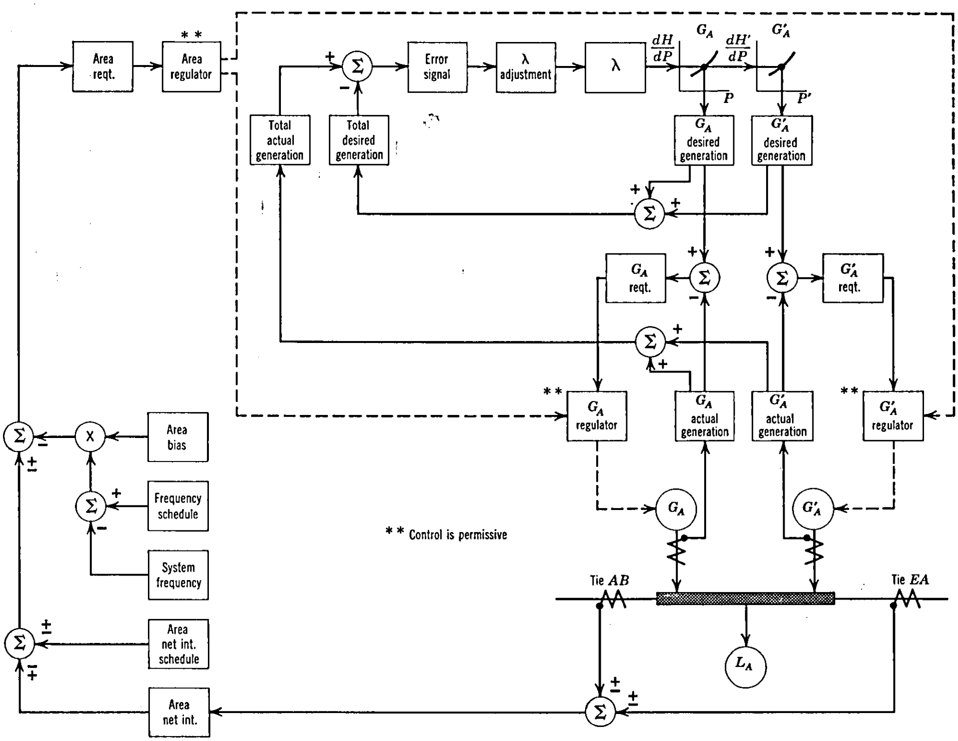


FIG. 51. Class III, Type 2 economy dispatch. Lambda adjustment is achieved by intermediate feedback from source outputs.

mental transmission loss or fuel factors and assumes that λ_A is the equivalent of incremental heat rate, is shown in Fig. 51. This method of λ_A determination uses an intermediate feedback from the output of participating sources, in the manner illustrated in Fig. 21. However, the λ_A reference thus determined corresponds only to the total generation already carried in the area and makes no provision for further area generation changes needed to reduce any prevailing area requirement to zero.

Block Diagram, with Transmission Loss and Fuel Factors. A block diagram for this type of control, expanded to include computation of incremental transmission loss and to provide for fuel cost adjustments, is shown in Fig. 52. The incremental transmission loss for each source is computed after the manner shown in Fig. 41.

As drawn, Fig. 52 shows the introduction of actual source outputs into the incremental transmission loss computation block. The desired source generations derived from the heat rate function generators could be used for this service, thereby providing a useful anticipating effect.

Equation for λ_A Balance. The equation for the λ_A reference adjustment in Figs. 51 and 52 is as follows:

$$\Sigma P_n = \Sigma D_n, \tag{43}$$

where P_n is the actual generation of each participating source, and
D_n is the computed desired generation for each participating source.

For a description of an installation of this type of control, see Ref. 32.

Class III, Type 3, λ Reference Adjustment Using Area Output Feedback and Area Requirement Feed-Forward. This is one of the newest of the economy dispatch techniques. Here the computation for λ adjustment is carried a step farther than in Class III, Type 2. In addition to the intermediate feedback from participating sources, the λ adjustment computation includes a feed-forward from prevailing area requirement.

Simplified Schematic. A simplified schematic diagram showing the general nature of a Class III, Type 3 economy dispatch system is shown in Fig. 53. The economy dispatch allocation is made on the basis of the total generation already existent in the area plus any prevailing area requirement.

Block Diagram, without Transmission Loss or Fuel Factors. A basic block diagram of a Class III, Type 3 control which has no transmission loss or fuel factors, assuming that λ_A is the equivalent of incremental heat rate, is shown in Fig. 54. The λ_A adjustment is based on an error signal derived by comparing the computed desired generation for all the participating sources with the total generation required of the area to carry its own load and reduce any prevailing area requirement to zero. The

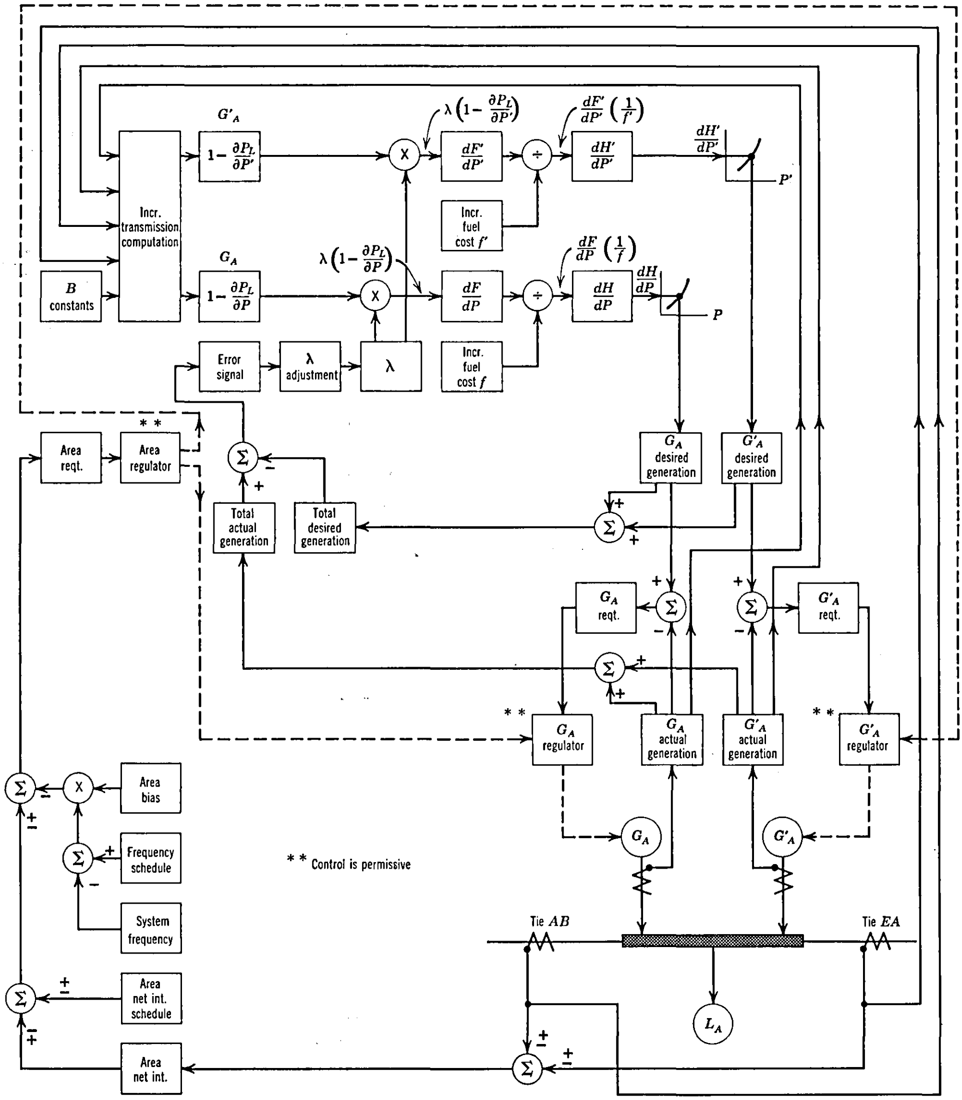


FIG. 52. Class III, Type 2 economy dispatch of Fig. 51 expanded to include incremental transmission loss computation and fuel cost adjustments.

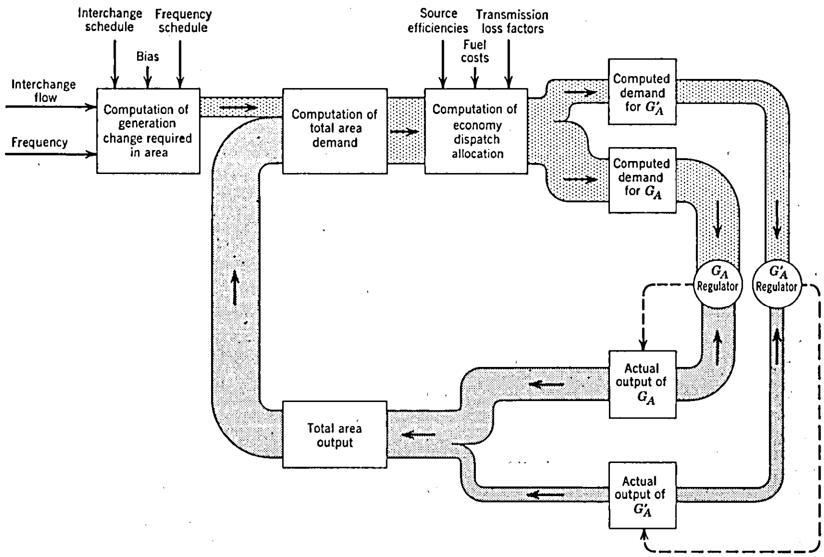


FIG. 53. Simplified diagram of economy dispatch computer control which matches total area supply to area demand.

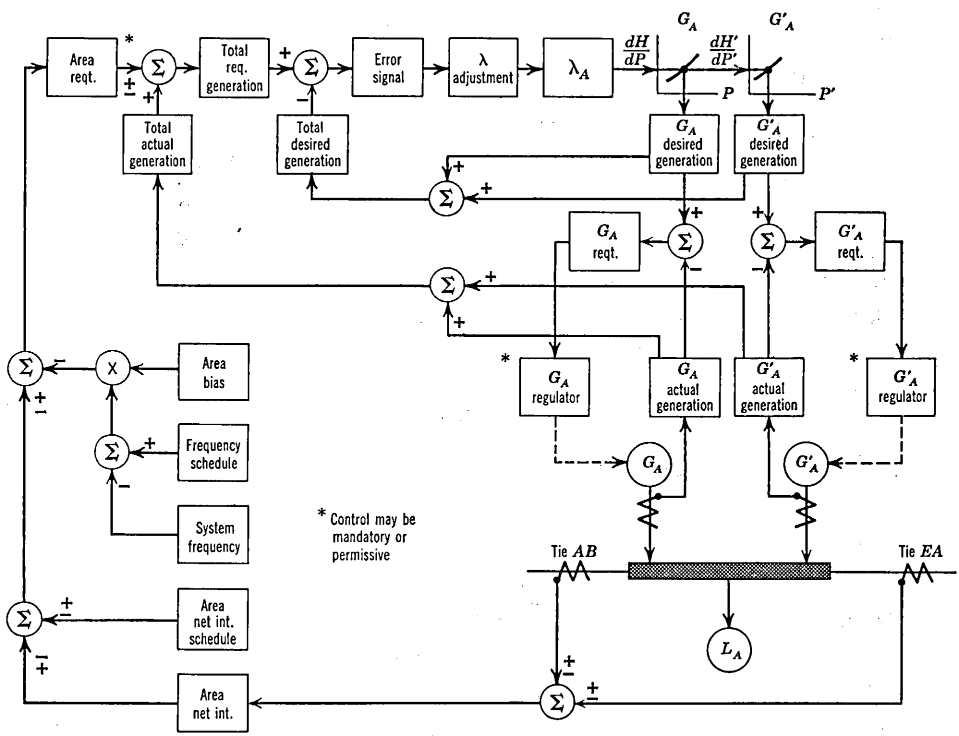


FIG. 54. Class III, Type 3 economy dispatch, based on eq. (45). Lambda adjustment is achieved by intermediate feedback from source outputs combined with feed-forward from area requirement and is independent of area control loop.

optimizing computation is independent of the control execution. The λ_A thus determined corresponds to the total area generation that will prevail after area requirement has been reduced to zero. Similarly, the desired generation computed for each of the participating sources represents the generation level that is to prevail for that source after area requirement has been reduced to zero.

A unique feature of this method of λ_A adjustment is that the λ_A thus established continuously represents the incremental cost that will prevail for the total generation then required of the area. It is based on direct rather than inferred knowledge of the total generation need of the area. Control execution is still a cascade, but the common λ reference is uniquely established independently of the action or response of individual source regulators or the controlled generators, and without interaction between the regulators.

Equations for λ_A Balance. The equations for λ_A adjustment in Fig. 54 is as follows:

$$E + \Sigma P_n = \Sigma D_n \tag{44}$$

or

$$E = \Sigma D_n - \Sigma P_n = \Sigma(D_n - P_n) \tag{45}$$

from which

$$E = \Sigma S_n. \tag{46}$$

Alternative Block Diagram, without Transmission Loss or Fuel Factors. The block diagram of Fig. 54 illustrates λ_A adjustment based on eq. (44). An alternative diagram showing λ_A adjustment by means of eq. (46) rather than eq. (44) is shown in Fig. 55.

Note that eq. (46) is identical to eq. (40) which represents the allocation relationship of the Class II, Type 3 control. This equivalence will be readily understood when it is realized that both control types use the same ultimate reference of feedback from prevailing outputs of participating sources combined with feed-forward from area requirement. For Class II, Type 3 control, the combination of these parameters is used as a direct reference for source allocation and yields source requirements whose algebraic sum is equal to area requirement. For Class III, Type 3 control, this same combination of parameters is used as a primary reference which results in the establishment of a secondary common λ_A reference at a value fulfilling the relation that the algebraic sum of source requirements equals prevailing area requirement.

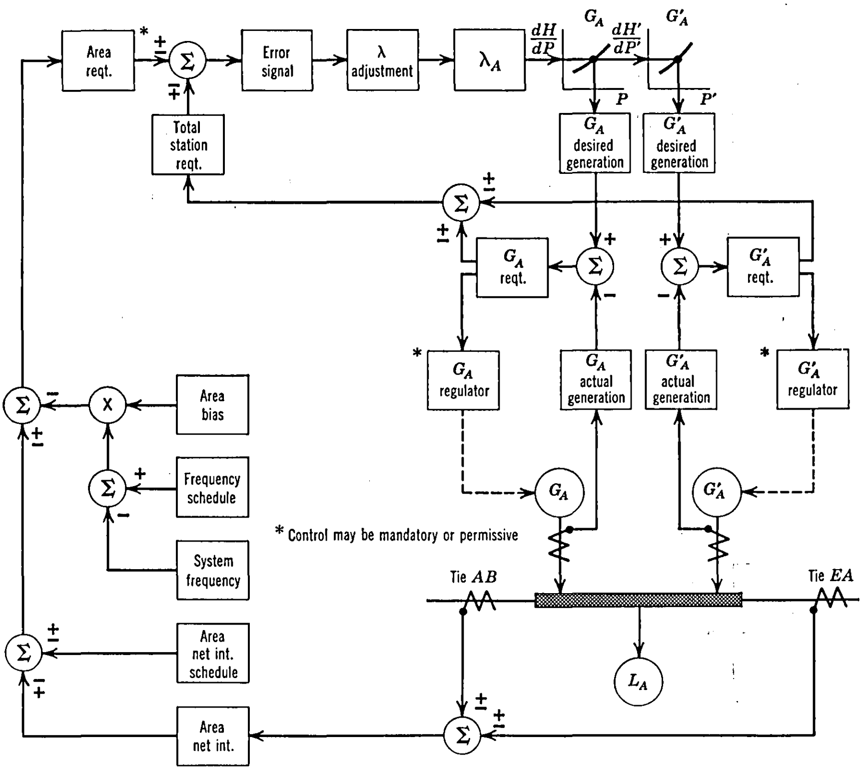


FIG. 55. Class III, Type 3 economy dispatch, redrawn to match eq. (46).

Block Diagram, with Transmission Loss and Fuel Factors. A block diagram of a Class III, Type 3 control, representing a full embodiment of eqs. (35) and (36), including incremental transmission loss and fuel cost factors, is shown in Fig. 56. Incremental fraction of power delivered for each source is automatically computed in the manner illustrated in Fig. 41, although computed desired generation for each source is used in place of actual source outputs. This provides a useful anticipating effect. Appropriate further computation for each source as shown in Fig. 56 then translates the common λ_A reference into incremental heat rate for that source. Desired source generation for each source is derived from its incremental heat rate versus source output function generator.

Independence of Response Rates. Like Class II, Type 3, and unlike Class III, Types 1 and 2, the desired generation computed for each participating source with a Class III, Type 3 system is unaffected by the rate at which other participating sources respond to their respective regulators.

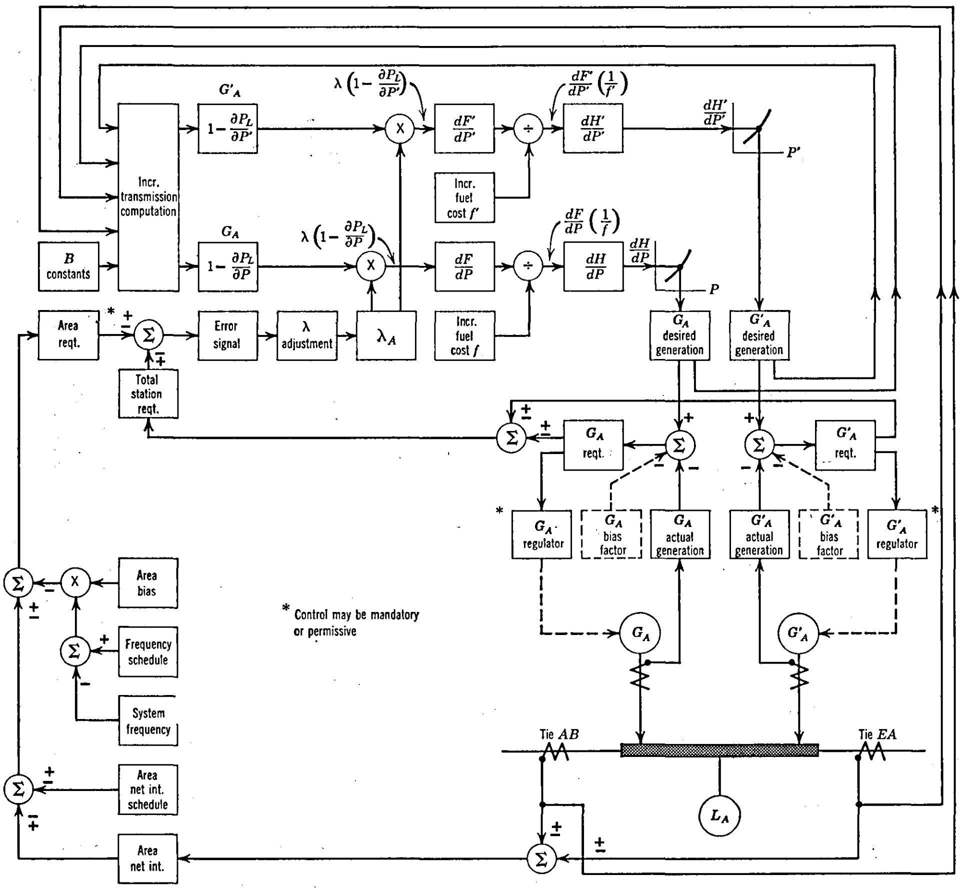


FIG. 56. Class III, Type 3 economy dispatch, based on eq. (46) and expanded to include incremental transmission loss computation and fuel cost adjustments.

For descriptions of typical installations of Class III, Type 3 controls made during the past two years, see Refs. 21, 33, and 34.

Independent Use of Computer. Since a Class III, Type 3 control system computes λ_A and desired generation for participating sources independently of actual control action, the computation portion of such a system can be isolated from actual control operation and used to derive data on area conditions that will prevail for various postulated operating situations. Typical uses for such computations are:

1. Determining the economics of buying power from or selling power to neighboring companies under various future system loading conditions.

2. Determining load allocation patterns, area efficiency, and fuel costs for new units before they are added to the system.

3. Determining load duration curves and fuel budgets for various units for future operating periods.

4. Exploring the economics of planned maintenance and outage schedules.

5. Determining whether extra expenditures might be justified for speedy completion of a new unit, on the basis of the fuel savings that may result from less use of older units.

9. CONTROL EXECUTIONS

General

This section considers facets of automatic control execution when applying the concepts of Sect. 8 to automatic economy dispatch and to area regulation. Many complexities beyond the simplified block diagrams of Sect. 8 are introduced when planning a control system to fulfill the operating requirements of an actual installation. The basic concepts, however, remain those shown in the diagrams.

Location of Equipment

In the practical application of automatic control systems there is need to consider whether sources are to be treated on a station or on a unit basis. The customary practice in present-day technology is for a central operating location to dispatch allocations and monitor performance on a station basis, whereas allocations within stations are of course on a unit basis. The block diagrams of Sect. 8 do not identify "sources" as either stations or units. The same basic concepts apply to both. The heat rate versus output relationships are derived fundamentally on a unit basis and must be utilized on that basis somewhere in the system.

Some portions of a control system will be located within the stations of the units that are to be regulated. Other portions of the control system will be located at a central operating office. Connections between the several portions will be made with appropriate direct wire, carrier, or microwave channels.

An illustrative schematic diagram of computing circuits at a load dispatcher's office for a Class II, Type 3 control is shown in Fig. 57. A companion diagram of the computing circuits for this type of control in one of the regulating stations is shown in Fig. 58.

For a discussion of typical equipment arrangements at a centralized dispatching office and at regulating stations for Class III, Type 1 controls, see Refs. 30 and 31; for Class III, Type 2 control, see Ref. 32; and for Class III, Type 3 controls, see Refs. 21, 33, and 34.

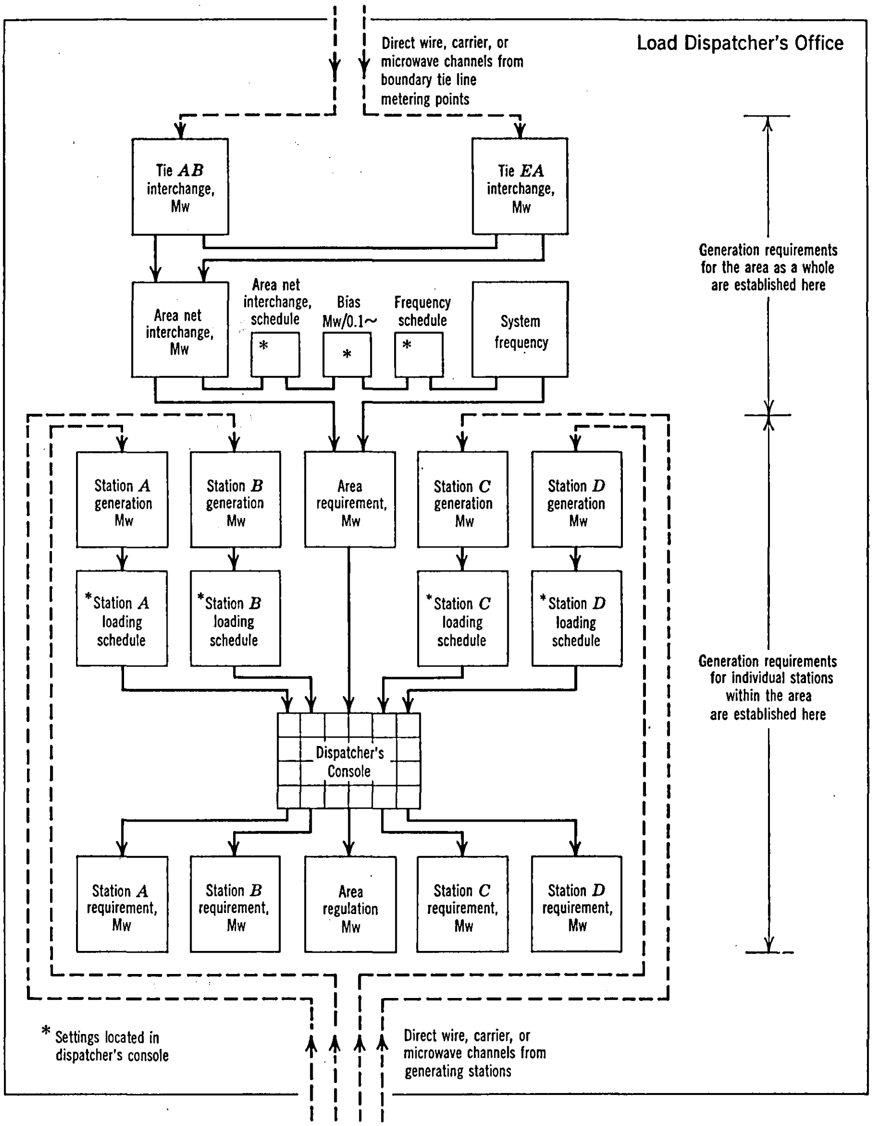


FIG. 57. Schematic of computing circuits in the load dispatcher's office for a Class II, Type 3 control. Dispatcher's console includes "base point" and "participation" settings for two segments of each station's loading schedule. Companion schematic of the computing circuits at one of the regulating stations is shown in Fig. 58.

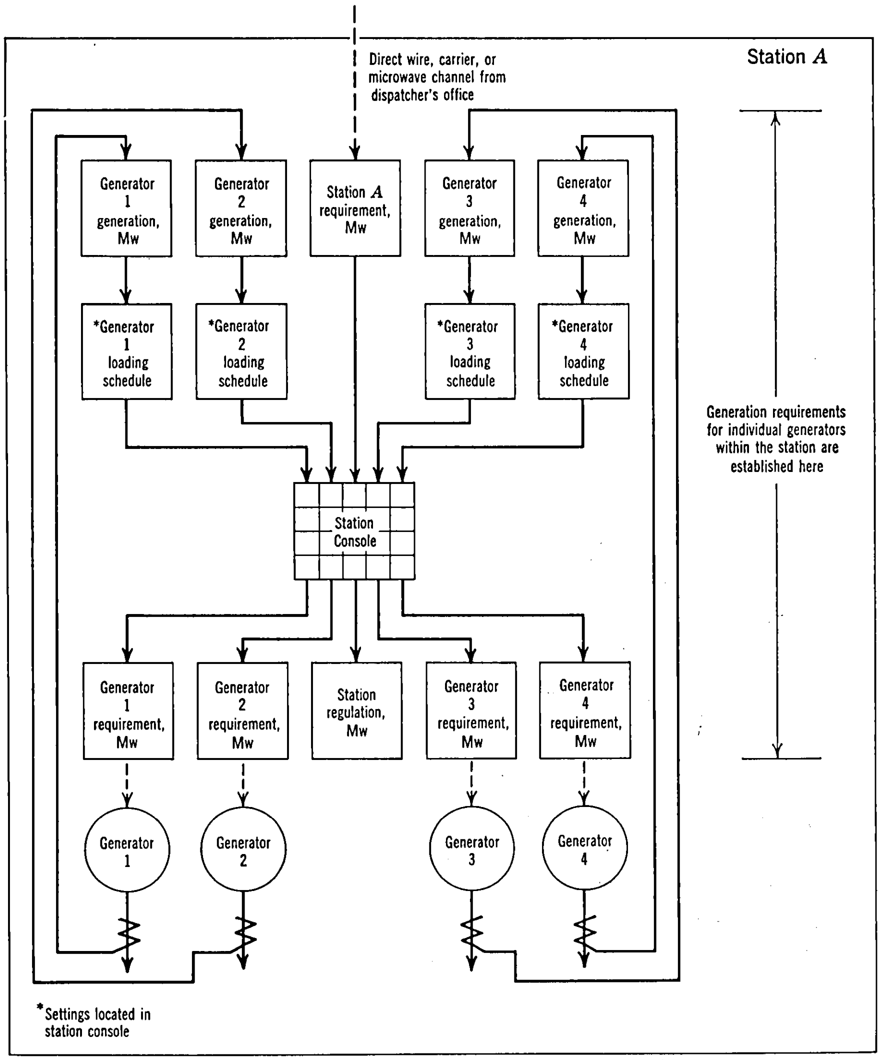


FIG. 58. Schematic of computing circuits at one of the regulating stations for a Class II, Type 3 control. Station console includes base point and participation settings for each generator's loading schedule. Companion schematic of the computing circuits at the load dispatcher's office is shown in Fig. 57.

Types of Control Execution

The block diagrams of Sect. 8 show various methods for calculating the change in generation required of sources participating in economy dispatch control. These source requirement blocks are shown feeding regulator blocks, which in turn operate on the respective sources to reduce any corresponding requirement to zero. No details are included on the Sect. 8 diagrams to show how the control is executed from the regulator blocks to the sources.

There are two basic methods of control execution currently in use: one is *permissive control,* the other *mandatory control.* The latter is also referred to as *forcing* or *command* control. Some economy dispatch methods can utilize either type of control execution. Others can utilize only the permissive approach.

The difference between the two control executions derives from the fact that there are error signals at two different operating levels which dictate the need for generation change of participating sources. One error signal is at the area level. It is the area requirement, which indicates a need for generation change for the area as a whole. The other signals are at each of the individual source levels. It is the source requirement which indicates a need for generation change at that particular source. The way in which the area error and source error signals are combined and utilized defines whether the control execution is of the permissive or the mandatory type.

Permissive Control. In this execution all corrective control action originates from area requirement. Source requirement regulators do not themselves initiate control action but are used to determine whether or not the prevailing control action from the area requirement regulator will or will not be applied to their respective sources. When the area error, for example, produces a *raise* signal, all sources which at that time have a *raise* requirement are permitted by their respective source regulators to respond to the area control signal. Similarly, when the area regulator produces a *lower* signal, all individual sources which then have a *lower* requirement are permitted by their respective source regulators to respond to the area control signal.

Circuits must of course be arranged so that there is always at least one source that will accept an area raise signal and at least one source that will accept an area lower signal.

Class II, Type 1, Class II, Type 2, and Class III, Type 2 controls, as noted in Figs. 44, 45, 51, and 52, all require the use of permissive control, since the area requirement does not enter directly into the computation that yields the source requirements, and only by permissive action can area requirement be reduced to zero.

Class II, Type 3, Class III, Type 1, and Class III, Type 3 controls, as noted in Figs. 49, 50, 54, 55, and 56, can use either permissive or mandatory executions.

An example of a permissive execution is shown in Fig. 59. This applies specifically to the Class II, Type 3 control of Figs. 57 and 58. It illustrates permissive action at both the load dispatcher's office and at one of the participating stations.

At the load dispatcher's office, station raise requirement contacts, R_A, R_B, R_C, and R_D, or station lower requirement contacts, L_A, L_B, L_C, and L_D, for stations A, B, C, and D, are respectively closed when the corresponding station has a raise or lower requirement. This permits the master area raise or lower control signals to be transmitted to the corresponding station. At the station, unit raise requirement contacts, R_1, R_2, R_3, and R_4, or unit lower requirement contacts, L_1, L_2, L_3, and L_4, for units 1, 2, 3, and 4, are respectively closed, depending on the prevailing computed requirement of the corresponding unit. This permits the raise or lower impulse received from the dispatching office to be routed to the generating units in the station that have a corresponding requirement.

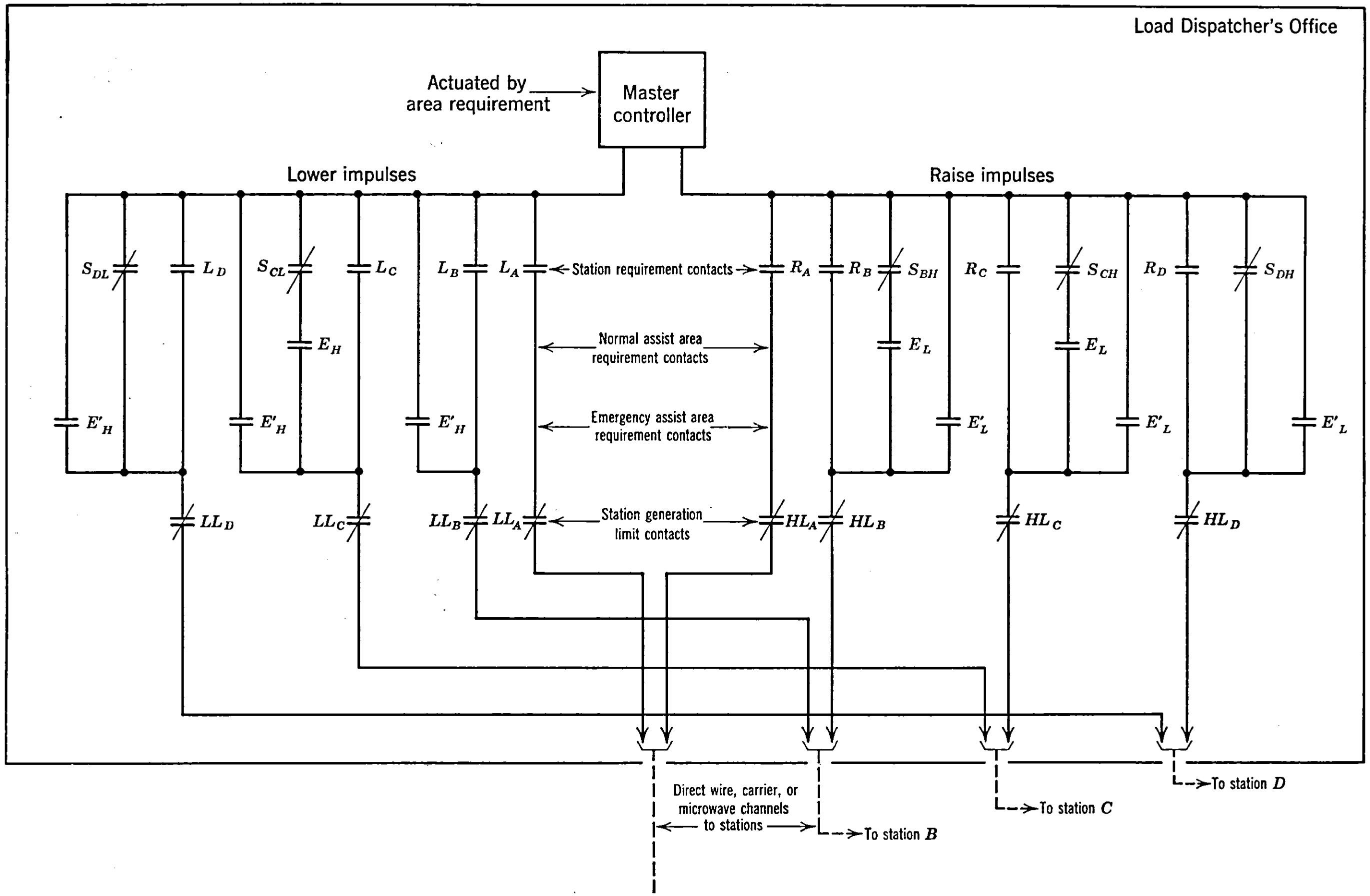
Thus, all regulating units in the area that have a raise requirement respond to an area raise signal; all regulating units of the area that have a lower requirement respond to an area lower signal.

By bearing in mind the implications of eq. (46), it will be apparent that for the Class II, Type 3 permissive execution shown in Fig. 59 there will always be at least one area source with a requirement of the same direction as the area requirement.

A virtue of a permissive execution for economy dispatch is that all control action is always in a direction to correct prevailing area errors. This is advantageous from the point of view of cooperative regulation of the area with respect to the rest of the interconnection.

In an economy dispatch system which, by automatic computation, uniquely assigns a discrete portion of each area load change to a specific participating source—as in Class II, Type 3 or Class III, Type 3 controls—each step of area error correction by permissive action makes a simultaneous appropriate economy dispatch allocation, the needs of the area and of the sources being thereby concurrently fulfilled.

Mandatory Control. In a mandatory execution, control action for each source is derived directly from its own source requirement regulator. A requisite premise for mandatory executions is that the economy dispatch computing or controlling circuits include any prevailing area requirement


Load Dispatcher's Office
Actuated by area requirement
Master controller
Lower impulses
Raise impulses
S_{DL}
L_D
S_{CL}
L_C
L_B
L_A
Station requirement contacts
R_A
R_B
S_{BH}
R_C
S_{CH}
R_D
S_{DH}
E_H
E_L
Normal assist area requirement contacts
E'_H
E'_L
Emergency assist area requirement contacts
LL_D
LL_C
LL_B
LL_A
Station generation limit contacts
HL_A
HL_B
HL_C
HL_D
Direct wire, carrier, or microwave channels to stations
To station B
To station C
To station D

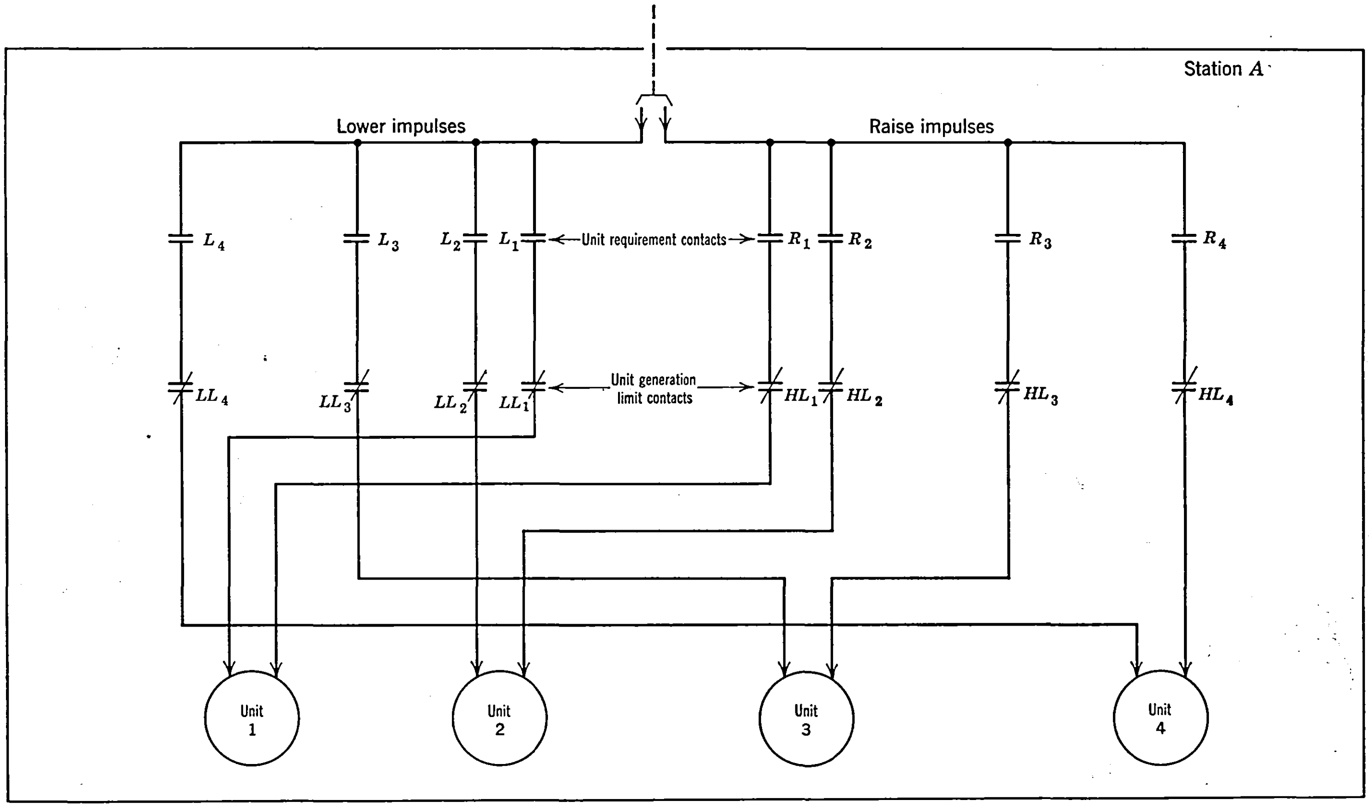


FIG. 59. Schematic of Class II, Type 3 economy dispatch at load dispatcher's office and a regulating station using permissive control. Also shown are normal assist and emergency assist actions at the dispatcher's office.

in such a manner that reducing source requirements to zero by mandatory control will also reduce area requirement to zero.

Class II, Type 3 controls, Fig. 46, Class III, Type 1 controls, Figs. 49 and 50, and Class III, Type 3 controls, Figs. 54, 55, and 56, fulfill this requirement and hence can use mandatory execution. In Class II, Type 3 control, the area requirement is used as a feed-forward in the source requirement computation; hence when each station requirement has been reduced to zero, area requirement will also have been reduced to zero. In Class III, Type 1 control, the area requirement continues to float the λ reference until area requirement is reduced to zero. In Class III, Type 3, area requirement is used as a feed-forward in the automatic setting of the

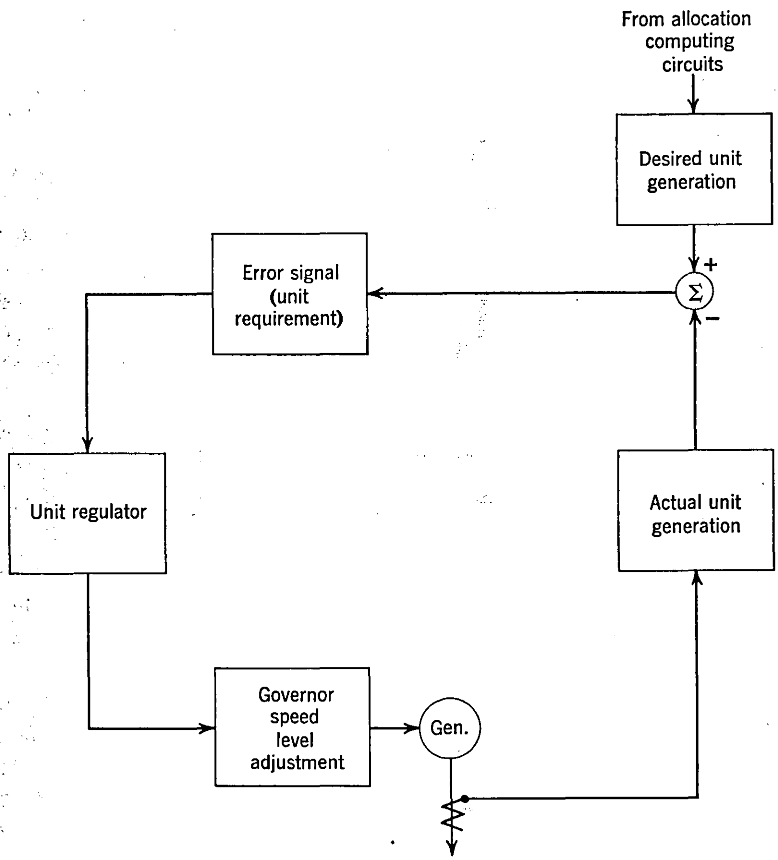


Fig. 60. Simplified schematic of mandatory execution. Unit regulator applies direct control action to the generator until the unit requirement, which is the error signal derived from a comparison of desired unit generation and actual unit generation, is reduced to zero.

λ reference, thereby yielding a λ reference which, when achieved, will have reduced area requirement to zero.

A simplified schematic of a typical mandatory execution from a source regulator is shown in Fig. 60. Any prevailing source requirement causes direct control action to be applied to the unit until the source requirement is reduced to zero.

A mandatory control permits adjustment of each source regulator to match the response characteristics of its respective source. In an economy dispatch system in which the economy dispatch assignment to each source depends on the response rates of other participating sources, care must be taken that a large disparity in response rates of participating machines does not result in ultimate overcorrection and undesirable departures of source loadings from economy dispatch. Where the economy dispatch computation yields an allocation for each source that is independent of the response rates of other participating sources, a mandatory response may be used without concern for interaction between participating sources.

Combinations

In this chapter control systems are classified by the basic concepts which they employ. Each of the classes and its subdivided types could utilize optional features and arrangements, and some installations might represent combinations of more than one class. One frequently used arrangement, for example, is to superimpose a Class I, Type 3 area regulation control on a Class II or Class III economy dispatch system, thereby permitting temporary participation in area regulation of units not then in line for economy dispatch response.

Area Assist Action. In executing step 3 of Table 1 on multiple area systems, a normal objective would be to assign generation changes to area sources so that area regulation is achieved at the same time that the economy dispatch objective is fulfilled. Such an execution, if feasible for a particular area, is advantageous since each area load change is assigned, as it occurs, to the area source that is to absorb it for economy dispatch. Each area load change is thus absorbed only once, and generation changes within the area are minimized.

In some areas, however, the permissible rate of change of the generation sources which are next in line to pick up load changes for economy dispatch may not be sufficiently great to satisfy fully the area regulation need, that is, step 2 for multiple area systems in Table 1. Rigid economy dispatch may not reduce area requirement to zero rapidly enough or maintain it within acceptable limits.

In such cases the control can be arranged to make an initial assignment of generation changes to units not then in line to retain them for economy dispatch and then reallocate them to the economy dispatch sources at rates permissible for those sources. This is the significance of the two arrows marked Rate of change of customer load and Permissible rate of change of sources shown entering the bottom of the Assimilation and computation block in Fig. 7. Such control action as noted in Sect. 2 has been identified as *area assist action.* It is achieved by a combination of a Class I, Type 3 approach with a Class II or Class III economy dispatch control.

Note that such area assist action by supplementary regulation applies to multiple area systems only. On a single area system, area assist action is already supplied by governor responses to system frequency variations, and no supplementary area assist action is required. On multiple area systems, area assist action can be achieved with either a permissive or a mandatory control execution.

With a Permissive Execution. For such a system, signals from the area regulator are routed to sources with available responsive capacity, even though their source requirements for economy dispatch may not at that time be in a direction to accept the area signal. Such area assist action

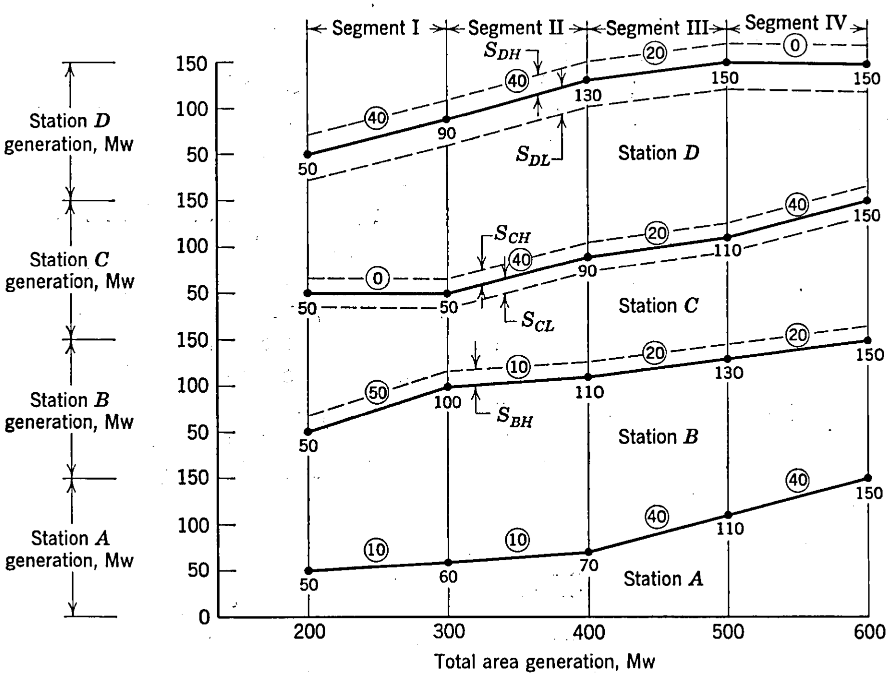


FIG. 61. Loading schedules of Class II, Type 3 control, Fig. 47, augmented to show normal area assist action at stations *B*, *C*, and *D*. The station *B* area assist is in the increase direction only. At stations *C* and *D* it is in both directions. See explanatory tabulation of Fig. 62.

may be in a single direction only, or it may be in both directions. It may be limited to a preset departure from economy dispatch, or it may be limited only by the extreme loading limits of the source. Some of the area assist action may become effective for any departure of area requirement from zero. More generally, it is utilized only when area requirement exceeds some preset setting or settings.

Figures 61 and 62 illustrate how various arrangements of area assist action can be applied with a permissive execution.

In Fig. 61, the four station schedules for a Class II, Type 3 control, as shown previously in Fig. 47, have been redrawn to show the assignment of area assist action within preset limits of economy dispatch allocation. The dash lines which run parallel to corresponding schedule lines define an *area assist band* for the respective stations. Load allocations are now made within these bands, instead of on the single line schedule if a predetermined area error is exceeded. For illustrative purposes, station A has no area assist band. Station B has an area assist band in the increase direction only. Stations C and D have area assist bands in both increase and decrease directions.

The points of area need, as reflected by magnitude of area requirement, at which area assist action becomes effective, are shown in Fig. 62. Area assist action which takes place *within* the schedule bands of Fig. 61 is

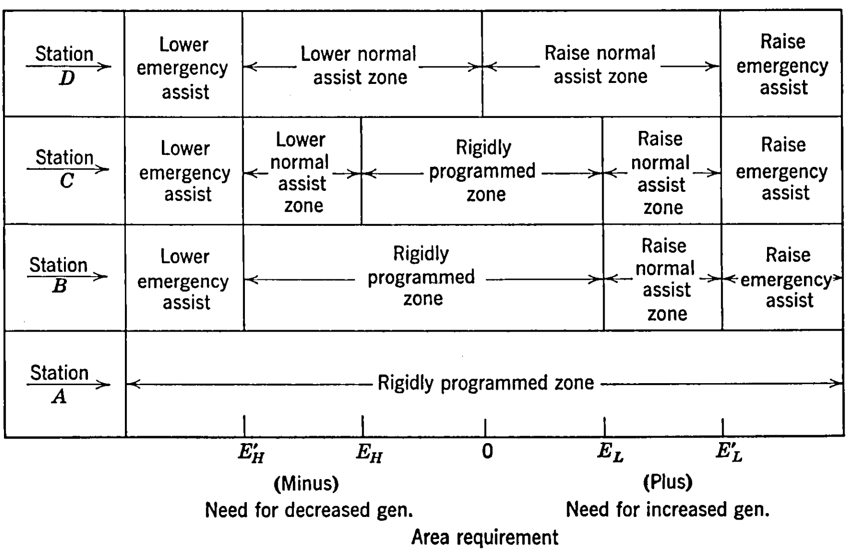


FIG. 62. Tabulation showing the points of area need, as reflected by magnitude of area requirement at which normal area assist and emergency assist actions become effective at stations B, C, and D, based on augmented schedules of Fig. 61 and circuits of Fig. 59.

identified as *normal area assist.* For station B, normal assist action is effective only when area requirement reaches magnitude E_L. For station C, normal assist is effective when area requirement reaches magnitude E_L or E_H. For station D, normal assist is effective for any departure of area requirement from zero.

In each case, normal area assist action is limited by a source requirement setting corresponding to the permissible departure of source loading from rigid economy dispatch schedule. This is illustrated in Fig. 59, where contacts S_{BH}, S_{CL}, S_{CH}, S_{DL}, and S_{DH} open when respective station requirement magnitudes reach preset band limits. In this figure contacts E_L and E_H close when area requirement reaches the corresponding low or high magnitude.

Area assist action *beyond* the schedule bands is identified as *emergency assist action.* Referring to Fig. 62, such emergency assist action takes place at stations A, B, and C when area requirement reaches either magnitude E'_L or E'_H. As illustrated in Fig. 59, the extent of the emergency assist contribution is limited only by the station and unit low and high limits, designated LL and HH with suitable subscripts.

With a Mandatory Execution. Where source control is of the mandatory type, area assist action can be assigned in the manner illustrated in Fig. 63.

This is a curve of area requirement versus desired generation for the source. When there is a zero area requirement, as at point I_0, the desired generation for the source as computed by one of the economy dispatch techniques discussed in Sect. 8 is D_n. When area requirement departs from zero, D_n as computed for economy dispatch, for the then prevailing area requirement, is augmented or reduced in proportion to the deviation of area requirement from zero. The amount by which D_n is thus augmented or decreased represents the area assist assignment for that source.

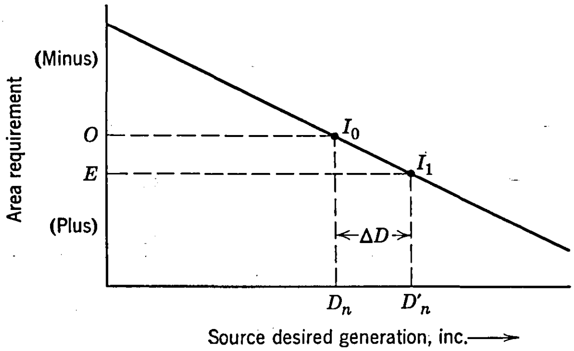


FIG. 63. Curve of area requirement versus source desired generation, illustrating how area assist action is assigned with mandatory executions.

In Fig. 63, for an area requirement of E, for example, D_n, the economy dispatch assignment to source n, would be augmented by ΔD yielding a desired generation of D'_n for the source. This is illustrated by point I_1 in Fig. 63.

This type of proportional area assist action is defined by the following equation:

$$D'_n = D_n + K_n(E), \tag{47}$$

where D_n is the desired generation for source n for economy dispatch,
D'_n is the desired generation for source n for economy dispatch plus area assist,
E is the area requirement, and
K_n is the proportionality factor for area assist action for source n.

For a control system involving both economy dispatch and area assist action, D_n is frequently identified as the *sustained assignment*, D'_n is the *initial assignment*, and $D'_n - D_n$, or $K(E)$, is the *area assist assignment*.

A block diagram of a Class I, Type 3 area regulation control, which is discussed in Sect. 3 and which can serve as the basis for mandatory area assist action, is shown in Fig. 64.

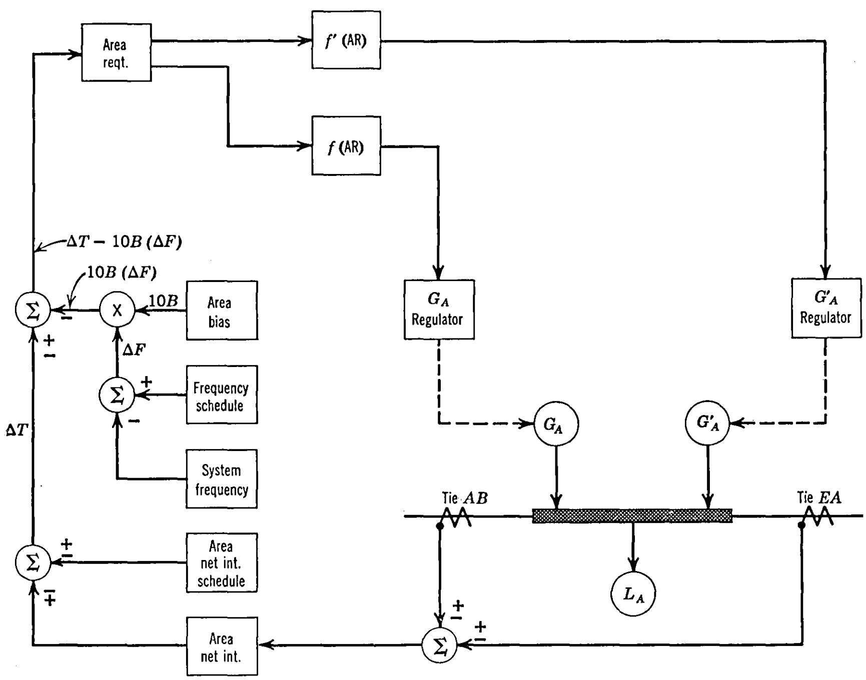


FIG. 64. Class I, Type 3 area regulation control (see Sect. 2) used as a basis for the mandatory area assist action illustrated in Fig. 63.

A block diagram showing a Class III, Type 3 economy dispatch system of Fig. 55, combined with such Class I, Type 3 area assist action, is shown in Fig. 65.

A group of curves illustrating the area assist action of a control system based on Fig. 65 is shown in Fig. 66. It is assumed, for purposes of illustration, that area A is small compared to the rest of the interconnection, that a load change of magnitude d is applied to area A at time T_2, that earlier than T_2 source G_A is fully loaded from the point of view of economy dispatch but has additional rapidly responding capability for area assist action, and that the new load d is to be fully accommodated for economy dispatch by slower-responding source G'_A. It is assumed also that the magnitude of the area assist proportionality factor, K_n, in eq. (47) is 1.0.

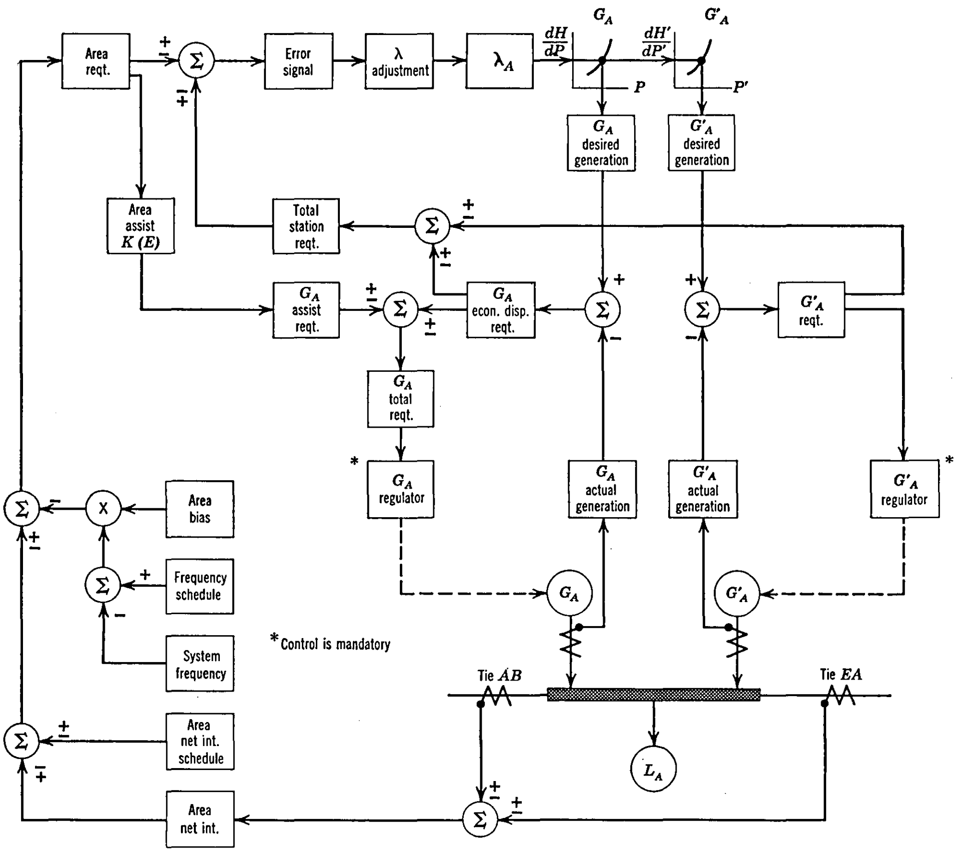


FIG. 65. Class III, Type 3 economy dispatch system of Fig. 55 augmented to include mandatory assist action per eq. (49). See Fig. 69 for further addition of unit bias factor.

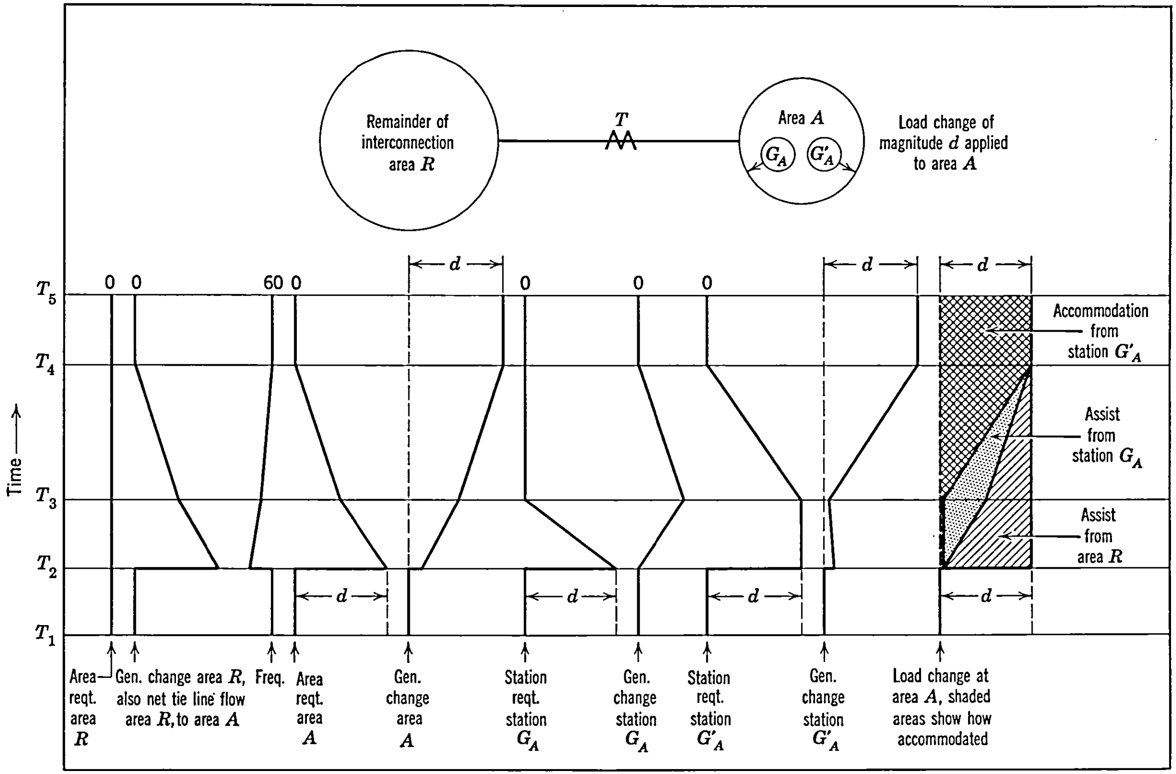


FIG. 66. Curves illustrating area assist action of a control system based on Fig. 65. A load change at area A which is small compared to the rest of the interconnection R is accommodated initially by the remainder of the interconnection, then by area assist action of station G_A, and finally by sustained economy dispatch action of station G'_A.

The ten curves of Fig. 66 relate to significant system and area parameters. From T_1 to T_2 the curves show initial balanced system and area conditions. At T_2 the load pickup of magnitude d occurs in area A and is instantaneously accommodated by system governing action. From T_2 to T_3 the control at area A causes source G_A to assist in the regulation, even though it is not next in line to do so insofar as economic dispatch is concerned. From T_3 to T_4 the slower responding source G'_A is able to pick up all the load change. At T_4 this economic reallocation is completed, and from T_4 to T_5 the system and area are again in balance, for the new conditions.

Coordinating Source Control with Governor Responses

In applying mandatory control executions, it is desirable to consider the coordination of unit control action with unit governor responses to remote load changes. This type of coordination at the area level for the area as a whole is reflected in the use of a tie line bias controller for the area. Although similar coordination at the source level for any *one* unit may not be regarded as too significant, such coordination for *all* units at the unit level is as important as the tie line bias coordination is at the area level for the area as a whole.

The nature of this problem is illustrated in Fig. 67. Frequency is plotted on the ordinate, and both desired unit generation D_n and actual unit generation P_n are plotted on the abscissa. It is assumed that D_n is computed by one of the economy dispatch methods of Sect. 8.

At point I_0 system frequency is 60 cycles, and the unit output is P_0, matching desired generation D_n. Assume that there is a remote load change resulting in a drop of system frequency by ΔF to F_1. Assume also that the local area in which the unit under study is located has a properly set area bias so that its area requirement stays on zero after the remote load change has occurred and system frequency has dropped.

The unit governor responds to the frequency drop, however, and unit generation is increased from P_0 to P_1, the latter generation corresponding to point I_1 on the unit governing characteristic *aa*.

If the desired generation remains at D_n, there will now be a mismatch between desired and actual generation and a source requirement will result, in accordance with the following relation:

$$S_n = D_n - P_n, \tag{48}$$

where S_n is the requirement of unit n,

D_n is the desired generation for unit n, and

P_n is the actual generation for unit n.

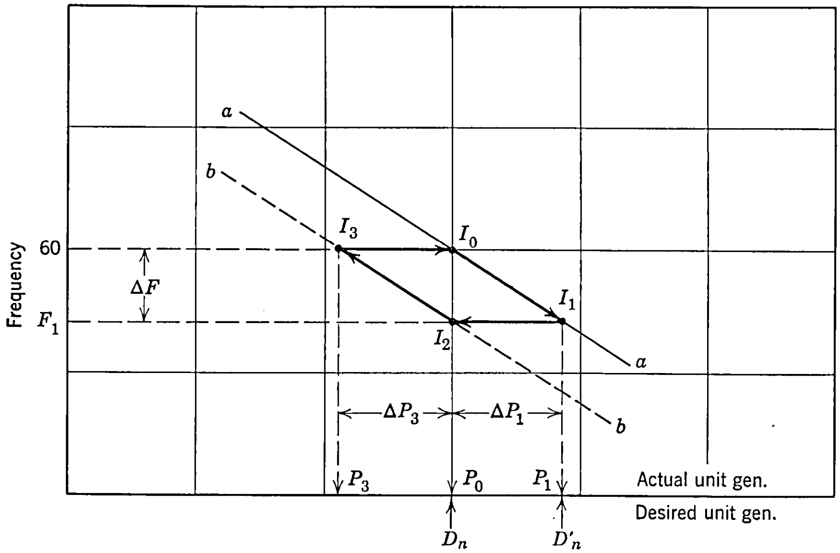


FIG. 67. Curves illustrating the need for unit frequency bias in order to properly coordinate governor responses to load changes in remote areas when mandatory control execution is used.

For the case cited S_n will be equal to $-\Delta P_1$, and there will be a resultant mandatory control action on the unit which will shift its governing characteristic from *aa* to *bb* and drop generation back to $P_0 = D_n$. This action, illustrated at point I_2 in Fig. 67, will cause the area requirement of the local area to depart from zero by an amount corresponding to $-\Delta P_1$ and will start a new cycle of economic dispatch allocations among the sources of the area.

Assume, however, that before this new cycle is begun by the local economy dispatch system, the remote area picks up generation in response to its previous load change and starts restoring system frequency to 60 cycles. The governor of the local unit will respond to this frequency correction and will carry the unit output from point I_2 to I_3 on the *bb* characteristic, reducing its generation to P_3. There is now again a mismatch between desired and actual generation, and source requirement is ΔP_3. Again, there is mandatory control action from the unit regulator, restoring unit generating to the original level P_0, as illustrated by point I_0 on the curve.

It is thus seen that the unit governing response to remote load changes is opposed by its mandatory controller, and for the sequence discussed the unit output is made to cycle as illustrated by the points I_0, I_1, I_2, I_3, and I_0. In addition, if the local economy dispatch system responds to this action of the local unit controllers, there will be an attempt at reallocation for economy dispatch of the generation temporarily picked up in the local area owing to a remote load change.

The fundamental objective in each area is to have its economy dispatch system respond only to its own local load changes, while permitting governor responses to remote load changes to persist without opposition until the remote area accommodates its load changes with generation changes of its own. Failure to coordinate the economy dispatch control at the unit level with governor responses can therefore, on occurrence of remote load changes, result in undesirable generation swings and resultant undesirable area errors in a local area. The solution to this problem rests in introducing a unit bias factor into the economy dispatch system, illustrated by augmenting eq. (48) with a bias term as follows:

$$S_n = D_n - P_n - B_n(F_0 - F_1) = D'_n - P_n, \tag{49}$$

where D_n is the desired unit generation at 60 cycles,
B_n is the bias factor for unit n, and is algebraically minus, and
D'_n is the desired unit generation at prevailing frequency.

Now, with the unit bias factor injected as in eq. (49), the occurrence of a remote load change and the resultant drop in system frequency yield a D'_n which is immediaely below P_1 in Fig. 67. This desired generation matches the new prevailing actual generation, S_n remains at zero, and the mandatory unit controller takes no action in opposition to the governing response.

The introduction of such a unit bias to coordinate source mandatory controls with source responses to remote load changes is shown by the dash line blocks in Figs. 21 and 56.

As noted in Sect. 4 and illustrated in Fig. 10, the governing characteristic of a unit is not a straight line over the full range of its operation. Since the unit bias factor imposes a definite predetermined frequency versus output characteristic on the source, its use will actually impart a linear governing characteristic to the unit which will override the variable characteristic and dead band of the governor itself.

For additional discussion on unit bias factors, see Ref. 35.

Combining Unit Bias and Area Assist

In a mandatory control execution the bias arrangement of eq. (49) and the proportional area assist action of eq. (47) may be combined so that a unit is linearly responsive to system frequency, is linearly responsive to area requirement, and has both of these responses superimposed on a sustained economy dispatch allocation. This in effect would assign an *area governor* characteristic to the unit, wherein it would take its share of sustained area load changes on an economy dispatch basis, would contribute additionally as desired for the prompt reduction in the magnitude of prevailing area requirement, and would not upset economy dispatch allocations of the area when there are area governing responses to frequency changes caused by remote load changes.

Such an area governing characteristic for an individual unit is illustrated in Fig. 68. Desired generation for economy dispatch at 60 cycles

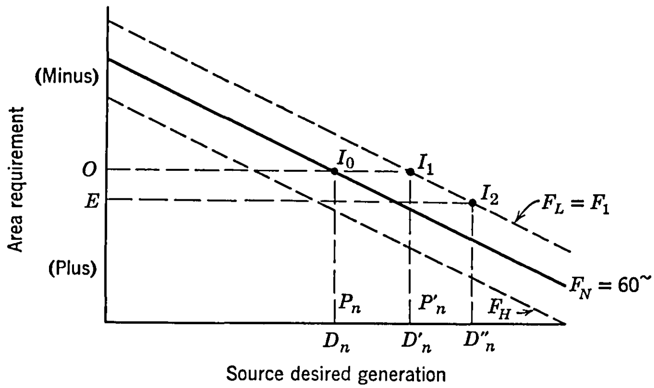


FIG. 68. Mandatory area assist curve of Fig. 63 augmented with unit bias. For normal frequency, F_N is the applicable curve. For a high frequency, F_H applies, and for a low frequency, F_L. For the example of response to a remote load increase given in the text, frequency is assumed to drop to F_1.

is D_n. A proportional area assist assignment would be added as in Fig. 63. A frequency assignment would be added by means of a unit bias factor. The F_N line in Fig. 68 represents the area requirement versus desired generation characteristic for normal frequency. The F_L and F_H lines represent this characteristic respectively for low and high frequency.

The equation for this combined response is as follows:

$$S_n = D_n - P_n + K_n(E) - B_n(F_0 - F_1) = D''_n - P_n, \tag{50}$$

where D''_n is the desired unit generation for prevailing frequency and prevailing area requirement.

To illustrate the effectiveness of this equation, assume, as at point I_0 in Fig. 68, that there is a starting frequency of 60 cycles, an area requirement of zero, and a source output P_n equal to the economy dispatch desired generation D_n. Assume then, as at point I_1 in Fig. 68, that there is a *remote* load change which results in a drop of system frequency to F_1. Local area requirement stays on zero, and source output P_n will be in-

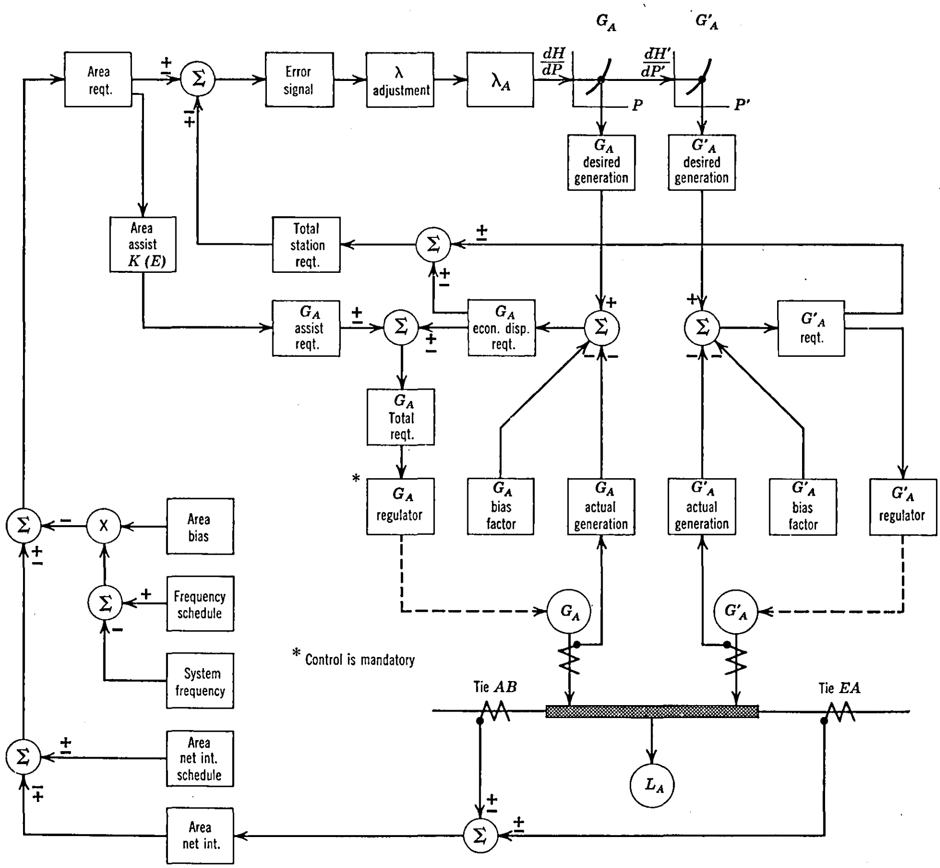


FIG. 69. Class III, Type 3 economy dispatch system of Fig. 55 augmented with both mandatory area assist action and unit bias factors in accordance with eq. (50). The unit bias factors provide coordination between action of the unit mandatory regulators and the governor responses to frequency changes caused by remote load changes.

creased by an amount dictated by the $B_n(F_0 - F_1)$ term of the equation. This new output P'_n matches the new desired generation D'_n. When the remote area picks up its load change, frequency will be restored to 60 cycles and unit output will return to P_n, as at point I_0.

If now there is a local load change, sufficient to reduce frequency to F_1, this unit will pick up any sustained economy dispatch assignment which may be allocated to it by a computed increase in D_n, will contribute additional generation on an area assist basis of an amount dictated by the $K_n(E)$ term of the equation, and will have an additional assignment for frequency response in accordance with the $B_n(F_0 - F_1)$ term. The total desired generation is D''_n, as at point I_2, in Fig. 68.

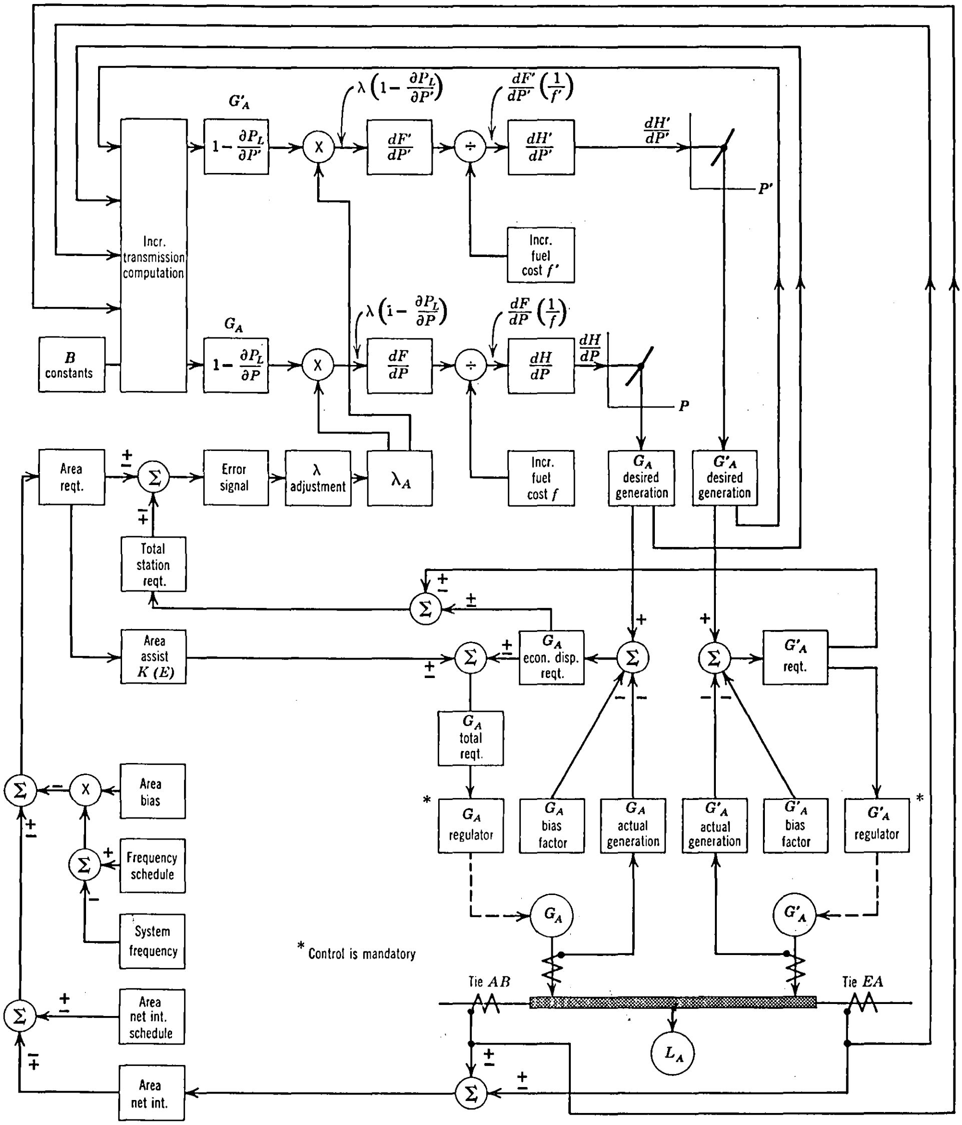


FIG. 70. Class III, Type 3 economy dispatch system of Fig. 56 expanded to include mandatory area assist and unit bias factors in accordance with eq. (50).

Thus, a mandatory execution, based on maintaing source requirement S_n of eq. (50) at zero, satisfies the economy dispatch allocation, provides an area assist assignment, and imposes a preset linear frequency versus output characteristic on the unit. A block diagram of the Class III, Type 3 economy dispatch system of Fig. 55 expanded to include both area assist action and unit frequency bias in accordance with eq. (50) is shown in Fig. 69. The Class III, Type 3 system of Fig. 56, complete with the transmission loss computation, and expanded to include area assist action and unit frequency bias in accordance with eq. (50), is shown in Fig. 70.

Protective Features

Since a typical integrated automatic control system is spread out over thousands of square miles of territory and is linked together by many long telemetering or control channels, it is desirable to incorporate into the system appropriate protective features to avoid improper operation of generating sources when components of the system fail or do not function properly. Although the protective features for each system will depend on the nature and characteristics of the installation, the following is a representative list of conditions for which protection should be supplied:

1. A high or low limit is reached on one of the sources.
2. A high limit is reached on one of the tie lines.
3. System frequency is abnormally high or low.
4. Normal voltage supplies are lost.
5. Telemetering or control channels are lost.
6. Normal telemetering or control signals are not received.
7. Abnormal telemetering or control signals are received.

The action to be taken when one or more of these conditions occur will vary with each installation and with the nature of the fault. Sometimes there will be an alarm only. Other times there will be partial or complete suspension of the automatic control. In any event, there should preferably be suitable visual and audible signaling, at the area dispatching office or at the station involved, so that the corresponding operator is promptly made aware of a fault condition for which it is his responsibility to initiate proper remedial action.

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COMPONENT SELECTION

F. COMPONENT SELECTION — A. S. Fulton, Editor

Basic Principles

A. S. Fulton

1. OBJECTIVES

Many times during the successive approximations making up the system design process the designer becomes concerned with filling out a block diagram skeleton with specific components and examining the system behavior resulting from the particular combination of functional elements. This process of selecting the components is inseparably fundamental to the system design process. However, it is our purpose in Part F to discuss the performance factors that are usually important for various kinds of components and to indicate the significant differences between the various ways of performing each kind of function—without regard to the overall system performance requirements. The material presented is, therefore, introductory in nature and limited in scope. One cannot expect to complete a system design using this material, but one can become familiar with typical component selection considerations.

This chapter is intended primarily as an introduction to the remaining chapters of Part F. Factors that apply more or less equally to the selection of all components are discussed. The more specific material regarding each component type is presented in later chapters of Part F.

Component Selection. Usually the designer of the system keeps his job a manageable size by considering a large number of elements of the system to be relatively standard items which he can employ in implementing system configurations of interest. Each system designer generally elects a different group of devices to be considered by him as components, and, consequently, it is very difficult to obtain an agreement whether a device is a component, a system, or a subsystem. We can, however, generalize certain classes of functions and delineate those properties of the devices performing functions that are of importance in the design of a system. The material of Part F has been organized on the basis of function rather than type of components. For example, the function of an amplifier is to produce a gain, but the component types discussed in the chapter on amplifiers are electronic, magnetic, hydraulic, pneumatic, etc. Tables and charts have been included where they are appropriate to illustrate the kinds of considerations to be given in the selection of components. These tabulations should not necessarily be considered as appropriate comparisons for direct selection of components.

Components in a Control System. The generalized control loop in the block diagram of Fig. 1 illustrates the definitions used in this chapter

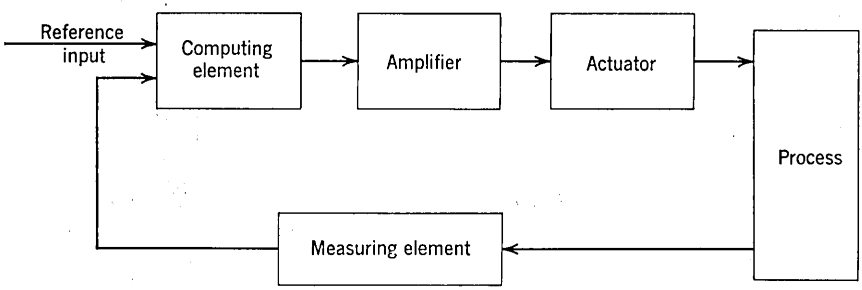


Fig. 1. Control system functions.

to separate the elements of a control system into various categories. It should be understood that many of the blocks often perform multiple functions. Usually there is a process being controlled. The process may be as simple as a shaft position or as complicated as a multistage chemical process. The measuring elements convert the process variables of interest to signals suitable for use in the control system. The converted signals are compared with reference or command signals and are modified appropriately by computing elements. The resulting signals are amplified by amplifiers. The term amplifier is generally used here to denote a device in which the output is in the same functional form as the input but at a higher power level. In control systems the computing elements and amplifier are often combined into one assembly. The actuator accepts the output signal of the amplifier and converts this to a quantity suitable for acting on the process. These definitions are not unique, but they serve to segregate components into the most practical categories from the standpoint of the purposes of Part F of this handbook.

2. GENERAL REQUIREMENTS

There are usually a number of factors arising from the intended application of the system which, together with the functional requirements, are important in the selection of components. A number of these factors which usually influence the selection of all components to a significant degree are discussed in this section.

Environmental Factors. Table 1 lists the variety of environmental

TABLE 1. ENVIRONMENTAL FACTORS—OPERATING AND NONOPERATING

Surrounding Atmosphere	Surrounding and Supporting Structures	Radiation Fields	Power Sources
Pressure	Temperature	Heat	Quantities available
Range	Range	Acoustic	Voltages
Cycles	Rate of change and gradients	Amplitudes	Nominal
Flow		Frequencies	Tolerance
Mass	Specific heats and conductivities	Electromagnetic	Frequencies
Specific heat and conductivity		Static	Nominal
Temperature	Shock	Alternating	Tolerances
Range	Transportation and handling	Nuclear	Waveforms
Rate of change (temperature and temperature gradients)		Neutrons	
	Installed	Small particles and rays	
Humidity	Vibration		
Contaminants	Amplitudes		
Corrosive	Frequencies		
Abrasive	Acceleration		
Explosive	Linear		
Conductive	Angular		
Dielectric strength			
Environmental cycles			

factors which must generally be considered in the selection of components. The environment has been arbitrarily separated into the four categories shown. These categories are not in general completely independent. *Examples:* (*a*) Those properties of the surrounding atmosphere, the surrounding and supporting structures, and the radiation fields which have to do with the environment's ability to deliver heat to the equipment or to conduct heat away from the equipment are interrelated in their influence on component selection. (*b*) Pressure and temperature, together with the flow velocity, determine the mass flow. (*c*) The dielectric strength will be a function of the pressure and density of the atmosphere surrounding the equipment.

The ways in which these factors influence the selection of particular kinds of components will be discussed in greater detail in Chaps. 21, 22, 23, and 24. Typical effects of the factors are:

1. *Permanent damage*
   a. Surrounding atmosphere can damage equipment through overpressure, by the action of corrosive and abrasive contaminants,

by interacting with the equipment to explode, and by excessive temperatures.

b. The surrounding and supporting structures can cause damage to equipment by imposing excessive shock, through inducing vibration resonances, and by temperature extremes.

c. Radiation fields can cause damage by inducing temperature extremes, by inducing vibration resonances, and by neutron bombardment of sensitive parts.

d. Power sources cannot generally cause damage except through extremes of overvoltage coupled with inadequate fusing.

2. *Errors and temporary malfunctions* can be caused by:

a. Temperature extremes caused by the interaction of heat generated in the equipment, the conduction heat transfer capabilities of the surrounding atmosphere, and the conduction and radiation heat transfer capabilities of the surrounding and supporting structures causing changes in dimensions and basic physical properties of materials.

b. Interactions of the environment with measuring elements such as the temperature of the supporting structure affecting a liquid temperature measurement.

c. Changes in insulation resistance due to high humidity, low dielectric strength, accumulation of water or ice during environmental cycles of pressure, temperature, and humidity.

d. Drifts, changes in gain or increases in noise levels caused by changes in power source voltage, frequency, or waveform.

e. Extraneous signals and drifts induced by radiation fields such as electromagnetic or acoustic fields and caused by unusual stresses due to temperature gradients and accelerations.

f. Cross-talk signals, usually present in measuring elements, due to interaction between variables such as apparent pressure changes due to acceleration or to coupling between axes in accelerometers.

Maintenance. The design of a system involves consideration of testing and repair. The choice of components, particularly of measuring elements, must often be considerably restricted to arrive at a system design that can be tested and maintained with the facilities, personnel, and supply system available. Detailed maintenance plans must be made to fit the particular conditions to be met in the field and will be different for each system.

The following component properties generally require special consideration:

1. *Test Equipment Required.* If the components of the system cannot be adequately tested using equipment which is available and familiar to

the user's maintenance group, special test equipment and training will be required. This will increase the cost of the system, increase the training time required to put the system in operation, and may also increase the level of maintenance skill required with attendant increases in operating costs.

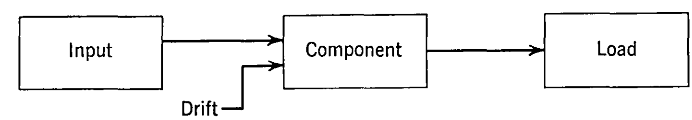
2. *Repair Methods.* If the user plans to repair system components whenever practical, additional test equipment requirements arise. If the user plans to repair the system by replacing components, some degree of interchangeability is required. Interchangeability requirements can be as simple as requiring only mechanical interchangeability plus adequate adjustments to obtain the necessary performance. On the other hand, complete electrical and mechanical interchangeability without adjustment may be required.

3. *Required Spares.* The spares required (either components or parts) will be determined by the component reliabilities, the permissible system "down-time," and the delivery times for reordering. The spares inventory represents a cost of operating the system, and careful consideration should be given to the relative merit of high reliability and high first cost as against lower reliability and lower first cost with the disadvantage of larger stocks of spares.

3. PERFORMANCE FACTORS AND DEFINITIONS

Table 2 is a list of the component parameters which are usually important. Other factors may be of importance, depending on the component function being considered. These factors are discussed in detail in Chaps. 21, 22, 23, and 24. Some of the properties of the input source

TABLE 2. PERFORMANCE FACTORS



Input Characteristics	Component Characteristics	Load Characteristics
Kind of variable	Range	Required signal
Range of variable	Input	Range and level
Impedance	Output	Input impedance
	Transfer function	
	Input impedance	
	Output impedance	
	Nonlinearities	
	Functional input-output relationship	
	Limiting amplitude	
	Limiting rate of change of amplitude	
	Limiting acceleration of amplitude	
	Dead space and hysteresis	
	Drift	
	Power source	

and the load are listed to emphasize the importance of considering the interactions between components.

Figure 2 illustrates a typical static input-output curve for a component. The nonlinearity and dead space have been exaggerated to illustrate the definitions.

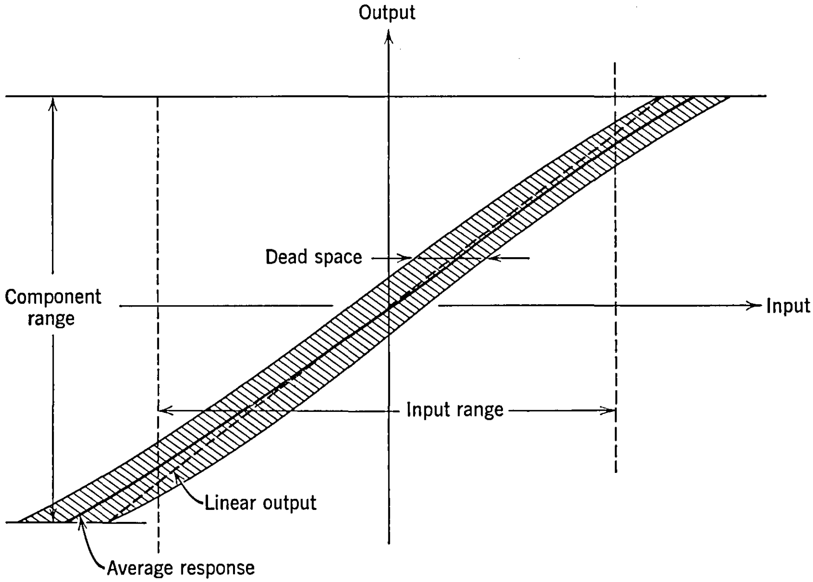


FIG. 2. Typical static input-output curve for a component.

The power source required by the component is listed because economical system design usually requires that all the components operate from the same power source. A considerable reduction can be made in the number of components considered for the system by restricting the possible sources of power.

Definitions

Input or Output Range. The total excursion of the input or output over which the component is required to perform accurately.

Component Range. The total excursion of the output (or input) over which the component is capable of responding accurately. Component accuracy is usually specified in terms of this quantity.

Dead Space. The total excursion of the input required to change the direction of the output. Some components, such as motors, have dead space only around zero input.

Transfer Function. The transfer function of a system or element is the relationship between the output and the input under specified conditions. In a linear system the transfer function is the ratio of the transform of the output to the transform of its input under the conditions of zero initial energy storage. It is a complete description of the dynamic properties of the system and may be represented as a mathematical expression, frequency response, or the time response to the specified input.

1. *Static Transfer Function.* The shape of the average response curve.

2. *Dynamic Transfer Function.* The ratio of polynomials in frequency terms which characterize the component response in its linear range. In some cases different dynamic transfer functions are required for different parts of the input range.

Nonlinearity. A nonlinear system is one represented by a nonlinear differential equation, i.e., the principle of superposition does not apply. For components it is often interpreted as the departure of the static transfer function from the theoretical curve.

Limiting. The presence of a large reduction in the slope of the average response of a component, called limiting, can be important as:

1. *Amplitude Limiting.* Based on the output magnitude.

2. *Velocity Limiting.* Based on the output time derivative.

3. *Acceleration Limiting.* Based on the second derivative of the output with respect to time.

Drift. The presence of an output which is unrelated to the input; an output which acts as if another input had been added.

Reliability

E. V. Bersinger

1. IMPORTANCE

Modern complex electronic equipment is notable not only for its almost unlimited capabilities but also for its inability to perform consistently. Electronic achievement in the future appears to be limited not by man's ability to conceive new electronic devices but by his inability to make them serve him faithfully.

Reliability as a Parameter. The importance of reliability has caused its consideration as a quality to give way to its consideration as a measurable performance parameter (Ref. 1). Reliability, as other performance parameters, may be predicted before construction of an equipment, and compliance to a reliability specification may be shown by engineering test on the finished product or field observation of it. Continuous reliability audits made during the design and production of a device or system form the basis for reliability control.

Reliability is a statistical quality which has been defined as the probability that an object will operate within given limits for a specified time under specified conditions. As a probability, reliability takes on values from zero, which is equivalent of never working, to a value of unity, which indicates perfection. Knowledge of the rudimentary principles of statistics and probability are necessary to an understanding of reliability (see Vol. 1, Chaps. 12 and 13).

2. FAILURES

Random Failure Law

The failure rate of a part or device during its life determines its reliability. An examination of the failure rate characteristics of a wide variety of complex items has shown that the failure given in Fig. 1 is applicable

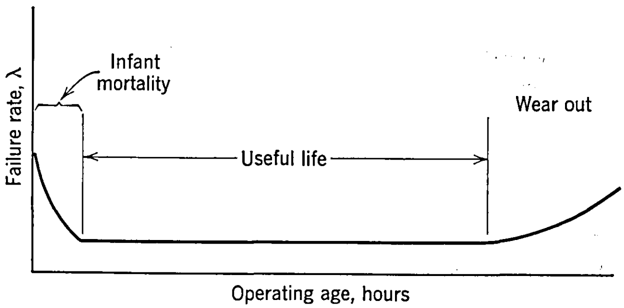


FIG. 1. Complex device failure rate.

to most complex items (Ref. 2). Failure rate (λ) is defined as the ratio of the number of devices failing during an interval of time to the number operating at the start of the interval. For those items for which this curve is not applicable, the low or near-zero failure rate portion of the curve must still represent the design goal toward which the designer must strive.

Failure Rate Curve. The failure rate curve of Fig. 1 may be conveniently divided into three areas:

1. *The Shakedown Period.* During this period, which usually represents less than 5% of the useful life of the equipment, the infant mortality due to the manufacturing process is in evidence.

2. *The Period When the Equipment Is in Useful Operation.* This period lies between the end of the shakedown period and the advent of wear out. During this time the failure rate of mature well-designed equipment is at a low, near-constant, value. If the failure rate is not constant, an assignable cause is present which may be removed by careful attention to the design.

3. *Wear Out.* Preventive maintenance procedures are generally employed at the beginning of this period to remove the equipment for renovation.

Mean Time between Failures. It is more meaningful to express failure rate as its reciprocal, mean time between failures (mtbf). Because this parameter has greater physical meaning, it has become the accepted reliability figure of merit. It is important to understand the difference between mean time between failure and average life. Average or usable life is taken to be the operating age at which the failure rate begins to increase as a result of wear out. Thus average life is a measure of the allowable length of time between overhauls. Mean time between failure is a measure of the operating time which may be expected between unpredictable or random failures when the equipment is properly maintained by overhaul at specific times roughly corresponding to the average life. The time of occurrence of failures resulting from wear out may be predicted, whereas the time of occurrence of failures during the useful life *cannot* be predicted. In properly maintained equipment the random failure rate during useful life is the factor which determines reliability.

Maintenance. Electronic equipment exclusive of the electromechanical elements does not exhibit a marked wear-out pattern. Moreover, overhaul of electronic equipment often restores the equipment to zero-time condition, complete with the failure causes which should be found by a shakedown procedure. There is evidence to indicate that mediocre maintenance of electronic equipment will increase the basic failure rate during useful equipment life. Unless there is absolute proof that portions of an electronic equipment are wearing out, it is unwise to remove that equipment for periodic maintenance.

Reliability Equation. During the period when the failure rate is constant, reliability R_i of any component i is given by the following function of operating time t and mean time to failure m_i, or failure rate λ_i by

$$R_i(t) = e^{-t\lambda_i} = e^{-t/m_i}. \tag{1}$$

The ratio of operating time to mean time between failures t/m_i is defined as normalized time. Table 1 gives values for reliability R_i as a function of normalized time. The third column of the table gives R_{red}, the theoretical reliability of a redundant pair of devices of reliability R_i. Redundancy is treated in greater detail in a later section.

For values of normalized time, less than 0.1 the approximation $R_i = 1 - t/m_i$ may be applied with negligible error (less than 0.6%).

Equation (1) may be used to describe equally well the reliability of an electronic system, component, or part as each of these has shown a random failure rate.

Series Failure Law

As a first approximation it may be said that electronic equipment depends upon successful operation of every part in order for the complete

Table 1. Reliability of Simple and Redundant Systems as a Function of Normalized Time, t/m_i

t/m_i	R_i	$R_{red} = 2R_i - R_i^2$	t/m_i	R_i	$R_{red} = 2R_i - R_i^2$
0.00	1.000	1.0000			
0.01	0.990	0.9999	0.51	0.600	0.8403
0.02	0.980	0.9996	0.52	0.595	0.8355
0.03	0.970	0.9991	0.53	0.589	0.8307
0.04	0.961	0.9984	0.54	0.583	0.8259
0.05	0.951	0.9976	0.55	0.577	0.8210
0.06	0.942	0.9966	0.56	0.571	0.8161
0.07	0.932	0.9954	0.57	0.566	0.8112
0.08	0.923	0.9940	0.58	0.560	0.8063
0.09	0.914	0.9925	0.59	0.554	0.8013
0.10	0.905	0.9909	0.60	0.549	0.7964
0.11	0.896	0.9891	0.61	0.543	0.7914
0.12	0.887	0.9872	0.62	0.538	0.7865
0.13	0.878	0.9851	0.63	0.533	0.7815
0.14	0.869	0.9829	0.64	0.527	0.7765
0.15	0.861	0.9805	0.65	0.522	0.7715
0.16	0.852	0.9781	0.66	0.517	0.7665
0.17	0.844	0.9755	0.67	0.512	0.7615
0.18	0.835	0.9728	0.68	0.507	0.7565
0.19	0.827	0.9700	0.69	0.502	0.7515
0.20	0.819	0.9671	0.70	0.497	0.7465
0.21	0.811	0.9641	0.71	0.492	0.7415
0.22	0.803	0.9610	0.72	0.487	0.7365
0.23	0.795	0.9577	0.73	0.482	0.7315
0.24	0.787	0.9544	0.74	0.477	0.7265
0.25	0.779	0.9510	0.75	0.472	0.7216
0.26	0.771	0.9475	0.76	0.468	0.7166
0.27	0.763	0.9440	0.77	0.463	0.7116
0.28	0.756	0.9403	0.78	0.458	0.7066
0.29	0.748	0.9366	0.79	0.454	0.7017
0.30	0.741	0.9328	0.80	0.449	0.6967
0.31	0.733	0.9289	0.81	0.445	0.6918
0.32	0.726	0.9250	0.82	0.440	0.6868
0.33	0.719	0.9209	0.83	0.436	0.6819
0.34	0.712	0.9169	0.84	0.432	0.6770
0.35	0.705	0.9127	0.85	0.427	0.6721
0.36	0.698	0.9086	0.86	0.423	0.6672
0.37	0.691	0.9043	0.87	0.419	0.6623
0.38	0.684	0.9000	0.88	0.415	0.6575
0.39	0.677	0.8957	0.89	0.411	0.6526
0.40	0.670	0.8913	0.90	0.407	0.6478
0.41	0.664	0.8868	0.91	0.403	0.6430
0.42	0.657	0.8823	0.92	0.399	0.6382
0.43	0.651	0.8778	0.93	0.395	0.6334
0.44	0.644	0.8732	0.94	0.391	0.6286
0.45	0.638	0.8686	0.95	0.387	0.6239
0.46	0.631	0.8640	0.96	0.383	0.6191
0.47	0.625	0.8593	0.97	0.379	0.6144
0.48	0.619	0.8546	0.98	0.375	0.6097
0.49	0.613	0.8499	0.99	0.372	0.6050
0.50	0.607	0.8451	1.00	0.368	0.6004

system to be reliable, and that part failures are independent of each other. Under these conditions the probability of successful equipment operation is the product of the probabilities of the individual parts operating successfully. This is based on the following theorem:

THE THEOREM OF COMPOUND PROBABILITY. *If a compound event is made up of a number of separate and independent subevents, and the occurrence of the compound event is the result of each of these subevents happening, the probability of occurrence of the compound event is the product of the probabilities that each of the subevents will happen.* This theorem has been termed the "series law," since the circuit elements are series aligned in a reliability sense. In order for the circuit to be successful, each of the components must operate successfully, and effectively there must be continuity of reliability through the series of parts.

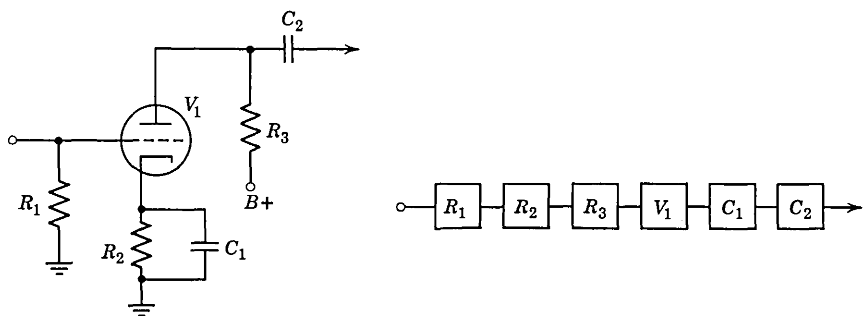


FIG. 2. Analytical model.

Total Reliability. Figure 2 shows an RC coupled amplifier and its reliability analog or analytical model. The total reliability R of the amplifier is given by

$$R = R_{R_1} \times R_{R_2} \times R_{R_3} \times R_{V_1} \times R_{C_1} \times R_{C_2}.$$

These analytical models are used in the systematic study of reliability and form the framework within which reliability may be predicted.

The *total reliability*, R of a system made up of parts of individual reliabilities R_i is given by the formula

$$R = \prod_{i=1}^{n} R_i \tag{2}$$

where R = total or system reliability,

R_i = reliability of the ith part.

Reliability Computations. For computational purposes it is preferable to compute the system failure rate λ as the sum of the individual part failure rates λ_i:

$$\lambda = \sum_{i=1}^{n} \lambda_i \tag{3}$$

where λ = the total or system failure rate,
λ_i = failure rate of the ith part.

If there are several like components, this computation can be carried out very successfully by multiplying the number of like components n_i by the class failure rate λ_i and summing this to obtain λ. Total failure rate λ is substituted into the exponential formula to compute the system reliability R for any period of time t:

$$\lambda = \sum_{i=1}^{n} \lambda_i n_i, \tag{4}$$

$$R = \exp\left(-t \sum_{i=1}^{n} \lambda_i n_i\right). \tag{5}$$

The failure rate is established by measuring the total number of hours of operation and the total number of malfunctions and dividing the former by the latter. This means of computing failure rate and its reciprocal m applies during periods when the failure rate is constant. If the evaluation of failure rate extends into infant mortality and wear out, the rate measured will be in excess of the actual rate during useful life. Figure 3 is a nomograph which may be used to compute failure rate m and reliability of both simple systems and two-unit redundant systems.

3. REDUNDANCY

Redundancy has been proved both in theory and in service as a means of improving reliability. Through redundancy it is possible to obtain high system reliability using components with relatively low reliability. If judgment is not exercised, however, a redundant system may well become more unreliable than its singular counterpart. The mathematical argument from which the theoretical gain in reliability resulting from redundancy is calculated is based on the independence of the two parallel components, i.e., failure of one component will not render the surviving one inoperative. If switching is used to isolate the two redundant paths, the reliability of the switches and of the decision-making system driving the switching must be considered. Aside from the technical aspects is consideration of increased size, weight, and cost which are unavoidable when redundancy is used.

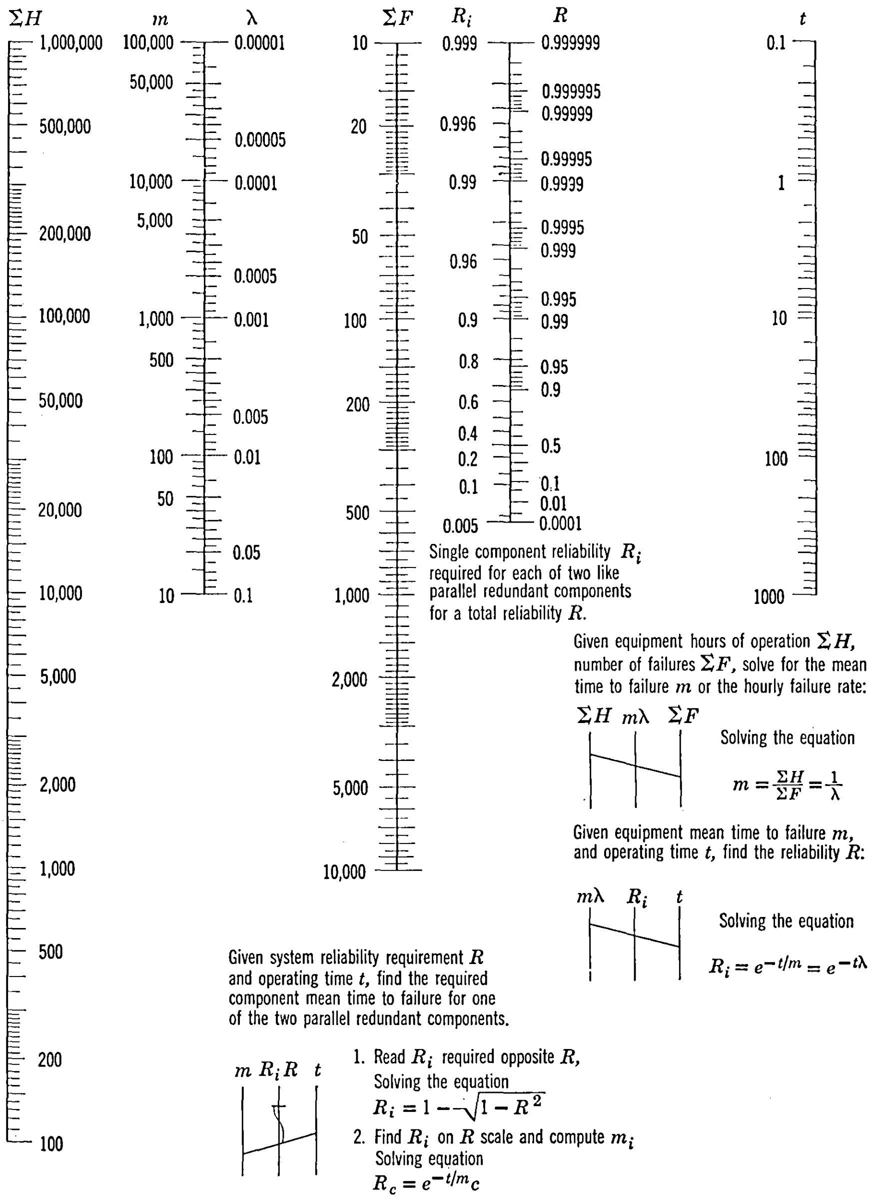


FIG. 3. Reliability nomograph.

Effectiveness of Redundancy. When two units are paralleled as shown in Fig. 4, in such a manner that both units must fail to cause system failure, system reliability is given by

$$R = R_1 + R_2 - R_1R_2. \tag{6}$$

If p identical units are used in parallel, the expression for system reliability becomes

$$R = 1 - (1 - R_i)^p. \tag{7}$$

Figure 5 shows the effect upon system reliability of n components in series and p components in parallel. The first redundant component results in a large increase in system reliability. As successive redundant elements are added, this increase diminishes.

Redundancy is usually restricted to two items in parallel. Figure 6 shows the reliability of a single component and of two parallel components as a function of normalized time. Redundancy will provide measurable gains in reliability in the region below a normalized time of 2. In the region of very short normalized time, redundancy will provide a considerable increase in reliability.

Another measure of the effectiveness of redundancy is the ratio of redundant pair mean time between failures m_{red} to component mean time between failures m_i as a function of normalized time. Figure 7 shows the relationship. For small values of t/m_i, m_{red}/m_i becomes the reciprocal of t/m_i.

The ratio of t/m_c may be made small by using parts with long times between failures. Normalized time may also be shortened by periodically checking both channels and replacing any channel found to be defective. In this situation the operating time t becomes the time between checks rather than the operational time of the equipment T, and the formula for system reliability is

$$R(t) = \exp(-T/Nm_i)[2 - \exp(-2T/Nm_i)] \tag{8}$$

where N = number of checks,
m_i = component mtbf,
$t = T/N$ = operating period = time between maintenance.

Figure 8 may be used to determine either the required component mtbf or the number of inspections N, given the system reliability R and the time period T over which this reliability applies.

Parallel Redundancy. Redundancy can be classified as parallel or standby by the manner in which the parallel pair is connected to the remainder of the system. Parallel redundancy requires no decision-making devices and consists of two channels, both of which must fail before a total

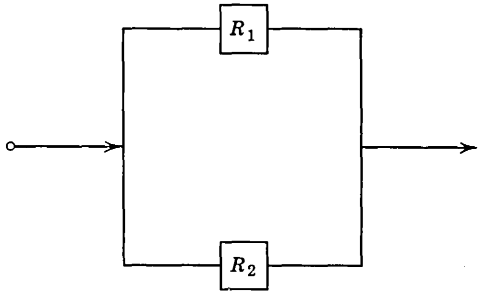


FIG. 4. Redundancy.

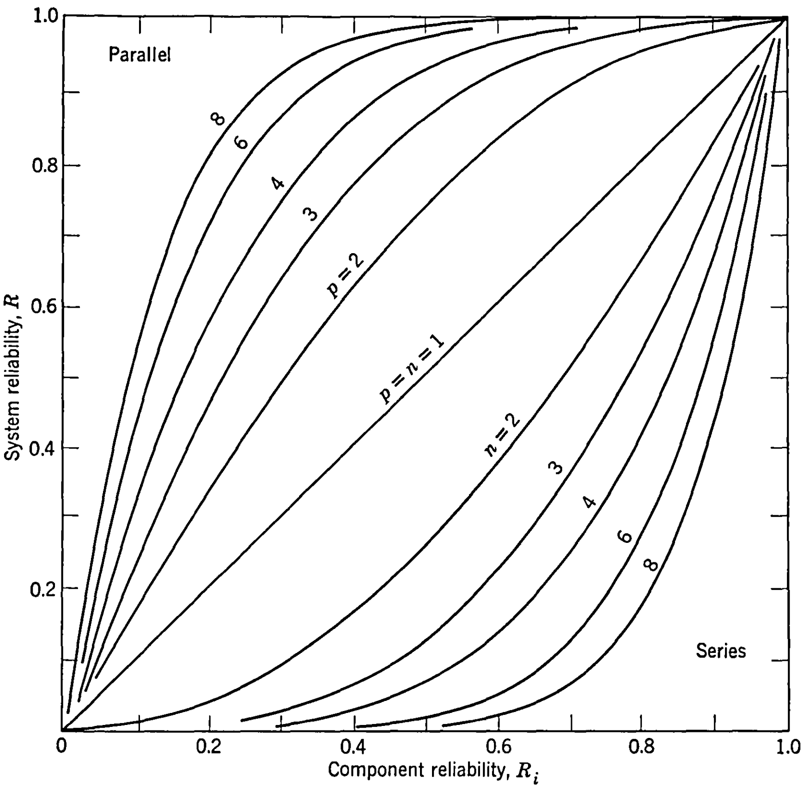


FIG. 5. Parallel and series system reliability.

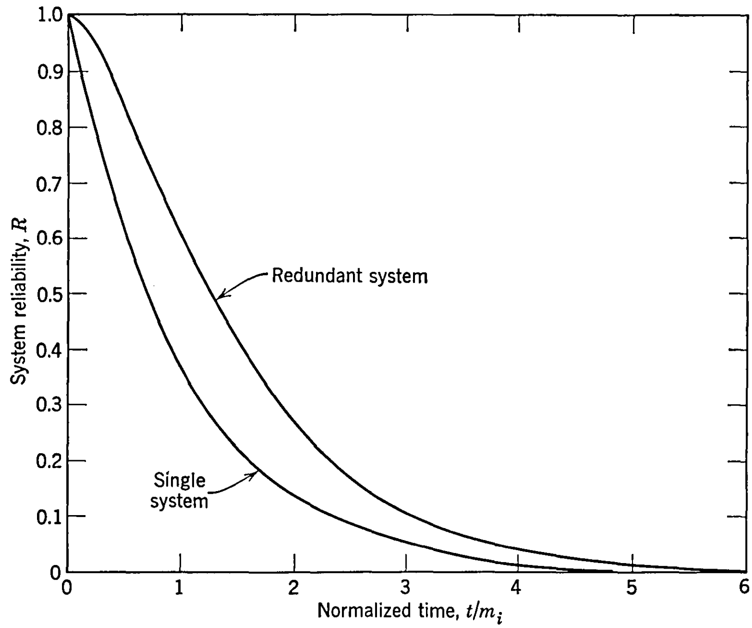


FIG. 6. Single and redundant system reliability.

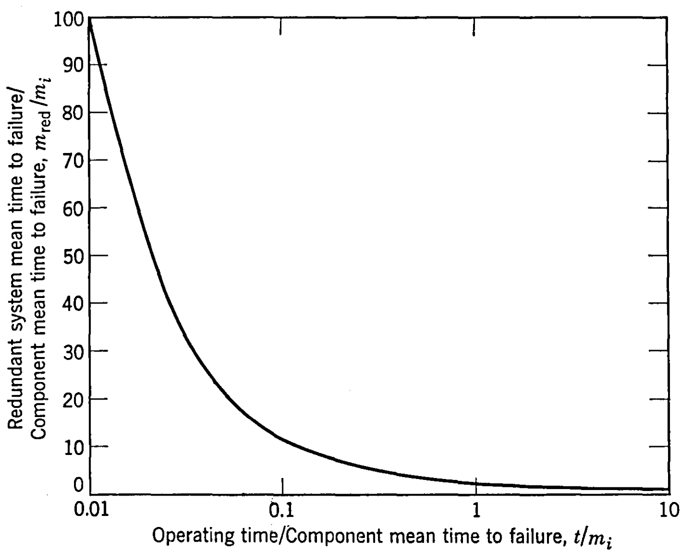


FIG. 7. Mean-time-to-failure improvement.

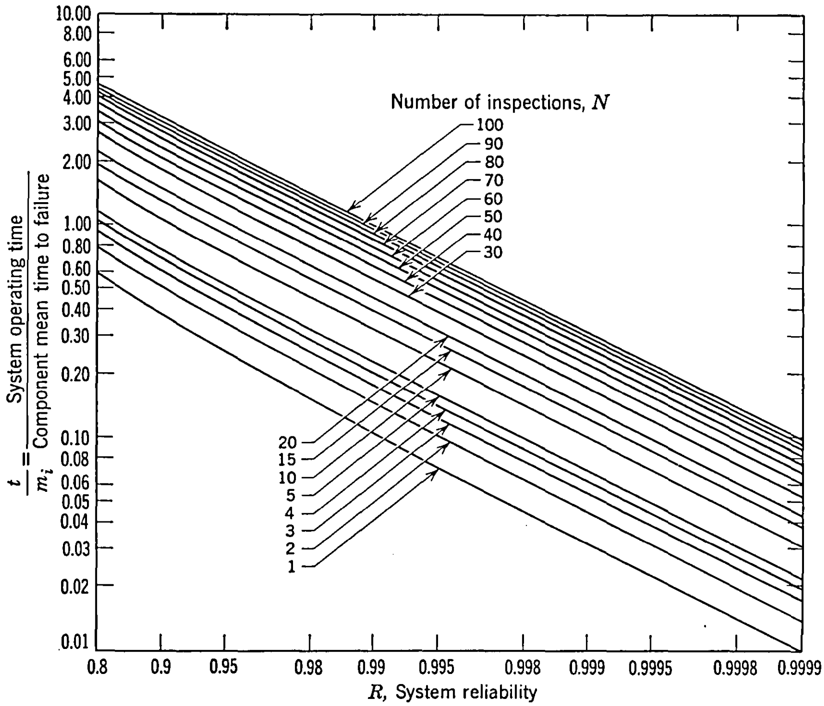


FIG. 8. Ratio of system operation hours to component mean time to failure, as a function of system reliability for a given number of periodic inspections in the case of two-unit parallel redundancy.

failure can occur. This is shown in Fig. 3. This condition is often difficult to put into effect, with its realization depending on how the inputs and outputs from the redundant elements are arranged. The successful arrangement is one that will allow either element to fail without jeopardizing the successful operation of remaining element. The reliability of this system can be computed directly from eq. (6).

Standby Redundancy. In standby redundancy a decision-making device and switching are employed to isolate the failed channel and to replace it with an operable active channel. This is represented graphically in Fig. 9. Often an operator is used as a decision-making device because of his ability to exert judgment. In any case, the reliability of the decision-making and switching devices must be considered. The intelligence input to the decision-making device may be the output of the primary channel or it may be some other quantity within the total system which is indicative of primary channel failure. If R_a is the reliability of a primary channel of Fig. 9, R_b the reliability of the secondary channel, R_x the probability of correct decision by the decision-making device when the primary

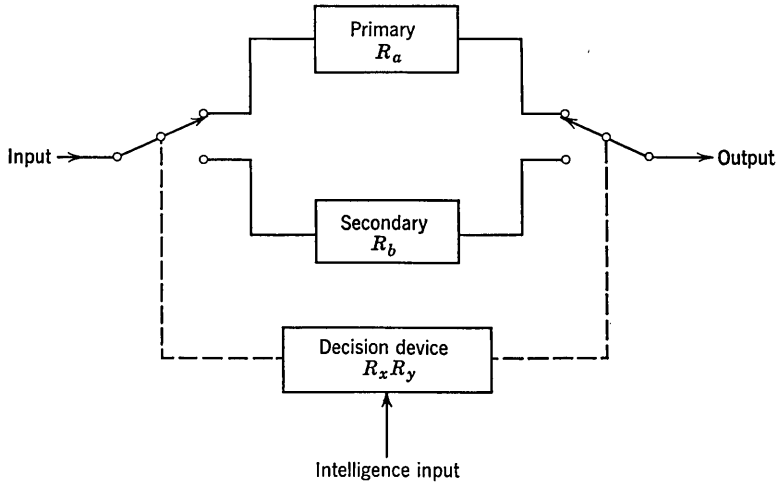


FIG. 9. Standby redunuancy.

channel is operating, and R_y the probability of correct decision by the decision-making device when the primary channel has failed, then the total reliability R is given by the equation

$$R = R_x(R_a - R_aR_b) + R_y(R_b - R_aR_b) + R_aR_b. \tag{9}$$

R_x and R_y are not usually independent, and the designer has a choice as to which he can make the greatest. Often the standby or secondary channel is less complex than the primary channel and hence more reliable. When such is the case, the designer should favor in his design R_y or the probability of correct decision following a primary channel failure.

4. RELIABILITY PREDICTION

Methods of Prediction. The reliability of an electronic system is predicted by substituting the expected part failure rates into the analytical models described in the preceding section. The expected failure rates are derived from an observation of the operation of analogous equipment. The prediction is usually made in two steps, the second being a refinement of the first. The first step in the prediction is based on a preliminary part count with no allowances made for the application or environmental stresses. As part test information and a clear definition of the equipment become available, the initial prediction should be modified to include this information.

Where parts of the system are redundant, this redundancy must be taken into account. The equivalent failure rate for a redundant pair is a function of either the operating time or the time between periodic checks and the part failure rates. Since it is a function of either operating time or time between checks, this must be specified before the reliability of a redundant part can be determined.

Failure Rates of Common Electronic Parts. Table 2 provides the expected failure rates of common electronic parts. These part failure rates apply as averages to a wide variety of parts under typical operating conditions, and considerable deviation may be expected within any classification. For example, power relays fail less frequently than general purpose relays because the power relays are not actuated as often.

The original thinking that electronic component failures were catastrophic and complete has been shown to be erroneous. The published failure rates include both catastrophic and degradation of performance or creeping type failures. Catastrophic failure rates are independent of the application since they so drastically change a part as to make it unusable in any possible application. This is not true for creeping failures since circuit tolerance to part changes varies from application to application. The best definition of the failure rates of Table 2 is that they are the result of part failures which were serious enough to cause corrective maintenance action in past equipment and will cause the same action in present equipment. These failure rates are a compendium of rates from various sources. Base environmental conditions for these rates are assumed to be a 40°C ambient and 100% rated load. A rather considerable improvement in the failure rate may be attained by derating, by protecting from the environment, and by choice of the best component within the class.

For other than the conditions of 40°C and 100% rated load, the failure rate may be obtained from Reference 3, which is a graphical tabulation of part failure rates as a function of their environmental and electrical stresses. In the absence of this document, an approximation of failure rate may be obtained by assuming that the failure rate varies as the fifth power of the applied environment above rated conditions and as the second power below rated conditions:

$$\lambda = \lambda_0 \left(\frac{T}{296}\right)^a \cdot \left(\frac{E}{100\%}\right)^b, \tag{10}$$

where λ = failure rate at existing conditions,

λ_0 = failure rate at 40°C (296°K) and 100% rated electrical stress,

$a = b = 2$ for less than rated, 5 for greater than rated,

T = component temperature, degrees Kelvin,

E = electrical stress, in percent of rated.

Note that the absolute temperature in degrees Kelvin is used.

When a parts manufacturer supplies derating factors, these should be used in preference to those of Table 2.

TABLE 2. EXPECTED FAILURE RATES FOR COMMON ELECTRONIC PARTS

Part	Failure Rate, %/1000 hr	Predominate Stresses
Small motors and generators (instrument servos)	5	Temperature, vibration, rate of rotation, brush or slip-ringed current
Sensitive relays (unsealed)	5	Temperature, dirt, vibration
Miniature tubes	3	Shock, temperature, element voltage and element dissipation, filament voltage
Wire-wound potentiometers	3	Temperature, rate of rotation, dirt, oil
Synchros	3	Temperature, vibration, rate of rotation
Clutches	3	Coil currents, temperature, duty cycle
Connectors	3	Dirt, number of engagements
Accelerometers (spring-retained mass)	3	Shock, vibration
Position potentiometers	2	Rate of rotation, dirt
Crystal diodes, selenium	2	Temperature, peak inverse voltage, maximum forward current
Vibrators and choppers	2	Contact currents, temperature, maximum voltage across the contacts
Relays, general purpose	2	Contact currents, dirt, shock
Transistors	2	Temperature, power source transients
Capacitors, tantalum	1	Voltage, temperature, heating from a-c component of a voltage
Switches	1	Number of actuations, contact current, back emf, mechanical abuse
Gear trains, instrument	1	Temperature, vibration, dirt, rate of rotation
Crystal diodes, germanium	1	Peak inverse voltage, maximum forward current, temperature
Lights	1	Voltage, vibration
Meters	1	Shock, vibration
Capacitors, mica and glass	0.9	Applied voltage, humidity, current
Switches, micro	0.8	Number of actuations, contact current
Resistors, wire-wound, power	0.7	Temperature, wattage dissipation
Capacitor, ceramic	0.7	Applied voltage, mechanical abuse, and installation
Resistors, wire-wound, accurate	0.6	Wattage dissipation, temperature
Transformers	0.5 per winding	Winding currents, temperature, potential to ground
Mag-amps	0.5 per core	Winding currents, temperature, potential to ground
Capacitors, high K	0.5	Voltage, temperature, current
Resistors, deposited carbon	0.4	Humidity, temperature, wattage dissipation, mechanical abuse
Capacitors	0.4	Voltage, temperature, rate of charging current
Circuit breakers and fuses	0.3	Number of actuations, surge currents
Resistors, composition	0.2	Wattage dissipation, temperature, humidity

Failure rate is a complex function of circuit tolerance to part variation and part parameter changes, which in turn are a function of the environment and the applied stresses. The relationships are not well known. Considerable effort is being applied to simplify the computation necessary to determine circuit performance as a function of part performance (Ref. 2) and to evaluate the changes that may be expected in part parameters (Ref. 3).

Confidence. Reliability measurement is often based on a limited number of failures. This leads to conclusions based upon such meager data that a large degree of statistical confidence cannot be placed in the results. The expression of confidence in a statistically derived mtbf requires the specification of an interval and of a probability that the interval will contain the true mtbf. Figure 10 provides a confidence factor by which an

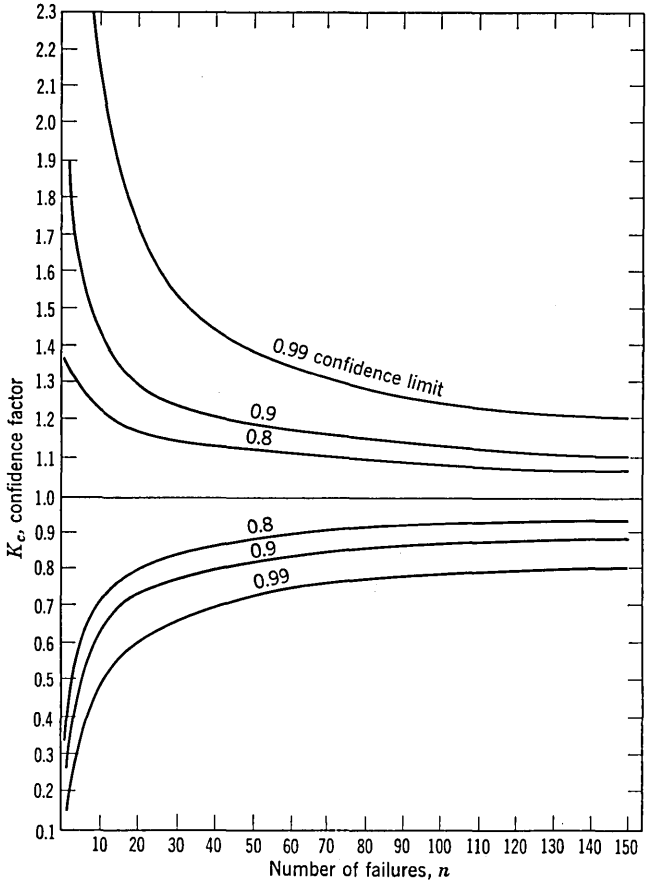


FIG. 10. Confidence factors.

evaluated m must be multiplied to obtain the confidence interval. The confidence factor is a function of the confidence limit chosen and the number of failures observed. A confidence limit of 0.8 is standard in reliability. With this limit there will be a 10% chance that the true mtbf will be below the interval and a 10% chance that it will lie above it. Often we are interested in the lower bound on mtbf, and in these cases there will be a 90% confidence that the true mtbf is greater than the lower bound of the confidence interval. For zero failures the lower bound on the confidence interval is obtained by dividing the number of hours of experience by 2.31. No upper bound exists for zero failures.

Table 3 is a tabulation of the confidence factors for the 80% confidence limit.

TABLE 3. CONFIDENCE FACTORS

Number of Failures	Confidence Factor		Number of Failures	Confidence Factor	
	Lower Bound	Upper Bound		Lower Bound	Upper Bound
0	See text		25	0.764	1.326
1	0.257	9.492	30	0.783	1.291
2	0.376	3.759	35	0.798	1.265
3	0.449	2.727	40	0.810	1.244
4	0.500	2.292	45	0.820	1.228
5	0.539	2.053	50	0.829	1.214
6	0.570	1.905	55	0.836	1.202
7	0.595	1.797	60	0.843	1.192
8	0.616	1.719	65	0.849	1.184
9	0.634	1.657	70	0.854	1.176
10	0.649	1.608	75	0.858	1.169
11	0.663	1.565	80	0.863	1.163
12	0.675	1.533	85	0.867	1.158
13	0.686	1.508	90	0.870	1.153
14	0.695	1.483	100	0.877	1.144
15	0.705	1.456	150	0.899	1.115
16	0.713	1.438	200	0.912	1.098
17	0.720	1.420	250	0.921	1.087
18	0.727	1.406	300	0.927	1.079
19	0.733	1.389	350	0.933	1.073
20	0.740	1.377	400	0.937	1.068

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3. *Reliability Stress Analysis for Electronic Equipment,* TR-59-416-1, Radio Corporation of America, Camden, N. J.

Measuring Elements and Sensors

M. E. Stickney

1. INTRODUCTION

A *measuring element* is the first element or group of elements which responds quantitatively to the variable in question to produce a signal suitable for transmission to intermediate, indicating, or control devices.

The total number of types of measuring elements available is very large. A number of publications (Refs. 1–43) have outlined the field of measuring elements and listed available instruments.

The measuring elements selected for discussion in this chapter are those that have electrical output and respond to the physical variables most often encountered in systems design. For other variables it is recommended that the references cited be consulted. Chapter 7, Instrumentation Systems, discusses pneumatic components in detail.

2. SYSTEM REQUIREMENTS

Analysis of objectives must be carried out before choosing system components. Requirements for measuring elements should be considered in terms of the noise levels to exist for the variable to be measured, the accuracy and reproducibility of subsequent demodulating and processing equipment, the environment in which it is to be used, and other restrictive conditions.

Check Lists. System analysis and preparation of specifications of measuring elements should include attention to the factors shown in the check lists of Tables 1, 2, and 3.

Special requirements associated with different types of measuring elements will be included with the discussion of those elements. It is important to evaluate transducers and their transmission lines separately with respect to environmental factors.

Accuracy *vs.* Reproducibility. Accuracy is frequently overspecified. Unless accuracy is required for accounting purposes or maintaining material balances, it is less restrictive to specify in terms of reproducibility. High accuracy requirements lead to extra problems in calibration and maintenance of standards and to higher costs. For purposes of process control, high reproducibility (repeatibility) is frequently the primary requirement as long as nonlinearity is not so serious as to lead to serious variations in loop gain. In some variables the measuring elements that are most reproducible have fundamental nonlinearities that are expensive to correct.

3. TRANSDUCER CHARACTERISTICS

A transducer (measuring element) in a data or control system must produce output energy suitable for transmission that is some reproducible function of a given variable. Transducer characteristics determine the limitations of a control or information processing system.

Static *vs.* Dynamic Measurement. There is a fundamental distinction in terms of transducer characteristics between static and dynamic measurements. Transducers for the measurement of static or slowly varying functions require externally applied excitation power. Where dynamic measurement only is required, power from the measured variable can frequently be converted to output signal through the use of self-generating transducer systems (electromagnetic, piezoelectric, etc.)

Errors. The output of a transducer is the sum of response to the desired variable plus response to the undesired variables or *error sources.* Error sources may be either random or nonrandom. The distinction is important since it is possible to compensate for nonrandom errors. Random errors can be reduced with respect to signal level only at the expense of time

TABLE 1. CHECK LIST OF OPERATIONAL FACTORS

Input-Output
- Variable measured
- Form of information presentation (ac, dc, pneumatic, digital, etc.)
- Range of values encountered

Transfer Characteristics
- Accuracy required (*vs.* reproducibility)
- Standardizing and calibration requirements
- Reproducibility required
- Response time required
- Dynamic range
- Resonant frequency
- Signal-to-noise ratio
- Linearity

Error Sources
- Temperature coefficient of zero
- Temperature coefficient of scale factor
- Response to acceleration

Excitation
- Power requirements
- Power frequency
- Power regulation

Installation and Maintenance
- Frequency of calibration and zero adjustment to obtain necessary accuracy
- Power line regulation requirements
- Maximum and minimum operating temperatures
- Heat dissipation
- Heating or refrigeration requirements
- Warmup time
- Attitude(s) while operating
- Materials of construction
- Lubrication requirements
- Transmission line requirements
- Environmental conditions (see Table 2)

TABLE 2. CHECK LIST OF ENVIRONMENTAL FACTORS THAT SHOULD BE CONSIDERED FOR NEW OR UNUSUAL APPLICATION PROBLEMS

Thermal
- Temperature range (for transducers)
- Temperature range (for transmission lines)
- Rate of temperature change while operating
- Heat sources (or absorbers) near sensor (which could lead to temperature gradients across sensor)

Atmospheric
- Barometric pressure ranges
- Rate of environmental pressure change while operating
- Humidity
- Corrosive liquids and gases
- Explosive atmospheres
- Airborne abrasive dust
- Airborne metallic dust

Mechanical
- Vibration (transducers)
- Vibration (transmission lines)
- Acceleration
- Radiation: alpha, beta, gamma, neutron flux

Magnetic fields
- Static
- Alternating, power supply frequency (for transducers)
- Alternating, power supply frequency (for transmission lines)
- Alternating, radio frequencies (for transducers)
- Alternating, radio frequencies (for transmission lines)

TABLE 3. CHECK LIST OF NONOPERATIONAL FACTORS

Weight	Storage conditions
Dimensions	Spare parts stock
Reliability requirements	Availability of replacements
Maintenance requirements	Delivery time
Personnel training for maintenance	Method of shipment
Personnel training for operation	Design life (rate of obsolescence)
Installation requirements	Price
Storage life before use	Availability

(bandwidth). A useful generalization in this respect is that the signal, S, to random error, N, ratio (signal-to-noise ratio) is proportional to the square root of the time period, P, over which a measurement is averaged. That is, $S/N = \sqrt{P}$.

Nonrandom Errors. Nonrandom errors result from response of the transducer system to environmental variables such as ambient temperature, supply voltage changes, etc., or from a change of characteristics of some component or components with time or use, e.g., the slow drift of bias voltage of a vacuum tube.

Random Errors. Random errors come from such sources as Johnson noise due to current flow in resistors, Shott noise in vacuum tubes, acoustic or thermal agitation noise in sensitive mechanical elements, response of microphonic elements to seismic shocks, etc.

Nonrandom Error Sources

Environmental Temperature. Response of transducers to ambient temperature change is probably the most common source of error in measurement systems. Typical examples of temperature effects which affect transducer accuracy are temperature coefficient of resistance (especially copper in transmission and excitation voltage supply lines), dielectric constant, expansion, viscosity, index of refraction, thermoelectric potentials, and change of transmittance of color filters with temperature. Two cures for the effects of temperature variation are compensation (mechanical or electrical) and thermostatting.

The upper and lower operational temperature limits for transducers have restricted their use in several fields. There is much important development work in process to eliminate temperature limitations. For this reason no product should be eliminated from consideration without first consulting the manufacturer.

Supply Voltage. Since the primary power for most electrically operated equipment comes from the regular a-c mains, line voltage variation is a common source of error. Typical examples of line voltage variation effects are change of amplifier gain, reference voltage change, light source intensity variation, etc.

Amplifiers are usually designed with negative feedback in order to make them independent of supply voltage variations. Where reference voltage stability is of extreme importance, batteries or standard cells are frequently provided.

The problem of line voltage variation is often avoided through the use of voltage-regulating transformers or electronic voltage regulation. Some instruments are affected by the harmonics introduced by the more commonly used voltage-regulating transformers. Models are available which have low harmonic content in the regulated output.

Vibration. Vibration is important since many components and transducers are microphonic. Special problems are sometimes introduced when there are 60-cycle vibration components, not an uncommon condition when the source of vibration is rotating machinery. Vibration problems are usually countered through the use of shock mounting or special filter networks to reject unwanted frequencies.

Acceleration. This error source is encountered primarily in aircraft and missile work. Minimizing response to acceleration usually calls for fundamental alteration in the design of the components involved. In some cases compensation is possible.

Magnetic Fields. Alternating magnetic fields lead to error signals from some low output level transducers. The solution may be reorienting with respect to the source of the field, magnetic shielding, filtering unwanted frequencies from the signal, or compensation.

Barometric Pressure. This is not usually a source of error, but it may be important in the aircraft and missile fields. Radiation instrumentation based on the use of ionization chambers may be affected by barometric pressure. Errors from this source are usually eliminated by compensation or by enclosing the affected elements in hermetically sealed chambers.

4. DISPLACEMENT MEASUREMENT

Basic Principles. Displacement is one of the most important primary physical variables. The displacement-to-electric output transducer is the main mechanical-to-electric conversion element in instruments for the measurement of acceleration, force, pressure, liquid level, density, flow, and several forms of temperature-measuring devices. Both *linear* and *angular* displacement will be considered here since they are usually electrically similar and each may be converted to the other by simple mechanical systems. Gyroscopes which are indirect displacement-measuring devices are discussed in Chap. 22.

TABLE 4. TYPICAL DISPLACEMENT-MEASURING ELEMENTS

	Type	Linear	Angular	Linearity (%)	Resolution (%)	Maximum Displacement
1. Resistive	Variable resistance					
Strain gages	Bonded or unbonded	x		0.1–1	0.1–1	500–2000 μin.
Potentiometers	Moving contact	x	x	0.05–1	0.02–1	0.1–250 in. (multiturn)
2. Magnetic	Variable air gaps or coupling					
Inductance	Moving coil or core	x		0.25	0.1	0.1 in.
Induction potentiometer	Rotating coil		x	0.5	0.03	±75°
Resolver	Rotating coil		x	0.1	0.1	360° (unlimited)
Reluctance	Moving magnet	x	x	0.5	0.5	±5°
Microsyn	Moving iron vane		x	0.1–1	0.05–0.25	±5°
3. Capacitance	Moving plates	x	x	2	essentially 0	180° angular indefinite linear
4. Piezoelectric	Deformation	x		5	essentially 0 determined by inherent hysteresis	low
5. Magnetostrictive	Rod and coil	x	x	1	essentially 0	low
6. Oscillations	Frequency modulation	x		1	essentially 0	10^{-3} in./in. length
Vibrating wire	Variable frequency	x				
7. Digital	Coded plates	x	x	10^{-4} in.	10^{-4} in.	360° angular indefinite linear

Displacement-to-electric transducers are based on the following principles of operation (see also Table 4):

1. Resistance change by stretching or compressing fine resistance wire or thin film conductors (bonded and unbonded strain gages) (Refs. 7, 8).
2. Magnetic effects in inductive devices by changing air gaps or coupling (differential transformers, variable reluctance pickups) (Refs. 9 to 11).
3. Capacitance variation by altering spacing between plates of capacitors (Ref. 5).
4. Phase shift by rotating armature of selsyn transmitter (Ref. 2).
5. Frequency variation by changing length of vibrating wire (Ref. 12).
6. Digital output by use of angular or linear switch code plates or geared switch trains (Refs. 13 and 14).
7. Potentiometer (Ref. 16).
8. Potentials generated from dynamic displacement by deforming piezoelectric materials such as quartz or barium titanate (Ref. 15).
9. Current generation by the relative motion of conducting coils in a magnetic field (Ref. 5).
10. Current generation in pickup coils by the deformation of magnetostrictive materials (Ref. 15).

Input Characteristics

Input Variable Requirements. Output from displacement transducers is a function of the relative motion between a fixed reference system (usually the body of the transducer) and a moving armature or contact. Full scale motion may range all the way from 100 μin. in the case of some vibrating wire transducers to many feet for some potentiometer, differential transformer, and variable reluctance transducers.

Mechanical Coupling to Transducer. The care which must be exercised in design of the coupling between the device whose position is to be measured and the displacement transducer depends on the accuracy and resolution required. One percent may usually be obtained with ease, 0.1% may be obtained through careful design, but when better than 0.1% is required rigorous system analysis is in order. *Example.* Many materials that would be used in the coupling to a displacement transducer change their length 0.01% for a change in temperature of only 6°C.

For high accuracy and reproducibility couplings should be as simple and short as possible, whether for linear or angular displacement. Where long displacement is involved, care is required in the alignment of the driving system with the driven element in the transducer. Tolerances required should be obtained from the manufacturer.

Input Force. The force required to drive displacement transducers is usually very small. Depending on the exact configuration the force to drive unbonded strain gage type instruments is from 1.5 to 80 oz full scale. Linear differential transformer (Ref. 10) and variable reluctance types (Ref. 9) require a few hundredths of a gram displacement to full scale. Potentiometers (Ref. 16) usually used as displacement transducers require from 0.001 to 3 in. oz starting torque, depending on the design selected. Running torques are substantially less. Forces required for linear types of potentiometers are generally comparable to angular models.

Transfer Characteristics

Resolution. With the exception of potentiometers and direct digital output types, the resolution of linear transducers is often determined by accessory equipment such as power supplies, amplifiers, and demodulators. Resolution of 0.01% or better can frequently be achieved by careful attention to detail.

The resolution of potentiometer type transducers is determined by the number of turns in the resistance windings. Depending on the application, the number of turns may range from less than 100 to 135,000. High resolution is always associated with high resistance (to 2½ megohms). There are potentiometers available in which the winding is continuous and the resolution is therefore not limited by the number of small turns of resistance wire.

Direct digital output transducers of the angular switch code plate style are available with a resolution of 1/2000. Optical code plates are available with resolution up to 1/16,000.

Distortion and Phase Shift Variation (AC). The following remarks refer only to the transducer types involved and do not include any phase shift problems which might result from the characteristics of the transmission lines.

Unbonded Resistance Strain Gage. Since the distributed capacity between the gage windings and the case is relatively small, serious phase shift does not occur below frequencies of 14 kc. Typical strain bridge resistance is 350 ohms.

Bonded Resistance Strain Gage. Since the strain bridge is bonded directly to a deflecting element, the distributed capacity is relatively high and phase shift will occur at lower frequencies. The manufacturer should be consulted when high-frequency excitation is to be used.

Differential Transformer. In this case phase shift is not a problem but harmonics are produced. The harmonics can be ignored where synchronous demodulation is to be used, but they lead to some zero offset where asynchronous demodulation is used. The inductance of a typical differential transformer is 12 mh in the primary coil and 5.2 mh in the secondary coil with a resistance of 22 ohms in each. Resonance occurs at approximately 10 kc.

Potentiometer. Depending on the resistance range, appreciable phase shift will become apparent in potentiometer type transducers from about 150 to 10,000 cps (Refs. 17 and 18).

Bandwidth. The response of transducers which require externally supplied power is usually limited by the frequency of excitation power except where d-c excitation is used. A limitation exists for the unbonded strain gage types in the resonant frequency of the strain wires. When signals are applied approaching this frequency, the wires may oscillate so violently as to break. Depending on the exact design, the resonant frequencies are usually in the range of 2000 to 10,000 cps.

Selfgenerating transducers (Ref. 15) now available have frequency response starting in the range of 1 to 10 cps and extending to the range of 10 to 120 kc.

Output Level. Output from externally excited transducers is generally quoted in terms of millivolt output per volt of excitation with maximum excitation voltage stated separately. For the strain gage transducers output is typically 2 to 3 mv/volt with maximum excitation ranging from 6 to 40 volts. A new type of unbonded strain gage transducer ("zero gage length") has output level up to 20 mv/volt.

Output of the magnetic types of displacement transducers varies from 37.5 mv/volt full scale output at 60 cycles excitation for a differential transformer to 0.5 volt/volt full scale for a variable reluctance type. For most types of magnetic transducers output level per volt of excitation increases with increasing frequency.

Drift. Drift factors may be separated into zero drift and sensitivity drift. Assuming constant environmental conditions, sensitivity drifts are usually negligible, i.e., on the order of or less than 0.01% of full scale. Zero drifts for systems which include displacement transducers will usually be found to be due to the method of mounting or to the system to which the transducer is attached. Strain gage type transducers will exhibit a permanent zero shift when overstressed.

Hysteresis. As in the case of drift, the appearance of hysteresis in a displacement measurement system will usually be due to factors external to the transducer. Since the strain gage types have stressed elements, they may exhibit some hysteresis. There is little information available, but indications are that if transducer elements are not overstressed, hysteresis will probably be less than 0.05% of deflection.

Mixing or Summing. Strain gage (Ref. 19), magnetic (Ref. 10), and potentiometer (Refs. 20, 21) type transducers may be used in mixing and summing circuits for analog computation.

Power Supply Requirements

Degree of Regulation. Whenever the output of the transducer is measured in units other than ratio to the power supply voltage, the degree of regulation of the excitation power source must be equal to or better than the precision of the measurement expected from the transducer. Regulated power supplies should be designed so that the initial surge that occurs when the transducer is first turned on does not damage it.

A-C Power Supplies. Where voltage-regulating transformers are used as power sources for magnetic type transducers, they should be selected to produce low harmonic content in the output. High harmonic content in the exciting voltage supply may lead to nonlinearities and zero offsets in the output of the transducers.

Total Power. Total rated excitation power requirements for most externally excited displacement transducers, except the variable capacitance types, is in the region of 0.3 to 0.5 watts.

Environmental Considerations

Factors Affecting Accuracy. Where accuracies of 1% or less are satisfactory, environmental variables do not usually present very serious problems. Where accuracies of the order of 0.1% are desired, environmental factors introduce serious problems, especially where long term stability is required. Accuracy or reproducibility in the region of 0.01 to 0.05% over even relatively short time periods requires close attention to compensation or control for environmental variables. An extensive list of environmental factors which may be important, depending on the application or accuracy and reproducibility required, is shown in Table 2 in Sect. 2.

Temperature. The temperature range within which displacement transducers can be operated continuously without permanent damage is typically from about −65°F to +250–+300°F. Special transducers are now becoming available which extend the upper temperature limit for continuous operation to 500°F.

The temperature coefficients of zero shift and sensitivity for uncompensated transducers are typically in the region of 0.04% of full scale per degree Fahrenheit. In the extreme these temperature coefficients may go as high as 0.08 per degree Fahrenheit. Temperature coefficients for compensated transducers are typically 0.01 or 0.02% of full scale per degree Fahrenheit for both zero shift and sensitivity (Ref. 10 and 22). Even with temperature-compensated transducers it is apparent that where stable operation at accuracies and reproducibilities of the order of 0.1% are desired, it is necessary to thermostat the transducers.

Where transducers are being used for the measurement of very small displacements, i.e., on the order of a few thousandths of an inch, care should be taken to see that no heat sources or cold heat sinks are near enough to the transducer to lead to large thermal gradients across the transducer.

Magnetic Fields. *Static Fields.* Strong static magnetic fields in the vicinity of some magnetic type transducers can lead to the generation of harmonics in the output signal which will affect linearity and zero setting.

Alternating Fields. Alternating magnetic fields at the excitation frequency can lead to out-of-phase components in the output signal which can be eliminated only by using synchronous demodulation techniques. This problem can frequently be avoided for strain gage transducers by using magnetic shielding. However, in the case of the electromagnetic transducers, the presence of adjacent magnetic material or conductors may seriously affect the linearity and cause zero shifts.

The presence of alternating magnetic field at other than excitation frequency is serious if subsequent demodulation equipment is asynchronous. The presence of an interfering frequency can sometimes lead to overloading of amplifiers in auxiliary equipment and to nonlinearities in the demodulated signal whether the output of the transducer at the excitation frequencies is linear or not.

Adjacent Materials. Where the electromagnetic types of transducers at high precisions and reproducibilities are to be used, it may be necessary to avoid the use of highly permeable magnetic materials in the immediate environment. For some transducers the presence of strongly magnetic materials within 3 in. of the transducer may affect the linearity of the output. For example, devices using linear differential transformers as the transduction element are usually fabricated from nonmagnetic stainless steels. The presence of excellent conductors in the immediate environment may sometimes also lead to nonlinearities.

Size, Weight, and Cost. Size, weight, and cost factors are determined primarily by the range covered, accuracy required, actuation power required, service life and reliability required, and environmental conditions which must be met. A premium must usually be paid for extreme miniaturization, accuracy of better than 0.5 to 1% of full scale, and extreme ruggedness of construction.

Prices for linear transducers for full scale ranges of a few thousandths of an inch or less or for angular transducers for full scale ranges of less than a degree start in the neighborhood of $125 to $150. Transducer sizes for these ranges vary from a diameter of approximately ½ in. and a length of 1 to 1¼ in. (weight approximately ½ oz) to diameters of ½ to 2 in. and a length of 3 to 4 in.

The greatest range of choice as far as price is concerned is in the linear transducer range of approximately $\frac{1}{10}$ in. to 3–4 in. and the angular range of about 200 to 3600°. If specifications are not stringent, prices may start at a very few dollars and go to several hundred dollars when difficult combinations of specifications are involved. Similarly in these ranges there is a great deal of variety of size and weight. The smallest start at approximately $\frac{3}{4}$ in. in all dimensions and go up to diameters and lengths of 3 to 4 in. Where no gear trains are used, the length of linear transducers is always equal to or greater than the full scale range involved.

5. PRESSURE AND FORCE MEASUREMENT

Basic Principles and Types of Pressure-Measuring Elements. A pressure-to-electric transducer consists of an element which deforms when there is a pressure difference across it and a displacement-to-electric transducer which converts the motion due to deformation to an electric signal. Virtually all forms of displacement-measuring elements discussed in the preceding section are used in pressure transducers. Types of pressure-measuring elements are listed in Table 5.

TABLE 5. PRESSURE-MEASURING ELEMENTS

	Type	Liquid	Gas	Comments	Range
1. Deformation					
Bellows, diaphragms, capsules, Bourdon tubes, etc.	Displacement	x	x	Displacement converted to electrical signals by elements of Table 4.	10:1 up to 100,000 psi
Pressure sensitive wire	Resistive	x		Useful for pressures beyond the range of Bourdon tube.	
Moving contact	Resistive		x	Variation in contact level of wires dipped in Hg.	1000:1 with servo readout
2. Thermal Conductivity					
Lamps	Resistance		x	Depend on rate of conduction in surrounding gas. Useful at low pressures.	
Pirani gage	Resistance		x		
Thermopile	Temperature		x		
3. Ionization					
Hot filament	Current		x	High-vacuum devices.	
Cold cathode	Current		x		
Alpha source	Current		x		

The pressure deformable element takes many forms (Ref. 5) such as flat diaphragms, spherical diaphragms, catenary diaphragms, single and multiple convolution bellows, pressure capsules (a combination of two corrugated diaphragms to form a completely enclosed chamber), Bourdon tubes, twisted Bourdon tubes, cylinders, and others. The range of displacement obtained from these elements varies from 100 μin. for a small flat diaphragm to 0.3 or 0.4 in. for a stack of capsules.

Pressure transducers will measure static and dynamic pressures or dynamic pressures only, depending on the nature of the displacement-sensing device. A dynamic pressure transducer based on principles not previously described (Ref. 23) is the electrokinetic transducer. This consists of a ceramic plate in the center of a cavity filled with a polar liquid which is enclosed on each end by a flexible metal diaphragm. A pressure difference across the cell causes fluid to flow through the ceramic disk, thereby generating a potential.

In the force balance type of transducer (Refs. 5 and 24) the differential pressure forces generated across the pressure-sensing element are balanced by electromagnetically generated forces. A displacement-sensing device, usually of the differential transformer type, is used to sense motion of the pressure element from a zero pressure position. The signal is amplified, converted to dc, and fed back into the windings of the electromagnet to produce the balancing forces. Deflection of the pressure element is thus determined by the total gain around the feedback loop. This system tends to eliminate nonlinearities which might otherwise be caused by the pressure-sensing elements.

Input Characteristics

Pressure Ranges. Pressures are specified as gage (referred to ambient atmospheric pressure), absolute (referred to vacuum), or differential (two pressure connections are provided so that the reference pressure may be at the customer's choice). Full scale pressure ranges are available from 0.01 psid to 100,000 psi. Several transducer manufacturers can provide instruments to measure pressures above and below these ranges on special order.

Pressure Media. The media to which a particular pressure transducer may be exposed are determined by the materials of construction. The materials that are best from the standpoint of corrosion resistance for a particular application are not necessarily best for high transducer performance. The characteristic most likely to be sacrificed to obtain corrosion resistance is hysteresis.

Transducers may be obtained to withstand exposure to most pressure media. However, there are some combinations of media and temperature conditions for which it is not practical to build transducers. In these cases it is frequently practical to isolate the transducer from the pressure medium through oil or other fluid-filled lines terminated by a suitable pressure-transmitting diaphragm. This method is frequently used to protect the reference side of differential pressure transducers from direct contact with the pressure medium. A properly designed isolation system may cause little or no loss of transducer performance.

Pressure Overload Factors. The first pressure limitation which should be mentioned is that of pressure overload without permanent zero displacement. Most transducers are designed with some safety factor so that the gage may be exposed to pressures above the nominal full scale value without requiring readjustment of zero. A typical overload figure in this respect is 150%. This pressure limitation cannot be exceeded even on a transient basis without requiring zero readjustment (Ref. 25).

Most manufacturers provide overload stops which limit the deflection of the pressure element, therefore allowing it to be subjected to higher pressures before suffering irreparable effects. Where an application requires that a transducer be subjected to pressures of several times the nominal rating, the requirements should be discussed with the manufacturers. It is possible when necessary to produce transducers with safety factors of ten or more in this respect, usually at some sacrifice in other phases of performance.

Of interest where hazardous materials or extreme pressures are involved is the bursting pressure of the pressure-sensing element and the outer case. With hazardous materials for which there may be pressure transients of sufficient magnitude to rupture the pressure-sensing element, the safest method of pressure measurement is to use an absolute type gage with an outer case that can be completely sealed and designed for the extreme pressure limit. With differential pressure measurements it is necessary that the transducer case be capable of withstanding the full pressure of the system being measured. In differential pressure measurements where the reference pressure is from corrosive media, the displacement-measuring element must be protected from the pressure medium by a pressure-transmitting seal.

For dynamic pressure transducers utilizing piezoelectric ceramics or crystals there is usually no permanent effect from overload until the element fractures.

Transfer Characteristics

All the comments made regarding transfer characteristics of displacement transducers apply to pressure transducers since pressure transducers always employ displacement-measuring elements for transduction. Resolution, saturation, distortion, and phase shift characteristics and output level and stability are all determined primarily by the displacement-measuring element.

Bandwidth. The frequency response of a pressure transducer is usually determined by the characteristics of the pressure element (except as limited by the excitation frequency for the displacement transducer; see Sect. 4). Since the pressure-sensing element is more compliant for the measurement of low pressures and stiff for the measurement of high pressures, the maximum frequency response will usually be related to full scale pressure for a given type of pressure transducer construction. At low pressures, frequency response is typically a few hundred cycles to 2 or 3 kc, and in the very high-pressure ranges it may extend to 20 kc and higher.

Dynamic pressure transducers usually have low-frequency response starting in the region of ½ cps to 4 or 5 cps and extending to frequencies from 10 to 100 kc.

Hysteresis. The hysteresis of a pressure transducer is usually determined by the choice of materials used in the pressure-sensing element, the treatment of the material received in the process of manufacture, and the stress levels to which it is used. An exception to this is the case in which the transduction element involves a sliding contact. Hysteresis figures range all the way from 0.02% of maximum deflection and below to as much as 1%. When the nature of the measurement permits, the effects of hysteresis may be minimized by approaching the readings from the same direction. For example, where the transducer may be returned to zero pressure between each reading, reproducibility on the order of 0.01% of full scale may be achieved in the presence of hysteresis of 0.15%.

Power Supply Requirements

Power supply requirements of pressure transducers are determined exclusively by the characteristics of the displacement-measuring element used, for which see Sect. 4.

Environmental Considerations

General. Environmental considerations for pressure transducers must include all the factors applicable to the displacement transducer utilized. A factor of importance in utilizing pressure transducers in which ambient pressure is the reference is the possibility of damaging the transducer element where there are corrosive gases in the atmosphere. In such environments, it may be necessary to isolate the reference side of the transducer by the means discussed under Pressure Media.

Acceleration. Most pressure transducers exhibit some response to acceleration. Acceleration response is greater in the low ranges and is usually negligible in the high-pressure ranges. Acceleration response may vary from several percent per gravity unit in very sensitive differential pressure transducers to 0.01% g or less in the high-pressure ranges. Pressure transducers involving linear transduction elements show response to linear acceleration with little or no response to angular acceleration, whereas pressure transducers involving the use of angular displacement transduction elements respond oppositely.

Temperature. Temperature affects the behavior of the pressure-sensing element as well as that of the transduction element. Since for most materials Young's modulus is a function of temperature, sensitivity is affected. Differential temperature coefficients between the case and the connection from the transducer element to the pressure sensitive element also contribute to overall pressure transducer temperature coefficients. Uncompensated temperature coefficients are typically 0.04% per degree Fahrenheit for both zero and sensitivity. Most manufacturers can introduce compensation to reduce the figures to 0.01% per degree Fahrenheit.

Size, Weight, and Cost

Size and Weight. Pressure transducers of an inch or less in all dimensions, and a weight of approximately an ounce, are commercially available in pressure ranges from 5 to 5000 psi. The miniature transducers are usually flush diaphragm types. Differential pressure transducers of roughly the same dimensions may be obtained in ranges from ± 1.5 to 150 psid. Such transducers are generally mounted directly in the wall of the vessel in which pressure measurement is to be made. At the other extreme are the transducers in which the case must withstand very high pressures for either direct high-pressure measurement or differential measurement at extreme pressures. Such transducers may range up to approximately 6 in. in all dimensions and weigh up to 12 to 15 lb for operation in line pressures up to 7500 psi. Transducers for the measurement of small differential pressures (1 psi and below) are usually large in diameter because of the necessity of using large diameter diaphragms or bellows in order to get high sensitivity.

Cost. The prices for pressure transducers in the moderate ranges and of moderate performance usually start somewhat above $100 each. For extreme pressure applications, high-pressure sensitivity, or other unusual performance specifications, premium prices range to $1000 and more per transducer, depending on the combination of specifications involved.

Force Measurement

Force-to-electric transducers consist of a combination of a displacement-to-electric transducer and an element which deforms owing to the application of force (Ref. 5). The force sensitive element is typically a proving ring, a metal bar, or some other form of spring. Force transducers are mechanically analogous to pressure transducers but are different in form. The discussions of displacement transducers and pressure transducers combined cover most of the aspects of the force transducers.

Force Ranges. Force ranges are available from a fraction of an ounce full scale to 100,000 lb full scale.

Overall Size, Weight, and Cost. Remarks regarding the cost of pressure transducers also apply to force transducers.

In the low-force ranges force transducers may have a variety of dimensions but will usually weigh from $1\frac{1}{2}$ to 5 oz. Dimensions and weight increase with range and may go up to 20×9 in. with a weight of about 5 lb.

6. SPEED MEASUREMENT

Speed is measured both for speed control and to provide stability control in a position control system. The measurement of speed in a speed control system requires precise measurement techniques. Providing stability control generally places less stringent requirements on the measuring element, although drift is still a significant problem.

Several commonly employed speed-measuring techniques are based on (*a*) induced voltage proportional to angular speed in a generator, (*b*) torque proportional to precession rate (angular speed) in a gyro, (*c*) Doppler shift proportional to linear speed, (*d*) centrifugal force proportional to the square of angular speed in mechanical governors, (*e*) number of events per unit time proportional to speed. Other types may be known to the reader, but these will serve to illustrate the properties important in speed measurement.

Tachometers. Devices that generate voltages proportional to angular speed are usually called tachometers. Direct-current tachometers consist of small, permanent magnet field generators. Their best accuracy is typically $\frac{1}{4}\%$ of full scale output. Direct-current tachometers can deliver considerable power (low impedance load) and are essentially free from drift. However, they operate poorly at low speeds owing to the commutation noise. Brush life limits the life and reliability of these units.

Alternating-current tachometers are essentially two-phase motors with special design features. In applications requiring high output and low internal impedance, squirrel cage rotor tachometers are indicated. However, variation in output with shaft angle, called "cogging," can be a problem. Drag cup rotors eliminate this problem at the sacrifice of output power. The precision of a-c tachometers can be 0.1% of full scale with reproducibility near high outputs even better. Residual output at zero speed can introduce drift, and quadrature output (90° out of phase with excitation) can be a serious problem in saturating following amplifiers and in reducing effective loop gain.

Gyros. Gyros are covered in detail in Chapter 22.

Doppler Shift. Devices employing Doppler shift are, in a sense, counting events per unit time. A separate category is employed to emphasize their typical use when the desired frame of reference is physically a long way from the system. Doppler speed-measuring devices are complex systems in their own right. For example, Doppler navigators for aircraft having an accuracy of 0.1 to 0.2% of full scale include in excess of 50 lb of complex electronic equipment.

Centrifugal Governor. Mechanical governors are included here because they are very common in industrial power sources. Since torque varies as the square of speed, and the sensing system deflects under load, linear output is usually not obtained. However, satisfactory performance in holding a desired speed within 5% or so is obtained with sufficient output power to actuate the throttle directly.

Counter-Tachometer. Devices that convert angular (or linear) motion into events (pulses, contact closures, etc.) and then produce an output proportional to the number of events per unit time are called counter-tachometers. Examples of this type are the capacitor tachometer, the speedometer or drag torque tachometer, and the frequency tachometer. The condenser tachometer in its simplest form transfers charge from a power source to a load once per revolution. The average load current is, then, a measure of speed. Refined examples of this type are capable of 0.1 to 0.2% precision but require a high-precision load, usually of high impedance.

The speedometer tachometer transmits a pulse of torque to a high-inertia spring-restrained system to produce a deflection proportional to average speed. Precision of about 1% is practical when the output is changed to an electrical signal by a low torque position measuring element.

One example of the frequency tachometer employs a synchronous generator together with a limiter-discriminator to produce an output proportional to speed variation from the center frequency of the discriminator. Precision at this center speed can be 0.01% with error curve linearity of 1%.

7. ACCELERATION MEASUREMENT

An electric output accelerometer (Refs. 5, 15, 26, 27, 28, and 29) consists of a combination of a displacement-to-electric transducer, a spring, a mass, and usually a damping system. Since accelerometers include linear transduction and force-deflecting elements, the discussion of linear transducers is applicable, and many of the characteristics of pressure transducer systems are similar. The differences in behavior occasioned by the mass and the damping system (Refs. 29 and 30) will be discussed later.

Natural Frequency. The natural frequency of an accelerometer in the region of ½ to 1 g full scale is normally 20 to 40 cps. Natural frequency increases with full scale range and will usually be above 1000 cycles at 1000 g. For angular accelerometers the natural frequency is typically in the region of 4 cps for a full scale range of 1.5 radians/sec^2 and goes to 150 to 225 cps at 3000 radians/sec^2. Damping is typically from 0.5 to 0.8 of critical.

Size, Weight, and Price. Accelerometer transducer sizes range from approximately 1 x 1 x 2½ in. up to approximately 3 to 4 in. maximum dimensions, and weights range from about 4 oz to 2–3 lb. Prices run in the region of $200 to $400 for typical instruments.

Environmental Considerations. *Temperature.* Since acceleration measurements are usually dynamic, temperature coefficient of zero shift is not a problem. However, where static acceleration is to be measured, special temperature compensation for zero shift may be required.

Acceleration Crosstalk. Response to transverse acceleration ranges from 0.003 g/g to 0.02 g/g, depending on the type of design and range of the instrument.

8. FLOW MEASUREMENT

Instrumentation for the measurement of flow is usually complex and expensive as compared with equipment for the measurement of most other primary variables. Flow meters employ many different combinations of physical principles as shown in Table 6 (Refs. 1–5, 32–35). There are

TABLE 6. FLOW MEASUREMENT

	Measurement	Liquid	Gas	Primary Output	Typical Accuracies (%)
Fixed orifice	Differential pressure	x	x	Displacement[a]	1 to 5
Variable orifice	Area	x	x	Displacement	2
Rotameters					
Piston type					
Mass flow	Angular momentum, torque	x		Displacement	0.25
Positive displacement[b]	Volume			Displacement	1.5
Pumps, meters		x	x		
Magnetic flowmeter	Magnetic field			Voltage	1
Drag disk flowmeter	Force	x		Displacement	
Turbine	Rotation	x	x	Displacement	0.25
Mass flowmeters					
Angular momentum	Rotation	x		Displacement	0.25
Gyroscopic	Torque	x		Displacement	
Boundary layer	Differential temperature	x		Voltage	
Rate-density acoustic	Sound velocity	x		Frequency	

[a] Linear or angular displacement.
[b] Volumetric displacement.

many excellent publications which summarize in some detail the various types of flowmeters available and their applications.

Differential Pressure Methods of Flow Measurement

One of the oldest and most commonly used methods of fluid flow is based on the basic physical fact that there is always a pressure drop associated with flow through a tube or an orifice (Refs. 3, 4, and 32). In order to employ this principle, the pressures at two points along a tube (either straight or of special design) or the pressures on either side of an orifice are transmitted through tubing to a differential pressure transducer.

Characteristics of the pressure transducers have been discussed previously in Sect. 5, Pressure and Force Measurement. The following discussion will refer to the characteristics of flow-measuring systems as determined by the primary flow-sensing element.

Error Sources. The diameter of the orifice or tube, density, viscosity, and the surface condition of the orifice or tube all affect the performance of differential pressure flow-measuring devices. All factors affecting these variables will cause variation of system performance. Since a change in pressure of compressible gases changes density, pressure can be an interfering variable in the measurement of gas flow by these techniques. The density and viscosity of almost all fluids are strongly temperature dependent, which makes temperature probably the most important interfering variable. When viscous forces are predominant in determining flow rate (viscous flow), the square root relationship between flow and differential pressure is no longer valid.

Accuracy. The accuracy of differential pressure flow-measuring devices under normal conditions of use is typically in the neighborhood of 5%. However, when a system is well maintained, when corrections are made for temperature effects on both the primary element and the differential pressure transducer, when fluid density corrections are made and a careful calibration procedure has been followed, accuracies of 1% can be achieved.

Orifice Plates. Because of the relative ease of installation and low cost, orifice plates are frequently preferred over other forms of differential pressure flow-measuring devices. The use of orifice plates is generally limited to fluids free of solids and to gases free of entrained liquids and solids. Where reproducibility of measurement over long time periods is required, it is necessary to provide adequate maintenance in order to keep the orifice plates clean. Building up of deposits on the orifice itself or on either side of the plate disturbs the flow pattern and leads to rapid deterioration in accuracy.

Eccentric orifice plates with the orifice located at the bottom of the pipe can be used to measure fluids which carry small amounts of nonabrasive solids and gases with some liquid entrainment since the solids are swept through the orifice by the flowing stream.

Other factors important in determining the performance of the orifice plates (and other types of differential pressure flow-measuring devices) are the exact placement of the taps in the pipe before and after the orifice which will conduct the pressure to the differential pressure transducer, and the length of straight pipe preceding the orifice. Typically it would be desirable to have 28 pipe diameters of straight pipe upstream from the orifice for optimum performance (Ref. 33). If it is not possible to provide a satisfactory length of straight pipe ahead of the orifice, the problem can sometimes be solved by introducing flow-straightening vanes in the pipe immediately ahead of the orifice plate. This reduces the cost advantage of the orifice plate.

Nozzles and Venturis. Flow nozzles will accommodate about 60% more flow than orifice plates with much smaller permanent pressure losses. Flow nozzles cost about eight to sixteen times more than orifice plates, and installation is more expensive (Ref. 33). However, more entrained solids can be tolerated in the fluid. The venturi tube will also handle more flow than the orifice plate with smaller permanent pressure losses and will accommodate high percentages of nonabrasive entrained solids. Venturi tube cost is about twenty times that of the orifice plate.

Pipe and Fittings. Where for one reason or another it is not possible to use an orifice plate, a nozzle, or a venturi for flow measurement and accuracy requirements are not high, it is frequently possible to use the pressure drop across a length of straight pipe or the pressure drop around an elbow. This is also a good method for flow-metering liquids high in entrained solids. Experimental calibration will be required for the most accuracy.

Other Methods of Flow Measurement

Variable Area Flow Metering. This type of flowmeter is based in principle on determining the annulus area required to maintain a constant pressure drop. The flowmeter of this type used most frequently is based on measuring the position of the float in a vertical tapered tube. Float design has been carried to the point of producing devices virtually insensitive to viscosity changes over a fairly wide range. In order to obtain electric output, optical or magnetic means are used to measure float position.

Drag Disk Flowmeters. This relatively new type of flowmeter is based on the formation of an annulus in the pipe in such a way that the forces on the disk caused by flow can be measured. One commercial form of this device utilizes a strain gage transducer as the output element.

Positive Displacement Flowmeters. A few positive displacement flowmeters (Ref. 4) provide output pulses which can be counted to provide flow rate or total flow information. Such flowmeters are among the most reproducible types available, but their use is restricted to streams free of suspended material.

Turbine Type Flowmeters. A propeller type rotor is suspended in the flowing stream (Refs. 2 and 4). A pickup is provided to produce output pulses which are counted to provide the desired rate information. Meters of this type provide accuracies from ± 2 to $\pm 0.5\%$ over a wide range of flows.

Electromagnetic Flowmeters. Electromagnetic flowmeters (Refs. 2 and 4) are based on the principle that potentials are generated by a conductor moving in a magnetic field. A magnetic field is provided such that the lines of force are at right angles to the direction of flow of the conducting fluid stream. The potentials generated are sensed by electrodes set in the wall of the pipe. This type of meter is independent of viscosity and density effects and is relatively unaffected by solids suspended in the stream. The dynamic range of measurement is unusually large since there are no moving parts.

Mass Flowmeters. Mass flowmeters (Ref. 35) are so designed that the signal produced is directly proportional to mass flow rather than flow rate. In order to obtain mass flow information from the typical flow rate meter, it is necessary to multiply flow rate by stream density. When there are suspended solids or bubbles in the stream, density may change rapidly and is difficult to measure accurately.

Gyroscopic Flowmeters. The flowing stream follows a path which simulates that of the rotor of a gyroscope forced to rotate simultaneously at right angles to the normal axis of rotation (Ref. 35). The torque produced is a measure of mass flow. Accuracy may be as high as $\pm 0.25\%$ of full scale. Output is an a-c voltage. This type of flowmeter is extremely bulky.

Angular Momentum Type Flowmeter. A motor-driven impeller imparts angular momentum to the fluid stream (Refs. 2 and 35). An angular torque proportional to mass flow is produced in a spring-restrained sensing wheel downstream from the driving impeller which opposes the angular momentum of the stream. An angular displacement transducer is used to sense the position of the sensing wheel. This is a relatively compact form of mass flowmeter. It is not well adapted to streams containing substantial amounts of suspended solids.

Boundary Layer Flowmeter. The boundary layer flowmeter operates on the basis of determining the amount of heat that must be supplied by a heater coil wrapped around a tube in order to maintain a constant temperature difference between two resistance thermometers located upstream and downstream from the heater (Ref. 36). No obstructions are introduced into the stream. Changes in the specific heat or viscosity of the stream introduce errors into the reading. If specific heat and viscosity increase and decrease together, errors tend to cancel out. The boundary layer flowmeter must be designed or calibrated to match the characteristics of the stream with which it is to be used. The output device is a wattmeter.

Rate-Density Product Mass Flowmeters. There are several mass flow measurement systems which combine flow rate and density information in a simple computer to produce an output signal proportional to mass flow (Ref. 35).

Acoustic Flowmeter. Acoustic transducers are disposed opposite each other in the walls of a pipe at an angle so that upstream and downstream sound velocity can be measured in order to obtain flow rate (Ref. 34). Another acoustic transducer is used to determine acoustic impedance which is a function of density. The product of flow rate and density is taken in a simple computer to produce an output proportional to mass flow.

Venturi and Turbine Type Mass Flow Systems. Mass flow measurement systems are available which combine venturi and turbine type flowmeters with float type density measurement to produce output directly in mass flow (Ref. 35).

9. LIQUID LEVEL MEASUREMENT

A variety of devices which have electrical signals as outputs are available for measuring liquid level. These methods may be classified as direct or indirect, according to whether they measure directly the position of the liquid surface or some other characteristic of the liquid from which the level may be calculated. Table 7 lists the types of liquid level detectors.

TABLE 7. LIQUID LEVEL DETECTORS

	Output	Accuracies	Remarks
Floats			
Ball, Magnetic	Displacement or force	⅛–¼ in.	
Pressure measurement	Displacement	½–2%	
Oscillator	Frequency change	0.001–0.125 in.	Limited range
Electrode contact	On-off contact	0.005 in./in. of span	
Capacitance	Capacity change	1%	Up to 3% errors for suspended solids or bubbles
Sonic detectors	Transit time (voltage)	0.01 in./ft	
Nuclear radiation	Pulse count	1%	Accuracy function of time constant used

Direct Methods. These techniques provide a measure of the surface position with respect to some reference point in the container. Typical examples are floats, surface contacts, and sonic reflection devices. In these measurements the accuracy is entirely a function of the measurement and is generally independent of other variables such as pressure and temperature.

Indirect Methods. Indirect measurements of liquid level involve measurement of some property of the liquid from which the level can be calculated. Two useful methods are based on measurement of hydrostatic pressure and electrical capacity. From hydrostatic pressure the height of the liquid column may be computed. For capacity measurement an electrode emersed in the liquid forms a condenser with the containing vessel as the other plate and the liquid as the dielectric. Changes in liquid level vary the capacity. Any change in density, composition, or electrical properties may produce errors in the measurements obtained with these devices. Hence other measurements are necessary to provide accurate calibration for these techniques.

10. TEMPERATURE MEASUREMENT

There is a great deal of literature available to help in the selection of systems of temperature measurement for the typical industrial and engineering applications (Refs. 1–6, 37, and 38) as well as for the problems involving extreme or otherwise unusual conditions. The methods to be discussed are those most often used for temperature-to-electric transduction. A more complete list is given in Table 8.

TABLE 8. TEMPERATURE-MEASURING DEVICES

Device	Output	Accuracy	Range of Application[a]
Thermocouples	Voltage	0.1–1%	−200 to 1450°C (−300 to 2650°F) Upper limit determined by properties of materials.
Pyrometers			
Radiation	Voltage from thermopile	See b	200 to 1750°C (400 to 3200°F)
Optical	Voltage from photocell	±2–6°F	750 to 2900°C (1400 to 5200°F)
Resistance thermometers	Resistance change	±1°F	−250 to 900°C (−400 to 1650°F)
Thermistors	Negative resistance change	1–5%	0 to 300°C (32 to 600°F)
Bulb thermometers			
Bourdon tubes	Displacement	See c	Liquid, −200 to 650°C (−300 to 1200°F)
Liquid and gas			Gas, 0 to 800°C (30 to 1500°F)
Bimetal strips	Displacement	1%	−200 to 550°C (−300 to 1000°F)
Noise thermometers	Noise voltage	0.1%	Laboratory instrument

[a] The span of a typical device will cover only part of the range.
[b] Calibration depends critically on determining emissivity of surface of interest. Repeatability good, but calibration sometimes difficult.
[c] See comments under Basic Principles and Types of Pressure Measuring Elements, Sect. 5.

Thermocouples. Thermocouples are unquestionably the most commonly used temperature-to-electric transducers. Their low output (from approximately 10 to 50 mv for the most commonly used thermocouples over their preferred range of use) is offset by their small size and ease of application. The problems involved in the selection and application of thermocouples are covered very extensively in the literature (Refs. 2–4). The electrical problems most frequently encountered in the application of thermocouples are ground loops and a-c pickup. Thermocouple transmission lines are frequently run long distances unshielded. It is not always appreciated that the commonly used millivolt recorders have filtering networks in the input circuitry for the rejection of a-c pickup. When amplifiers are time-shared at the input to data handling systems, it is usually necessary to allow one second per input thermocouple channel in order for the input filter necessary for a-c pickup rejection to reach equilibrium to 0.01%.

Where many thermocouple leads must be run for long distances, it is now frequently the practice to use remotely located temperature-regulated multiple cold junction boxes from which copper leads can be carried back to the central measurement point.

Resistance Thermometers. A commonly used electric output temperature transducer is a resistance of high temperature coefficient (Refs. 2, 3, and 4). The temperature coefficient of most common conductors is in the range from 0.35 to 0.65% per degree centigrade. Alloys are available with higher temperature coefficients. Resistance thermometers (platinum) may be used up to 1100°C.

Resistance thermometers with high sensitivity over small temperature ranges are being made from thermistor materials. Typical thermistor materials have negative temperature coefficients in the region of 4 to 8% per degree centigrade and may be used at temperatures up to 300°C.

Radiation Pyrometers. When conditions are such as to preclude the use of other forms of temperature measurement or when it is necessary to measure the surface temperature of rapidly moving objects, radiation pyrometers may be used. The radiant energy emitted from the surface in question (usually infrared) is optically focused on a thermocouple or bolometer radiation detector (Refs. 2–4). Accuracy is low at low surface temperatures, but there is no upper temperature limit. Calibration is affected by any changes in surface conditions that alter emissivity.

Photoelectric detectors for radiation pyrometry may be used at temperatures above 450°C.

11. NUCLEAR RADIATION MEASUREMENT

Instruments with electric output for the measurement of alpha, beta, and gamma radiation and neutron flux are most frequently based on the ionization of gases (Refs. 1, 39, and 40). Such instruments take many forms, only a few of which will be discussed. The measurement of gamma radiation and neutron flux are the most important in the operation of nuclear reactors (Refs. 40 and 41). Neutron flux is a measure of the power level of the reactor.

Radiation Chambers. Radiation chambers are gas-filled chambers with two electrodes. Usually the voltage applied to the electrodes is sufficient to collect all the available ions, thereby making the output essentially independent of the biasing voltage. Rate of change of biasing voltage must nevertheless be limited, since the impedance of amplifiers used is very high and rapid voltage variations in the chamber supply are coupled to the amplifier input through the distributed capacitance of the chamber. Depending on the design and application, the useful current output from chambers ranges from 10^{-16} to 10^{-3} amp. Response time of ionization chambers is determined by the chamber capacitance and associated distributed capacitance and by the input impedance of the amplifier.

Compensated Ionization Chambers. Measurement of neutron flux in nuclear reactors must usually be made in the presence of large amounts of gamma radiation. Compensated chambers are built in two sections, one of which is lined with boron to sensitize it to neutrons; the other is unlined and therefore sensitive only to gamma radiation (Refs. 40 and 41). The chamber has three electrodes, one of which is common to the two sections of the chamber. Voltages on the other two electrodes are equal but opposite in polarity, so that in the presence of gamma radiation only the net output current is very near zero. Chamber output is current from neutron flux only.

Proportional Counters. Proportional counters are ionization chambers in which one electrode is the usual distributed type and the other is a fine wire (Refs. 39 and 40). Voltage applied is high enough to produce pulses. Different types of radiation can be determined on the basis of pulse height. Proportional counters are sometimes filled with boron trifluoride (BF_3) gas which sensitizes them to neutrons.

Fission Chamber. In the fission chamber a coating is used which is enriched in U^{235}. Neutrons cause fission in the U^{235}, which in turn causes ionization of the surrounding gas (Refs. 40 and 41).

Geiger-Mueller Counter. The Geiger-Mueller tube consists of coaxial cylindrical and fine wire electrodes in an envelope at a gas pressure of a few centimeters of mercury (Refs. 38 and 39). Gas mixtures are used to make the tube self-quenching. Voltages applied are commonly 900 to 1000 volts, although there are some tubes available for operation at 300 volts. Count rate becomes nonlinear with radiation at high levels. Geiger-Mueller tubes are used primarily for measurements at low radiation levels.

Neutron Thermopile. The neutron thermopile consists of 40 thermocouples, alternate junctions of which are coated with boron (Ref. 40). Boron releases heat on being radiated, with neutrons causing the thermopile to generate a voltage proportional to neutron flux. Output is about 10 mv in a neutron flux of 10^{12} neutrons/cm^2/sec. The response time is on the order of seconds, and therefore the neutron thermopile cannot be used directly in reactor control circuits. Output impedance is low so that the shielding and insulation problem is not as severe as for ion chambers.

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Amplifiers

R. O. Maze

1. INTRODUCTION AND DEFINITIONS

This chapter contains a discussion of the various techniques available for amplifying signals for use in control systems. Since frequency conversion is often useful in achieving the required performance, a section on modulators and demodulators is included.

Definitions

Amplifier. An amplifier is a device capable of producing an output signal conforming to an input signal but at a higher power level. This increase in power level is always accomplished by modulating power from an external supply to provide output power. The amplifiers discussed in this chapter will be restricted to those having outputs of the same general form as that of the inputs.

Modulator. Although every amplifier is a modulator in the general meaning of the word, modulator as used here refers specifically to a device which produces an alternating-current output of a given (carrier) frequency corresponding in amplitude and phase with an input signal of a lower frequency.

Demodulator. A demodulator is the inverse of a modulator in that it recovers the signal as represented by the phase and amplitude of the carrier.

Electronic Amplifier. This amplifier of electrical signals employs some principle of operation in which electrons, ions, or other particles are the only moving elements.

Electromechanical Amplifier. This amplifier contains a mechanical element which moves in response to the input signal. The output is produced by modulation of the supply power by virtue of the motion of the mechanical element. For example, a relay is a simple, nonlinear electromechanical amplifier.

Rotary Amplifier. The external power source for this amplifier is a rotating shaft.

Fluid Amplifier. This amplifier has fluid conditions as input and output signals and usually employs fluid under pressure as the power source. Examples include hydraulic and pneumatic devices.

Gain. The gain of an amplifier is the ratio of output power to input power. In a linear amplifier this is a constant. It is common to specify gain either in decibels or simply as a ratio, that is,

$$\text{Gain} = \frac{\text{Power out}}{\text{Power in}}$$

or

$$\text{Gain, decibels} = 10 \log_{10} \frac{\text{Power out}}{\text{Power in}}.$$

Voltage gain. Voltage gain is the ratio of output voltage to input voltage. (Current gain has a similar meaning.) By common usage, particularly in communications circles, the term gain is used to mean the ratio of output to input signal without regard to impedance levels. Such gains are often given in decibels equal to $20 \log_{10} \frac{\text{Voltage out}}{\text{Voltage in}}$. Note that this is normally not the same as the power gain and can therefore cause confusion.

2. GENERAL PROPERTIES

The characteristics of amplifiers may be specified in terms of the performance factors of Table 2, Chap. 12, by the input or output characteristic of the amplifier, or by the characteristics of the amplifier itself. It is common to classify amplifiers according to the following characteristics: (1) the *type of input* which is acceptable, that is, electrical, mechanical, or fluid, and whether the input is a direct representation of the signal function or a modulated carrier, (2) the *type of output,* (3) the *gain,* (4) the *bandwidth,* or (5) the *power levels* involved. In this chapter the primary classification is according to principle of operation, and the other characteristics are dealt with in the discussions in the respective sections.

Comparison of Characteristics

The characteristics of the various types of amplifiers may be compared by reference to Table 1. The values in this table are those generally ac-

TABLE 1. GENERAL CHARACTERISTICS OF AMPLIFIERS

Type	Input Impedance, ohms	Response	Linearity	Cost: Development	Cost: Production
Vacuum tube	10^6 to 10^{12}	Excellent	1–10%	Low	Low
Semiconductor	100 to 5000	Satisfactory	5–20%	Moderate	Low
Dielectric	High	Good	Fair	(No data)	
Ion flow	Nonconducting—high Conducting—low	Good	Poor	Low	Low
Magnetic	10 to 10^3	Limited by carrier	Good	Moderate	Moderate
Electro-mechanical	10^2 to 10^4	10^{-1} to 10^{-3} sec	Poor	Moderate to high	Low to moderate
Rotary	10^2 to 10^4	1 to 10^{-2} sec	Good	High	High
Pneumatic	Not applicable	1 to 10^{-1} sec	Fair	Moderate to high	Moderate
Hydraulic	Not applicable	10^{-2} to 10^{-3} sec	Fair	Moderate to high	Moderate to high

cepted as typical in control systems design, and there may very well be exceptions to the ranges specified in each case. The tabulation should nevertheless prove useful in the preliminary process of selecting an amplifier type that best fits a given system requirement.

Typical applications of amplifiers in control and automation are diagramed in Fig. 1. In Fig. 1*a* an amplifier is used as a means of amplifying a control signal from some sensing device or transducer such as a thermocouple, a strain gage, or a differential transformer to increase its power level so that it can be read on a meter, recorded, or used as input to a computer. In Fig. 1*b* the amplifiers designated A and B provide buffering without a significant increase in the voltage level so that the resolver R and loads applied to outputs e_3 and e_4 do not affect the sources represented by e_1 and e_2. Another application which is more or less typical of what is referred to here as a control application is shown in Fig. 1*c*. Here the amplifier receives an error signal e_e from the comparison of the output angle θ_o with the command signal e_i and raises its power level to that necessary to drive the motor against the loading of the output shaft. In

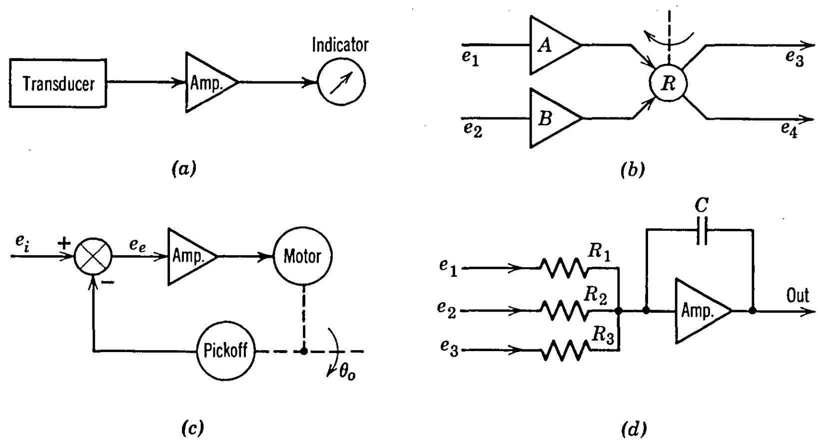


FIG. 1. Applications of amplifiers: (*a*) instrument amplifier, (*b*) buffer amplifiers drive resolver, (*c*) servo with mechanical differential, (*d*) computer (integrator).

Fig. 1*d* an amplifier is employed as an element of an analog computer. The particular circuit shown is arranged to integrate the sum of voltages e_1, e_2, and e_3 with proportional and integration constants as determined by R_1, R_2, R_3, and C. The four amplifier applications described here are referred to as instrumentation, buffering, control, and computation.

Unique Requirements of Amplifiers for Automation and Control

The requirements for control amplifiers differ from those for amplifiers intended for other purposes. Even when compared with audio amplifiers of the same input levels, power output, and frequency ranges, the most satisfactory design for the audio amplifier may not represent the optimum for control applications and vice versa.

Frequency Response, Bandwidth. Requirements for control amplifiers rarely extend beyond 100 cps because of the characteristics of the physical systems involved.

Gain Stability. Gain stability is usually of relatively less importance in the control amplifier; the use of feedback stabilization within the amplifier is rare.

Linearity. Waveform distortion in carrier frequency amplifiers may exceed 10% in most cases without reducing control performance. Amplifiers employed to drive the control phase of a two-phase servo motor are particularly noncritical.

Reliability, Serviceability, Ease of Maintenance. The actual importance of these factors for control amplifiers versus amplifiers for other applications depends on the specific use for which the control is intended, but usually these are critical factors for the control amplifier.

Computer Amplifiers. Amplifiers intended for analog computer service must meet requirements that are much more severe than those for control amplifiers. Extreme gain stability, wide bandwidth, low distortion, and freedom from noise are commonly part of the requirements for computer amplifiers. If the computer requires the amplifiers to be capable of amplifying direct current, the amplifier may need special stabilizing circuitry (see Vol. 2, Chap. 22, Linear Electronic Computer Elements).

D-C Amplifiers. Design of d-c amplifiers for servo, computer, instrument, and other applications requires unusual treatment because the amplifier will amplify its own input drift as discussed later.

Environmental Factors

The capability of various amplifiers to continue to perform within specification limits is a function of the specific components incorporated in their design. Certain general comments regarding the relative environmental tolerance of the respective types can be stated, however.

The effects of the surrounding atmosphere can be almost completely eliminated from consideration if the necessary special design features to accomplish this are economically justified by the application. However, the hermetic sealing of entire units such as complete amplifiers or even larger assemblies carries with it certain penalties in cost and ease of maintenance that preclude such construction in many commercial and industrial applications.

Even without hermetic sealing, the use of water repellent varnishes and suitable components can greatly reduce the effects of humidity; these same varnishes can be made fungus resistant by the addition of antifungicides; dielectric encapsulation can overcome the problem of arcing caused by reduced pressures at high altitudes; and dust and abrasives can be eliminated from consideration by these same techniques.

The electronic amplifier can be most easily designed for adverse environmental conditions. Semiconductor amplifiers are compact and thus easy to encapsulate, but semiconductors are very susceptible to deterioration of performance owing to high temperatures and to radiation. The vacuum tube amplifier, on the other hand, is temperature and radiation resistant but is usually somewhat more sensitive to shock and vibration. The electromechanical devices are available in sealed units but are relatively sensitive to shock and vibration, even when special design features are incorporated to reduce such effects. Hydraulic amplifiers are generally limited in temperature and radiation resistance by the fluids employed, whereas pneumatic units are not limited by the fluid (gas) but by the effects of high temperature and radiation on the materials of which they are fabricated. With sufficient care in choice of materials, pneumatic amplifiers that operate at temperatures above 500°C and in the presence of high radiation intensities can be constructed. Such amplifiers are not shock and vibration resistant to the same extent that electronic amplifiers are, however.

The rotary amplifier is severely handicapped by adverse environmental conditions. The necessity for bearing lubrication can be overcome for high-temperature operation by employing gas or solid lubricated bearings, and the difficulties of hermetic operation of commutators can be reduced by operating at reduced ratings. In general, the rotary amplifiers are very suitable for installations for which the environmental conditions are not extreme.

Temperature Specifications for Electronic Amplifiers

Temperature specifications are generally based on the temperature of the heat sink provided, which may be the surrounding air, the coolant in a liquid or gas system, or perhaps the frame or base to which the unit is to be attached. In examining the temperature the entire thermal situation including conduction, convection, and radiation must be considered. The temperature of a given component is affected by all of these.

Electronic amplifiers have been produced which can operate with heat sink temperatures as high as 500°C, although this is accomplished at a high cost in terms of component price and size and weight. The limiting factors standing in the way of still higher temperature operation appear to be the Curie temperatures of the materials involved. Silicon steel is usable to 650°C; certain dielectrics such as ceramics and conditioned mica are also usable. None of the very high dielectric constant materials is of value in this temperature range. Vacuum tubes are the only electronic amplifying devices capable of providing useful performance above 200°C.

More practical high-temperature specifications call for maximum ambient temperatures of 125°C. A wide range of components are available off the shelf, although most of the components cannot be selected from "commercial" stock. Certain components either are fundamentally not workable at these temperatures or must be derated considerably. Examples of components which are marginal or not available for 125°C ambients are piezoelectric transducers, germanium transistors, aluminum electrolytic capacitors, choppers, and electric cells.

Equipment for industrial applications should be designed to withstand ambient temperatures of about 60°C (140°F) unless special restrictions are placed on its application, since the grouping of apparatus may cause ambients of this order to occur on hot days.

The operation of amplifiers at very low temperatures also poses certain problems. Semiconductors are not generally useful below −75°C, although this will depend on the particular semiconductor considered. Electrolytic capacitors and batteries are not operable below temperatures that depend on the type of component considered, but trouble may be encountered from 0 to $-60°C$ in various cases. Again the vacuum tube proves to be the most versatile amplifying device. Even the requirement for cathode heat may be overcome by the use of cold (field) emission cathode units.

The ability of an amplifier with a given set of components to survive a specified high-temperature environment may be optimized by proper attention to cooling. All amplifiers produce heat since they are not 100% efficient; the removal of this heat to the heat sink provided reduces the temperature rise between component and sink.

The removal of heat by metallic conduction is probably the most effective technique if this is possible. The use of convection cooling has been generally overrated and is not effective for small components unless entire assemblies are arranged to encourage convective flow. The rate of cooling by conduction, convection, and forced air may be calculated with appropriate equations (Refs. 2 and 4).

Construction Techniques for Electronic Amplifiers

The choice of construction technique for electronic amplifiers depends on the following factors.

1. Number to be produced.
2. Maintenance requirements.
3. Environmental conditions.
4. Miniaturization.
5. Reliability requirements.

The following discussion of construction techniques takes up the topics of mechanical construction, wiring techniques, and fabrication by mechanized methods.

Mechanical Construction. The basic elements of the electronic assembly are the mechanical supporting structure, the heat transfer elements, the electrical connectors, the component mounting elements, and the components and connecting wiring. To these might be added handles and/or fastening devices; electrical, dust, and/or radiation shields; hermetic enclosures; and also controls and indicators in special cases.

Materials and Methods. The most common supporting structure consists of sheet metal, usually aluminum, fabricated from sheet stock by cutting, bending, and then welding or riveting. The use of extruded, stamped, or drawn supporting structure is effective if production is sufficient to warrant the construction of necessary dies or if existing shapes can be adapted. Machined structure should be avoided if possible because of the increased cost over other fabrication techniques. Die castings can take the place of machined parts if production units are involved, but the replacement of a machined part with a stamped or drawn part will sometimes result in lighter weight as well as lower cost.

Aluminum has both advantages and disadvantages as a construction material. It is reasonably low in density and high in electrical and thermal conductivity, and has sufficient mechanical strength for most applications. However, aluminum has a high electromotive force with respect to common metals such as iron, copper, tin, nickel, and silver. It corrodes rapidly in air until a layer of nonconducting oxide is formed that makes it necessary to take special care to obtain good electrical ground connections and thermally conductive joints. If special surface chemical treatment is employed to reduce corrosion for applications in which salt spray may be encountered, additional special attention must be given to electrical and thermal conductivity of joints. Various platings for aluminum, particularly silver, although difficult to apply, may sometimes be warranted.

Other materials used when weight is not important include copper and brass. Nickel-plated or silver-plated copper is particularly suited to low-noise applications because of the ease with which good electrical connection can be made to these metals for grounding. Nickel, silver, or cadmium over mild steel may be used to obtain corrosion resistant magnetic shielding.

Potting. The use of potting, that is, completely filling the voids with liquid or plastic, sometimes has advantages for cooling, reducing corona in high-voltage equipment, sealing components from humidity or other contamination, and, in the case of solid potting, increasing the mechanical stability of the assembly. With potting the effects of the increased dielectric constant must be considered, as well as possible injury to components and leads owing to shrinkage during setting of solid potting materials.

Wiring Techniques. Wiring techniques for electronic equipment may be broken down into the following classifications, or, for a given unit, to combinations of these classifications.

1. *Point to Point.* Small components and leads are wired with flexible insulated wire between terminals of the larger, chassis-mounted, components.

2. *Cable and Terminal Board.* All small components are wired to suitable terminals on insulating terminal boards. Larger components are chassis-mounted. Lead wires running between terminal boards and chassis-mounted components are cabled together and clamped to the chassis.

3. *Rigid Wiring.* Rigid wiring includes printed circuits, stamped wiring, plated wiring, etc. Wiring is firmly bonded to insulating base. All components are soldered directly to the wiring. Heavier components may be screwed to the base, and metallic reinforcements such as eyelets may be used at component attachment points.

Fabrication of Wiring by Mechanical Methods. Although hand wiring of electrical equipment is the most common technique, particularly for developmental equipment, there are certain methods of fabrication of wiring which are primarily mechanical and which result in a high degree of reproducibility. Mechanically fabricated wiring may be preferable for various reasons, including reduced cost of fabrication, reduction of quality control problems, reproducibility of electrical characteristics, and increased mechanical strength. Mechanical wiring also improves the internal appearance of the equipment.

Printed Wiring. Perhaps the most widely used technique for mechanized wiring is that known as printed wiring. A laminated sheet is first made up from a core of insulating material such as Bakelite, ceramic, or impregnated fiber glass with a sheet of conducting material such as copper or silver bonded to one or both sides. The wiring is formed by etching away that portion of the conducting material not used in the circuit. The preparation of the blank for the etching process may be done photographically, by printing or by silk screening. Eyelets, inserts, and other mechanical anchoring devices may be added to the board, and then components are applied by passing their leads through holes in the board and dip-soldering the entire assembly.

In other methods of mechanizing wiring that do not require the use of the laminate, the conductor is applied to the base only in those areas where conductors are desired. The conducting material may be stamped out and then bonded, or it may be sprayed, electroplated, or printed directly onto the insulating base.

Care must be taken to insure good bonding of conductors to the base material. Some insulating materials expand more with temperature than the metallic conductors, and this must be tolerated by the structure.

Certain components such as capacitors up to about 100 $\mu\mu$f, inductors up to about 100 μh, and nonprecision resistors of any value may be formed by the same techniques as an integral part of the circuit.

The automatic assembly and soldering of conventional lead-mounted components into printed circuits have been developed. Components need only be loaded into hoppers in sufficient quantity, and the process is completely automatic from that point.

The decision to employ mechanized wiring techniques is usually made for economic reasons. The use of automatic assembly methods increases the setup cost for a given production run, particularly if the amortization of the production equipment is included. The degree of flexibility attainable with a given process tends to vary directly with the cost of equipment; that is, equipment capable of rapid changeover from one circuit to another radically different circuit is more costly than equipment intended for production of a family of similar circuits.

The cost of circuit wiring makes up only a fraction of the cost of the finished electronic equipment, and the use of mechanized wiring techniques may not always be justified on an economic basis. The initial design cost is usually greater for printed circuits than for hand wiring.

D-C Amplifiers

The amplification of the continuous or direct component of a control signal, particularly where the input signal power available is very low and where freedom from drift must obtain for long periods of time, is a problem that has received considerable attention from the control and instrument designer for many years. The electronic amplifier is the only amplifier capable of effective operation at these low power levels, so the discussion of direct-current amplification is essentially a question of how to design electronic amplifiers for direct coupled operation. The difficulties encountered in the design of such amplifiers are due to:

1. The requirement for stable ungrounded voltage sources, or special biasing networks to provide proper initial conditions for the amplifying element.

2. The presence of direct coupling which allows the amplifier to amplify its own long term drift. All amplifying devices have some long term drift, although it may be small for the rotary and electromagnetic-mechanical devices.

Biasing. Three typical biasing arrangements are shown in Fig. 2. It should be noted that the biasing networks of Fig. 2*b* cause degeneration, but if nonlinear devices such as neon tube, another vacuum tube, or Zener diodes are used instead of R_1, R_2, and R_3, this loss can be greatly reduced. The circuit of Fig. 2*c* uses balanced circuitry and allows some compensation for drifts caused by supply voltage variations, tube warm-up, etc. Many schemes intended to reduce the drift of d-c amplifiers have been described in the literature, and vacuum tube amplifiers can be built that are stable to about 10-mv equivalent input voltage over reasonable periods of time if sufficient care is exercised.

Stabilization. The use of feedback cannot overcome the effects of drift in the input circuit since this is equivalent to a change in the input voltage. This can be seen by reference to Fig. 3 in which an amplifier of overall gain A is shown with feedback applied to the cathode of the input stage. ΔE represents the effects of a change in the input stage. The output signal,

$$e_o = \frac{(e_i - \Delta E)A(R_1 + R_2)}{R_1 + R_2 + AR_2},$$

or if A is large

$$e_o = \frac{R_1 + R_2}{R_2}(e_i - \Delta E).$$

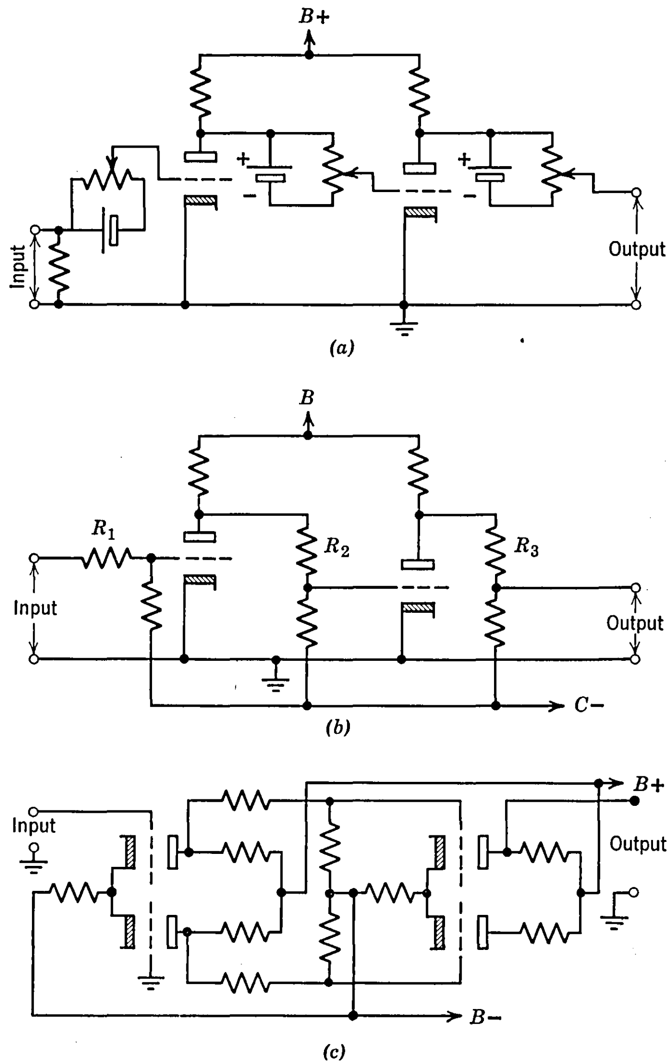


FIG. 2. Methods of providing biasing for d-c amplifiers: (a) biasing batteries, (b) biasing networks, (c) balanced amplifier.

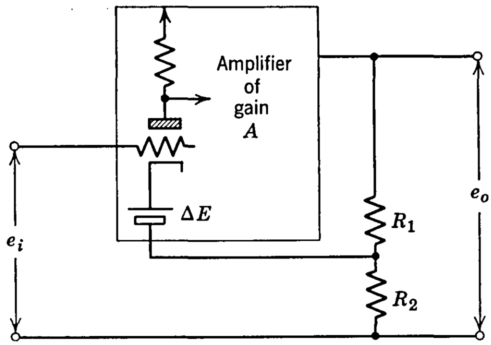


FIG. 3. Effect of changes in input circuit of feedback amplifier.

An effective means of stabilizing d-c amplifiers is the use of a comparatively drift-free d-c amplifier of the modulator-demodulator type to be described in Sect. 4 in a circuit similar to the one shown in Fig. 4*a*. Another technique is to use a stable comparison device, such as one of the electromagnetic-mechanical devices, to compare input and output as shown in Fig. 4*b*. The circuit shown is a current-multiplying amplifier. Actually the mechanical device functions as a modulator in this circuit, and therefore this is equivalent to the arrangement of Fig. 4*a*.

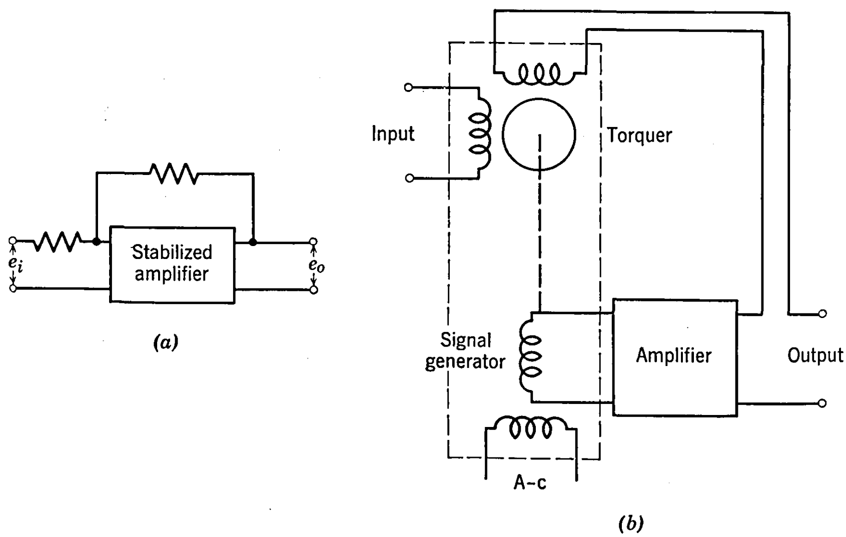


FIG. 4. Stabilization of d-c amplifiers: (*a*) modulator-demodulator feedback amplifier, (*b*) amplifier with electromechanical modulator and feedback.

3. MODULATORS AND DEMODULATORS

As pointed out in the preceding sections, d-c amplifiers suffer from very serious limitations because they amplify the drift in their own input conditions. This drift can be overcome to a large extent by the use of techniques in which the signal is first modulated, then amplified, and subsequently demodulated. This reduction of drift is possible because of the nature of the noise found in electronic amplifier devices: the noise level in a given band of frequencies centered on a carrier such as 60, 120, 400 cps or comparable frequencies is always lower than the same bandwidth if it includes zero (d-c). For this reason the level of spurious signals in the output can be greatly reduced by modulation before amplification. Further reduction of the spurious signal level is afforded by phase sensitive detection, or *coherent* detection as it is often termed.

As an example, in a typical vacuum tube d-c amplifier the drift might be of the order of several millivolts. If modulation at, say, 400 cps is employed, the equivalent input noise level may be reduced to a few microvolts. With coherent detection this can be further reduced. Noise levels of 10^{-8} volts with 0.1 cps signal bandwidth are possible. The degree of phase coherence between the input modulator and the output demodulator, and the stability of phase changes in the modulator and demodulator and in the amplifier proper, are significant in establishing the minimum attainable noise level.

Types. A tabulation of typical modulators is given in Table 2, and these devices are discussed briefly in the following paragraph. Demodu-

TABLE 2. MODULATORS FOR CONTROL APPLICATIONS

Type	Zero Stability: Volts	Zero Stability: Watts	Input Impedance, ohms	Remarks
Contact chopper	10^{-8}	10^{-22}	10^{6}	Can attain this performance only if bandwidth is reduced to decrease noise
Capacitor chopper	10^{-3}	10^{-20}	10^{14}	Suitable only for high-impedance, well-shielded input devices
Alternator	1	10^{-4}	10^{4}	Estimated performance
Semiconductor diode	10^{-2}	10^{-9}	10^{5}	
Vacuum tube diode	10^{-2}	10^{-10}	10^{6}	Requires stabilized heaters and matched tubes for this stability
Balanced triode modulator	10^{-1}	10^{-14}	10^{12}	Transistor triode superior to vacuum tube
Magnetic	10^{-3}	10^{-9}	10^{3}	Most rugged and simplest type of modulator

lators are tabulated in Table 3, and a discussion of these devices is also included.

TABLE 3. PHASE SENSITIVE DEMODULATORS

Type	Input Impedance, ohms	Usable Levels: Volts	Usable Levels: Watts	Remarks
Vacuum triode	10^7	0.01–1	10^{-11} to 10^{-7}	Power gain obtained. Output power *m* may be several watts or more
Transistor triode	10^4	0.01–1	10^{-8} to 10^{-4}	Power gain possible. Outputs of several watts
Chopper	10^7	10^{-4} to 10	10^{-15} to 10^{-5}	High efficiency. Limited frequency range

All the modulators described here are of the *suppressed carrier* type. The carrier is reintroduced in the demodulator as the reference phase in phase sensitive demodulation. Suppressed carrier modulation is preferable for control purposes since the zero or null amplitude is well defined, whereas in ordinary amplitude modulation the zero position is dependent on the carrier level.

Phase modulation is of interest as a possible type of modulation for some control applications, particularly where devices such as thyratrons and relays are involved. Phase modulation is normally introduced in the output stage itself, and the modulation circuit may be quite different from those discussed here.

Modulator Classification. Modulators may be classified according to whether (1) the nonlinear parameter is varied at the carrier frequency, (2) the nonlinear parameter is varied by the signal, or (3) the signal and the carrier are applied to a common nonlinear impedance. All the more significant modulators for control applications fall into category 1 or 3 with category 1 as exemplified by the chopper providing greatest null stability, and category 3 in the form of magnetic and semiconductor devices providing maximum flexibility.

Contactors. The mechanically driven contactor is often referred to as a *chopper* since it effectively chops the input waveforms by first presenting a very low impedance and then an open circuit. The output waveform produced for a continuous d-c input is a square wave, although the amplitude varies according to which circuit is employed. Two typical modulator circuits are shown in Fig. 5. The circuit in Fig. 5*a* is adapted to operation from high-impedance sources in which the high input imped-

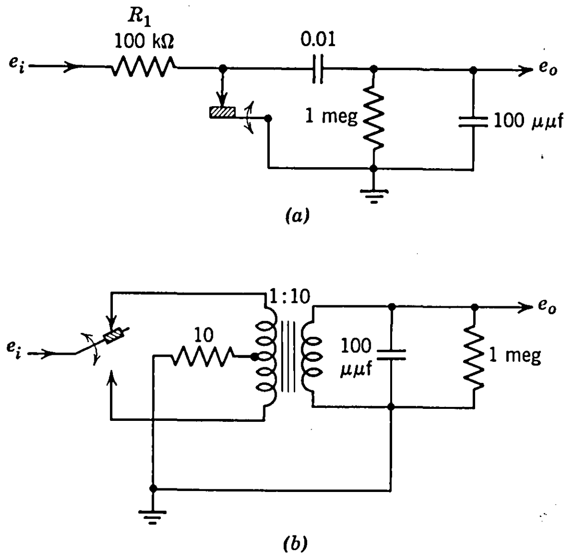


FIG. 5. Typical circuits for chopper modulator: (*a*) high impedance asymmetrical, (*b*) low impedance balanced.

ance keeps the loading on the source of e_i within allowable limits. The low-impedance circuit is suitable for use with thermocouple or other low-impedance sources in which a voltage stepup can be obtained in the input transformer.

The mechanical contactor may also function as a demodulator. Here, as well as in the modulator application, it is outstanding by reason of its very high ratio of open to closed resistance ($>10^{10}$). The circuit of Fig. 5*a* requires only one-half of a double throw chopper, and the second half may function as a demodulator as shown in Fig. 6*a*. Another demodulator circuit is shown in Fig. 6*b* where the chopper functions as a full-wave phase sensitive detector. The half-wave modulator and demodulator introduce a time lag caused by the intermittent grounding of input and output, which is avoided by use of the full-wave circuit.

The contact modulator does not modulate or demodulate on a sinusoidal basis but produces a square wave output as a modulator. If this square wave is amplified with reasonable fidelity, the demodulator then operates at a low ripple level.

The mechanical chopper may be driven either directly by an a-c excited electromagnet and armature arrangement or from a rotary motor through a cam arrangement.

The chopper is the obvious choice for precision applications in which the very high open-closed ratio and the high open circuit impedance are

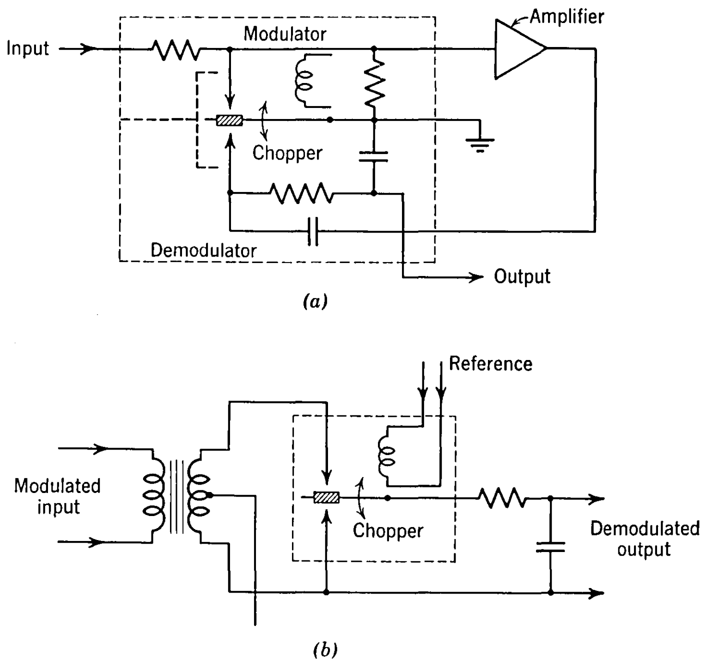


FIG. 6. Chopper demodulators: (*a*) single chopper used as modulator and demodulator, (*b*) full-wave phase sensitive demodulator.

necessary. The compactness of the electromagnetic unit makes it quite suitable for portable and airborne equipment.

Despite very substantial improvement in chopper reliability in recent years, it is not potentially, or practically, capable of equaling the magnetic or the semiconductor modulators in this respect, and this is its most serious handicap for applications requiring long life without replacement. Another problem encountered in the application of electromagnetically driven choppers is sensitivity to vibration and acceleration, and in critical applications the phase stability with respect to the driving voltage may not be sufficient.

Other Mechanisms. Modulators and demodulators have been constructed based on other than mechanical motion for producing a resistance variation at the carrier frequency. Among the mechanisms that have been utilized are (1) pressure sensitive resistors such as carbon, (2) the Hall effect, and (3) photoconductive materials. None of these devices compares favorably with the chopper for precision applications, although they overcome to a certain extent some of the objections to the chopper which are based on sensitivity to acceleration and vibration.

Variable Capacitance and Inductance. If a constant voltage is applied to a capacitance (Fig. 7*a*) which is varied in synchronism with some carrier frequency, the current into the capacitor will represent a modulated signal. The capacitor might be a motor-driven air variable capacitor or a pressure sensitive capacitor driven by piezoelectric or magnetostrictive forces. Similarly, if the signal is represented by a current through an inductance varied in synchronism with the carrier, the voltage across the inductor is a modulated signal. The output from the variable inductance may be taken either directly from the terminals of the inductor or from a secondary winding as illustrated in Fig. 7*b*. Another possible technique is to vary the mutual inductance between the primary and secondary by rotating one with respect to the other.

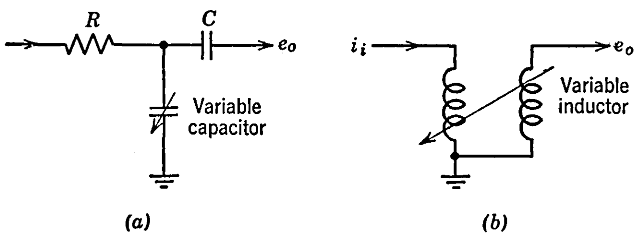


FIG. 7. Reactance modulation: (*a*) capacitance modulator, (*b*) inductance modulator.

These modulators have not seen extensive application except for laboratory devices.

Diode Modulators. If two signals are introduced to the same nonlinear impedance, the amplitude of one will affect that of the other, and a degree of modulation is obtained. A simple diode modulator is shown in Fig. 8*a*. The output consists of the rectified waveform at the right. It should be noted that this modulator is capable of being modulated in one direction only and cannot produce suppressed carrier modulation.

Elimination of the carrier from the output for zero signal can be effected by use of the diode switch principle in which two diodes are biased by the carrier and the signal is introduced to their common point. The ring modulator may be used as shown in Fig. 8*b* to make a circuit which is actually two of these switches in series. The output obtained is now of either polarity according to the signal but is still half-wave in that for a given polarity of signal there is no output during one-half of the carrier cycle.

The delay caused by half-wave operation may sometimes be of importance so that full-wave operation is desirable. The circuit of Fig. 8*c* has two diode switches arranged so that they switch alternate half-cycles

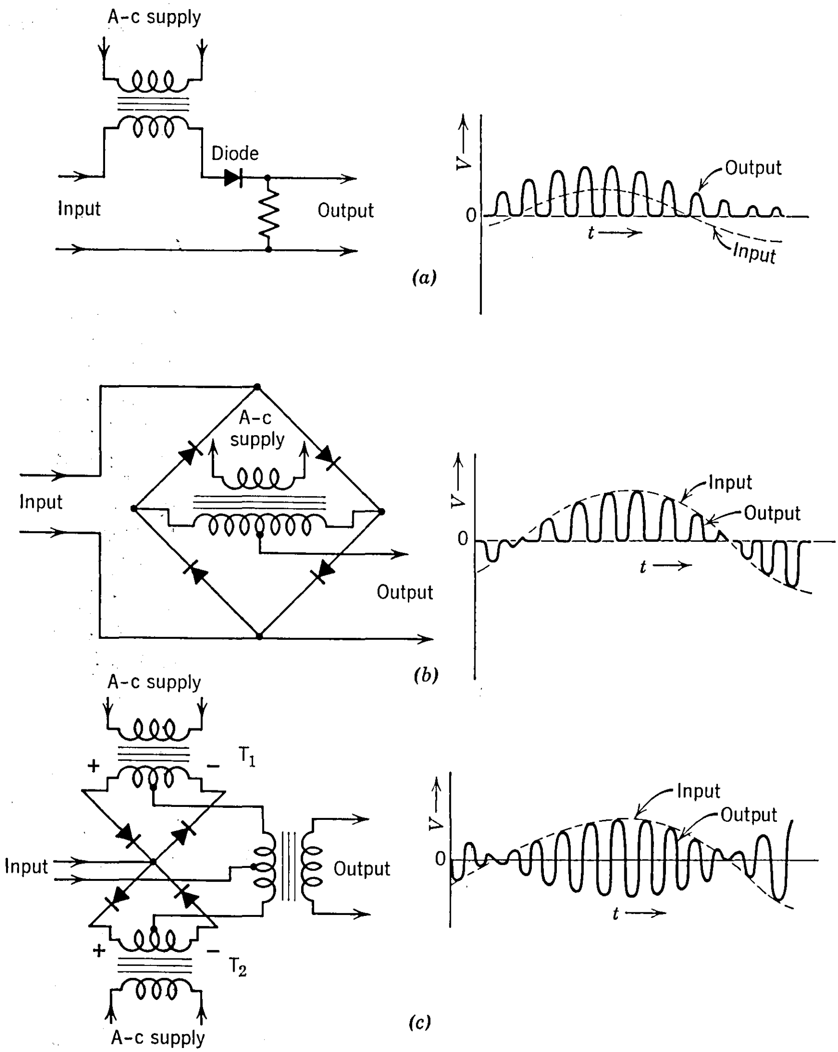


FIG. 8. Diode modulators: (*a*) simple diode modulator, (*b*) ring modulator, (*c*) full-wave diode modulator.

of the input across alternate halves of the output transformer and thereby obtain full-wave modulation. A circuit employing the ring modulator for full-wave operation can be devised, but this is not so effective in the separation of the carrier and signal circuits as the double diode switch arrangement.

Various other circuits can be devised and may have advantages in a given application, depending on input and output impedances, type of diodes available, harmonics allowable in the output, etc.

Thermionic diodes have large ratios of back-to-front resistance and are ideal for high-impedance modulators at high voltages. At low voltages the variation between diodes of the potentials produced by the cathode escape velocities of the electrons (contact potential) causes drifts. Semiconductor diodes are better with respect to low-level operation. External series and parallel resistances are often added to semiconductor diodes to swamp out variations in front and back resistances, respectively, in corresponding diodes. Positive bias on the diodes can improve the conversion efficiency. Since semiconductor diodes are temperature sensitive, the mounting of all diodes in a given modulator in good thermal contact with a metallic (heat conducting) portion of the structure is recommended.

Semiconductor diode modulators have been constructed with input impedances of the order of 100,000 ohms with zero stability of less than 10 mv for a wide range of temperatures. Vacuum tube modulators require regulated heater supplies to approach the same stability level.

Triode Modulators. Vacuum tube and semiconductor devices having three or more terminals can also be employed as modulators. Both vacuum tube and transistor triodes have been used extensively as modulators. Vacuum triode modulators suffer from drifts caused by contact potential as is true of vacuum diodes.

Typical circuits for vacuum tube and transistor modulators are shown in Fig. 9. The transistor circuit is notable in that the input signal supplies both the bias and the collector voltage for the transistor.

The value of the triode modulator for control applications is in its ability to produce relatively high-level output voltages in a single circuit and thus to reduce the effect of spurious power supply coupling. The transistor modulator, in particular, appears to have considerable advantage over the equivalent diode circuit both for conversion gain and for zero stability.

Magnetic Modulators. If a ferrous-cored inductor is driven into saturation on both the positive and negative peaks of the a-c supply voltage, the current through the coil will consist of the fundamental plus the third and higher odd harmonics. If a small bias (signal) is applied to the field either by adding dc to the supply or by the use of a second coil, the second and other even harmonics will be represented in the output. This is the second harmonic modulator, the simplest and most stable form of magnetic modulator. The even harmonics can be extracted by bucking the output of two cores. The zero stability of the second harmonic modulator is in the order of 10^{-9} watts. A conventional second harmonic

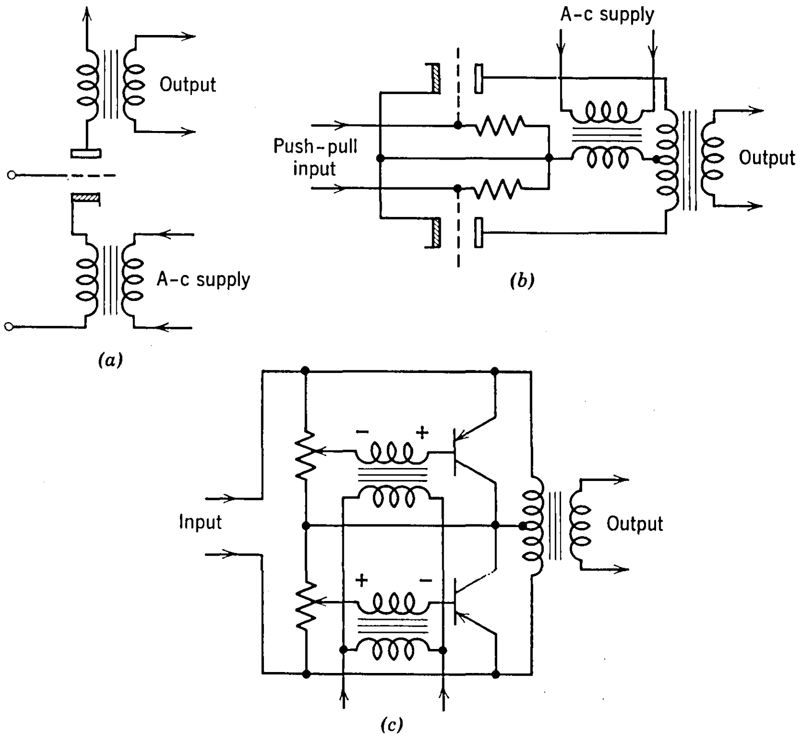
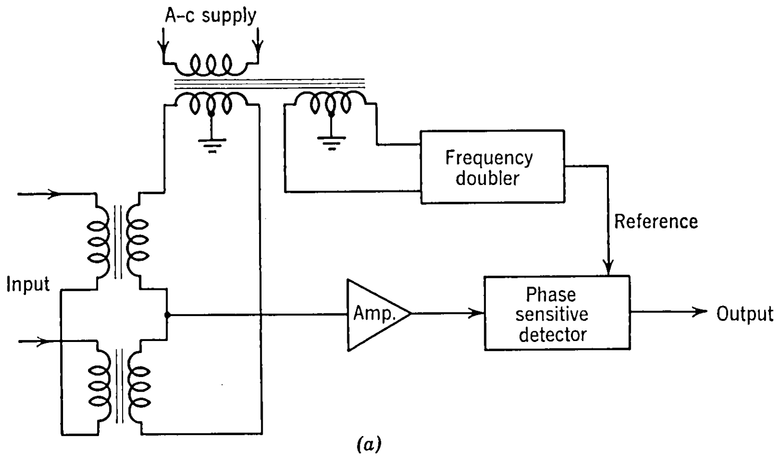


FIG. 9. Typical triode modulator circuits: (*a*) basic triode modulator, (*b*) balanced modulator, (*c*) transistor modulator.

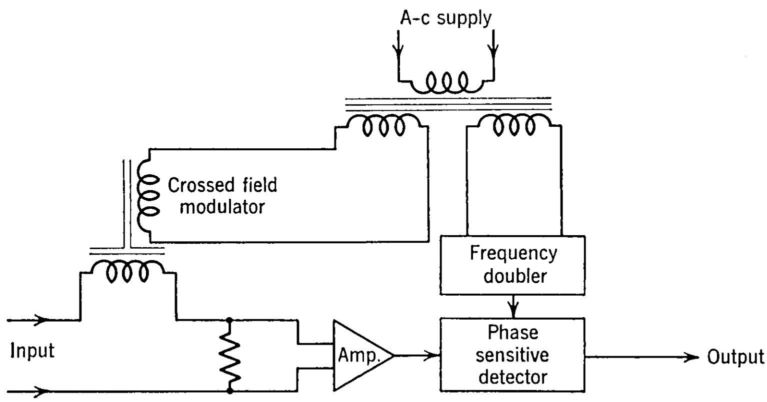
modulator and amplifier is shown in Fig. 10*a*. A single core magnetic modulator employing crossed signal and a-c supply fields is shown in Fig. 10*b*.

Phase Sensitive Demodulators. Since suppressed carrier modulation is commonly used in carrier frequency servos, it is necessary to provide a phase sensitive device at the output or power end of the amplifier. The actuator, such as the two-phase servo motor, may be phase sensitive, but very often it is necessary to provide a phase sensitive demodulator so that an actuator which is not phase sensitive may be employed.

Choppers are sometimes used in a modulator-demodulator arrangement similar to Fig. 6*a* and may also be used in other demodulator circuits, but they are not suitable for many demodulator applications. The phasing of the mechanical contactor is not particularly stable. Where it is used as both modulator and demodulator, the instability is tolerable within limits. Variations in the on and off times with respect to the driving voltage can result from changes in driving voltage, from mechanical in-



(a)



(b)

FIG. 10. Magnetic modulators: (a) second harmonic modulator and amplifier, (b) single core crossed field modulator.

stability including temperature effects, and from contact wear. The contacts are generally limited to low currents. The frequency range over which the mechanical contactor is usable is limited by mechanical resonances to frequencies from 100 to 1000 cps, depending on the design.

Vacuum tube and "dry" diodes are both useful in the demodulator circuits of Fig. 11. The similarity of Figs. 11*a* and *b* to the frequency discriminators of communications circuits is apparent. The circuits are essentially similar when used as demodulators, except that the one in

Fig. 11*a* is generally preferable since it uses fewer components and produces a larger output for a given voltage from the signal transformer. Circuits in Figs. 11*a* and *b* are satisfactory when simplicity is required and the load has a high impedance, but more complex circuits such as that in Fig. 11*c* are required when it is desired to supply power to the load

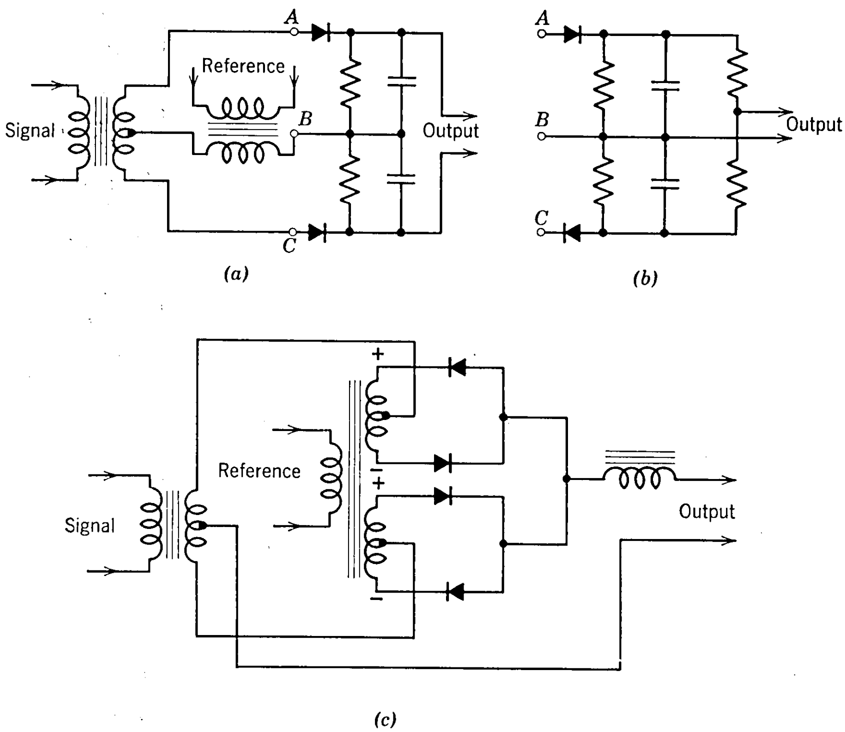


FIG. 11. Diode demodulator circuits: (*a*) two-diode detector, (*b*) alternative arrangement of (*a*), (*c*) four-diode demodulator.

with reasonable efficiency. Note the similarity of this circuit to the modulator of Fig. 8*c*.

Circuits for dry and vacuum diodes are similar except that the impedance levels, back voltages, and forward currents must be chosen for the particular rectifier employed. Cathode heater power must, of course, be supplied for the vacuum diode. Transistors and vacuum triodes do not approach interchangeability to the same extent, owing to the radically different reverse voltage characteristic of the transistor collector as compared to the vacuum triode plate; however, a diode in series with the collector will block the reverse current flow and allow similar circuits to be employed.

The reversible full-wave diode demodulator has the disadvantage that it cannot be used with a capacitive load since the capacitance could always discharge through one pair of diodes at the time when the reference and signal voltages are zero. The triode demodulator overcomes this disadvantage to a considerable extent, for the triode can be biased near cutoff for zero signal voltage, thus minimizing the leakage. Such a circuit is shown in Fig. 12.

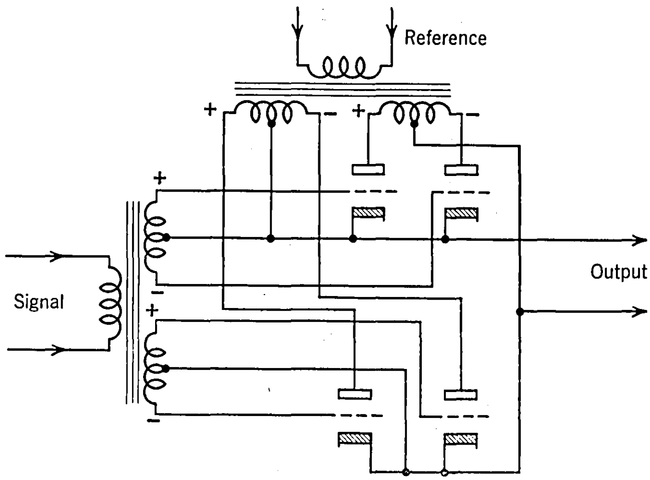


FIG. 12. Full-wave vacuum triode demodulator.

4. ELECTRONIC AMPLIFIERS

Vacuum Tube Amplifiers

The vacuum tube amplifier is by far the most versatile type of amplifying device. At the frequencies considered here it is essentially unilateral; that is, it neither loads the input appreciably nor reflects spurious currents back into the input (grid) in operation. Also at these frequencies its response is effectively instantaneous and, unless time delays are deliberately introduced in the related circuitry, the vacuum tube amplifier can be represented as a pure gain (no delay) element in the control analysis. Other advantages of the vacuum tube are its low cost, the availability of a wide range of types, the reproduceability of its characteristics, and the ease with which the associated circuitry can be constructed.

Amplification. Because of the unilateral nature of the vacuum tube, the gain calculations may be made stage by stage. In carrier frequency amplifiers the midband conditions will usually apply, that is, the reactances in the circuit are of little importance at the operating frequency;

however, the Miller effect may require consideration in triode stages with large values of amplification factor and load resistance. (The capacitances of tube socket, wiring, and components must be considered as well as the plate-to-grid capacitance in calculations of Miller effect.)

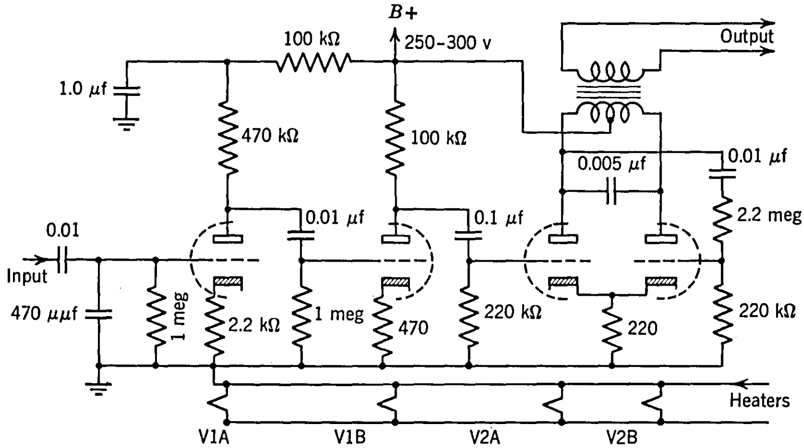
Hum Pickup. Where a-c heaters are used, the hum pickup at the input grid may be as great as several millivolts. If the amplifier is to operate at lower zero signal than this, special attention must be given to the shielding and dress of input leads, and unshielded high impedance wires should be avoided. For example, a high-impedance lead 1 ft long might pick up 10 to 20 volts of power frequency signal in a typical equipment location. A single layer of braided sheathing will reduce the pickup greatly, but double braid may be required in some cases. Heater leads should be twisted (or run in tubing), and power transformers should not be placed close to the input stage. Several means of reducing hum by grounding the heater leads in various ways have been suggested. Merely tying one heater lead to the same ground as that of the input grid resistor is simple and effective. With care the a-c pickup may be reduced to the order of 50 μv across a 1-megohm grid resistor. If a lower hum level is required, the use of dc on the heater of the input stage is justified.

Triodes and Pentodes. Both triodes and pentodes are suitable for control amplifiers, but triodes are generally preferred. The choice should be based on the highest degree of simplicity for the accomplishment of a given function. Typical triode and pentode control amplifiers are diagramed in Fig. 13. Triodes are preferred for critical input stages since both the shot noise and the microphonic noise are less than in the comparable pentode; also the lower plate resistance of the triode couples less of the power supply noises into the stage.

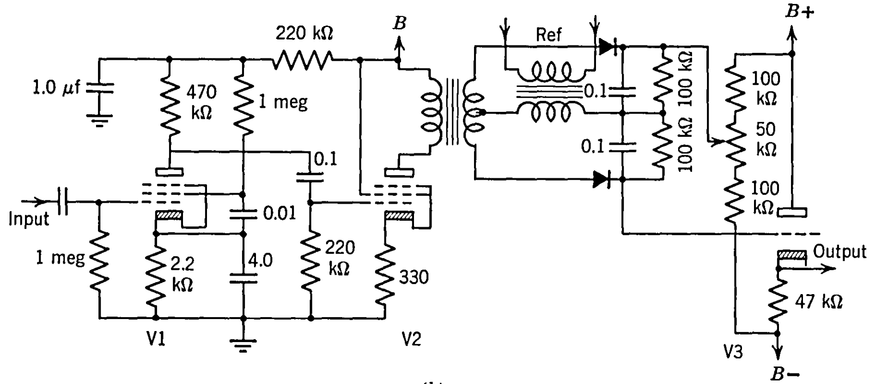
Low-level stages after the input may be either triode or pentode, depending on the overall gain requirement as well as other considerations. The twin triode provides a large gain per envelope. Cathode bias resistors that are not bypassed may be employed.

Design Considerations. Output and driver stages should be designed together since the loading of the output stage often requires the selection of a low-impedance driver load resistor, coupling capacitor, etc. The output stage may be a tetrode or pentode if the output power requirements are high, but triodes are preferable for instrument servos because of their lower output impedance.

Interstage coupling almost invariably consists of the simplest *RC* networks, since transformers are heavy and bulky considering the slight improvement in gain that can be obtained. High-power output stages may be coupled to the load by a transformer, and transformer coupling between driver and output stage may be employed.



(a)



(b)

FIG. 13. Examples of vacuum tube amplifiers: (*a*) typical triode amplifier for control application 400 cps carrier frequency—V1 is high-mu unit, V2 medium-mu unit; (*b*) pentode amplifier with phase detector and direct coupled output stage.

A transient may cause blocking of the amplifier if *RC* coupling is employed, owing to the charging of the coupling capacitor by grid rectification. For this reason the minimum coupling capacitor size compatible with the overall phase shift requirement should be employed. Blocking can be avoided entirely by the use of direct coupling, of course, and in a given amplifier a combination of *RC* and direct coupling may be the best arrangement.

Reliability. The reliability obtained for vacuum tubes may be increased by careful application well within their ratings. There are several categories of long-life and "ruggedized" tubes. The most distinctive features of these tubes are their mechanical design and the extra care that goes into their assembly and inspection. The decision whether to go to a ruggedized tube in a given application is ultimately an economic one. In vehicular equipment where vibration is a problem there is no alternative to the use of ruggedized tubes. In stationary equipment, regulation of the heater supply voltage against line voltage extremes and, in some cases, uninterrupted heating of the cathodes should be given priority over the use of special tubes.

Semiconductor Amplifiers (See also Chap. 27, Transistor Circuits)

General Principles. Although the vacuum tube employs the relationships between fields and electrons in a vacuum to obtain amplification, transistors and related devices depend on the relationships between fields and holes and electrons which provide current flow in a semiconductor material for their amplification (see Chap. 26, Semiconductor Devices). In general, vacuum tubes are characterized by very high input impedances as compared to output impedances, whereas the transistor is just the reverse and typically has an output impedance of twenty or more times the input impedance. Although the vacuum tube is nonlinear in its operation, the grid current is so small in most cases that the nonlinearity of the current-voltage relationship in the input can be ignored. In contrast, the transistor device has low input impedances, and the nonlinearity of the current-voltage relationship must be considered in any amplifier handling signals of appreciable magnitude.

Although a number of variations have been devised having interesting and sometimes useful characteristics, the semiconductor amplification device of greatest use to the control designer is the three-element transistor having leads connected internally to the semiconductor base, emitter, and collector respectively. The basic mechanism involves the injection of current carriers (holes or electrons) into the base region by way of the emitter. These carriers influence the conditions at the collector-base junction so that current flow is possible in the reverse-biased collector junction. The impedance of the collector junction is high, typically 20,000 ohms to a megohm, depending on the transistor. The output is essentially a current proportionate to the emitter current. A detailed explanation of the physical basis for transistor action can be found in various references (Refs. 7 and 8).

Comparative Characteristics. The characteristics of commercially available transistor units continue to change rapidly as evidenced by the thousands of types which have become obsolete since the advent of the transistor. In general, the control engineer will find junction transistors of the alloyed variety most suitable for his purposes. Either germanium or silicon units may be employed, and both *p-n-p* and *n-p-n* types are available. The choice between available types will, in any given case, be based on such factors as cost, availability, environmental conditions, reliability, compatibility with power supply, and other circuit elements.

Advantages. Transistors in control amplifiers have the following advantages over vacuum tubes.

1. Higher reliability, greater ruggedness.
2. Lower power consumption, higher efficiency.
3. Smaller size.

Disadvantages. Some relative disadvantages of transistors which may or may not be significant in a given instance are:

1. Poorer temperature stability.
2. Bidirectional impedances.
3. Variation in parameters with aging.
4. Lower impedance.
5. Higher noise levels.
6. Nonlinearity.
7. Frequency limitations.

Certain advantages and disadvantages pertain to the transistor as a production component. Other characteristics are more important in development engineering. For example, the bidirectional impedance characteristic is not a disadvantage after a design has been reduced to production, but it complicates the use of the transistor in laboratory apparatus and necessitates more development time for a transistor amplifier than for an equivalent vacuum tube unit.

Reliability. Potentially the transistor is highly reliable, and mean time to failure in millions of hours has been predicted. There is no cathode aging as in a vacuum tube, although some modification of the junction may take place owing to temperature, radiation, and other effects. The removal of harmful substances from the capsule and the establishment of a permanent hermetical seal are essential to the attainment of longest possible life from the transistor.

Because of mechanical scaling effects the small transistor is capable of withstanding the most severe acceleration and vibration conditions without damage, although microphonic noises may occur. Although the power transistor has size and mass comparable to those of a vacuum tube, the structure of the semiconductor can be integrated with the mounting elements of the capsule, so that it is far superior to the vacuum tube in its resistance to mechanical damage.

Power Consumption. The vacuum tube requires that power be supplied to the cathode heaters continuously whether plate current is flowing or not. Typical heater power is in the order of 2 watts per tube envelope for low-level vacuum tube amplifiers, even though the signal power may be measured in microwatts. In transistor amplifiers there is no need for such wasteful heating. Also the vacuum tube operates poorly at very low plate voltages, whereas the transistor is effective with collector voltages about one-tenth typical plate voltages of the vacuum tube.

These two factors give the transistor a tremendous advantage where a large number of low-level amplifier stages are required and where cooling is difficult. However, the transistor may not withstand the high temperatures the vacuum tube is able to endure, so it must be cooled to a lower temperature. Thus the cooling requirement for transistor amplifiers may be either more or less than for vacuum tubes, depending on factors such as the temperature of the heat sink, the presence of other dissipative devices close to the transistors, and, of course, on whether germanium or silicon transistors are employed.

The high efficiency of the transistor reduces the demand on the primary power source considerably. This makes transistors ideally suited for applications such as airborne digital computers and portable radios and testing instruments. Low primary power demand may be of little importance in other applications such as industrial control circuits.

Temperature Stability. When transistors are subjected to elevated temperature, the short circuit current multiplication factor α increases, the collector saturation current I_{CO} increases, and the collector resistance r_C decreases. By careful design it is possible to construct germanium transistor circuits which operate satisfactorily as high as 85°C, but penalties must be paid in terms of additional components or overdesign of components and in terms of performance reduction. Operation to ambient temperature of 70°C is more practical.

Silicon transistor parameters also vary with temperature, but limiting conditions do not usually occur below 150°C. Unfortunately the direct replacement of germanium units by equivalent silicon units is not possible, even when the additional cost is not prohibitive. Parameters are different enough that circuit changes are required. Silicon units are not available with gains and power ratings as high as those of the best germanium types. Improvement in this situation may be expected, but there are fundamental reasons why silicon units will continue to differ from germanium units, posing unique problems for the circuit designer.

Circuit Calculations (see also Chap. 27, Transistor Circuits). Exact analysis of performance in any given application can be made with the aid of curves for the particular transistor. Approximate analysis based on small signal conditions is usually employed followed by consideration of effects of nonlinearity where this is significant, as for power amplifiers.

The equations of Table 4 are given as exact equations based on the equiv-

TABLE 4. TRANSISTOR PERFORMANCE EQUATIONS

	Grounded Base	Grounded Emitter	Grounded Collector
Current gain	$-\dfrac{\alpha r_c + r_b}{r_c + r_b + R_L}$	$\dfrac{\alpha r_c - r_e}{r_c - \alpha r_c + r_e + R_L}$	$-\dfrac{r_c}{r_c(1-\alpha) + r_e + R_L}$
Voltage gain	$\dfrac{(r_b + \alpha r_c)R_L}{(R_L + r_c + r_b)(r_e + r_b) - r_b(r_b + \alpha r_c)}$	$-\dfrac{R_L(\alpha r_c - r_e)}{(R_L + r_c)(r_e + r_b) - r_b(\alpha r_c + r_e)}$	$\dfrac{r_c R_L}{r_c(R_L + r_e) + r_b(R_L + r_e + r_c - \alpha r_c)}$
Power gain	$\dfrac{R_L(r_b + \alpha r_c)^2}{(r_e + r_b)(R_L + r_c + r_b)^2 - r_b(r_b + \alpha r_c)(R_L + r_c + r_b)}$	$\dfrac{R_L \alpha r_c - r_e)^2}{[r_c(1-\alpha) + r_e + R_L]^2 r_b + [r_c(1-\alpha) + r_e + R_L]r_e(R_L + r_c)}$	$\dfrac{r_c^2 R_L}{[r_c(1-\alpha) + r_e + R_L]^2 r_b + [r_c(1-\alpha) + r_e + R_L]r_c(r_e + R_L)}$
Input impedance	$r_e + r_b\left(1 - \dfrac{r_b + \alpha r_c}{R_L + r_c + r_b}\right)$	$r_b + \dfrac{r_e(R_L + r_c)}{R_L + r_c + r_e - \alpha r_c}$	$r_b + r_c\left(1 - \dfrac{r_c - \alpha r_c}{r_c - \alpha r_c + r_e + R_L}\right)$
Output impedance	$r_c + r_b\left(1 - \dfrac{r_b + \alpha r_c}{r_e + r_b + R_s}\right)$	$r_c - \dfrac{(\alpha r_c - r_e)(r_b + R_s)}{r_e + r_b + R_s}$	$r_e + \dfrac{(r_c - \alpha r_c)(r_b + R_s)}{r_c + r_b + R_s}$

alent circuit of Fig. 14*a*. Only the external circuit for the grounded base arrangement is shown, but the corresponding diagrams for grounded emitter and grounded collector are obvious. The factor α is defined as the ratio of the change in collector current to the change in the emitter current where the collector voltage is held constant. Other symbols are as shown in the equivalent diagram. Note that the current gain in the

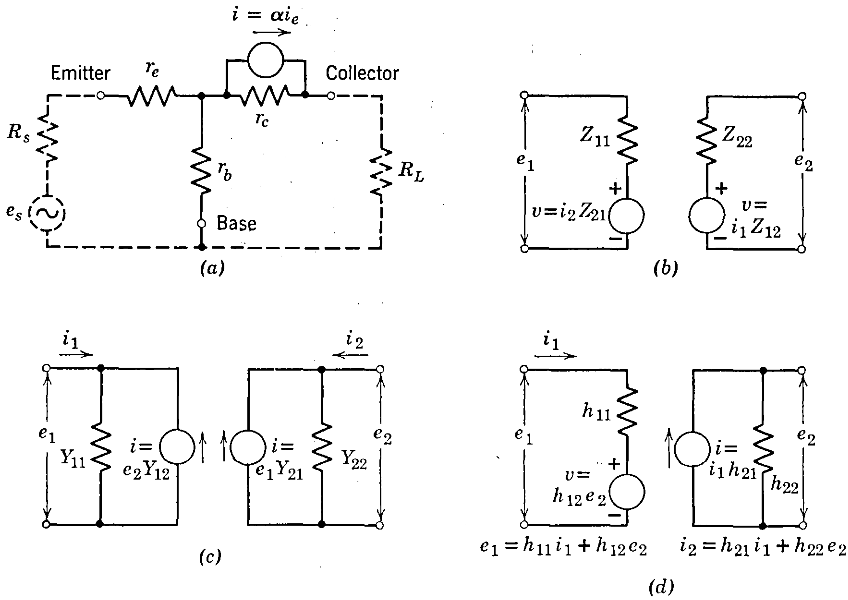


FIG. 14. Equivalent circuits: (*a*) physical equivalent, grounded base connection, (*b*) open circuit impedances, (*c*) closed circuit admittances, (*d*) hybrid.

grounded base condition can be reduced to $-\alpha \dfrac{1}{1 + R_L/r_c}$ if $r_c \gg r_b$; similarly the current gain for the grounded emitter is $\dfrac{\alpha}{1 - \alpha + R_L/r_c}$ if $r_c \gg r_e$; the grounded collector current gain is approximately equal to $-\dfrac{1}{1 - \alpha + R_L/r_c}$ when $r_c \gg r_e$.

In control circuitry the grounded emitter connection finds wide usage because of its higher power gain and because the ratio of output impedance to input impedance is not nearly so large as that for the grounded base arrangement. The grounded base arrangement may be found more suitable in some cases, however, for reasons of frequency response, bias stability, and load isolation. The grounded collector connection is useful as an impedance matching scheme similar to the cathode follower in vacuum tube circuitry, but it does not provide any significant isolation of the input from variations in the load impedance.

The parameter β has been defined as $\alpha/(1 - \alpha)$ which is an approximation of the current gain of a grounded emitter stage where $r_c \gg r_e$ and $r_c \gg R_L$.

The use of transistor parameters specified in terms of other than the physical equivalent parameters, r_e, r_b, r_c, etc., has become common since this simplifies the equations involved. Typical *artificial* systems include open circuit impedance, closed circuit admittance, and hybrid systems as shown schematically in Figs. 14*b*, *c*, and *d*. The values of these black box parameters are generally specified by the manufacturer for the particular circuit configuration normally used in a given application, so the equations for grounded emitter, grounded base, and grounded collector are the same (although the values of the parameters obviously are not). The hybrid parameters are based on the relation between voltage input and current output in the forward and the reverse direction respectively. The subscript on the h symbol normally represents the direction of measurement and the grounded terminal: thus h_{fe} is the forward hybrid parameter for grounded emitter connection, h_{ie} is the inverse hybrid parameter for the same configuration, h_{re} is the feedback parameter, and h_{oe} is output conductance. In some cases upper case subscripts are employed to designate total (d-c) rather than incremental relationships. Numerical subscripts are also employed as shown in Fig. 14*d*.

The calculations involved in multistage transistor amplifiers are quite complex owing to the impedance coupling of the transistor. This is sometimes referred to as the bilateral characteristic. A change in load impedance does not merely mismatch the last stage but causes impedance changes to be reflected throughout the amplifier. Multistage coupled amplifiers can be designed readily by the use of computing machines, but experimental verification is to be recommended. Transformer coupling reduces the work involved since impedance levels can be arbitrarily determined by the transformer design. Sensitivity to load variations can be reduced by feedback, but only at the expense of gain.

The practical limitations on the use of silicon transistors at the present time are (1) the cost and (2) the limited range of collector power and β available in silicon types. Improvement in this situation may come with time.

Input Circuits. The input impedance of typical transistor amplifiers lies in the range of 100 to 1000 ohms for grounded base circuits and 1000 to 5000 ohms for grounded emitter circuits. This impedance may vary considerably with signal level, temperature, the particular transistor used, and the load impedance. Higher impedances may be obtained with grounded collector input but, of course, gain is sacrificed.

Noise. Transistor noise power varies inversely with frequency at low frequencies and then approaches a constant level at higher frequencies. The break frequency varies with the transistor and with the operating bias voltages. For low-level input stages it is possible to operate in the region of zero collector to base voltage. When operated at these low bias values the transistor may equal or surpass the vacuum tube for low noise if a proper impedance match can be obtained.

A transistorized circuit which has been used as an instrument servo amplifier is shown in Fig. 15. This circuit has an overall power gain of 87 decibels and will supply an output power of 2 watts.

Dielectric Amplifiers

Certain materials such as barium and strontium titanate have not only very high dielectric constants but also a dielectric constant which varies with the applied voltage. The possibility of producing an amplifier utilizing the low-frequency input signal to bias the nonlinear dielectric material in a high-frequency circuit, thus producing amplification, has been considered, and a number of experimental units have been built. Although no extensive use has been made of this type of amplifier, it is included here in the interest of completeness.

The basic element of the dielectric amplifier might be considered to be a modulator. In Fig. 16*a* the voltage sensitive capacitor C_v changes its impedance with the low-frequency input voltage. A greater sensitivity of the modulating impedance to the capacitance value can be obtained by using the capacitor in an *LC* circuit close to resonance at the high-frequency power supply. A practical circuit utilizing the resonance principle is shown in Fig. 16*b*. The series capacitor C_L has a high impedance at the operating frequency so that the resonant circuit is fed with an essentially constant current.

Although power gains as high as 40 db per stage have been attained experimentally, there are several reasons for the lack of acceptance in the control amplifier field. (1) A special high-power–high-frequency source is required. (2) When the resonance principle is used, the output is quite sensitive to the load impedance. (3) The voltage sensitive capacitance materials are also sensitive to temperature. Voltage sensitive capacitors find application in systems for automatic tuning of resonant circuits and in the generation of frequency-modulated signals.

Ion Flow (Plasma) Amplifiers

As compared to the vacuum triode, or tetrode which it resembles structurally, the gas-filled tube is characterized by much lower impedances

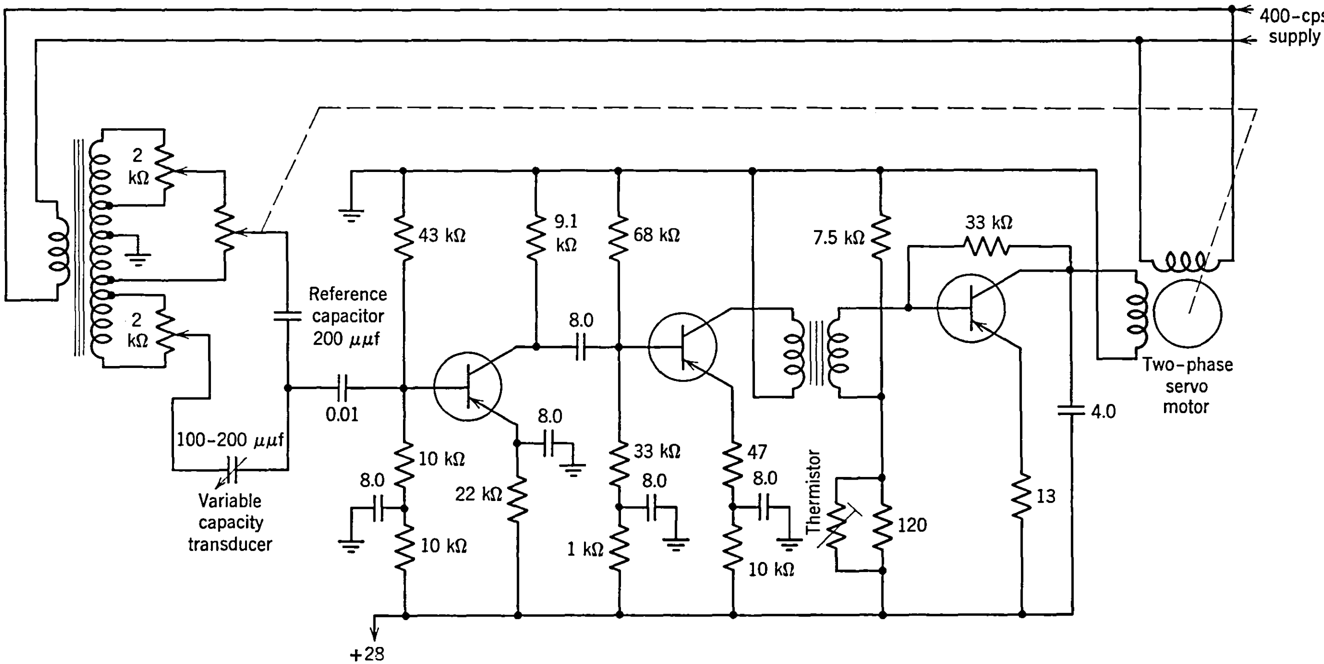


FIG. 15. Schematic of transistorized instrument servo used in a fuel gage. (Courtesy of Minneapolis-Honeywell Regulator Company.)

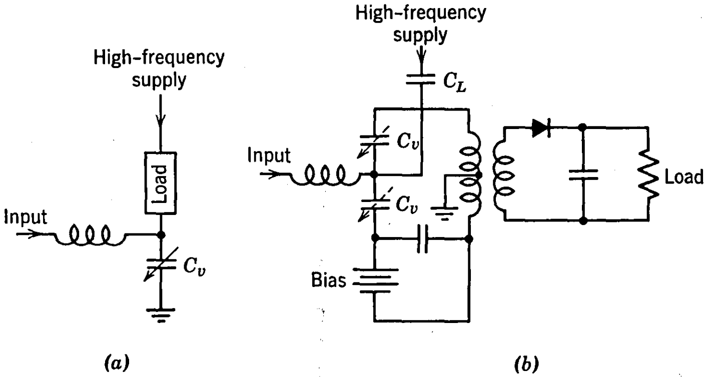


FIG. 16. Dielectric amplifiers: (a) basic element, (b) practical circuit.

and greater current-handling capacity. The conventional thyratron is not capable of proportional amplification since the establishment of the gas discharge condition effectively blocks the grid, and the discharge must be quenched by the removal of the anode voltage. Because of this limitation it is employed either as an electronic relay or with ac applied to the anode.

In general, the thyratron is less reliable than the vacuum tube and in most applications must be considered to have a limited lifetime. It is somewhat sensitive to ambient temperature and is a generator of noise because of the sudden rise of current upon the initiation of the discharge. There are limitations on both the ionization time and the deionization time, although only the latter is long enough to be of any concern in control applications. Its chief advantage lies in its large power-handling capacity for a given envelope size and its low internal voltage drop.

A typical relay circuit is shown in Fig. 17*a*. Resetting of the device must be accomplished manually. Phase modulation of the a-c voltage on the grid allows the firing time to be varied in the circuit of Fig. 17*b* so that the duration of the plate current pulse is varied. The output may be filtered if dc is required, or it may be applied directly to the field windings of a servo motor or of a rotary amplifier.

Special gas-filled tubes of various kinds have been devised, including those that function as a vacuum triode until they fire, and the plasmatron in which the plasma is kept alive (ionized) continuously by the use of auxiliary electrodes. Roughly proportional control can be obtained by the plasmatron.

Application of the thyratron amplifier is in industrial controls at high-power levels where high efficiency is of importance and reliability is not a primary consideration. At lower power levels the power transistor has

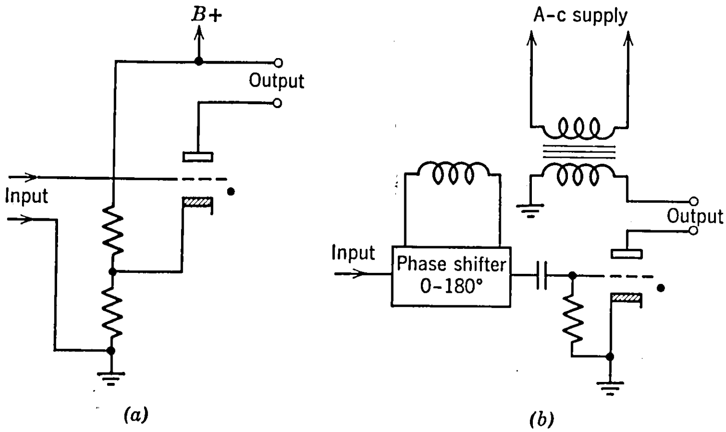


FIG. 17. Basic thyratron control circuits: (*a*) relay circuit, (*b*) conduction angle control.

definite advantages for control applications. Thyratron output stages should have adequate shielding and filtering to prevent radio noise from being radiated.

Magnetic Amplifiers (See also Chap. 25, Magnetic Amplifiers)

Magnetic amplifiers are based on the modulation of the supply power (a-c) by a d-c signal by virtue of the nonlinear characteristics of certain ferrous materials. They are characterized electrically by low input impedance, large power gain per stage, and slow response. Physically they are large, heavy, and rugged. The reliability of magnetic devices is high, although they may not be as tolerant of high temperatures as vacuum tube amplifiers.

The Saturable Reactor. The basic element of the magnetic amplifier is the saturable reactor shown schematically in Fig. 18*a*. Additional windings may be provided for multiple inputs, for bias, and for feedback as shown in Fig. 18*b*. The bias winding allows for adjustment of the operating point near the knee of the saturation curve where the gain is highest. Either positive or negative feedback may be employed: negative feedback affords gain stability, increases linearity, and improves response at the cost of gain; positive feedback increases gain but reduces stability and can produce triggering if carried far enough.

Construction. The magnetic amplifier is outstandingly rugged and reliable if constructed of high-quality materials and with due attention to adequate mechanical design. It is not sensitive to shock or vibration, and maintenance requirements are so low that it may be placed in a less accessible location and bolted into the structure if desired.

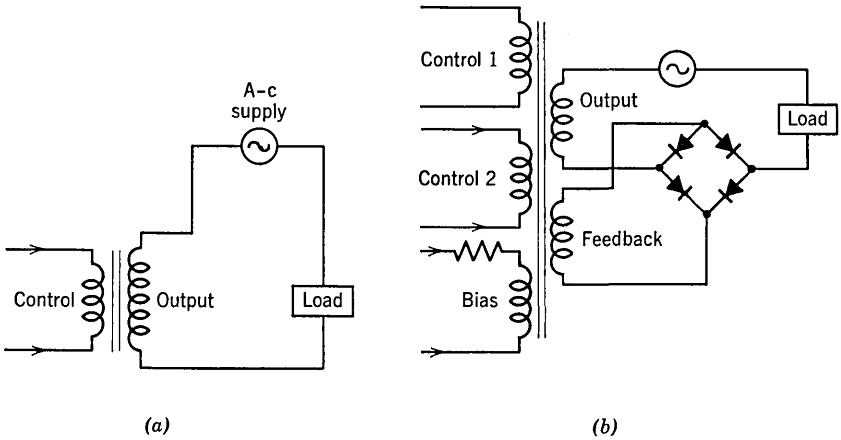


FIG. 18. Basic winding configurations: (*a*) saturable reactor, (*b*) saturable reactor with bias and feedback windings, d-c input, a-c output.

Magnetic amplifiers may be fabricated from commonly available materials for use at any desired output power level and to match a-c loads of a wide range of impedances. Low-impedance d-c loads are readily matched through the use of suitable dry rectifiers. Although the magnetic amplifier suffers by comparison with transistor and vacuum tube amplifiers in size and weight of low-power units, it is more competitive at higher power levels. The relatively high efficiency and the ability to handle high power levels make the magnetic amplifier the obvious choice for output power levels over about 10 to 20 watts in which the response is adequate, particularly in fixed installations.

Circuits. The magnetic amplifier requires a d-c input and produces a modulated carrier output. Auxiliary circuits such as phase sensitive detectors for operating from a-c inputs or rectifiers to provide d-c outputs may be required. The circuits of Fig. 19 employ self-saturation in that the output current is caused to flow in unidirectional pulses by the use of rectifiers. The circuit in Fig. 19*a* can be used in cascade since it provides d-c output. Four cores are required since a core can be driven into saturation in only one direction by the d-c control current, and bipolar full-wave output is desired. In Fig. 19*b* a circuit is shown which produces half-wave pulses suitable for driving the control phase of a two-phase servo motor, in a somewhat less efficient manner than if a sinusoidal driving voltage were used, but allows bipolar a-c output to be obtained with only two cores.

The input resistance of the magnetic amplifier is characteristically low and may be in the order of 10 to 500 ohms, depending on the type of

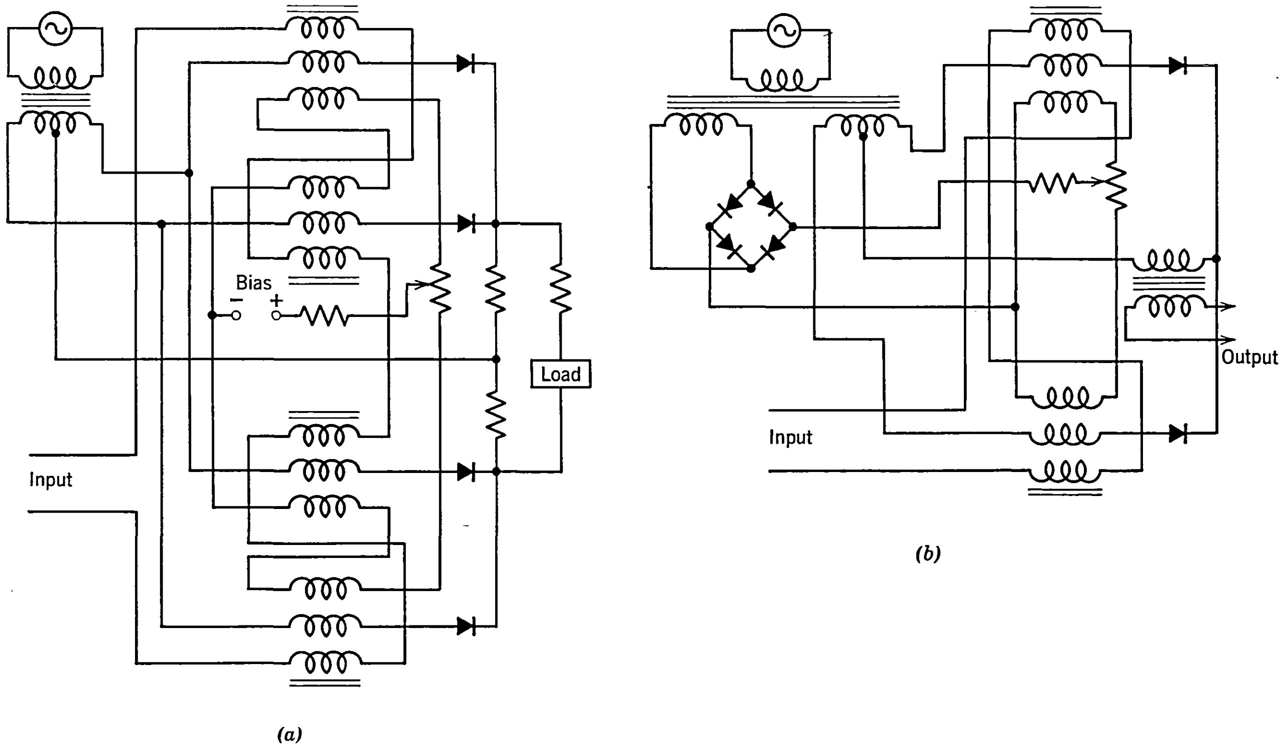


FIG. 19. Typical magnetic amplifier circuits: (*a*) complete magnetic amplifier with d-c input and output, (*b*) half-wave magnetic amplifier with d-c input and bipolar transformer-coupled a-c output.

amplifier. Rectifiers must have low forward resistance to function efficiently, so silicon junction diodes or selenium diode units are preferred to vacuum diodes.

Stability. The null stability of the magnetic amplifier may be improved by matching of core and rectifier characteristics. The actual stability attainable is a function of the time constant, core, control winding, etc., but it may be better than 10^{-9} watts for a typical unit over fairly wide temperature variations. In view of the low input resistance, the voltage stability is not outstanding, although it is better than that of the vacuum tube amplifier but poorer than that of the chopper–vacuum tube combination.

Figure-of-Merit. A figure-of-merit, defined as the ratio of the gain to the time constant, is useful in determining how much gain can be obtained in a single amplifier and still provide acceptable response. This figure-of-merit can be derived in the idealized situation as

$$M = kfA_cN_s^2RL/R_S^2(1 + R_L/R_S)^2,$$

where f is the supply frequency, A_c the core area, N_s the number of turns in the supply winding, R_S the resistance of the supply winding, R_L the load resistance, and k an empirical constant depending on factors such as core material, shape, space factor, etc. Typical values for M are on the order of 10^4 to 10^5 for 400 cps units. If the desired response cannot be obtained in a single stage, cascading will produce improvement.

The figure-of-merit criterion should be applied with due consideration for the basic limitations on response of the magnetic amplifier imposed by (1) the supply frequency and (2) the core material. The amplifier is not capable of responding during the period when the supply voltage is near zero, and this can be generalized in the practical case by stipulating that the response time cannot be less than one-half the period of the supply. The core material limits the maximum frequency at which acceptable gain and performance can be obtained to the order of 100 to 10,000 cps, although operation as a modulator or as a computer element with less stringent requirements on gain and efficiency is possible to much higher frequencies.

Hybrid Amplifiers with low-level transistor amplifier stages followed by a magnetic output stage allows the higher input impedance and smaller size and weight of the transistor to complement the power-handling capacity and low output impedance of the magnetic amplifier. Vacuum tubes may also be used in hybrid units.

Advantages and Disadvantages. The relatively large outlay of development effort required for obtaining optimum performance from magnetic amplifiers severely limits their use in experimental apparatus, and it is recommended that the prepackaged units available from various manufacturers be utilized in preference to special designs wherever possible.

The properties of magnetic amplifiers of interest in control applications may be summarized as follows.

Advantages.

1. High reliability. Mechanically rugged. Low maintenance requirements.
2. Wide range of output power and impedance levels.
3. Inputs and output may be isolated for dc.
4. High efficiency.

Disadvantages.

1. Excessive size and weight for low-power applications.
2. Temperature limitations of rectifier and some core materials.
3. Low input resistance.
4. Relatively expensive, high development cost.

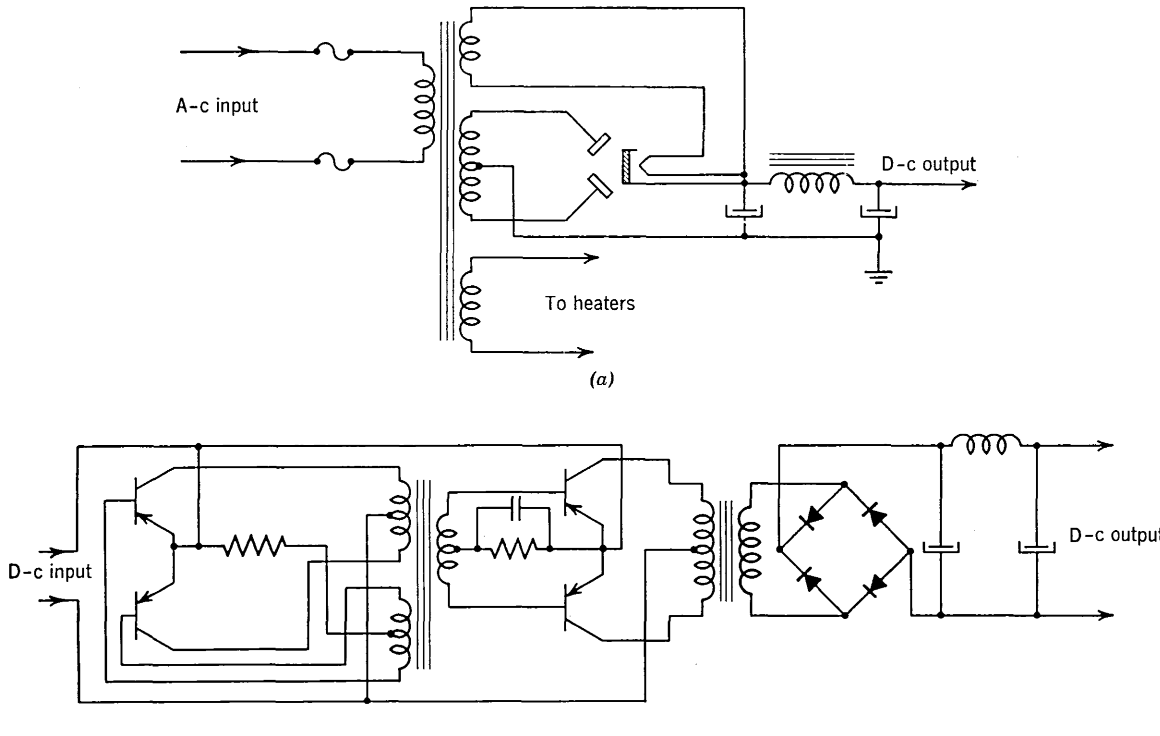
Power Supplies for Electronic Amplifiers

Many control applications are such that the performance requirements placed on the power supply are not at all demanding. Voltage, ripple, and internal impedance may vary through fairly wide limits without seriously reducing the performance. Reliability is usually the single most important requirement.

Power supplies which operate from an a-c source include vacuum or dry rectifiers with *RC* or *LC* filters. A dynamotor or transistor power converter might be used when the primary supply is ac. Batteries are suitable for portable equipment when other primary power is not available. Some examples of power supplies are shown in Fig. 20.

Carrier frequency servo amplifiers are tolerant of high ripple levels, particularly in the output stages. Output stages that do not require d-c supplies can be designed for carrier frequency servos where the output stage functions as a demodulator, or where it drives a two-phase servo motor. If only the low-level, low-current stages are supplied, the drain on the filter is greatly reduced and the use of *RC* filter elements is more practical.

For computer applications the performance requirements may be quite stringent. The regulation of the supply voltages for amplifiers, particularly unstabilized d-c amplifiers, may be held to less than 1 volt for wide variations of input and load, and superregulators which hold their output to within 0.01 volt may be used for signal biases, limits, initial conditions, and similar critical applications. The basic configurations of these sup-



(b)

Fig. 20. Power supplies for electronic amplifiers. (*a*) Vacuum tube supply; dry diode may replace vacuum tube, resistor sometimes used instead of inductor. (*b*) Transistorized d-c power converter; use of separate driver permits driving output transistors at maximum power.

plies are similar to that shown in Fig. 21; the main differences are in the stability and gain of the d-c amplifier and in the type of reference used. A regulator of moderate stability and high reliability can be constructed, employing solid-state devices.

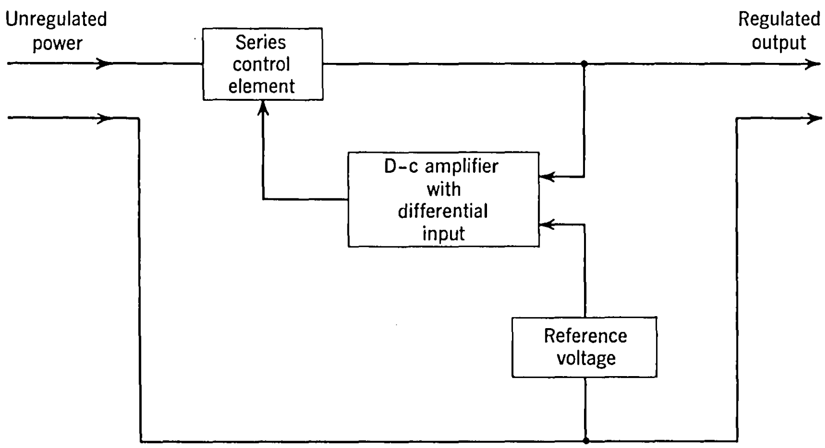


FIG. 21. Basic regulator diagram.

5. ELECTROMECHANICAL AMPLIFIERS (See also Chap. 7, Instrumentation Systems)

Description. This classification as used here is intended to include the amplifiers that amplify an electrical input by first producing a mechanical motion through motor action, and then producing a modulation of the supply power by variable inductance, movable contacts, or other mechanically actuated control. The term electromagnetic-mechanical has also been applied to these devices.

Types. The electromechanical amplifier is sometimes overlooked because of preoccupation with more complex electronic devices, but it has widespread application in the industrial control field and has found its way into a number of military applications. The electromechanical relay is a very common example of this category. Some other examples include the combination of moving coil galvanometer and potentiometer; voice coil torquer and moving coil pickoff; and microsyn torquer and signal generator. These units are diagramed in Fig. 22.

The electromechanical amplifier employing moving contacts is capable of large power gains. A sensitive relay requiring 10 mw of input power can control 100 or more watts of output power. Isolation of the various inputs so far as direct potentials are concerned can be accomplished by the use of separate input coils.

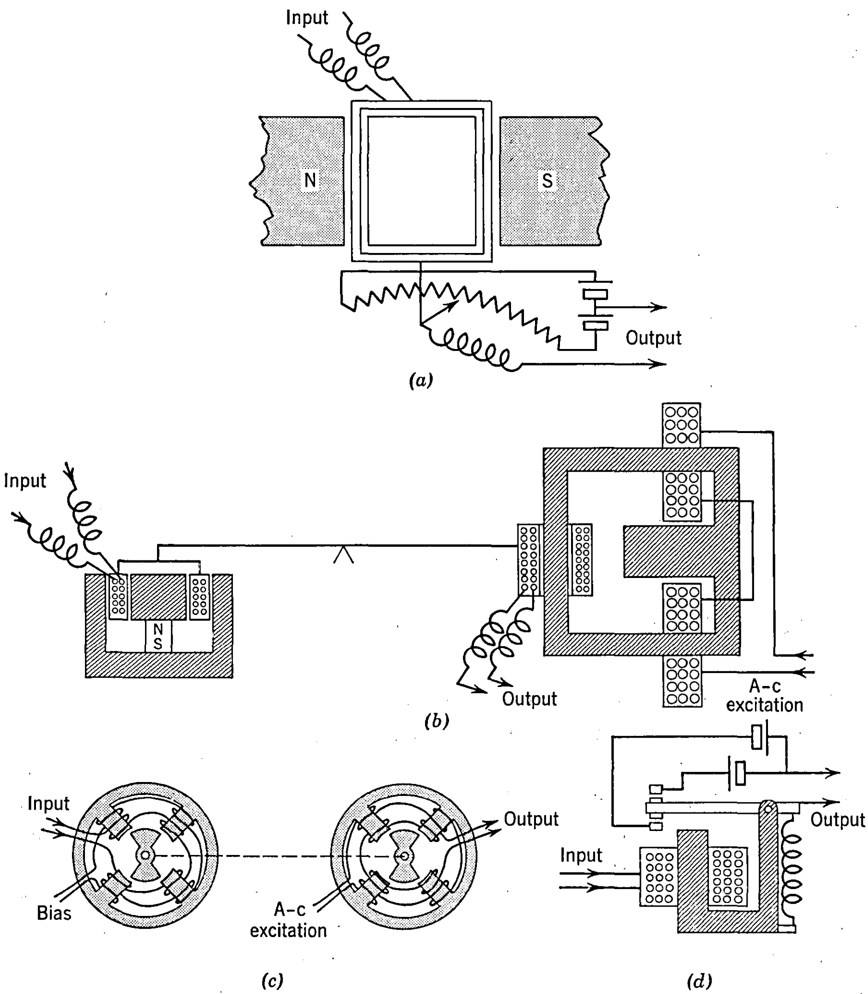


FIG. 22. Electromagnetic mechanical amplifiers: (*a*) moving coil galvanometer-potentiometer output, (*b*) voice coil (dashpot) torquer, moving coil pickoff, (*c*) variable reluctance (microsyn) input and output, (*d*) relay off-on electromechanical-mechanical amplifier.

Perhaps the units shown in Figs. 22*b* and *c* should be classified as modulator-demodulators since they are more useful in this capacity than directly as an amplifier. The torque produced is a direct function of the input current, and very little zero drift is present. The application to closing a feedback loop around electronic amplifiers to stabilize their gain was mentioned in the discussion of d-c amplifiers in Sect. 2 of this chapter.

Response. Because the input current of these devices produces a magnetic field by flowing through a coil, the input characteristics are similar to those of magnetic amplifiers in that they have resistances in the order of 100 to 10,000 ohms and relatively high inductance. They respond to d-c inputs generally, although a phase sensitive carrier device could be produced.

The response of the electromechanical amplifier is typical of devices incorporating electromechanical elements and is slow in comparison to electronic amplifiers. The actuation (closing) time for a relay may be of the order of a millisecond. Longer delays can be incorporated if they are desired by the provision of shorted turns surrounding the magnetic paths. Response of heavily damped units such as the microsyn is rapid compared to other electromechanical devices, particularly where they are used in the feedback of a high-gain amplifier. Response times of less than a millisecond are theoretically possible and have been approached in practice.

Nonlinearities. The relay is basically nonlinear. It requires an input current of some minimum value before it closes. The current must then be reduced considerably below this value before it opens again. These characteristics are due primarily to the nonlinearity of the force-displacement relationship. Magnetic hysteresis plays a part in this action also and contributes to the effect. After the field increase to a sufficient value to overcome the spring and pull the armature in, the force increases as the armature approaches the pole owing to increased magnetic field. In opening, the relay must reduce the field to the point where the spring pulls the armature away, even though the armature is in contact or close proximity to the core. This hysteresis of relay action can be reduced by reduction of the travel by stops or spacers; however, the nonlinearity has a useful function in providing rapid and forceful action of the contacts.

Other electromechanical devices also exhibit nonlinear characteristics owing to stiction, magnetic hysteresis, or other such effects. Sensitive galvanometer suspensions employ torsional supports, and they can be used to overcome the nonlinearities of pivot and jewel or other bearings, although they impose an additional spring restraint which may interfere with the desired operation. Vibration or tapping is beneficial in freeing bearings of electromechanical devices, although they can also produce undesired mechanical rectification effects.

Linearizing Performance. There are a number of techniques for linearizing the performance and lowering the threshold of basically nonlinear amplifiers such as relays. The introduction of a dither current of sinusoidal or triangular waveform superimposed on the input causes the relay to cycle with the time period, in each condition, a function of the signal level. Frequencies of 10 to 100 cps might be employed for dither. The effect on the transfer function is to establish a band where the output is proportional to or at least a continuous function of the input when averaged over several cycles of the dither frequency. The gain of the relay is less than if it were operated without the dither, and the response time is effectively reduced, although both of these effects depend on the level of dither employed and are usually tolerable in typical applications. Additional wear due to dither is negligible in signal and low-power applications; in fact, it may actually promote good contact because of polishing action. At high power levels such dither may not be tolerable owing to reduction of contact life, and the deliberate introduction of time delay by means of a shorted turn may be necessary to prevent undesired hunting.

It is possible to construct feedback systems which are quasi-stable in that they hunt back and forth across the null or zero condition in the absence of an input signal. This condition is effective in linearizing relay action similar to the effects of superimposed dither.

Other Characteristics. The input threshold level for electromechanical units vary with the construction. The threshold level for a sensitive relay may be as low as 10 mw, and the fluid-floated microsyn has been shown to be sensitive to power levels as low as 5×10^{-10} watts. However, this still represents only a questionable improvement over the threshold of the magnetic modulator.

Output characteristics of relays include low impedance and enormous power-handling capabilities. A given relay will break a much larger a-c load than d-c load since the arc upon opening can be extinguished when the current passes through zero. Inductive loads are the most difficult to handle, although the arc may be controlled by employment of quenching networks made up of resistance-capacitance or resistance-capacitance-diode combinations.

Where low-level signals are to be amplified, the electromechanical amplifier devices must be operated at reduced loads. Even the relay must be rated for reduced loads in the more sensitive designs. Reaction torques occur in the variable transformer type devices when the output is heavily loaded.

The cost of development of electromechanical units is relatively high, and development of a unit for a single application will rarely be economic.

Standardized designs are available from a number of sources, however, and these may be adapted to many applications at a cost comparable to that of electronic amplifiers.

6. ROTARY AMPLIFIERS (See also Chap. 22, Actuators)

Description. A rotary amplifier is a mechanically driven device which is capable of delivering electrical power derived from the mechanical drive to its output terminals in response to an electrical input. The simplest device of this type is the d-c generator with field current control. The metadyne and Amplidyne are other examples.

Applications. Rotary amplifiers are suitable for producing high power levels such as might be required to drive a large d-c motor. If a suitable sized unit is available, they are readily applied; if a special unit must be designed, the cost of development may be greater than that of a magnetic amplifier for the same application. The energy storage in the rotary mechanical elements and the heat capacity of the windings allows large peak power to be drawn from the amplifier, provided the average power is kept within the ratings. The devices are mechanically simple and can be maintained by relatively untrained personnel. The useful life can be made quite long if reasonable attention is given to regular maintenance of the bearings and commutator.

Rotary amplifiers have been replaced by hydraulic systems in many applications in recent years. If the rotary amplifier is to be used to drive a d-c motor, the combination of hydraulic pump, servo valve, and actuator is smaller and lighter for a given power level and can be made to have faster response than the rotary amplifier-motor combination. If electrical output is required from the system, or if the use of hydraulics is objectionable from the standpoint of noise or cleanliness, the rotary amplifier is to be preferred.

Characteristics. The input signal requirements for rotary units is similar to that for the magnetic and electromechanical devices, although their resistance and inductance is often somewhat higher since more space is used for the input windings. This is more a consequence of the size of the typical unit than of any basic difference. The input impedance can, of course, be modified by the use of feedback from the output.

The input threshold is related to the magnetization curve of the machine considered. Sufficient current must flow in the input winding to cause a change in the magnetization of the field in order to produce a change in the output. The gain at these very low levels is small owing to the shape of the magnetization curve; if feedback is involved, the input and output impedances of the unit for small signals differ from those for large signals. The introduction of superimposed dither or the use of quasi-stable systems is not so attractive as for relays since the power levels involved are such that continual hunting at the null condition wastes sizable amounts of power. The incorporation of nonlinear electronic devices to compensate for the nonlinearities for small signals is a possibility.

Figure-of-Merit. The response of the rotary amplifier can be described by a figure-of-merit M based on the ratio of power amplification A to the time constant T, as in the case of the magnetic amplifier. For a given amplifier with constant mechanical supply speed, this can be shown to be a constant dependent on the machine design. For a separately excited generator this reduces, in the simplified case, to

$$M = A/T = R_f^2/\alpha R_L,$$

where R_f is the field winding resistance and R_L is the load resistance. The constant for a given generator α relates the field inductance L_f to the voltage gain G by the equation $L_f = \alpha G^2$. The time constant T is related to the field winding characteristics by the expression $T = L_f/R_f$.

Configurations. Various rotary amplifier configurations have been devised, but basically they may be considered as modifications of two configurations, the separately excited generator shown schematically in Fig. 23*a* and the metadyne shown in Fig. 23*b* of the same figure.

The separately excited generator is capable of providing an output voltage proportional to the field excitation. The output impedance is low, i.e., the output voltage is relatively independent of the load current, so that feedback windings are sometimes employed to limit the load current.

The metadyne is in effect two generators with a common armature. If the output brushes of a simple generator are shorted, the current which

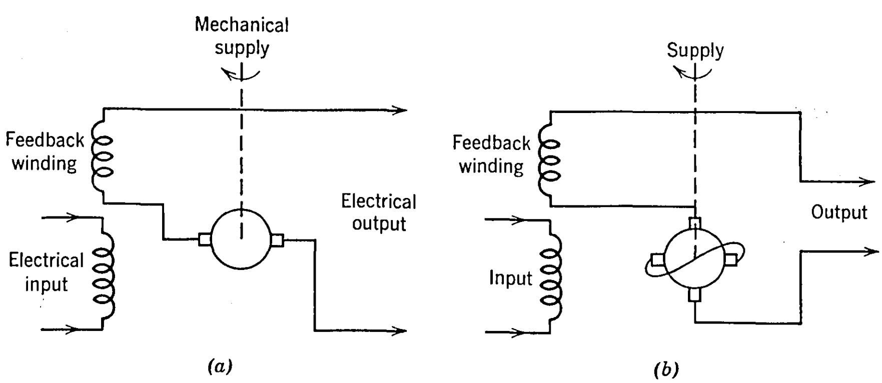


FIG. 23. Rotary direct-current amplifiers: (*a*) separately excited generator, (*b*) metadyne.

flows produces a magnetic flux at right angles to the external field flux. If a second set of brushes is added at positions 90° electrically from the first, an output current proportional to the input excitation can be obtained. Since the demagnetizing force of the output current directly opposes the external field, the metadyne behaves as a current rather than as a voltage generator. A feedback winding can be employed, and if the feedback winding is made to compensate exactly for the output current demagnetizing effect, the amplidyne configuration is obtained. The amplidyne is essentially a constant voltage generator.

Various types of rotary amplifiers may be used in cascade, and feedback may be applied across more than one amplifier. A combination of one or more generators with a d-c motor is referred to as a Ward-Leonard control.

7. PNEUMATIC AND HYDRAULIC AMPLIFIERS (See also Chap. 7, Instrumentation Systems)

Pneumatic Amplifiers have been applied widely in industrial control devices, particularly in chemical processing. They are useful wherever a pneumatic supply is available, and the limitations on speed of response are acceptable. Pneumatic controllers have been devised to control temperature flow, level, pressure, and other process variables. A pneumatic controller may respond to mechanical motion, force, or fluid pressure input from the sensor. This signal is compared to the reference level, and the error signal is amplified by a pneumatic amplifier. The output of the pneumatic controller is a pneumatic pressure to operate a valve or other control device. The controller may be provided with a pen and chart arrangement to record the value of the controlled (input) variable.

Hydraulic Controllers can also be constructed, but hydraulic fluid is not as clean to work with as air and requires return piping. On the other hand, the hydraulic fluid transmits impulses more rapidly and provides a stiffer actuator action. Hydraulic controllers have been applied widely in power steering and braking on automobiles and in hydraulic power systems for tractors and earth-moving equipment, but they are not common in the low-power control field. Perhaps the output stage of the two-stage hydraulic valve described in Chap. 7 is the best example of the use of a hydraulic amplifier in a high-performance control device.

A pneumatic controller with reset adjustable proportional band and rate action is diagramed in Fig. 24*a*, and its equivalent electrical diagram is given in Fig. 24*b*. The amplifiers A_1, A_2, and A_3 are computer amplifiers required to simulate the proportional band operation; A_4 simulates the flapper-nozzle, and A_5 simulates the pilot valve. The dynamics and nonlinear characteristics are not represented in this equivalent circuit.

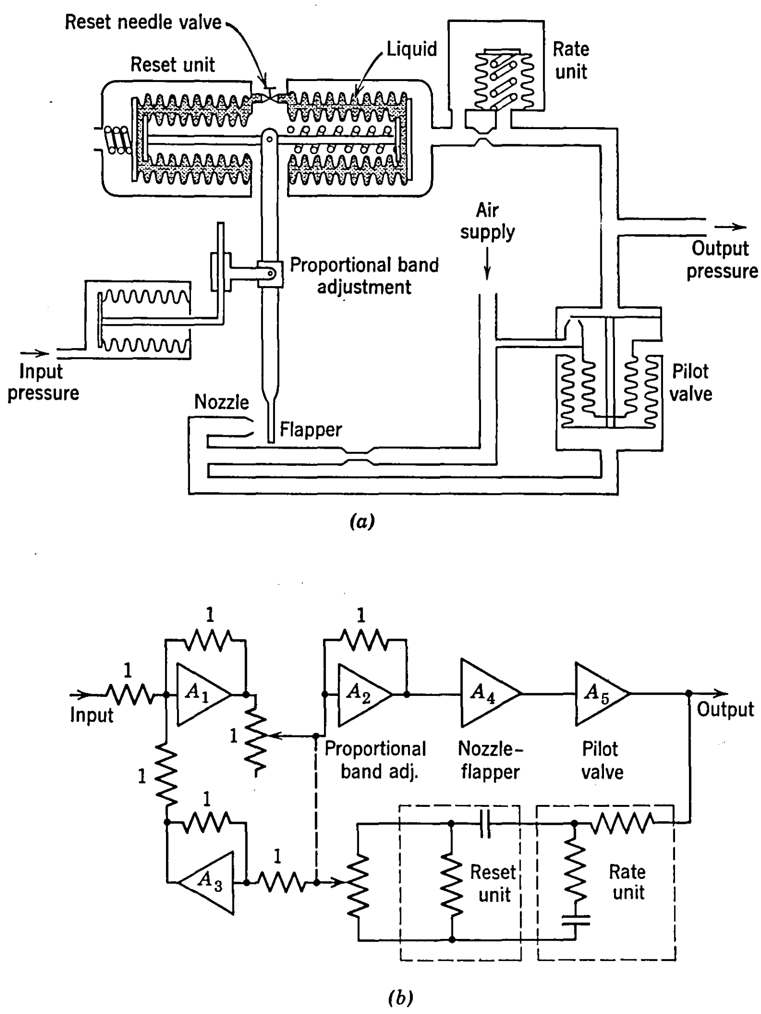


FIG. 24. Pneumatic amplifier with reset and rate feedback: (*a*) pneumatic amplifier, (*b*) electrical equivalent.

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Actuators

Adam G. Kegel and George S. Axelby

1. INTRODUCTION

Definition. A standard definition of an actuator has not been proposed by any of the national engineering societies which have been active in the field of automatic control systems. Therefore, an actuator will be defined by describing commonly accepted usage of the term and by comparison with amplifiers.

The principal distinction made here between an *amplifier* and an *actuator* is that the former develops an output having the same functional form as the input, but at a higher power level, whereas the latter converts from one form of energy to another form suitable for controlling the process. This results in a somewhat broad concept of what is meant by an actuator. For example, this definition can include such devices as electrohydraulic valves wherein an electric signal produces a fluid motion, and indeed valves must be considered in any discussion of actuators. Pneumatic, electropneumatic, and electrohydraulic valve actuators are covered in some detail in Chap. 7, Instrumentation Systems.

However, in the usual case the term actuator implies a device whose output is mechanical motion, and most of the actuators described in this section fit into this category. Devices such as heaters and blowers are not described, and pumps are described only insofar as they are used in conjunction with hydraulic motors.

It should be noted that the power level of the command signal to an actuator may be large or small relative to the mechanical output power, and that in some applications an actuator may be considered as a power amplifier as well as a power converter.

Actuators and Rotating Amplifiers. There are some control components having mechanical motion which are not actuators, although they are usually associated with actuators. In the particular field of electric control, such devices are the Amplidyne, the Rototrol, and the Regulux. (These are trade names of particular machines made by the General Electric Company, Westinghouse Electric Corporation, and Allis-Chalmers Company, respectively.)

The Amplidyne, Rototrol, and Regulux are essentially electric generators with special feedback windings to increase the power and voltage gain between the input and output signals. Although mechanical motion, i.e., generator rotor rotation, is involved, the useful output signal is basically electric. These rotating amplifiers, as they are frequently called, are used to provide signals at a high power level for actuators. Since these devices are used with various standard actuators and have individual specifications as power amplifiers, they will not be discussed further in this section.

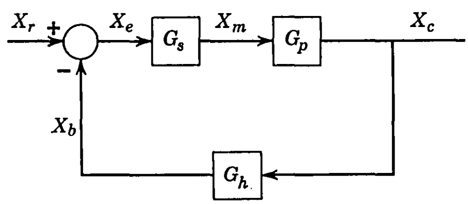
The Ward-Leonard system contains an actuator, usually a d-c motor, and a generator, which may be one of the rotating amplifiers previously mentioned driven by another motor, acting as a power amplifier. Although Ward-Leonard systems contain an amplifier, they are manufactured in various sizes and by several companies as a unit and are used as actuators. Therefore, their overall characteristics will be mentioned briefly, but the details about the actuator may be found in the discussion on d-c motors with armature control.

Actuator Combinations. In many control systems, an actuator may be used to control another actuator. For example, an actuator may be used to (1) move a throttle on an engine which in turn is an actuator controlling speed or position; (2) turn a rotatable transformer which controls a voltage level; (3) turn a valve which controls the flow of a liquid or gas; (4) move a vane or spool valve controlling the flow of oil to a piston which itself may be another actuator controlling speed or position.

In this section system configurations and combinations of various actuators will not be presented. Instead, basic characteristics of actuators and their use will be discussed so that the performance of actuators in combination can be derived according to the special application.

However, the last example (4) of actuator combinations is one often used in hydraulic control devices. The vane or spool valves are usually not considered to be separate actuators because they are designed as integral units with the output piston. The input is usually an electric signal at a low power level, and the one accessible output is mechanical motion at a higher power level. These devices will be treated as one actuator.

Use of Actuators in Feedback Control Loops. A typical feedback control loop is shown in Fig. 1. From the previous discussion it is obvious



Transfer functions:

G_s for elements at low signal levels,
G_p for elements at high power levels,
G_h for elements in feedback path.

Signals:

X_r for loop input or reference,
X_c for loop output or controlled variable,
X_b for loop return or primary feedback,
X_e for loop actuating signal,
X_m for internal actuating signal, or manipulated variable.

FIG. 1. Typical feedback loop in which actuators may be used.

that actuators can be used in G_s, G_p, or G_h and that they may appear in all of these parts of the same control loop. In many control loops the only actuator is in G_p, at the highest power level in the loop, directly controlling X_c, the variable which is being controlled as a function of the input. Although the discussion of actuators should not be limited to actuators that can be used only in this capacity, the choice of such an actuator in the preliminary design of a control is particularly important for the following reasons.

1. At the high power level, the size, weight, and cost of an actuator are usually a large portion of the size, weight, and cost of the entire control loop.

2. The performance of the control loop is often limited by the actuator.

3. It is usually difficult to modify the actuator to improve loop performance because of the expense. However, special actuators and modifications of standard models frequently can be obtained and should be considered when the actuator is chosen.

Some of the characteristics that must be considered when choosing an actuator will be given in Sect. 2, Actuator Specifications.

Standard vs. Special Actuators. There are several actuators such as a-c motors, d-c motors, hydraulic valves and actuators, and mechanical clutches and brakes, which are made by different manufacturers with many similar characteristics. They are sometimes interchangeable, and they can be used for a wide variety of control purposes.

Methods of measuring, analyzing, and using these actuators are well established; therefore these actuators may be referred to as *standard actuators,* and the emphasis in the discussion of actuators will be given to standard actuators. *Note.* There is no definite distinction between standard and special actuators, and the choice of classification has been influenced by the detail of the manufacturer's data as well as the availability and general use of the actuator.

2. ACTUATOR SPECIFICATIONS

Basically, the actuator characteristics must be determined from the system specifications, the nature of the input signals, output signals, the driving impedances, and the load impedances with which it is to be used. The system may or may not be a feedback control loop, although the emphasis will be given to the use of actuators in feedback control loops.

Requirements

Power and Torque. Since an actuator produces mechanical motion it must be capable of providing sufficient torque and speed to power the load, the output device it is moving, under all operating conditions. For example, an actuator may be required to accelerate a load against an external restraining force and change from one velocity to another in a particular time interval. It is required that the actuator simultaneously provide the necessary power and torque and be capable of exceeding the maximum range of velocity and accelerations of the load. (In a position servo with unity feedback, $G_h = 1$, the output motion should be nearly equal to the input, and the actuator must be capable of providing the maximum combination of the loop input speed and acceleration.) The method of determining these requirements numerically is discussed in Sect. 4, Selecting Actuators.

The inertia and friction in the actuator should be small with respect to the load inertia and friction because some of the actuator power is used to accelerate its own inertia (or mass) and to overcome its own friction forces.

Figure of Merit. Although the values of the actuator torque and inertia vary considerably from actuator to actuator, a figure of merit is often used to describe the accelerating capability, effectively the rapidity of the actuator response. The figure of merit is the torque-to-inertia ratio or more significantly the torque-squared-to-inertia ratio. The torque is the actuator stall torque at rated voltage. The first ratio indicates the actual acceleration capabilities of the actuator without a gear train; the second ratio represents a number which is independent of a gear train ratio (see Refs. 1 and 2 for further discussion).

The units of the torque-squared-to-inertia figure of merit vary. It may be expressed in oz-in./sec^2 for small actuators or in lb-in./sec^2 for larger actuators, or in lb-ft/sec^2 for very large units. The number itself varies widely. It may vary from 1×10^2 to 2×10^5 lb-in./sec^2 for electric servo motors, and from 1×10^7 to 2×10^8 lb-in./sec^2 for hydraulic piston-actuated servo motors, depending on the size and power range. However, for the same power range and type of actuator, the one having the largest torque-squared-to-inertia ratio is chosen, assuming that other actuator characteristics are nearly the same. *Note.* It is assumed here that the actuator and load torques referred to the same shaft are nearly equal. However, if the gear train should have a large reduction and the load inertia is much less than the motor inertia referred to the same shaft, the actuator with the largest torque-to-inertia ratio may be chosen. If the load inertia is much larger than the motor inertia, the actuator with the largest maximum torque will be selected when maximum load acceleration is required.

Travel. Some actuators such as linear actuators and single vane actuators have output members with a limited amount of travel. Of course, it is a requirement that the overall motion of the actuator be great enough to include the specified travel of the load.

Gain. The gain of an actuator is usually specified as the slope of the curve relating its steady-state output velocity (or position) to its input command signal. A typical gain curve is sketched in Fig. 2. Generally, it is required that this slope be linear within 1 to 20%, depending on the application, until the velocity exceeds the specified maximum.

Another gain characteristic of the actuator may be specified as the torque gain. This is the slope of the curve relating the actuator output torque to the input actuating signal. In the usual case, this should be linear within the operating range of torques.

Dynamic Characteristics and Time Constants. As gain characteristics describe the steady-state operation of an actuator, the speed-torque-

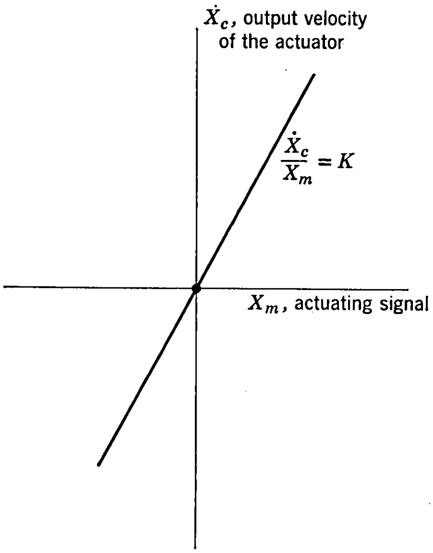


FIG. 2. Typical linear actuator gain curve, load torque = 0; actuator connected directly to the load.

voltage relationships, also obtained by steady-state measurements, may be used to predict the approximate dynamic operation of the actuator. A plot is made of the actuator output speed for corresponding torque loads on the actuator output member with various values of actuating signals applied. *Note.* Hydraulic actuator speed-torque curves are sometimes plotted in units of flow and pressure rather than of speed and torque. However, for a given actuator, flow and pressure are proportional to speed and torque. Typical, idealized speed-torque curves are shown in Fig. 3. Their use in determining the principal time constant of the actuator is shown in Fig. 5, in which a simple inertia load is assumed. The transfer function becomes more complex when spring-mass loads are used. (See subsection on Loading Effects on Actuator Characteristics in Sect. 4.) To prevent objectionable variations in loop operating characteristics, it is usually desirable to have the slopes of these curves parallel within $\pm 25\%$, although this is not always possible.

In addition to the equivalent time constant which may be calculated from speed-torque curves, there are usually other time constants associated with the actuator characteristics. If a simple inertia load is used,

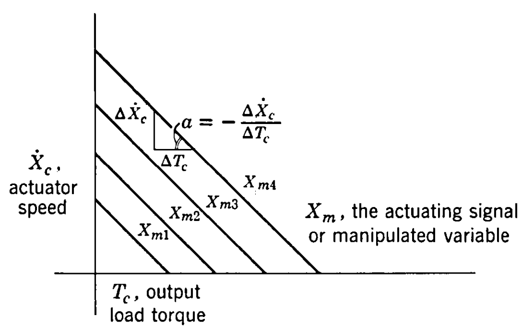


FIG. 3. Typical linear actuator speed-torque curves (for feedback control systems). Actuator connected directly to the load.

the other time constants are associated with input components which receive the actuating signal. These time constants are usually one-twentieth to one–two hundredth of the principal time constant and may be neglected when analyzing a closed loop containing the actuator. The possibility of secondary time constants should always be considered, particularly in designing high-performance control loops, and if the value of the secondary time constant for an actuator being evaluated is not known or specified, it should be obtained from the manufacturer, calculated, or measured (see Actuator Measurements in Sect. 3).

Nonlinearities

Although the characteristics of an actuator are often considered to be linear relationships, the gain characteristic always has some degree of nonlinearity, and the time constant generally varies as the operating conditions change. Common nonlinear gain characteristics are shown in Fig. 4. Many of the nonlinearities have been given names.

Dead Zone. Figure 4*b* illustrates dead zone or dead band. This indicates that the actuator does not develop an output until the magnitude of the actuating signal exceeds a certain level. This is usually caused by stiction, a relatively constant frictional force which opposes motion. In many actuators the value of the stiction force is very small relative to the maximum force which the actuator can produce, and it is often neglected. However, for high-performance control systems even a small dead zone must be considered with respect to the error it may produce. This is considered in more detail in Sect. 4, Selecting Actuators. The dead band for actuators used in high-performance loops is usually 1 to 2% of the proportional band.

Saturation. Another form of nonlinearity, shown in Fig. 4*c*, is referred to as saturation or limiting. This characteristic is present in all actuators. The linear portion of the curve is referred to as the proportional band, and the maximum value of the actuating signal within the proportional band is sometimes referred to as the dynamic range of the actuator. The point of transition between the linear slope and the zero slope of the curve is known as the knee of the curve. The knee of the curve may not be a sharp corner (it may be approximated by drawing straight lines if the actual gain characteristic has curvature), but many actuators do have an abrupt change in slope because the limited value of the output speed is caused by a mechanical limit, effectively a limit stop in an internal actuator.

Nonlinear Output. Another gain characteristic is illustrated in Fig. 4*d*. The gain of the actuator is the ratio of the output speed (in some actuators this could be position, acceleration, or torque of the output

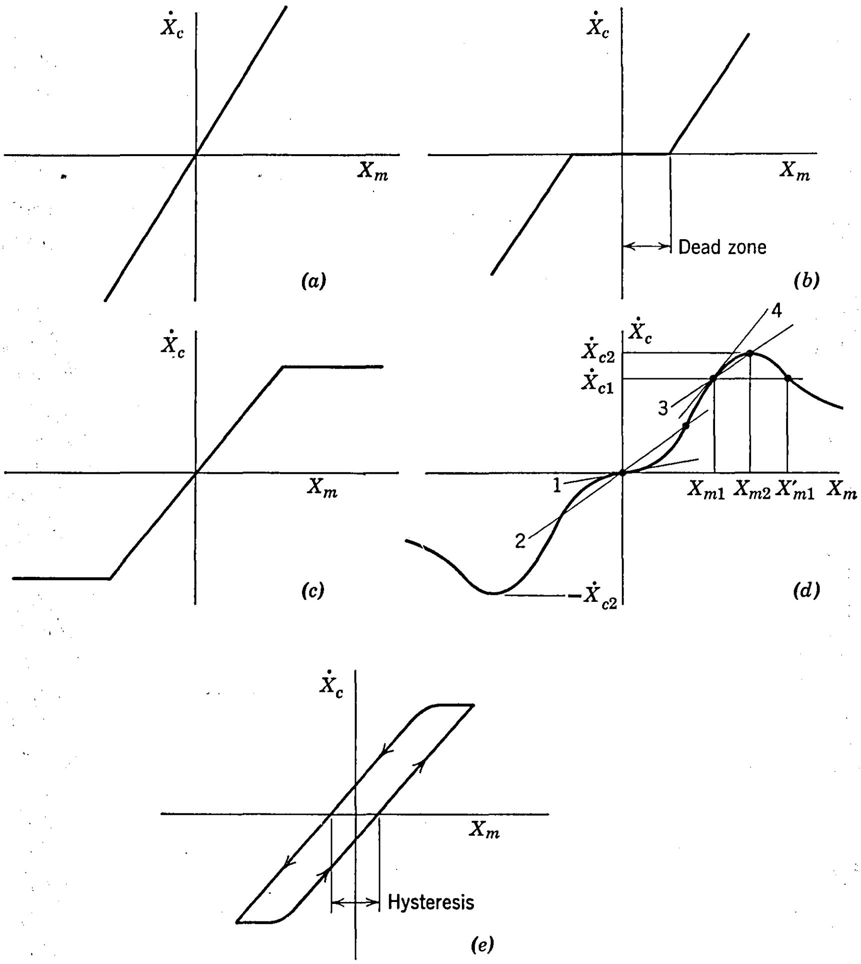


FIG. 4. Typical actuator gain curves, load torque = 0. Actuator connected directly to the load: (*a*) linear, (*b*) dead zone, (*c*) saturation, (*d*) nonlinear, (*e*) hysteresis. X_m is the actuating signal, $\dot{X}_c$ the output velocity of the actuator; $\frac{\dot{X}_c}{X_m}\frac{X_s}{X_a} = K$.

for a given actuating signal) to the corresponding actuating signal where the output speed and actuating signal are referred to the desired operating point. For small variations in input signal the gain is the slope about the desired operating point. For example, slope 1 represents the gain about the operating point where zero speed is desired. In a feedback control system this would be typical of the gain in a position loop. For large variations about the desired position the temporary, equivalent gain would be represented by slope 2 in Fig. 4*d*.

In a feedback control system controlling the velocity of the output member, the operating point could be anywhere on the curve between the points $+\dot{X}_{c2}$ and $-\dot{X}_{c2}$. At a constant output speed $\dot{X}_{c1}$, slope 3 would represent incremental gain and slope 4 would represent a temporary equivalent gain if the output speed should deviate from the desired value $\dot{X}_{c1}$.

If a speed $\dot{X}_{c2}$ is required, the incremental gain of the actuator will be zero. This will make the performance of a control loop very poor, and the actuator gain characteristic must be specified to avoid this condition. In fact, the slope of the gain curve for any operating point affects servo loop gain, performance, and stability directly, and it should not deviate more than 20 to 50% for most control systems. For high-performance systems, this allowance may only be 5 to 10% and if the actuator is to be used in a feedback path (G_h in Fig. 1) the accuracy of the actuator essentially becomes the accuracy of the system, which might be 0.1% for a servo loop in a computer. For this reason actuators, especially large actuators, are not used in feedback paths because it is usually difficult to design an actuator with an accurate, linear gain curve.

The actuator gain characteristic shown in Fig. 4*d* has two values of actuating signal, X_{m1} and X'_{m1}, which produce the same output speed. In a feedback control loop the point $\dot{X}_{c1}$, X'_{m1} could not be an operating point because the slope is negative and represents an unstable region. Consequently the usable value of the actuating signal in a control loop will always be less than $\pm X_{m2}$. *Note.* It should be observed that phasing the control loop in an opposite manner would cause the stable region to exist where the slope is negative, and the actuating signal would always be greater than X_{m2} and the output speed of the actuator could not be reduced to zero.

Hysteresis. Another important property of the actuator gain characteristic which must be specified is its hysteresis. This is sketched in Fig. 4*e* and is similar to hysteresis in magnetic materials in that the actuator speed may be different for the same value of actuating signal, depending on the direction in which the signal is changing. Actually, the slope of the gain curve may be relatively constant if the actuating signal is changed in the same direction. However, for sinusoidal signals hysteresis changes the effective gain and phase shift in a feedback control loop, and this may seriously affect the performance and stability of the loop. The hysteresis in an actuator should be specified to have as low a value as possible. Usually it should be at least as narrow as the allowable dead band, generally 1 to 2% of the proportional band.

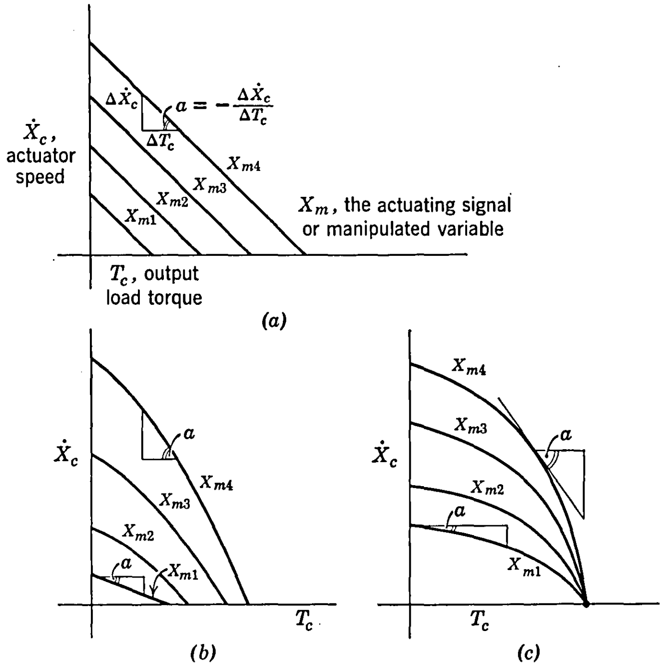
Symmetry of Gain Characteristic. It is usually desirable to have the gain characteristic of an actuator, whether linear or nonlinear, symmetrical about the origin, the point of zero speed and zero actuating signal, particularly when actuator saturation is an important factor. Symmetry assures equal performance in either direction of motion. Of course, in special applications an asymmetrical characteristic may be deliberately introduced to make performance vary in opposite directions of motion or position.

Input Impedance. The input impedance of an actuator, particularly an electric motor, may vary considerably over the operating range of the actuating signal. It may vary owing to saturation effects of the control circuit inductors or the actuator velocity. It is sometimes impossible to match driving source and actuator input impedances. Consequently, the impedance of a voltage source is usually specified to be much lower than the actuator input impedance, and the impedance of a current source is made much higher than the actuator input impedance. Otherwise, a variation in the actuator transfer function and operating characteristics may occur for different levels of input actuating signals. This effect can be determined by measurement if the actual input impedance is simulated when the measurements are made. For more details on actuator measurements see Sect. 3 on Actuator Measurements.

Speed-Torque Curves. Another important nonlinear characteristic which must be considered in actuator specifications is illustrated in Figs. 5*b* and *c*. These show speed-torque curves which are not linear. Since the time constant (*Ja* in Fig. 5) of the actuator is proportional to the slope of the speed-torque curve at any operating point, it can be seen that the time constant may have a wide range of variation. (The operating point is determined by a particular loop input velocity command signal.) As previously stated, the slopes of the speed-torque curves should not vary more than $\pm 25\%$ but for many actuators this variation may be $\pm 50\%$. This large variation does not always affect loop stability very seriously, however, for two reasons.

1. The gain of the actuator may vary with load torque in proportion to the variation in time constant. *Note.* The gain curves sketched in Fig. 4 were assumed to be derived from zero load torques. Actually, for different load torques the slope of a linear actuator will remain the same but the curve will be displaced to the right or left of zero proportional to the load torque. In a nonlinear actuator the gain curves will be displaced also, but not necessarily in proportion to the load torque, and the slopes of the curves will vary.

If the gain of the actuator varies in proportion to the time constant and the time constant is greater than the reciprocal of the system bandwidth (corner frequency is less than the bandwidth), the system bandwidth and the time response or the frequency response will not be changed apprecia-



Use of speed-torque curves in determining the approximate dynamic transfer function:

$$\dot{X}_c = f(X_m, T_c)$$

$$d\dot{X}_c = \left(\frac{\partial \dot{X}_c}{\partial X_m}\right)_{T_c \text{ constant}} dX_m + \left(\frac{\partial \dot{X}_c}{\partial T_c}\right)_{X_m \text{ constant}} dT_c$$

$$\Delta\dot{X}_c = b\Delta X_m + (-a)\,\Delta T_c$$

Where $\Delta\dot{X}_c$, ΔX_m, ΔT_c are small variation about an operating point, b is the actuator gain, $-a$ is the slope of the speed-torque curve.

For linear curves or small variations:

$$sX_c = bX_m - aT_c$$

For inertia load:

$$T_c = Js^2X_c$$

$$\frac{X_c}{X_m} = \frac{b}{s(Jas + 1)}$$

FIG. 5. Typical actuator speed-torque curves (for feedback control systems): (a) linear, (b) and (c) typical nonlinear.

bly. The accuracy of the system for low-frequency components of the input will be affected, however, in proportion to the gain change.

2. The operating point of the actuator in a feedback control loop may not vary greatly. The operating point of the actuator changes slightly, but only along the zero-speed axis if the actuator is used in a position control loop, when constant load torques are present, and the stability and accuracy of the loop may be designed for small signal variations

around the origin of the plot. *Note.* Constant load torques are usually not greater than 10% of the maximum torques expected owing to dynamic operation.

During transients the stability of the loop may change on an instantaneous basis, but this will not be serious in most cases. However, in a speed control system the operating point of the actuator may be varied all along the speed axis as well as away from the axis by any applied torque and by a constant load torque caused by the rotation and a consequent viscous friction force. For this type of actuator, the time constant variation can be rather large, and the stability of the loop will vary significantly. The loop is designed to be stable for the operating point with the least favorable combination of gain and time constant. Again this discussion assumes an actuator that produces a constant velocity for a constant value of actuating signal. The same reasoning applies to other kinds of actuators, however.

3. ACTUATOR MEASUREMENTS

The Need for Measurements. All the important actuator characteristics just discussed may not be available from the manufacturer. Sometimes if data are available they may not apply to a particular application, or the conditions under which the data were taken may have been unknown or different from those desired. For these reasons the actuator characteristics are often measured by the control system designer to obtain detailed data in the region of interest with the same measuring instruments, power supplies, and techniques that will be used in determining the control system performance.

There are many different ways of measuring actuator characteristics. If the actuator is to be used in a high-performance feedback control loop, several different measurement techniques may be used in an attempt to determine the actuator characteristics in different ways. If the data do not correlate after rechecking the measurements, the theory of the actuator operation must be reconsidered. Sometimes a malfunction or a poor design will produce inconsistent operations and make measurement difficult. These are the qualities that must be known if the actuator is to be used satisfactorily; the data must be repeatable if the actuator is to be used with confidence in a high-performance system.

Although all the methods of measuring actuator characteristics may not be used, especially in a preliminary evaluation, most of the measurements should be used to correlate the various data and to obtain a complete understanding of the actuator before it is used in a high-performance system, for an expensive actuator has characteristics, not easily changed, that largely determine and limit overall system performance.

Measurement Equipment. The measuring equipment may be quite simple or very elaborate, depending on the complexity of the actuator and the type of control system in which it is to be used. Some measuring equipment commonly used is listed in Table 1. Many of these devices

TABLE 1. MEASURING EQUIPMENT

A. Function generators
   1. Hewlett-Packard 202-A.
   2. Servoscope.
   3. Solartron transfer function analyzer.
B. Devices for measuring magnitude and phase of sinusoidal signals
   1. Oscilloscope (frequencies greater than 5 cps).
   2. Servoscope.
   3. Solartron.
   4. Strip chart recorders
      a. Brush.
      b. Sanborn.
C. Devices for measuring speed
   1. Tachometer.
   2. Stroboscope.
   3. Electronic counter.
   4. Stop watch (for very low velocities).
   5. Strip chart recorders.
D. Devices for measuring torque or force
   1. Spring scale.
   2. Torque watch.
   3. Pressure gauge (hydraulic or pneumatic).
E. Modulators for converting low-frequency signal variations into suppressed carrier signals (for a-c–operated actuators).
F. Devices for applying calibrated load torques
   1. Prony brakes.
   2. Electric dynamometers.
G. Devices for recording time responses
   1. Strip chart recorders (must have chart speeds greater than 5 in./sec and frequency responses of at least 100 cps for high-performance actuators).
   2. Oscilloscopes, usually with long persistence screens.
H. Devices for measuring position
   1. Potentiometers.
   2. Synchros.
   3. Resolvers.
I. Devices for measuring acceleration (accelerometers of various kinds).

are covered in Chaps. 7, Instrumentation Systems; 20, Measuring Elements and Sensors; and 23, Computing Elements.

Adequate power supplies with low source impedances are needed as well as accurate ammeters and voltmeters. Frequently, special measuring devices are constructed from a combination of the equipment considered or from basic engineering principles. Ingenuity and engineering resourcefulness are usually found to exist in any measurements laboratory, particularly if the equipment being measured is unique.

Actuator Gain Measurements. Basically, measurements are used to determine the actuator gain and speed-torque characteristics in Figs. 4 and 5. The gain of an actuator should never be described by a one-point measurement, i.e., one value of actuating signal and its corresponding output speed. A plot must be made using various values for input current or voltage (pressure and flow for hydraulics or mechanical motion for mechanically controlled actuators) and the corresponding output velocities. The velocity can be measured with any of the suggested measuring equipment, but the source of the actuating signal should not be much different from the driving source which is to be used in the control system, and the speed-measuring equipment must not introduce added friction. The measurements are usually repeated several times for several actuators if the preliminary measurements appear promising or if the actuator is to be used in production. From the data plots the actuator characteristics of Fig. 4 can be determined quantitatively.

In addition to the single gain curve of Fig. 4, a family of gain curves can be made by repeating the gain measurements for various load torques which may be applied with a Prony brake or electric dynamometer. These curves indicate the range of gain variations, the variable b in the equation of Fig. 5. The data for these curves may be replotted as speed versus torque for constant values of the actuating signal to check the speed-torque curves, which are considered next.

Speed-Torque Measurements. In a preliminary analysis the single gain curve of Fig. 4 is determined, and the end points of the speed-torque curves in Fig. 5*a* are determined by measuring the output speeds with zero load torque, and the stall torques with zero speed, for various values of actuating signal. The speed-torque curves are assumed to be straight lines between these points. This would lead to considerable error in estimating the curves for Fig. 5*c* which are typical of a hydraulic valve with zero leakage. Therefore, if the preliminary measurements appear to be promising for the intended application, other points are determined by measuring the speed for various load torques and input signals. If the curves of Figs. 4 and 5 are obtained from different measurements, the data should be checked for correlation.

From the curves in Figs. 4 and 5, the transfer function of the actuator can be estimated as indicated in Fig. 5 if the inertia is known. However, there are other methods of determining the actuator transfer function. Frequency responses and transient responses of the actuators can be made. These responses should be made for various magnitudes of input signals. In the linear regions, between the dead zone and saturation region, the responses will have similar appearances. For very large and very small amplitudes the responses will be different and will indicate where the nonlinear regions are and how the transfer function varies.

Frequency Response Measurements (see also Vol. 1, Chap. 22, Relation between Transient and Frequency Response). Frequency responses are obtained by applying sinusoidal functions at low frequencies (the frequencies must be modulated if the input has an a-c carrier) to the input and by measuring the amplitude and phase of the corresponding output velocity by conventional techniques. It is necessary to use a speed-detecting device which has a very small inertia with respect to the actuator inertia. A small accelerometer may also be used to give more accurate measurements in the high-frequency regions where the effects of secondary time constants may appear. Of course, the measured transfer functions will include the transfer function of the accelerometer and tachometer which must be known and removed from the actuator transfer function. *Note.* Usually an accelerometer is represented by s^2 and a tachometer by s; the primary time constants are much smaller than the secondary time constants of the actuator.

The output velocity or acceleration, magnitude, and phase are compared with the corresponding actuating signal magnitude and phase. The results are plotted as magnitude versus frequency on semilog paper or log-log paper (the latter is more convenient), and phase versus frequency on semilog paper, and the actuator time constants are found by:

1. Approximating the magnitude plots with straight lines using integer slopes; multiples of 6 db/octave are used on semilog paper, multiples of unity on log-log paper.
2. Determining the corner frequencies at which the straight lines intersect.
3. Calculating the time constants as the reciprocals of the corner frequency values.
4. Checking the theoretical phase calculated from the corner frequencies with the measured phase.
5. Readjusting the corner frequencies until the theoretical and measured data agree (for more details see Ref. 3, pp. 344–375).

Transient Response Measurements (see also Vol. 1, Chap. 22, Relation between Transient and Frequency Response). The transient responses are obtained by applying different magnitudes of steps to the actuator input. A velocity-measuring device with negligible inertia is used to determine the resulting variation of speed with time. A photocell is used to convert a beam of light, reflected from a rotating shaft with black segments, into pulses which are counted electronically as an indication of speed for actuators with very small inertias. The equivalent principal time constant is equal to the time required for the output velocity to reach 63% of maximum steady-state velocity. Secondary time constants are difficult to detect with transient response measurements. For actuators with very fast responses it is even difficult to obtain a time response sufficiently accurate to determine the primary time constant with confidence. However, the transient response can be made to provide an indication of actuator response easily and quickly. The response to an impulse input may also be used to determine the equivalent transfer function of an actuator, if it can be measured accurately (for further information see Ref. 3, pp. 375–390).

Actuator Component Measurements. Actuator characteristics may be determined by measuring the characteristics of the individual parts of the actuator. For example, the inertia and friction of the moving parts may be measured, the inductance and resistance of control coils may be determined, and the pressure flow characteristics within a hydraulic or pneumatic actuator may be evaluated. These parameters, coupled with the differential equations that determine the operation of the actuator, can then be used to derive the transfer function on a semitheoretical basis.

Note that if the speed-torque curves in Fig. 5 are used to compute the transfer function of the actuator, the inertia of the output shaft and actuator rotor must be known. The inertia can be measured by any of several techniques. One successful method consists of comparing the period of oscillation of the unknown inertia with that of a known inertia (a disk for example) when suspended from a long thin wire. The ratio of inertias is equal to the square of the ratio of the respective periods. When the actuator is used to control a load inertia which is of larger or of comparable size with respect to the actuator inertia, the inertia of the actuator is revised to be the sum of the rotor inertia and the load inertia referred to the actuator shaft.

Actuator, Size, Weight, Cost, and Environment

In addition to the linear and nonlinear actuator-operating characteristics, there are other factors that must be considered in the design of an actuator. These involve the actuator size, weight, cost, environment, reliability, and life. Because these factors have different importance for different applications, they can be discussed only briefly and in general terms.

Accuracy versus Dynamic Range and Cost. Usually, if high accuracy is desired, the actuator is more expensive, particularly if the accuracy is desired over a wide range of output velocity or position. The achievement of great accuracy over a wide range may involve special designs and special assemblies at extra cost. For a particular actuator design it may be impossible to attain great accuracy over an extended range at any cost. A precision actuator must be made with extreme care; the mechanical clearances, the bearings, the end play, the rigidity, and the type of materials must all be given careful consideration.

Speed of Response versus Power, Size, and Cost. Speed of response is governed largely by a high torque and low inertia. High viscous friction also contributes to fast response time, but it also causes a loss of power, so it is not usually increased to improve the response time.

For a given type of actuator and a given load the speed of response can be obtained only by increasing the size and power of the actuator. This results in a larger torque-to-inertia ratio, a larger torque-squared-to-inertia ratio, and a larger torque to accelerate the load. Of course, a larger actuator needs greater power, and it has a larger size, with increased weight and cost. High-performance systems with rapid response actuators are expensive.

Environment versus Cost and Reliability. Special environmental conditions apply to actuators in military applications, process control, and some industrial control problems. Perhaps the largest variety of environmental conditions, extreme temperature, shock, vibration, and atmospheric changes are specified by military control systems. In some applications an actuator may be enclosed from the atmosphere, it may be heated to avoid temperature variations, it may be mounted on shock absorbers or vibration mountings, or it may be constructed from special materials impervious to a corrosive environment. This illustrates that the size, cost, and complexity of an actuator may be increased if it is to operate under all possible environmental conditions. The size, weight, and cost for operation under a wide range of environmental conditions must be considered with the frequency of use and the required life of the system because some special control systems, such as those used in missile guidance, must be light, inexpensive, and reliable but only for a relatively short time. During the life of the equipment the actuator characteristics may not have time to change appreciably even if environment changes are extreme.

4. SELECTING ACTUATORS

The selection of an actuator to operate satisfactorily in a control loop is closely connected with the desired loop performance. The actuator power, torque, speed, and time constants are intimately related to the power, torque, and speed necessary to control the load, including the actuator inertia and friction, in a prescribed, stable manner as a function of time. The choice of a gear train, the selection of an amplifier, and the compensation necessary to meet the required accuracy and stability all depend on the actuator selection.

Power Capabilities. Fundamentally, the actuator must be capable of furnishing enough power to move the load as specified. Therefore, the motion of the output member, its inertia, expected speed, acceleration, and force loading including gravity forces on unbalanced structures must be specified, deduced, or assumed from the system requirements.

When the required output position is known as a function of time, the corresponding velocity and acceleration can be found. Since $T_i = Ja_i$, where T_i equals the instantaneous torque required for accelerating the load, J the inertia of load and actuator, and a_i the instantaneous acceleration of load and actuator inertia (all quantities are referred to the output shaft), the required actuator torque can be estimated from the acceleration curve. *Note.* Combining the inertia of load and actuator indicates that the actuator requirements are influenced by the choice of the actuator itself. Usually the actuator inertia is made equal to the load inertia when reflected to a common shaft. Using this relationship, the actuator inertia can be assumed and the torque estimated.

Instantaneous power P_i is expressed as the product of the instantaneous torque T_i and velocity V_i:

$$P_i = T_i V_i.$$

From the torque and velocity curves and the external load torques such as gravity unbalance, wind loading, added weights, etc., the total required output power of the actuator as a function of time can be found from the following relationship:

$$P_t = Vq + VJa,$$

where P_t equals total power, V the required load velocity, q the load torque, and a the acceleration of load and actuator inertia.

It is usually considered advisable that the power capabilities of the actuator output exceed the required load power by at least 10% in high-performance systems.

Frequently, it is not possible to predict whether maximum expected load forces will occur when the maximum velocity or acceleration is desired, and the assumption is often made that the most severe combinations of torque, speed, acceleration, and load forces will occur simultaneously and that the actuator must be capable of providing the desired motion under these conditions. For example, in an airborne radar system operating in the search mode, the antenna motion with respect to a space reference may be required to change abruptly from maximum velocity in one direction to maximum velocity in the reverse direction as quickly as possible. At the same time, the airplane may swerve suddenly in the opposite direction. The actuator will be required to supply the torque and speed to counteract the airplane motion, and, in addition, move the antenna in space as desired.

It can be shown that maximum power is delivered to the load in a valve-controlled hydraulic system when the load pressure is two-thirds of the supply pressure (valve pressure equal to one-third of supply pressure). (See Ref. 8, p. 408.)

In addition to choosing an actuator that will produce the required load power, it is necessary to select an actuator that will not overheat while supplying the required speed and torque. For example, it does not require power at the load to hold an unbalanced member stationary, but it does require a constant torque or force. If this torque is supplied by an electric motor, power is dissipated as heat because of the currents flowing in the armature and field to produce the torque. This heating effect in the motor can be greater at standstill if the stalled torque is high than it is at high speeds, especially if the motor has an integral cooling fan.

In an hydraulic motor the heating conditions are reversed; for high speeds the hydraulic flow is greater and the heating effect is increased (for a more complete discussion of actuator power ratings see Ref. 4, pp. 95–106).

From the foregoing discussion it is evident that the power rating and temperature rating of the actuator must be considered, especially for high ambient temperatures, in addition to the power capabilities. In missile applications where the life of the actuator may be short, this may not be an important consideration; however, it is apparent that the selection of an actuator on the basis of the power required and the power rating depends on the application and the system specifications.

Torque and Speed Capabilities. In addition to being able to supply the required load power without overheating, the actuator must be able to furnish the required instantaneous torque and speed simultaneously. The simultaneous torque and speed can be found from the time plots of acceleration and velocity previously described. If the required load speed and torque cannot be realized by connecting the actuator directly to the load, it may be possible to use a gear train, usually a reduction, between the actuator and the load, especially if the actuator speed is much larger and the torque is much smaller than that required by the load. Therefore selection of an actuator indirectly leads to a selection of a gear train, which will now be discussed.

Selection of a Gear Train. Because the factors that determine the selection of the gear train are interrelated with the factors that influence the selection of an actuator, it is necessary to consider a gear train when considering actuators.

The gear ratio N is determined first. Expressed as the ratio of actuator speed to load speed, it is usually a number greater than one. It is determined in the following manner.

1. From the time plots of desired output speed, acceleration, and power, the regions of maximum power are selected for investigation. This may be only one well-defined region. If the output function is not well defined, it may be the maximum power produced by the most detrimental combination of velocity, load torques, and acceleration torques that could occur at any time.

2. An actuator is selected that will produce the required peak power according to its rating, and the speed-torque curve for the maximum value of actuating signal is obtained.

3. The required load torque T_l is plotted as a function of the gear ratio N from the following equation:

$$T_l = T + a(J + N^2 J_0),$$

where T_l is the total required load torque in the region of interest,
T is the load torque in the region of interest,
a is the load acceleration in the region of interest,
J is the load inertia,
J_0 is the actuator inertia,
N is the gear ratio, the independent variable.

T_l increases parabolically as a function of N. This is sketched in Fig. 6*a*.

4. The available load torque produced by the actuator is plotted as a function of N from the speed-torque curves, as shown in Figs. 6*b* and *c*. The speed-torque curves can be quite nonlinear.

5. The plot of the required load torque is superimposed on the plot of available load torque to determine the usable range of gear ratios as shown in Fig. 6*d*. If there is no intersection, the actuator cannot be used.

6. The process is repeated for other peak power regions to determine the narrowest band of possible gear ratios.

7. If the narrowest band of possible gear ratios is not zero, the gear ratio is selected as follows (the selection of a gear train is discussed in more detail in Ref. 1, pp. 223–228, Ref. 4, pp. 92–118, and Ref. 8, pp. 320–328):

a. The largest possible gear ratio is chosen if it is desirable to move the output load at very slow, smooth velocities with maximum resistance to load torques.

b. The lowest possible gear ratio is chosen if slow velocities are not required and load torques are not severe, to make the gear train simple and economical.

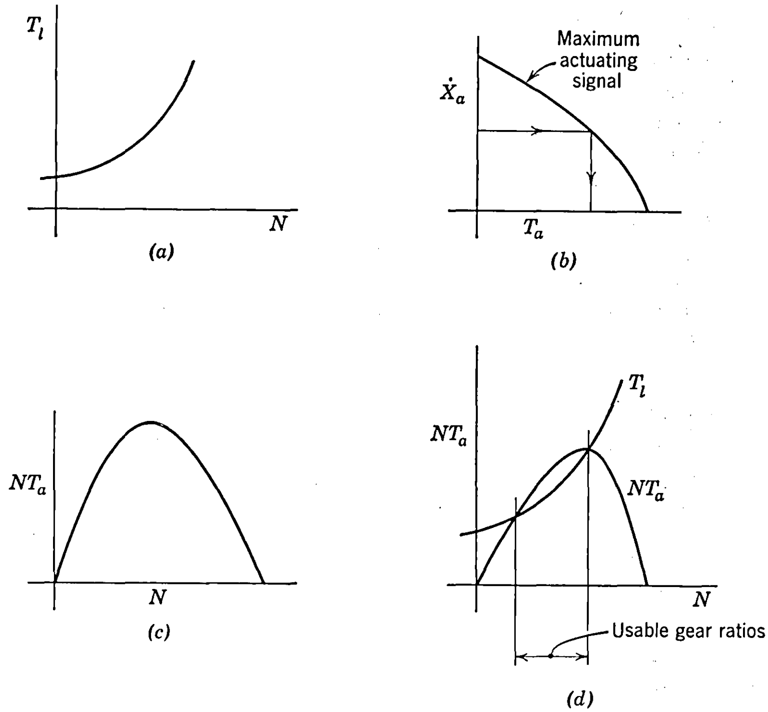


FIG. 6. Graphs for determining usable gear ratio: (*a*) required torque, (*b*) speed and torque available for maximum actuating signal, (*c*) available torque, and (*d*) available torque and required load torque, where T_l is the required load torque, N the gear ratio, T_a the actuator torque, $\dot{X}_c$ the required load speed, and $\dot{X}_a$ the actuator speed. To determine the gear ratio, plot (*c*) from (*b*) by calculating $\dot{X}_a = N\dot{X}_c$ for various values of N and given values of $\dot{X}_c$, by deriving values of T_a from (*b*) and by multiplying T_a by the assumed value for N. Then superimpose curves (*a*) and (*c*). The region of intersection indicates the allowable range of gear ratios.

c. The gear ratio $N = \sqrt{J/J_0}$ is chosen if the maximum load acceleration is desired.

d. The gear ratio $N = (\dot{X}_a)_{\text{max}}/(\dot{X}_a)_l$ is chosen if a large friction load is to be overcome. [$(\dot{X}_a)_{\text{max}}$ is the actuator speed at maximum actuator power, $(\dot{X}_a)_l$ the maximum desired load speed.]

Gear Meshes. After the overall gear ratio is selected, it is necessary to choose the number of gear meshes and the ratio of each mesh. This will not be discussed in detail in this section because it is beyond the scope of actuator selection. (For more information on selecting the number of gear meshes and ratios see Ref. 1, pp. 241–242, Ref. 4, pp. 114–118, and Ref. 5, pp. 130–133.) However, several points should be noted.

1. The inertia of the gear train must be considered as part of the load or actuator inertia.

2. The first mesh reduction ratio should not necessarily be as large as possible.

3. Generally the gear train should be reversible in a feedback control system.

4. The gear train generally must be made with precision; low backlash and low eccentricity must be attained, especially for high-performance systems (see Ref. 1, pp. 239–258).

Loading Effects on Actuator Characteristics. As indicated in Fig. 5, the transfer function of an actuator is intimately associated with the characteristics of the load it moves. In Fig. 5 a simple inertia was assumed for the load. Usually the load is not a simple inertia, especially for airborne equipment in which the lightweight load structures may have appreciable compliance. Structural compliance coupled with the load inertia produces rather complex expressions which lead to possible quadratic terms in numerator and denominator of the actuator transfer function. See Ref. 14 for more details.

Quadratic terms represent resonant frequencies which always limit the performance of control loops. Often they are neglected or assumed negligible, especially if the load structure appears to be rigid. This neglect in preliminary analyses often results in an unsatisfactory choice of actuator and undesirable system performance. Generally, the lowest resonant frequency in the actuator transfer function, which must include the load, should be greater than five times, preferably greater than ten times, the control system bandwidth.

System Error Specifications and Actuator Selection

The selection of an actuator for use in a control loop is influenced not only by the load but by the allowable loop error and the amplifier which transforms the error signal, or loop actuating signal, into the actuator input signal. The particular actuator characteristics which must be considered in connection with other system components are (1) dead zone, (2) saturation of input signal elements in actuator, (3) principal or largest time constant, and (4) input impedance.

Each of these characteristics will be discussed separately in more detail. However, the discussion is not intended to be a design manual for control loops; rather it is a brief presentation of some of the related characteristics which must usually be considered in selecting actuators.

Dead Zone Limitations. The loop actuating signal is related to the error between the loop output and input signals. In fact, if G_h equals unity in Fig. 1, the actuating signal is the error. It is always desirable to keep the actuating signal from exceeding specified limits in steady-state operation, and it is always desirable to keep the actuating signal as close to zero as possible when it is the system error.

The actuating signal, or error signal, is modified by an amplifier to supply an adequate control signal to the actuator. If the actuator has no dead zone (discussed in Sect. 2, Actuator Specifications), it will start to move for any value of actuating signal, no matter how small its magnitude, and it will continue to move until the magnitude of the actuating signal is reduced to zero. The actuator will continue to move at a constant velocity if a velocity is produced when a constant input signal is applied to the actuator. This is the common type of actuator which effectively integrates the input.

With an actuator dead zone, however, the magnitude of the loop actuating signal must be greater than E_m, the actuator dead zone divided by the amplifier gain which precedes the actuator. See Fig. 7. (The

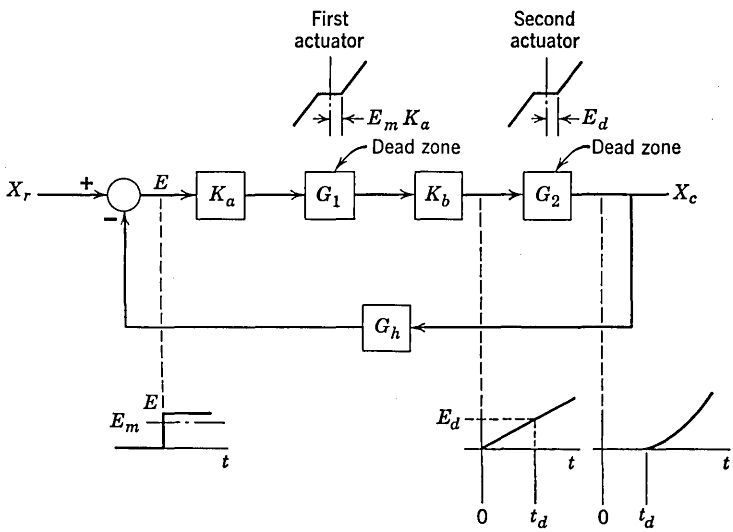


FIG. 7. Relative time responses of loop signals.

amplifier may consist of several units, possibly of different types. In this discussion the ratio of the actuator input signal to the loop actuating signal represents all the amplifying elements, and it is referred to as the amplifier.) The actuator will not move and the output of the feedback control loop will not be directed to reduce the magnitude of the loop actuating signal or error until it has exceeded the value E_m. If E_m is not exceeded, the integration of the actuator, on which expected system performance may depend, is effectively removed and a steady-state error as large as E_m may exist at any time.

If the allowable error signal and the total amplifier gain are both known, the maximum allowable actuator dead zone is specified. It is simply the product of the allowable error signal and the overall amplifier gain.

If two or more actuators are cascaded in a feedback control loop, the dead zone of the first actuator may be determined as a function of the amplifier gain and the allowable magnitude of the error signal as described. However, the allowable dead zone of the second actuator must be determined in a different manner.

If the first actuator has an integration, it will move continuously as soon as the input signal exceeds its specified dead zone. Therefore the second actuator will move whenever the first actuator responds, but only after the first actuator has moved sufficiently to produce a signal greater than the dead zone of the second actuator. This causes a finite time delay in operation.

This time delay is related to (1) the amount that the signal to the first actuator exceeds its dead zone, (2) the gain of the first actuator, and (3) the gain of the amplifier between the first and second actuators.

The relationship between these factors can be expressed by a steady-state equation:

$$[(E - E_m)K_aG_1]t_dK_b = E_d \qquad E \geq E_m,$$

where the quantities in the equation are shown in Fig. 7 and are described as

E = loop actuating signal,
E_m = maximum allowable loop actuating signal (from specifications),
K_a = gain of amplifier preceding the first actuator,
G_1 = gain of first actuator (steady state),
t_d = allowable time delay,
K_b = gain of amplifier between the first and second actuators,
E_d = the dead zone of the second actuator.

From this equation is can be seen that the allowable dead zone of the second actuator must be determined by the application; no general rules exist other than the equation itself. It should be emphasized that the relationship given is a steady-state relationship; a nonchanging input was assumed. For dynamic, time-varying inputs the performance of the loop with one or two actuators would have to be analyzed on the basis of stability and time-varying errors as discussed in Vol. 1, Chap. 25, Nonlinear Systems.

Saturation of Actuator Input Elements. The method of selecting an actuator to provide the required power, speed, and torque has been discussed. However, the performance of some actuators is limited by the input elements which transform the input signal into motion. For example, a valve controlling the flow of fluid into an actuator has definite limits of travel, and the current controlling the valve may be limited before the capabilities of the actuator are realized. Very often this saturation effect is included in the actuator gain curves and speed-torque curves, but this must be checked to insure that the actuator will operate as expected.

Principal or Largest Actuator Time Constant. The principal time constant, or largest time constant, of the actuator is usually larger than the reciprocal of the required system bandwidth. To provide adequate loop stability it is necessary to include compensation, usually an electrical network, which offsets the phase lag, produced by the principal time constant, with a phase lead (see Sect. 2, Actuator Specifications).

Any network or equivalent which produces phase lead with increasing frequency must also increase the magnitude of the output-to-input signal ratio as the signal frequency increases. In a similar manner, the initial magnitude of the time response is considerably larger than its final value if a sudden change occurs in the input signal to a phase lead network. These increases in signal amplitude are applied to the actuator to offset the normal loss of amplitude or speed of response which results from a constant amplitude input signal. From this standpoint, a compensating network is a device which provides an extra-large command signal, when it is needed, to force the actuator output into a desired motion. Of course, this can happen only as long as (1) the larger amplitudes do not saturate the actuator input driving elements, and (2) the larger amplitudes do not excite the natural resonant frequencies of the actuator load and its structure.

Generally, the first consideration does not seriously affect the system performance if the actuator is selected to have the necessary speed, torque, and power to follow the specified input (see Power Capabilities earlier in this section). The compensating network merely adjusts the actuator input in a manner that will enable the actuator to provide the desired speed and torque at the appropriate time. System performance for inputs other than those specified may not be satisfactory if saturation occurs, and if the input magnitude is larger and more quickly applied than the specified input, it is likely that saturation will occur.

The second consideration, that the signal may excite the natural resonant frequency of the load, is usually more important because, if the resonant frequencies of the actuator load are excited in a feedback loop, they may be returned to the actuator input in a positive way to reinforce the natural vibrations and produce a distinctive, high-frequency vibration characteristic of loop instability. The methods of analyzing loop stability are discussed in Vol. 1, Chap. 21, Stability. Stability has been mentioned here only on a qualitative basis to illustrate that an actuator cannot be selected or used independently, especially in a feedback control loop, without considering the characteristics of other loop elements.

Input Impedance. The input impedance of an actuator varies, depending on the type of actuator and sometimes on the operating conditions of the actuator itself. For example, the input impedance of an electric motor, the ratio of the input voltage to the input current, varies with the motor speed because the input current changes as the motor speed varies, owing to the back emf (see Ref. 1, p. 139). On the other hand, the input impedance of a hydraulic actuator usually varies very little because the actuator is controlled with an integrally mounted flow valve which isolates mechanical motion from the electrical inputs.

The input impedance may be calculated, measured, or obtained from manufacturer's data. The input impedance is the load on the preceding element in the loop, usually an amplifier, and it must be considered in the amplifier design.

Although it is desirable to match the power amplifier output impedance to the actuator input impedance for maximum power transfer, this cannot be done for actuators in which the input impedance varies. Therefore it is necessary to make the amplifier output impedance lower than the smallest input impedance value, perhaps with internal feedback, by a factor of 10 or more.

The output impedance of the driving amplifier should be low for some actuators because if may affect the performance detrimentally. For example, if an amplifier with a large impedance is used to drive an a-c servo motor, it can cause the motor to move on single-phase power without benefit of the loop control signal.

Actually, the speed-torque curves given by the manufacturer to describe the operation of the motor are changed by a large input impedance, and the inherent damping of the motor is decreased. Although a capacitor is sometimes placed across the winding of an a-c motor to increase its impedance and gain, it may also change the motor characteristics and it must be used with caution.

Concluding Remarks. The foregoing discussion has illustrated that an actuator cannot be selected without considering the required overall system performance, the load, the input impedance, the impedance of the driver, and the number of actuators to be used in the control system. This has been a general discussion; it was not intended to give detailed examples of design methods, for the procedures are different for every system in which an actuator must be selected. The basic principles to be considered when selecting an actuator have been given, but the ingenuity of the designer must be applied to use them and provide the most efficient actuator operation.

5. ELECTRIC ACTUATORS

For most of the low-power applications, in which the output power may vary between a fraction of a watt and 100 watts, two-phase a-c induction motors are usually selected. Where higher power outputs are needed, d-c motors may be used to provide output power from fractions of a horsepower to hundreds of horsepower. The two types of motors are discussed separately.

Two-Phase A-C Control Motors

The two-phase control motor has two input windings. One winding is excited by a fixed line voltage usually of 60 or 400 cps, and the other winding is connected to a control voltage. The phase of the current in the control winding is nominally adjusted to be 90° out of phase with the current in the line phase, and the speed of the motor shaft, for any constant value of torque on the shaft, is approximately proportional to the magnitude of the control current. The direction of rotation is governed by the phase of the control current which may lead or lag the reference current by 90°.

The operation of the two-phase motor may be described mathematically using symmetrical component theory (see Ref. 6). Although the resulting equations give considerable insight into the design and operation of the motors, they are simplified for practical purposes to give the same transfer function as that derived in Fig. 5 from the speed-torque curves. For a given motor, this transfer function is usually adequate for control loop design.

Construction of Two-Phase Motors. Linear speed-torque curves are provided by a high-resistance rotor. This also provides damping and reduces the possibility that the motor will run on a single phase as it might do with a low-resistance winding. If this should occur, the ability of the control winding current to vary the speed would not exist, and as a control motor the unit would be of no value.

Several rotor types, such as the drag cup, squirrel cage, and solid rotor, are used, depending on the application. Drag cup rotors are very light, provide constant torque at all rotor angles, but produce relatively low output torque for a given motor size. Squirrel cage rotors achieve the highest torque in a given size and the highest torque-to-inertia ratio, but the output torque varies with rotor angle (called cogging), which is undesirable in some applications. The solid rotor is little used since its performance is similar but inferior to the drag cup. The solid rotor is more rugged mechanically, however, and may be required in severe environments.

Efficiency of Two-Phase Motors. All the electric two-phase motors are very inefficient power transmitters. At either maximum stall torque or maximum output velocity with no load, there is no output power produced, but power is always used in the fixed phase winding. Therefore the efficiency of the motor under these conditions is zero. The maximum efficiency of 20 to 25% usually occurs when the motor is turning near half-speed with a load torque about half of the rated maximum stall torque.

The power loss in the motor causes considerable heat rise, and when the motor power rating exceeds 10 watts or when unusually high ambient temperatures are encountered, the motor is usually air-cooled by a fan mounted within the motor frame in line with the rotor. The fan is driven by a separate, constant speed motor to provide maximum cooling under all operating conditions. Although the blower volume may be as large as the motor itself, the overall motor size and weight are much less than they would be if the fan were not used.

Application of Two-Phase Motors. Two-phase motors are used in low-power control systems where a-c power is available and where relatively fast, smooth motion is desired.

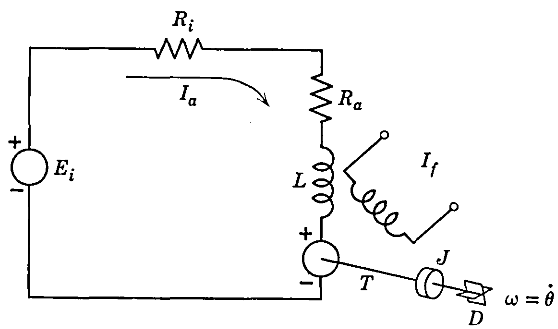
The control voltage or current may be produced by transistor amplifiers, vacuum tube amplifiers with cathode follower or transformer-coupled outputs, or magnetic amplifiers. Because of their inherent high velocity, electric motors are coupled to load members with a speed-reducing gear train.

D-C Control Motors

The d-c motor may be controlled by varying the field current while maintaining a constant armature current or varying the armature current while supplying a constant field current. The latter method is usually used in automatic feedback control loops for the following reasons: (1) the power loss due to the constant field current is much less than it is when constant armature current is used; (2) a feedback control loop with variable field current is inherently unstable, and considerable loop compensation may be necessary.

The transfer function of the d-c motor may be obtained from speed-torque curves as illustrated in Fig. 5. The speed-torque curves for d-c motors are not usually given, however, because the transfer function can be derived from the basic motor circuit equations as shown in Figs. 8 and 9.

Construction of D-C Control Motors. Direct-current motors contain brushes and armature windings. Some small motors have permanent magnet fields, but large control motors have stator windings, usually with enough field current to saturate the core material, both when armature



Descriptive equations:

$$E_i = I_a(R_1 + R_a) + I_a sL + k_1 s\theta = I_a R + I_a sL + k_1 s\theta$$

$$T = k_2 I_a = Js^2\theta + Ds\theta = s(Js + D)\theta$$

$$\frac{\theta}{E_i} = \frac{k_2}{s[JLs^2 + (DL + JR)s + k_1k_2 + DR]} \qquad \text{usually } D \approx L \approx 0$$

$$\frac{\theta}{E_i} = \frac{1}{k_1 s[(JR/k_1k_2)s + 1]}$$

k_1 = constant with units, volts/(radians/sec)
k_2 = constant with units, torque/amp

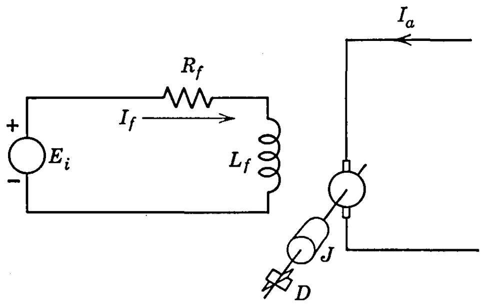
Note. $k_1 = k_2$ (numerically) if mks units are used.

$k_2 = 0.7378k_1$ (numerically) if torque is in foot-pounds and current is in amperes.

FIG. 8. D-c motor transfer function, fixed field current.

control is used and to prevent changes in magnetic flux due to rapid starting, stopping, reversing, and possible change in field voltage. Although a motor with a permanent magnetic field is less expensive, smaller, and lighter than one with a wound stator, the latter is usually more reliable and has a longer life because the field of a permanent magnet is subject to change under control loop operating conditions.

Application of D-C Motors. Although d-c motors can be obtained in small sizes, they are not often used in low-power instrument servomechanisms because (1) the inertia of the rotor is larger than in equivalent a-c control motors; (2) the friction drag of the brushes may result in a relatively large dead zone; (3) interpoles to minimize commutation problems cannot be used on smaller motors; (4) the brushes require adjustment



Descriptive equations:

$$T \approx kI_f = Js^2\theta + Ds\theta$$

$$E_i = I_f(R_f + sL_f)$$

$$\frac{\theta}{E_i} = \frac{k}{s(Js + D)(R_f + sL_f)}$$

$$\frac{\theta}{E_i} \approx \frac{k}{R_fJs^2[(L_f/R_f)s + 1]}$$

$$D \approx 0$$

$$k = \text{torque/amp}$$

FIG. 9. D-c motor transfer function, fixed armature current.

and replacement rather often, especially under military environmental conditions, because control loop motors often operate near zero speed and stall torques where initial currents are large.

The d-c armature control current for large motors is usually furnished by thyratrons, magnetic amplifiers, rotating amplifiers such as the Rototrol, Amplidyne, or Regulux, or by a d-c generator. The latter combination of a d-c generator and armature controlled motor is usually referred to as the Ward-Leonard system, and it often has many modifications in practice. The Ward-Leonard system is not generally used in high-performance systems because the other motor controllers of the same size have a faster response time.

Direct-current motors capable of supplying hundreds of horsepower usually have several auxiliary control circuits to prevent overloads and to provide interlocks with other control circuits and thus induce automatic stoppage in an emergency. Basically, the operation for the large motors is the same as it is for the smaller motors, and the problems of selecting an actuator for a particular load and input still apply; but the selection of special auxiliary equipment deserves special attention which will not be discussed here.

6. FLUID ACTUATORS

Fluid actuators are a broad class encompassing both liquid (hydraulic) actuators and gas (pneumatic) actuators. Although hydraulic control is the most widely used form of fluid control, pneumatic control is an important form having many useful applications for which hydraulic fluids are impractical or undesirable. For example, in chemical and petroleum processing plants, when time constants are of the order of minutes, pneumatic controllers and actuators are generally used (see Chap. 7, Instrumentation Systems).

Hydraulic Actuators

Hydraulic actuators are perhaps the most versatile of all the actuators. They may be used in low-power applications in the 1-watt range or in high-power applications involving thousands of horsepower to produce either linear or rotary motion with smooth, positive action.

Hydraulic actuators are particularly useful when (1) hydraulic power is readily available, (2) high power with great efficiency is needed with a minimum size and weight, and (3) smooth, stepless operation and high acceleration are required over a large velocity range.

Advantages of Hydraulic Actuators

1. Extremely high output power, torque, and velocity for a given size, weight, and amount of auxiliary equipment.
2. Smooth, stepless, positive motion over a great velocity range, perhaps greater than 1000:1.
3. Great efficiency: 85 to 90% at full torque or full speed, even 80% with 25% load.
4. Peak output power as much as twice the continuous power rating.
5. Usually self-lubricating because of the hydraulic fluid which flows through the actuator.
6. Inherently fast action; high corner frequencies, or low time constants, and exceptionally high torque-squared-to-inertia ratios.
7. Maximum load acceleration obtainable even at high velocities.

Disadvantages of Hydraulic Actuators

1. Need for an oil supply and a high-pressure source, usually 1000 to 5000 psi.
2. Need for tubes or pipes to conduct the flow of oil, with attendant problems of physical location and relocation for servicing.
3. Need to keep volume of oil under pressure at a minimum to reduce undesirable compressibility effects, particularly with large load masses or inertias.
4. Possibility of entrapped air destroying positive control action.
5. Possibility of hydraulic fluid becoming contaminated with micro-

scopic dirt particles which may clog valve orifices and cause excessive wear (special filters may be required).

6. Possibility of oil leaks occurring under high pressure, causing damage to surrounding equipment, malfunction of the hydraulic actuator, fire hazard.

7. Oil viscosity changes with temperature causing a variation in actuator performance.

8. Need for precise machining and careful assembly, with attendant expense, to obtain the performance advantages of hydraulic control.

Principle of Operation. Generally, hydraulic actuators fall in one of two categories: (*a*) the *pump-controlled* or originative system and (*b*) the *valve-controlled* or dissipative system. Each system will be discussed separately.

Pump-Controlled Actuators

These actuators have high efficiency, and they are used on high-power control applications from 20 to 2000 hp. Basically, the unit consists of a hydraulic pump and a motor. The pump power is furnished by an external, constant speed drive, usually an electric motor. The direction and amount of fluid flow through tubes connecting the motor to the pump govern the speed and torque of the motor. The fluid flow is controlled by the position of a mechanical lever on the pump. On small units the lever may be moved by a solenoid, but on larger models the lever is moved by a small hydraulic actuator. Either method provides pump control from electric signals, a requirement of most control systems. Because the amount of flow and pressure is controlled by the pump, only the power needed by the load at any time must be provided.

Axial Piston Pump Construction. Although the construction of the pump may have variations, the most common form is the axial piston type shown schematically in Fig. 10. In this type of pump an odd number of pistons are aligned in parallel cylinders about a center shaft. Nine pistons are often used. An odd number eliminates even harmonics. Action would be smoother with a larger number of pistons, but the expense would be greater. The barrel containing the cylinders and pistons is rotated about the shaft by an electric motor. Extending outside of the barrel, the free ends of the pistons are joined to a cylindrical ring with ball and socket bearings. The ring rotates about the shaft with the pistons in a stationary ring bearing which may be tilted at an angle from its nominal position, perpendicular to the rotating shaft.

When the ring bearing is perpendicular to the shaft, the rotating pistons have no linear motion in their cylinders, and the oil in the cylinders does not move axially. However, when the ring is tilted by the control lever, the rotating pistons move a linear distance proportional to the tilt angle, during a half-revolution of the barrel, and push the oil out of the cylinder. On the next half-revolution the pistons reverse their direction of motion and pull oil into the cylinders. The oil forced from each cylinder is directed into an oil line which is connected to the motor. The oil from the motor is returned to the other side of the pump barrel in another oil line. Since the speed of the motor is proportional to the flow of oil, it is also proportional to the tilt of the ring bearing or the position of the control rod in steady-state operation.

Axial Piston Motor Construction. The construction of the motor illustrated in Fig. 10 is similar to that of the pump. However, the tilt of the ring bearing is fixed. Note that in some units the external, rotating shafts may be perpendicularly attached to the ring bearing, and the cylinder barrel may be tilted with respect to the shaft. However, the operation of the pump and motor is the same as described. Oil from the pump forces the piston connecting rods against the inclined ring and causes it to move. This in turn produces rotary motion of the cylinder barrel, the shaft, and the load.

Vane Motor Construction. Another type of motor sometimes used with variable pump control is the vane motor. Essentially, this motor is constructed like a water wheel with vanes protruding from a central drum about the output shaft. The flow of oil from the pump against the vanes causes the drum and the attached load to rotate.

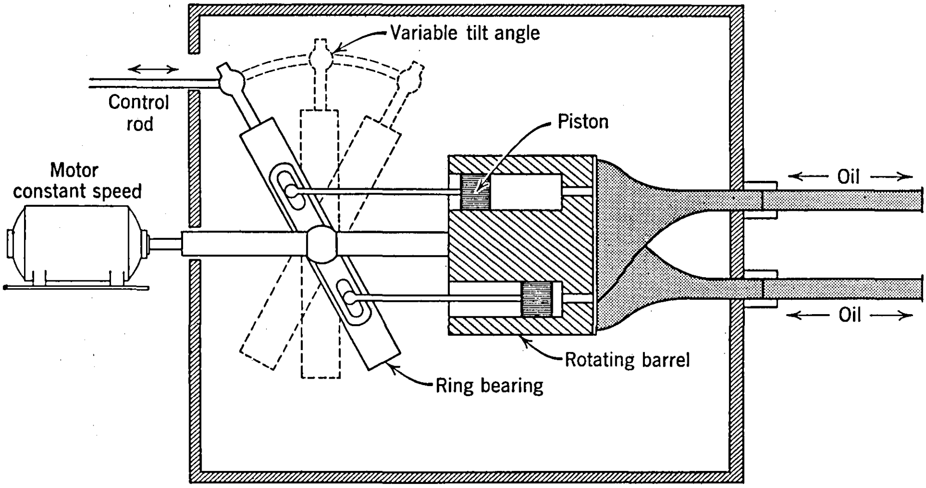
Characteristics of Axial Piston and Vane Motors. Typical characteristics of the axial piston motors and vane motors are given in Table 2. It should be noted that these are representative characteristics. The full range of motor sizes is not given. Actually, motors may be obtained up to 2000 hp.

Because the velocity of the output shaft is much greater than that needed for most applications, it is usually necessary to use gearing with

TABLE 2. TYPICAL CHARACTERISTICS OF AXIAL PISTON MOTORS AND VANE MOTORS [a]

Typical Characteristics	Axial Piston Motors Operating at 3000 psi		Vane Motors Operating at 1000 psi	
	Small	Large	Small	Large
Maximum continuous speed, rpm	7000	2000	2200	1200
Maximum torque, lb-in.	45	1800	250	2000
Moment of inertia, lb-in.-sec^2	1.4×10^{-4}	400×10^{-4}	6×10^{-4}	100×10^{-4}
Torque^2/inertia, lb-in./sec^2	1.7×10^7	8.1×10^7	10×10^7	40×10^7
Principal time constant with no load, sec	0.002	0.005	0.0006	0.0006
Oil displacement, in.^{3}/rev	0.1	3.7	1.7	14
Approximate volume oil under compression, in.^{3}	0.3	11	7.0	50
Oil leakage at 1000 psi, in.^{3}/sec	0.04	1.5	3.0	20

[a] Adapted from Ref. 15.



Pump

the motors described. Unfortunately, some of the inherent advantages of the smooth control possible with liquid flow are lost when axial piston pumps and motors are used because of (1) the piston action which causes torque pulsations at low velocities, and (2) the mechanical gearing with characteristic friction and backlash. These disadvantages are eliminated in the valve-controlled linear and rotary actuators discussed later under Valve-Controlled Actuators.

Hydraulic Position Control (Pump Preamplifier). The hydraulic position actuator, or hydraulic preamplifier, needed to convert electric signals into pump control rod motion may be constructed in a variety of ways. One simple configuration is given in Fig. 11. As shown, the device has direct mechanical feedback to make output position rather than output velocity proportional to the input current.

An electric signal in the solenoid causes the core to rotate the feedback lever about the pivot at the end of the power piston, which cannot move readily because the oil on each side of the piston cannot move initially. Motion of the lever causes the lightweight valve to travel in the same direction and permits the high-pressure oil to flow to one side of the power piston while allowing the oil on the other side of the power piston to escape to the pump or oil reservoir. The low-pressure fluid line is connected through the pump and reservoir back to the high-pressure fluid line.

As the power piston moves, it forces the feedback lever to revolve about the pivot on the solenoid arm, and this returns the valve to its initial position where it prevents further movement of the power piston. Thus move-

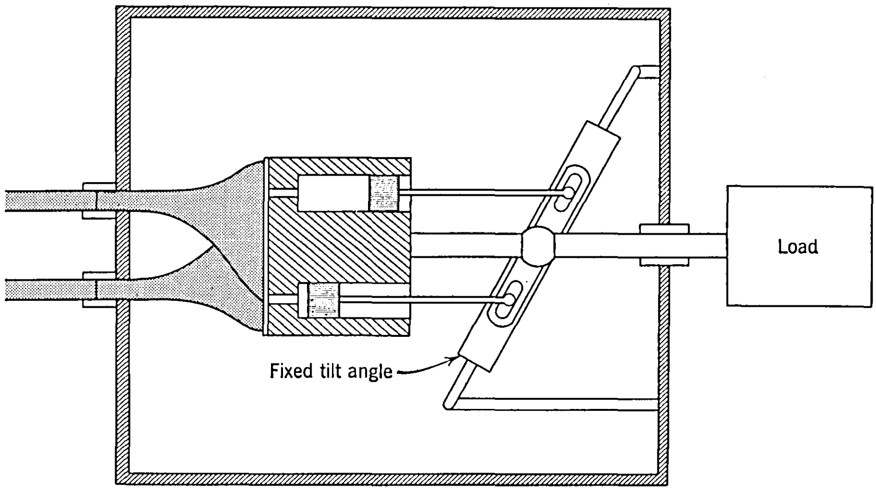


FIG. 10. Descriptive sketch of a pump-controlled hydraulic actuator.

ment of the solenoid core produces a proportional movement of the power piston.

Valve-Controlled Actuators

Valve-controlled actuators operate from a constant pressure hydraulic supply (pump, reservoir, and regulator) and require a variable orifice valve to modulate or control the flow of fluid to the actuator. Since the valve delivers only a portion of the pressure and flow of the supply to the load, valve-controlled servomechanisms are not as efficient as pump-controlled servomechanisms. Because the valve in large part determines the operating characteristics of a valve-controlled servomechanism, and since the valve may be considered as an actuator itself, in that it produces a flow of fluid in response to an input signal (usually electric), various types of control valves will be described before the actuators that are usually used in conjunction with them.

Valves. The basic valve elements which are commonly used are the spool, the flat plate, the poppet, the jet pipe, and the flapper.

Spool Valves. Figure 12 shows schematic diagrams of spool valves. These diagrams are not intended to indicate actual construction. The spool is moved axially, usually by an electromagnetic device such as a solenoid, to port fluid from the supply to the load or from the load to the return.

The three-way valve is so called because it requires three hydraulic connections, supply, return, and load. It is capable of forcing the load

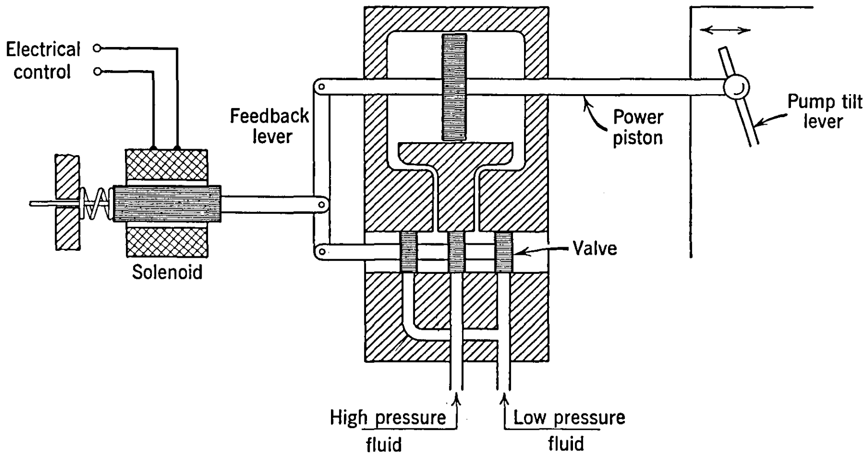


FIG. 11. Diagram of a hydraulic position actuator as a preamplifier.

in only one direction, so that some restoring means, such as a spring, must be provided at the load. Two three-way valves may be used in combination to give two-directional control.

The four-way valve requires four hydraulic lines: supply, return, and both sides of the load. It permits direct control of the load in both directions.

Both of the spool valves shown in Fig. 12 are "closed center" valves; that is, with the spool in the center position, there is no "leakage" flow from supply to return. This results in high efficiency, but it has the disadvantage of a possible dead zone if the spool overlaps the port. Further-

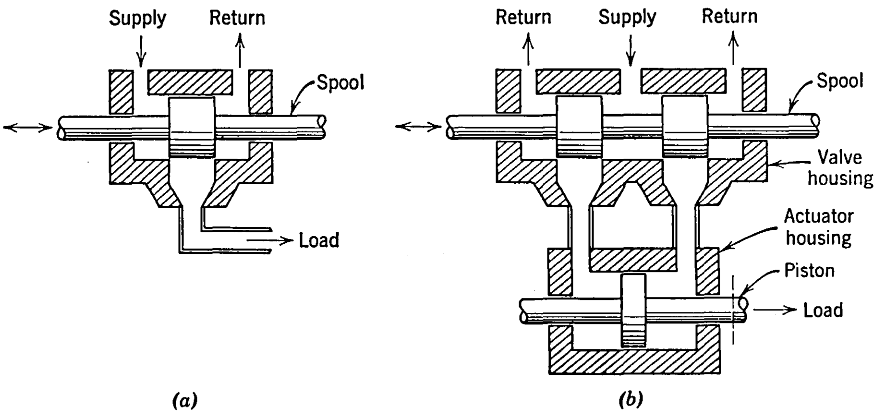


FIG. 12. Schematic diagrams of spool valves: (*a*) three-way valve, (*b*) four-way valve and actuator.

more, the absence of leakage means that, for any value of spool movement, under locked load conditions, the full supply pressure is delivered to the load. This causes all the flow-pressure curves (analogous to the speed-torque curves of electric servo motors) to intersect at the zero-flow, full-supply-pressure point, as shown in Fig. 13*a*. This characteristic may be undesirable in some control applications.

"Open center" operation results when the spool underlaps or is narrower than the corresponding port, thus permitting some leakage flow to occur.

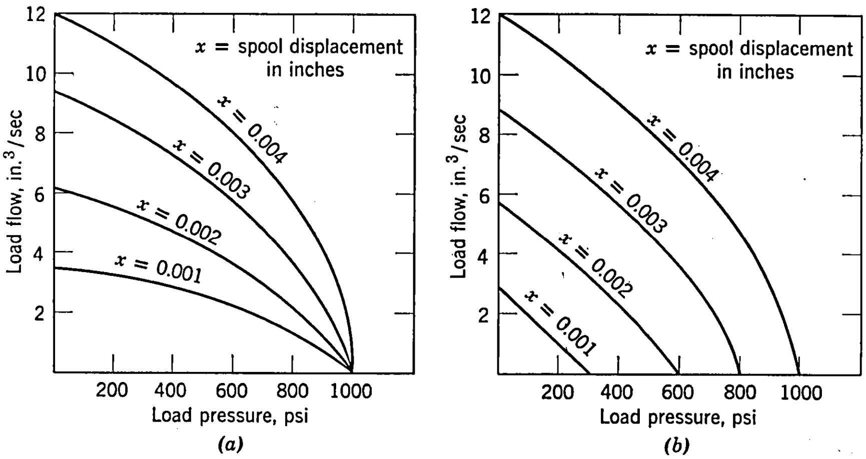


FIG. 13. Flow-pressure curves of four-way spool valves: (*a*) closed center operation, (*b*) open center operation.

Although this operation is less efficient than closed center operation, it helps to avoid the possibility of dead zone and yields more linear flow-pressure curves as shown in Fig. 13*b*.

Spools are commonly on the order of 0.25 in. in diameter and require relatively small motion for control, on the order of 0.01 in. or less.

Spool valves are often used as the second stage of a two-stage valve in which the first stage is a low-force, low-inertia device such as a flapper or a jet pipe.

Flat Plate Valves. The flat plate control valve is shown schematically in Fig. 14. The basic principle of operation is similar to that of the spool valve, except that the movable element is a flat plate, which may slide axially or pivot about a point midway between the ports. The "sandwich" construction used in these valves reduces the manufacturing problems associated with the spool valve, resulting in low internal leakage, zero dead zone, and less susceptibility to clogging by dirt particles

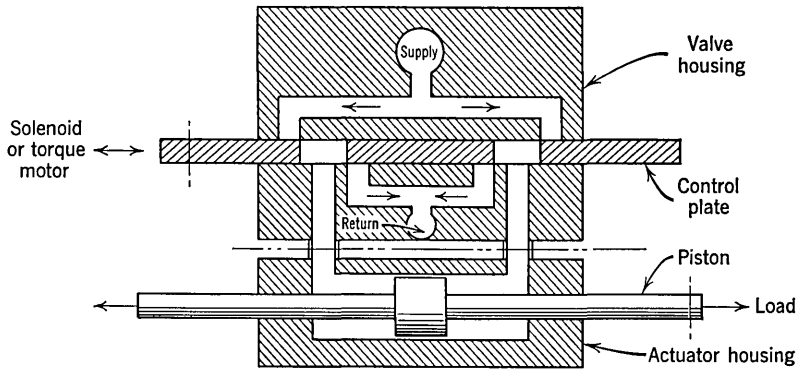


FIG. 14. Schematic diagram of flat plate flow valve and actuator.

present in the fluid. These advantages are often offset by the need for external manifolding with attendant bulky construction and external leakage.

Poppet Valves. The poppet valve is illustrated in Fig. 15. Although this valve has many applications, it has limitations when applied to precise controls, owing to the likelihood of dead zone and the difficulty of balancing static forces.

Jet Pipe Valves. A schematic diagram of a jet pipe valve is shown in Fig. 16. The nozzle is swiveled between the two load pipes so as to develop a differential pressure across the load (Ref. 7).

Flapper Valves. The valve shown in Fig. 17 is a two-stage valve employing a flapper as the first stage and a spool as the second stage.

Current through the electromagnetic torque motor causes the flapper to restrict one nozzle while opening the other. With the flapper in the center, the hydraulic circuit is balanced; the flows through both nozzles are equal; the pressure drops through the two upstream orifices are equal; and the pressures at each end of the spool are equal. When the flapper restricts one of the nozzles, the pressure drop across the corresponding orifice is reduced and the pressure drop across the opposite orifice is increased, resulting in a differential pressure across the spool. The spool moves until the spring force equals the hydraulic force, and the result is fluid flow to one side of the load and return flow from the other side of the load.

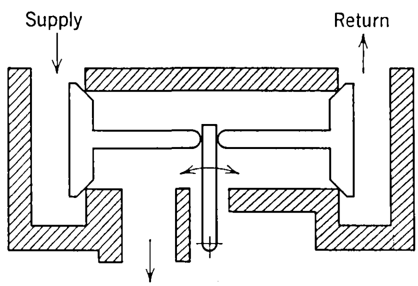


FIG. 15. Poppet valve (three-way).

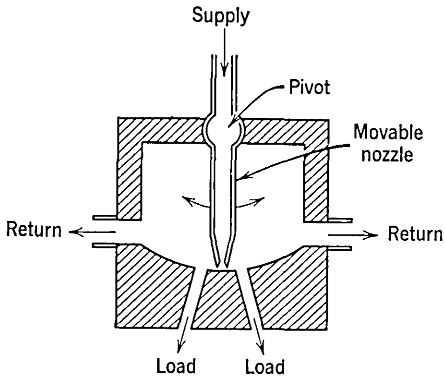


FIG. 16. Schematic diagram of jet pipe valve.

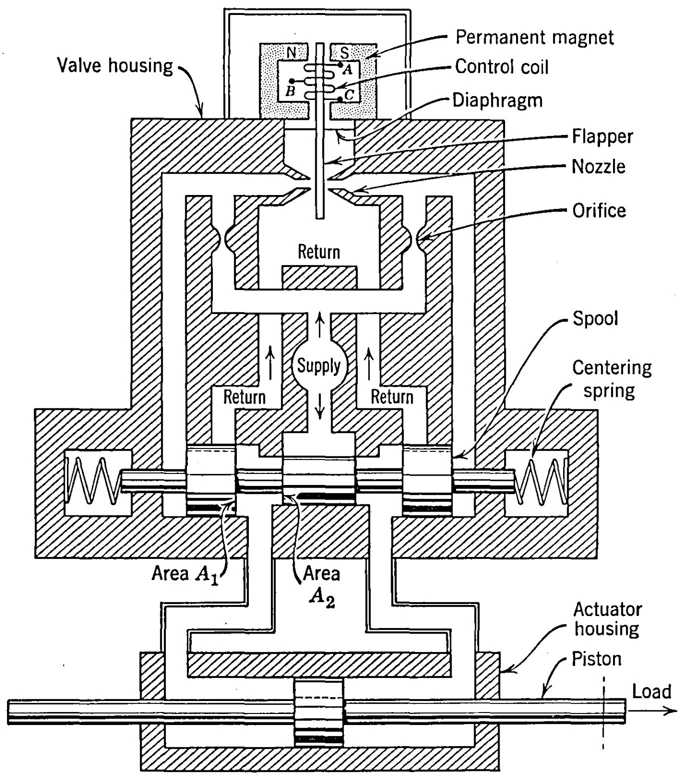


FIG. 17. Schematic of two-stage flapper valve. It is a flow valve when $A_1 = A_2$, a pressure valve when $A_1 > A_2$.

This type of valve has high gain and good response (low time constant), making it well-suited for high-performance servomechanisms.

Flow Valves. In Fig. 17, if the areas A_1 and A_2 are equal, the valve is known as a flow control valve. Under this condition the load pressure forces on the second stage spool are balanced, so that there is no tendency for the load to be reflected to the spool.

Pressure Valves. If area A_2 in Fig. 17 is intentionally made smaller than A_1, the effect is to introduce negative feedback proportional to the differential pressure across the load. This effectively makes the valve a pressure control valve instead of a flow control valve, and introduces the advantageous feature of damping any oscillatory tendency of the load (caused by load inertia, mechanical compliances, and fluid compressibility). Thus, it is often possible with pressure control valves to overcome the obstacle of resonant load characteristics that limit the performance attainable with flow control valves.

The addition of suitable hydraulic "networks" inside the valve can be used to produce a pressure derivative control valve, which acts like a flow control valve under static conditions and has the damping effect of the pressure control valve under dynamic, or high-frequency, conditions.

Actuators Controlled by Valves. Valve-controlled actuators are principally of two types: the single piston or ram, and the single vane or rotary ram. It is possible to control axial piston motors and rotary vane motors by means of valves, but to do so would be to lose the inherent efficiency of the pump-motor type of drive while retaining the disadvantages of the pulsating torque and gear drives associated with such devices.

The single piston actuator and the single vane actuator respond smoothly to the controlled flow from the valve and can be designed to deliver the torque demanded by the load without requiring any mechanical gearing. Smooth motion over a 1000-to-1 speed range is easily attainable with these direct drives.

With the ram type of actuator directly coupled to the load, the predominant mass or inertia is usually that of the load, and the actuator torque and speed capabilities are the dominant factors in their design or selection.

Single Piston Actuators (Rams). Figures 12*b*, 14, and 17 show single piston actuators, or rams, and the method of operation is obvious. The force developed is the product of the differential pressure and the active piston area, and the velocity is the flow rate delivered by the control valve divided by the displaced volume per unit motion of the piston. Thus, for a given supply, valve, and load, the size and power capability of the actuator are interrelated.

Single Vane Actuators (Rotary Rams). Figure 18 shows a single

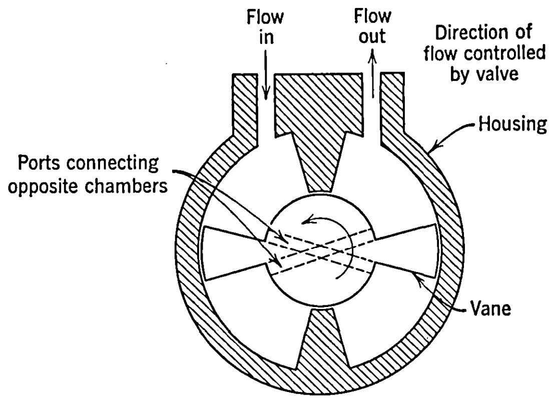


FIG. 18. Single vane (rotary ram) actuator.

vane actuator, or rotary ram. Fluid delivered to either port will cause a corresponding rotation of the vane in either direction. Single vane actuators have displacement factors (displaced volume per unit angular motion) ranging from 0.04 in.^{3}/radian to 1.50 in.^{3}/radian, with 0.60 in.^{3}/radian being typical. They are commonly designed for operation at pressures of 1000 psi and range in weight from a few ounces to several pounds. Angular travel is usually no more than about 140° for the type shown. Removing one-half of the vane and one of the partitions from the chamber will permit angular travel as high as 320° but raises the problem of unbalanced pressure forces on the vane.

Pneumatic Systems (See also Chap. 7, Instrumentation Systems)

Pneumatic devices are similar to their hydraulic counterparts in that pressure and flow of a fluid are varied to transmit information and power in a system, but the fluid in each type of system has widely different characteristics which influence the choice and use of actuators in a control system. Of course, the basic difference is that the liquid fluid used in hydraulic systems is much less compressible in a normal environment than the gaseous fluid used in pneumatic systems.

Pneumatic actuators are particularly useful when (1) light weight and minimum expense are desired, (2) extreme speed of response and great accuracy are not required, (3) the ambient temperature variation is large, and (4) a large torque-to-weight ratio is not required (larger torques can be produced by relatively small, light actuators if high, but hazardous, supply pressures are used).

The advantages and disadvantages of pneumatic actuators are considered separately in more detail, and then their operation, construction, and application are discussed.

Advantages of Pneumatic Actuators

1. The gaseous fluid used in the actuators is desirable because

a. It costs nothing if air is used, whereas fireproof hydraulic fluid is very expensive.

b. It does not constitute a fire hazard.

2. Lighter weight systems, important in missile and aircraft, result because

a. The fluid is very light in weight.

b. No fluid return line from the actuator is needed.

3. Fluid leaks are not important because

a. The fluid does not damage surrounding equipment.

b. The system can continue to operate if the leak is not large.

4. The fluid viscosity does not vary widely with temperature changes, and the actuator damping remains relatively constant.

5. Fluid contamination is not a serious problem if reasonable precautions are taken.

6. Most pneumatic actuators are relatively inexpensive and require little maintenance.

Disadvantages of Pneumatic Systems

1. A satisfactory power supply may be difficult to obtain because

a. Work and time are required to compress the gas and maintain a desired pressure.

b. Relatively large and heavy accumulators may be required to maintain a constant pressure when rapid or frequent changes in load occur.

c. A volume of compressed gas can be very dangerous; a rupture can cause an explosion.

2. High-performance pneumatic actuators and systems are difficult to realize because

a. Time is required to develop control pressure in an actuator when the fluid is highly compressible.

b. Power transmission through pneumatic connecting lines is relatively slow. This produces transport lags which make fast system response impossible.

c. High-compressibility properties of the gaseous fluid result in low-frequency resonances which greatly reduce stability margins and make fast control loop responses impossible to achieve.

3. Although not mechanically important, the noise developed in pneumatic systems, from normal leakages, may be objectionable.

4. Pneumatic flow and pressure are influenced by factors other than the input control signal because the compressibility depends on the nature of the fluid flow (see Ref. 8). The compressibility may be

a. A function of the gas temperature and depends on whether the flow is an isothermal or an adiabatic process. Generally, the flow is assumed to be isothermal.

b. A function of flow velocity, defined in terms of the Mach number. Compressibility increases with the Mach number and pipe length.

c. A function of laminar or turbulent conditions usually determined from the value of the Reynolds number R. Generally $R < 1500$ insures a desirable laminar flow.

Applications of Pneumatic Actuators. Pneumatic actuators have a unique status in control systems because they may be used as error detectors, amplifiers, compensating networks, and power actuators. It is possible to construct a complete control system using only pneumatic actuators with no electronic control equipment. For example, in a pressure control loop the input and output pressures can be compared with a piston or a bellows which moves when the pressures are not equal. This motion can be used to slide a valve controlling the flow of gas to a power actuator, a bellows, or piston, which moves an output pressure control valve until the output pressure changes to become equal to the desired input pressure.

Although pneumatic actuators are evidently quite versatile with considerable potential advantage, they have not been used as much as hydraulic actuators in continuous feedback control systems because the speed of response is limited by the fluid compressibility to about one-third to one-quarter the response of a hydraulic system using similar components. However, the largest use for pneumatic actuators has been either in open loop control or in closed loop control where a relatively slow speed of response can be tolerated. Therefore, pneumatic control has been widely used in the process control industry, with safe, low operating pressures under 100 psi, where the speed of response is not an important factor (see Chap. 7, Instrumentation Systems, and Part D, Chemical Process Control Systems).

Pneumatic actuators may be used as on-off devices which control valves or brakes from one extreme position to another. There is little danger of breakage from the abrupt motion because it is softened by the compressibility of the fluid; and because there are definite limits in position travel, stability is not a problem. Of course, truck and train brakes have been controlled by pneumatic actuators for many years.

Another important application of pneumatic actuators is in the control of missiles. Here large stability margins may not be needed, the supply pressure may be of large magnitude, up to 1000 psi or more, because the danger of an explosion is not important, and the supply pressure need only be available for a short time from a precharged container or from the rocket gases. In this connection, gases with very high pressures and temperatures may be used, and with the resulting low Mach numbers the fluid becomes less compressible, more like a liquid. Of course, actuator materials must be selected, designed, and fabricated to withstand these severe operating conditions, and considerable work is being done to develop reliable actuators for these "hot gas servos."

General Characteristics of Pneumatic Actuators. Dynamic characteristics of actuators are influenced by the components with which they operate. For example, the connecting lines and orifices have resistance to flow, and volumes, including the actuator volume, have effective capacitance or the ability to store a volume of fluid under pressure as a potential flow. Because actuators have capacitance, several may be used at low power levels in conjunction with resistive lines to produce compensation networks equivalent to relatively complex RC networks (see Ref. 8, pp. 448–452).

Unlike resistance in electric circuits, the resistance in pneumatic devices, including actuators, varies with the shape of the orifice through which the fluid flows, as well as with the pressure drop over the orifice. For example, a round orifice has a resistance of about $8 \times 10^{-4}(\Delta P/D^2)$ lb-sec/in.^{5} where D is the orifice diameter, and a square orifice has a resistance of about $4 \times 10^{-4}(\Delta P/A)$ lb-sec/in.^{5}, where A is the area of the orifice and ΔP is the orifice pressure drop between 0.5 and 1.5 psi. On the other hand, a long thin tube has a resistance of about $10^{-7}/D^4$ lb-sec/in.^{5} per inch of length.

Although capacitance is slightly dependent on whether the fluid expands isothermally or adiabatically, it is independent of temperature change. The capacitance is dependent on the pressure, however, and this must be considered. For example, capacitance C_p, in in.^{5}/lb, may be expressed approximately as

$$C_p = 0.9V/P$$

where V is the volume in cubic inches and P is the pressure in lb/in.^{2}. Under normal conditions, P is about 15 lb/in.^{2} and $C_p \approx 0.06V$.

These properties essentially define the dynamic characteristics of the pneumatic actuators. The amount of flow and torque that must be provided depends on the required load velocity, disturbance, torque, and accelerations, and these capabilities can be determined in a manner similar to that previously described.

Although the viscous friction of the fluid in pneumatic control systems may be neglected, the mechanical friction is usually quite important because lubrication is not normally provided by the fluid as it is in hydraulic systems. Therefore, to minimize the friction and stiction, which produce errors and wear in actuators with rotating or sliding members, adequate lubrication must be provided. Supplying proper bearing lubrication and spraying droplets of lubrication into the gaseous fluid will take care of this problem.

Types of Pneumatic Actuators. Many pneumatic actuators are very similar to hydraulic actuators, and, if proper lubrication is provided, some hydraulic actuators will operate with a gaseous fluid. However, other types of pneumatic actuators which have no rotating or sliding parts are commonly used, and they are also described.

Single Piston Actuators. These actuators are similar to those used in hydraulic systems, and problems of selecting the required actuator are similar except that the capacitance, compressibility of the gas, and lubrication must be given special consideration in their effects on system performance. Actuators of this type may be used to provide unlimited two-directional linear motion provided the volume of gas in the actuator does not become large enough to cause instability for the speed of response required.

Single Vane Actuators. These actuators, also used as hydraulic actuators, provide limited rotary motion. As in the piston actuator, the lubrication, the volume, the pressure of the gas, and the corresponding load torque must be considered in the selection of an actuator of this type.

Rotary Vane Actuator. This is similar to the continuously rotating hydraulic vane actuator. Lubrication must be provided to reduce the friction and wear of the rotating vanes. The speed-torque curves of this device show that it is essentially a pressure or torque controller, because they vary significantly with flow or load velocity changes.

Axial Piston Actuators. This type of actuator was described in the discussion of hydraulic actuators. It is not used as a pneumatic actuator, however, because lubrication must be suspended in the gas and the mixture is likely to explode in the hot cylinders (see Ref. 8, p. 476).

Gear Actuators. Although this type of actuator is widely used in hydraulic systems, it is not often used in pneumatic systems largely because of the lubrication problem. However, with proper lubrication, this type of actuator could be expected to provide satisfactory performance.

Bourdon Tube Actuators. This actuator is influenced by temperature changes as well as by pressure changes. It is essentially the closed, curved tube used in barometers which tends to straighten as the internal pressure is increased. Thus, by controlling the pressure, it may be made to exert a force over a limited distance. The output force is a very linear function of the internal pressure. The proportionality factor between the pressure and force depends considerably on the material, usually a metal, from which the tube is made, and the spring constant is usually quite large. Repeatability is excellent but, because of natural hysteresis in the metal, this is an actuator variable which must be determined. Of course, the metal must not be overstressed in operation or its life and accuracy will be seriously affected. Therefore, mechanical stops or pressure limits should be provided when this type of actuator is used.

Bellows Actuators. The bellows type of actuator has features quite similar to those of the Bourdon tube. Although it is a spring-loaded device, it may be constructed with a large number of flanges for relatively large motions. Bellows actuators may also be made in a variety of configurations, with levers and other bellows to provide motion or force as the difference of two or more pressures.

To maintain a long life and accuracy, bellows as well as Bourdon tubes are often operated at half their rated travel and pressure limits. If this is done, millions of cycles of operation may be obtained.

7. MECHANICAL ACTUATORS

All actuators have mechanized parts, but the distinction made here between *mechanical actuators* and other actuators is that the input signal to the actuator is amplified into useful output power, torque, and motion through mechanical leverage and friction, rather than by a fluid or by electric flux linkages. There is not a sharp division between mechanical and other classes of actuators because the mechanical action is usually initiated by electric signals. Ordinary gear trains, levers, and cams are not considered as actuators since they do not amplify the input power or convert it to other forms of energy. In this discussion the actuators considered as mechanical are rotary solenoids, brakes, and clutches. There are many other special mechanical actuators, such as the Graham drive, which are essentially variable speed devices, and they are mentioned briefly below in the section on Special Actuators.

Rotary Solenoids

These actuators are available in various sizes with output torques ranging from 0.1 to 100 in.-lb. In this range they weigh less than 6 lb; they are less than 4 in. in diameter and less than 2.5 in. thick. Basically, they are solenoid coils with an output shaft through the center of the coil. When the coil is energized with a d-c voltage, the shaft is pulled into the coil. However, a disk on one end of the shaft, resting on ball bearings in slightly inclined grooves on the solenoid housing, forces the shaft to rotate on the ball bearings as it moves axially into the coil. The slope of the incline is not made constant because the axial force is not linear, but the result is that the output torque is linear with respect to the angular motion and the control voltage. Angular motion is limited to about 100°, but only 45° is normally provided. Therefore it can be seen that, although they are very compact, rotary solenoid actuators can produce a relatively large torque through wide angular motion.

These actuators are usually used for remote positioning control and for the shafts of rotary switches, but they can be used in feedback loops to provide position control.

Clutches and Brakes

Clutches and brakes are perhaps the most widely used form of mechanical actuator, although they are not used as widely as electric or hydraulic actuators. They may have various designs, but the design is based on the simple clutch principle of connecting two shafts with friction disks. The difference between the *clutch* and a *brake* is that the clutch connects one moving shaft to another, whereas a brake connects one moving shaft to a stationary member.

Clutches are particularly useful when

1. Light weight and simplicity are desired.
2. Extreme accuracy and smooth motion are not required for long periods of time. (This is a function of the type of clutch used.)
3. Rapid response is necessary.
4. Large power gains are needed.
5. Sudden shock loads are expected.
6. Large ranges of temperature and pressure are encountered.
7. Extremely long life is not required.

Brakes are usually used to supplement clutch operation. They are used to stop rapidly moving loads quickly or to prevent shafts from moving when disengaged from the clutch. The advantages and disadvantages of clutches are considered in more detail, with a discussion of their operation, construction, and applications. Because brakes are quite similar in construction and operation, they are not discussed separately.

Advantages of Clutch Actuators

1. Light weight and small size for a large power output may be realized.
2. Extremely large power amplifications of several hundred may be obtained.
3. Input signal levels are often low enough to be provided directly from detecting devices without electronic amplification.
4. High torques and very fast accelerations provide output response times in the order of a few milliseconds.
5. Smooth output motion can be obtained.
6. Sudden load torques do not damage the actuator because slippage can occur naturally between the input and output shafts (unless a crown tooth coupling is used to prevent slippage).

7. The effects of stiction and friction can be eliminated.

8. High efficiency and reliability can be realized, depending on the design.

Disadvantages of Clutch Actuators

1. Some types of clutches have high wear rates.

2. Heat is generated by the slippage between the input and output shafts.

3. Stability problems may be introduced because of chatter, bounce, and grabbing.

4. Intermittent performance, inherent in some types of clutches, may prevent smooth position tracking.

5. The performance of clutches in feedback control loops is difficult to predict and to calculate. Thus, the prediction of performance and the determination of specifications for a satisfactory clutch are difficult.

General Characteristics of Clutches. As previously stated, clutches transmit power through mechanical friction. These are the common friction clutches. Magnetic particle clutches also fall in this class. However, there are many devices called clutches which do not use mechanical friction as a coupling medium. These are known as *eddy current clutches* and *hysteresis clutches* in which power is actually transmitted through magnetic flux linkages similar to the way in which electric motors transmit power. Electric motors, however, use rotating electric fields to produce rotary mechanical motion, but the clutches use a rotating shaft as a source of power, and the magnetic fields are used to induce connecting torques between the input and output shafts. The magnetic fields are developed by control signals, and they do not in themselves produce rotary motion. Although these devices are not strictly mechanical in nature, they are discussed in this section because they are clutches.

The torque versus input control current and the speed-torque curves are needed for clutches, as they are for all actuators, when one is being selected for a particular application. The slope of the torque versus input control signal curves provide the effective clutch gain, and the speed-torque curves are an indication of the damping, operating ranges of torque, speed, and power. As in all types of actuators, these curves vary in shape and magnitude with different designs and types of clutches, and there are no universal curves to which a designer can refer. The characteristic curves of clutches do have one common feature—all are quite nonlinear. Most of the clutches have curves with square law characteristics, and they are frequently linearized to provide uniform, predictable performance by using two clutches in a push-pull arrangement to connect the input and output shafts. This general design, illustrated in Fig. 19, is the most often used because it also provides bidirectional rotation of the output shaft

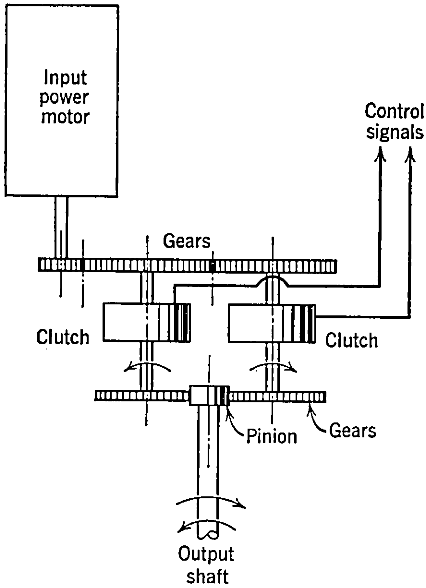


FIG. 19. General design of clutches used in push-pull to provide linearity and bidirectional motion.

from a single direction rotation of the input shaft. Thus, only one input motor is needed, and it need not be reversed to change the direction of the output shaft. The inertia of the drive motor, operating at a high constant velocity, acts like an energy accumulator, and its speed does not change with suddenly applied loads.

Because clutch slippage, always present to some extent, dissipates power in the form of heat, it is important to consider the temperature conditions, the heat rise, and the cooling facilities when a particular clutch actuator is being selected. The power dissipated because of slippage is a function of the input and output speeds and the power transmitted to the load as shown in the following relationship:

$$P_s = (\omega_i/\omega_L - 1)P_L,$$

where P_s is the power dissipated because of slippage,
ω_i is the input velocity,
ω_L is the load velocity, and
P_L is the power transmitted to the load.

This dissipated power becomes large when thousands of horsepower are transmitted to the load, as it may be with eddy current clutches. Therefore, the slippage must be kept as low as 3 to 5%. The heat loss may still be high, however, and air cooling or water cooling may be necessary to prevent large temperature rises and loss of efficiency.

Friction Materials for Clutches. Clutches are sometimes designed to have continuous slippage between the input and output shafts. Soft materials provide smooth motion as the friction surfaces are pressed more tightly together to increase the output torque. Soft metals, cork, and composition materials have been used. However, these materials wear rapidly, especially if the average pressure between the surfaces is not properly adjusted, the performance deteriorates, and it is difficult to obtain clutches with equal and consistent characteristics for smooth push-pull operation for long periods of time.

For these reasons friction clutches are frequently used in on-off operation. In this way maximum torque is transmitted only as long as needed to change the output shaft position or velocity to a desired value. The resultant operation is not smooth, although it can be effectively linearized by dither (Ref. 9) or with velocity feedback. For this on-off operation, different clutch materials are used to give reliable, consistent operation for millions of cycles. It has been found that case-hardened steel against cold-rolled steel is the most suitable, although polished chrome-plated steel against cold-rolled steel and stainless steel against case-hardened steel are also satisfactory (Ref. 10).

Types of Clutches

Friction Disk Clutch. This type of clutch is perhaps the most widely used. The principle has already been described. It is capable of slightly higher accelerations than a hydraulic actuator of comparable size, which is about five times faster than an equivalent-sized electric motor. Power ratings usually do not exceed 200 to 300 watts, but, when operated from a current source, the time delay may be only 3 to 10 msec and the power gain may be as high as 200 to 1.

The speed-torque curves of a friction clutch show that the torque does not vary with speed. Therefore, it is essentially a torque source with no inherent damping. Effectively, it has a double integration in its transfer function and, if it is to be used as a continuous control actuator in a closed loop control system, adequate damping must be provided to obtain a stable system.

The Magnetic Fluid Clutch. This type of clutch has face plates which are permanently separated; they do not come in contact and transmit torque through surface friction. Instead, the permanent gap between the clutch faces is filled with an oil in which magnetic particles are suspended. When a magnetic field, proportional to a control current of less than 10 ma, is applied to the clutch with axial flux lines, the magnetic particles attract one another, the viscosity changes, the fluid becomes more like a solid, and more torque is transmitted from the input to the output shaft with power gains up to 200 to 1. The fluid provides cooling, lubrication, fairly uniform distribution of the magnetic particles, a linear torque with control current when used in push-pull operation, and very smooth power transmission with relatively little wear. Theoretically, large amounts of power could be transmitted with little no-load drag, but, actually, the amount of power is limited to about 300 watts, and the practical usefulness of this type of clutch is curtailed by the lack of suitable seals to prevent the fluid from leaking out of the clutch. For this reason, a dry powder magnetic clutch may be selected in preference to a magnetic fluid clutch.

The Dry Powder Magnetic Clutch. This clutch is similar in operating principle and in construction to the magnetic fluid clutch, except that the fluid is removed. (Sometimes this clutch is constructed somewhat like the hysteresis clutch described later.) Only the magnetic particles are left to provide torque between the clutch face when the magnetic field is applied to them. Because the fluid is not used, the problem of obtaining leak-proof seals is nonexistent, but other disadvantages occur. Wear may be greater because the particles can act as abrasives, the power transmission can be nonuniform because the powder may not distribute itself evenly in the space between the clutch faces, and corrosion may become a more serious problem in certain environments. In spite of these disadvantages, however, the dry powder clutch normally has the smooth transmission characteristic of the fluid clutch, and several hundred horsepower may be transmitted through it.

Hysteresis Clutch. The face plates in this type of clutch are in the form of concentric cylindrical surfaces. One face plate is a solid cylinder with a cylindrical slot cut in one end, concentric with the axis, and with an electric coil inside of it to produce a radial magnetic flux perpendicular to the slot when a small control signal is applied to the coil through slip rings. Actually, the slot does not have smooth surfaces. Instead, it has a number of staggered axial grooves on each side of the slot which form pole faces of the same polarity when the control flux passes through them. The other clutch face is a thin cylindrical shell made of permanent magnet material which is placed into the slot between the pole faces of the other clutch member. The magnetic clutch face is attached to the output shaft, and the other member is attached to the input shaft.

When the control signal energizes the coil, flux is produced across the pole faces through the magnetic shell between them. As a result, magnetic poles are generated in the magnetic material by the flux, but these induced poles lag the pole faces angularly because of hysteresis. Therefore, the induced poles are attracted to the pole faces which rotate with the input shaft, and the torque transmitted is nearly proportional to the control current. Enough torque may be produced to make the output shaft follow the input shaft at full-rated torque without slippage and without the resultant heat dissipation. However, if the input speed is increased, the lag angle of the induced poles with respect to the stator pole faces increases until slippage does occur. The resultant temperature rise increases rapidly as the speed is further increased, although the torque transmitted to the load does not increase greatly for a given control current. Thus the maximum velocity of the hysteresis clutch is limited.

The hysteresis clutch produces very smooth operation, it has a fast response time of less than 5 msec, it is not subject to wear, it generates no heat caused by slippage, but its power gain is less than 20 to 1, and it is only used in low-horsepower applications.

The Eddy Current Clutch. Like the hysteresis clutch, the eddy current clutch has no surfaces which come into direct contact. The eddy current clutch has a cylindrical rotor with several pole faces on the input shaft and an internal coil which may be excited by a d-c control current through slip rings. A thin shell made of conducting material, an integral part of the output shaft, is placed over the rotor. When an input current is applied to the rotor coils, *and* the conducting shell is moving relative to the rotor, eddy currents are induced in the conducting material of the shell. The eddy currents induce a magnetic flux which produces a drag torque on the output shaft. Because slip must occur to produce output torque, heat dissipation must be carefully considered if an eddy current clutch is selected for applications in which the power transmitted is large, for example, several thousand horsepower. In these large power ranges, with only a few per cent slippage, air or water cooling must be provided. Cooling improves the efficiency of the clutch, and with water cooling the efficiency of the clutch approaches that of an induction motor.

The speed-torque curves of the eddy current clutch are usually quite nonlinear. They may have a slightly positive slope at low velocities and then a large negative slope as the speed approaches the maximum at low torques. This indicates that the viscous damping is widely variable. The control characteristics are also changed widely with temperature because of the temperature coefficient of the winding and conducting rotor. Therefore, because of its variability, some form of feedback must be designed around an eddy current clutch to make it have reliable and consistent operation.

The eddy current clutches are similar to induction motors, and as in induction motors, the rotor resistance must be considered when selecting an eddy current clutch. Clutches with low-resistance rotors have less slip and greater efficiency but low starting torques, whereas clutches with a high rotor resistance have less efficiency but higher starting torques.

Although the eddy current clutch is capable of controlling large power smoothly and continuously with no wear, its power gain is less than 20 and it has longer response time than the other types of clutches.

Special Actuators

The actuators discussed in the preceding paragraphs are made by many manufacturers, and the operating characteristics are basically the same. However, there are many actuators, each of which has been developed by only one company, sometimes as a combination of the more basic actuators previously described. Although they are useful in many special applications, they have specifications and characteristics which can best be determined from the manufacturer's data and recommendations.

Because of the large number of special actuators which are commercially available, all of them cannot be discussed or even listed here. However, a few are mentioned to indicate what special actuators are. They are often known by trade names and, although many are standard production items, they can be designed for special applications upon order.

Special actuators are often the results of unusual applications for which new devices had to be developed to meet design specifications. Sometimes they find wide application, the demand for them increases, and they are produced by several manufacturers and cease to be special actuators. However, whether actuators are special or not, they cannot be selected without a thorough study of their particular characteristics and their reliability over the necessary operating ranges in the type of environment in which they will be required to operate.

Graham Drive. This is basically a variable speed control. It is a mechanical actuator which uses conical pulleys to drive an output shaft through an idler which is moved with a small control force along the pulleys to change the velocity of the output shaft. The manufacturer indicates that the speed is controlled within 5% at maximum rated torque. The drive is available in compact sizes from $\frac{1}{4}$ to 3 hp. Speed ranges range from 0 to ± 9000 rpm. Power is transmitted with 85% efficiency. Units are made by the Graham Drive Company of Menomone Falls, Wisconsin.

Servotran. This is a speed control device similar to the Graham drive. It is available in sizes from $\frac{1}{50}$ to $\frac{1}{4}$ hp, and it can be reversed from full speed forward to full speed backward in 50 msec. The output torque is constant, and the efficiency is between 85 and 95%. This device is available from Humphrey, Inc., in San Diego 6, California.

Electrostatic Clutch. This is a clutch which works on the principle that adhesion between a metal clutch plate and the surface of a semiconductive material covering the second clutch plate varies with the applied voltage. The facing of one clutch surface is made of a special rubber base plastic with good homogenous resistivity, dimensional stability, and wear resistance. A small amount of a liquid lubricant is used to reduce residual drag and clutch wear. Torques may be produced up to 80 in.-lb in a few milliseconds. Speeds of 2500 rpm may be obtained with 30 ma of current at 150 volts. This type of clutch has been designed by the IBM Product Development Laboratories, Endicott, New York.

"Inchworm" Actuator. This actuator, developed for precise linear motion of machine tool components, operates on the principle of magnetostriction. In this design the actuating rod is made of nickel with two split ring clamps about its circumference separated by an accurate spacer and an electric coil. The split ring clamps are closed alternately by air pistons in synchronism with an intermittent control signal applied to the coil. When the control signal is applied, one clamp is released and the bar shrinks toward the end securely held by the other clamp. Then the first clamp is actuated, and the bar is held securely in its shortened position. The control signal is removed, the second clamp is deactivated, and the rod expands with great force a few millionths of an inch. The second clamp is again activated, the first clamp is de-energized, the control signal is again applied, and the operation is repeated. In this way the bar is accurately advanced through the clamps with a great force in very accurate increments of motion, and the detrimental effects of static friction and stiction become virtually nonexistent (Ref. 11).

The "inchworm" actuator principle has been developed by the Airborne Instruments Laboratory, Inc., in Mineola, New York.

Cam Piston Motor. This is a special pneumatic actuator. It has three opposed pistons aligned axially in one cylinder. One face of each piston is actually a cylindrical cam and, as the pistons are moved back and forth along the ball-splined output shaft with ported air pressure, they are rotated by the rollers on the stationary housing acting upon the cam surfaces. As the pistons turn, the output shaft turns. Air is ported to the pistons through rotors on the output shaft. These devices operate without lubrication with low air supply pressure up to 200 psi in temperature ranges from -65 to $+1000°F$. Stall torques of 750 lb-in. are available in 9.8 hp units which are about 1 ft long and 3 in. in diameter. Rotational velocities can be varied smoothly from 0 to 2500 rpm. The cam piston actuators are produced in different sizes by the Garrett Corporation, Air Research Division, in Phoenix, Arizona.

Step Servo Motor. The step servo motor is becoming more important in control work in which digital computation is used because it is a direct digital-to-analog converter and power amplifier. The motor resembles an ordinary induction servo motor, but it operates on discrete d-c pulses. Each separate positive pulse advances the motor through a 45° angle, and eight pulses are needed to make one revolution. Of course, the number of pulses per second that can be used to operate the motor is limited by the motor response, but up to 120 pulses per second can be used to produce a rotation of 15 rps or 900 rpm.

Small step motors have three to six times as much stall torque as the same size a-c servo motor, but they require two to three times more power per pulse than the a-c motor normally uses.

Step motors are available in several sizes, and they are being produced by several manufacturers such as Induction Motors of California in Maywood, California.

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Computing Elements

A. S. Fulton

1. INTRODUCTION

In addition to making measurements of the process variables, amplifying suitable signals, and controlling certain processes, it is often necessary to perform additional operations on the signals of interest in order to obtain the desired result. Components employed to accomplish these additional operations are identified as *computing elements.* These components may operate to approximate mathematical operations such as addition, multiplication, division, integration, and differentiation. In addition, arbitrary functional relationships may be introduced between variables. This chapter is concerned with discussing briefly the important properties of components commonly employed to produce these operations.

Computing Combined with Control Components. It is often possible to combine a computing function with some other function in the control process. For example, a pressure transducer with a potentiometer output can be used to produce a signal proportional to the product of the pressure and some other variable by exciting the potentiometer with a voltage proportional to the second variable. These combinations are not explicitly discussed here. Also, when the process becomes sufficiently complex, it becomes economical to employ a digital computer to perform the computations. Discussions of applications of this type are given in Chap. 13, Computer Control, and Chap. 14, Data Processing. Essentially this chapter will be devoted to analog computing elements. For a more detailed discussion of analog computer elements see Vol. 1, Part E, Feedback Control, Chaps. 21–26.

Computing Methods. Ways of performing these computing functions are discussed in two categories, electrical inputs and outputs and mechanical inputs and outputs. Many combinations of electrical and mechanical signals are possible, but they are generally reduced to one or the other of these types before computing is done. For example, when one variable is a shaft angle and the other a voltage, a measuring element can be used to convert shaft angle to a voltage and the electrical operation can then be carried out. Conversely, the voltage can be converted to a mechanical motion with a servo and the mechanical operation carried out. Electrical computing means seem to find widest use, probably because of the greater ease with which (1) changes can be made in an electrical system and (2) signals can be transmitted. Most system designs are reached via evolution as test data indicate the need for changes, and flexibility is quite important.

Controllers. Certain special devices such as pneumatic controller units are available; these provide addition, integration, and differentiation, with independently adjustable gain. These units operate to approximate these three functions by means of complex valving and throttling of air. These and other types of controllers are discussed in detail in Chap. 7, Instrumentation Systems.

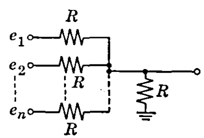
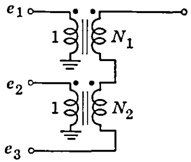
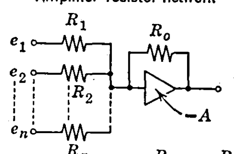
2. ADDERS

Electrical. Properties of electrical adders are shown in Table 1. The environmental effects are about the same for all three types. Temperature is the principal problem and the problems associated with amplifiers are next. The general remarks made about each type indicate the considerations that are usually important.

Resistance Addition. When the source of the voltages is of low internal impedance and the load is either a very high or a constant resistance, resistance adding is usually employed. Additional gain to make up for the attenuation can usually be found in some component of the system. On the other hand, if the resistance values used must be high, noise pickup or stray capacity may cause difficulties.

Transformer addition is often very attractive because of the precise turn ratios possible with high-quality toroidal cores. Core losses can be kept low so that the load impedance is accurately reflected to the input. If the load varies widely, reflecting the load impedance to the input will not be desirable.

TABLE 1. ELECTRICAL ADDERS

Typical Configuration	Properties: Source	Component	Load
Resistor network  $e_o = \frac{e_1 + e_2 + \cdots + e_n}{n+1}$	1. Impedance must be resistive. 2. Directly affects scale factor. 3. Range limited by power rating of resistors.	1. Handles ac or dc. 2. Attenuation severe for many inputs. 3. Simple, reliable.	1. Must be resistive. 2. Directly affects scale factor.
Transformer network  $e_o = e_1N_1 + e_2N_2 + e_3$	1. Load impedance reflected to source as N^2. 2. Range limited by core saturation.	1. Handles ac 2. Scale factor controllable. 3. Simple, reliable. 4. Some problem with phase shift owing to distributed and shunt capacitance of windings. 5. Provides a degree of isolation between input and output.	1. Usually not critical owing to multiplication by N^2 in interaction with input.
Amplifier-resistor network  $e_o = \frac{R_o}{R_1}e_1 + \frac{R_o}{R_2}e_2 + \cdots + \frac{R_o}{R_n}e_n$	1. Impedance must be resistive. 2. Impedance usually not critical since R_n can be made much greater. 3. Range usually limited by amplifier range.	1. Handles ac or dc. 2. Scale factor controllable. 3. Complex, less reliable. 4. Very precise. 5. For d-c applications drift must be considered (see Chap. 16).	1. Usually not critical owing to low output impedance. 2. Possibility of difficulty because of stability requirements of feedback amplifier.

Amplifier-resistor addition, although complex, is usually easy to apply because the amplifier provides effective isolation of the impedance effects. When the amplifier is already present for some other purpose, and sufficient loop gain can be obtained, this type usually provides the greatest flexibility and precision.

Mechanical. In addition to purely mechanical adders, synchros find wide use in systems in which angular shaft positions that are physically widely separated must be added. Synchro components that will add shaft positions and produce a sum shaft position without additional electronic devices are available. Operating in a self-energized manner, accuracies of 1° can typically be obtained. When the indicator (sum) synchro is servo-driven to a null, precision of 1 milliradian can be obtained.

Three types of mechanical adders are the gear differential, the tape differential, and the beam differential (see Fig. 1). Of these, the gear differential offers the greatest flexibility because of its unlimited travel. Blacklash can be a problem, although this can be reduced to values consistent with precision gearing with careful design. Abrasive and corrosive atmospheres can permanently damage all these mechanical units, and high temperatures can decompose the lubricants, but the gear differential continues to be linear until catastrophic failure.

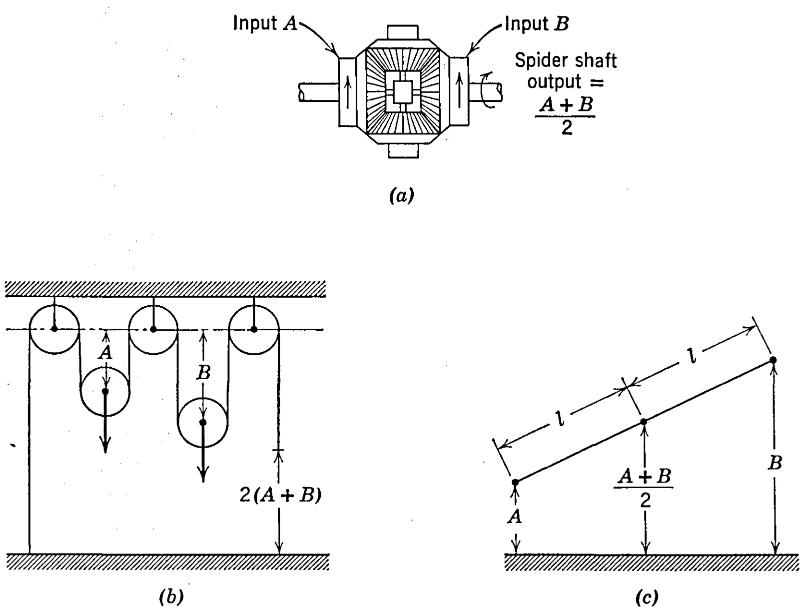


FIG. 1. Mechanical adders: (*a*) bevel gear differential, (*b*) tape differential, (*c*) beam differential.

The tape and beam differentials, although potentially eliminating backlash, are plagued by other failings such as stretching of the tape, changing of effective beam pivot distances owing to contamination in the pivots, etc. Under these circumstances errors in scale factor can creep in. In addition, their input and output ranges are limited. If approximate addition is satisfactory, the beam differential can provide addition in a simple, low-cost configuration.

3. INTEGRATORS AND DIFFERENTIATORS

Integrators and differentiators are logically discussed together because the same collection of parts can generally be changed, by a simple rearrangement, from one function to the other. Integration and differentiation are usually done with respect to time. For more details see Vol. 2, Chap. 22, Linear Electronic Computer Elements, and Chap. 27, Mechanical Computer Elements.

Two different types of applications exist for these devices. First, simple approximate devices find widest use for stabilization of control loops, etc. Second, precision devices find use in special applications and in simulation. Table 2 summarizes various configurations. Most precise mechanical computations are generally best implemented by rearranging the equations to eliminate differentiation, since this process becomes very cumbersome and inaccurate without servo-controlled components. Precise electrical computations can also usually be made more easily without differentiation, for precise electronic differentiators involving feedback amplifiers tend to be unstable because of the additional high-frequency rolloff caused by the feedback network.

The ball and disk integrator provides a way of integrating with respect to an independent variable. This can be a considerable convenience in some specialized problems, but in most applications the input representing *du* (see Table 2) is driven at a constant rate to provide integration with respect to time.

4. MULTIPLIERS AND DIVIDERS (See Vol. 2, Chap. 23, Nonlinear Electronic Computer Elements, and Chap. 27, Mechanical Computer Elements)

Electrical Multipliers. When electrical quantities are available for computation, either multiplication or division can generally be accomplished with the same device. Electrical multiplier-dividers are of three main types, servo, time division, and square law or functional (see Table 3). The servo and time division types can provide either multiplication or division directly, whereas the square law or functional types are usually used in the feedback path of an operational amplifier to provide division.

TABLE 2. INTEGRATION AND DIFFERENTIATION

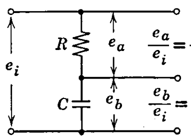
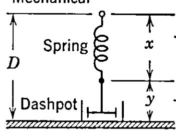
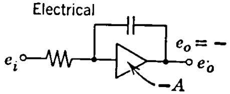
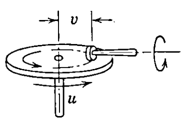
Element	Properties: Source	Properties: Component	Properties: Load
Approximate Devices Electrical $\tau = RC$  $\frac{e_a}{e_i} = \frac{\tau p}{1 + \tau p}$	1. Should be pure resistance and included in R. 2. Low impedance permits some reactive source impedance.	1. Approximates differentiation up to angular frequency equal 0.1τ. 2. Phase shift curve usually most significant control of performance.	1. High resistance. 2. Low-C-distributed capacities should not be a problem.
$\frac{e_b}{e_i} = \frac{1}{1 + \tau p}$	1. Should be pure resistance and included in R.	1. Approximates integration above angular frequency 10τ. 2. Phase shift curve should be considered.	1. High resistance, changes τ somewhat.
Mechanical  $\frac{x}{D} = \frac{\tau p}{1 + \tau p}$ $\frac{y}{D} = \frac{1}{1 + \tau p}$	1. Displacement input, low impedance source. 2. Range limited.	1. Dash pot may not be linear, giving more error. 2. Same general rules apply as for electrical unit. 3. Very simple and reliable.	1. High load impedance so as not to disturb motions. 2. Friction errors most serious.
Accurate Devices Electrical  $e_o = -\frac{1}{\tau}\int e_i\,dt$	1. Impedance not critical, R can be made high. 2. Range limited by amplifier.	1. Precision 0.1% with chopper stabilized amplifiers. 2. Drift a problem in some applications.	1. Load not critical unless for output limiting or amplifier stability.
Mechanical—ball and disk integrator  $\int v\,du$	1. Low impedance dead space can be a problem.	1. Precision 1% in standard unit. 2. Some slippage and drift. 3. May be some dead zone near zero.	1. High impedance load necessary. Loading can cause slip and wear.

TABLE 3. ELECTRICAL MULTIPLIER-DIVIDERS

Type	Input	Component	Output
	Properties		
Servo (See Fig. 2)	1. Impedance can be relatively high. 2. Signal can be either unmodulated or carrier.	1. Precision depends essentially on potentiometers, 0.1% common. 2. Bandwidth low, 10 cps good. 3. Most commonly used with functional outputs (see next section). 4. Drift associated with servo.	1. Load affects potentiometer linearity. 2. Many outputs possible by adding potentiometers to servo shaft. 3. Output can have different frequency than input.
Time Division (See Fig. 3)	1. Impedance can be relatively high. 2. Unmodulated signals.	1. Precision depends essentially on gating elements, 0.2% common. 2. Bandwidth high, 1 kc/sec good, 10 kc/sec marginal. 3. For wider bandwidth integrator linearity may be a problem. 4. Drift in gating and in cancellation for bipolar outputs.	1. Load impedance usually not critical. 2. Many outputs possible by adding gating circuits. 3. Output essentially unmodulated.
Square Law	1. See adders. 2. Unmodulated signals.	1. Precision depends on shape of functional curve, 5% good. 2. Bandwidth very high, limited by distributed capacity. 3. Virtually impossible to test at very high frequencies.	1. Load not critical when amplifier used.
Functional (See Fig. 4)	1. Unmodulated signals.	1. Generally the same as square law. 2. Temperature stability a problem.	1. Usually not critical.

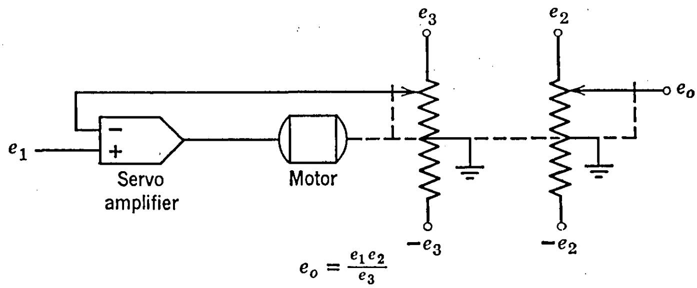


FIG. 2. Servo multiplier-divider.

Servo Multiplier-Dividers. The block diagram of Fig. 2 illustrates a servo type. A brief discussion of this configuration will indicate the kind of factors of importance in considering its use. As shown, the multiplier-divider will work as a multiplier with a constant value of e_3, within the range of values of $|e_1| \leqq |e_3|$ and for all values of $|e_2|$ within the dissipation ratings of the potentiometer. The loading on the output potentiometer must be made equal to that imposed on the feedback, or e_3, potentiometer by the adding network in the servo amplifier. When e_3 is permitted to vary to provide division, care must be taken to be sure that the relation $|e_1| \leqq |e_3|$ is preserved and that $|e_3|$ remains sufficiently large to ensure adequate sensitivity in the servo. Some form of automatic gain control is usually required in the servo amplifier when division is intended.

Time Division Multiplier-Divider. The time division type takes many forms. To illustrate the nature of the process, one form is shown in the block diagram of Fig. 3.

The pulse generator starts the integrator by opening the switch shorting the condenser. When the output of the integrator equals e_2, the com-

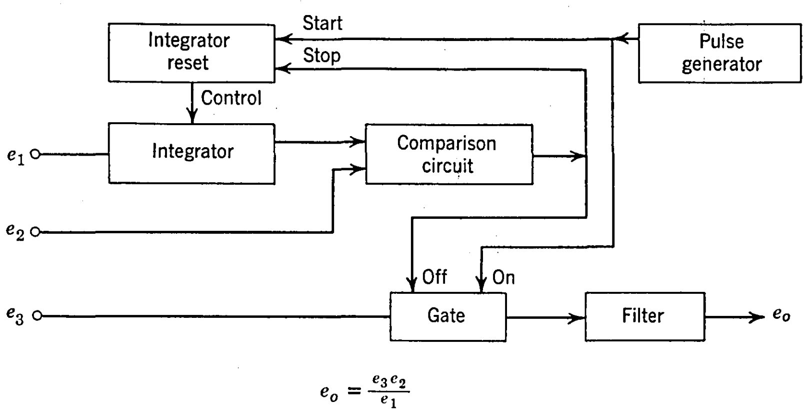


FIG. 3. Time division multiplier-divider.

parison gate generates a pulse which stops and resets the integrator by closing the switch. The process generates a time interval proportional to e_2/e_1. This time interval is used to gate e_3 to the output. The output filter is usually required to suppress the higher frequencies generated as a result of the repetitious nature of the process.

The block diagram of Fig. 3 represents a simplified unit which is restricted to e_1 and e_2 having fixed polarities. More complex switching arrangements can provide for e_1 and e_2 to be bipolar, but these arrangements usually imply some additional drift owing to the attendant cancellation circuitry. The bandwidth is usually determined by considering the phase properties of the product, since these are usually of the most significance in a control system. Phase shift can usually be held small to frequencies corresponding to about one-tenth the pulse repetition frequency.

Square Law Multipliers. This type of multiplier employs a nonlinear device having essentially a square law characteristic. Vacuum tubes are often used. Adders are required to mechanize the equation.

$$4ab = (a + b)^2 - (a - b)^2.$$

Since the output is derived by subtracting two quantities, some combinations of inputs will produce very poor accuracy. For example, if $a = 0.1b$, the output is obtained by subtracting two large numbers:

$$4 \times 0.1b^2 = 1.21b^2 - 0.81b^2.$$

Consequently, 1% errors in forming $1.21b^2$ and $0.81b^2$ will produce errors of 4% or so in the product. Often this condition can be avoided, however, and this type of multiplier is simple and capable of processing wide band signals.

Division employing a square law device is obtained by placing the device in the feedback path of an amplifier. Stability in this feedback system is generally a problem.

Functional Multiplier-Divider. This type is similar to the square law in making use of nonlinear properties of certain elements or combination of elements. Elements having logarithmic response are employed. The block diagram of Fig. 4 illustrates this type. Drifts in the function gen-

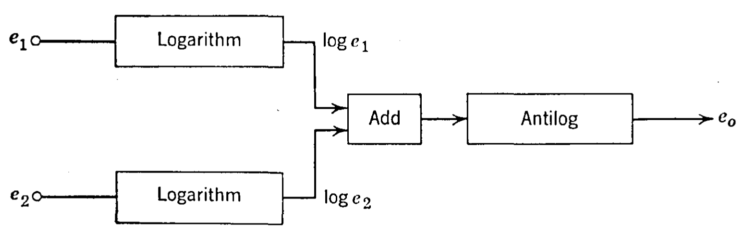


FIG. 4. Functional multiplier-divider.

erators can cause serious errors, depending on the range of variables to be accommodated. This type of multiplier is very useful when the ranges of e_1 and e_2 are such that range of the product is less than 10 to 1. Function generators are discussed in the next section.

Modulation is sometimes employed to produce multiplication. However, modulated carriers of different carrier frequencies are required, and these are rarely present in control systems. This type operates on the principle that if voltages

$$e_1 = A \sin \omega_1 t$$

and

$$e_2 = B \sin \omega_2 t$$

are presented to a mixer and then to a bandpass filter centered at $\omega_1 + \omega_2$ or at $\omega_1 - \omega_2$, an output

$$e_o = AB \sin [(\omega_1 \pm \omega_2)t]$$

will result.

Mechanical Multipliers-Dividers. Many forms are available for combined mechanical-electrical computation; however, most of them depend on function generators of one form or another. Servos employing mechanical multiplication in the feedback path can, of course, divide, but this is usually not an economical arrangement. Function generators are discussed in the next section.

Purely mechanical multiplication can be performed in at least two ways, integration and similar triangles. Integration makes use of the identity

$$uv = \int u \, dv + \int v \, du.$$

Since mechanical ball and disk integrators permit integration by independent variable, this form provides a direct solution. Unlimited range is available with this mechanization, but these integrators are relatively expensive.

The similar triangles method of mechanical multiplication can be employed where limited ranges are adequate. This method also provides division. The configuration shown in Fig. 5 is not intended to illustrate a practical configuration. It does, however, indicate a method of multiplication or division which can provide very accurate results over the range provided.

5. FUNCTION GENERATORS (See Vol. 2, Chap. 23, Nonlinear Electronic Computer Elements, and Chap. 27, Mechanical Computer Elements)

In many control systems special mathematical functions or experimentally derived arbitrary functions relate the variables available for

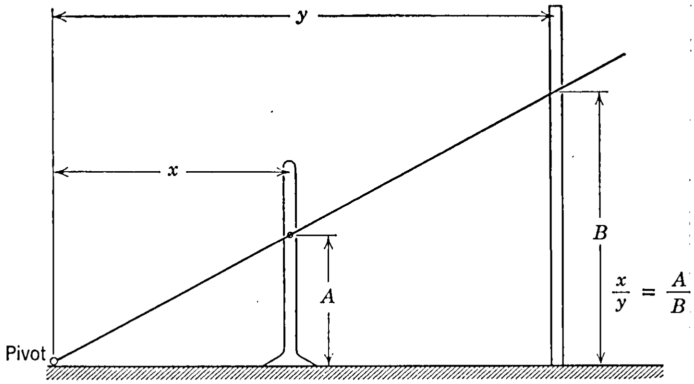


FIG. 5. Similar triangles multiplication.

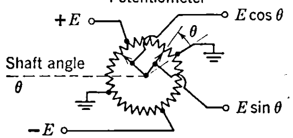
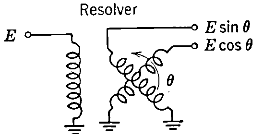
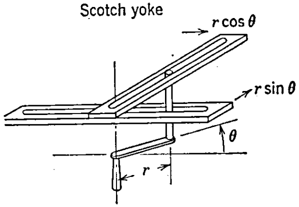
measurement and the actuation required for proper control. In other systems, the generation of functional relations is sufficiently simple to permit their implementation by function-generating components. This is the type of component discussed here. The complex implementations arising in some problems such as digital control of processes (see Chap. 13, Computer Control, and Chap. 14, Data Processing) are essentially system designs in themselves and are beyond the scope of this chapter. The problems of deriving or measuring the proper functions are discussed in Vol. 1, Chap. 19, Methodology of Feedback Control, and Chap. 20, Fundamentals of System Analysis. It is assumed that the problems have been reduced to generating functions of one or two variables.

Sine and Cosine. A large number of applications exist for generating sine or cosine of an independent variable. Typical components employed are (1) functional potentiometers, often called sine-cosine pots; (2) functional variable transformers, called resolvers; and (3) a mechanical analog, called a scotch yoke. Table 4 contains a summary of these generally used types. Special combinations are sometimes employed. For example, the scotch yoke principle combined with a ball and disk integrator can be used to generate the orthogonal components of velocity. However, these common types serve to illustrate applications.

The other trigonometric functions approach infinity for certain inputs. The generation of one of these functions can usually be handled best as the generation of an arbitrary function. They can be generated from sine and cosine, but it is usually simpler and just as accurate to employ the techniques available for generating the arbitrary functions to be discussed.

Arbitrary Functions. Electrical techniques for generating arbitrary functions are limited to functions of one variable for reasonable component complexity. So-called three-dimensional cams can produce arbitrary

Table 4. Sine-Cosine Function Generators

Type	Properties		
	Input	Component	Output
Potentiometer 	1. A-c carrier or d-c. 2. Both plus and minus voltage required. 3. Low impedance source. 4. Unlimited shaft rotation θ, voltage E, limited by designation.	1. Precision typically 0.5% in small size. 2. Good life, 10^6 shaft rotations or better. 3. Phase shift may be a problem at high a-c frequencies and high resistances. 4. Environmental performance good. 5. $d\theta/dt$ limited owing to brush wear.	1. Impedance critical, potentiometer designed for load.
Resolver 	1. A-c carrier only. 2. Single polarity. 3. Low impedance source.	1. Precision typically 0.1% in small size. 2. Phase shift can be a problem at frequency extremes. 3. Very long life, slip rings only sliding contact. 4. Environmental performance good. 5. High $d\theta/dt$, limited by armature strength and balance.	1. Impedance must be high resistance, but not critical.
Scotch yoke 	1. Mechanical inputs, output loads input. 2. r value not readily adjustable. 3. Unlimited θ, r limited by yoke travel.	1. Precision good, 0.1% readily obtained. 2. Corrosion and abrasion, also wear, a problem. 3. Good life. 4. Low speed due to inertia of parts.	1. Power available from input.

functions of two independent variables. Functions of a greater number of independent variables require some form of approximation to reduce the problem to a combination of functions. For example, if one of the approximations

$$
\begin{aligned}
F(x, y, z) &= f(x, y) \times g(z), \\
&= f(x, y) + h(z), \\
&= f(x)g \times (y) \times h(z),
\end{aligned}
$$

or

$$F(x, y, z) = f(x) + g(y) + h(z)$$

can be shown to be sufficiently accurate, standard types of function generators can be used. If not, the function probably cannot be mechanized.

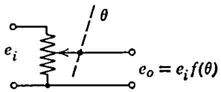
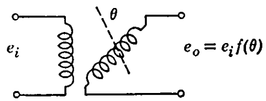
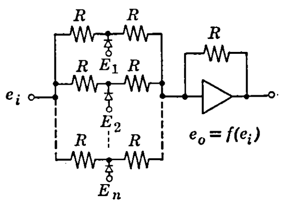
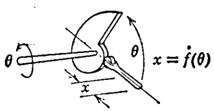
Potentiometers, variable transformers, biased diode assemblies, electrical and mechanical cams, and special linkages are commonly used for generating arbitrary functions. The properties that are generally significant in selecting this type of component are summarized in Table 5. In considering the problem of generating functions, we are usually concerned with the slopes present in the function as well as the value of the function. The slope is important because changes in slope are usually the significant factor in selecting the type of function generator.

Functional Potentiometers produce the output by generating a series of straight lines of different slope. The slope can be controlled by varying the resistance element itself—such as mandrel size, wire size, and winding pitch—or by providing taps and connecting resistors in parallel with the resistance element. Controlled loading can modify these straight lines, but in a limited number of ways. Potentiometers have the merit of wide availability and the flexibility that comes from being able to change potentiometer curves readily.

Variable Coupling Transformers inherently produce functions which are generally sine-cosine in shape. They are generally used for this type of function, and their use has been limited by the considerable expense of changing the shape of the function. This comes about because of the complexity of producing stator and rotor windings with a specified functional relationship between the transformation ratio (coupling coefficient) and shaft angle.

Biased Diodes. If a source of suitable reference voltages is available, and the d-c signals are present, biased diodes are often useful. Their most serious limitation is that they cannot produce a product of one voltage times the function of another variable, as can the potentiometers and variable transformers. On the other hand, the generation of the function can be performed at very high frequencies since no mechanical motions are involved. It is theoretically possible to generate a function of two

TABLE 5. ARBITRARY FUNCTION GENERATORS

Type	Properties: Input	Properties: Component	Properties: Output
Potentiometer 	1. Same as Table 4 except (*a*) θ travel limited to less than 360°, (*b*) polarity required depends on $f(\theta)$.	1. Same as Table 4 except precision about 0.25% attainable.	1. Same as Table 4.
Variable coupling transformer 	1. Same as Table 4.	1. Same as Table 4.	1. Same as Table 4.
Biased diodes 	1. Low impedance. 2. Range limited by diode ratings. 3. Dc only.	1. Precision depends on number of diodes used and functional shape; 0.1% reasonable from diode performance. 2. Handles wide band signals. 3. Amplifier required for precision. 4. Drift in diodes and amplifier. 5. Large number of low impedance reference voltage required.	1. With amplifier, not critical.
Cams 	1. θ travel < 360°. 2. x travel limited by space available.	1. Same as Table 4 comments on scotch yoke.	1. Same as Table 4.

independent variables with a biased diode network by supplying the reference voltages E_1, E_2, . . ., E_n from individual function generators. A very broad band signal might require such heroic measures, but the design would be quite complicated.

Cams provide a simple, direct mechanical function generation. Where a mechanical output is useful and the bandwidth of the independent variable is narrow, this method is recommended. The time required to produce new cam shapes sometimes discourages system designers, and if the system equations change frequently owing to changes in requirements, other function generators may be better. However, system engineers tend to pass cams by many times when they are the best choice.

Linkages can provide practical functional relationships. However, they are even more difficult to apply in early designs than cams. Since the pace of system design often requires the choice of speedy methods rather than those that are most suitable from long range cost and performance considerations, linkages rarely receive consideration. Considerable information is available regarding their design (Refs. 4 and 5) for those interested in their application.

Three-Dimensional (3-D) *Cams* provide a method for generating a function of two independent variables. These cams essentially consist of a large number (approaching infinity) of r, θ cams stacked up in z with a way of moving the cam follower (output shaft) in z. They are widely used for reproducing ballistic tables and other applications in which the frequencies of interest are within the capability of mechanical devices and the functions cannot be specified as the product or sum of two independent functions of the variables.

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Continuous End Point Analyzers

J. K. Walker and C. W. Bailey

1. INTRODUCTION

Throughout the process control industry there are requirements for measuring special properties of materials that cannot be met by employing simple, direct measuring concepts. Consequently, a number of types of units which generally operate by making secondary measurements and deriving the desired information have been developed. These devices are fairly complex systems in their own right, and yet many process control people would consider them components, generally of the measuring element class.

The analysis instruments available to monitor the numerous process streams in use today number well into the hundreds. These instruments use such physical properties as heat and light absorption, mass difference, refractive index, and thermal conductivity. This chapter contains a brief summary of the more commonly used types of analyzers from the standpoint of principles of operation.

2. OPTICAL ANALYZERS

Types and Applications. Under the heading of optical analyzers there are two general classifications, namely dispersive and nondispersive analyzers. These analyzers use infrared, visible, or ultraviolet light to accomplish the analyses. The infrared and ultraviolet analyzers (*dispersive analyzers*) rely on the phenomenon that different compounds absorb different wavelengths of light in the spectrum. By comparing a sample with known materials, certain analyses may be made. The *refractometer* (*nondispersive analyzer*) uses visible light and measures the amount that a compound will deflect a beam of light directed at some angle incident to the compound. Again, comparing with known compounds enables an analysis to be made. Table 1 shows some typical applications of optical analyzers.

TABLE 1. APPLICATIONS OF OPTICAL ANALYZERS

Analysis	Analyzer
Ethylene purity in 90–100% range	Infrared
Methane and ethane in ethylene	Infrared
Carbon dioxide in hydrocarbon streams	Infrared
Trace water in liquid benzene and Freon	Infrared
Phosgene in hydrogen chloride	Infrared
Butadiene in hydrocarbon streams	Ultraviolet
Oxides of nitrogen (N_2O_4 and NO_2)	Ultraviolet
Ozone	Ultraviolet
Acetone in water	Refractometer
Alcohol in water and water in alcohol	Refractometer
Styrene in ethylbenzene	Refractometer

Molecular Spectroscopy. Many analyses such as those shown in Table 1 have evolved from extensive *molecular spectroscopy* studies in laboratories exploring molecular structure and methods of analysis. Certain fundamental laws have evolved and may be used for purposes of calculating analyses. Probably the most versatile and widely used law is the Beer-Lambert law, which in essence says that the amount of light absorbed at some wavelength is a function of the extinction coefficient of the compound and of the concentration and the length of the cell containing the sample. Given as an equation the law expresses the absorbance A as

$$A = \log (I_0/I) = eCX = -\log T, \quad (1)$$

where I_0 = initial radiation intensity,
I = incident radiation intensity,
e = extinction coefficient,
C = concentration,
X = length of cell, and
T = transmittance.

Other variables such as temperature, pressure, solvation, etc., affect the apparent chemical concentration, but these effects will not be discussed here.

Equation (1) holds only for monochromatic light, so calibration of known mixtures permits a quantitative analysis for as many components as there are monochromatic absorption frequencies. At each of these frequencies, or wave numbers, the sum of the absorbancies of each compound in the mixture is equal to the whole. By knowing the extinction coefficient e and absorbancies A, the concentration of each component may be determined by substitution into the simultaneous equations:

$$A_{ij} = \Sigma e_{ij}C_{ij} \quad (2)$$

Infrared Analyzers

Dispersive. In Figure 1 is shown a schematic diagram of the optical components of a versatile laboratory type, automatic scanning, double beam infrared spectrophotometer. Two beams from the Nernst glower source S_0 are focused so that one, S, passes through the sample

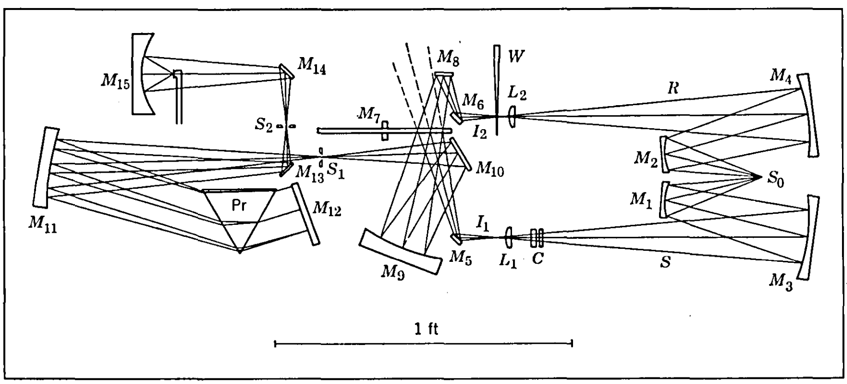

FIG. 1. Schematic diagram of an automatic scanning double beam infrared spectrophotometer. (Courtesy of Perkin-Elmer Corporation.)

cell C, and the other, R, passes through a calibrated optical wedge W. The beams are then alternately combined by a rotating sector mirror M_7 to transmit one beam at a time. If the two pulsed beams are equal, no output is observed at the detector; if the beams have unequal intensities, a pulsating voltage is used to drive the optical wedge attenuator into a null position. This principle of operation is used to compute and record automatically the transmittance (I/I_0) as the spectrum is being scanned.

A salt prism Pr disperses the radiation into a spectrum of monochromatic wavelengths, and the movable Littrow mirror M_{12} brings the different wavelengths into focus at the detector D, which is a sensitive thermopile. The prism and sample cell windows are made of such things as rock salt, potassium bromide, and lithium fluoride. These materials are hygroscopic and therefore subject to damage by high humidity. Consequently, the instrument is operated at slightly elevated temperatures. Temperature, of course, affects the refractive index of the components, so there are compensating devices to correct for this.

Figure 2 (Ref. 1) is a photograph of the basic instrument. The overall dimensions are 40 in. × 20 in. × 22 in. without the external power supply and amplifier. The recording drum and wavelength drive are mounted on top of the monochrometer cover. The main control panel is over the housing, with additional controls on the front of the base.

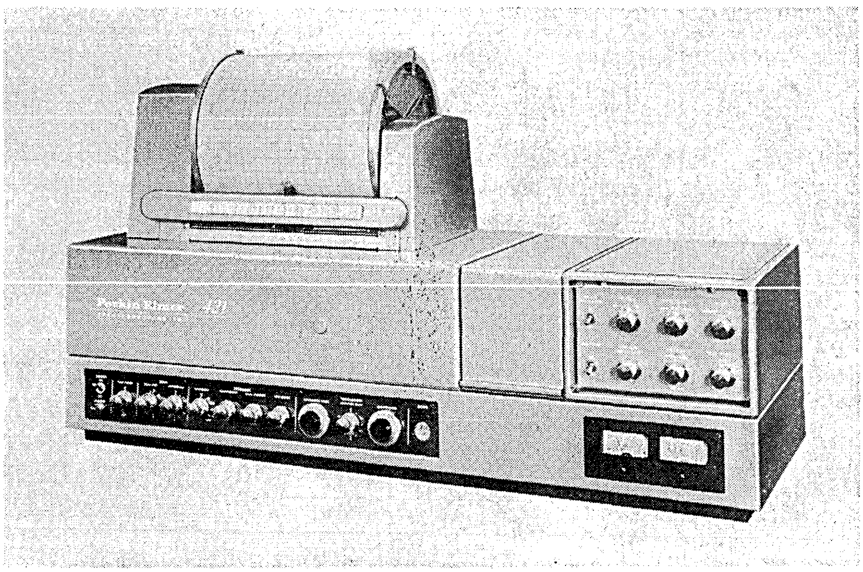

FIG. 2. Photograph of infrared spectrophotometer described by Fig. 1. (Courtesy of Perkin-Elmer Corporation.)

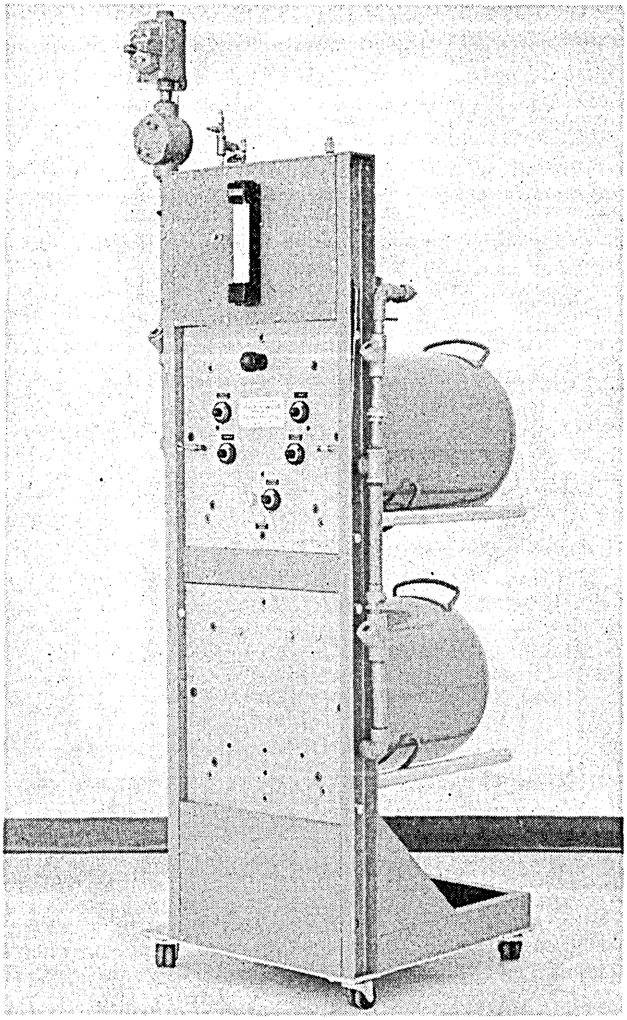

FIG. 3. Two-wavelength dispersive analyzer. (Courtesy of Consolidated Electrodynamics Corporation.)

To design a dispersive infrared analyzer for continuous end point analysis, it is necessary to sacrifice versatility and performance for reduced size, explosion proofing, and stability. One manufacturer has designed a two-wavelength dispersive analyzer (Fig. 3) for continuous operation using an optical null principle to eliminate effects of amplifier gain changes, source deterioration, and optical interference. Fixed, rugged optics are used, relying on dispersion, interchangeable detectors, prisms, and sources for versatility. Provisions for thermostating, explosion-proof housing, and an inert gas purge are designed into the instrument.

The optical design shown in Fig. 4 is very similar to the laboratory model, except that it employs a single split beam. The two wavelengths are selected on vertically mounted Littrow mirrors and are cyclically chopped out of phase with the detector focal point. As an energy difference is detected, an attenuator consisting of an optical wedge is driven into the beam of the reference wavelength to restore the output to balance. The position of the attenuator is then a measure of the concentration of the desired component.

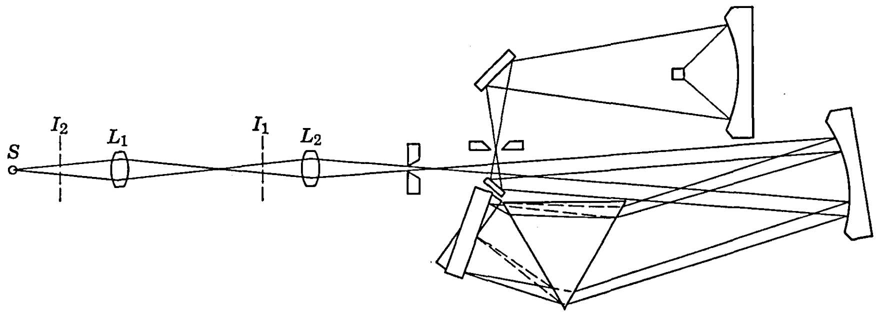

FIG. 4. Two-wavelength split beam dispersive analyzer. (Courtesy of Perkin-Elmer Corporation.)

The greatest advantage offered by such a dispersive instrument is the reduced application engineering required for sensitizing the instrument for unicomponent analyses in which complex interference is encountered by variations in the sample composition. An almost direct relationship exists between the operation of the monitor and that of the laboratory type dispersive instrument. It is therefore possible to sensitize the unit from readily available tables and spectra. Sensitivity, cell geometry, and applications can be predicted without time-consuming tests. Analysis of both liquids and gases now possible in the laboratory may be translated directly to the process instrument wherever two frequencies or wavelengths are sufficient to define the problem.

Nondispersive. With petroleum cracking, hydrocarbon separation, and purity demands on chemical products, continuous process analyzers are needed. To meet this demand, numerous simplified nondispersive analyzers have been developed. In contrast to dispersive monochromatic analyzers, the nondispersive instruments make use of selective emission, selective filters, selective detectors, or combinations of these techniques. Selective emitting sources of infrared radiation have not been found practical. Several negative filtering and selective (positive) detector types of analyzers have been developed commercially and differ primarily in the radiation detector used.

The *negative filtering types* of infrared analyzers are composed of a single or double nichrome spiral source and various sizes of optical gas and liquid cell arrays with optical windows made of materials with the desired infrared transmission characteristics. The radiation detectors used are nonselective devices, such as bolometers and thermopiles. These instruments must then depend on selective optical filters or sensitization by gas-filled cells. This principle of operation is limited in its threshold of detection (minimum detectable concentration), since it must detect a small change in the high intensity of radiation incident upon the detector. This limit is governed ultimately by electrical noise, stability, and optical cleanliness.

Figure 5 shows a typical optical arrangement of a negative filtering type infrared analyzer and related electrical components. The source S con-

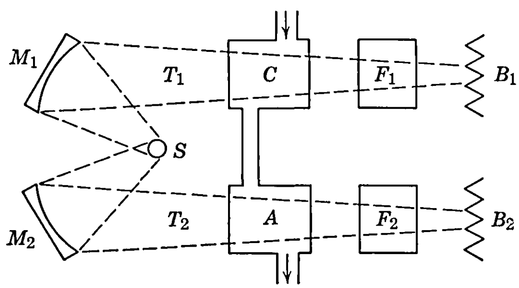

FIG. 5. Negative filtering type infrared analyzer.

sists of a coil of nichrome or chromel wire operated at 6.3 volts and 4 amp, approximately 400–800°C, to emit radiation over a sufficiently wide wavelength band for most gas analyses. The radiation is split by two front-surfaced plane concave mirrors M to transmit through two optical trains. Sample cells A and compensator cells C of varying dimensions generally extend across both radiation beams. Filter cells F are supplied for each beam for sensitization to desired radiation. Optical trimmers T are generally located in each beam near the source to balance the incident radiation on the two matched bolometers or thermopiles B. The detectors are nothing more than thermal sensitive resistors forming two arms of the familiar Wheatstone bridge. Since this resistance is proportional to the temperature determined by its radiation, an unequal radiation unbalances the bridge. The amount of unbalance is then calibrated in terms of chemical concentration, as previously discussed.

The usual operational mode consists of introducing the gas mixture into the sample cell S, which absorbs radiation equally in both beams. The pure component gas of interest is then introduced into one of the filter cells. The filter cell absorption is representative of 100 per cent compound A, and the difference between the radiation with the sample cell filled and empty is a portion of the calibrated range. The compensator cell is used to blank out radiation changes due to variations in composition of gases other than the component of interest. Ratios of gases may be determined by filling both filter cells with different gases. More than one gas may be determined in a series of samples of different composition, etc. The concentration range and sensitivity can be varied by changing the cell lengths and arrangements.

Positive filtering types of continuous infrared analyzers differ from the negative filtering type in the use of the detector. These selective detectors are based on early designs of Luft (Ref. 2), Pfund (Ref. 3), and Veingerov (Ref. 4). The detector is made up of a sealed chamber filled with an infrared-active gas and provided with a window to admit radiation. A sensitive microphone is incorporated to detect minute pressure changes in the gas as the temperature changes with radiation absorption. The radiation is chopped (10–20 cycles/sec) as it enters the cell, causing the gas to be cyclically heated and cooled. The resulting pressure pulses are detected by the microphone and converted to an a-c electrical signal suitable for stable amplification and recording. The two detectors are physically connected through a pneumatic leak to allow for ambient pressure equalization. The detector is sensitive only to the rapid pressure changes detected at the frequency of the chopper, resulting in a rapid response to small changes in the incident radiation. Dynamic crystal and condenser microphones have been used, the former offering the greater sensitivity and stability.

Figure 6 shows that the operation of the microphone instrument differs only in sensitization technique. The detector P acts as the filter cell previously described, and the sample cells S_a, compensator cell C_o, source S, and trimmers T remain essentially the same. Using the detector cell as a positive gas-filled filter, the microphone detects only the wavelengths or frequencies absorbed by the particular gas of interest. Should overlapping bands exist in gas mixtures, these frequencies are absorbed in either the compensator cell or a mixture of gases used in the detector.

In theory, the problem of sensitizing the analyzers for a particular application appears simple. In actual practice, this is only true for applications in which no overlap or interference of absorption frequencies occurs between the desired component and other compounds in a mixture. For practical use of these instruments on process streams, the complete range of variation in composition of the gases or liquids must be taken into

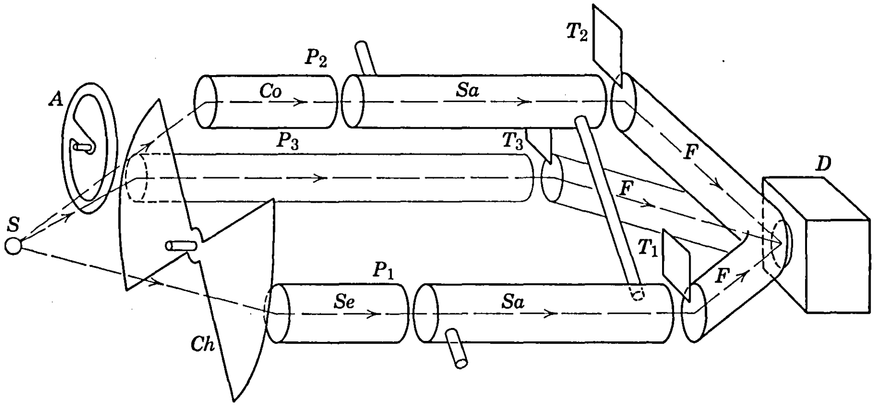

FIG. 6. Positive filtering type infrared analyzer.

consideration, so that the analyzer output may be representative of the desired component only. In highly complex mixtures of infrared active gases, sensitization can become a large scale research project. In practice, a dispersive infrared analysis is necessary to predict interferences at the various frequencies. To apply this information involves experimenting with negative filtering materials and gases to arrive at an optimum cell arrangement with sufficient energy change incident on the detectors for adequate sensitivity and discrimination.

Various *optical materials* which permit partial infrared transmission are used to advantage. Table 2 lists the more commonly used materials and their respective transmission regions. Synthetic filters of germanium-deposited silicates are available with very specific transmission characteristics. For example, one filter employed in the region of water and hydrocarbon absorption for the O–H and C–H frequencies transmits a center wavelength of 3.4 microns with a half bandpass of 0.15 micron and 70% transmission.

TABLE 2. OPTICAL MATERIALS USED IN INFRARED FILTERS

Infrared Filters	Wavelength, microns
Quartz	0–4.4
Sapphire	0–5.3
Lithium fluoride	0.11–6.0
Calcium fluoride	0.12–9.0
Arsenic glass	13.0
Silver chloride	1–30
Sodium chloride	0.2–15.0
KRS-5	0.5–40
Potassium chloride	0.38–21

Chemical and physical stability of these filters are very important when operation is automatic and unattended. For this reason quartz, sapphire, and calcium fluoride are the most commonly used materials.

Applications. Applications for these highly competitive instruments are becoming widespread. In general, the positive type instruments are recommended for minor and trace quantity detection and the negative for major component and liquid phase applications. Table 3 lists some of

TABLE 3. APPLICATIONS OF NONDISPERSIVE INFRARED ANALYZERS

Compound-Analyzed Gases	Concentration Range, %
Carbon dioxide	0.0061–100
Carbon monoxide	0.0002–100
Sulfur dioxide	0.001–100
Water vapor	0.0001–0.4
Hydrogen cyanide	0.00002
Methane	0.01–100
Acetylene	0.005–100
Ethylene	0–50
Ethane	0–10
Propane	0–2
Isobutane	0–10
n-Butane	
Liquids	
Water	0.1–2.0
Alcohol	0–2.0
Toluene	0–5.0

the common gases analyzed. In very specialized cases parts per billion detection threshold have been claimed.

Limitations in the use of infrared nondispersive analyzers are primarily determined by sample-handling requirements and the infrared activity and specificity of detection.

1. Gas sample must be free of solid matter and condensable vapors.
2. Homogeneous mono- and diatomic molecules are inactive to infrared radiation.
3. Instrument response time is limited by volume of sample cells, lines, and flow rate of gases used.
4. Economics of sample size may necessitate recycling back to the process.
5. Liquid samples can be used only on the negative filtering type analyzers or on analyzers whose positive detector can be sensitized with an interfering gas.
6. Liquid cells must be less than 0.2 in. thick, with a resultant slow sampling purge time.

7. Materials of construction are limited in use at higher temperatures and in corrosive atmospheres.

8. Sensitization is costly and maintenance slow when explosion hazards exist.

9. Unpredicted contaminants can be registered as components of interest.

10. Frequent zero and span standardization are required for confident operation.

To overcome these limitations several companies offer completely explosion-proof or explosion-resistant models with numerous accessories, such as sample vaporizers, filters, flowraters, regulators, multiple stream sample programmers with solenoid valves, and automatic zero, span standardization, and telemetering equipment. Competition among six or more commercial models has become so great that completely automatic installations are available at a very modest cost.

Ultraviolet Analyzers

Dispersive. Development of ultraviolet analyzers has followed much the same path as that previously described for infrared analyzers. Early work was devoted to the exploitation of the principle of spectrophotometers, or laboratory dispersive analyzers. Numerous models have been designed and built commercially. One of the instruments in this field is pictured in Fig. 7. This instrument is usable over a range of wavelengths from 2000 A to 2.5 mμ. The schematic optical diagram, Fig. 8, shows the path of ultraviolet and visible radiation through the instrument. Radiation from the hydrogen lamp A or tungsten lamp C enters the double monochromator through slit D. It is dispersed by the prism F and grating J. H is the variable width intermediate slit. Monochromatic radiation leaves slit L and at 30 cycles/sec passes alternately through the reference cell T' and sample cell T by means of a rotating semicircular mirror used as a chopper. It alternately reflects the light beam to the mirror R and allows it to pass on through to the mirrors P and R'. The beam through the reference cell, consisting of 30-cycles/sec pulses of monochromatic light, and a similar beam through the sample cell are directed to the photocell X by the mirrors V, V' and W, W'. The light pulses of the two beams are out of phase with each other so that the photocell receives light from only one beam at a time. The two beams are measured and compared electronically, to record directly in terms of linear absorbance or transmittance.

Synchronized with the square wave pulses of radiation incident on the receiver is a system of photoelectric timing signals which introduce the

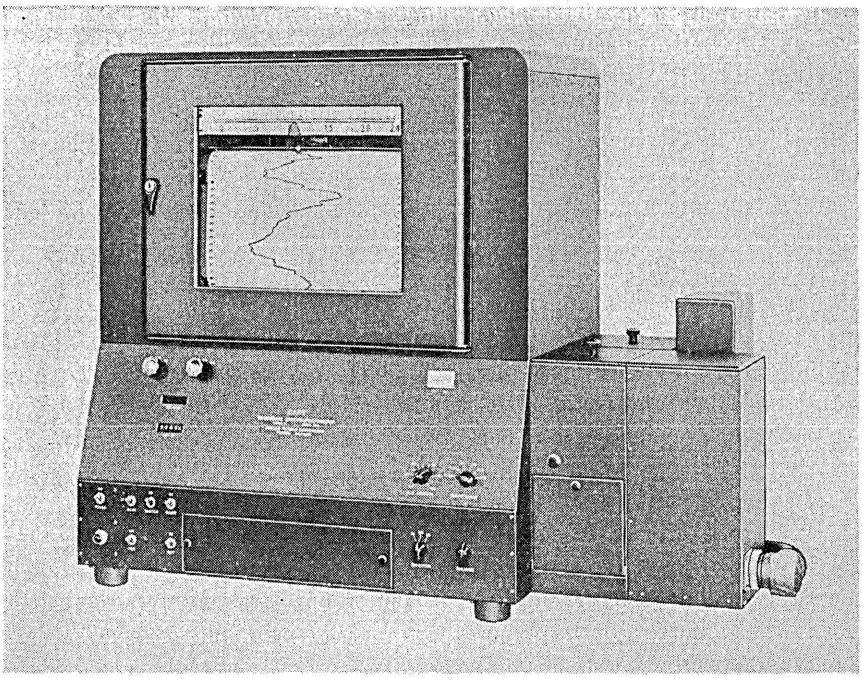

FIG. 7. Dispersive ultraviolet analyzer. (Courtesy of Applied Physics Corporation.)

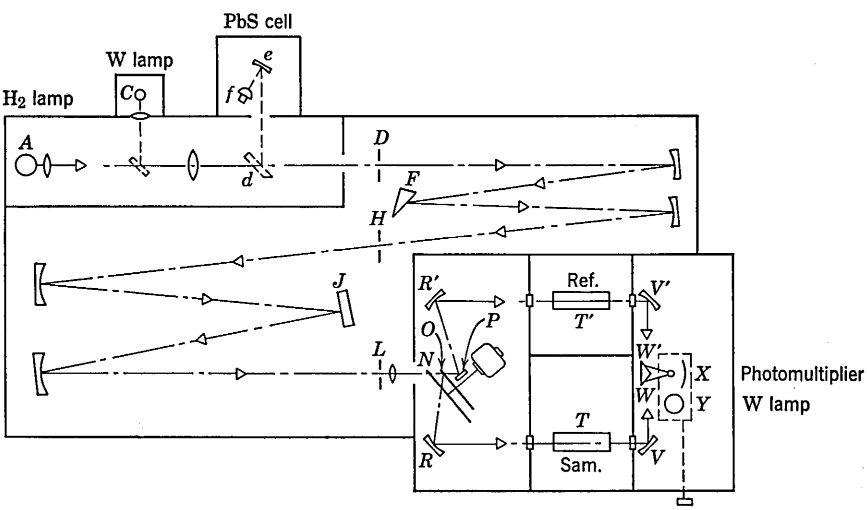

FIG. 8. Path of ultraviolet radiation through ultraviolet analyzer. (Courtesy of Applied Physics Corporation.)

attenuation of the recorder slidewire during the reference interval, but the sample signal passes to the comparison circuit unattenuated. A null-balancing servo is used to adjust the position of the pen and slidewire. The accuracy with which the pen and ink recorder on the instrument can be read is equivalent to plus or minus 0.002 in the 0 to 1 absorbance range and plus or minus 0.005 near an absorbance of 2. With proper sample handling techniques, an analytical accuracy of 0.2% may be obtained on concentration determinations. Stray light is less than 0.0001%.

Nondispersive. Two types of continuous ultraviolet analyzers are available commercially. One analyzer is an industrial type designed

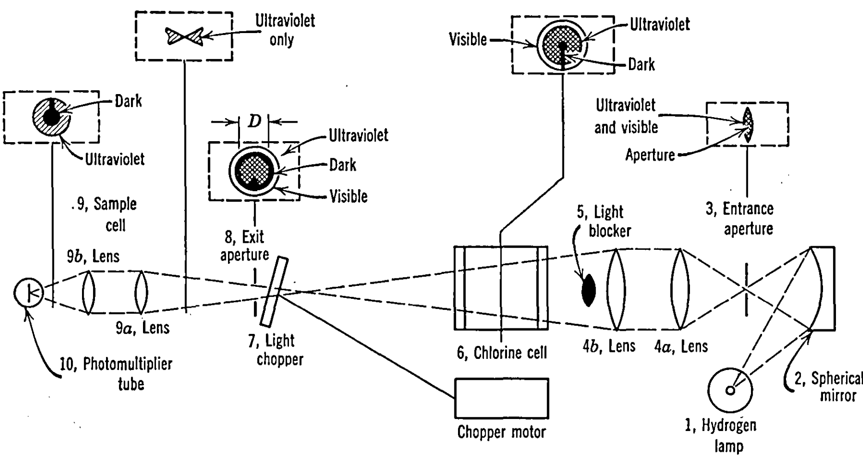

FIG. 9. Radiation path in nondispersive ultraviolet analyzer.

to be explosion-proof for continuous process control. The principle of operation is that of a single chopped beam, filter photometer. The second type is a two-cell, chopped beam, filter photometer which provides for both a sample and a reference cell comparison.

The optical system of the former instrument passes radiation of wavelengths from 200 to 280 mμ as follows. Radiation from the hydrogen lamp 1 passes to the photomultiplier tube by the path shown in Fig. 9. The spherical mirror 2 focuses the radiation on the entrance aperture 3 which serves as a point source. Collimating lenses 4 also serve as windows for the chlorine filter cell. The energy distribution of the radiation incident on the cell is the same as the hydrogen lamp spectrum, which consists essentially of a continuum from 200 mμ to approximately 360 mμ with some radiation in the visible region. Since chlorine absorbs energy from

280 to the short visible region, the energy leaving the chlorine cell consists of a band from 200 to 280 mμ plus some visible radiation.

The unwanted visible energy is removed by focal isolation of energy leaving the cell and lens 4*b*. The dispersion characteristics of the lens cause long wavelengths to focus at a greater distance from the cell than the short wavelengths do. The exit aperture 6 is a flat disk with a round opening in the center. It is so placed that the short wavelength radiation passes through the opening, and the long wavelengths fall on the disk and are blocked. The paraxial rays of energy are not refracted sufficiently to permit separation by the focal isolation method. It is necessary to eliminate these rays by means of light blocker 5. The energy reaching the chopper 7 contains only wavelengths from 200 to 280 mμ. The light chopper 7 is a circular disk which is composed half of quartz and half of Vycor; these are alternately interposed in the beam of energy at the rate of 30 cycles/sec. Since the short wavelength cutoff of Vycor is 230 mμ, the energy transmitted to the sample cell is limited to the band from 230 and 280 mμ. When the quartz half is in the beam, almost all the energy reaching the chopper is transmitted to the sample cell. The energy incident on the sample cell and photomultiplier tube 10 is ideally a square wave. The instrument as described is especially designed for the measurement of butadiene in a mixture of 1.3 butadiene and *cis*- and *trans*-2-butene. The output signal from the analyzer is fed to a pneumatic recorder-controller, which is used to set the control index of a rate-of-flow recorder-controller.

Numerous other materials with ultraviolet absorbance in this particular region can be recorded with this analyzer. Some of these materials include hydrogen sulfide, acetylenes, mercaptans, benzene, methylvinylpyridine, and styrene. By changing the filters and material used in the light chopper, energy bands other than that specified may be obtained to increase the versatility.

The second analyzer is very similar in principle to that just described. It consists of a light source, lens, and rotating mirror chopper. The chopper is a half-silvered mirror which alternately permits radiation to pass through the reference cell and sample cell for comparison of the absorbance of the transmitted radiation at the photomultiplier. Various light sources, filters, and sample cell arrangements are available for the determination of ultraviolet-sensitive materials.

Process Refractometers

Refractive Index. The refractometer is a nondispersive optical analyzer which uses the physical property of liquids and solids known as refractive index. The *refractive index* of a material is determined by measuring the divergence of a light beam in passing from one medium to another, usually air. This is shown in Snell's law of refraction, as seen in the equation

$$\frac{n_2}{n_1} = \frac{\sin i}{\sin r} = \frac{V_1}{V_2},$$

and if n_1 is air, then

$$n = \frac{\sin i}{\sin r}$$

where n = refractive index,
i = angle of incidence,
r = angle of emergence, and
V = velocity of the light.

Snell's law is described graphically in Fig. 10. A ray of light originating in a medium M_1 with a refractive index n_1 is obliquely introduced at a velocity V_1 and angle i to a medium M_2, velocity V_2, and refractive index n_2. In Table 4 ranges of refractive indexes are listed for some of the more common applications.

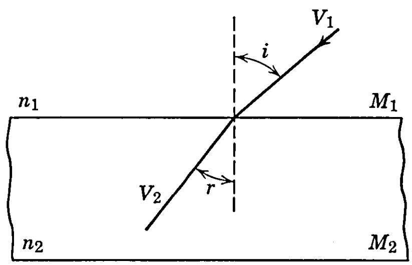

Fig. 10. Snell's law.

The factors which affect the refractive index of a material are primarily temperature, pressure, and wavelength of the incident light; therefore, it is necessary to compen-

TABLE 4. REFRACTIVE INDEXES OF TYPICAL MATERIALS

Refractive Index	Material
1.30	
	Water
1.35	Alcohols (C_1–C_4)
	Alkanes (C_4–C_{12})
1.40	Nitric acid
	Fats and waxes
1.45	Vegetable oils
	Sugar solutions (0–85%)
1.50	Alkylbenzenes
	Plastics
1.55	Styrene
	Naphthalenes
1.60	
	Carbon disulfide
1.65	
	Anthracenes
1.70	

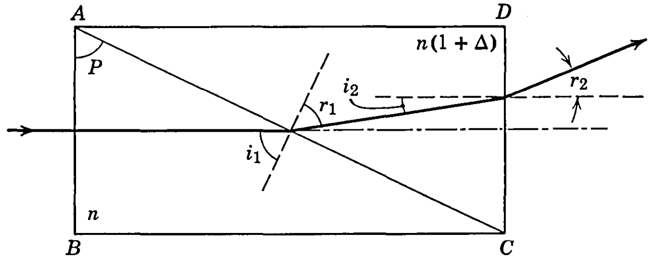

FIG. 11. Differential prismatic cell of a refractometer.

sate for these variables in order to use the high resolution available. A sodium vapor lamp is taken as a standard of comparison, using the D line at 5893 A at 20°C, designated by the symbol n_{D}^{20}.

Because the principles of operation and design of the standard refractometer are well known, they will not be discussed further here. In process applications in which a change in refractive index is to be measured or the refractive indexes of two streams are to be compared, the differential refractometer is used. This technique is extremely sensitive and less subject to the variations in its environment just discussed. Figure 11 shows a typical arrangement of a differential prismatic cell.

If a collimated light beam is normally incident on the cell window AB, no refraction takes place. When the light strikes the interface AC, at an angle i_1 to the normal, refraction occurs only if the mediums in the two halves of the cell differ.

The sensitivity of the instrument can be varied by changing the angle P, which proportionately affects the angle r_2. With optimum selection of angles, a difference in refractive index of 0.000012 can be detected per division of the recorder chart.

A schematic diagram of the sampling system is shown in Fig. 12. A

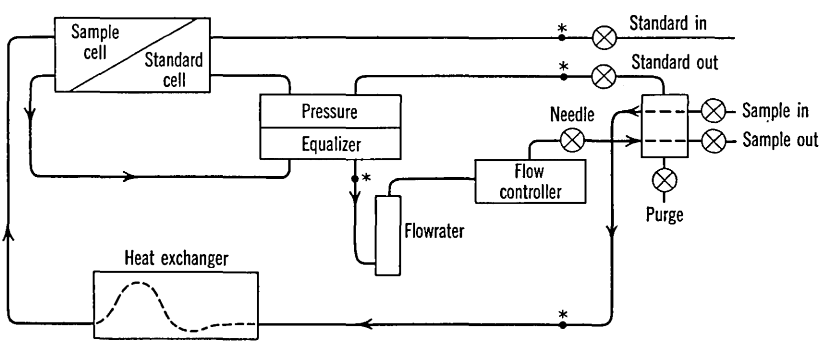

FIG. 12. Schematic diagram of a sampling system for a refractometer. (Courtesy of Consolidated Electrodynamics Corporation.)

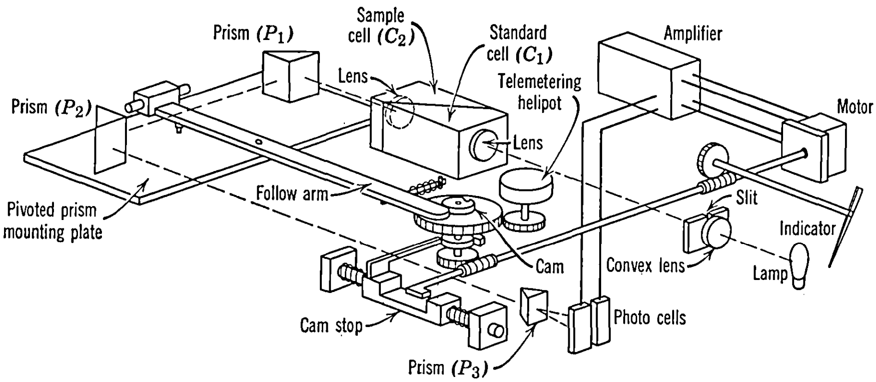

FIG. 13. Refractometer optical servosystem. (Courtesy of Consolidated Electrodynamics Corporation.)

heat exchanger and pressure equalizer are used to insure that both cells operate under equal conditions. Constant flow of the sample liquid is maintained by a rotameter.

The optical servo system is shown in Fig. 13. Light from an incandescent bulb illuminates the entrance slit through a convex lens. The first cell window also acts as a collimating lens. The slit image is focused on the apex of the prism P_3, dividing the image into two beams, each of which falls on one of the two photocells. A null balancing servo system is used to equalize the signals from the two photocells connected in opposition. The balanced position of the tracking cam is a measure of the difference in refractive index.

A typical refinery control application is shown in a schematic flow chart, Fig. 14.

3. MASS SPECTROMETER ANALYZERS

Perhaps the most outstanding advantages that the mass spectrometer has over other types of analyzers are those of speed, versatility, and ability to resolve multicomponent samples. The mass spectrometer has been used since about 1942 in petroleum and chemical laboratories to analyze batch samples to determine set points for process controllers. In recent years smaller mass spectrometers that may be attached directly to a process stream and monitor certain components continuously have been developed. Indeed, the loop can, in many cases, be closed so that the mass spectrometer controls certain parameters automatically.

Mass spectrometry entails essentially three processes in one: (1) ionization of the gas, (2) mass separation of the ions, and (3) collecting, recording, and computing the results. Ionization is accomplished by subjecting the sample gas at low pressure to a beam of electrons of sufficient

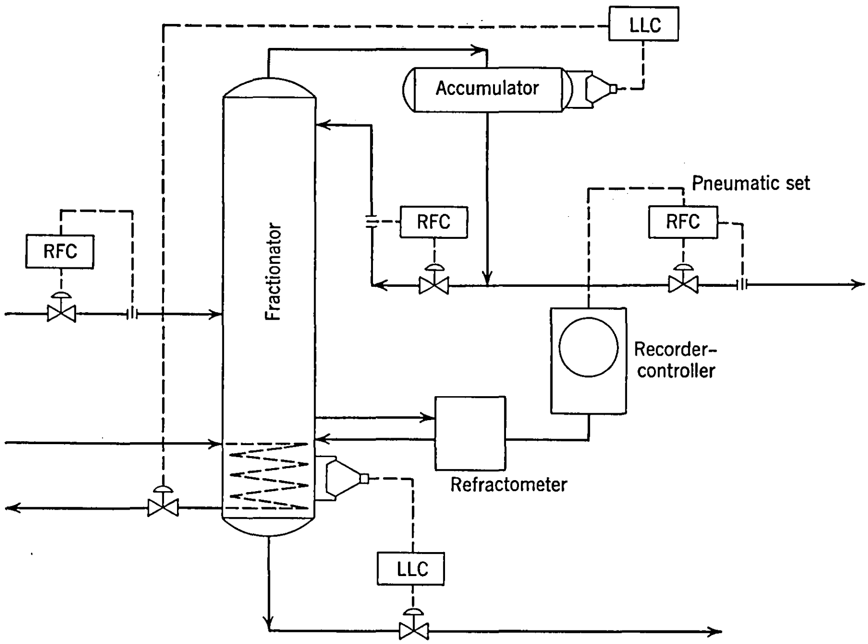

FIG. 14. Schematic diagram of a typical refinery control application of a refractometer; LLC = liquid level controller, RFC = radio-frequency controller.

energy to remove an electron (or electrons) from the sample and/or break a molecular bond (or bonds) yielding fragment ions. The ions formed are predominantly positively charged particles. The mass separation of these charged particles is accomplished in one of two ways, either magnetically or by time of flight. These will be discussed later.

The mass spectrometer is capable of rapidly analyzing multicomponent streams, ranging from light gases to quite heavy liquids which have fairly high vapor pressure or have been vaporized by heating. It can detect components that are present in only a few parts per million. Applications for which mass spectrometers are being used successfully are given in the following list.

1. Monitoring isotope ratio separations of uranium in Atomic Energy Commission plants.

2. Analyzing exhaust gases and air-fuel ratios for studies of internal combustion, gas turbine, and jet engines.

3. Controlling furnace atmospheres in heat-treating furnaces and high-vacuum metallurgy plants.

4. Monitoring and controlling ethylene oxide process streams.

Magnetic Analyzers

Four general types of mass spectrometers have been developed for magnetic mass separation for process monitoring. These are (1) 60° magnetic sector (Nier type), (2) 160° low resolution, (3) 180° low resolution, and (4) 360° cycloidal focusing, medium resolution.

The principle of operation is based on the formula

$$MV/e = K \times 10^{-5} H^2 R^2$$

where M = the mass of the particle,
e = the charge on the ion,
V = the accelerating potential in volts,
K = a constant,
H = the magnetic field strength in gausses, and
R = the radius of curvature in centimeters.

Resolution or separation of the adjacent beams differing in mass by ΔM is then represented by the formula

$$D = R\, \Delta M/M$$

Since the ions produced in a mass spectrometer ion source follow distinct curved paths in the magnetic field, it can be seen from the preceding expression that the particles of different mass traverse paths, the radii of which are determined by the strength of the magnetic field and the magnitude of their initial energy (accelerating voltage). The radius of the analyzer must, in practice, be fixed so that the ions are registered only as they travel a predetermined path. In order for an ion of any mass to be collected, it must travel a path proportional to the square of the magnetic field and inversely proportional to the accelerating voltage. On striking the collector the positive charge is collected, causing a current to flow. The current, proportional to the number of ions of any one species, is then amplified and recorded for analytical purposes.

To record the relative abundance of the various ions produced in the mass spectrum, it is customary to scan the ion currents by varying either the magnetic field or the accelerating voltage.

Nier Type. The oldest and most common of the magnetic sector instruments was developed by A. O. Nier for use, during the war, in the uranium gaseous diffusion plants. These mass spectrometers used conventional 60° magnetic sector analyzers having a radius of 3 in. The instruments determined the ratios of two adjacent mass isotopes and monitored certain constituents of the process stream to maintain purity of the final product. Figure 15 is a graphic diagram of the analyzer used in the 60° instruments.

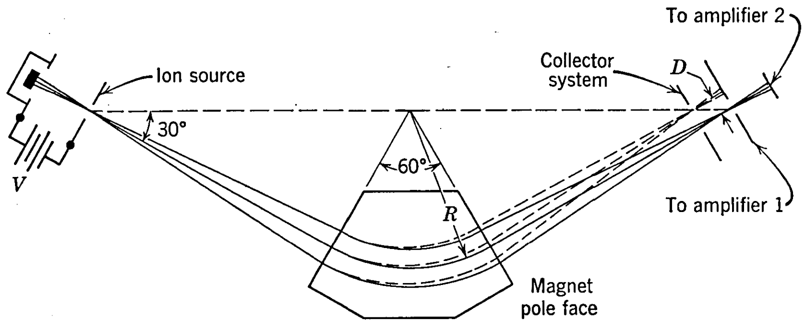

FIG. 15. Ion optics of Nier type mass spectrometer. (Courtesy of Consolidated Electrodynamics Corporation.)

Mass Spectrometer, 160° Sector. The second type of instrument developed for industrial process monitoring had a 160° magnetic sector analyzer with a 2½-in. radius. The resolution of this instrument limited its use to materials having a mass of 70 or less. It is housed in a vapor-tight housing and has completely automatic programming of the various streams and periodic standardization with a reference sample. Only two of these instruments were actually built and used in refinery processes. Several smaller ones were developed, using a commercial model of a helium leak detector. The resolving power of these instrumnts was limited to mass 20, and they were designed for semiautomatic operation. Since that time, however, a commercial model of such a unit has been designed and is described in the next paragraph.

Mass Spectrometer, 180° Sector. The first commercial process mass spectrometer, pictured in Fig. 16, used a conventional 180° magnetic analyzer, with a radius of 1 cm., and a source with an octal plug base 3 in. in overall length (see Fig. 17). A tungsten wire filament emitted the bombarding electron beam. The gas sample was introduced at atmospheric pressure through a capillary tube (0.006-in. i.d.) and gold foil leak arrangement producing a pressure in the source of approximately 0.1 micron of mercury.

It is essential that the pressure be low in both the ionization and analyzing regions so that the mean free path of the gas molecules is great enough to avoid scattering of the positive ions. The pressure was kept low by a dynamic pumping system consisting of a charcoal trap instead of the usual cold trap, oil diffusion pump, and mechanical pump in series. The trap was provided with a heater for charcoal regeneration. The sampling and pumping system is shown in block form in Fig. 18.

Mass Spectrometer, Cycloidal Focusing. To extend the useful mass range of the instrument just discussed, a rather unique principle of focus-

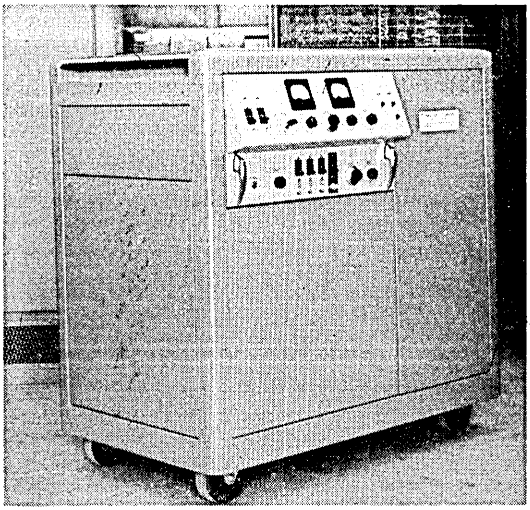

FIG. 16. Mass spectrometer, 180° low resolution type. (Courtesy of Consolidated Electrodynamics Corporation.)

ing the ion beam was employed, called the cycloidal focusing principle. This is the fourth type of magnetic analyzer developed. By using essentially the same mass spectrometer pictured in Fig. 16, a new source was developed employing a 360°, 1.1-in. radius, as shown in Fig. 19. This instrument has a mass range of 2 to 150.

The principle of operation of the cycloidal focusing mass spectrometer is not new. It was first invented by Bleakney at Princeton approximately twenty years ago. In such an instrument the focal point of the ions is independent of either the energy or direction of motion of the ions when

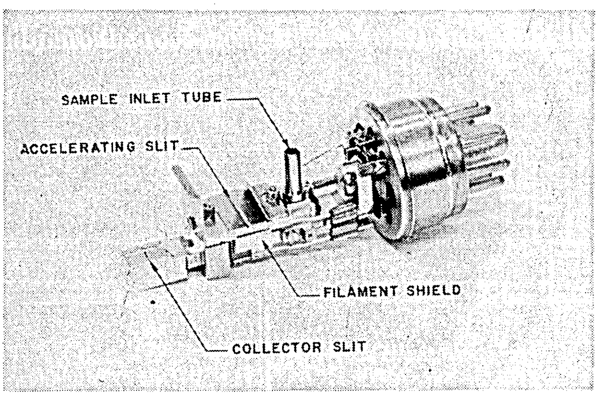

FIG. 17. Analyzer section of mass spectrometer, 180° low resolution type. (Courtesy of Consolidated Electrodynamics Corporation.)

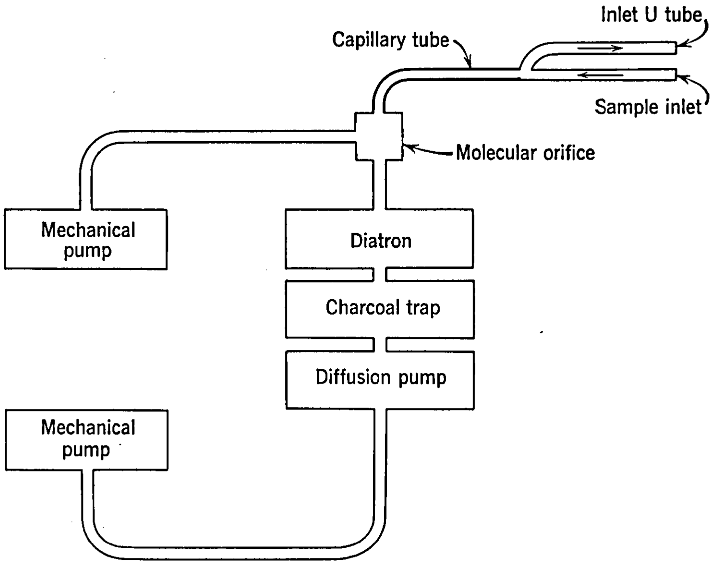

FIG. 18. Sampling system for mass spectrometer of Fig. 16. (Courtesy of Consolidated Electrodynamics Corporation.)

they are injected into the crossed magnetic and electrostatic analyzer. Therefore, no stops are needed to limit the angular divergence of the ion beam, and there is no theoretical lower limit on the ion energy or, therefore, on the size and weight of the magnet.

In a conventional 180° mass spectrometer, there is a natural angular aberration effect at the focal point because the ions leaving the source normally leave with a slight angle divergent to the normal, as shown in

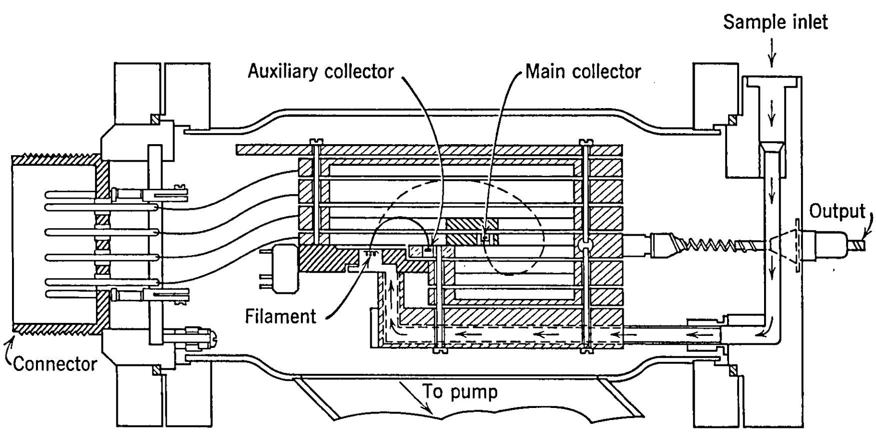

FIG. 19. Analyzer section of mass spectrometer, cycloidal focusing. (Courtesy of Consolidated Electrodynamics Corporation.)

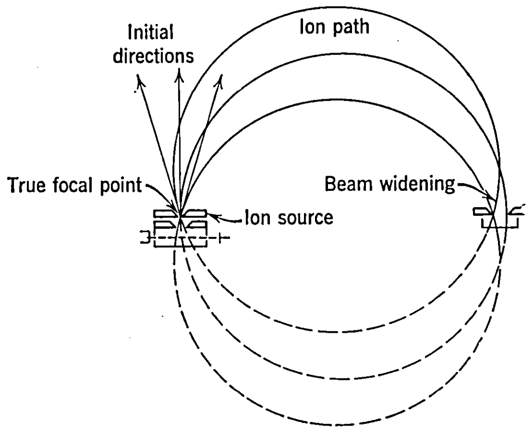

FIG. 20. Ion path of mass spectrometer, natural angular aberration effect at focal point. (Courtesy of Consolidated Electrodynamics Corporation.)

Fig. 20. The only perfect focal point in such a system is 360°, or at the origin of ionization. In the cycloidal instrument the crossed electrostatic and magnetic fields are controlled so as to remove the 360° focal point physically from the origin, as illustrated in Fig. 21. The spectrum is scanned by causing the electrostatic field strength to decay with the accelerating voltage.

A sample spectrum of the isotopes of Xenon, recorded on the cycloidal mass spectrometer, is shown in Fig. 22. Because of the nearly perfect focus attained with this system, it is not necessary to have deep valleys between adjacent peaks to obtain completely quantitative separation.

Still another unique principle of mass spectrometer operation has recently been exploited as a commercial instrument. The unit is called an ion resonance mass spectrometer (Omegatron) and is essentially a minia-

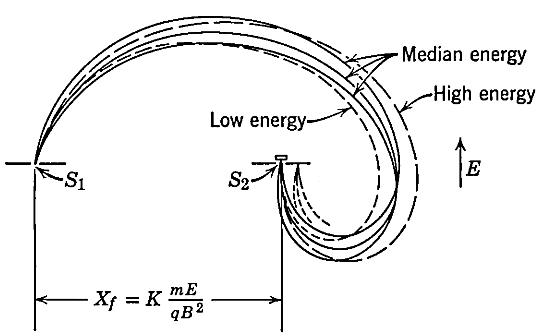

FIG. 21. Ion path of cycloidal instrument, removal of 360° focal point. (Courtesy of Consolidated Electrodynamics Corporation.)

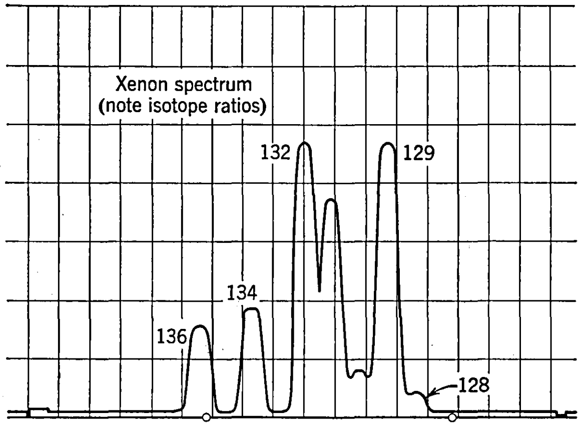

FIG. 22. Resolution in cycloidal mass spectrometer. (Courtesy of Consolidated Electrodynamics Corporation.)

ture cyclotron. An alternating electric field of radio frequency is employed at right angles to a fixed magnetic field. Therefore, ions having a cyclotron resonance frequency equal to the frequency of the applied electric field gain energy continuously and spiral out to a collector (Fig. 23). All other ions fail to gain sufficient energy and are not collected. Although this device has a potentially high efficiency, since it requires no slits or baffles limiting the collection of desired ions, it is fundamentally limited because the ionizing and analyzing regions are inseparable. Therefore, an extremely low background pressure is required in order to insure a sufficiently high sample-to-background ratio. A resolving power of 90 is reported, as shown in the spectrum of n-heptane containing some toluene (Fig. 24).

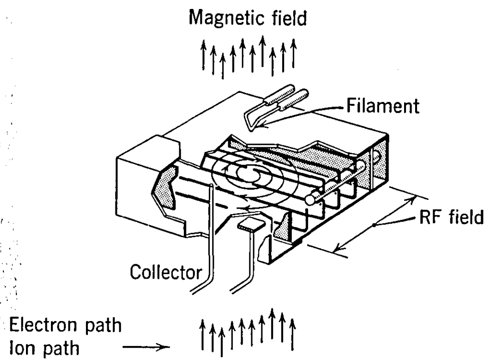

FIG. 23. Ion resonance mass spectrometer operating principle.

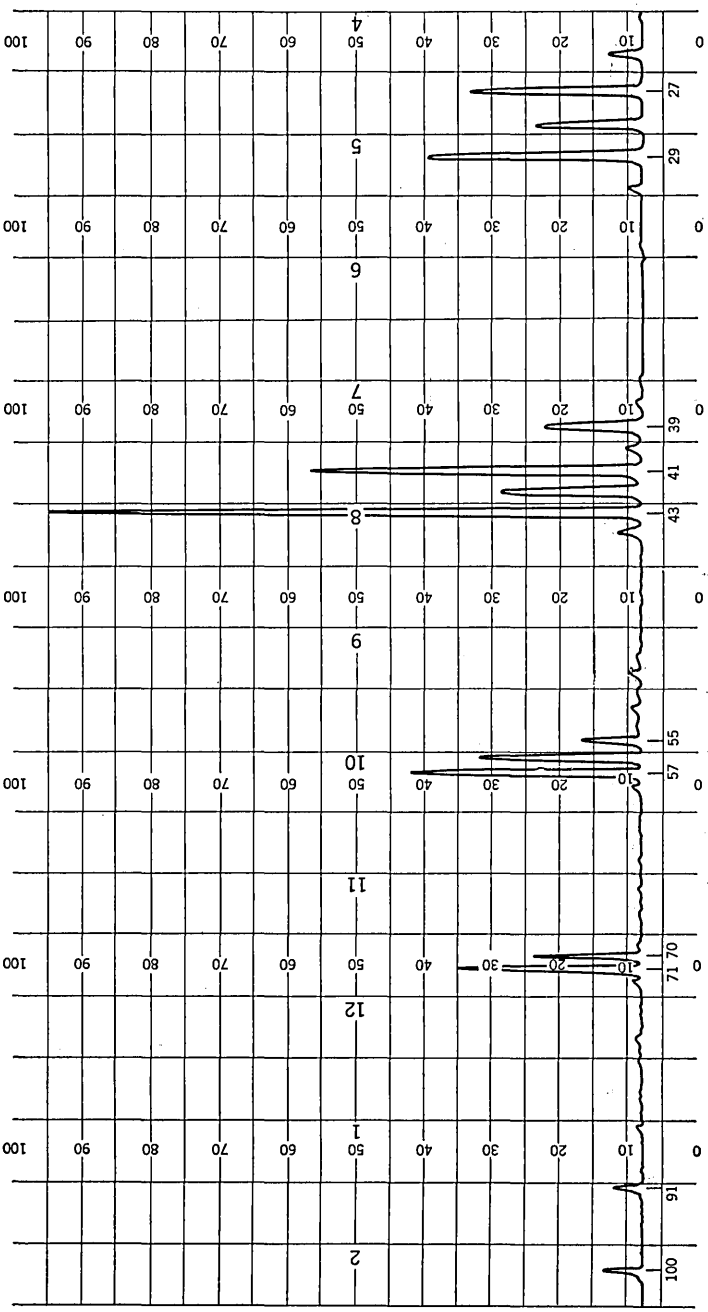

FIG. 24. Spectrum of n-heptane containing some toluene.

Time-of-Flight Analyzers

A nonmagnetic radio-frequency mass spectrometer, first invented by Bennett, has been developed as a commercial instrument by two separate manufacturers. In this device ions are injected into the end of a tube containing a series of transverse grids (Fig. 25). By applying a radio-frequency voltage to certain of the grids and by properly proportioning static electric fields, ions of a selected mass can be made to gain more energy in traversing the tube than ions of any other mass. The use of a potential barrier in front of the collector selects the desired ions. The reverse of this principle has also been described as a "linear decelerator" mass spectrometer. Ions are shot through a series of grids, the spacing of which becomes progressively smaller. Alternate grids are connected together, and between the two sets thus formed a radio-frequency potential is applied. Ions of the correct mass lose a maximum of energy, as compared with either heavier or lighter ions, and are separated at the end by a parallel plate electrostatic analyzer. The resolving power of these units increases with the total number of grids used and is reported to be 100 for 20 grids.

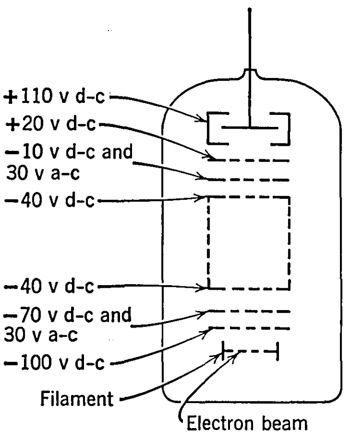

FIG. 25. Analyzer section of nonmagnetic radio-frequency mass spectrometer.

A linear time-of-flight mass spectrometer, based on the early work of Stephens, also employs a radio-frequency drift tube, in which short pulses of ions are injected into the analyzer. Since all ions have the same energy, their velocities vary inversely as the square root of the mass, so that resolution in time and space occurs as the pulses traverse the drift tube. The sequence of separated pulses is collected and observed on a cathode-ray oscillograph, which gives a continuous display of the spectrum or any portion thereof. Resolving power of the order of 100 has also been reported for this instrument.

4. GAS CHROMATOGRAPHY ANALYZERS

Some fifty years ago the technique of chromatography was developed to separate and identify chemical compounds according to their colors; hence the term *chromatography*. This technique was expanded to use other physical properties of the separated compounds to identify them. One such property was refractive index. These methods are still being used on a routine basis. The most recent development along these lines is the method of using gas and vapor samples and measuring the thermal conductivity of the compounds emerging from the separation column.

Of the several systems of gas and vapor chromatography used today, the system of elution from a packed column of some inert material such as crushed firebrick coated with some low-vapor-pressure oil such as dinonylphthalate seems to be the most versatile and widely used. Basically, the chromatographic system operates as follows (see Fig. 26).

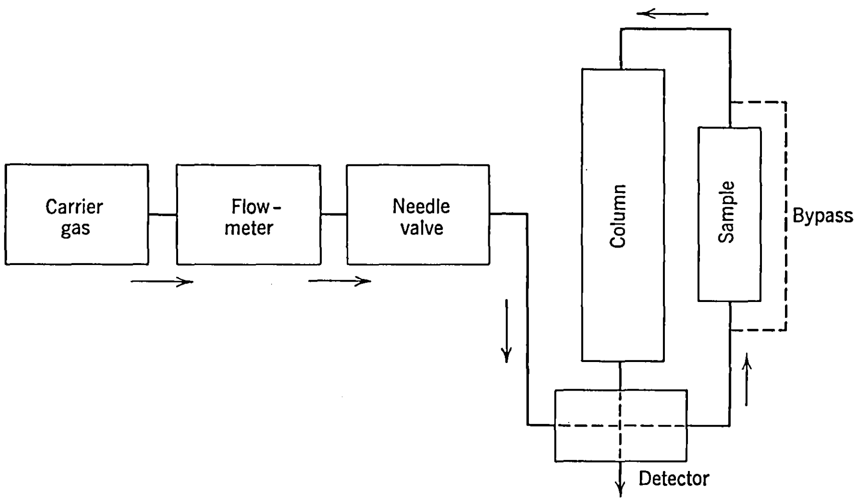

FIG. 26. Schematic diagram of gas chromotography apparatus.

A stream of some inert gas, such as helium, is made to flow through the packed column mentioned and into one side of a detector cell. The helium passes through the other side of the detector cell before passing into the column. The detector cell is composed of two compartments, each having a means of detecting thermal conductivity (see Fig. 27). When only helium is flowing, both cells detect the same thermal conductivity, so the Wheatstone bridge, of which the two detectors are arms, can be balanced and the recorder will read zero. If now a sample to be analyzed is injected into the helium stream just ahead of the column, the components will be dissolved in the oil coating and preferentially eluted by the helium according to the solubilities of the components. As each component emerges from the column into the detector cell, there will be a change in thermal conductivity of the resulting binary mixture. This difference is detected and noted by the recorder as a peak. As the first component emerges from the cell, the recorder approaches the zero line until the next component emerges from the column to give another peak.

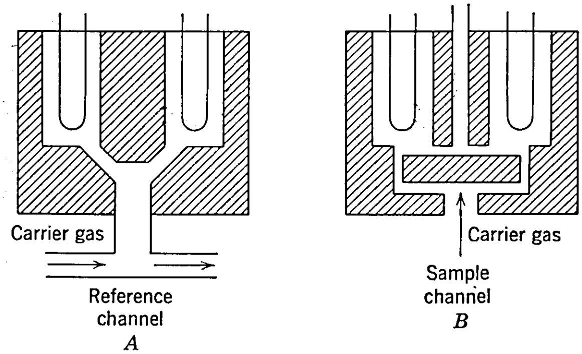

FIG. 27. Thermal conductivity detector. (Courtesy of Consolidated Electrodynamics Corporation.)

The area under each peak is proportional to the concentration of the component which it represents in the original mixture. The time that it takes for each component to pass through the column (retention time) can be used as a means of qualitative identification of the component. The retention time has to be determined using pure materials.

Probably no other method of analysis has been adopted quite as rapidly as chromatography. There are very few laboratories not using the method. One of the largest users is the gasoline industry. They use chromatography for such things as evaluating an oil field, checking the flue gas from boilers to determine the air-fuel ratio as well as the polutants being emitted into the atmosphere, checking end products for purity, and making sure maximum production of some material is being obtained. The information gleened from chromatography helps to increase production, improve quality, reduce cost of production, and control the process of production.

Some of the applications involving chromatography are (1) control of alkylation unit, (2) control of a sulfur recovery plant, (3) control of a butane-isobutane recovery unit, and (4) continuous analysis of a synthesis gas for synthetic ammonia production.

These are but a few; as the techniques of chromatography develop and improve, many more processes will undoubtedly be controlled by this method. Work is already being done to develop methods in the plastics industry, food and pharmaceuticals industries, and various other chemical processing plants.

5. SPECIALIZED ANALYZERS

A few specialized analyzers have been developed to monitor either a single component or one type of compound in gas streams. Of these probably the most widely used are the oxygen analyzers and the sulfur analyzers.

Oxygen Analyzers

There are three general types of oxygen analyzers, those utilizing the paramagnetic properties of oxygen, those employing oxidation and reduction chemical reactions of oxygen, and those employing the electrolysis control effect of oxygen.

Oxygen is paramagnetic (readily drawn into a magnetic field), whereas other common gases with few exceptions are slightly diamagnetic (repelled by a magnetic field). This principle is the basis for the specific measurement of uncombined oxygen in various background gases, since most industrial gases do not affect the measurement. If a hot wire is suspended in a closed chamber cell containing a paramagnetic gas such as oxygen, and a magnetic field is placed in proper relation to the wire, a magnetic convection current is generated in the gas. This current cools the heated wire in proportion to the paramagnetic intensity of the gas. The resulting temperature change of the wire can be used as a source for the measurement of oxygen content. The measurement actually made is the resistance of the wire, which is a function of its temperature. This value is recorded as per cent oxygen.

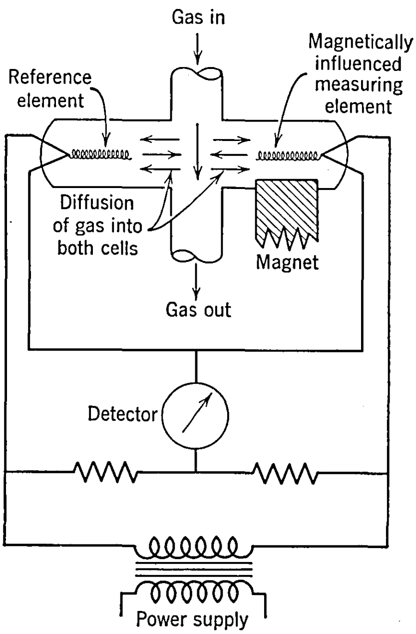

Fig. 28. Thermomagnetic bridge circuit of oxygen analyzer. (Courtesy of Leeds & Northrup Company.)

The schematic diagram in Fig. 28 illustrates a basic thermomagnetic bridge circuit. Two cells are shown, one a reference cell and the other a measuring cell, each containing an electrically heated resistance element. Thermal convection currents cool the reference element, and thermal and magnetic convection currents cool the measuring element. With no magnetic field, or in a gas stream with no oxygen present in the measuring cell, no difference is detected. With a magnetic field and oxygen present, there will be an additional cooling because of magnetic convection currents.

Changes in rate of heat loss from the measuring element caused by the combined effect of the thermal and magnetic convection currents will alter the temperature of this element, producing a resistance change which is detected and measured by an unbalance in a conventional Wheatstone bridge. Factors affecting the accuracy of the system are changes in static and barometric pressure. These effects are canceled by means of a second bridge circuit.

A single magnet is used to envelop both bridge elements to cancel effects of variations in its field strength. The magnet is movable for zero calibration purposes, and air is used to adjust the instrument span. The accuracy of the instrument can be maintained to plus or minus 0.15% by frequent calibration against a standard gas mixture. The sensitivity is plus or minus 0.05% in a recommended range of operation from 0 to 10%.

In another oxygen analyzer which depends on the paramagnetic properties of oxygen for its operation, the magnetic effect of a gas stream is measured by noting changes in the susceptibility of a test body suspended in a nonuniform magnetic field. The test body is a light (dumbbell-shaped) cell made of hollow glass and suspended on a quartz fiber (see Fig. 29). A galvanometer type mirror is fixed on the suspension to indi-

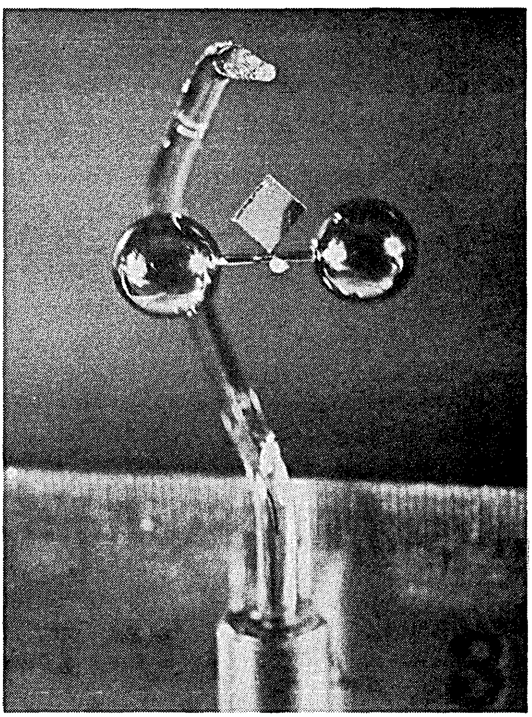

FIG. 29. Paramagnetic oxygen analyzer. (Courtesy of Beckman Instruments.)

cate the frictionless rotation in the magnetic field. The gas stream is played in contact with the test cell where the paramagnetism of the various gaseous components affects the equilibrium position of the mirror.

Deflection Type Analyzer. The deflection type analyzer uses a torsional restoring voltage to balance the magnetic rotational force. A beam of light from the mirror is deflected proportional to the oxygen concentration. To operate the instrument on a null-balance principle, the test body is maintained in the equilibrium position by means of an applied electrostatic field. The beam of light is divided through a split mirror between two photocells. Any unbalance in the radiation incident on these cells is used to produce an electrical restoring voltage on the test cell. The glass test body is metal coated and converted to a variable source of electrical potential. Two electrodes near the test body serve to affect a heterostatic electrometer system. The potential required to maintain the cell in equilibrium is then a measure of the paramagnetic gas concentration. Any null-balance servo system or recording potentiometer circuit may be used to operate the instrument.

Factors affecting the proper operation of this principle are ambient temperature, gas flow, and vibration. It is necessary to control the temperature thermostatically and shock-mount the test body.

Colorimetry Type Analyzer. A second type of instrument, which is available commercially, uses the principle of chemical oxidation and reduction. This instrument operates on the principle of colorimetry. Oxygen in the sample gas continuously oxidizes an oxygen-sensitive reagent (alkaline solution of sodium anthraquinone beta-sulfonate), which is continuously reduced back to its original deep-red color by passing it over a zinc amalgam. Barrier layer photocells measure the differential absorption between the completely reduced (red) reagent and the partially oxidized reagent of a lighter color. The photocell measuring circuit is continuously balanced by a motor-driven potentiometer in a null-balance type servo system.

A schematic diagram of the chemical system is shown in Fig. 30. Starting at point A, the fully regenerated reagent is drawn up by the metering pump and passed through the reference cell and into the reactor where the liquid comes in contact with the sample gas. The reagent, now partially oxidized and lighter in color, passes through the sample cell and is rejected at constant level at the top of the overflow pipe at B. The level of reagent in the reactor is thus determined by the level of B. The liquid then runs down the inside of the overflow pipe and is collected in the regenerator above the surface of the zinc amalgam. The reagent flows through the amalgam bed where regeneration occurs and is then drawn up by the metering pump to repeat the cycle. In the regeneration process small amounts of gas are produced and collect just below the wire screen which supports the zinc amalgam. To remove this gas without allowing oxygen to contact the fully reduced reagent, a bypass pump continuously removes a small amount of liquid and any gas formed from a point just

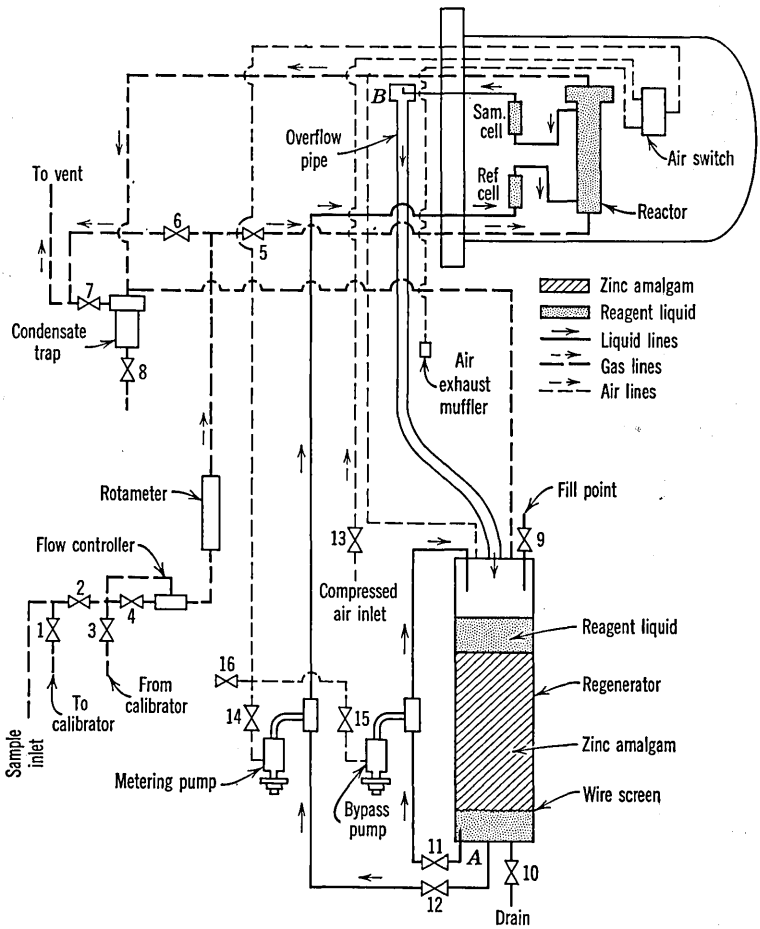

FIG. 30. Chemical system of a colorimetry type oxygen meter. (Courtesy of Consolidated Electrodynamics Corporation.)

below the screen and passes it to the top of the regenerator. Valves 9 and 10 provide means of adding or removing reagent from the system. The sample gas enters the analyzer at the sample inlet, and passes through valves 2 and 5, through the differential flow controller and rotameter, into the reactor. From the reactor the sample gas passes through the condensate trap and the vent.

The optical system is arranged as shown in Fig. 31. Light from the source is focused into two beams by the collimating lenses. Each beam passes through a green filter and sample cell before falling on the photocell. A machine screw trimmer is placed in each beam to provide a means of equalizing the output of the cells when zeroing the analyzer. Both sample cell and reference cell are constructed of stainless steel with glass windows. The cell thickness is 1 cm. Each cell is provided with a circular

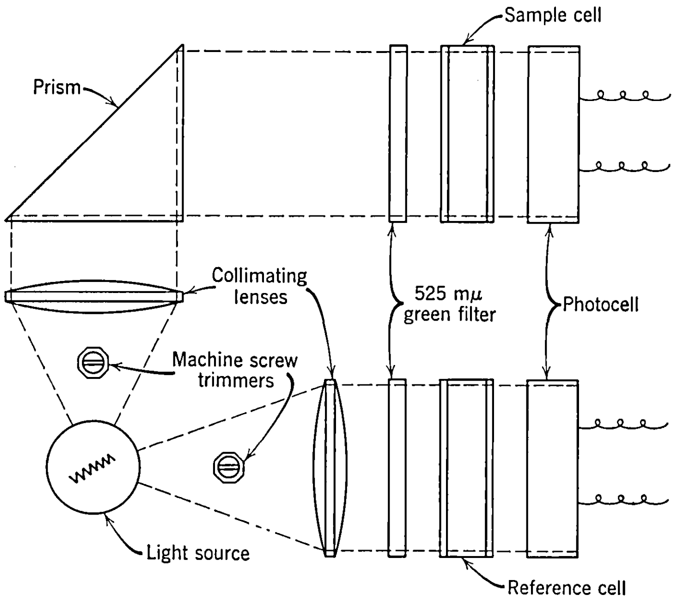

FIG. 31. Optical system of oxygen analyzer. (Courtesy of Consolidated Electrodynamics Corporation.)

mask at the edges to prevent collection of bubbles. The photocells are mounted in a single aluminum block to equalize their temperatures. The electrical system consists of a simple power supply servo system and telemetering circuit.

Electrolytic Type Oxygen Analyzer. A third type of oxygen analyzer operates on an electrolytic reaction principle. The heart of the instrument is the detector cell which is essentially a primary cell with a metallic and a hollow carbon electrode immersed in an electrolyte (oxylite). Figure 32 is a flow diagram of the instrument. The sample is introduced through a filter, pump, and pressure regulator to a heat exchanger coil and then into a hollow carbon electrode of the detector cell. Hydrogen evolved at the carbon electrode in the cell causes polarization of the cell, which manifests itself in a decrease of the generated voltage and current.

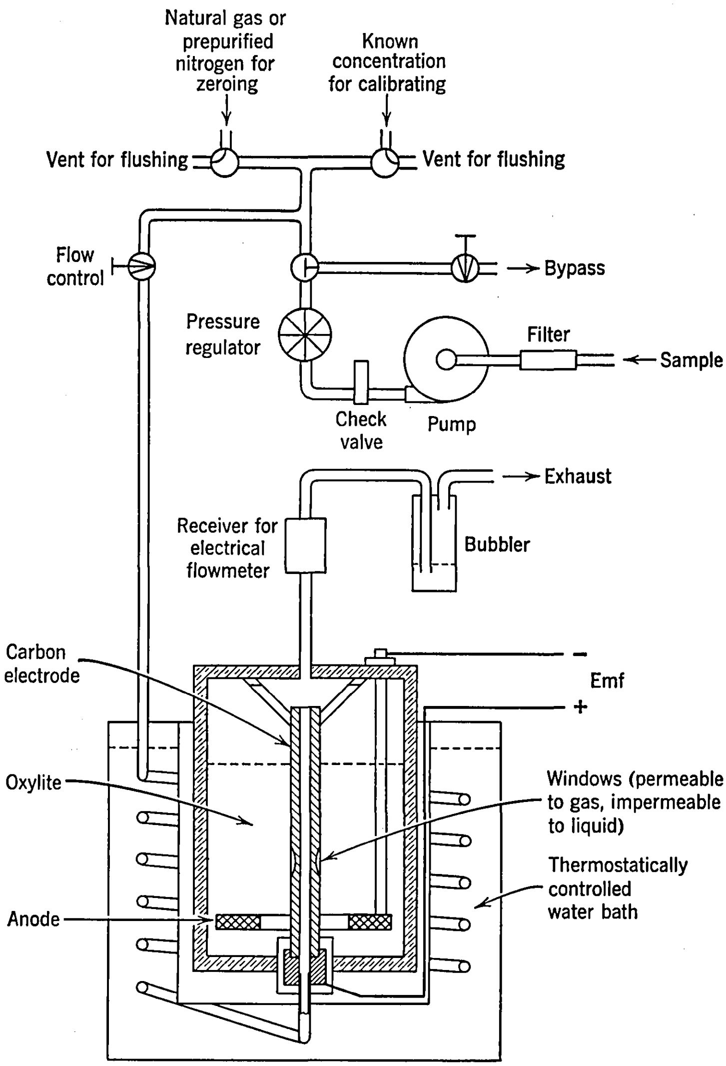

FIG. 32. Flow diagram of electrolytic type oxygen analyzer.

Oxygen from the sample diffuses through the wall of the electrode and combines with the hydrogen. Removing the hydrogen from the immediate vicinity of the electrode depolarizes the cell, generating more electrical current by the cell. The current generated is then a measure of the oxygen concentration of the gas stream.

Factors influencing the accuracy and stability of operation of the instrument are ambient temperature changes and oxidants in the gas stream, such as acid or corrosive gases other than carbon dioxide. A thermostatically controlled water bath is used to temperature-compensate the gas sample. Scrubbers are required when oxidants are to be expected.

Sulfur Analyzers

Numerous instruments for the continuous determination of hydrogen sulfide and sulfur compounds have been described in the literature. Most of the instruments are especially suited for applications in which these compounds are present in extremely low concentrations, and conventional methods are not sufficiently sensitive to obtain quantitative measurements. All these methods are based on classical chemical methods, such as colorimetry, potentiometric titration, and conductivity.

Colorimetric Sulfur Analyzers. The colorimetric type analyzer meters the flow of gas through a restricted area of a permeable white reactive tape which has been impregnated with lead acetate. Assuming stoichiometric conversion of the hydrogen to lead sulfide, the brown coloration intensity then becomes a measure of the H_2S concentration. The area of the tape subjected to the gas is then illuminated with light of appropriate spectral quality and reflectance, as compared to a similar area of tape which has not been exposed to the gas. By means of a photometer consisting of two photovoltaic cells in a bridge type circuit feeding a null-balance potentiometric recorder, the difference in transmission is recorded and calibrated in terms of concentration. A continuous, permanent record can be obtained. By proper variation of the

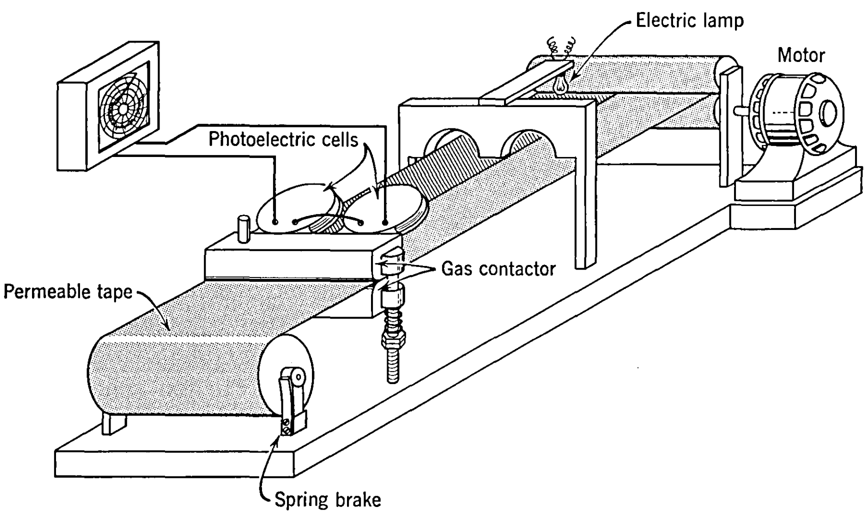

FIG. 33. Continuous colorimetric hydrogen sulfide detector.

length of exposure, the instrument can be used for a concentration of 0.005 grains per 100 cu ft of gas. The instrument is calibrated by varying both the flow rate of gas (0.4 cu ft/hr) and the duration of sampling interval. At the conclusion of a preselected test period, the photometer is automatically disconnected from the recorder whose indicator remains at the position corresponding to the reflectance of the tape at the end of the test period. At the same time a motor releases the tape and advances it sufficiently to expose a new section. The period of change requires approximately 1 sec. The recorder is reconnected to the photometer output and zeroed, and then the test is repeated. Each period of test results in a peak on the record indicative of the integrated hydrogen sulfide concentration during the test.

For continuous operation of the instrument, the revolving roller draws the tape at a fixed rate through the opening between the gas contactor and collector, and beneath the photocells.

Figure 33 is a perspective view of the continuous colorimetric hydrogen sulfide detector. The permeable tape is impregnated with lead acetate (2% by weight lead acetate, 2% sodium hydroxide, and 1% glycerin). The gas cell is provided with a spring-loaded slot through which the tape may be moved between samples. Photoelectric cells are fixed on the gas contactor in a position to absorb light from the electric lamp source as reflected from the tape.

Potentiometric Sulfur Titrimeter. Two commercial models of a continuous potentiometric sulfur titrimeter have been developed. One is a carrying case model of the standard fixed model shown in Fig. 34. This instrument is capable of continuously recording trace quantities of oxidizable sulfur compounds in the range of 0.1 to several hundred parts per million by volume in the gas phase. Coulometric titration with electrolytically generated bromine is used. The end point of the titration is electronically controlled in a feedback d-c amplifier.

Figure 35 is a block diagram of the titrimeter and amplifier circuit. The feedback amplifying system is composed of a d-c amplifier, a source of adjustable d-c reference voltage, and the titration cell. A recording milliammeter is included to measure amplifier output current. There are two sources of voltage in the input circuit. The first is a reference voltage from a stable source and the second is that of the bromine-sensitive electrode and reference electrode in the titration cell. The net voltage from these two sources acts as a control on the amplifier output which is used to control the generation of bromine. The reference voltage is set equal to the voltage of the electrolytic cell to keep the bromine concentration fixed at the stoichiometric point of the titration. If the bromine concentration falls below the desired level, the sensor voltage will fall below the

FIG. 34. Continuous potentiometric sulfur titrimeter. (Courtesy of Consolidated Electrodynamics Corporation.)

reference voltage, producing an input to the amplifier. The amplifier in turn increases the current to the cell, generating more bromine, until the feedback system maintains the set point. Because of the continuous flow of gas through the titration cell, a small amount of bromine is always being swept out. A small signal is, therefore, maintained to the amplifier so that the generating current never actually equals zero.

According to Faraday's law, in the process of electrolytic changes equal quantities of electricity charge or discharge equivalent quantities of ions at each electrode. Therefore, the quantity of bromine generated is a function of the current and an indication of the extent of the titration. In an oxidation reduction reaction of this type, the voltage developed at the electrode is a logarithmic function of the concentration. Therefore, at the point of equivalence, the change in voltage is large, resulting in an extremely sensitive and rapid response to changes in concentration.

The titration cell is a glass vessel containing an electrolyte of dilute sulfuric acid (7.5N) and potassium bromide (0.1N), from which the

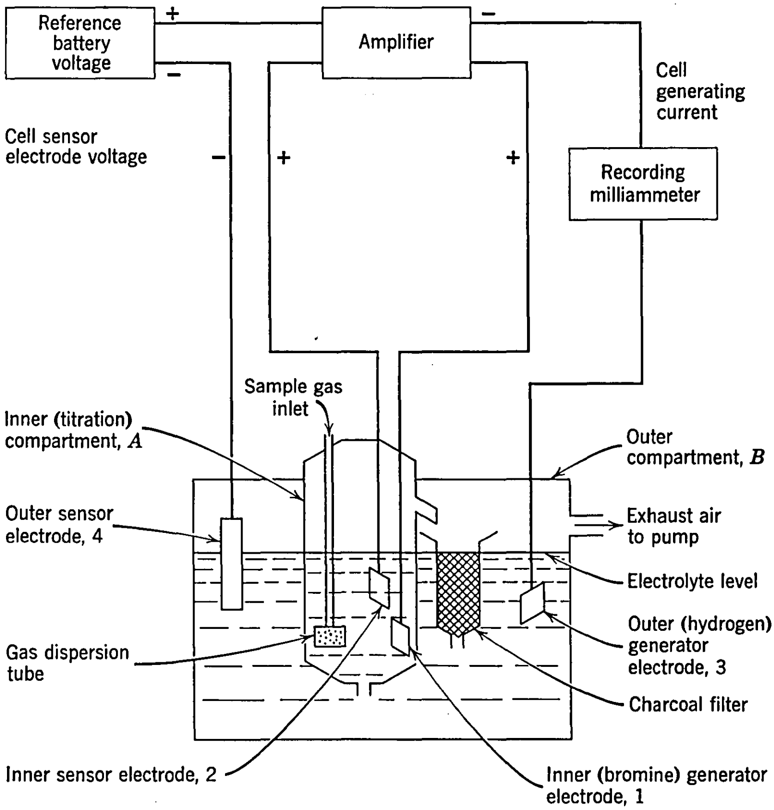

FIG. 35. Block diagram of a sulfur titrimeter and amplifier circuit. (Courtesy of Consolidated Electrodynamics Corporation.)

bromine is generated. The reaction takes place in the cell *A*, where the sample gas is continuously introduced. Bromine is generated from electrode 1. Electrode 2, in the same compartment, is platinum and is sensitive to changes in the bromine concentration. The outer chamber *B* acts as a reservoir for electrolyte and contains two electrodes, 3 and 4, to complete the circuit. Hydrogen is generated at electrode 3, and electrode 4 is a calomel half-cell used for a reference for 2. Mercurous bromide is used instead of the conventional chloride.

Sulfate Ion Sulfur Analyzer. A third type of instrument devised for the continuous determination of sulfur and sulfur compounds depends on chemical conversion of the sulfur to the sulfate ion. The sample is pumped counter-currently to the flow of a hydrogen peroxide–sulfuric acid solution in an absorbing column, where the SO_2 is oxidized to the sulfate ion. The resulting change in electrolytic conductivity of the solution is detected by a conductivity cell and recorded directly in parts of SO_2 per million parts of air. For applications of gas streams containing other compounds of sulfur, the gas is first burned to SO_2 before passing into the detector cell.

Limitations of Sulfur Analyzers. There is no single method or instrument available which is absolutely specific for any one type of sulfur compound. All these instruments present some limitations with respect to selectivity for hydrogen sulfide and high sensitivity and applicability over the full range of concentrations of sulfur compounds likely to be encountered in gaseous materials. In choosing the proper end point analyzer for sulfur, a definite distinction must be made between the various concentration ranges of interest: 1 grain per 100 standard cubic feet, 1 to 1000 grains per 100 standard cubic feet, and the high range extending from 1 to more than 90 mole %. For accurate determination

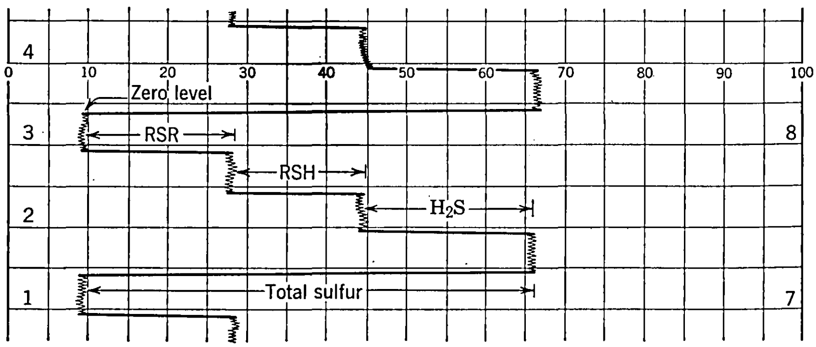

FIG. 36. Record from a titrimeter analyzer used with hydrogen sulfide, mercaptans, and sulfides. (Courtesy of Consolidated Electrodynamics Corporation.)

of hydrogen sulfide in the low concentration range, the colorimetric reactions and potentiometric titrimeter principles appear to be the most suitable. The intermediate- and high-concentration ranges must be analyzed on a batch basis or on a continuous monitoring mass spectrometer. Most of the methods discussed suffer from interference from various contaminants such as humidity, suspended solids, and sensitivity of the lead sulfide paper. The titrimeter technique is not specific for any one sulfur compound. Figure 36 illustrates its use with hydrogen sulfide, mercaptans, and sulfides, employing selective absorption. Olefinic hydrocarbons also present interference with the determination of sulfur because bromine reacts with the unsaturated double bonds. The conductivity analyzer is subject to interference from any acid gas soluble in water, such as oxides of nitrogen, hydrogen chloride, hydrogen sulfide, and ammonia. Hydrocarbons and carbon dioxide do not influence operation of the instrument.

6. VISCOSIMETERS

In the last ten years considerable progress has been made toward the development of viscosimeters, which promise instruments for continuous process monitoring and control. Because of the high sensitivity to changes in temperature, unusual care in temperature control is required in the use of any such device. Viscosimeters of four general types have been developed, only the general principles of which will be discussed here.

Capillary Tubes. Capillary tubes are probably the oldest type of viscosimeters in use. Viscosity measurements are made in terms of time required for a definite volume of liquid to travel through a short section of capillary tubing. This same principle, used under closely controlled conditions of temperature and flow rate, has been applied to continuous recording viscosimeters. The principle of operation of such a device is based on Poiseuille's law:

$$\text{Viscosity} = Pr^4/8VL^2$$

where P is the pressure drop across the capillary,
r is the radius of the capillary,
V is the flow rate of the liquid, and
L is the length of the tubing.

Constant flow of liquid through the tube is maintained by means of a gear pump and constant speed motor. Critical components of the apparatus are the pump, capillary tube, temperature, and pressure control. The temperature must be controlled within $\pm 0.1°F$ in order to determine the viscosity to an accuracy of $\pm 1\%$.

Drag Type Viscosimeters. Three commercial models of a viscosimeter employ a principle of measuring viscosity in terms of drag on rotating spindles, cylinders, and bobbins. Typical of this type of device is one (Fig. 37) which employs a synchronous motor equipped with reducing gears and a torsion spring to drive a disk-like spindle submerged in the liquid under test. The resistance to the rotating spindle exerts a lag in the drive, causing the torsion spring to move until a balance is reached between the driving and retarding torques. The angular relationship between the drive motor and the spindle is calibrated in terms of viscosity of the fluid. These instruments are usable in the range of 2 to 50,000 centipoises with a sensitivity of 1% of the full range.

Plummet Type Viscosimeters. A second principle used for viscosity measurement employs two tapered glass or metal plummets, which are operated suspended in a flowing liquid. The physical positions of the two plummets are used to indicate viscosity. One of the tubes is designed to be sensitive to rate of flow, and the other is specific to viscosity. The

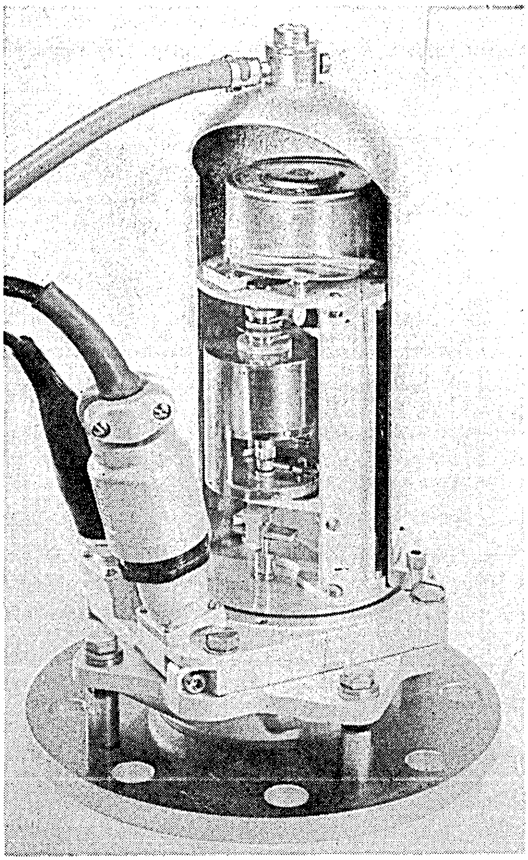

FIG. 37. Drag type viscosimeter. (Courtesy of Brookfield Engineering Laboratories, Inc.)

plummet positions are measured by means of an external impedance coil for transmission to a remote recording device. With a temperature control unit accurate to $\pm 0.5°F$, viscosity can be measured with an accuracy of $\pm 3\%$.

Falling Piston Viscosimeters. A falling piston principle is used as a viscosimeter (Fig. 38) with the rate of fall through a liquid column taken as a measure of viscosity of the liquid. A discontinuous measurement is made every 3 minutes by raising a piston of stainless steel and allowing it to fall through a distance of 4 in. The piston moves through a precision bore metal tube submerged in a flowing reservoir of liquid. A sampling valve allows the samples to be successively trapped within the cylinder. After the piston is raised and released, a microswitch is actuated, causing the recorder pen motor to drive upscale at a constant speed. When the piston trips a second switch at the end of the stroke, the recorder is shut off so that the deflection of the pen is proportional to the time of fall of the piston and the related viscosity. Accuracies of $\pm 1\%$ are claimed in the range of 0.5 to 200,000 centipoise.

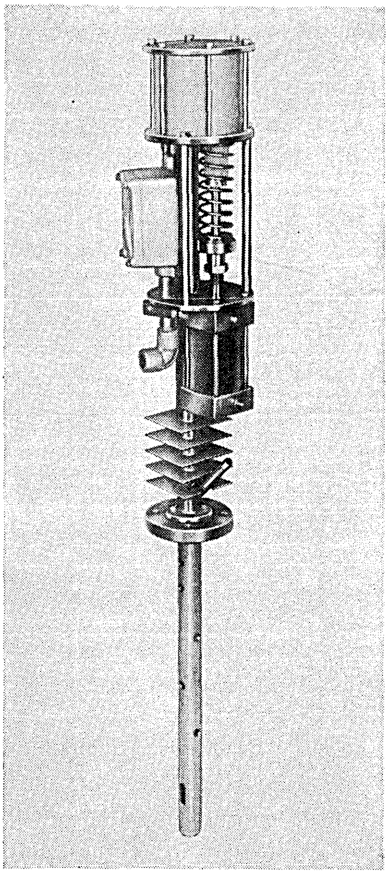

FIG. 38. Falling piston viscosimeter. (Courtesy of Norcross Corporation.)

Ultrasonic Viscosimeter. An ultrasonic device has been developed commercially as a continuously monitoring viscosimeter. This device employs a high-frequency vibrating probe (Fig. 39) which measures viscosity in terms of the vibration dampening of the probe in contact with a liquid. The dampening effect on the vibrating blade is then calibrated in terms of the product of viscosity and the density of the liquid medium. The probe consists of a thin steel alloy blade (6 in.) having a resonant frequency of 23,000 cycles/sec.

A pulse generator is used to feed the vibrator by means of a coil around the enclosed portion (4 in.) of the blade. Upon submerging the probe in a liquid, the frequency at which energy pulses are received from the oscillator increases in proportion to the viscosity of the liquid. The temperature of operation is controlled to $\pm 5°F$, permitting operation in the range of 0 to 50,000-centipoise-density units under conditions of high temperature and pressure and remote operation. Ac-

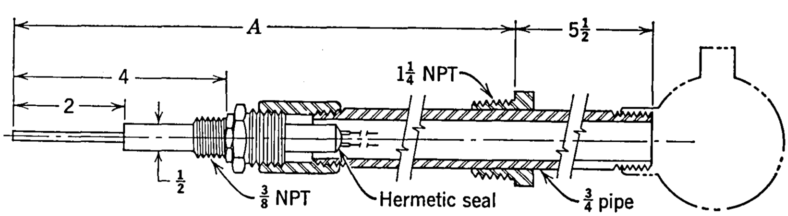

FIG. 39. Ultrasonic viscosimeter. (Courtesy of Bendix Aviation Corporation.)

curacy of measurement is limited to approximately ±2% of reading.

This method is limited to materials which have no suspended matter in them, for the method measures the dampening effect by the first monolayer, and deposits would seriously affect the reading.

7. THERMAL CONDUCTIVITY ANALYZERS

Gases. Thermal conductivity is one of the many physical properties of gases and liquids which offers a potentiality for the development of simple gas analyzers and flowmeters. Numerous instruments have been developed for the detection of a single gas in simple binary mixtures. These instruments have been in use for a number of years for monitoring and control of combustion gas, synthesis gases, and rare gas purification. Gas analysis by thermal conductivity is based on the principle that each pure gas has a characteristic heat capacity. Therefore, for a flowing gas stream the *thermal conductivity* of a particular gas is defined by the following equation:

$$Q = K(t_2 - t_1)aT/d$$

where Q is calories,
t_1 and t_2 are temperature in degrees centigrade,
a is area in square centimeters,
T is time in seconds, and
d is thickness of the cell in centimeters.

Thermal conductivity, as defined by this equation, is the time rate of transfer of heat by conduction through a unit thickness, across a unit area for a unit difference in temperature; it is expressed in units of cal/sec-cu cm/°C $\times 10^6$. Analyzer design is based on the fact that if a wire, surrounded by a flowing gas, in a closed chamber is connected to a source of constant voltage, the temperature of the wire changes until the continuous dissipation of thermal energy is equal to the electrical energy supplied to the wire. Dissipation of the energy is accomplished by radiation, thermal convection, and conduction through the surrounding gas. If the wire used has a high-temperature coefficient of electrical resistance, this resistance will have a value corresponding to the thermal conductivity and composition of the surrounding gas. By comparing resistances of two such wires surrounded respectively with a reference gas and unknown gas mixture, a quantitive measure of the variable component can be determined.

Changes in the gas flow rate over the filaments in the cell are known to cause marked errors in analysis by virtue of variable convection currents, etc. For this reason the cells are usually designed to route the gas flow past the filaments, allowing diffusion of the gas into the filament chamber to minimize the flow effect. The physical arrangement of the cell is shown schematically in Fig. 40. Four temperature-sensitive wire fila-

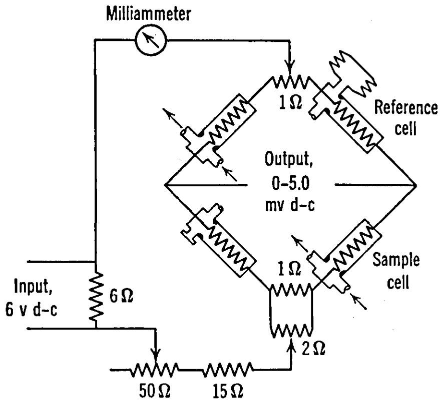

FIG. 40. Schematic of thermal conductivity analyzer for gases.

ments are arranged in a Wheatstone bridge circuit. Two of the filaments are suspended in a confined gas, and the other two filaments are exposed to the gas being analyzed. A constant d-c input is applied to the bridge. All the filaments are cooled in accordance to the thermal conductivity of the gas in the cell. Resistances of the two filaments in the reference cell remain constant, but the resistance of the filaments in the sample cell changes with its temperature, producing a bridge unbalance that is measured in terms of gas composition.

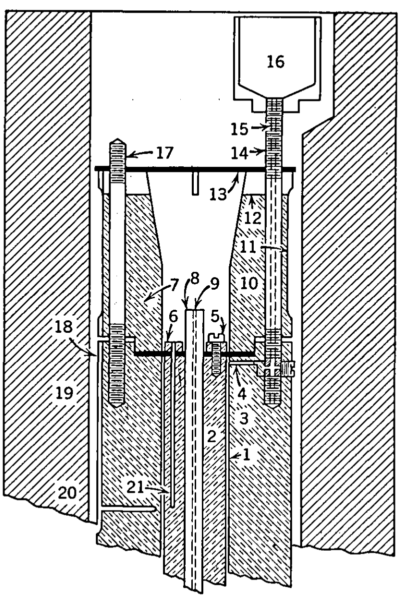

FIG 41. Test cell for liquid thermal conductivity analyzer.

Factors affecting cell application include ambient temperature fluctuations, convection rate within the cell, and water vapor which has a high thermal conductivity and pressure variation in the sample cell. The recommended sampling rate for such a cell is 1 cu ft/hr.

Liquids. Measuring the thermal conductivity of liquids has been a subject of some interest for the last thirty years. The most recent instrument developed for this purpose was designed to measure the thermal conductivity of liquids with an accuracy of 1% in the range of 0 to 100°C. The apparatus is limited to liquids which flow readily at these temperatures and which will not attack copper, solder, or Teflon.

The test cell, shown in cross section in Fig. 41, uses a thin (0.016 in. $\times$ 5 in.) annular layer of liquid as the test specimen. Nonmetallic closures and fine lead-in wires are used to reduce thermal effects. The liquid layer 1 surrounds a heater cylinder 2 of polished copper. Its diameter is 0.37 in., and it is 4.99 in. long. The single wire heater is 0.004-in. enamel-covered Nichrome V, made 1% longer than the cylinder to provide extra energy for heat loss at either end. Current leads of 0.026-in. copper are drawn through 0.020-in. bore 9, in 0.125-in. copper capillary 8. Over this fits a Teflon disk 7, which serves to seal the annulus.

Before the annulus is filled with test liquid, air is removed through opening 4, into a hollow stud 14, connected to a vacuum pump. The temperature difference between heater and receiver cylinders is measured by copper-constantan thermocouples of 0.0045 in. enamel-covered wire.

At any given temperature level it is essential that the temperature variation of the cell be held to a minimum. This is accomplished by a constant temperature bath of mineral oil.

8. DIELECTRIC CONSTANT ANALYZER

The dielectric constant as a physical measurement is becoming increasingly useful as an analytical tool. Several of the major oil companies have been investigating its application in determining the aromaticity of hydrocarbons, moisture in solids, interface distribution in flowing streams, and chemical purity.

The dielectric constant of a substance is defined as an electrical property which affects the capacitance between the electrodes of a condenser. The dielectric constant is determined by direct comparison to a known standard such as air, according to the formula

$$C = \frac{0.2416KL}{\log (r_1/r_2)}$$

where C is capacitance,
L is length of annular space,
r is the radius of the cell, and
K is the dielectric constant.

With flowing liquids a sample cell consisting of a cylindrical annular space between an outer cylinder and an inner insulating rod is used as shown in

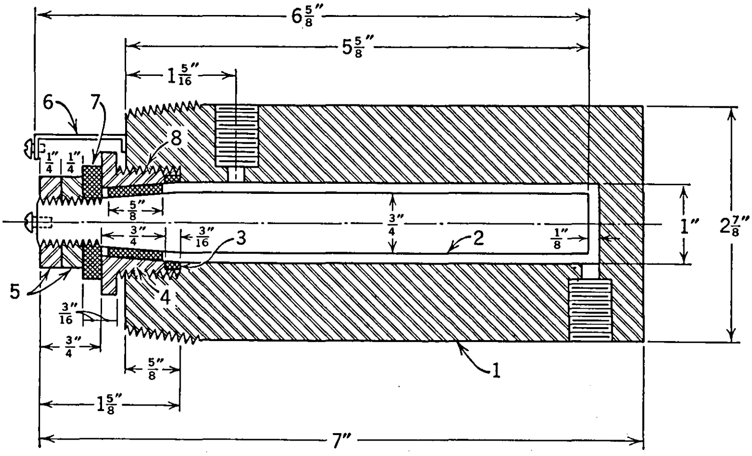

FIG. 42. Dielectric constant analyzer.

Fig. 42. The sample cell 1 is made of stainless steel. The center electrode is insulated by means of Teflon washers 3, 7, and 8. Metal washers 5 are used to provide a pressure-tight seal. External threads on the open end of the outer cylinder 1 permit attachment of the cell to a Crouse-Hinds conduit box for electrical shielding, explosion proofing, and auxiliary oscillator. Capacitance measurements can be made with standard capacitance measuring instruments or recorders.

9. VAPOR PRESSURE ANALYZERS

Since many petroleum fuels are produced to certain vapor pressure specifications, a continuous vapor pressure recording instrument is needed. Such an instrument (Fig. 43) has been developed for commercial use. The instrument is shown schematically in Fig. 44. Since the instrument is designed to measure vapor pressure, it is necessary to control the temperature ($\pm 0.002°F$) by means of a constant temperature bath. The sample is fed into the valve chamber through a proportioning pump and preheater coil, consisting of a coil of copper tubing, immersed in the bath. The height of the liquid in the valve chamber is controlled by a float valve and atmospheric discharge sump. The vapor pressure is measured by a pressure gage which is connected to the liquid portion of the chamber. To prevent the measurement of any partial pressure of air contamination, a vent line is provided to bleed off a small amount of vapor and air continually. The vapor pressures of gasolines have been measured to ± 0.1 psi, which permits accurate blending of products to meet desired specifications.

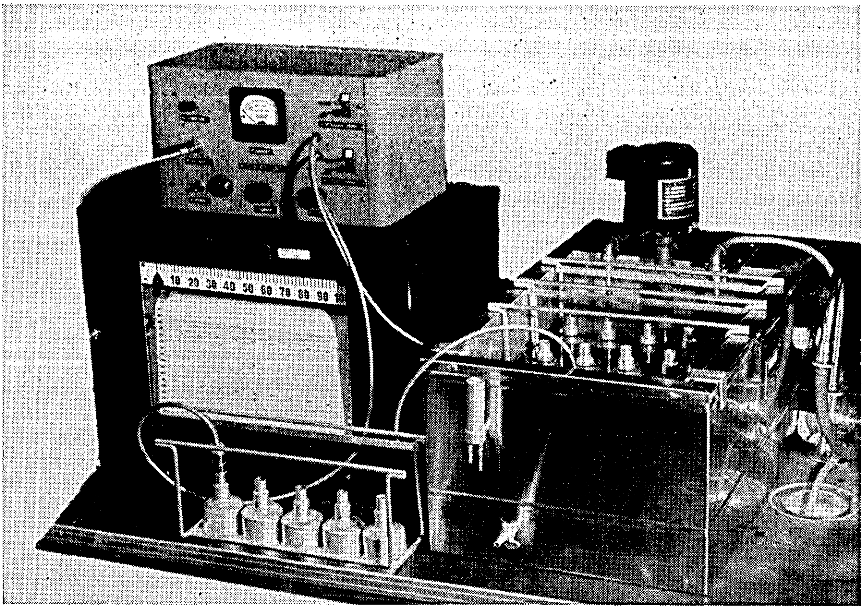

FIG. 43. Vapor pressure analyzer.

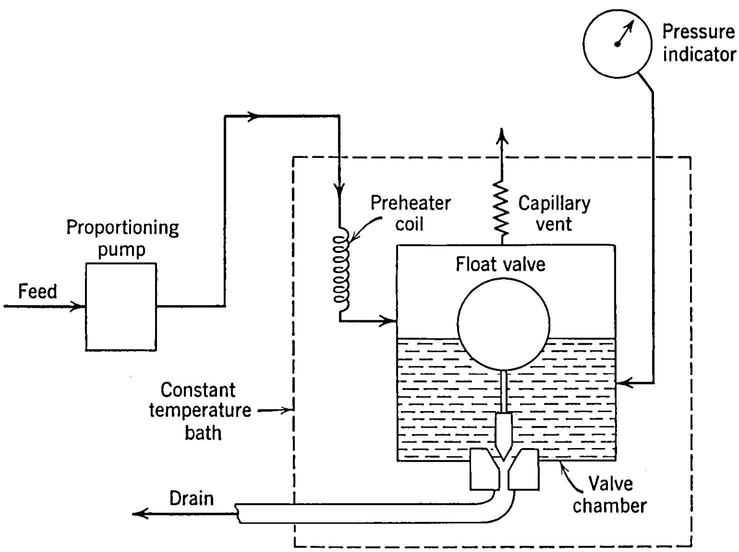

FIG. 44. Vapor pressure analyzer schematic.

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DESIGN OF COMPONENTS

G. DESIGN OF COMPONENTS

Magnetic Amplifiers

G. F. Pittman, Jr., and R. O. Decker

1. INTRODUCTION

A magnetic amplifier controls the power delivered to a load from an a-c source by employing a controllable, nonlinear reactive element or circuit, generally interposed in series with the load. The power required to control the element can be made far less than the power controlled; hence power amplification is achieved.

Advantageous features of magnetic amplifiers include the following:

1. The physical structure lends itself to ruggedness and resistance to severe environmental conditions.
2. Electrical isolation between input and output and between multiple inputs is readily obtained.
3. Relatively high efficiency is possible—80 to 90% in larger sizes.
4. Because operation is directly from an a-c power source, no special power supply is usually required.
5. High-power amplification is possible (over 10^6 per stage) if slow response can be tolerated.

Because of these and other features of magnetic amplifiers, they have found wide application in such diverse fields as airborne servomechanisms and central power station generator regulators in utility systems. They have been used to amplify signals from thermocouples, photocells, and ionization chambers, as well as to control the speed of the largest steel mill drives. Magnetic amplifiers have been built to amplify signals at micromicrowatt power levels and also to control powers in excess of a megawatt.

The basic components of magnetic amplifiers are saturable magnetic core reactors and semiconductor rectifiers. The advances which have taken place in the fields of high-quality magnetic materials and semiconductor devices since the late 1930's have been directly responsible for the rapid growth of magnetic amplifier applications in recent years.

2. MAGNETIC AMPLIFIER FUNDAMENTALS

Saturable Reactors. A *saturable reactor* consists of one or more wire windings linked together by a closed magnetic circuit, or core, constructed of a readily saturable ferromagnetic material. In some cases, particularly for amplifiers capable of controlling several hundred watts or more, conventional transformer type materials and construction are employed for saturable reactors. Use of high-quality nickel-iron alloy materials, however, makes possible much higher performance amplifiers of smaller size and weight. In order to realize the advantages of these materials, cores must be constructed essentially without air gaps. This is commonly accomplished by using a toroidal core configuration which is made up of a thin ribbon of the magnetic material wound in a spiral fashion (see Sect. 3).

Idealized saturable reactor characteristics are shown in Fig. 1 in the form of hysteresis loops analogous to those frequently presented for transformer cores. The important features of the ideal saturable reactor characteristics are:

1. Sharp saturation at some well-defined saturation flux, ϕ_s.
2. Very low differential permeability in the saturated region.
3. Very high differential permeability in the unsaturated region.

As a result of these characteristics, a saturable reactor has the capability of presenting a relatively high impedance when operating in the unsaturated region ($-\phi_s < \phi < \phi_s$), comparable to the magnetizing impedance of a transformer, and of presenting a very low impedance when operating in the saturated region ($\phi = \pm\phi_s$). By exercising control over the region in which the saturable reactor operates, its effective impedance may be varied and the flow of power controlled.

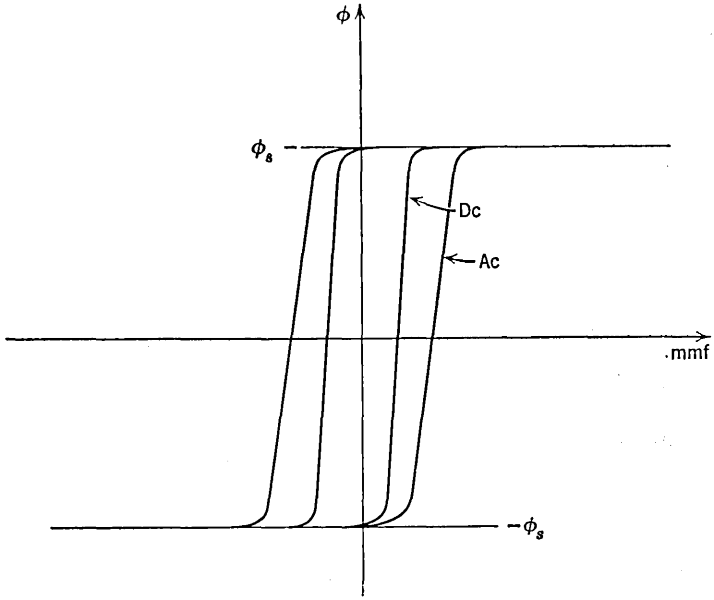

FIG. 1. Idealized saturable reactor characteristics.

Control of Power. The mechanism of power control is illustrated by the elementary magnetic amplifier load circuit of Fig. 2. The presence of the rectifying element in series with the saturable reactor gate winding identifies this as a *self-saturating* magnetic amplifier circuit.

Assume that the magnitude of the a-c voltage source is adjusted so that, with the rectifier short circuited, the core flux excursion is exactly $2\phi_s$. That is,

$$E_{ac} = 4.44 f N_g \phi_s \times 10^{-8},$$

where E_{ac} and f are the root-mean-square (rms) value and the frequency

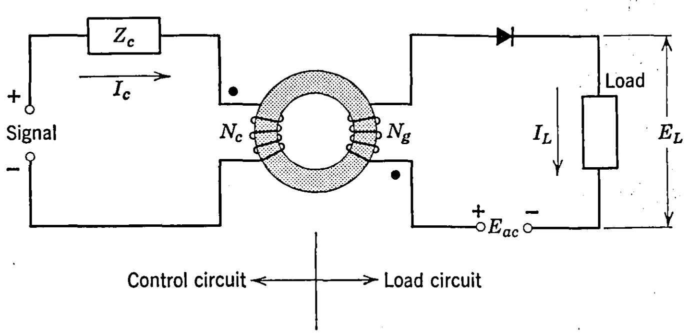

FIG. 2. Elementary half-wave magnetic amplifier.

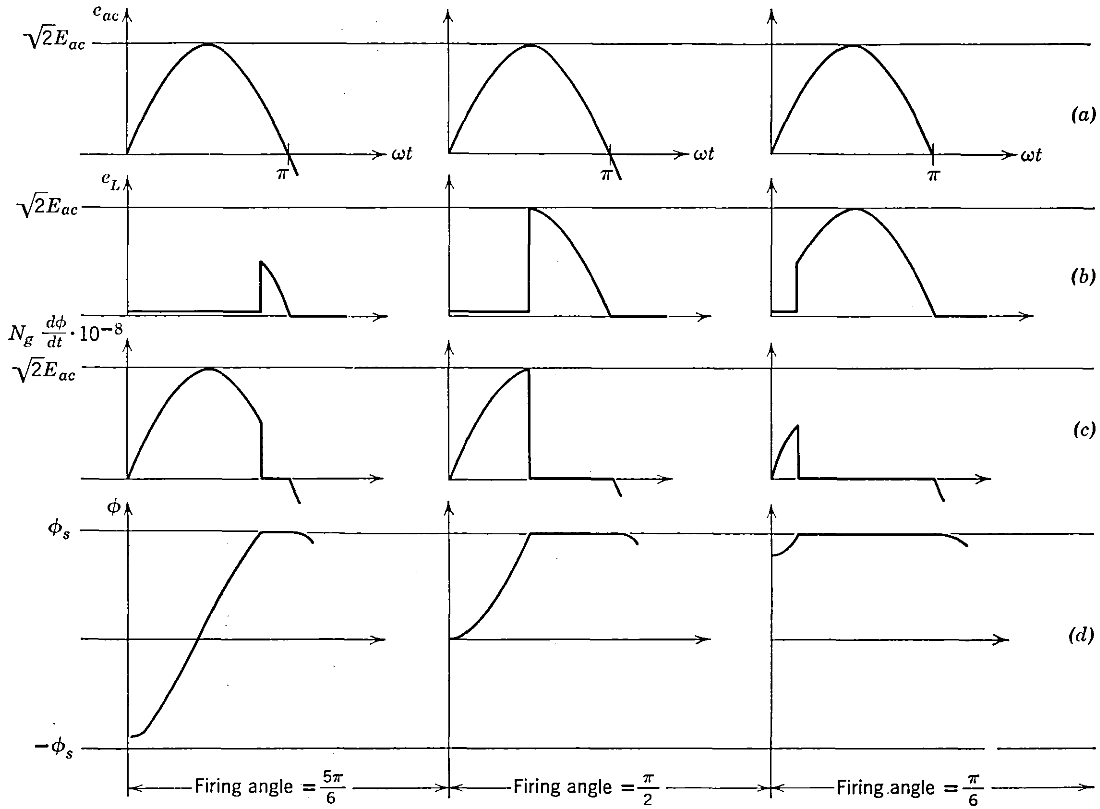

FIG. 3. Waveforms for various preset flux conditions in the magnetic amplifier of Fig. 2: (*a*) applied line voltage e_{ac}, (*b*) load voltage e_L, (*c*) gate winding voltage $N_g d\phi/dt$, and (*d*) flux in the reactor, ϕ.

of the applied voltage, N_g is the number of gate winding turns, and ϕ_s is the saturation flux of the core in Maxwells. This is the condition of *normal excitation.* With the rectifier in the circuit, power can be delivered to the load only on alternate half-cycles when the a-c source has the polarity indicated. The amount of power delivered is determined by the flux conditions of the core at the beginnings of these half-cycles. Figure 3 shows waveforms corresponding to various conditions of preset flux, ϕ_p, at the beginning of the half-cycle. In the early part of the half-cycle the core is operating in the unsaturated region $(-\phi_s < \phi < \phi_s)$, and the gate winding presents a high impedance in the load circuit. Because of this, virtually the entire a-c source voltage appears across N_g and practically none at the load. During this interval, core flux increases from its preset level, ϕ_p, toward ϕ_s in accordance with the relationship:

$$e_{ac} = N_g \frac{d\phi}{dt} \times 10^{-8}.$$

When the core saturates $(\phi = \phi_s)$, no further flux change is possible and the a-c source voltage can no longer be supported across N_g. At this point, therefore, the gate winding suddenly changes its impedance from a very high value to a very low one, and the a-c source voltage appears largely across the load for the remainder of the half-cycle. The similarity to thyratron waveforms is evident and similar terminology is applicable; the half-cycle is termed the *gating or firing half-cycle,* and the angle at which the core saturates and the voltage appears across the load is termed the *firing angle.*

The Volt-Second Concept. During a gating half-cycle, the core flux changes from ϕ_p to ϕ_s; the voltage, e, appearing at the gate winding terminals is related to this change by:

$$\int e\,dt = N_g(\phi_s - \phi_p) \times 10^{-8}$$

This indicates that the change in flux from ϕ_p to ϕ_s is associated with the appearance at the winding terminals of a definite volt-second integral or area. The operation of the saturable reactor may thus be interpreted as absorbing or subtracting from the volt-seconds of the source, a volt-second area proportional to ϕ_p, and permitting the remainder to appear across the load. The load voltage, E_L, averaged over the half-cycle, is thus:

$$E_L = 0.9E_{ac} - 2fN_g(\phi_s - \phi_p) \times 10^{-8}.$$

For the normal excitation conditions assumed $(E_{ac} = 4.44\, fN_g\, \phi_s \times 10^{-8})$, this becomes:

$$E_L = 2fN_g(\phi_s + \phi_p) \times 10^{-8}.$$

This relationship is shown graphically by the solid line of Fig. 4*a*.

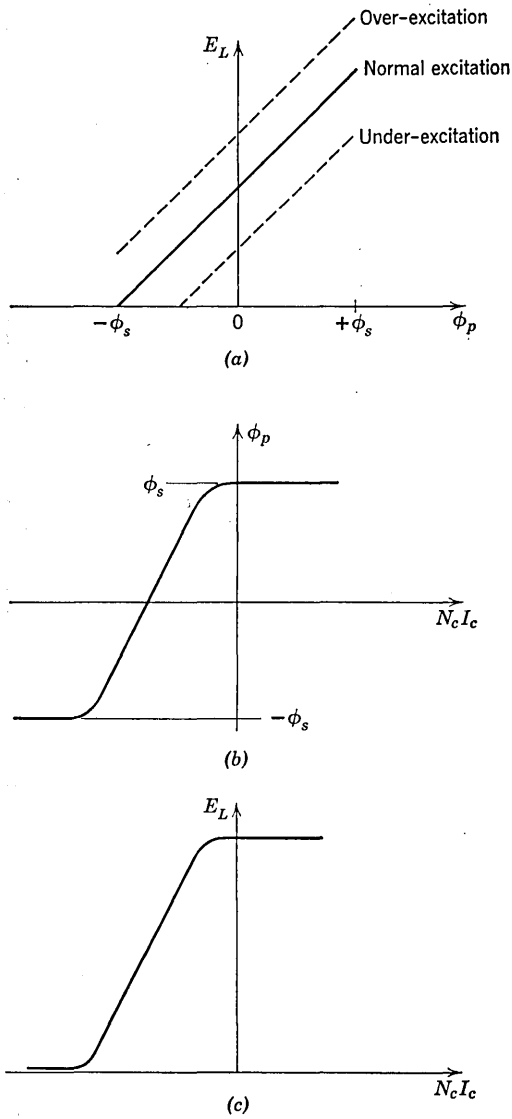

FIG. 4. Magnetic amplifier operation relationships: (a) load voltage E_L versus preset flux ϕ_p, (b) preset flux ϕ_P versus average control magnetomotive force N_cI_c, (c) transfer characteristic, load voltage E_L versus average control magnetomotive force N_cI_c.

In cases of other than normal excitation, the curve is shifted vertically by an amount equal to the difference between the actual half-cycle average value of the a-c source and the value for normal excitation, as shown by the dashed lines in Fig. 4*a*. For over-excitation ($E_{ac} > 4.44fN_g\phi_s \times 10^{-8}$) it is seen that even with $\phi_p = -\phi_s$ voltage still appears across the load, i.e., the amplifier cannot be cut off. For under-excitation ($E_{ac} < 4.44fN_g\phi_s \times 10^{-8}$) the load voltage is zero for a range of values of ϕ_p near $\phi_p = -\phi_s$ and no control of load voltage is exercised in this range (negative values of E_L are not possible). The under-excitation condition is normally avoided because of a "triggering" phenomenon which results in a bistable region in the characteristic of Fig. 4*a* in the vicinity of the $E_L = 0$ point.

Resetting. Since the output of a magnetic amplifier during a particular gating half-cycle is determined by the value of ϕ_p at the beginning of that half cycle, it is evident that control of the amplifier may be exercised by controlling ϕ_p. The process of establishing the initial flux ϕ_p at the beginning of each gating half-cycle is termed *resetting* and is accomplished during the intervening or *resetting half-cycles.* Resetting is conveniently achieved by applying a signal to a second signal or control winding on the saturable reactor as shown in the half-wave magnetic amplifier of Fig. 2. The magnitude of the signal determines the change in flux which takes place during the resetting process; hence the value of ϕ_p at the beginning of the subsequent gating half-cycle and the resulting output.

The form of the relationship between preset flux level ϕ_p and the average magnetomotive force applied to the core during the resetting half-cycle N_cI_c is shown idealized in Fig. 4*b*. This type of curve is a convenient way of expressing the significant characteristics of a core for magnetic amplifier purposes. In particular, the slope of the curve is an important design parameter of the core (see Sect. 3). It is often expressed as a core gain, K_{AT}:

$$K_{AT} = 2f\left(\frac{\Delta\phi_p}{\Delta N_c I_c}\right) \times 10^{-8}$$

having the dimensions of volts per turn per ampere-turn, or, in terms of core material characteristics, as a differential permeability, μ_d:

$$\mu_d = \left(\frac{\Delta B}{\Delta H}\right) = \frac{l_c}{0.4\pi A_c}\left(\frac{\Delta\phi_p}{\Delta N_c I_c}\right)$$

where l_c is the mean magnetic path length of the core in centimeters and A_c is the core cross-sectional area in square centimeters.

The transfer characteristic giving output (current or voltage) as a function of input control current for the half-wave amplifier of Fig. 2 is obtained by combining the relationships of Fig. 4*a* and 4*b*. A typical transfer characteristic is shown in Fig. 4*c*. The condition of full output at zero signal is a result of the high remanence of the magnetic material assumed, which results in the core flux remaining at the $+\phi_s$ level during the resetting half-cycle in the absence of any applied magnetomotive force. In practice the transfer characteristic with respect to a given control winding can readily be shifted to the right or left by applying a constant bias magnetomotive force to the core by means of a separate bias winding. The amount of resetting is determined by the net magnetomotive force acting on the core, that is, the algebraic sum of the resetting ampere-turns of all control and bias windings present on the core. It is this feature which also makes magnetic amplifiers well adapted to summation of a multiplicity of signals.

It is important to recognize that the resetting process requires that a finite amount of signal power be supplied to the amplifier. In the amplifier of Fig. 2, the resetting of core flux from ϕ_s to ϕ_p implies that a volt-second integral:

$$\int e\,dt = -N_c(\phi_s - \phi_p) \times 10^{-8}$$

be induced in N_c during the resetting interval. At the same time, the required exciting ampere-turns must also be supplied through N_c. Either the voltage applied to N_c or the current passed through it may be considered as the independent input variable; they are in any case interrelated by the core characteristics.

The impedance element in series with the control winding of the amplifier of Fig. 2 is necessary in order that substantial currents in the signal circuit do not result from the voltage induced in N_c during the prefiring interval of the gating half-cycle. Such currents are reflected into the gate winding with undesirable effects on the amplifier performance. The necessity for a high signal circuit impedance is a restriction on the usefulness of the simple half-wave amplifier. This type of amplifier, especially combinations of several such elements to obtain reversible output, does find application, however, in many situations where the impedance requirement is tolerable and where fast response is important.

The signal circuit impedance restriction is not present in full-wave amplifiers and is also circumvented by the Ramey type amplifier, Fig. 5*a*, which effectively blocks the signal circuit during the gating half-cycle by the combined action of the a-c source and rectifier in series with the control winding.

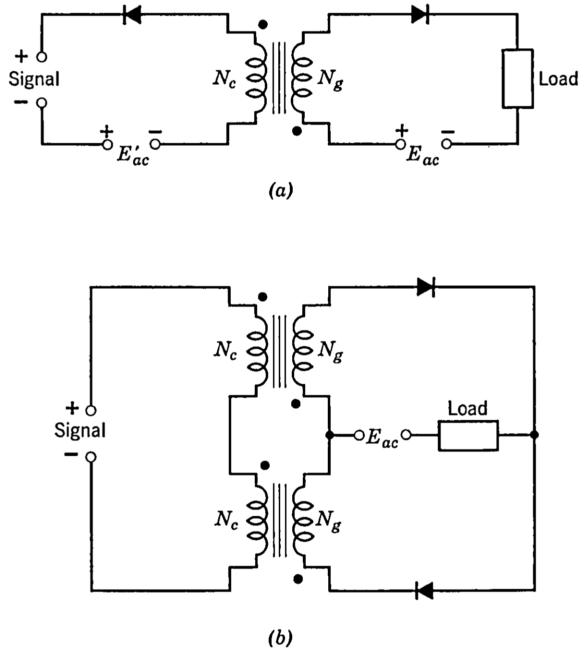

FIG. 5. Magnetic amplifier circuits not restricted by high signal circuit impedance. (a) Ramey type amplifier and (b) full-wave amplifier.

Full-Wave Self-Saturating Amplifier. Full-wave amplifiers employ two saturable reactors interconnected in such a way that the gating half-cycle of one is the resetting half-cycle of the other. Connecting the control windings of the two reactors in series effectively causes the resetting reactor to serve as the required high signal circuit impedance for the gating reactor. The load circuit is arranged so that the two reactors supply power to a common load on alternate half-cycles. The net result is higher voltage and power gains which, however, are obtained at the expense of slower response.

A typical full-wave, self-saturating amplifier circuit for a-c output is shown in Fig. 5*b*. It is seen that the control circuit is connected in such a way that identical control magnetomotive force is applied to both cores. As a result, identical control of the preset flux level of both cores is exercised on alternate half-cycles. In steady-state operation, the average voltage induced in the control circuit due to the firing of one reactor is exactly the voltage required to reset the other. These two voltages, therefore, cancel each other, and the signal source is required only to supply the d-c voltage corresponding to the resistive voltage drop of the signal current flowing in the circuit. The interaction between the two control windings results in a kind of inherent positive feedback within the amplifier. This has the effect of increasing gain but at the expense of slower response to a change in input signal voltage.

The transfer characteristic of a full-wave amplifier in terms of half-cycle average output as a function of control current is essentially the same as that of a half-wave amplifier using one of the cores. In the full-wave amplifier, however, output occurs on each half-cycle, and, in the case of the circuit of Fig. 5*b*, is ac with no d-c component.

Response Time. In the case of the single-reactor, half-wave amplifier, the output during a given gating half-cycle is a function only of the signal during the preceding half-cycle. Thus a change in signal introduced at the beginning of a resetting half-cycle will have its full effect upon the output during the following gating half-cycle. The dynamic performance of this type of amplifier is, therefore, closely approximated by a one-half cycle delay; yielding a transfer function of the form

$$G(s) = K_v e^{-TS},$$

where K_v is the amplifier voltage gain, and T is an interval of time approximately equal to one-half cycle of the supply frequency.

The output of a full-wave amplifier on a given half-cycle is a function not only of the signal during the preceding half-cycle but also of the *output* during that half-cycle, because of the inherent positive feedback effect mentioned above. This results in a longer delay in the amplifier output reflecting the full effect of a step change in signal. If the delay extends over many cycles of the supply frequency, the amplifier output change during the transient can be closely approximated by a continuous exponential change. This implies a transfer function of the form:

$$G(s) = K_v \left[\frac{1}{1 + \tau s}\right]$$

where K_v is the amplifier voltage gain, and τ is a time constant (analogous to that of a linear inductive circuit) characteristic of the amplifier.

The time constant of a full-wave amplifier is a function of many of the parameters of the amplifier circuit, but it can be conveniently expressed in terms of the amplifier voltage gain:

$$\tau = \frac{N_c}{2fN_g} \cdot K_v \quad \text{(seconds)},$$

where N_c is the number of control winding turns per core, N_g is the number of gate winding turns per core, and K_v is the amplifier voltage gain. In this form, it is readily apparent that the response of an amplifier may be varied over a wide range by modifying the gain of the amplifier, e.g., through changes in signal circuit parameters N_c or R_c or through application of feedback around the amplifier.

Figure of Merit. Another useful way of relating the amplifier time constant to other amplifier characteristics is through the *figure of merit M* of the amplifier:

$$M = \frac{K_p}{\tau},$$

where K_p is the power gain of the amplifier. The figure of merit can be shown to be independent of the signal circuit parameters N_c and R_c; hence adjustments of these parameters to modify response time have a direct effect upon power gain.

The above considerations are based upon the assumption that any other circuits linking the cores in the same manner as the control windings (bias windings, additional control windings, etc.) display a very high reflected impedance as seen from the signal circuit compared with the impedance of the signal circuit. Coupled circuits which do not meet this condition result in further increases in amplifier response time. More detailed treatments of this effect may be found in the literature (see Ref. 11).

Feedback. Because control of a magnetic amplifier is exercised by the net average resetting magnetomotive force acting on the core during the resetting half-cycle, feedback may be conveniently applied through an additional set of feedback windings on the cores connected in the same manner as the control windings. By passing a direct current proportional to output voltage (or current) through the feedback windings, feedback is obtained while still maintaining electrical isolation of the signal circuit from the output circuit. This is referred to as *magnetomotive force feedback.*

Feedback may also be applied by the conventional methods of adding a feedback voltage electrically in series with the signal voltage in the signal circuit. Through use of these various types of feedback, and combinations of types, desirable effects can be achieved in such amplifier characteristics as gain, input and output impedance, response time, stability, and sensitivity.

Simple Saturable Reactor Circuit. The *simple saturable reactor* circuit shown in Fig. 6*a* is an amplifier employing saturable reactors alone without requiring rectifiers (unless output rectification to obtain dc is desired). In effect, the circuit configuration without rectifiers results in an amplifier which has a strong inherent negative magnetomotive force feedback. The transfer characteristic of the amplifier of Fig. 6*a* is shown in Fig. 6*b*. It is convenient to consider the amplifier as a current amplifier

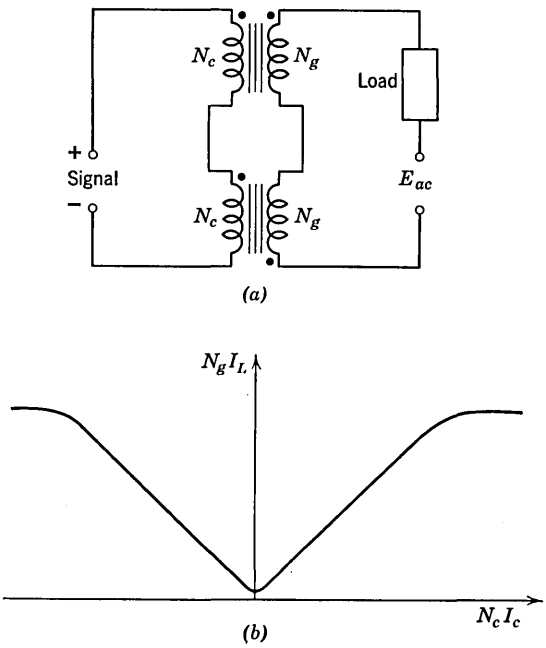

FIG. 6. Simple saturable reactor: (a) circuit and (b) transfer characteristic.

with a current gain of

$$K_i = \frac{N_c}{N_g}.$$

Because of the inherent negative feedback effect of the circuit, this type of amplifier is quite stable in its characteristics but has relatively low gain, slow response, and poor figure of merit. It has found application primarily in instrumentation (e.g., metering of large direct currents) and in very high-power applications where the absence of rectifiers is an advantage.

3. MAGNETIC AMPLIFIER COMPONENTS

The operation of a magnetic amplifier is based on the nonlinear magnetic characteristics of saturable reactor cores. In the case of the self-saturating amplifier circuits, the nonlinear electric characteristics of the incorporated rectifiers are also essential. Important properties of magnetic amplifiers, such as gain, response time, and figure of merit, depend greatly on these components. Amplifier size, weight, and power-handling capability are dictated by them. Accordingly, special core materials and rectifiers have been developed for use in magnetic amplifier circuits.

Cores

Core Material Properties of Importance. In Fig. 7*a*, a typical dynamic hysteresis loop is shown. It is measured under specified conditions, for example, by exciting the core with a sinusoidal alternating voltage of a certain frequency. The figure serves for a discussion of some of the core characteristics of particular importance in magnetic amplifiers.

Saturation Flux Density (B_s). As with transformers, a high saturation flux density results in reduced size and weight of the cores and tends to increase the efficiency of an amplifier. It is, therefore, a desirable property of the core material in output stages handling appreciable power.

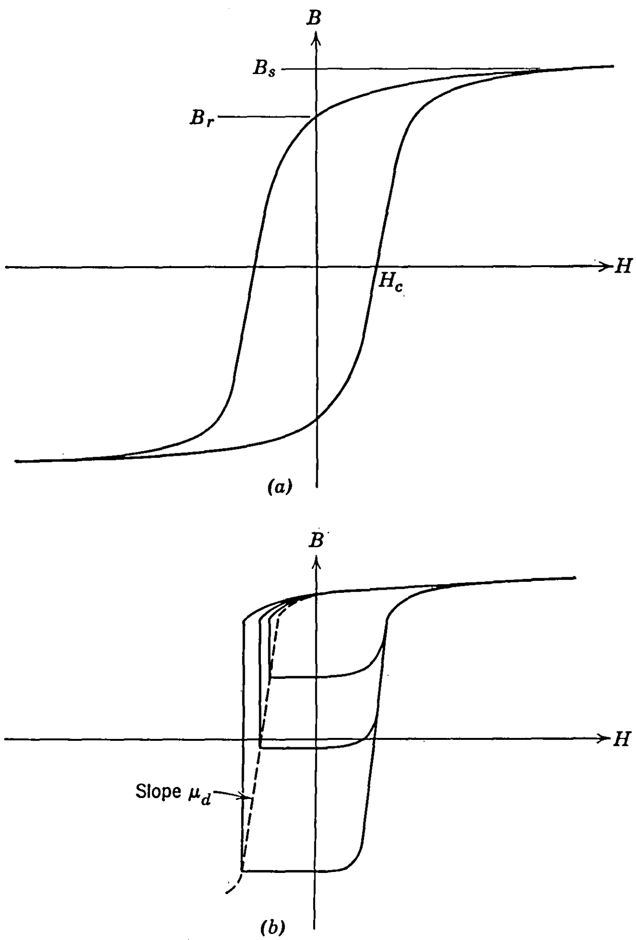

FIG. 7. Magnetic core hysteresis loops: (*a*) typical dynamic hysteresis loop and (*b*) typical minor loops traversed in magnetic amplifier operation.

In low-power (preamplifier) stages, however, high saturation flux density is often exchanged against the advantages of a narrower hysteresis loop or a higher permeability.

Squareness Ratio (B_r/B_s). The ratio of remanent flux density B_r to saturation flux density B_s is termed the *squareness ratio*. The higher B_r/B_s, the more linear the amplifier transfer characteristics and, due to more nearly complete saturation, the better the utilization of the core material will be. Maximum squareness ($B_r/B_s \rightarrow 1$) is obtained with materials having a high degree of crystal orientation.

Hysteresis Loop Width (H_c). The width of the dynamic hysteresis loop may be indicated in terms of the coercive force H_c. It determines the magnetizing current of the reactors and, hence, the output current of the amplifier at the cutoff point. The loop width affects also the core losses and, with self-saturating circuits, the operating range of the control current; the latter is of interest when matching cores, for example, for circuits employing multiple cores.

Differential Permeability (μ_d). The gain of a self-saturating magnetic amplifier depends on the relation between the flux change during the reset period and the applied reset ampere-turns. It may be expressed in terms of a *differential permeability* (μ_d), which is defined as the ratio of the incremental change of preset flux density (ΔB) to the incremental change of average resetting magnetizing force (ΔH) when the core is operated in a cyclic manner similar to that occurring in self-saturating magnetic amplifier circuits. This is illustrated in Fig. 7*b*. The differential permeability varies somewhat with the actual resetting conditions, i.e., with the different minor hysteresis loops described with the various amplifier circuits. As a first approximation, however, the differential permeability may be considered a property of the core itself.

Types of Core Materials. The commonly used magnetic amplifier core materials are marketed under many trade names, but they are variations of four essentially different types of materials. Some quantitative information is contained in Table 1.

Silicon-Iron. Cold-rolled and heat-treated anisotropic transformer steel containing about 3–3.5% silicon is most frequently employed in output stages where high saturation flux density and relatively low cost are of importance. The material possesses a high degree of grain orientation which provides easy magnetization in the rolling direction. Silicon-iron strip is produced in gages ranging from 0.002 to 0.025 in. thickness.

50% Nickel-Iron. These materials are most frequently used in high-performance magnetic amplifiers up to several kilowatts rating. The upper power limit is mainly prescribed by cost considerations. The material is produced in two variations, the isotropic and the oriented aniso-

TABLE 1. REPRESENTATIVE CHARACTERISTICS OF COMMERCIAL MAGNETIC MATERIALS

Class	Trade Name	Approximate Composition	B_s (kilogauss)	H_c at 60 cps (oersted)	μ_d (kilogauss/ oersted)	B_r/B_s	Resistivity (micro-ohm-cm)
I	Hipersil Magnesil Selectron Trancor	3 Silicon 97 Iron (oriented)	18	0.5	25–100 (5-mil strip)	0.75	45–50
II	Allegheny 4750 Carpenter 49 48 Alloy Hipernik	50–53 Nickel 47–50 Iron (low orientation)	15	0.08–0.16	50–100	0.5–0.8	50
III	Deltamax Hipernik V Orthonik Orthonol Permenorm 5000Z	50 Nickel 50 Iron	15	0.15–0.25	100–200	0.90–0.96	50
IV	Mumetal	5 Cu, 2 Cr 77 Ni, 16 Fe	6	0.03–0.06	100	0.6	6
V	4-79 Permalloy Hy-Mu 80 Mo-Permalloy Square mu	4 Mo, 79 Ni 17 Fe	7	0.03–0.06	200–500	0.5–0.8	55
VI	Supermalloy	5 Mo, 79 Ni, 16 Fe	7	0.015–0.04	250–1000	0.50–0.75	65
VII	Type H Ferrite	—	3.4	0.15–0.3	5–10 at 100 Kc/sec	0.4–0.5	104

tropic type. The former has a narrower hysteresis loop whereas the latter has a much squarer loop. The saturation flux density of both is relatively high. Punched laminations and toroidal strips ranging from less than 0.001 to 0.006 in. thickness are readily available.

65–79% Nickel-Iron. These core materials of the Permalloy type are most suitable for sensitive low-level amplifiers. Their saturation flux density is only about one-half that of the 50% nickel-iron types, and the hysteresis loops are less square. However, their differential permeabilities are extremely high and their hysteresis loops are very narrow. The addition of other components, such as molybdenum, increases the resistivity considerably; this reduces the losses and makes the cores applicable for audio and low radio-frequency applications. The materials are produced in punched laminations and in toroidal strip rolled as thin as 0.00012 in.

Ferrites. The outstanding feature of the nonmetallic ferrites is their resistivity, which is up to several orders of magnitude higher than that of the metallic core materials; this makes ferrite cores useful far into the megacycle frequency range. The other characteristics, saturation flux density, squareness, and hysteresis loop width are inferior compared with those of the magnetic metals at lower frequency. A wide field of applications for ferrite cores is the storage of information in high-speed digital computers and control apparatus.

Core Construction.

Toroids. A truly airgap-free reactor can be constructed by stacking punched rings of a nonoriented magnetic material and threading a winding through the center hole of the core. However, because of the restriction to nonoriented material and the wasteful shape of the punching, it is much more common to form toroids by spirally winding a continuous magnetic strip which may possess magnetic orientation in the lengthwise direction. Owing to the continuous transition of the magnetic flux through the spiral airgap along its entire length, there is a negligible airgap effect. After the cores have passed their final anneal, they are quite strain-sensitive when made of nickel-iron. Therefore, they are placed in ring-shaped boxes made of micarta, nylon, or aluminum. The boxes are often filled with damping liquids, such as silicon oil. Cores with very small cross section or those made of thin strip are wound and annealed on ceramic or stainless steel bobbins.

Windings can be applied to toroids by special winding machines. Since the cores are usually required in pairs for magnetic amplifiers, two partially wound toroidal reactors may be stacked, and a common winding may be placed through the holes of both cores. Figure 8 shows a cross section through a pair of finished toroidal reactors.

Although the good properties of the core materials are most fully preserved with toroidal reactors, cost may be high because of the complicated winding operation, especially in the larger sizes. A compromise consists of wrapping the iron strip on a rectangular mandrel and staggering the strip width so that the cross section of the core approaches a circular shape. A bobbin consisting of two half-cylinders is fitted around each of

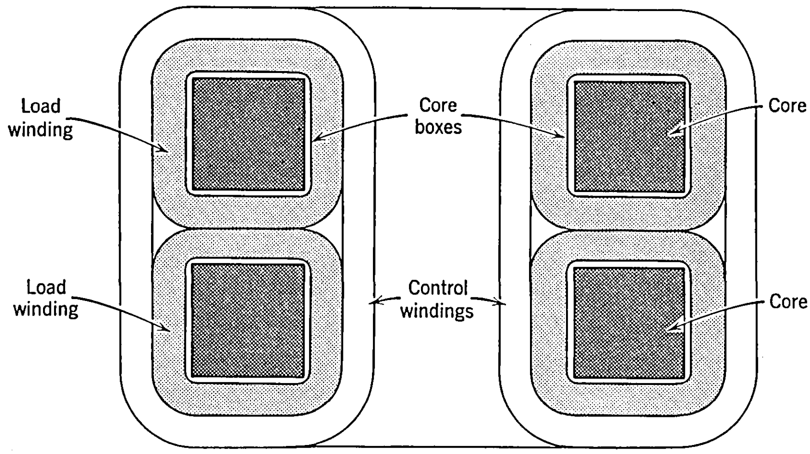

FIG. 8. Cross section of a pair of toroidal reactors.

the longer legs and rotated, while the wire is placed on it. This method appears to be advantageous in the medium power range.

Cut Cores (C Cores). This core construction is most suitable for oriented silicon-iron. It is more economical than the toroidal construction because the coils can be premanufactured on high-speed multiple coil winders. The core is made from iron strip; it is wound on a rectangular mandrel, bonded and cut to obtain two C-shaped sections. The cut surfaces are lapped, and the two sections are placed through the coils and are banded together. Cut cores show a noticeable airgap effect when compared with toroids.

Stacked Cores. Ordinary U-I or E-I laminations may be used for magnetic amplifier cores. However, because of the airgap effect at the butt joints and the local saturation in the neighboring laminations, high performance cannot be expected. Special laminations have been developed for small and medium power reactors resulting in a performance approaching that of toroids. Figure 9 shows such U laminations, having a yoke width twice the leg width. By stacking the punchings alternately

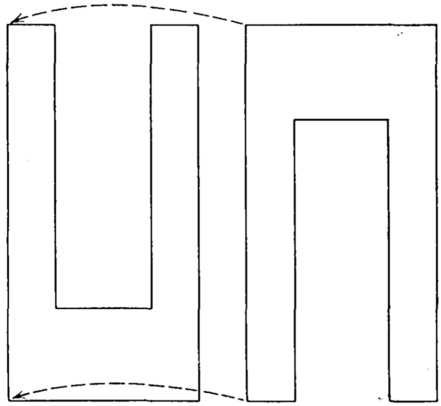

FIG. 9. U-laminations with yoke width twice the leg width.

from one or the other side of the coil, enough overlap is produced to avoid noticeable airgap effects and local saturation. These laminations require the use of nonoriented or doubly oriented material such as 50% nickel-iron. It is important to protect the core from mechanical stresses.

Singly oriented material, like silicon-iron, again calls for different techniques. A method found to be very effective is as follows. The overlapping U laminations, pictured in Fig. 9, are formed by joining three correctly oriented strips along diagonal cuts through the corners of the U. By alternately stacking the three-piece U's, airgap and local saturation effects are avoided.

Rectifiers

Solid-state rectifiers represent the nonlinear electric element, essential in most magnetic amplifier circuits. Besides good electrical properties (high voltage and current ratings, low leakage current, and forward voltage drop), ruggedness, sufficient temporary overload capacity, and long life are important features, since these are major attributes of magnetic amplifiers.

Selenium rectifiers are manufactured in a large variety of sizes from miniature rectifiers for printed circuits, carrying a few milliamperes up into the 100-amp range. Smaller cells for magnetic amplifier applications are usually rated with convection cooling at 200–300 ma/in.^{2} average forward current in half-wave operation and 25–60 volts/cell peak reverse voltage. Cells can be operated in series and parallel in order to obtain suitable overall voltage and current ratings. A typical leakage current for a 60-cycle rectifier with 45-volts/cell peak voltage is less than 6 ma/in.^{2} average. If the rectifiers are used as self-saturating rectifiers, it is advisable to use lower peak reverse voltages (20–30 volts/cell) in order to reduce the leakage current. The forward voltage drop is approximately 1 volt/cell at rated current, increasing as the rectifier ages. Selenium rectifiers are usable at temperatures from $-55°C$ to $+80°C$. The relatively high capacitance of the cells ($0.01\ \mu f/in.^2$) restricts their application to frequencies below 1000 cycles.

Germanium rectifiers are produced as point contact diodes for current ratings below 1 amp and as diffused junction rectifiers for 150 amp and more, with the latter being cooled by forced air or water. The peak inverse voltage ratings of the smaller units are as high as 350 volts, but are normally below 150 volts, especially with the larger cells. Germanium rectifiers have very low forward drop (less than 1 volt/cell) and lower leakage current than selenium rectifiers. The maximum ambient temperature of germanium rectifiers is about $+60°C$. Germanium point contact diodes have excellent high-frequency characteristics which make them applicable up to about 10 Mc.

Silicon junction rectifiers are the most recent development. They are outstanding for their high operating voltages (up to 800 volts/cell), their high current ratings (up to several hundred amperes per cell), and their small size. Silicon rectifiers show extremely low leakage and low forward voltage drop (1–1.5 volts); they can be operated at ambient temperatures between $-60°$ and $+105°C$. Hole storage in the rectifier junction limits the use of silicon rectifiers to the audio-frequency range. In Fig. 10 the forward and reverse d-c characteristics of a commercial ¼-in.-diameter silicon rectifier cell and a 7-cell 5-in. by 5-in. selenium rectifier stack are compared. The curves show clearly the advantages to be gained by the use of silicon rectifiers.

4. MAGNETIC AMPLIFIER DESIGN

Choice of Core Material and Core Construction

The first steps in magnetic amplifier design are the choosing of core construction and core material. As discussed previously, materials are important in terms of characteristics such as differential permeability, squareness ratio, and saturation flux density. A high differential permeability is necessary to obtain a high voltage gain and a low time constant. A high squareness ratio is desirable to obtain complete saturation and maximum output with minimum internal voltage drop in the amplifier. The linear operating region is also increased. A high squareness ratio is necessary in cascading Ramey circuits if full output is to be maintained.

High saturation flux density together with a high squareness ratio is important in high-power stages if power capacity per unit weight is an important consideration.

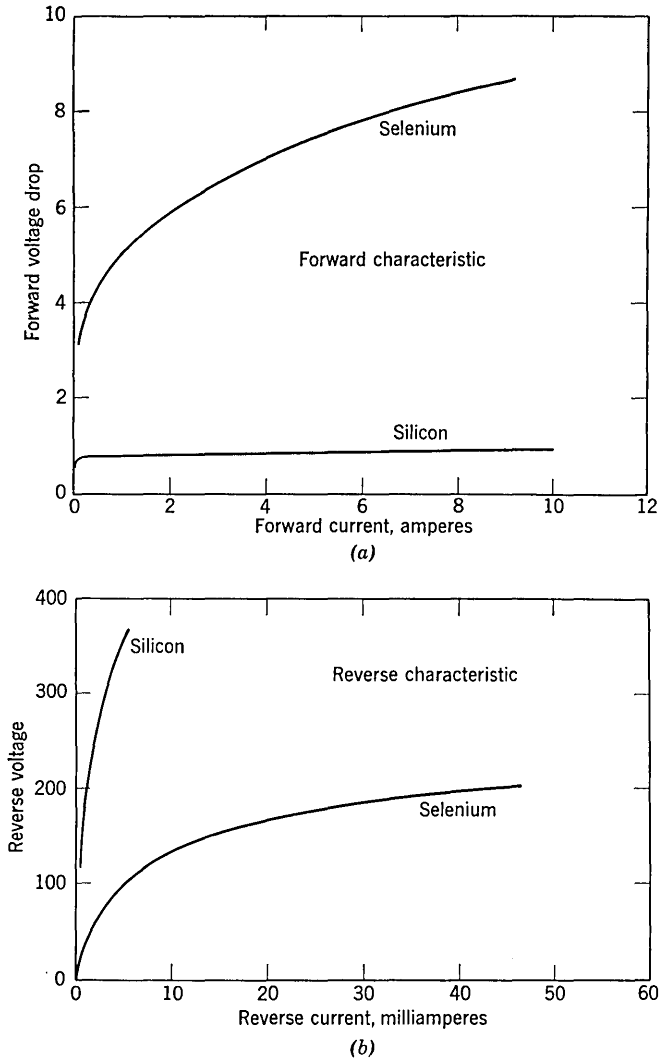

FIG. 10. Comparison of forward and reverse characteristics of a commercial ¼-in. diameter silicon rectifier cell and a 7-cell 5-in. by 5-in. selenium rectifier stack.

A high value of differential permeability is necessary for low-level sensitive stages.

When high performance multistage magnetic amplifiers are desired, it is good practice to use materials of classes V or VI (Table 1) in the low-level stages and materials of class III (Table 1) for the output or power stage.

Load Circuit Design and Core Size Selection

A magnetic amplifier can be designed only when the functions it is to perform are specified. These specifications must include output voltage and power requirements, maximum allowable time constant, and allowable loading on the source or input device, i.e., the input power.

In practice, the design begins with the output stage. In addition to the output voltage and power requirements, a minimum efficiency and maximum weight are sometimes specified. Even though maximum weight or volume is not specified, it is good practice, because of cost considerations, to use the minimum size core that will meet the output voltage and power requirements and any other specifications that are stated.

The core material and core construction are chosen on the basis of cost and performance in accordance with the discussion in the previous section. The problem of load circuit design thus becomes one of choosing the core size and the number of gate winding turns. In the design, adequate space must be left for the control and bias windings.

Design Procedure. This problem can be solved in many ways, all of them based upon empirical data and cut-and-try procedures. The following procedure for designing full-wave single-ended amplifiers has proved to be successful and can serve as a guide to those relatively unskilled in the art. Many experienced designers have developed alternate methods based upon experience which include extensive charts, tables, and nomograms. Frequently, however, these graphical aids are based directly on winding techniques and manufacturing methods peculiar to a particular organization.

The design proceeds from the basic formula for a sinusoidal supply voltage:

$$E_{ac} = 4.44 B_s N_g A_c f \times 10^{-8}, \tag{1}$$

where E_{ac} = supply voltage in volts rms,
B_s = saturation flux density in gauss,
N_g = number of gate winding turns,
A_c = cross-sectional area of the core in square centimeters,
f = supply frequency in cycles per second.

The first step is to solve eq. (1) for $N_g A_c$.

$$N_g A_c = \frac{E_{ac} \times 10^8}{4.44 B_s f}. \tag{2}$$

N_g and A_c are the unknown quantities since it is assumed that E_{ac} and f are selected from power supply considerations. It is also assumed that the core material has been chosen; thus B_s is fixed. To eliminate one of these unknowns it is desirable to relate N_g to the core dimensions which can be done by recognizing that the number of turns N_g that can be wound on a core is a function of the portion of window area occupied by the winding, the wire size, and the spacing factor which varies somewhat, depending upon how tightly the wire is wound on the core. Thus,

$$N_g = s_g \frac{A_g}{a_g}, \tag{3}$$

where A_g = area occupied by the winding in circular mils,
a_g = cross-sectional area of bare wire in circular mils,
s_g = spacing factor.

The spacing factor under ideal conditions for square wire with no insulation would be unity, i.e., one turn of wire one circular mil in cross section would occupy one circular mil of winding area. In practice, however, on account of the use of round wire, wire insulation, layer insulation, and winding practices, this factor is less than 0.6. For toroidal cores, practice has shown that a good average value of s_g would lie between 0.4 and 0.6. Therefore,

$$N_g \approx 0.5 \frac{A_g}{a_g}.$$

Substituting into eq. (2) gives

$$A_c = \frac{E_{ac} \times 10^8 a_g}{2.22 B_s f A_g}. \tag{4}$$

By multiplying both sides of eq. (4) by A_w, the total window area in circular mils, an expression related to core dimensions is obtained:

$$A_w A_c = \frac{E_{ac} \times 10^8}{2.22 B_s f} \left(\frac{A_w}{A_g}\right) a_g. \tag{5}$$

$A_w A_c$ is the product of window area in circular mils and cross-sectional area of the core in square centimeters. This product, in circular mils-square centimeters, is frequently given in catalog data of core manufacturers.

The unknown factors on the right-hand side of this equation are:

A_w/A_g = ratio of window area of the core to area occupied by the winding turns N_g,
a_g = cross-sectional area of the bare wire in circular mils.

The cross-sectional area of the wire is determined by the allowable current density, which in turn is limited by the allowable temperature rise of the packaged reactor. For power stages it is good practice to allow 1000 circular mils per ampere, although this may vary from 500 to 1500, depending upon packaging and allowable temperature rise.

A_w/A_g is a quantity which the designer is free to choose within limits. The ratio can never equal unity because some of the window area will be occupied by control and bias windings and insulation between windings. Most important for toroidal cores is the fact that a shuttle is used for winding, and space must be left for the shuttle. The practicing designer soon learns approximate values for this ratio, and the experienced designer has a wealth of charts, tables, and nomograms which give him this information.

For toroidal core power stages A_w/A_g has a value between 1.2 and 1.5, depending upon the number of control and bias windings. For low-power stages where a greater percentage of the window area is taken up by control windings, the ratio is between 2.0 and 3.0.

Further consideration of eq. (5) can yield some useful information concerning the power-handling capacity of the core directly.

If a value, say 10^{-3} amp per circular mil, is assigned for current density of the gate winding, eq. (5) can be manipulated to give a relation between A_w, A_c, and the product of E_{ac} and gate current.

$$A_w A_c = \frac{10^{11} \times E_{ac} I_g}{2.22 B_s f} \left(\frac{A_w}{A_g}\right). \tag{6}$$

The quantity $A_w A_c$ can be calculated, then, directly from the considerations of power required by the load. When $A_w A_c$ is known, the designer can select, from a core manufacturer's table, a core size which has a value of $A_w A_c$ equal to or greater than the value obtained from eq. (6). Experience will play some part in this selection because of the uncertainty regarding A_w/A_g, a_g/I_g, and s_g. Once the core size has been chosen the value for A_c is fixed, and N_g can be calculated from eq. (2).

$$N_g = \frac{E_{ac} \times 10^8}{4.44 B_s A_c f}. \tag{7}$$

The resistance R_g of the winding should be calculated next. $I_g{}^2 R_g$ represents the major power loss in the reactor at power frequencies and thus determines the temperature rise and efficiency of the output stage. Also, $I_g R_g$ is a voltage drop which must be taken into account along with the saturated reactive drop in the core in order to determine the voltage at the load. Usually, the saturated reactive drop is much less than R_g; therefore at power frequencies,

$$E_L \approx E_{ac} - I_g R_g. \tag{8}$$

If R_g is too large, the current density must be decreased and $A_w A_c$ recalculated. If R_g is very small, it is highly likely that too large a core has been chosen.

R_g can be calculated if the average length of turn of the winding is known.

The average length of turn, l_g, depends upon the core box dimensions, the number of turns in the winding, and the tension applied to the wire during winding. Due to varying winding tension, l_g will vary as much as $\pm 10\%$ among wound cores in mass production where the number of turns and core box dimensions remain constant.

The minimum length of turn is the perimeter of the cross section of the core box. The maximum is the perimeter of the cross-sectional area of the complete winding. Thus

$$l_{\min} = (\text{O.D.} - \text{I.D.}) + 2h,$$

where O.D. = outside diameter of the core box,
I.D. = inside diameter of the core box,
h = height of the core box.

From experience it is possible to assign an average value, l_g. This average value is between 1.5 to 2.0 times the minimum length of turn, depending upon the number of turns.

$$\therefore\ 1.5(\text{O.D.} - \text{I.D.}) + 3h \leq l_g \leq 2(\text{O.D.} - \text{I.D.}) + 4h.$$

If N_g and l_g are known, the resistance R_g can be calculated by the following formula:

$$R_g = N_g \times l_g \times \frac{\rho}{a_g}, \tag{9}$$

where ρ = resistivity of the wire (= 0.864 ohm circular mil per inch at 20°C for copper wire),
a_g = cross-sectional area of the wire in circular mils.

To summarize, it is necessary to emphasize that gate winding design and core size selection are dependent upon many empirical factors. The designer learns, by experience, approximate values for these empirical factors.

Control Circuit Design

The load circuit design and core size selection have been carried out without consideration for time constants and the input power required to control the core. The basic premise has been that the power stage must be designed to meet the voltage and power requirements of the load. If the core size selected to meet these requirements does not satisfy input power and time constant requirements, a preamplifying stage or stages must be added.

With half-wave circuitry the problem of juggling power gain and time constants is simplified because of the fixed response time. For full-wave stages, the product of time constant and input power is directly related to the core volume and core material, as shown in the following equations.

From the equations

$$\frac{N_c \Delta I_c}{1} = \frac{\Delta H l}{0.4\pi} = \frac{\Delta B A_c 2f \times 10^{-8}}{K_{AT}}, \tag{10}$$

$$\tau = \frac{1}{2f}\frac{N_c}{N_g}K_v = \frac{N_c^2}{R_c}\frac{K_{AT}}{2f} = \frac{N_c^2}{R_c}\left(\frac{\Delta B A_c}{N_c \Delta I_c}\right) \times 10^{-8}, \tag{11}$$

$$\Delta\phi = \Delta B A_c, \tag{12}$$

where ΔH is in oersteds,
l is in centimeters,
$\Delta\phi$ is in lines,
ΔB is in gauss,
K_{AT} is in volts per turn per ampere-turn,
K_v is the voltage gain,
N_c is the number of control winding turns,
I_c is the control current,
R_c is the total control circuit resistance,

the expression for the product of input power and time constant can be calculated.

$$P_i\tau = [(\Delta I_c)^2 R_c] \times \left[\frac{N_c^2}{R_c}\left(\frac{\Delta B A_c}{N_c \Delta I_c}\right)\right] \times 10^{-8},$$

$$\therefore P_i\tau = (N_c \Delta I_c)(\Delta B A_c) \times 10^{-8}, \tag{13}$$

$$\therefore P_i\tau = \frac{(\Delta B)(\Delta H)}{0.4\pi} \times A_c \times l \times 10^{-8}. \tag{14}$$

ΔB and ΔH are functions of the chosen core material while $A_c l$ is the volume of the chosen core. To minimize drift, the amplifier should be worked over its full range. That is $\Delta B = 2B_s$, and ΔH should represent the coercive force required to reset the flux over the linear control range, as represented in the idealized core characteristics, Fig. 4*b*.

In designing the control circuit for a full-wave amplifier, it is necessary to determine whether the input power and time constant requirements can be met by the selected core size. This check can be accomplished by using eq. (13), if the ampere-turns required for full control are known.

The ampere-turns required for control can be calculated either from eq. (10), by using appropriate values for ΔH, l, or K_{AT}, or from experimental core test data which are usually tabulated directly in ampere-turns for different core materials and core sizes.

Once the ampere-turns $N_c \Delta I_c$ are known, the quantity N_c^2/R_c is fixed by eq. (11). If the maximum value of ΔI_c is limited by the current characteristics of the source, N_c is determined from eq. (10). If the maximum value of ΔI_c can be varied, then N_c and R_c can be varied within the limits imposed by eqs. (10) and (11).

The wire size of the control winding has a lower bound imposed by the allowable current density and an upper bound imposed by the allotted winding space. The current density of the control winding can be varied over a greater range than the gate winding. A common value for control winding current density is 2×10^{-3} ampere per circular mil.

If the input power-time constant requirements cannot be met by the core size selected on the basis of power-handling capability, a preamplifying stage will be needed. This design can proceed in the following manner.

Calculate the input power required to obtain a 1- or 2-cycle delay from the output stage. Design the preamplifying stage load circuit and select a core size to meet this power requirement. Design the control windings on this stage to determine whether the allowable time delay is met. (The overall time delay of two stages is roughly equal to the sum of the time constants of the individual stages.) If the requirements cannot be met another preamplifying stage will be needed.

Feedback networks around stages can be used to alter the dynamic response of full-wave magnetic amplifiers and alleviate somewhat the need for more than two stages. However, a discussion of this subject is outside the scope of this chapter. (See Ref. 12.)

Sample Design Problem

Given:

Supply voltage frequency	$= 60$ cps
Desired load voltage	$= 100$ volts (rms)
Load resistance R_1	$= 100$
Desired efficiency	$\geqq 90\%$
Time constant	$\leqq 0.1$ sec
Input power—P_i for full control	$\leqq 5 \times 10^{-2}$ watt
Max. control current	$= 10^{-2}$ amp

From the foregoing, the following quantities are calculated:

Load power	$= 100$ watts
Supply power	$= 110$ watts
Supply voltage	$= 110$ volts
R_g	$\leqq 10$ ohms
$P_{i}T$	$\leqq 5 \times 10^{-3}$ watt-sec

Load Circuit Design. As a starting point, it is assumed that the material to be used is 2-mil strip Hypernik V in a toroidal core. From eq. (6) if A_w/A_g is assumed to be 1.5, and current density is taken to be 10^{-3}

amp/circular mil,

$$A_w A_c = \frac{10^{11} \times 110 \times (1.5)}{2.22 \times 14{,}000 \times 60}$$
$$= 8.9 \times 10^6 \text{ circular mil-cm}^2.$$

From core tables this indicates a $2 \times 3 \times 1$ core, where $A_w = 3.6 \times 10^6$ circular mils and $A_c = 2.742 \text{ cm}^2$.

$$N_g = \frac{110 \times 10^8}{4.44 \times 1.4 \times 0.6 \times 10^6 \times 2.742} = \frac{110 \times 10^8}{10 \times 10^6} = 1100T,$$

$$R_g = 1100 \times 1.5(1 + 2) \times \frac{0.865}{1000} = 1.1 \times 4.5 \times 0.865 = 4.3\ \Omega.$$

The wire size from tables is #20 AWG.

Control Circuit Design. The first step is to determine the ampere-turns required for full control. This can be calculated as follows.

From core size tables, the mean length of magnetic path is 19.67 cm. A typical value of ΔH for Hypernik V is 0.2 oersted.

$$\therefore N_c \Delta I_c = \frac{\Delta H l}{0.4\pi} = 3.1 \text{ ampere-turns.}$$

Next the product of input power and time constant can be calculated:

$$P_i\tau = (N_c \Delta I_c) \Delta B A_c$$
$$= 3.1 \times 1.4 \times 2.742 \times 10^{-4} \text{ watt-sec.}$$
$$P_i\tau = 1.2 \times 10^{-3} \text{ watt-sec.}$$
$$(P_i\tau) \text{ available} = 5 \times 10^{-3} \text{ watt-sec.}$$

Thus, it can be seen that the input power and time constant requirements can be met by the selected core size.

The next step is to determine N_c.

$$N_c = \frac{3.1}{10^{-2}} = 310 \text{ turns.}$$

The wire size can be calculated by using a current density of 2×10^{-3} amp/circular mil.

$$\frac{I_c}{A_g} \leqq 2 \times 10^{-3},$$

$$a_g \geqq 5 \text{ circular mils.}$$

The space required for this winding can be calculated from eq. (3).

$$A_2 = \text{area occupied by control winding,}$$
$$= \frac{N_c}{s_c} a_g = 3100 \text{ circular mils.}$$

The total window area of the core is 3.6×10^6 circular mils; thus, it can be seen that the control winding occupies negligible space compared to the load winding.

5. COMMONLY USED CIRCUITS

Full-Wave, Single-Ended

Three basic self-saturating single phase magnetic amplifiers are shown in Fig. 11. They are classified as full-wave because power is delivered

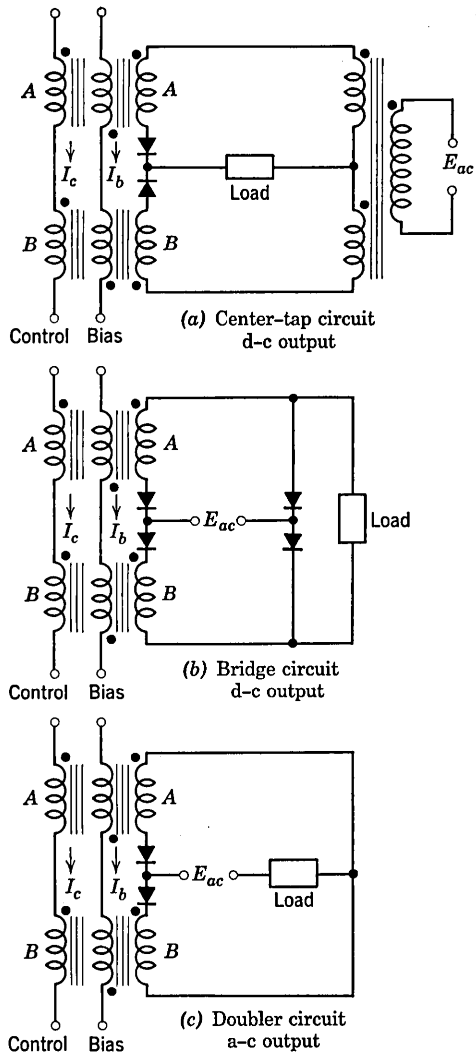

FIG. 11. Basic full-wave, self-saturating single phase magnetic amplifiers.

to the load on each half-cycle when the reactors are saturated. They are classified as single-ended because in the d-c circuit the load voltage does not reverse polarity, and in the a-c circuit the load voltage does not reverse phase. The same reactors may be used in any of these circuits, although the rectifier requirements are different for each circuit. The center-tap circuit, Fig. 11*a*, requires a transformer, and unless a transformer is needed to step the voltage up or down, this circuit has little or no advantage over the bridge circuit shown in Fig. 11*b*. The center-tap circuit requires only two rectifiers, but the required voltage rating of each is twice the rating of each rectifier in the bridge circuit. Figure 11*c* shows the widely used doubler circuit which gives full-wave, a-c output.

Full-Wave, Push-Pull

Push-pull circuitry is of two types. Polarity reversal circuitry is used when a reversible polarity d-c load voltage is required. A typical load for this type of circuit is the field winding of a machine. Phase reversal circuitry is used when phase reversal of the a-c load voltage, with respect to a reference voltage, is required. When the load is a two-phase servo motor, for example, 180° phase reversal of the control winding voltage with respect to the reference winding voltage causes reversal of rotation.

Full-wave push-pull circuitry requires a minimum of four saturable reactors and four rectifiers. Full output voltage of the polarity reversible circuitry is a full-wave rectified d-c voltage. Full output voltage of the phase reversal circuitry is an a-c voltage of the same form as the supply voltage.

Polarity Reversal Load Circuitry. A widely used circuit to obtain polarity reversal is that employing mixing resistors, as shown in Fig. 12. Although this circuit uses two doubler circuits with full-wave rectifiers, similar circuits can be obtained using bridge or center-tapped circuits.

The mixing resistor push-pull circuit has low efficiency; thus, it is usually limited to applications where the output power requirement is below 1 kw.

A higher efficiency push-pull circuit is shown in the book by Storm (Ref. 1, p. 440).

Phase Reversal Circuitry. Three common circuits used to obtain phase reversal are shown in the following figures. The circuit shown in Fig. 13 uses a transformer to provide two separate a-c sources. The circuit shown in Fig. 14 uses a transformer to couple the amplifier output to the load. The circuit shown in Fig. 15 uses two load windings on each of four cores, *A*, *B*, *C*, and *D*.

The resistor R_1 is usually required to limit circulating currents after a pair of cores, say *A* and *D*, or *B* and *C*, saturate in one half-cycle.

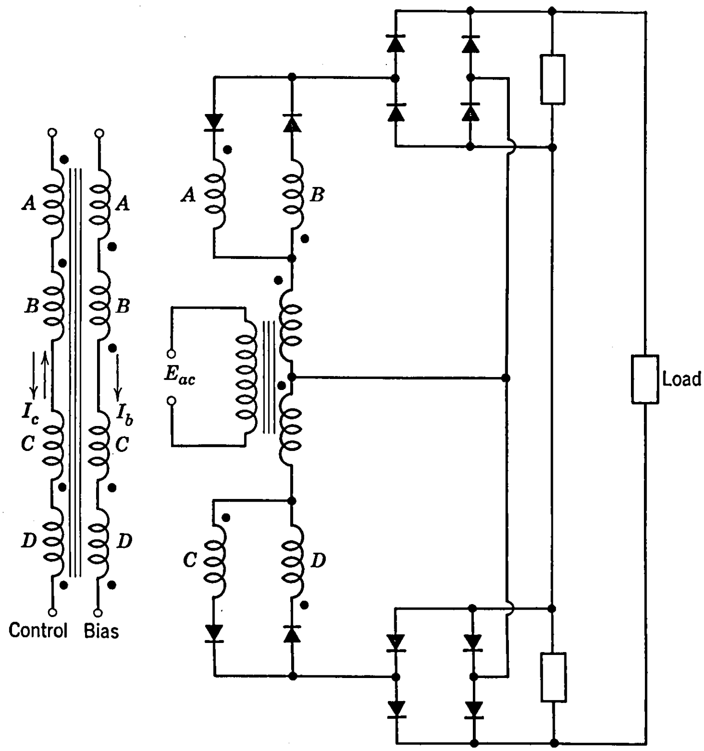

FIG. 12. Full-wave push-pull circuit, polarity reversal load type.

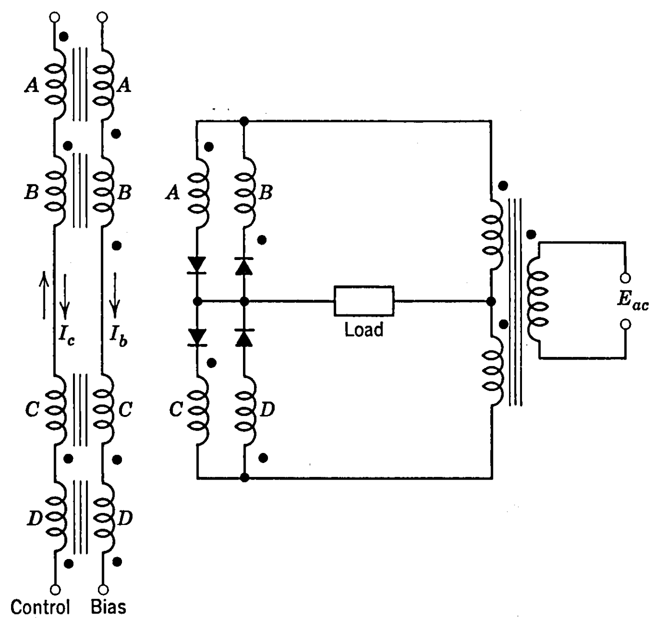

FIG. 13. Full-wave push-pull circuit, phase reversal type using a transformer to provide two separate a-c sources.

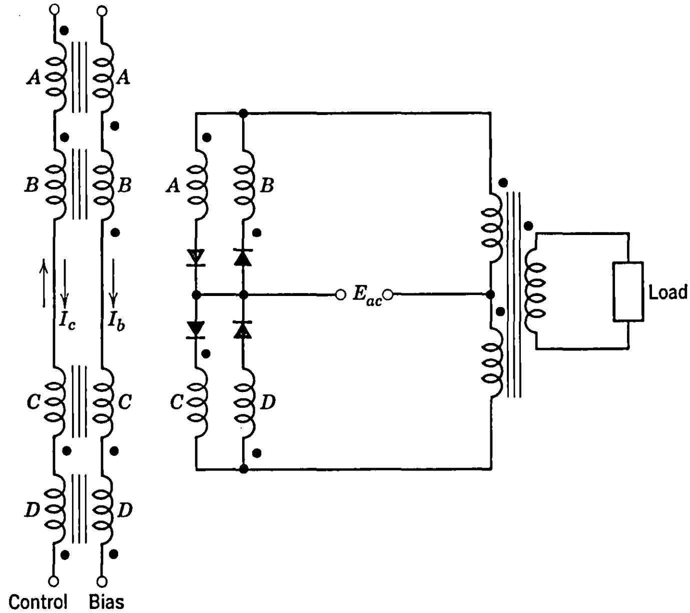

FIG. 14. Full-wave push-pull circuit, phase reversal type using a transformer to couple the amplifier output to the load.

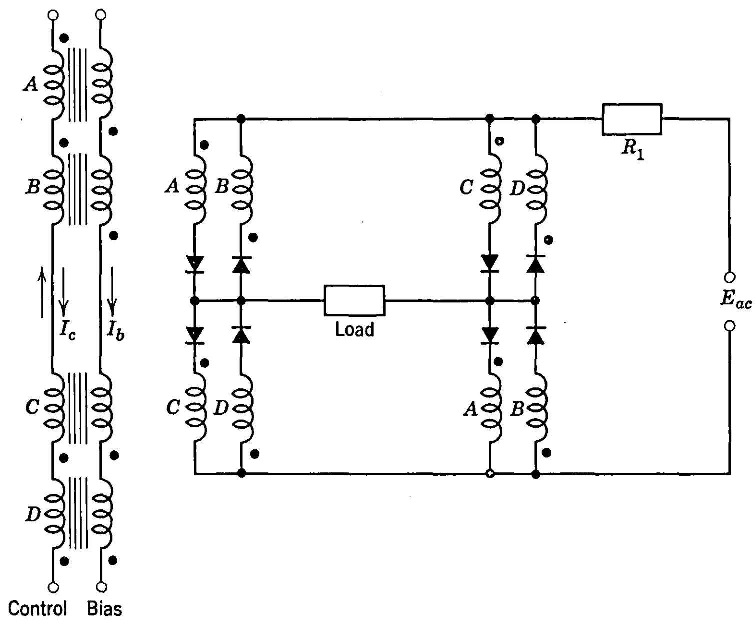

FIG. 15. Full-wave push-pull circuit using two load windings on each of four cores.

Bias Considerations. The push-pull circuitry is such that the amplifiers may be biased at any point, although circulating currents usually make it necessary to bias for half output or less with zero signal.

The approximate composite gain curve is obtained by graphically adding the individual curves as shown in Fig. 16. For this set of curves the

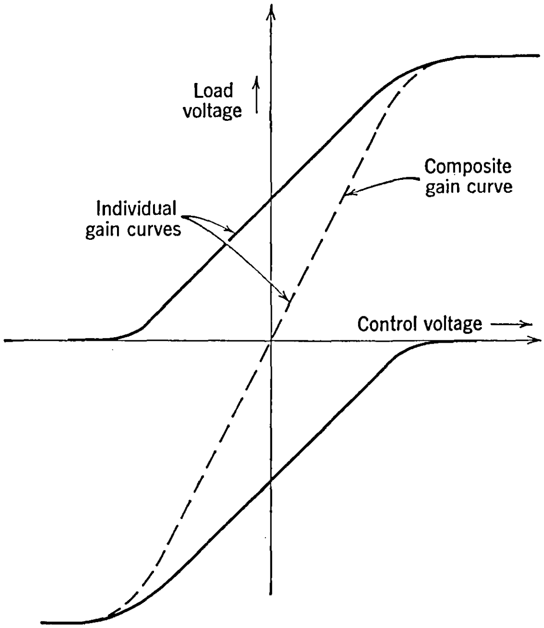

FIG. 16. Composite gain curve for full-wave push-pull circuits.

bias on each pair of cores is such that the firing angles are 90°. The composite gain is twice that of each individual stage.

Half-Wave, Push-Pull

Two common types of half-wave push-pull circuits are the NOL and Geyer bridge circuits.

NOL Bridge Circuit. The NOL bridge circuit was developed by the Naval Ordnance Laboratory as a fast-response magnetic amplifier for use in a-c servo systems where the output device is a two-phase servo motor. The circuit in its simplest form is one-half of the push-pull doubler circuit, as shown in Fig. 17.

The cores A and B have the same gating half-cycle. The output waveform, for full control signal applied to the cores, is a half-wave d-c output which reverses polarity when the control signal reverses. Typical wave-

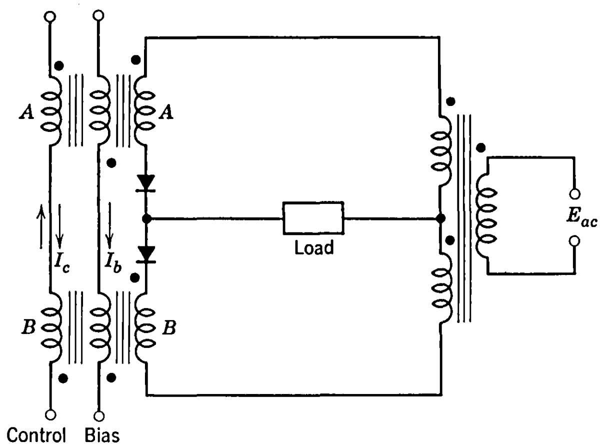

FIG. 17. Half-wave push-pull magnetic amplifier circuit. Simplified NOL bridge type.

forms for a resistive load are shown in Fig. 18. The cores are assumed to be biased for a 90° firing angle with zero signal.

These waveforms contain a fundamental component which is proportional to the control signal, although higher harmonics are also present. When the load is the control winding of a two-phase servo motor, the torque will be proportional to the fundamental component. Usually in

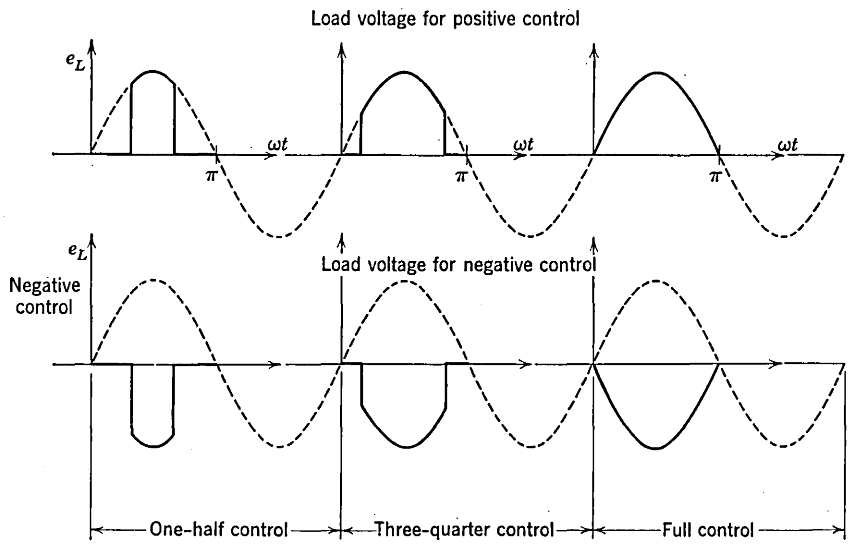

FIG. 18. Typical waveforms for a resistive load. Magnetic amplifier of Fig. 17.

this application a tuning capacitor is used in parallel with the motor windings.

The circuit, as shown, will not work very well because the supply transformer must carry a direct current. The practical circuit uses split magnetic amplifier gate windings as shown in Fig. 19.

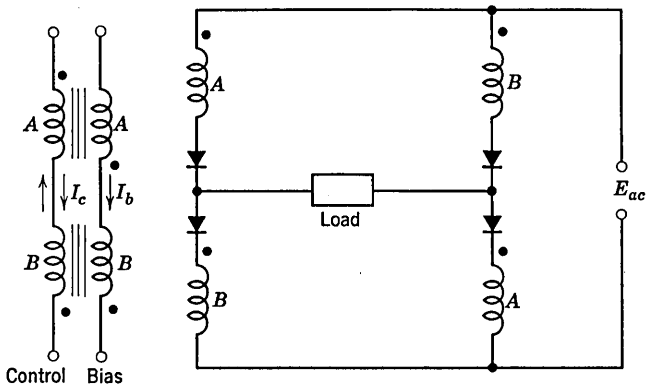

FIG. 19. Half-wave push-pull magnetic amplifier circuit. Practical NOL bridge type.

The response time of this circuit, as in the case of the basic half-wave circuit, is one-half cycle for resistive loads. If the load is highly inductive, the load current does not decay to zero instantaneously. If the delay time due to the inductive load is great enough, the load current will continue to flow into the next resetting half-cycle and, since this will affect resetting, it results in an increase in the response time of the amplifier. For this reason, it is advisable to use a tuning capacitor in parallel with the motor.

A severe limitation on biasing the cores exists because of circulating currents; therefore, a current limiting resistor is frequently used in the load circuit.

Geyer Bridge Circuit. A second magnetic amplifier circuit which has useful characteristics in controlling a-c servo motors is a half-wave bridge circuit as shown in Fig. 20.

This circuit differs from the NOL bridge in several major points. The first major difference is that the voltage across the load does not change d-c polarity, but the fundamental component changes phase by 180° as the signal polarity changes. The second major difference is that cores *A* and *B* gate on alternate half-cycles of the supply voltage.

This circuit is sometimes used with no bias, which is feasible since quiescent currents flow through the load. For this condition the wave-forms are as shown in Fig. 21.

For no bias, the current has a variable damping effect on a two-phase

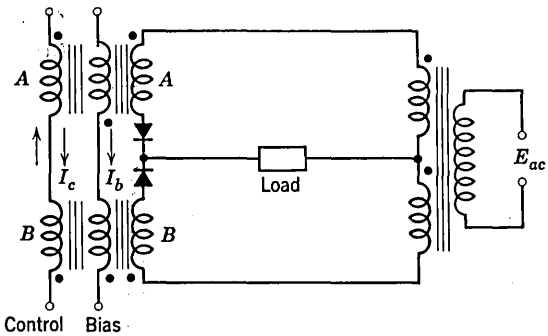

FIG. 20. Half-wave push-pull magnetic amplifier. Geyer bridge circuit.

servo motor because the d-c current in the winding (and the damping) decreases with an increase in the torque-producing fundamental component.

For a quiescent firing angle less than 90° the amplifier again has a variable damping effect, but the direct current and the damping increases with an increase in the torque-producing fundamental component.

The decreased damping with increasing torque is usually desirable in systems because of the effects on transients. However, for this condition the motor is carrying large direct currents at null. In a position servo the motor must be derated to allow for the increased losses.

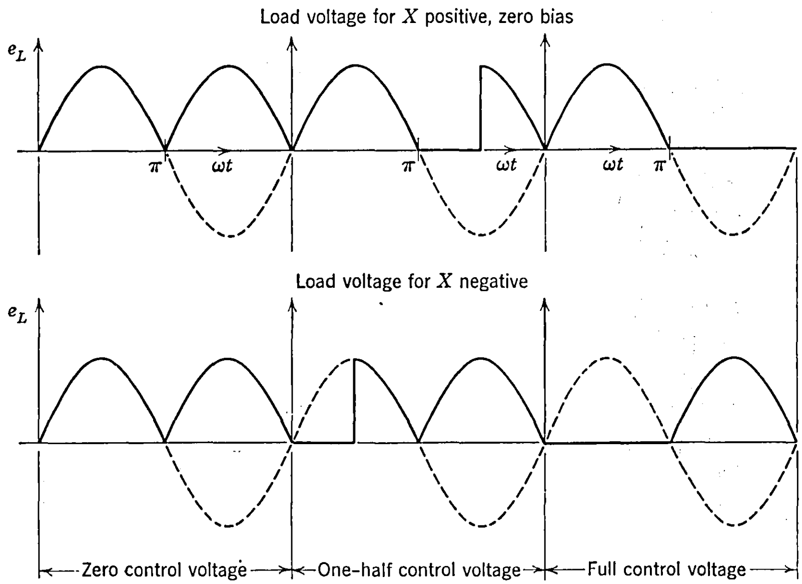

FIG. 21. Waveforms for magnetic amplifier, Geyer bridge circuit of Fig. 20.

Relation to Full-Wave Circuitry. The half-wave push-pull circuitry has inherently lower power gain than full-wave circuitry because of the half-wave outputs and absence of internal positive feedback. However, the circuits are very useful where an absolute minimum response time is required, and where moderate power gains per stage will meet system requirements.

Three-Phase Circuitry

Three-phase circuits are used when maximum power output for minimum weight is desired. These circuits find wide use in aircraft voltage regulating systems. Three-phase circuitry is also desirable for obtaining low ripple with d-c output and balanced load conditions in high-power applications.

Several typical circuits are shown in Figs. 22 and 23. Figure 22 shows a three-phase half-wave circuit which requires only three reactors. Fig. 23 shows a three-phase full-wave bridge circuit.

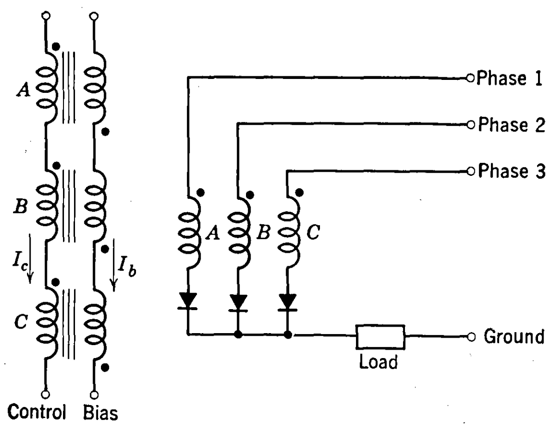

FIG. 22. Three-phase half-wave magnetic amplifier circuit.

Ramey Logic Circuitry

The basic Ramey circuit, shown in Fig. 5*a*, is useful as a logical circuit element. All functions normally provided by relays can be duplicated at much greater speed and with greatly increased reliability by Ramey logic circuitry. The logic variables are represented by half-cycle voltages. To supplement the brief explanation of operation of the basic circuit given for Fig. 5*a*, consider the *and* circuit, Fig. 24. If variables E_A and E_B do not exist during a resetting half-cycle, the voltage E'_{ac} will reset the core through R_1, R_2, and E_x, and on the next gating half-cycle no output will occur. The voltage E_x and the resistor R_1 are of such magnitudes that in

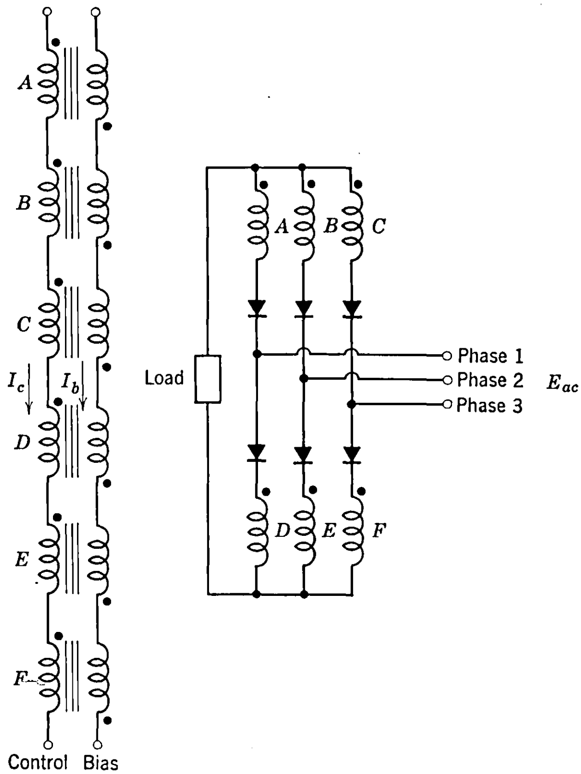

FIG. 23. Three-phase full-wave magnetic amplifier bridge circuit.

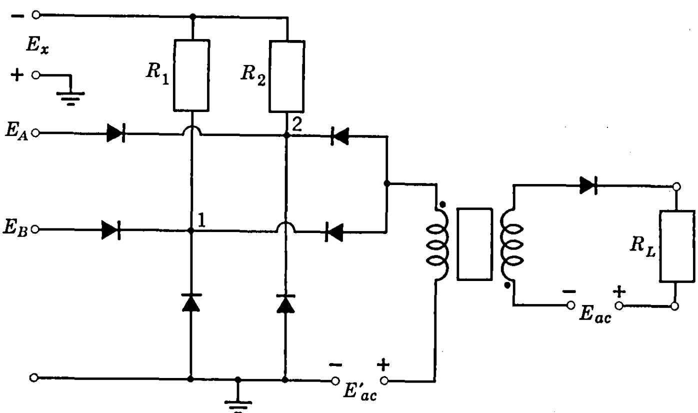

FIG. 24. Ramey type *and* circuit.

the absence of signal E_A point 1 is slightly negative with respect to ground. Similarly, R_2 is of the proper magnitude such that point 2 is also slightly negative with respect to ground in the absence of E_B.

The magnitudes of E_A and E_B are such that points 1 and 2 become positive at voltages greater than E'_{ac} when E_A and E_B occur. Thus, E'_{ac} cannot reset the core, and full output occurs on the next gating half-cycle. If only E_4 or E_B exists during the resetting half-cycle, the voltage E'_{ac} can reset the core through the path provided by R_2 or R_1.

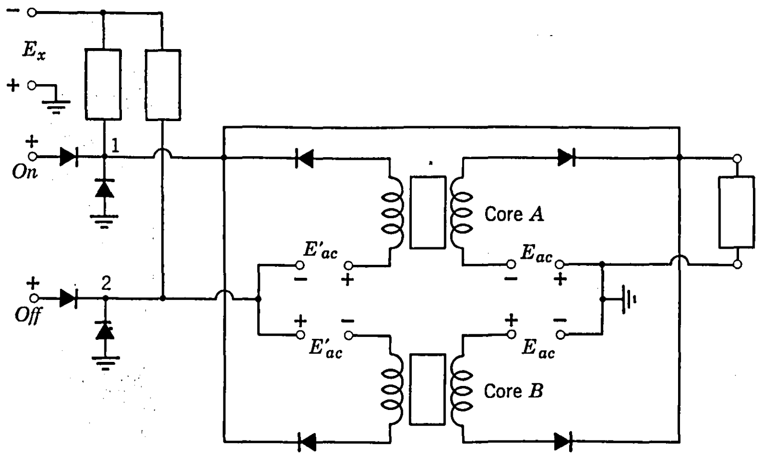

FIG. 25. Ramey type flip-flop circuit.

The flip-flop circuit, Fig. 25, is a combination of two basic Ramey amplifiers operating on alternating gating half-cycles. That is, the gating half-cycle of one amplifier is the resetting half-cycle of the other. The control circuits are arranged so that when point 1 due to the *on* input becomes positive with respect to ground, core A cannot be reset and has an output during its next gating half-cycle which causes point 1 to remain positive with respect to ground. Thus, core B cannot be reset, and the flip-flop remains on. When an *off* input occurs, point 2 becomes positive with respect to ground and this voltage plus E'_{ac} is sufficient to reset the core which has a resetting half-cycle coinciding with the *off* half-cycle.

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Semiconductor Devices

John S. Saby

1. INTRODUCTION

Objectives. Semiconductor devices have many advantageous features such as low power dissipation, size, reliability, and ruggedness that make them useful for computation and control systems. Successful systems applications require general understanding of the basic properties of semiconductor devices and their skillful use in applications for which these properties are needed. The objectives of this chapter are:

1. To provide sufficient information on the principles of operation and the characteristics of typical semiconductor devices to enable the electronic system design engineer to recognize the applications which need semiconductor devices to achieve the required design objectives.

2. To aid in determining the very important differences between those semiconductor circuit characteristics which on one hand arise from immutable properties "built into" the semiconductors themselves, and those,

on the other hand, which can be controlled or altered by choosing the proper component or by further development of the optimum circuit. For general information on circuits, see Chap. 27, Transistor Circuits.

Definitions. The accepted definitions of semiconductor terms, adopted as Standards by the Institute of Radio Engineers (see Ref. 1) are listed for reference in Sect. 8, Terminology.

2. PRINCIPLES OF OPERATION OF SEMICONDUCTORS

Metals, Insulators, and Semiconductors. Metals are good conductors because of the presence of "free" electrons in the crystal lattice. Insulators on the other hand have virtually no "free" electrons and are normally poor conductors. At high temperatures or high voltages electrons are torn free from their lattice bonds and the insulator temporarily becomes a conductor. Many semiconductors have crystal structures like insulators, but electrons are loosely bound so that a noticeable amount of conduction occurs even at room temperature.

Basic Properties

The attainable range of circuit characteristics of semiconductor devices, and the variation of these characteristics with temperature, operating point, ambient illumination, etc., are all directly dependent upon certain basic properties of semiconductors (see Refs. 2–8). These factors determine the nature as well as the ultimate limits of performance and reliability of semiconductor devices. Table 1 lists the properties of semiconductors (Refs. 37, 86, and 87).

Current Carriers. In semiconductor devices, the current is ordinarily carried by a relatively few high-mobility charge carriers moving in the solid semiconductor material. Their density ranges from 10^{11} to 10^{20} per cubic centimeter, or roughly one carrier per trillion atoms, to one per thousand, respectively. This is in contrast to conduction in most metals, where there are very many carriers, say roughly one "free" electron per atom, but where the *mobility,* i.e., average terminal drift velocity per unit electric field, is much lower. Thus even though most metals have high electrical conductivity, their conductivity per electron is lower than that of the useful semiconductors. It is ordinarily impractical to attempt to control the conductivity of a metal by temporarily changing its electron population, whereas controlled conductivity changes of a million to one are sometimes possible in semiconductor devices.

Current Densities. In the useful semiconductors, there are mobile charge carriers of both positive and negative charge, and whenever they are free to do so, they distribute themselves so as to achieve approximate total charge neutrality in any general region. Hence, extremely high

TABLE 1. SEMICONDUCTOR PROPERTIES

	Units	Germanium		Silicon		Silicon Carbide (some values highly tentative)	
		Value	Temperature	Value	Temperature	Value	Temperature
Density	gm/cm^3	5.32	—	2.32	—	3.22	—
Melting point	°C	936		1420		2600 (subl. d.)	
Thermal linear expansion coefficient	per °C	6×10^{-6}	0–400°C	4×10^{-6}	25°C	5×10^{-6}	0–1000°C
Thermal conductivity	cal/sec cm °C	0.14	25°C	0.20	0.25°C	≈ 0.03	250–1000°C
Specific heat	cal/gm °C	0.74	0–100°C	0.181	20–100°C	0.15	0–100°C
Dielectric constant (relative), ϵ_r	—	16	—	12	—	7	—
Energy gap	ev	0.72	—	1.1	—	2.8	250°C
Impurity ionization energy	ev	0.01–0.03	—	0.05–0.08	—	0.1–0.3	—
Mobility of electrons, μ_n	cm^2/volt sec	3600	25°C	1400	25°C	~ 40	250°C
Mobility of holes, μ_p	cm^2/volt sec	1700	25°C	500	25°C	~ 8	250°C
Diffusion constant of electrons, D_n	cm^2/sec	90	25°C	35	25°C	~ 1.0	250°C
Diffusion constant of holes, D_p	cm^2/sec	45	25°C	13	25°C	~ 0.2	250°C
Intrinsic number, n_i	per cm^3	2×10^{13}	25°C	2×10^{10}	25°C	$\sim 10^{10}$ (extrapolated)	250°C
		$9.2 \times 10^{15}\, T^{3/2}$	$\exp \dfrac{-6500}{T}$	$5.1 \times 10^{15}\, T^{3/2}$	$\exp \dfrac{-4400}{T}$	$\sim 2 \times 10^{18-20}$	1000°C
Intrinsic resistivity, ρ_i	ohm cm	59	25°C	160,000	25°C	$\sim 10^{9-10}$ (extrapolated)	250°C
						~ 100	1000°C

densities of carriers can be employed, resulting in high conductivity and low device impedance, without resulting in space charge limitations at high currents, up to thousands of amperes per square centimeter in some cases. This is in contrast with vacuum tube operation where, even though the electron velocity is limited only by the speed of light, high currents are ordinarily space charge limited. Tubes thus require fairly high anode voltages to achieve high current by electron acceleration, since the absolute electron density achievable in most electron beams is much less than in semiconductor devices. Even in gas tubes, the attainable ion and electron densities are much smaller than the carrier densities characteristic of semiconductor devices.

Crystal Structure. In the useful semiconductors, there are just enough valence electrons to fill the interatomic bonds which hold the solid material together. For example, in germanium and silicon, the "diamond cubic" crystal lattice has four "shared pair" (covalent) interatomic bonds connecting each atom with its four nearest neighbors, each bond requiring one of the four valence electrons per atom, effectively tying up all valence electrons in interatomic bonds. This is shown in Fig. 1. Thus silicon and germanium would be insulators if it were not for the free charge carriers resulting from thermal vibration.

Intrinsic and Extrinsic Carriers. Thermally excited carriers are termed *intrinsic* carriers, since their formation is an inherent property

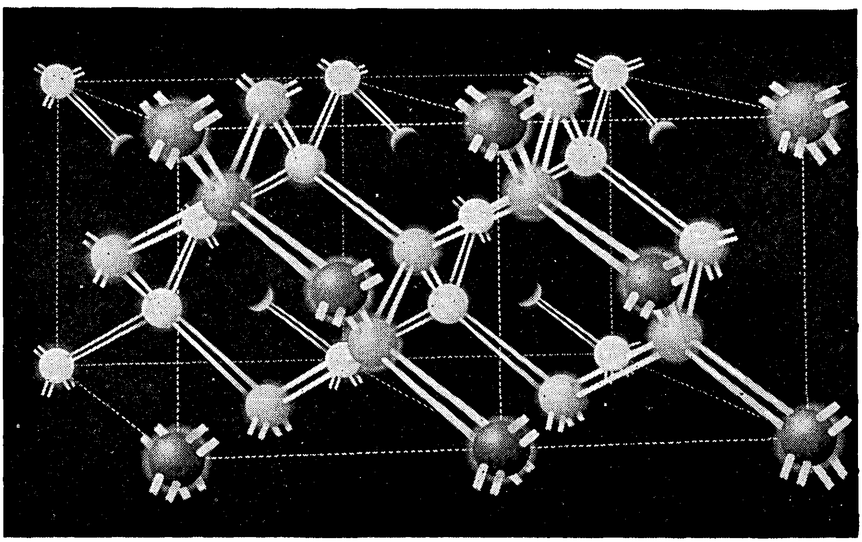


FIG. 1. Diamond cubic lattice of the common semiconductors. Each atom has four nearest neighbors, and each interatomic bond involves two electrons.

of the pure single crystal material. On the other hand, conduction due to impurity centers, as described below, is termed *extrinsic.*

The intrinsic number n_i, i.e., the probable number of thermally free electrons, is

For silicon $\quad n_i = 9.2 \times 10^{15} T^{3/2} \exp(-6500/T),$

For germanium $\quad n_i = 5.1 \times 10^{15} T^{3/2} \exp(-4400/T),$

where T is the absolute temperature. At room temperature (300°K) for silicon n_i is about $2 \times 10^{10}/\text{cm}^3$; for germanium n_i is about $2 \times 10^{13}/\text{cm}^{+3}$. The energy (energy gap) required to free the valence electrons for conduction is 0.72 ev for germanium, 1.1 ev for silicon. This difference is responsible for the much smaller value of n_i for silicon, leading to greater usefulness of silicon devices at high temperatures.

Extrinsic Carrier Types. *N Type.* Conduction electrons can also be supplied by certain impurity atoms called *donors,* which have five valence electrons and fit into the semiconductor lattice substitutionally as indicated schematically in Fig. 2 with one electron left over. The extra elec-

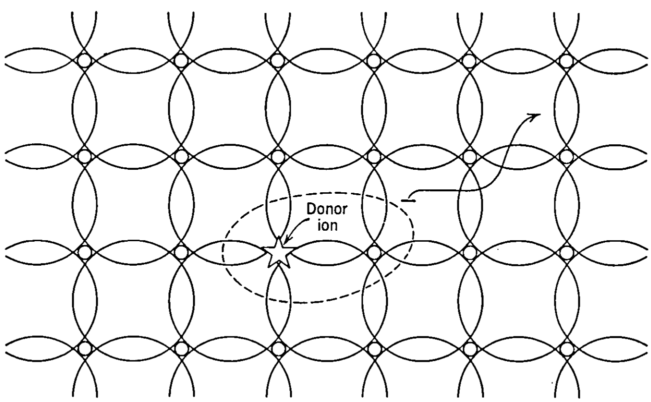


FIG. 2. Donor impurity in lattice (Ref. 3).

tron is bound to the extra positive charge in somewhat the same manner as the electron is bound to the proton in a hydrogen atom. However, the binding energy of such a combination in a medium of dielectric constant ϵ_r will be smaller by a factor of ϵ_r^2. For silicon ($\epsilon_r = 12$), the binding energy comes out less than 0.1 ev on the basis of this very crude calculation. The accurate experimental binding energy turns out to be still smaller, and silicon's donor electrons are essentially free at room temperature where the thermal energy kT is about 25 electron millivolts. At lower temperatures, however, silicon's donor electrons are not free to conduct.

Germanium ($\epsilon_r = 16$) has a smaller binding energy for donor electrons than silicon, and they are free to conduct at lower temperatures than for silicon. Conductivity resulting from these negative electrons is called *n type conductivity,* and semiconductors in which this conduction mechanism predominates are called *n type semiconductors* in which the electrons are often referred to as *majority carriers.*

P Type. Impurity atoms having three valence electrons can also fit into the semiconductor lattice as indicated in Fig. 3. The incomplete bond

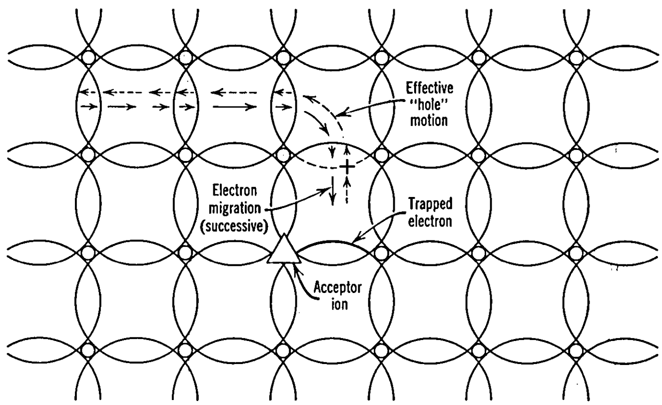


FIG. 3. Acceptor impurity in lattice (Ref. 3).

resulting need not stay in one place, since the lattice has practically the same energy if an electron from some nearby bond occupies the site, i.e., is trapped or "accepted" there, leaving the incomplete bond or mobile *hole* at another location. For this reason, such trivalent atoms are called *acceptors.*

It is convenient to consider the motion of this hole, even though it is really the resultant of the motion of many electrons. Since the hole represents an electron which is not there, it turns out to be equivalent in many respects to a mobile positive charge. It is attracted to the extra negative charge, now trapped at the site of the acceptor atom, and the binding energy is similar to that of the electron attracted by the donor ion. Hence the holes are free to move at room temperature and conduct current. Both the Hall effect and the thermoelectric coefficient have the algebraic sign corresponding to conduction by positive charge carriers. Conductivity resulting from this mechanism is therefore termed *p type conductivity,* and semiconductors in which it predominates are called *p type semiconductors,* in which the holes are often referred to as *majority* carriers.

Energy Levels. It is useful to represent the bound states and conduction states in a solid on a diagram such as Fig. 4 in which the ordinate

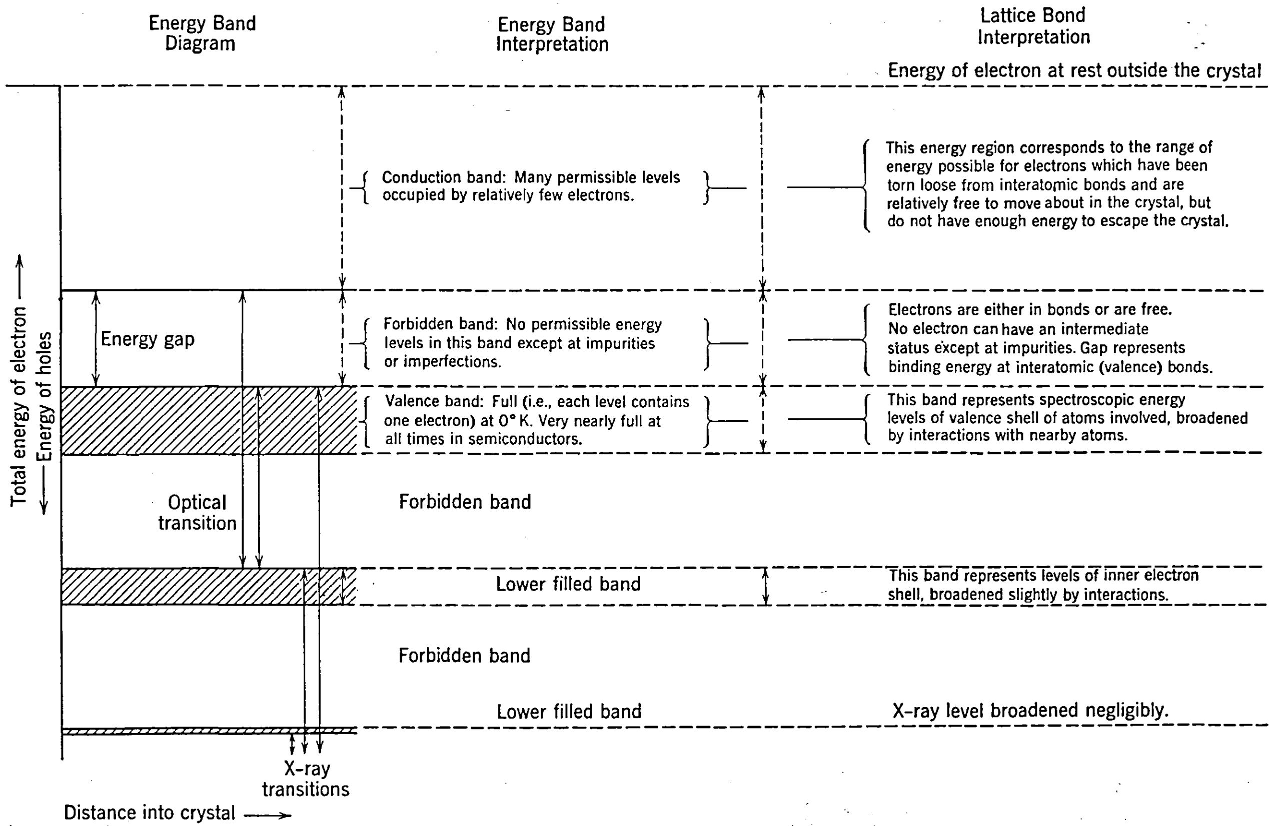


FIG. 4. Energy level diagram for a semiconductor.

represents the total energy of an electron in the solid. The top line represents the energy needed to escape from the solid. The abscissa represents a physical dimension in the crystal. The diagram as drawn represents a semiconductor in which the lower cross-hatched bands are roughly related to the filled electron shells or X-ray levels of the atom. They are bands, not single lines, because of interactions between atoms in the solid. The upper cross-hatched band is the "valence band," representing the same outer shell of electrons which forms the interatomic bonds in our discussion above.

In this energy level diagram for a semiconductor, there are just as many valence electrons as energy levels in the valence bond. When all the levels are full, electrons can only exchange places, a process which does not contribute to carrying current. This corresponds to the case discussed above in which all the electrons were tied up in interatomic bonds.

Energy Levels and Lattice Bonds. In the lattice bond discussion, an electron either was involved in these bonds or was free to move about in the lattice—nothing in between. On energy level diagrams, this is taken care of by the "forbidden" bands. The width of the forbidden band between valence band and the conduction band is called the *energy gap* and corresponds to the energy required to "break the interatomic bonds." A valence electron either is in the conduction band, relatively free to move about, or is in the valence band. For every electron raised into the conduction band, a hole appears in the valence band, which can move by exchanging levels with electrons. If the electrons in the conduction band were given sufficient additional kinetic energy (electron affinity, related to work function), they could escape the solid by thermionic emission.

The interatomic bond representation is convenient for introduction to the subject, but the energy level diagram is far more useful for quantitative considerations, since it is based upon the correct quantum statistical treatment of these subjects.

Charge Neutrality. Each intrinsic electron leaves behind one "hole," or incomplete lattice bond. Thus, intrinsic semiconduction is due to hole-electron pairs characteristic of the semiconductor lattice whose density increases rapidly with temperature. Certain "insulators," for example, diamonds, are actually intrinsic semiconductors wherein the electrons are so tightly bound that conduction is negligible at ordinary temperatures. The density of electrons is equal to the density of holes in intrinsic semiconductors, so that charge neutrality is maintained. If holes and electrons were equally mobile, intrinsic conduction would be neither n type nor p type. However, the electrons are more mobile, and conductivity is n type when both carriers are equal in density, or even when the holes slightly outnumber the electrons.

The additional carriers in extrinsic semiconductors do not result in charge unbalance. Each electron in n type semiconductors is neutralized by a positive donor ion which has one more charge than the surrounding germanium atoms; the holes in p type semiconductors are balanced by the electrons trapped by the acceptor atoms.

Minority Carriers. Minority carriers (electrons in p type material, holes in n type material) are always present to some extent, even in highly extrinsic semiconductors, owing to generation of intrinsic carriers (hole-electron pairs). Consider a nearly intrinsic semiconductor which contains a few donors and is therefore slightly n type. The extra supply of electrons present increases the probability that individual holes will be filled, i.e., the donors not only add electrons but also suppress holes. In like manner, acceptors add holes and suppress electrons. It turns out that, over virtually the entire range of hole and electron concentration of interest, $n_0p_0 = n_i^2$ where n_0 and p_0 are equilibrium electron and hole concentrations, respectively, and n_i is the intrinsic number.

In highly doped germanium and silicon the majority carrier density is essentially equal to the net donor or acceptor concentration and is practically independent of temperature, whereas the minority carrier density is seen to increase rapidly with increasing temperature since it is equal to n_i^2 divided by a constant.

Mobility of Electrons and Holes. Since the electrons and holes are charged, their motion results in electric current. In an electric field conduction electrons drift from minus to plus with an average drift velocity given by $\mu_n E$ where μ_n is the mobility of electrons and E the electric field. Mobilities in silicon are 1400 cm^2/volt sec for electrons, 500 for holes; in germanium, mobilities are somewhat higher, 3600 for electrons, 1700 for holes. (See Table 1.) If the density of electrons is n per unit volume and e is the electric charge, the resulting drift current density is $ne\mu_n E$. Holes drift in an analogous fashion giving a drift current density of $-pe\mu_p E$ where p is the hole density per unit volume and μ_p is the hole mobility.

Diffusion and Drift. Since the thermal velocity is high (10^7 cm/sec for electrons in germanium at room temperature), electrons and holes wander around erratically and diffuse in the direction of decreasing concentration. The diffusion constant D is proportional to the mobility, as given by the Einstein relation $D = \mu kT/e$, where e is the electronic charge. Since a diffusion flow is given by the concentration gradient times the diffusion constant, the diffusion current density for electrons is $eD_n \nabla n$ and for holes is $-eD_p \nabla p$.

Both diffusion and drift usually occur simultaneously, and in such cases the total current is the sum of the electron current density I_n and hole current density I_p, and each of these is the sum of the corresponding drift current and the diffusion current. To summarize:

$$I_n = ne\mu_n E + eD_n \nabla n,$$

$$I_p = pe\mu_p E - eD_p \nabla p.$$

Since thermal velocities are usually much higher than drift velocities, the actual motion of electrons and holes is usually a very erratic "random walk" with a relatively small drift component in the direction of the electric field, unless the field is of the order of 10^4 volts/cm or greater.

Algebraic Signs in Current Equations. When a field E is applied, the electrons will move in the opposite direction from the holes in any material, so at first one would expect the drift terms to have opposite sign. But the electrons and holes moving in opposite directions are also carrying opposite charges, so they actually carry *current* in the *same* direction, and the *drift* terms have the *same* sign in both equations.

For the diffusion terms, suppose that there exists a gradient of both holes and electrons decreasing from left to right. [This occurs, for example, in certain (*p-n-m*) power rectifiers.] Both holes and electrons will tend to diffuse to the right in this case. But since the holes and electrons carry charges of opposite sign, the *electric current* is carried in the opposite direction, by the two types of carriers (conventional current (+) of holes left to right, conventional current (+) for electrons right to left). So the *diffusion* terms have *different* algebraic signs in the two equations.

The sign convention for E is such that electric current is plus when E is plus. Sign convention for ∇ is plus in the direction of increasing concentration. Hole current flows away from higher concentration so ∇ diffusion term is negative in the I_p equation.

P-n Junctions

When a p type and an n type region occur within a continuous semiconductor body, the boundary between these regions is called a p-n junction. A p-n junction is in itself a rectifier whose conductances and capacitances, and their temperature and frequency variations, form the basis for semiconductor electronics (see Refs. 6 and 8). The rectifying action of a p-n junction may be described in terms of the hole and electron densities on both sides of the junction.

A p-n junction is shown in Fig. 5a. Electrical charge neutrality is normally maintained in the p type material by equal densities of positive mobile holes and negatively charged acceptor atoms (see Sect. 2). Similarly with mobile electrons and ionized donor atoms in the n type material. Because of thermal motion, the mobile holes tend to diffuse from the p region across the p-n junction into the n type material, leaving behind

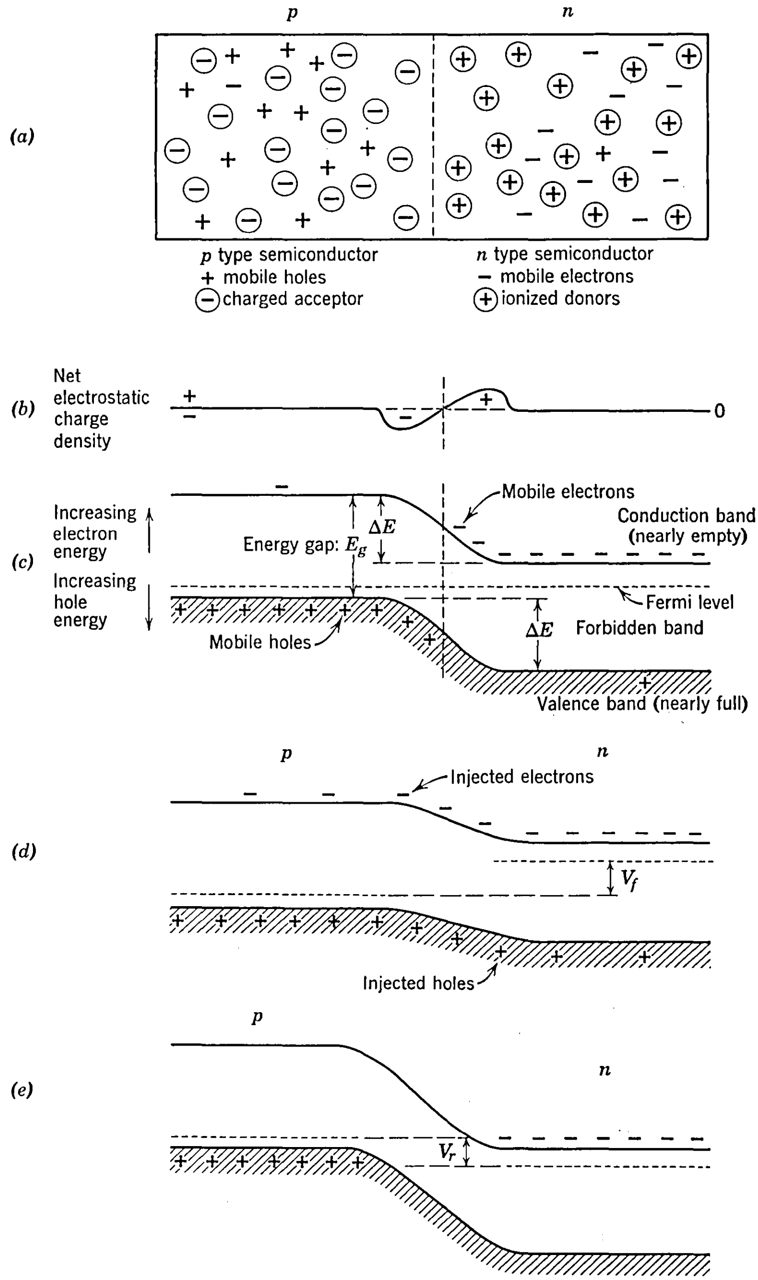


FIG. 5. *P-n* junction: (*a*) physical picture, (*b*) electrostatic charge distribution showing equilibrium charge dipole layer, (*c*) potential energy with no bias at equilibrium (only mobile charges shown), (*d*) with forward bias V_f (only mobile charges shown), (*e*) with inverse bias V_r (only mobile charges shown).

some acceptor atoms, each charged negatively by its trapped electron. The mobile electrons similarly diffuse across the junction, leaving uncovered some positively charged ionized acceptor atoms on the n type side of the junction.

The uncovered charges fixed in the lattice create an electrostatic charge dipole layer along the junction, with the n type side positively charged and the p type side negatively charged as shown in Fig. 5*b*. This dipole layer results in an electrostatic potential difference across the junction, whose polarity opposes the outward diffusion by pulling electrons back into the n type region, holes back into the p type region. If it were not for the built-in potential difference appearing across the junction because of the fixed charges, the holes and electrons would fill all parts of the crystal to the same density. At thermal equilibrium the electrons flow back into the n region at exactly the same rate as they are diffusing out; the same is true for the holes. The concentrations of holes and electrons in the semiconductor regions far from the junction are not affected by the presence of the junction except for the built-in contact potential difference due to the uncovered charge dipole layer.

The potential energy situation is illustrated in Fig. 5*c* where the built-in contact potential difference across the p-n junction has erected exactly the same potential barrier ΔE to the diffusion of electrons out of the n region as it has to the diffusion of holes out of the p region. Electron energy is normally being measured upward, hole energy downward on energy band diagrams, so that the behavior of holes and electrons can be inferred from the same diagram. The Fermi level indicated here is the equilibrium level for the system. It has the same value everywhere for all parts of a system in equilibrium.

Forward Bias. If the p region of a p-n junction is biased positively with respect to the n region by a potential difference V_f, as shown in Fig. 5*d*, the potential barrier ΔE for holes is reduced by the amount of V_f, and holes will now diffuse into the n region faster than they are drawn back by the decreased potential barrier. The resulting net flow of holes into the n region is called *injection*. Electrons are likewise injected into the p region at the same time because the electron potential barrier ΔE is also lowered simultaneously by the same amount V_f. The situation is diagramed schematically in Figs. 6*a* and 6*b*, in which only the mobile carriers are shown.

The excess minority carriers (over the equilibrium concentration) recombine with majority carriers. The holes and electrons thus move toward each other and recombine on both sides of the junction. This is the "forward" bias, corresponding to easy flow of current, which in the ideal case is limited only by the supply of excess holes and electrons and the

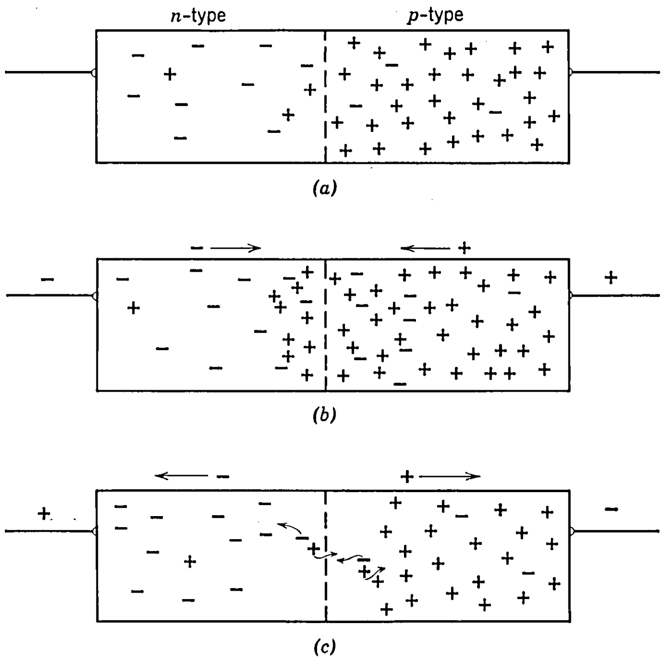


FIG. 6. Current carriers in a p-n junction: (a) equilibrium, (b) forward bias, (c) inverse bias. Only the mobile carriers are shown.

rate of recombination. Each recombination is equivalent to one electron having passed through the system. Charge neutrality is maintained to a good approximation during this process since an extra electron appears in the n region for every hole injected into this region so that $n - n_0 = p - p_0$. Similarly with the p region. Strictly speaking, it is therefore more appropriate to regard the excess population as excess hole-electron pairs, rather than excess holes or excess electrons. However, if the density of excess pairs is small compared to that of the majority carriers, the resulting phenomena are mostly due to the additional minority carriers, and we commonly speak of "holes injected into n type germanium," etc.

Inverse Bias. When an inverse bias is applied to a p-n junction, electrons are pulled into the positively biased n region, i.e., toward the left in Figs. 5a and 5e, also 6c, uncovering more positively charged donor ions on the p side of the junction, and increasing the pre-existing built-in contact potential difference. Similarly an equal number of trapped electrons are uncovered on the p side of the junction, and there results a larger dipole layer of fixed charge (positive in the n region, negative in the p region) whose magnitude and distribution depend upon the density and position of donor and acceptor atoms near the junction. The relation between the charge density (ρ), the geometry, and applied potential V is given by Poisson's law $\nabla^2 V = \rho/\epsilon$, where ϵ is the dielectric constant. The inverse current depends on the rate of thermal generation of hole-electron pairs in the regions on both sides of the junction, since the diffusion of electrons out of the n region and holes out of the p region becomes negligible when the potential barrier has increased approximately 0.1 volt.

As the applied inverse potential is increased, more fixed charge is continually uncovered. The variation of charge with voltage corresponds to a small signal a-c *depletion layer capacitance* c_j given by $c_j = dQ/dV$, where Q is the total charge at voltage V. For abrupt junctions, c_j is proportional to $V^{-1/2}$ (see Refs. 2 and 9); for graded junctions, c_j is proportional to $V^{-1/3}$ (see Refs. 8 and 10). Depletion layer capacitance is associated with all inversely biased junctions. Its magnitude is likely to be of the order of 1 to $100 \mu\mu f/cm^2$ junction area, depending upon carrier density and distribution, but it does not vary appreciably with frequency or temperature.

Injection Levels. When excess minority carriers are injected into a region in a semiconductor and charge neutrality is maintained, no high electrostatic potentials arise. Therefore there is essentially no space charge limitation to injection. At low injection levels, the few additional majority carriers are not noticeable, and the useful effects are due primarily to the additional minority carriers. At high injection levels, the additional hole-electron pairs may be so much more numerous than the original carriers that the population of holes can be assumed to be approximately equal to the population of electrons, regardless of the initial type of the material.

Carrier Lifetme. The additional hole-electron pairs recombine independently and randomly, resulting in an exponential decay. The time interval required for the excess carriers density to reduce by the factor e (root of natural logarithms) is called the *lifetime* τ of the minority carriers. The lifetime varies somewhat with resistivity, temperature, and injection level, but it is directly controlled by the number of "recombination centers" in the crystal. Typical lifetimes for low injection levels in germanium and silicon devices lie in a controllable range from less than a microsecond to more than a millisecond.

The volume recombination rate is given by $(n - n_0)/\tau$ or $(p - p_0)/\tau$, where $n - n_0 = p - p_0$ is the density of excess hole-electron pairs. If the actual minority population is less than n_0, i.e., near an inversely biased junction, then the rate of thermal generation is $(n_0 - n)/\tau$ or $(p - p_0)/\tau$ per unit volume. Since the minimum electron population is zero, the maximum thermal generation rate for minority carriers is n_0/τ for p type material, p_0/τ for n type material.

Inverse Current. The inverse current in a *p-n* junction is made up of holes (minority carriers) thermally generated in the *n* region moving across the junction into the negatively biased *p* region plus the electrons (minority carriers) thermally generated in the *p* region moving across the junction in the opposite direction into the positively biased *n* region, as in Fig. 6*b*. Since the minority carriers near an inversely biased junction flow across the junction as rapidly as they are formed, their population there is nearly zero and the inverse current is proportional to their maximum rate of thermal generation which in turn is proportional to n_0 as noted above for the *p* type material, p_0 for the *n* type material near the junction.

Junctions made with lower resistivity material (higher equilibrium majority carrier density because of higher doping with impurities) therefore have lower equilibrium minority carrier density ($n_0p_0 = n_i^2$) and hence lower inverse current density than otherwise similar junctions made with high-resistivity material.

Since minority carrier density is given by n_i^2 divided by majority carrier density as noted earlier, inverse current is proportional to n_i^2 and thus increases rapidly with temperature. Silicon junctions may have very roughly a million times smaller inverse "leakage" current than comparable germanium junctions because n_i for silicon is roughly 1000 times smaller than n_i for germanium.

High-Temperature Characteristics. The exact upper design limit of high-temperature operation of silicon has not yet been determined quantitatively. The practical limit depends upon the application as well as the device, but at best silicon is probably only capable of useful operation at temperatures below 350°C. Operation of future semiconductor devices at much higher temperatures, say up to 500°C or more, would depend on the use of materials other than silicon or germanium, for the intrinsic carrier generation is characteristic of each material and cannot be fundamentally altered by device design.

The semiconductors such as silicon carbide (see Ref. 84), potentially useful at higher temperatures, have larger energy gaps, i.e., a larger energy is required to free electrons from the valence bonds in the lattice. Semiconductor materials with higher energy gaps ordinarily have lower dielectric constants (see Ref. 7), and therefore extrinsic conduction electrons are more tightly bound to the donor ions, as noted above in the description of the action of donor atoms. Consequently new semiconductor materials developed for very high temperatures may not be efficient at room temperature because the extrinsic carriers may not be free for conduction (see Table 1).

Because of the larger contact potential difference across silicon *p-n* junctions, related to silicon's larger energy gap, the forward junction voltage (usually less than one volt) at useful operating currents is several tenths of a volt higher for silicon than for germanium. This applies to the forward voltage drop in junction diodes and rectifiers, as well as to the hold-on voltage for switching transistors (see Ref. 6). This difference between germanium and silicon is fundamental and cannot be overcome by device design. In general, semiconductor materials suitable for higher temperature operation will require slightly higher forward junction voltages.

Other Devices. Certain metal-semiconductor contacts have rectifying properties very similar to *p-n* junctions (see Refs. 11, 12, 14, and 53), but this action in devices is seldom under the quantitative control possible with *p-n* junctions. In some cases, such contacts are very useful because of the small size which can easily be achieved with a pointed wire or a small plated contact.

3. DIODE CHARACTERISTICS

The semiconductor diode is a two-terminal device containing a *p-n* junction of more or less ideal characteristics (or a metal-semiconductor contact having similar characteristics). Figure 7 shows typical outlines of commerically available semiconductor diodes. Many types pass a high forward current with much less voltage drop than comparable vacuum tubes, a characteristic highly desirable in the clamping, gating, and logic circuits essential to computation and control (see Chap. 27, Transistor Circuits, Sect. 7, Switching Circuits). Currents from microamperes to amperes can be handled in this manner. The high-current types are also useful as rectifiers in power supplies or for the operation of d-c machinery. Silicon or germanium junction rectifiers are more efficient than vacuum tube or selenium and copper oxide rectifiers. In addition their compact construction often permits installation at the point of use and thus simplifies distribution and control systems.

Since the semiconductor diode or rectifier cells contain one *p-n* junction, the volt-ampere characteristic of all types are quantitatively related to the characteristics of the *p-n* junction itself, namely $I = AI_0$ [exp $(qV/kT) - 1]$, when I is the current, A is the cross-section area, I_0 is the inverse leakage current density (for the ideal *p-n* junction), k is Boltzmann's constant, T is the absolute temperature, q is the electronic charge, and V is the applied voltage. For positive V, the current increases exponentially with V whereas the current density rapidly approaches $-I_0$ for negative V. In practical terms, the characteristics to be expected may be described by reference to Fig. 8. The inverse current is nearly independent of voltage below the breakdown voltage BV. Near the origin, the impedance is practically the same for forward and inverse voltages, so that

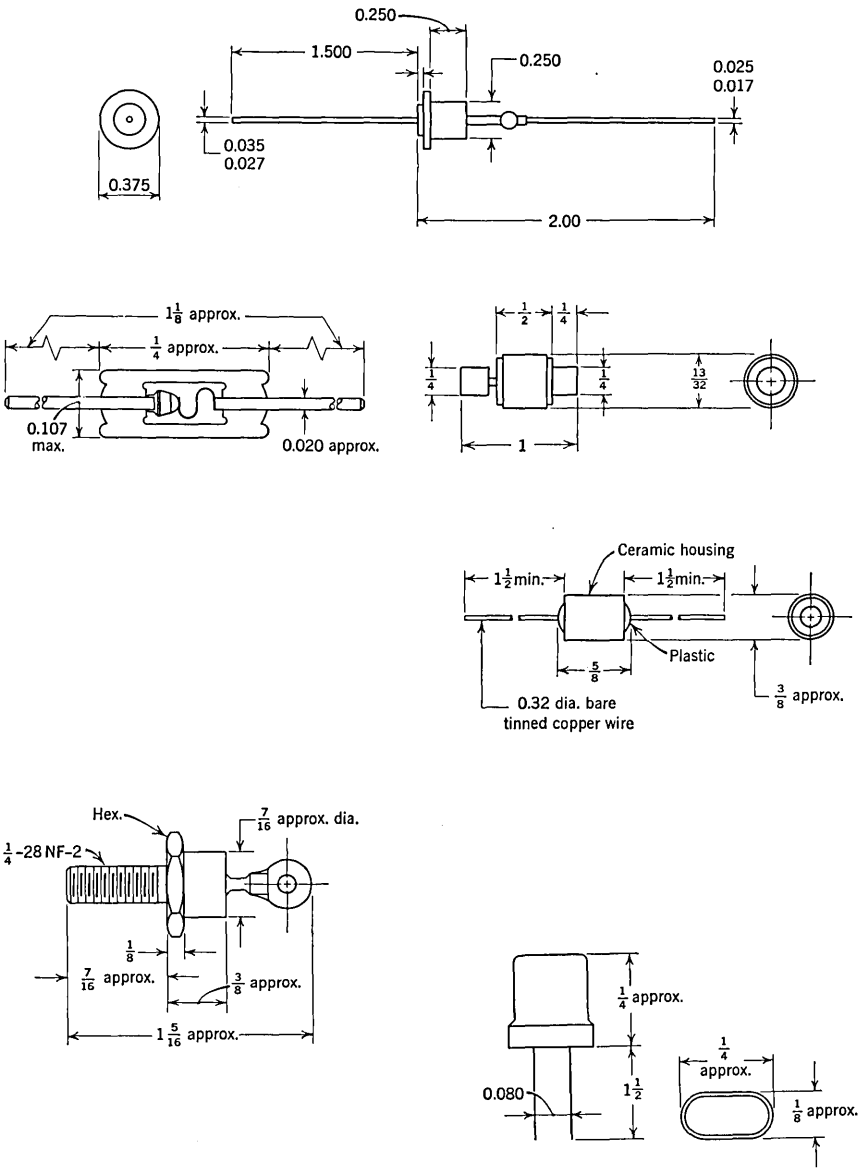


FIG. 7. Typical commercial semiconductor diode and rectifier outlines.

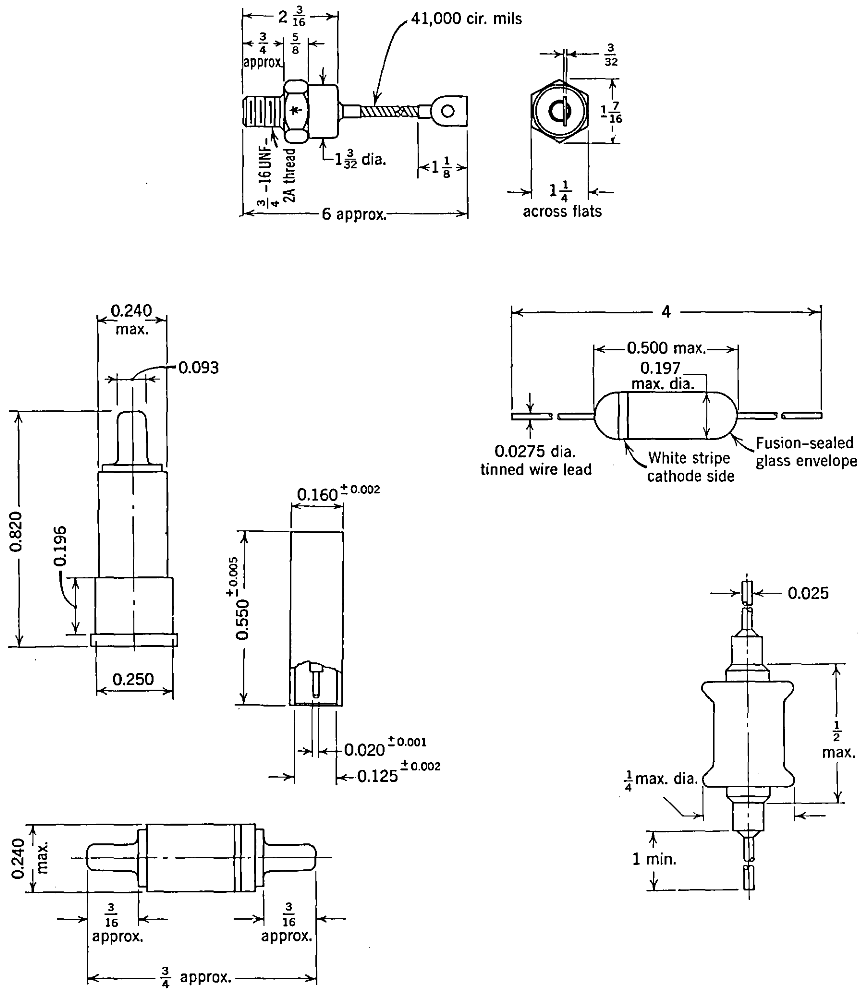


FIG. 7—(*Continued*)

there is very little rectification if the junction is operated at low voltage, i.e., less than a few hundredths of one volt applied.

Grown Junction Rectifiers. The simplest type of junction rectifier structure is the grown junction (Refs. 13, 14, and 15) shown in Fig. 9*a*, whose low-level current voltage relation is that of the simple *p-n* junction above, if V is taken to represent the junction voltage. In practical grown junction devices the junction is actually in series with a semiconductor region of finite resistance, and the forward voltage drop across the entire device also includes the voltage drop across the semiconductor body. The

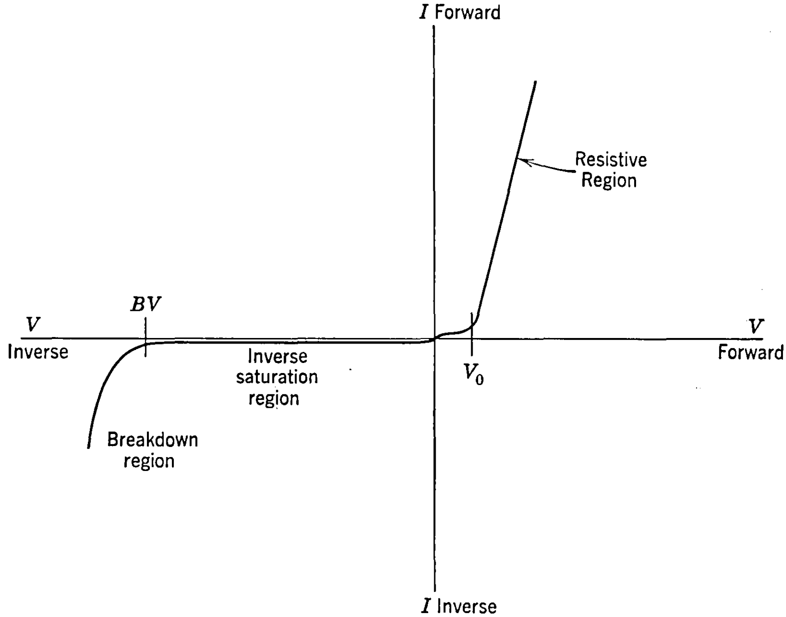


FIG. 8. Generalized semiconductor diode characteristics. Voltage and current are plotted to arbitrary units on a linear scale.

shape of the characteristic shown to linear scale in Fig. 8 is typical: at very low forward voltages, very little current flows. Then as the exponentially increasing current becomes noticeable there is a rapid increase in current. The series resistance eventually dominates the characteristic, and at high currents the device may be represented approximately as a resistor R_S in series with a fairly constant junction voltage drop V_0, which is likely to be of the order of ¼ volt for germanium, ¾ volt for silicon (see Fig. 9*b* and 9*c*). The effective series resistance for grown junctions usually ranges from a few ohms to a few hundred ohms. At all currents, this type of rectifier may be represented fairly accurately by R_S in series with a *p-n* junction.

Thin Wafer Diodes. A large number of junction rectifiers or diode structures may be considered together as a general classification, sketched in Fig. 10. The device consists of two good metallic contacts sandwiching a thin semiconductor wafer containing regions or contacts of appropriate electrical properties fabricated by alloying, diffusion, plating, etc.

Figure 10*b* indicates two possible semiconductor configurations for thin wafer diodes, the *p-i-n* and the *p-n*-ohmic, (or *n-p*-ohmic).

***P-i-n* Rectifier.** The *p-i-n* configuration consists of an intrinsic (*i*) region separating a high-conductivity (strongly extrinsic) *p* type region

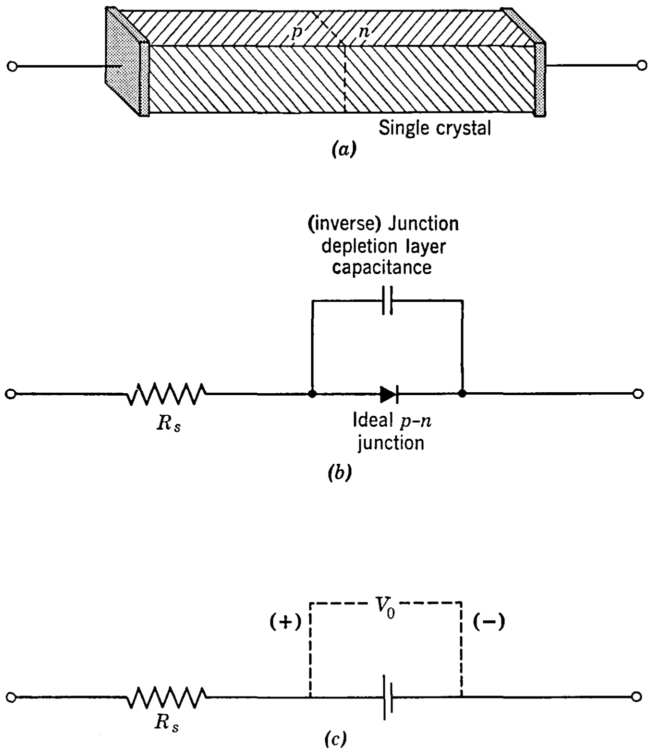


FIG. 9. Grown junction semiconductor diode: (*a*) grown diode structure; (*b*) equivalent circuit; (*c*) approximate equivalent circuit valid for forward bias region only, above approximately 0.2 volt for germanium, and above approximately 0.6 volt for silicon.

($p+$) and a high-conductivity region ($n+$) (see Refs. 19 and 20). In this structure, electrons from the $n+$ region and holes from the $p+$ region are injected in equal numbers in the center region where they recombine, each recombination being equivalent to one electron passing through the device in the forward direction. In practice, the wafer is actually mildly extrinsic (weakly doped) n or p type instead of being precisely intrinsic. Under an inverse bias the inverse voltage appears across the p-n junction (see Refs. 19 and 20).

***P-n*-Ohmic Rectifier.** In the p-n-ohmic configuration shown in Fig. 10*b*, holes are injected by a forward bias into the n region where space charge balance is maintained by an approximately equal excess of electrons. They diffuse toward the ohmic contact where, owing to the properties of this type of contact, excess carriers immediately recombine, each recombination again being equivalent to one electron passing through the device in the forward direction (see Refs. 17 and 21).

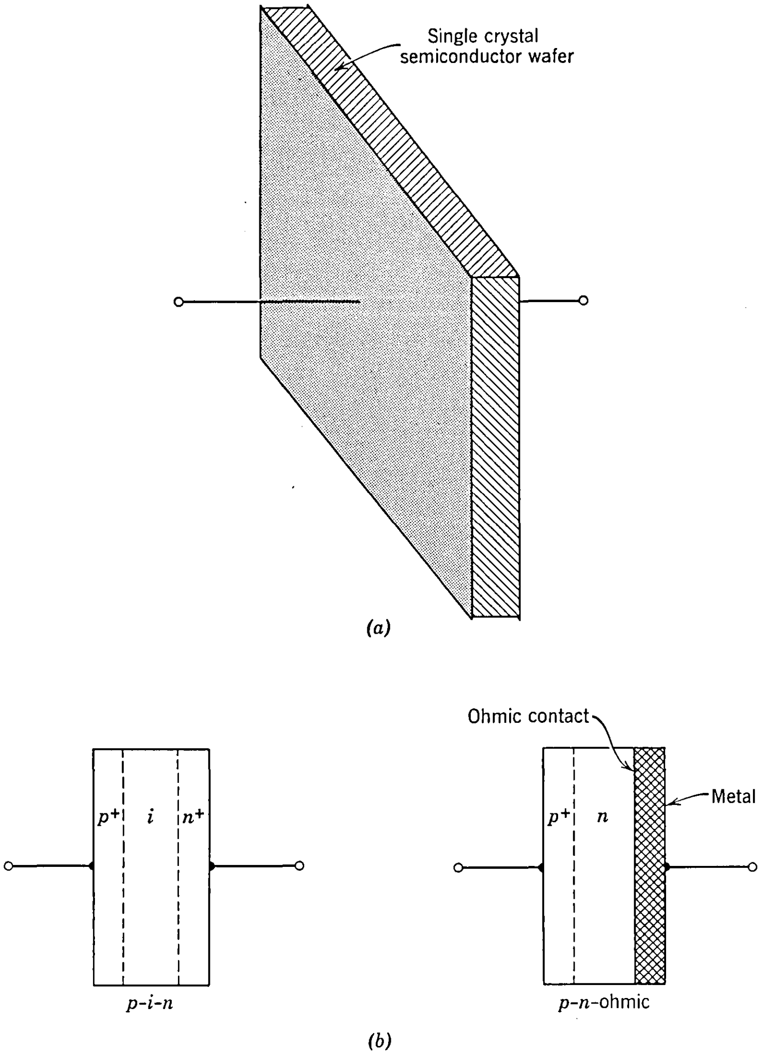


FIG. 10. Thin wafer semiconductor diode: (*a*) thin wafer semiconductor diode or rectifier structure, (*b*) semiconductor configurations for thin wafer diodes.

Differences between Thin Wafer and Grown Junction Diodes. In spite of the differences in mode of operation between these thin wafer types, leading to different design relations between semiconductor properties, geometry, and device characteristics, their current voltage characteristics are remarkably similar (and similar to intermediate types involving imperfectly ohmic contacts, etc; see Ref. 17). They differ in several respects from the characteristics commonly observed for grown junction diodes.

1. Thin wafer diodes are characterized by much lower series resistance than grown junction, R_S being less than 0.1 ohm for most high-current types and decreasing noticeably at high currents. The thin wafer class therefore includes practically all high-current devices.

2. Thin wafer diodes ordinarily have larger I_0 than grown junction diodes having the same area (i.e., having similar small signal inverse bias capacitance). This means that the current density, forward or inverse, is higher for the same voltage.

3. In grown junction diodes, the forward current increases as exp (qV_f/kT) or approximately exp $(39V_f)$ until the voltage drop across the series resistance becomes appreciable. On the other hand, thin wafer diodes have this same exponent at very low voltages, but in the normal operating range (above about 0.1 volt for germanium and above 0.5 volt for silicon) the exponent is commonly $qV_f/2kT$ or $19.5V_f$ (owing to injection effects) until series resistance dominates, or until the voltage becomes high enough to change the injection characteristics of the junctions. In practical devices of both types, the forward current-voltage exponent tends to decrease as current increases, and the effects of resistance, injection, etc., thus tend to obliterate this difference in many cases.

4. The junctions of thin wafer alloy junction rectifiers are likely to be abrupt. Therefore the small signal capacitance at inverse voltages is more likely to vary approximately as $V^{-1/2}$, whereas the graded junctions more typical of grown junction diodes lead to capacitance varying as $V^{-1/3}$.. For rectifiers made by impurity diffusion, usually fabricated in the wafer geometry, the variation of capacitance is likely to be intermediate between $V^{-1/2}$ and $V^{-1/3}$, depending on the degree of junction gradation.

Miscellaneous Types. The small junction diodes are similar to a class of diodes which may be termed "bonded" diodes, in which the active portion of the device is a small junction formed under a wire contact to a semiconductor wafer. This type of junction differs from other junctions principally in the techniques of fabrication and in the detailed geometry. The characteristics are superior to the older point contact diodes, and the area is smaller than most junction diodes, making capacitances smaller; however, the characteristics of bonded diodes *per unit area* are usually intermediate in quality between point contact diodes and junction diodes, with regard to inverse leakage and forward conductance. This implies that present bonded types are evolving in the direction of small area true *p-n* junction characteristics as rapidly as it is technically and economically feasible.

Rectifying contacts on semiconductors can also be made by pointed wires or by plated metal areas using appropriate materials. The resulting diodes act in fundamentally the same way as the types described above.

Function Generators. Both grown junction and thin wafer diodes at low forward voltages (below about 0.3 volt for germanium and below perhaps 0.6 volt for silicon) can be used to generate exponential or logarithmic functions, since the low forward current in a well-designed diode is proportional to exp V_f below these voltages, the proportionality factor being AI_0. Values of AI_0 in the range of 1–100 amp may be encountered for low-current germanium devices, values for silicon being about 10^6 times smaller. Good experimental operation is possible only at forward currents larger than the inverse leakage current (Refs. 17 and 88), so that clean inverse characteristics are essential for these function-generating applications, even though the diode is actually used in the forward direction. Step functions can also be developed by using the abrupt change in current near zero forward voltage, or using Zener diodes at predetermined inverse voltages.

High-Temperature Derating. All practical diodes have some inverse leakage current in excess of theoretical, mostly because of surface effects, but also resulting from imperfections in the junction. Many of these excess conductances are nonlinear and increase with voltage (avalanche effect) and temperature. Also the theoretical leakage (inverse junction current) increases very rapidly with temperature. Most available diodes are actually limited by irreversible destructive thermal runaway due to increase of leakage with temperature resulting from self-heating at high voltages rather than by the ultimate avalanche effect used for voltage reference diodes. The avalanche effect is not very temperature sensitive and is normally nondestructive unless too much energy is dissipated in the device.

It is therefore characteristic of practical rectifiers as with transistors that maximum voltage and current ratings must usually be decreased for high-temperature operation. Since it is junction temperature, not merely ambient temperature, which causes runaway, power rectifiers are designed for minimum internal temperature rise from self heating.

Germanium diodes and rectifiers are ordinarily designed to operate at maximum junction temperatures of 100°C or below. Some commercial silicon diode types are rated for normal operation up to 200°C. Developmental diodes of gallium arsenide have been operated successfully above 400°C, silicon carbide diodes (Refs. 37 and 88) above 1000°C.

Voltage Reference Diodes. The maximum inverse voltage which can be applied to a semiconductor junction will never exceed that corresponding to the reversible avalanche current multiplication effects (Refs. 18, 22, 23, and 35), which cause the current to increase rapidly with voltage. The voltage at which inverse current begins to increase rapidly is called the *breakdown voltage* (*BV*) and depends ideally upon the impurity density of the semiconductor and its distribution throughout the junction region. In practical rectifiers it also depends critically upon surface conditions on the device near the junction, conditions which often lead to breakdown voltages smaller, less reproducible, less abrupt, and noisier than avalanche breakdown. The breakdown voltage can be controllably designed from a few volts up to several thousand volts. In junctions where the breakdown is sufficiently "clean," i.e., where current above *BV* increases rapidly, continuously, and reversibly with voltage, the low impedance in the breakdown region makes it practical to employ such diodes as voltage regulator devices (see Ref. 24). The inherent small rate of change of breakdown voltage of the junction itself with temperature is comparable to that of voltage regulator tubes and can be compensated almost completely by connecting one or more junction diodes in series with the voltage regulator diode, but in the opposite direction. The slow increase of inverse breakdown voltage with temperature in the regulator diode is thus compensated by the decrease in forward voltage with temperature in the other diode(s). Voltage reference diodes, sometimes referred to as Zener diodes or avalanche diodes, have been made with breakdown voltages from 10–20 volts up to more than 100 volts.

High-Frequency Effects. At high frequencies, the diode junction depletion layer capacitance becomes a serious problem (see Chap. 27, Transistor Circuits, Sect. 4, High-Frequency Amplifiers). The small signal junction capacitance decreases slowly with junction inverse voltage, being proportional to $V_r^{-1/2}$ (for abrupt transition p to n, i.e., *step junctions*) and proportional to $V_r^{-1/3}$ (for gradual transition, i.e., *graded junctions*). The junction capacitances are likely to be found in the range 5–100 $\mu\mu$f, with somewhat smaller values available in certain small area point contact or bonded types.

Although the depletion layer capacitance must be charged as the inverse voltage is applied, a more serious turnoff limitation for switching application arises from "charge storage" effects (Refs. 26 and 27) whereby the carriers injected during the forward current operation do not disappear instantly when the forward bias is removed. They remain near the junction where they may permit high inverse transient currents. Thus the initial inverse current may be of the same order of magnitude as the previous forward current.

This additional current will continue to flow until all the excess carriers previously injected have flowed back across the junction (contributing to the excess inverse transient current) or have recombined without crossing the junction (not contributing to the inverse transient current). The excess conduction decays exponentially to the ultimate steady-state current, the decay constant being determined by the lifetime of the semiconductor material near the junction, by the semiconductor geometry, and by the mode of operation. The recovery time is shorter in circuits where it is possible to drive the diode "off" by a high inverse voltage, and it is shorter for thin diodes constructed of low-lifetime material.

In silicon it is known that lifetime can be higher at high injection levels (see Ref. 16) so that the high-current recovery time may be still slower.

The recovery time for charge storage effects in diodes can be faster for thin wafer types, since stored carriers will diffuse out of a thin wafer faster than they would disappear because of normal recombination. The approximate relation between cleanout time t, carrier diffusion constant D, and wafer thickness W is $t \doteq W^2/D$. In silicon, the diffusion constants ($D = \mu kT/q$) for injected holes and electrons are approximately 13 and 35 cm^2/sec, respectively. For germanium, the corresponding values are about 45 and 90, respectively. Germanium alloy junction rectifiers designed for high-power operation are often 10–15 mils thick, and the cleanout time corresponds to operation at 50 kc or less. Smaller units with W less than 0.001 in. can be made to clean out in much less than 1 μsec, depending upon the circuitry. The total quantity of stored charge in junction devices is proportional to the active volume of the device and to the forward current density. It also depends upon certain other construction details in the device (Refs. 26 and 27).

It is possible (Ref. 28) to obtain even faster cleanout, where speed is more essential than circuit simplicity, by biasing the diode to an operating point just below the breakdown voltage as indicated in Fig. 11. A small increase in voltage causes a great increase in current (forward bias). A small increment of the opposite polarity (inverse bias) reduces the junction current to its inverse leakage value (in the same direction as the "forward" current). Silicon rectifiers having low leakage and a very sharply defined reversible breakdown region are useful for this type of operation. Since the junction is operated at inverse bias over the entire range of operation, the resulting high electric fields sweep out carriers faster than conventional operation.

Semiconductor diodes are used in small signal high-frequency applications such as detectors, mixers, converters, etc., where the principal

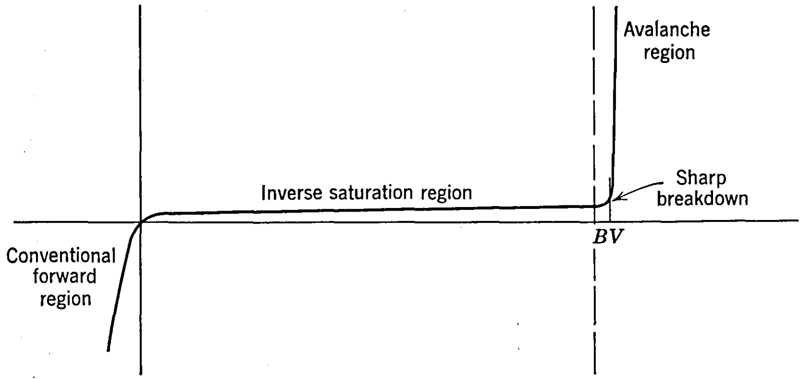


FIG. 11. Avalanche rectification.

requirement is a useful degree of high-frequency nonlinearity. The required high-frequency characteristics are related to low-frequency characteristics such as capacitance, resistance, noise figure, etc., but the correlations are not simple. High-frequency diodes are commonly developed for, specified, and tested in circuits closely related to the final circuit application (see Refs. 11 and 25). For a discussion of amplification effects, refer to the next section.

Complete specifications of currently listed types have been published annually by *Electronic Industries* magazine, Chilton Company Publishers, Philadelphia.

4. AMPLIFICATION BY SEMICONDUCTOR DIODES

Reactance Amplification. Semiconductor diodes, like other nonlinear circuit components, can be used to generate output currents of various sum and difference frequencies when an incoming signal is mixed with that of a local or "pump" oscillator which is producing a time variation of the impedance parameters in the nonlinear circuit element at the pump frequency. If the nonlinear impedance is suitably reactive, as is true of a junction diode, rather than acting merely as a variable resistance, then it is possible for the pump oscillator to supply enough energy so that one of the generated output signals is larger than the incoming signal, and amplification can thus be achieved. If the pump frequency is twice the signal frequency, it is possible to have the amplified signal output at the same frequency as the incoming signal. By shifting the pump frequency, it is also possible to have the output at a different frequency from the input and thus combine the functions of frequency conversion and amplification.

Early radar crystal converters sometimes exhibited this effect, under conditions discussed in Ref. 11. This amplification is the direct antecedent of the more recently developed semiconductor *parametric amplifier* or *reactance amplifier* which derives its more reproducible action from the nonlinear variation of *p-n* junction capacitance with voltage (see Sect. 2, Principles of Operation of Semiconductors). The nonlinear impedance of *p-i-n* junctions at high frequencies can also be used for microwave-switching purposes. One of the most important features of semiconductor reactance amplifiers is their operation as low-noise microwave amplifiers (see Ref. 92).

Reactance amplification by semiconductor diodes is a special case of a much more general method of amplification by means of nonlinear or time-varying reactance (see Ref. 93). One well-known previous example of parametric amplification by a time-varying capacitance is the vibrating reed or "dynamic capacitor" electrometer for measuring small voltages from high-impedance sources.

Parametric Computer Components. Digital computer components called parametrons have been developed to make use of subharmonic parametric oscillation which can represent binary digits by stable operation in either of two phases in a manner analogous to the resonant string in Melde's experiment (familiar to elementary physics students), which vibrates at half the frequency of the driving tuning fork and which can "lock in," either at a given phase of the fork or at one fork cycle (i.e., one-half string cycle) later. Tristable operation can also be obtained in some cases by introducing a stable "no oscillation" mode in addition to the two phases of oscillation. Although ferrites have been employed predominantly in present commercial use, semiconductor junction diodes have been used to provide the nonlinear circuit elements for parametrons (see Ref. 94).

Semiconducting Switching Diodes. Carrier multiplication due to avalanche operation at high electrical fields can result in negative resistance characteristics and has been used to make switching diodes (see Refs. 81 and 82). Negative resistance due to carrier multiplication by transistor action using *p-n-p-n* structures can also be used in switching (see Ref. 80). In many cases the device is technically a diode because of its two-terminal construction, even though its action requires transistor action (see Sect. 5, Transistor Characteristics) (see also Refs. 38 and 39).

Tunnel Diodes. Amplification can also be accomplished in special semiconductor diodes, using the quantum mechanical "tunnel effect" to control the forward conductance of abrupt *p-n* junctions.

In very highly doped semiconductors, the "built-in" potential difference across the junction can slightly exceed the energy gap, as shown in Fig. 12. The electrons moving toward the junction in the conduction band would normally be reflected back by the potential barrier. However, in

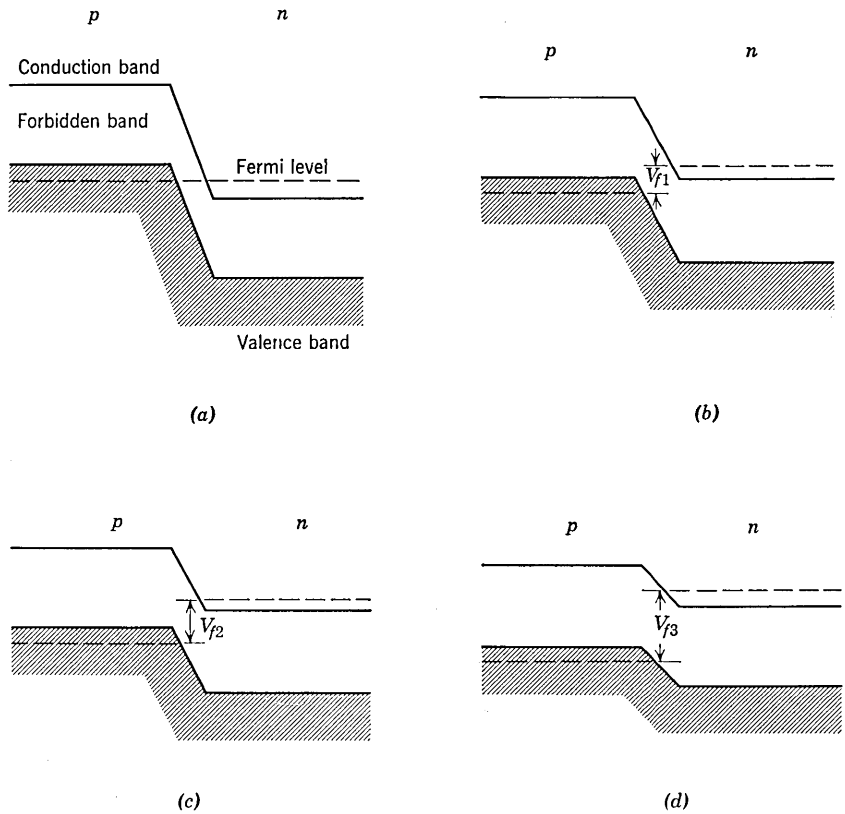


FIG. 12. Energy band diagrams for tunnel diode: (*a*) equilibrium semiconductor on both sides of *p-n* junction so heavily doped that bands overlap slightly; (*b*) low forward bias V_{f1}, high tunnel current, bands still overlap; (*c*) intermediate forward bias V_{f2}, tunnel current is cut off since bands no longer overlap; (*d*) high forward bias V_{f3}, normal *p-n* junction forward current.

this case, since the conduction band of the *n* type material overlaps the energy of the valence band of the *p* type material, the electrons could conceivably cross the forbidden band and fill any of the numerous empty valence band levels (holes) in the highly doped *p* region without appreciable energy change. An electron cannot exist in the forbidden band, of course, but if the junction is narrow enough, the laws of quantum mechanics permit the electron to disappear on one side of the thin barrier and to reappear instantly on the other side by the so-called "tunnel effect," even though it does not have sufficient energy to surmount the barrier. Although the probability of an individual electron "tunneling" across the junction is very small, the electrons are so numerous that currents of the order of 1000 amp/cm^2 could in principle be tunneled across thin junctions in present materials.

At equilibrium many electrons cross the junction continually in both directions. When a small forward bias is applied, as in Fig. 12*b*, electrons flow from the *n* region into the *p* region, and the tunnel effect provides a very high conductance. When the forward voltage is increased as in Fig. 12*c* sufficiently to remove the overlap (of the order of a few tenths of a volt for germanium), the electrons from the *n* type material can no longer tunnel across to empty levels at the same energy in the valence band, and the current decreases. At still higher forward voltages, the current increases as holes and electrons are injected across the built-in potential by normal diffusion, but this ordinary current does not become appreciable until the applied voltage approaches within a few tenths of a volt of the total built-in potential drop across the junction.

For inverse voltages, tunneling can take place freely, and the inverse conduction of a tunnel diode is ordinarily much higher than the forward conduction (see Ref. 95).

The resulting current-voltage relation shown in Fig. 13 shows a negative-resistance region which is responsible for the amplification and switching characteristics of this device. As implied by the explanation above, the tunnel diode is fundamentally a high-current–low-voltage device.

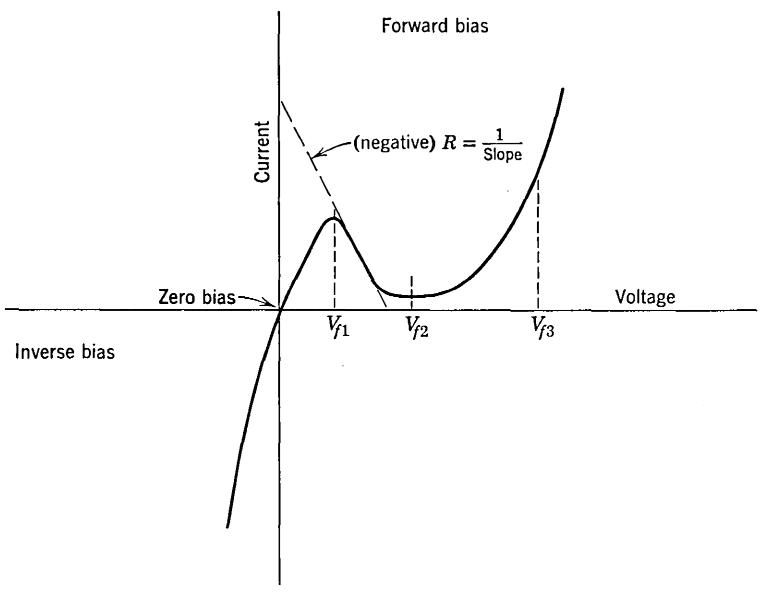


Fig. 13. Voltage-current relations for a tunnel diode. V_{f1}, V_{f2}, V_{f3} correspond to bias conditions shown in Fig. 12.

The voltage swing available in the negative resistance region energy is proportional to the energy band overlap which can be developed across the junction, which is a basic property of the semiconductor. In addition to germanium, with which most of the early tunnel diodes were made, successful tunnel diodes have also been made using silicon, as well as gallium arsenide, gallium antimonide, and indium antimonide.

Since the current in tunnel diodes is carried by majority carriers moving rapidly in electric fields, and since the carriers cross the junction itself by the instantaneous tunneling process, rather than by the much slower process of diffusion, the tunnel diode has great potential use of high-frequency circuit applications (see Ref. 96).

Tunnel diodes have achieved amplification above 1000 megacycles and show promise of operation at much higher frequencies. Although slightly noisier than parametric amplifiers and masers, tunnel diodes are much less noisy than transistors and vacuum tubes, and require far less operating power than any of these devices. Introductory quantities of tunnel diodes are commercially available.

5. TRANSISTOR CHARACTERISTICS

The operation of a transistor is based on the semiconductor principles described above. Minority carriers are injected by the forward-biased emitter junction into the base region. They diffuse toward the collector where they are collected, i.e., pass across the inversely biased collector junction into the collector circuit. The three fundamental processes of transistor electronics then are injection, diffusion, and collection (see Refs. 2, 6, 8, and 34). Transistor types are covered in Sect. 6, Transistor Types.

Transistor characteristics will be described with the purpose of emphasizing their physical origin in semiconductor processes. These processes within the transistor are the basis for the equivalent circuits described in Chap. 27, Transistor Circuits, Sect. 1, Basic Circuit Considerations and Symbols, and Sect. 4, High-Frequency Amplifiers, and are amenable to very useful approximate analysis in terms of small signal parameters (see Ref. 30). Just as with vacuum tubes, the large signal analysis is in a somewhat primitive state and must often be considered in terms of the limiting physical phenomena rather than in terms of the algebraic relations between small signal parameters. Typical junction transistor characteristics are displayed in Figs. 3 and 5 of Chap. 27, Transistor Circuits.

Transistor Action. The action of ordinary junction *transistors* (i.e., active semiconductor devices with three or more electrodes) is illustrated in a general way by Figs. 14*a* and 14*b* which show a conventional common base bias arrangement for *p-n-p* and *n-p-n* junction transistors. The energy diagrams are shown for unbiased and biased connections in Figs.

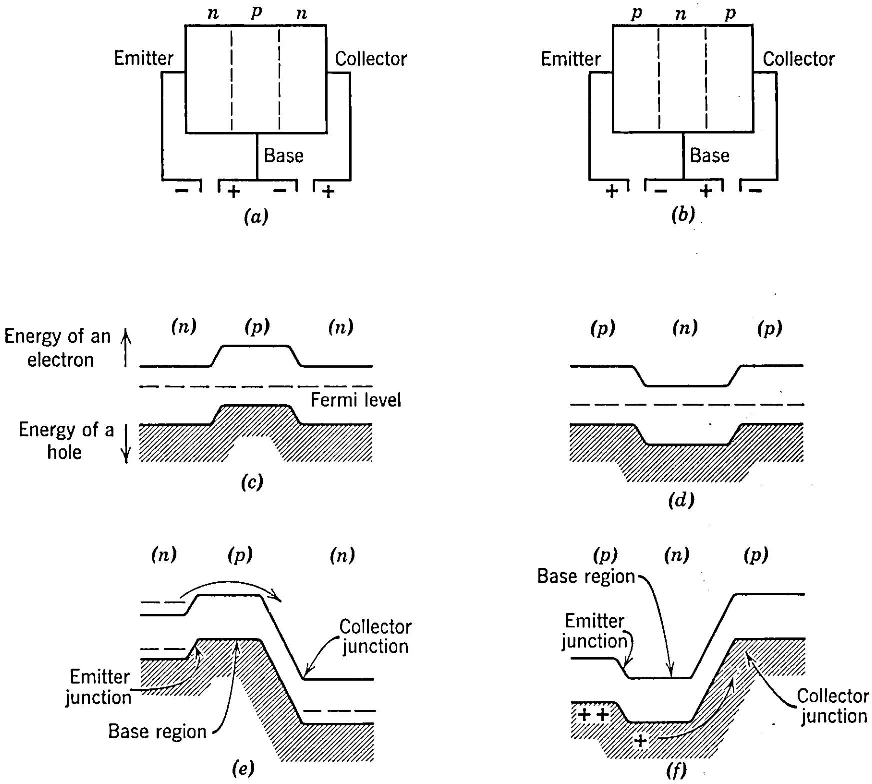


FIG. 14. Transistors: (*a*) *n-p-n* configuration, (*b*) *p-n-p* configuration, (*c*) energy band diagrams with no bias, (*d*) energy band diagrams with no bias, (*e*) energy band diagrams showing effects of bias applied in (*a*) and (*b*), (*f*) energy band diagrams showing effects of bias applied in (*a*) and (*b*).

14*c* to 14*f*, where it can be seen that as a result of the built-in junction potentials in the *n-p-n* transistor (Fig. 14*c*) the base region provides a potential barrier to the passage of electrons from emitter to collector. With biases applied, as in Fig. 14*e*, a forward emitter bias causes electrons to be injected into the base region. If the base region is thin enough, many of the injected electrons can diffuse across the base to the collector junction, where they will be drawn into the collector region by the collector junction potential which is larger than the built-in potential because of the inverse bias. Similarly the base of the *p-n-p* transistor provides a potential barrier across which holes may be injected from the emitter region to the collector region as indicated in Figs. 14*d* and 14*f*.

Unlike the vacuum tube, in which a voltage is employed to control a current, the transistor uses one current to control another. When biased as shown, the emitter circuit operates at relatively low voltages. Less than 1-volt emitter-base voltage (V_{EB}) normally covers most of the range of current from microamperes to amperes. Any change in emitter current causes a nearly equal change in collector current, i.e., nearly all the emitter current flows into the collector instead of into the base. Since the collector base voltage V_{CB} can be much higher than the emitter-base voltage (V_{EB}), up to more than 100 volts in some cases, it is clear that power gain can be achieved by using the input signal to change the emitter current and by inserting a load impedance in the collector circuit. Typical characteristics are shown in Fig. 3 of Chap. 27, Transistor Circuits.

***N-p-n* and *p-n-p* Transistors.** The fundamental difference between *n-p-n* and *p-n-p* transistors from the point of view of the user is that they employ exactly opposite polarity biases. The collector of the *n-p-n* is biased positively, in a manner similar to the anode of a vacuum tube, whereas the *p-n-p* collector is biased negatively with respect to the base. The availability of both types results in a freedom of transistor circuit design analogous to what would be possible for vacuum tubes if tube types which employed positive electrons also existed.

Mode of Operation. The common base connection described above results in voltage gain but not current gain. In fact, for junction transistors the ratio of collector current changes to emitter current changes (with common base and at zero frequency), called α_{b0}, is slightly less than unity. A unit change in d-c emitter current corresponds to a collector current change of α_{b0} and a base current change of $1 - \alpha_{b0}$. If the transistor is used in a common emitter connection, as shown in Fig. 15, the input signal may be applied as a change in base current, where the resulting low-frequency collector current change is $\alpha_{b0}/(1 - \alpha_{b0})$ or α_{e0} times larger than the input. The value of α_{e0} approaches infinity as α_{b0} approaches unity. Since the collector-emitter voltage V_{CE} can be much higher than the base-emitter voltage V_{BE}, voltage gain is possible in addition to the current gain α_{e0} for the common emitter connection. Practical forms of these circuits and quantitative circuit relations are to be found in Chap. 27, Transistor Circuits, Sect. 3, Low-Frequency Amplifiers, for common emitter and common base as well as common collector connections. Typical characteristics are shown in Fig. 5 of Chap. 27, Transistor Circuits.

Unfortunately, the simple mode of operation described above does not apply strictly at higher frequencies, where alpha decreases, where there is a lag between emitter and collector currents, and where the effective built-in capacitances between the electrodes become significant. Also, at high temperatures an increase of collector current occurs which is not controlled by the emitter current.

In order to design semiconductor circuits around these inherent char-

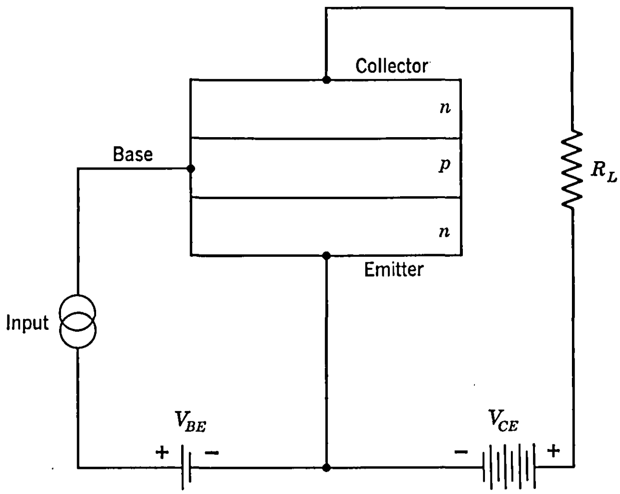


FIG. 15. Common emitter connection for *n-p-n* transistor.

acteristics, it is desirable to have some understanding of how they depend upon semiconductor principles.

Injection. For simplicity, consider the *n-p-n* transistor in Fig. 14*a*, and assume that the emitter-base junction is biased as shown in the forward direction, i.e., with the *n* region negative, and also that the collector-base junction is biased inversely, i.e., *n* region positive. The forward current results not only in electrons being injected from the emitter region into the base region but also, simultaneously, in at least a few holes being injected from the base region into the emitter region. Part of the emitter current is thus carried by the electrons injected into the base, the rest by the holes injected from the base into the emitter. It is desirable to have nearly all the emitter current carried by the electrons injected into the base. The fraction of emitter current carried by electrons is called the *injection efficiency* gamma (γ), and its value (which may be well above 0.99 at low currents) depends upon physical constants of the emitter and base regions built into the transistor. It is given by $\gamma = 1/(1 + \sigma_B W_B/\sigma_E L_{PE})$, where σ_B is the conductivity of the base region, W_B the base width, σ_E the conductivity of the emitter region, and L_{PE} the *diffusion length* of holes in the emitter, i.e., $L_{PE} = (D_P \tau_E)^{1/2}$ where D_P is the diffusion constant for holes and τ_E the lifetime of holes in the emitter. The injection efficiency decreases at high currents largely owing to increase of σ_B caused by injected carriers (see Ref. 31). The injection efficiency increases slightly with collector voltage because the collector depletion layer widening causes a decrease of the effective value of W_B.

Diffusion. The injected electrons move about by random thermal motion (corresponding at room temperature to thermal velocities of about 10^7 cm/sec), so that they would tend to fill all the base region to a uniform density, along with the holes which are present to neutralize them. However, the collector junction is inversely biased, i.e., it is positive with respect to the base. The injected electrons can pass across the high-voltage collector junction without difficulty and into the collector circuit. They cannot return because of the inverse voltage. The passage of electrons from emitter to collector is therefore ideally a case of diffusion from a source to a sink.

The *collector cutoff current* I_{CBO}, which flows even in the absence of emitter current, is (for an *n-p-n* transistor) made up of holes moving from collector to base and electrons moving from base to collector. The hole current from the collector to the base is always designed to be small by using a low-resistivity collector region which suppresses hole generation, and the collector current due to electrons in the *p* type base is also normally small in the absence of emitter current.

Each injected electron wandering about in the base region is matched by an excess hole supplied by the base lead to maintain charge neutrality, and some of them recombine. Each electron recombining with a hole in the base region thus results in a net flow of base current. Each electron crossing the collector junction contributes to collector current, and in this case the extra hole stays in the base to neutralize another new electron or returns to the base circuit without causing any net d-c base current.

The fraction β of injected carriers which has not recombined and therefore reaches the collector is called the *transport efficiency*. It is given by $\beta = \text{sech}\ (W_B/L_B) \doteq 1 - W_B^2/2L_B^2$ where L_B is the diffusion length of minority carriers in the base (see Ref. 8 and 34). It thus depends principally upon factors which are built into the transistor. At high current the lifetime of electrons in the base may change, and the result will be in variations in β, but ordinarily transistors can be designed with W sufficiently small that β is nearly unity at all currents. The transport efficiency increases slightly with collector voltage because of the decrease in W_B as a result of collector junction depletion layer widening.

Strictly speaking, the flow of charge across the base region is due to the combined effects of diffusion and drift. At high currents there is a small electric field which reinforces the diffusion effect as though the diffusion constant were increased somewhat (less than doubled) (see Refs. 31 and 32). This effective increase in diffusion constant also improves the high frequency and transient behavior at high currents (Ref. 33).

Collection. The minority carriers are collected by diffusion across the inversely biased collector junction where they are swept away by the electric field which is high enough to overcome back diffusion. All of them contribute to the collector current. However, some hole-electron pairs are formed and result in carrier multiplication due to avalanche effects occurring in the high electric fields (up to 10^5 volts/cm and higher) in the collector depletion layer. The collector multiplication factor M is given approximately by $M = 1/[1 - (V_C/BV)^n]$ where V_C is the collector voltage and BV is the avalanche breakdown voltage of the collector junction, and where n is an exponent characteristic of the material, ranging from $n = 1.4$–2 and 3.4–4 for silicon *n-p-n* and *p-n-p*, respectively, and higher values of $n = 3$ and 6 for germanium *p-n-p* and *n-p-n*, respectively (see Refs. 23, 35, and 36). It follows that M is essentially unity for low collector voltages but increases rapidly as V_C approaches V_B.

Current Transfer Ratio. The net change in collector current per unit change in emitter current is called the current transfer ratio, or alpha, and is the product of injection efficiency, transport efficiency, and collector multiplication, i.e., $\alpha_{b0} = \gamma\beta M$. It is clear that α_{b0} should be expected to decrease at high currents owing to "γ fall off," and to increase at high voltages owing to increase of M. The total collector current I_C is given by $I_C = I_{CB0} + \alpha_{b0}I_E$ where I_E is emitter current. Alpha is normally slightly less than unity, permitting high current gains in the common emitter connection which are stable as long as alpha does not exceed unity. At high collector voltage, however, alpha can exceed unity owing to collector multiplication, and instability may result. The region of α_{b0} greater than unity has also been used for regenerative switching purposes (see Refs. 38 and 39).

As collector voltage increases, the collector junction depletion layer encroaches upon the base region and reduces its effective width. This has the effect of increasing alpha by increasing both γ and β. Carrier multiplication due to avalanche effects also causes alpha to increase with V_{CB}.

Collector Conductance. It might appear that the output incremental conductance of the collector would be essentially the same as the inverse conductance (dI/dV) of an inversely biased *p-n* junction, which is practically zero. In addition to the effect of leakage conductances, however, an appreciable conductance arises from the increase in alpha with collector voltage mentioned above (see Ref. 40). Since the a-c collector conductance is $(\partial I_C/\partial V_C)_{I_E}$, if the collector voltage is increased by ΔV_C resulting in a change in α_{b0} of $\Delta\alpha_{b0}$, the collector current will be increased by an amount $\Delta I_C = I_E\Delta\alpha_{b0}$, so that the average conductance thus introduced is $\Delta I_C/\Delta V_C$ or $I_E(\partial\alpha_{b0}/\partial V_C)_{I_E}$. This conductance, called the Early effect, is commonly the dominant factor in collector output conductance. The effect also results in a pronounced increase in collector conductance at high emitter current and at high collector voltage.

Although the discussion above has referred to the *n-p-n* transistor as an example, the *p-n-p* types operate upon the same principle, except that all the polarities are reversed.

Temperature Effects. The most pronounced temperature effect upon transistor characteristic is the variation of I_{CBO} which increases more than 5% per degree centigrade (i.e., doubles every 10 degrees centigrade, increases by one decade every 30 to 40 degrees centigrade) in the normal range of operating temperature (see Fig. 13 of Chap. 27, Transistor Circuits). This is the same effect which limits rectifier performance at high temperatures. In general, there are two practical approaches to handling this high-temperature problem: (1) Choose a transistor whose cutoff current is so small that its variations have no appreciable effect upon the operating point. It is for this reason that silicon transistors are important for high-temperature operation. (2) Design stabilization circuitry so that changes in I_{CBO} are compensated or otherwise prevented from affecting the operating point. This is extremely important for d-c amplifier design. But I_{CBO} will eventually become so large that the dynamic range of the device or the capabilities of the power supply are exhausted in merely supplying this junction leakage current, so that the useful operating temperature range can be extended only to a certain extent by stabilization. In addition to shifts in operating point due to I_{CBO}, the transistor current gain, impedances, and power gain also vary with temperature. Negative feedback is often useful to minimize the effect of such temperature variations as well as to make the circuit insensitive to interchanging transistors with slightly different individual characteristics (see Chap. 27, Transistor Circuits, Sect. 2, Temperature Effects and Bias Stabilization).

Alpha sometimes increases with temperature because of the increased number of minority carriers in the collector region, but this effect can be avoided by proper design (see Ref. 41). Owing to these and other variable effects, the exact temperature dependence of alpha must be examined for each type of transistor wherever small variations are to be considered.

High-Frequency Effects. It is clear that any process of diffusion is really an average "wandering" or "leakage" of a quantity (as by random thermal motion) into regions where its concentration is lower. Characteristic of this process, there is a diffusion time delay t_d for a signal diffusing across the base width from emitter to collector given by $t_d \doteq W_B^2/D$. This results in a phase lag which is roughly proportional to frequency.

At higher frequencies, when the phase lag may become comparable to the time interval required for a quarter cycle, or more, some of the injected carriers can actually be delayed, canceled, and lost by backward diffusion during parts of the cycle, i.e., the maxima tend to diffuse into the minima.

This mechanism sets an upper frequency limit to diffusion processes even in ideal transistor structures.

In addition to the dispersion in transit time introduced by the diffusion process itself, the construction of nonideal transistors may be such that W_B is not constant, i.e., the emitter and collector junctions may not be parallel with each other and a geometrical dispersion of the transit time results. Since the diffusion transit time is longer for those regions of the transistor with wider W_B (usually around the edges), parts of the emitted signal may arrive at the collector out of phase, and partial cancellation can occur at very high frequencies. In addition, even at moderate frequencies, the high-frequency collected signal will be weaker for the wide base portions of the transistor, and part of the emitted signal is thus lost by attenuation (see Ref. 42).

The Alpha Cutoff Frequency. The frequency at which the value of α_b is decreased to 70% (i.e., $2^{1/2}/2$) of its low-frequency value α_{b0} is called the alpha cutoff frequency f_{ab} (see Chap. 21, Sect. 4). The alpha cutoff frequency is very roughly equal to the reciprocal of the diffusion time delay. But the diffusion time delay (equivalent to a phase shift which increases with frequency) causes $1 - \alpha_b$ to increase even before α_b has dropped perceptibly. This effect may be visualized circuit-wise by recalling that $1 - \alpha_b$ is proportional to the magnitude of the "vector" difference between emitter current and collector current. Physically it is related to the fact that the injected carriers passing through the base must be neutralized instantaneously, while they are in the base region, by carriers which move in from the base lead, and then move back out into the base lead as the carriers are collected. This reciprocal charge motion is, in fact, an alternating current in the base circuit proportional to $1 - \alpha_b$.

The increase in $1 - \alpha_b$ causes the common emitter current gain proportional to $1/(1 - \alpha_b)$ to cut off, i.e., drop to 0.7 of the low-frequency value, at a much lower frequency f_{ae} than the alpha cutoff. As a useful approximation, $f_{ae} = (1 - \alpha_{b0})f_{ab}$ (sec Refs. 43 and 44).

Power Gain and Frequency. The actual behavior of power gain with frequency depends upon the detailed interaction of these factors with the distributed resistive and capacitative impedances of the transistor and may be calculated from the appropriate equivalent circuit (see Figs. 47 and 48 of Chap. 27, Transistor Circuits, and Sect. 1, Basic Circuit Considerations and Symbols, and Sect. 4, High-Frequency Amplifiers).

In general, the power gain of a reasonably ideal transistor at low frequency is independent of frequency out to the point at which high-frequency effects begin to decrease gain. The rate of decline of available gain with frequency is then about 6 db per octave for abrupt junction transistors, and also for *p-n-i-p* and drift transistors; i.e., gain is proportional to f^{-2}. For graded junction transistors, such as grown n-p-n types, the base resistance feedback interactions which lower the gain often result in a rate of decrease of gain of about $4\frac{1}{2}$ db per octave, or gain proportional to $f^{-3/2}$ (see Chap. 27, Sect. 4, High-Frequency Amplifiers).

For each transistor, a frequency limit f_{max} is finally encountered beyond which the power gain falls below unity. This limit can be measured conveniently by connecting the transistor in a suitable oscillator circuit and noting as f_{max} the frequency beyond which it is not possible to obtain oscillation. Since this limit can be expressed approximately by $f_{max} = 0.2 (f_{ab}/r'_bC_c)^{1/2}$ where r_b is small signal base resistance and C_c is collector capacitance, this expression is often used as a figure of merit for high-frequency applications (see Chap. 27, Transistor Circuits, Sect. 6, Oscillators, Modulators, Mixers and Detectors).

A reasonably accurate idea of the possible variation of gain with frequency can be obtained graphically as in Fig. 16 by determining f_{max} for the transistor in question, either from direct observation or by calculating the figure of merit. On semi-logarithmic paper, with gain in decibels as linear ordinate, the frequency response in the declining range may be represented approximately by a straight line through 0 db at f_{max} having a slope representing 6 db decrease per octave for most transistors (except grown junction transistors, usually n-p-n, $4\frac{1}{2}$ db per octave). The low-

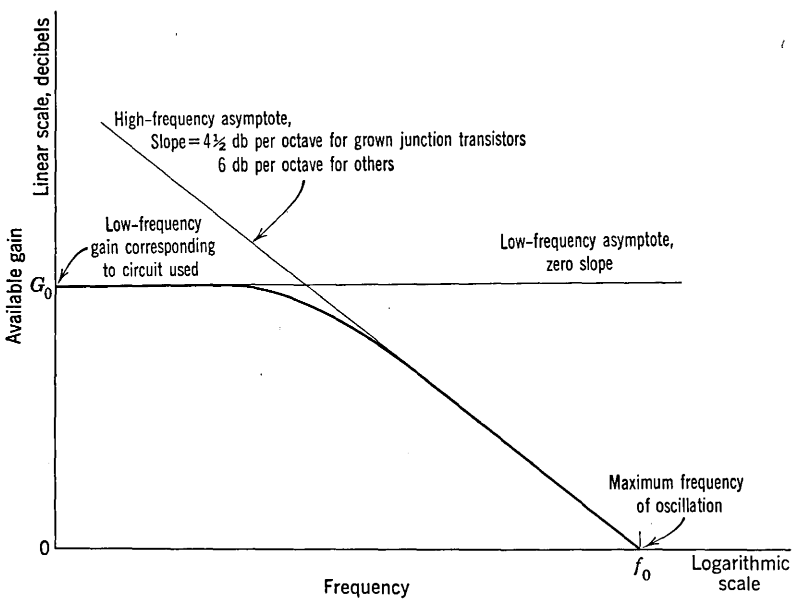


FIG. 16. Graphical representation of typical gain-frequency behavior of transistors.

frequency gain asymptote is drawn as a straight line with zero slope representing the appropriate low-frequency power gain G_0 observed or calculated for the circuit used. The actual available gain for the transistor as a function of frequency is ideally a continuous curve joining the two asymptotes.

High-Frequency Circuit Design. This idealized representation ignores the detailed effects of the circuit and indicates in an approximate manner what can reasonably be expected in stable amplifier circuits as limited by the transistor. The circuit can be designed to have lower gain in any frequency range. It is also possible to obtain more gain by regenerative feedback, but at the expense of stability. As an extreme case, an oscillator can be regarded as an amplifier with infinite gain. Unfortunately, the transistor contains certain feedback impedances built into its characteristics, and these must be taken into account together with the amplifier circuit in defining available gain in a careful manner. In fact, conventional amplifier circuits are often unstable and may even oscillate owing to their feedback effects if not properly "neutralized" (see Chap. 27, Transistor Circuits, Sect. 4, High-Frequency Amplifiers).

High-Frequency Transistor Design. The above frequency limitations are fundamental results of the diffusion process whereby carriers move from emitter to collector. It is possible to improve this process somewhat by designing the base region with a built-in emitter-collector electric field which can be produced by introducing a gradation or step in the normal carrier density in the base region. Such transistors permit use of thinner bases having less difficulty with high base resistance than would result from uniform thin bases designed for the same maximum collector voltage. The drift transistor (Refs. 46, 47, and 49) and the *p-n-i-p* transistor (Ref. 48) are examples of the successful application of this technique which permits up to several octaves improvement in frequency response.

Charge Storage. The phenomenon of charge storage limits the large signal behavior of transistors, just as it does with rectifiers; i.e., a transistor carrying a high current will not cut off until the stored charge due to injected carriers has diffused out of the base region. The normal cleanout time for grounded base circuits is the order of $1/f_{ab}$, i.e., the time required for the previously injected carriers to diffuse to the collector. This time can be shortened in circuits where the emitter junction can be biased inversely, since the carriers can then diffuse toward the emitter as well as the collector.

The higher gain of the grounded emitter circuit often carries with it the penalty of longer switching time. The approximate rise time t_r is given by

$$t_r = \frac{1}{2\pi f_{\alpha e}} \ln \frac{\alpha_e I_{b1}}{\alpha_e I_{b1} - 0.9 I_s},$$

where I_s is the current to be switched, I_{b1} is the base current used to switch the transistor "on," i.e., from cutoff to saturation in the circuit employed. The approximate fall time t_f may be expressed similarly,

$$t_f = \frac{1}{2\pi f_{\alpha e}} \ln \frac{I_s + \alpha_e I_{b2}}{0.1 I_s + \alpha_e I_{b2}},$$

where I_{b2} is the base current used to switch the transistor "off," i.e., from saturation to cutoff. The value of α_e used here is the large signal average value effective over the total current swing. It is seen that both rise time and fall time can be decreased by driving the transistor harder, i.e., by using larger I_{b1} and I_{b2}, but at the expense of gain. The fall time will also generally be somewhat larger than the time constant of the collector circuit $R_L C_c$. The speed of switching is higher for high-gain transistors. The current gain with emitter and collector interchanged is a factor in some circuits.

The total stored charge depends upon the current originally flowing; since the current is due to diffusion, higher currents result in more stored charge. The amount stored for a given current depends upon the construction of the transistor, being smaller for thin base units (see Refs. 45, 49, 50, and 69; also Figs. 77 and 78 of Chap. 27, Transistor Circuits, and Sect. 7, Switching Circuits).

Diffusion Capacitance. In addition to the large signal charge storage effects in transistors and rectifiers, there is a small signal charge storage capacitance which increases the collector capacitance of junction transistors at high currents by a mechanism which has no direct analogy in rectifier operation. The charge accumulated, or "stored" in the base region in order to maintain diffusion at high current, is proportional to the base width and proportional to the current. When the collector voltage increases, the collector junction depletion layer spreads further into the base region, decreasing the base width. This in turn changes the amount of stored charge in the base region. The change in stored charge per unit change in collector-base voltage is equivalent to an incremental capacitance, in parallel with the collector depletion layer capacitance, and is called the *diffusion* capacitance, since the charge is stored as a consequence of the diffusion of carriers. It is proportional to collector current and is independent of frequency at low frequencies (see Ref. 51). It decreases with frequency in the vicinity of the alpha cutoff frequency, becoming negligible at higher frequencies.

Since it is proportional to the rate of change of depletion layer with voltage, diffusion capacitance can be minimized by proper junction design. Fortunately the same distribution of resistivity which improves high-frequency gain in the drift and *p-n-i-p* transistors also minimizes collector diffusion capacitance to the extent that it is likely to be much smaller than the collector depletion layer capacitance.

In simple alloy or surface barrier types with homogeneous base material, the diffusion capacitance can be significantly large. For example, a silicon *n-p-n* alloy transistor whose base width is 0.001 in., made of material having a homogeneous resistivity of 1 ohm cm, will have a diffusion capacitance of about 1500 $\mu\mu$f at 1 volt on the collector and 1 amp collector current. The diffusion capacitance is independent of the area of the transistor junctions, since it depends upon total current. The collector capacitance of the same silicon *n-p-n* transistor due to collector junction depletion layer alone would be 400 $\mu\mu$f/mm^2 of collector junction area. In this transistor, both diffusion capacitance and depletion layer capacitance decrease inversely as the square root of the collector voltage.

High Power. The operation of a transistor at high power implies operation at high voltage and/or higher current in addition to higher dissipation. Many of the "high power" problems are really merely indications that applications exist which tax the capabilities of presently available transistors.

High Voltage. The maximum permissible collector voltage of a transistor ordinarily depends upon features built in by the designer but is seldom greater than 100 volts, more often less than 50 volts. Although momentary pulse operation at collector voltages greater than specified is sometimes possible, failures are often associated with momentary overvoltages in the collector circuit.

High Current. The high-current problem on the other hand is usually one of gain rather than failure, since the current gain normally passes through its maximum at a fairly low current and then decreases monotonically, until the practical maximum collector current is determined by that current beyond which the current gain is insufficient for the application in question. High current gain at high current and low series resistance are the principal objectives encountered in high-current switching. Unless current gain holds up, the transistor will "saturate" at high collector currents, particularly when the voltage drop across the series resistance reduces the effective collector voltage to a value too low for proper operation.

The design of a high-power transistor involves much more than scaling up the dimensions of a low-power structure to handle the increased dissipation. For one thing, the base thickness cannot be increased without degrading high-frequency performance. And a complicated emitter-base geometry must usually be provided for high-current operation in order to provide reasonable current gain at high currents (see Refs. 52 and 89).

The average output and load impedances at maximum output are determined from the voltage and current swing just as they are for vacuum tubes.

Thermal Design for High Power. With regard to dissipation, the ultimate limit to high dissipation is ordinarily high junction temperature, resulting in high I_{CBO}, in still further dissipation, and ultimately in thermal runaway. The higher the existing ambient temperature, the more care should be devoted to installing the transistor in such a manner as to minimize the *external thermal resistance* (temperature rise, case to ambient, per watt dissipation) between the transistor and the ambient. A low thermal resistance contributes directly to thermal stability at all temperatures, permitting higher stable dissipation. A well-designed transistor should have a low *internal thermal resistance* (i.e., low temperature rise, junction to case, per watt dissipation). Germanium transistors are commonly limited to specified maximum operating junction temperatures, usually below 100°C; silicon transistors have similar limitations but can usually be operated at 150°C or higher.

The internal thermal resistance of a transistor (degrees per watt) is usually published by the manufacturer. Lower dissipation ratings are usually published for higher case or study temperatures. Even when the thermal resistance is not published, it is often possible to obtain a useful estimate of this quantity, noting that it is roughly equal to the rate of decrease of permissible case or stud operating temperature with power (degrees lower stud temperature per watt additional dissipation). For example, consider a power transistor whose dissipation rating by the manufacturer decreases 5 watts for a 20-degree centigrade increase in case temperature. This implies a probable internal thermal resistance of about 4 degrees centigrade per watt. In an application where such a transistor is to be bolted to a heat sink with an estimated thermal resistance to ambient of, say, 2 degrees per watt, there would be little to be gained in thermal stability by further elaboration of the external heat sink. On the other hand, redesign of a heat sink with 10 degrees centigrade per watt thermal resistance to ambient could probably be justified if thermal stability were a problem. In general, it is better to use actual values of thermal resistance, where possible, rather than to depend on general rules of thumb regarding recommended fin thicknesses, area, etc.

All transistors contain a certain amount of passive electrical resistance in series with emitter, collector, and base. These resistances are not completely modulated by transistor action within the device. In operation, voltage drops across these resistances in the output may seriously decrease the dynamic range of the device. Those in the input circuit increase the difficulty of driving. Even one-quarter of an ohm anywhere in the emitter or collector circuit of a high-current transistor will result in a passive voltage drop of $2\frac{1}{2}$ volts when the device is operated at 10 amp, and this will cause a serious reduction in dynamic range when the supply voltage is limited. In switching circuits, excess series resistance may lead to unwanted crosstalk, feedback, or lack of isolation between separate circuits.

6. TRANSISTOR TYPES

Transistors are feasible in either *p-n-p* or *n-p-n* configurations for any of the types to be described. Typical outlines of commercially available small signal and low intermediate power transistors are shown in Fig. 17, higher power structure outlines in Fig. 18. Although there are some differences in present commercial availability, both *p-n-p* and *n-p-n* types should eventually become available with nearly equivalent characteristics as required for complementary circuit design or by power supply polarity.

Structures. The many semiconductor structures developed for various *n-p-n* and *p-n-p* transistor types can be roughly segregated into two general classes of construction: the bar geometry illustrated in Fig. 19*a* usually associated with grown junctions and the wafer geometry of Fig. 19*b* usually associated with alloy, surface barrier, and diffused types. Some configurations are obtained by etching into one side of a rigidly supported wafer, leaving the active portion of the device standing in low relief above the final surface of the wafer as a flat-topped plateau with well-defined edges. These are sometimes referred to as *mesa* geometries. Figure 20 shows various methods of forming semiconductor junctions and contacts.

Bar Geometry. The bar geometry arises from the fact that grown junction transistors are cut from crystals containing the junctions as grown (Refs. 8, 14, 29, 30, and 54), or are made from bars with junctions introduced after cutting by processes which involve melting back the end of the bar and allowing it to resolidify (Refs. 55 and 56). Bar geometry transistors have been made in both *n-p-n* and *p-n-p* types, but for reasons related to practical technology, the available bar transistors are *n-p-n* types. Because of their geometry, some types of bar transistors have noticeable series resistance in the emitter and collector which can seriously limit their usefulness for high-current operation, even though they may be capable of dissipating the required power. High-current operation for bar types is usually limited also because of the relatively small cross-section area which can be controlled by the small base contact at the outer edge of the *p* region.

Bar transistors can be conveniently fabricated into rugged structures with smaller cross section and lower capacitances than the ordinary wafer types, an advantage for high-frequency applications. It is also possible to

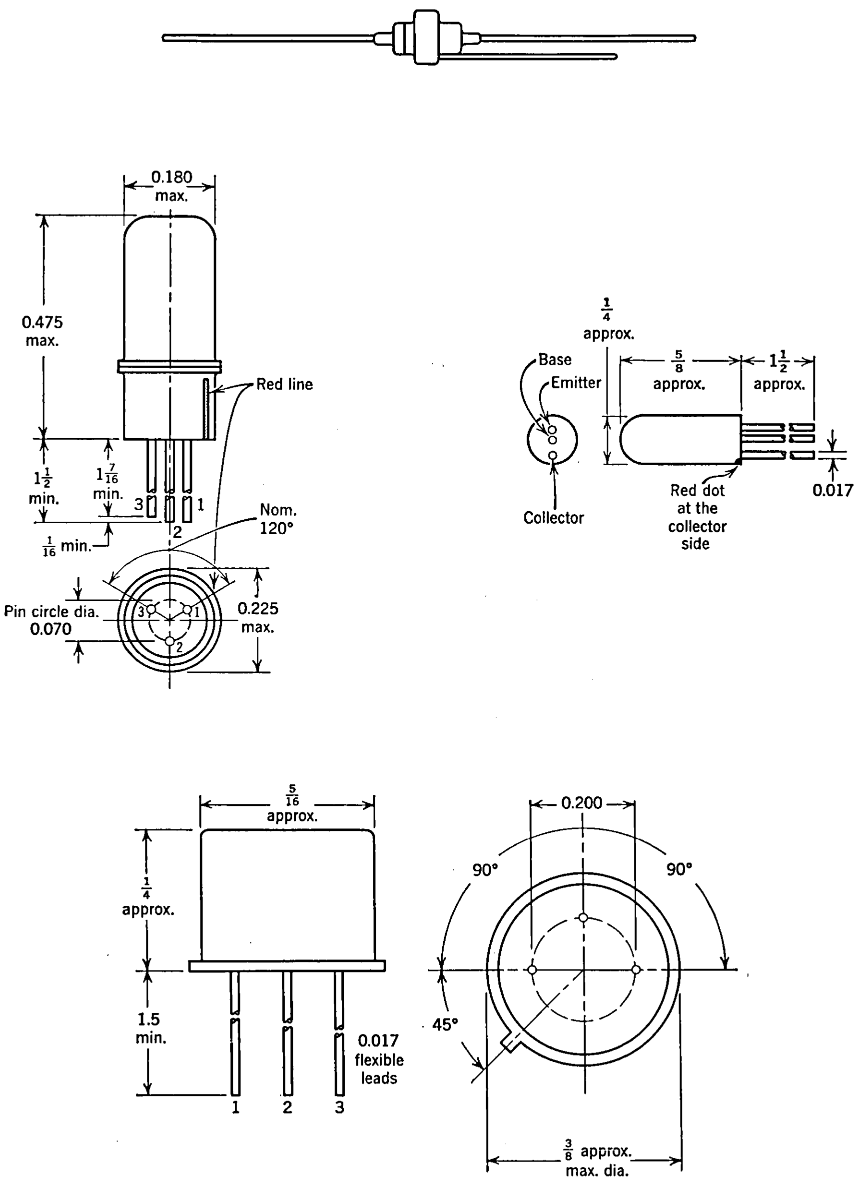


FIG. 17. Typical commercial small signal transistor outlines.

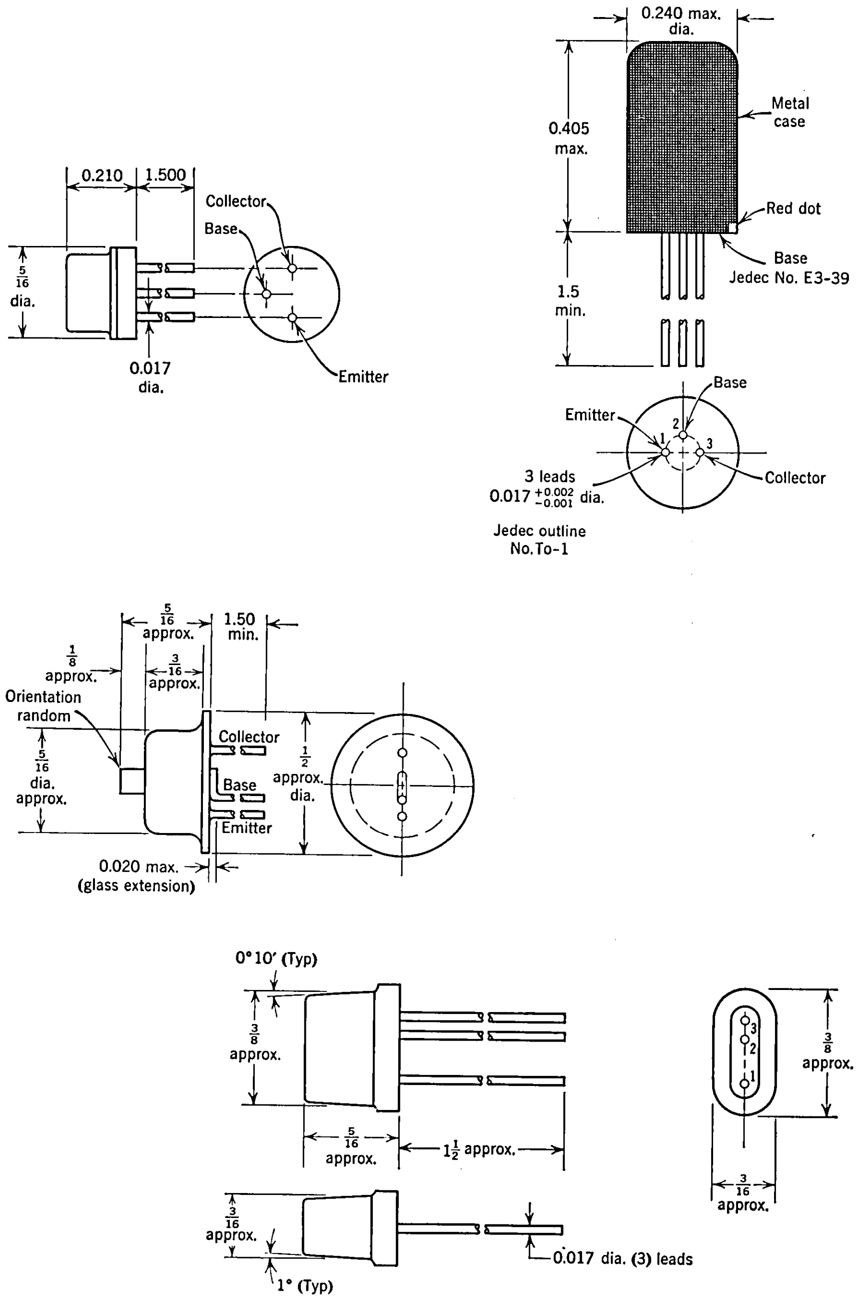
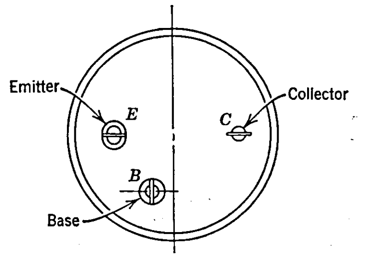
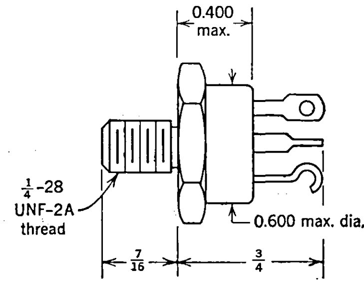
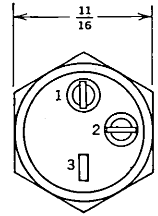
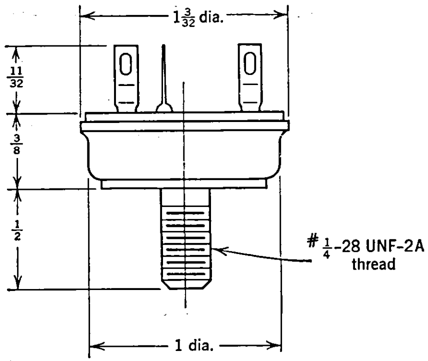


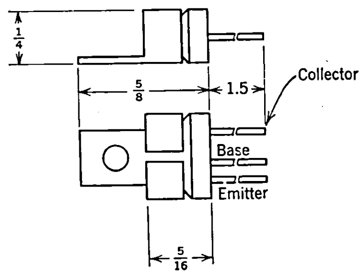
FIG. 17—(*Continued*)

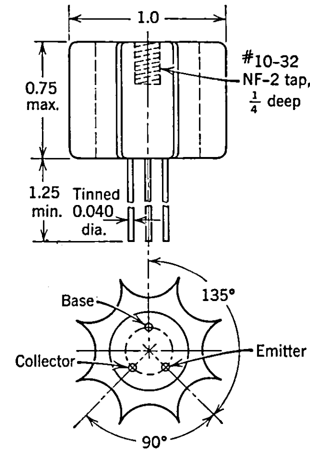

Emitter
E
C
Collector
B
Base


0.400
max.
1/4-28
UNF-2A
thread
0.600 max. dia.
7/16
3/4


11/16
1
2
3


1 3/32 dia.
11/32
3/8
1/2
1/4-28 UNF-2A
thread
1 dia.


1/4
5/8
1.5
Collector
Base
Emitter
5/16


1.0
#10-32
NF-2 tap,
1/4 deep
0.75
max.
1.25
min.
Tinned
0.040
dia.
135°
Base
Collector
Emitter
90°

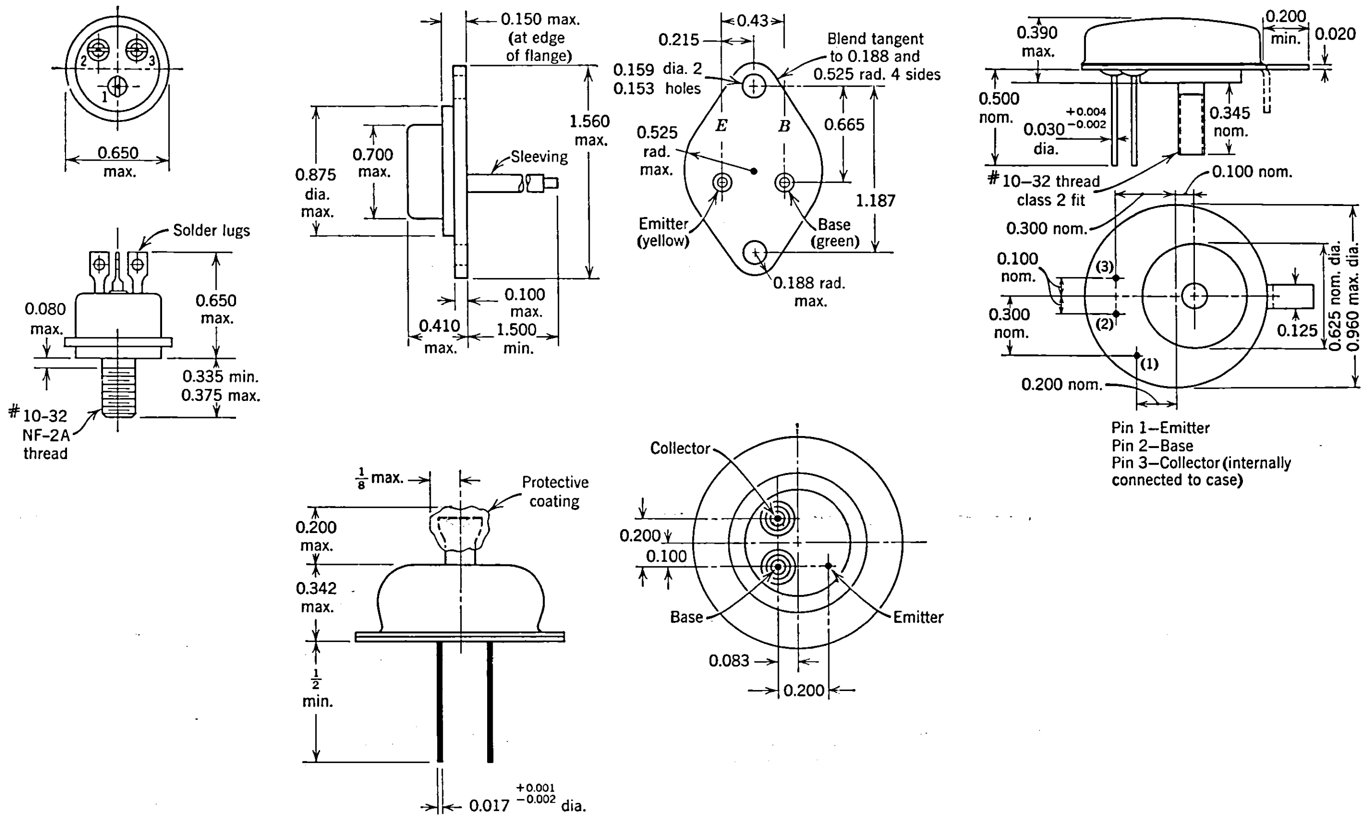


FIG. 18. Typical commercial power transistor outlines.

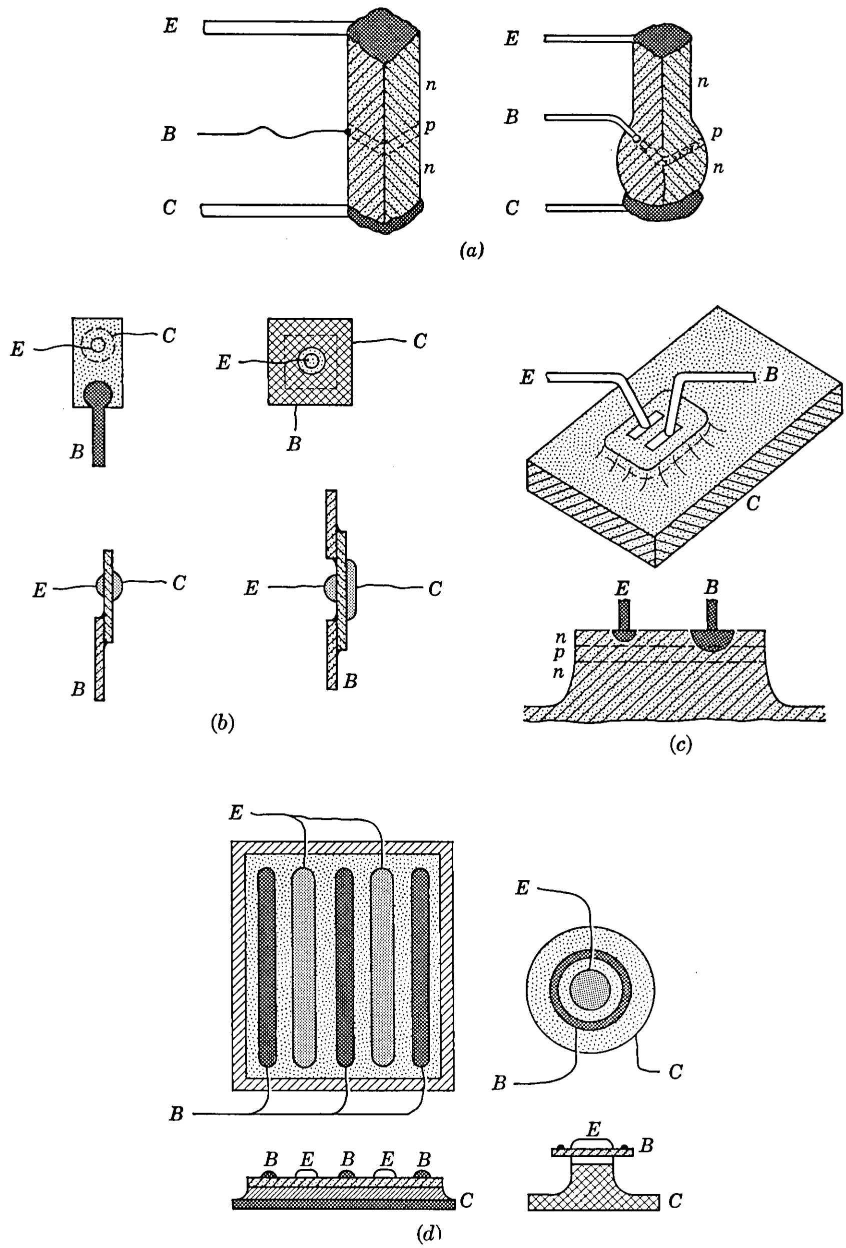


FIG. 19. Junction transistor configurations: (*a*) bar geometries, (*b*) low-power alloy wafer geometries, (*c*) high-frequency diffused wafer (mesa) geometry, (*d*) high-power alloy wafer geometries.

grow the emitter-base and collector regions in the crystal with resistivity distributions (i.e., abrupt emitter junction, graded collector junction) approximating the ideal *p-n-i-p* or drift transistor distributions.

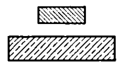
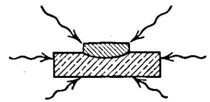
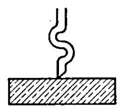
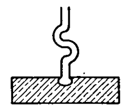
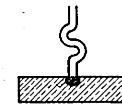
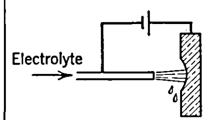
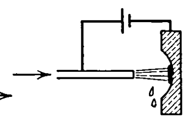
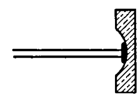
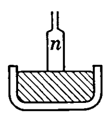
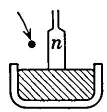
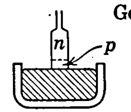
Wafer Geometry. The wafer geometry is usually associated with transistors made by creating the emitter and collector junctions on a semiconductor wafer previously cut to the desired size. Such structures are rugged, capable of sufficient elaboration in the emitter-base geometry for high-current operation, and are characterized by very low series resistance in the emitter and collector contacts. Furthermore, they are widely available in both *n-p-n* and *p-n-p* types. The emitter and collector can be made by alloying or plating which produces abrupt junctions, or by solid diffusion, an extremely versatile junction process which can be controlled to produce almost any degree of gradation (see Refs. 8, 14, 41, 45, 57–64).

The wafer geometry has also been approximated using grown junctions in the "remote base" power transistor (see Ref. 65).

The distinction between the bar and wafer geometries is not fundamental but is made here because of practical considerations related to types available at present. Transistor technology is becoming more independent of available structures as the various configurations are gradually being optimized.

Semiconductors. *Materials.* Germanium and silicon types are feasible in most ranges of current, voltage, and frequency. Germanium is now available in more types, chiefly because of the more highly advanced state of germanium technology. Silicon types have characteristics generally similar to their germanium counterparts except for lower I_{CBO} operability to higher temperatures and somewhat higher hold-on voltage drop for switching types. Semiconductors other than germanium and silicon have been used to make developmental transistors, but as yet none are available.

Commercial Types. *Low Power, Low Frequency.* There is a wide range of germanium and silicon transistors available in the low-dissipation (0–150 mw) low-frequency (0–200 kc) range, suitable for audio, etc., also medium-frequency types (200 kc–5 mc) suitable for low-frequency RF amplification and switching. Early types of higher frequency transistors have generally sacrificed dissipation ratings to achieve the small dimensions and low capacitances essential to high-frequency gain. On the other hand, early power transistors have usually sacrificed high-frequency gain in favor of rugged construction and efficient heat flow. With improving technology, the present trend is to optimize the design for more output at a required frequency, or for a wider useful frequency range for the required power rating.

Alloy	 Semiconductor and alloy in contact.	 Alloy metal is melted, dissolves some semiconductor.	Ge or Si  Semiconductor recrystallizes.	Wafer geometry devices are prepared individually. Many can be alloyed simultaneously.
Bonded	 Fine pointed wire, e.g. gold, in contact with semiconductor.	 Pulse of current causes end of wire to melt into semiconductor, forming eutectic at fairly low temperature.	Ge or Si  Semiconductor recrystallizes on cooling, creating tiny alloyed junction or contact, complete with lead wire.	Individual fabrication is commonly used for wafer geometry diodes or contacts to bar geometry devices.
Surface Barrier	 Wafer is electrolytically jet-etched using salt of metal to be plated in next step.	 Polarity is reversed and metal contact is plated onto freshly etched surface.	Ge or Si  Lead wire is soldered to plated contact. Heat may be applied prior to soldering to produce microalloy junction.	Individual etching and plating. Wafer geometry.
Grown Junction	 Growing n type crystal is slowly withdrawn from melt.	 Sufficient acceptor impurity is added to melt so that remainder of crystal will be p type.	Ge or Si  P-n junction bars are later sawed out of crystals. N-p-n transistor junctions are also possible by second (donor) doping.	Many bar geometry pellets containing junctions are cut from the junction region of each crystal, contacts are applied individually.

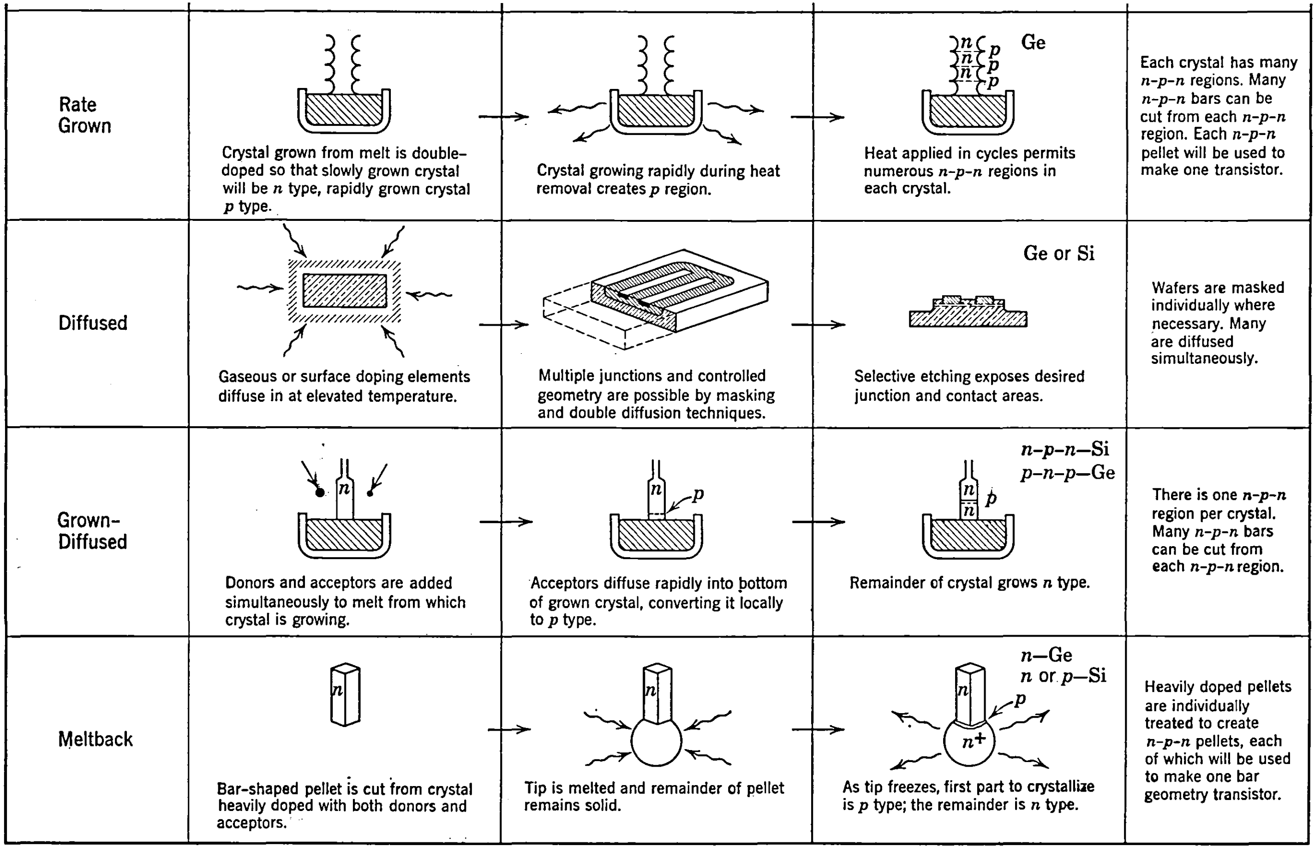


FIG. 20. Formation of semiconductor junctions and contacts. Many combinations and modifications of these processes are in commercial use.

The typical germanium or silicon low-power audio transistor operates with 50–150-mw dissipation, at a maximum collector voltage of 15–60 volts, and at a maximum collector current of 25–100 ma.

Intermediate Power. There are numerous available germanium intermediate power types, mostly *p-n-p,* all of the wafer geometry, capable of maximum collector currents ranging from several hundred milliamperes to several amperes, with maximum collector voltage typically ranging from 25 to 80 volts. A few medium current transistors are listed for use in the 100–200-volt range of maximum V_{CB}.

High Power. Since transistor circuitry typically involves fairly low-voltage power supplies, high-power operation is commonly limited by the maximum current for which there is useful gain. Germanium *p-n-p* transistors are available for operation to 30 amp, the *n-p-n* types being limited to 3 amp. Silicon *n-p-n* power transistors have been listed for operation as high as 5 amp; however, at present no *p-n-p* silicon transistor is listed for operation above 200 ma. Transistors usable above 30 amp should soon become available in both *p-n-p* and *n-p-n* types, probably first in germanium, later in silicon.

Intermediate Frequency. Transistors for operation in the low RF range, for 456-kc IF stages, and for RF amplification up to a few megacycles are available in alloy type units, both *p-n-p* and *n-p-n,* and in *n-p-n* grown junction units in silicon and germanium. The generally available RF units are mostly limited to 50-mw dissipation or less. The standard AM broadcast radio receiver band is thus accessible to available transistors, and transistors are widely available in matched sets or kits which will provide specified performance in various radio receiver circuits. Interchangeability charts covering many transistor types are available from manufacturers. Characteristics have not yet been standardized to the same extent as vacuum tubes, and when replacing individual transistors it is often necessary to replace by exact duplicate from the same manufacturer.

High Frequency. Transistors for use in the frequency range from 10–30 up to 600 Mc are available. In the frequency range above a few megacycles, transistor types include the surface barrier transistors with very small plated metal rectifying contacts instead of junctions (Ref. 58), and grown junction tetrodes (Ref. 67) which use an electrical bias to confine the transistor action to regions of favorable dimensions for high-frequency amplification. Some tetrodes have been operated above 1000 Mc. Point contact transistors have been operated at several hundred megacycles.

The *p-n-i-p,* drift, and diffused base transistors make use of base region impurities in more sophisticated distribution than do the uniform wafers normally used at lower frequencies. Germanium and silicon transistors of types used above 250 Mc have utilized various techniques in order to achieve reproducibly and useably the required base widths in the range of 0.1 mil (2 microns). These optimized types are also useful at lower frequencies.

High Power vs. High Frequency. The high-frequency transistors now available are low-power devices just as present high-power transistors are low-frequency devices, and at any foreseeable state of the art, frequency may have to be limited somewhat for the highest power devices, and vice versa. However, the objectives of high gain and high frequency are not entirely incompatible, since many refinements in present transistor structures are improving both high-frequency and high-power performance. For example, very thin base regions are essential to both, and both must employ emitter-base geometry leading to low base resistance and high power gain (see Refs. 45, 66, 69, and 70). The ultimate limitations on transistor design depend upon the semiconductor material used and have not yet been approached very closely by any available types (see Refs. 71 and 72).

Miscellaneous Devices. Discussion here has emphasized the *n-p-n* and *p-n-p* devices together with closely related transistors because these comprise the preponderant majority of types in use today. Other types have been developed which deserve consideration for special applications particularly suited to their unique characteristics.

One-Junction Transistors. It is possible to obtain transistor action using a structure containing only one *p-n* junction as illustrated in Fig. 21.

One use of this geometry is in the *filamentary transistor* in which a pulse of carriers injected at the forward-biased junction *j* (originally a point contact, see Ref. 73) is swept toward one end of the bar. The presence of the extra carriers in the bar modulates its resistance (i.e., more current will pass between *A* and *B* until the pulse is swept out). This modulation effect can be used to provide control of steady current as well as of pulses.

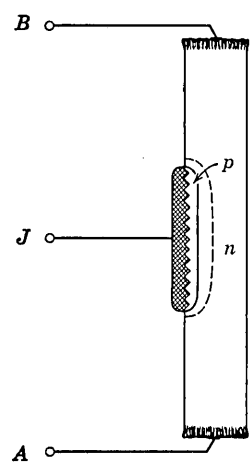


FIG. 21. Single junction transistor geometry.

Another use of one-junction geometry is in the *unipolar* or "field effect" transistor (Ref. 74) in which the junction at *J* is biased inversely. Current can pass between *A* and *B* only through the narrow region behind the junction. As the inverse voltage is increased, the depletion layer widens and the effective width of the conducting path from *A* to *B* is narrowed or "pinched off." A voltage change at *J* can therefore be employed to change the current through the bar. The characteristics are analogous in many respects to those of a pentode vacuum tube (see Ref. 75). This mode of operation is interesting in that the controlled current is carried by the action of majority carriers which can move rapidly in the electric field, and thus the unipolar transistor is not limited at high frequencies by diffusion effects as is the ordinary transistor. One factor which limits frequency response relates to the *RC* time constants involved in changing the bias on the junction capacitance.

A third use of this single junction geometry has been in the development of a regenerative switching device called the *unijunction transistor* (originally called the double base diode) (see Ref. 76). In this application of the single junction geometry, the potential gradient along the bar due to the bar resistance and a bias current between *A* and *B* controls the local potential in the vicinity of the *p-n* junction. If the bias potential is such that the junction is biased inversely, very little current flows, and the device remains nonconducting. If any potential is momentarily shifted to permit forward current at the junction, the resulting modulation of conductivity holds the local potential down so that the junction is locked in a forward bias and the device can remain conducting. The action is analogous to a thyratron except that it conducts with a lower voltage drop across it. On the other hand the thyratron will hold off higher voltages. Its flip-flop action is of interest for circuit simplicity because one unijunction transistor can be used in certain applications in place of two *n-p-n* or *p-n-p* transistors.

These three devices have been described using the same figure, and in fact all three modes of operation have been achieved on the same structure. Naturally the optimum form is different for each. All have been made on both *n*-type and *p*-type bars, although the contact and geometry requirements for the regenerative unijunction transistor are somewhat more critical on *p*-type bars.

Point Contact Transistors. Transistor action was originally discovered using point contacts instead of junctions on a wafer or "base" of germanium as illustrated in Fig. 22 (see Refs. 8, 14, and 77). High-frequency operation is possible with extremely close spacing between the two point contacts (see Ref. 68). Although no longer in general use for small signal amplification (since the junction transistors function in a more nearly ideal and reproducible manner), the point contact transistors have one feature that is of interest. A mechanism for current multiplication is physically formed in the vicinity of the collector contact (Refs.

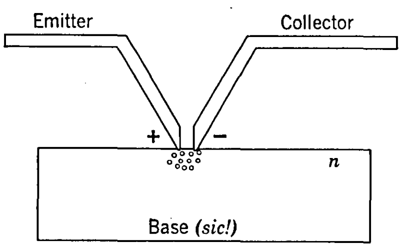


FIG. 22. Point contact transistor.

78 and 79), and alpha therefore is usually larger than unity. This permits one-transistor flip-flop circuits, etc., resulting in a circuit simplicity for some switching applications.

***P-n-p-n* Transistors.** Regenerative switching action can also be obtained by building an extra *p-n* junction into the device near the collector junction. The carriers collected are trapped by the potential minimum or "*p-n* hook" resulting from the extra junction. Their presence in this region causes injection from the collector back into the base, resulting in current multiplication (see Ref. 80). Transistors with a hook collector can be grown in bar form, or the hook region can be created by some combination of grown alloyed diffused junction. Hook transistors have been made to switch several amperes (see Ref. 65).

Other Types. One *p-n-p-n* device, the silicon "controlled rectifier," has been developed to switch kilowatts of power (see Ref. 66).

The *p-n-p-n* devices, regarded as overlapping *p-n-p* and *n-p-n* structures, have high impedances available in the "off" condition for both directions of current flow, which makes them easier than power transistors to incorporate in a-c control circuits. The multiplication which results in high forward conductance in the "on" condition may be initiated by a pulse sufficient to bias one of the junctions beyond its V_B, by momentary current flow into one of the intermediate regions through a third electrode, or by other methods analogous to those used to pulse thyratrons.

Magnetic fields have also been used to control semiconductor devices. For example, the Hall effect is proportional to the product of magnetic field and current, and hence a semiconductor can be used as an analog device to multiply two quantities.

Control of semiconductor properties by charges in the surface layers has also been achieved in the "fieldistor" (Ref. 83).

Arrays of regenerative single junction transistor elements have been arranged on a single slab of semiconductor in such a manner as to provide an entire ring counter in a single device (Ref. 84).

One special class of transistor devices utilizes injection of carriers into the high field created by the depletion layer of an inversely biased *p-n* junction. Triode operation is possible using an emitter contact carefully located within the depletion layer, as in the *depletion layer transistor* indicated schematically in Fig. 23 (Ref. 90). Tetrode operation is also possible, as in the *spacistor tetrode* indicated in Fig. 24 (Ref. 91) where the additional electrode is used to modulate the potential in the depletion layer. One feature of these depletion layer devices is that the injected carriers move by drift in the electric field, thus achieving high-frequency performance not limited by the relatively slow process of carrier diffusion.

Many different types of semiconductor devices (Ref. 29) have been de-

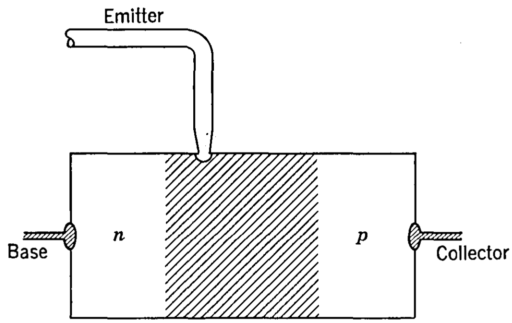


FIG. 23. Depletion layer (triode) transistor.

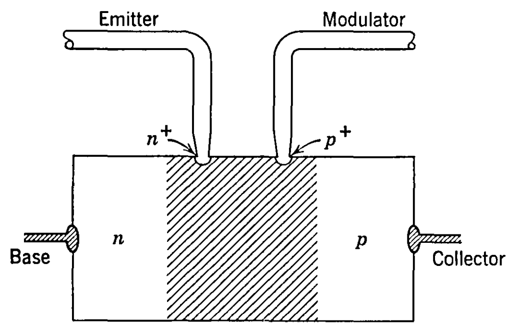


FIG. 24. Spacistor tetrode transistor.

veloped, at least to the extent of proving feasibility. The eventual availability of each will depend upon the alertness of possible users to recognize needed components, as well as the technical success of further development work.

No numerical table of available transistor characteristics has been included in this section because the transistor art is developing so fast (see Ref. 85). Complete specifications of currently listed types have been published annually by *Electronic Industries* magazine, Chilton Company Publishers, Philadelphia.

7. PHOTOELECTRIC SEMICONDUCTOR DEVICES

Types. Radiation in the visible, ultraviolet, infrared, or X-ray spectra can increase the current through passive semiconductor devices. Semiconductor devices can also be used as photovoltaic cells to convert radiant energy into electrical energy for purposes of control or as a source of electrical energy. Several general types of photo devices are outlined in Table 2.

Principle of Operation. All these devices operate on the principle that a quantum of light absorbed by the semiconductor can create a free

TABLE 2. PHOTODEVICE CLASSIFICATION

Type	Use	Examples	Quantum Efficiency	Remarks
Junction photodiode	Control	Ge or Si *p-n* junction	≤ 1	Response speed comparable to circuit frequency response of diode. Sensitivity to incandescent light about 30 ma per lumen absorbed.
Phototransistor	Control	Ge or Si *n-p-n* bar	$\gg 1$	Speed of *n-p-n* comparable to grounded emitter frequency response of corresponding transistor.
Photoconductive materials	Control	PbS, CdSe, PbTe, InSb	$\gg 1$	Full response often very slow (seconds or minutes) but speed can be gained by using only part of d-c response amplitude.
Photovoltaic cells	Power	Si solar cells Lightmeter cells	<1	100 watts per square meter available from sunlight using silicon cells.

electron, a free hole, or a hole-electron pair if the quantum energy is sufficient. The minimum amount of energy required per quantum is usually roughly equal to the energy gap of the material.

Einstein's relation between wavelength and energy $E = h\nu$ can be expressed $E\lambda = 12{,}400$ where E is the quantum energy in electron volts and λ is the wavelength of the quantum in angstrom units. Thus silicon devices (energy gap 1.2 ev) are sensitive to quanta having wavelengths less than about 10,000 A, or 1 micron. Similarly, lead sulfide cells will respond to wavelengths below 3 microns, indium antimonide cells out to about 7 microns, etc. Warm surroundings create additional dark current in cells sensitive to the longer wavelengths. Infrared cells intended for sensitive response beyond a few microns are usually refrigerated to minimize these difficulties.

Photodiodes and Phototransistors. In junction photodiodes and phototransistors, the hole-electron pair created by a quantum absorbed near a junction is equivalent to a minority carrier injected near the junction. In operation, the junction is inversely biased, and the additional minority carriers increase the current. Since one hole-electron pair separated in this manner is equivalent to one electron passing through the device, the simple *junction photodiode* may be regarded as having a maximum "quantum efficiency" of approximately unity. Since ordinary incandescent illumination contains wavelengths not used efficiently, germanium and silicon devices of this type may yield 20–30 ma per lumen absorbed in the sensitive region.

It is possible to multiply this current sensitivity by transistor action, using avalanche multiplication at high fields or "hook" multiplication using *n-p-n* bars, or by point contacts. With some increase in noise and dark current, and with lower frequency response, gains in effective quantum efficiency roughly equivalent to the current gain of the corresponding transistor can be obtained, and such devices are called *phototransistors.*

Since the dark current of these cells is inverse junction current, it increases rapidly with temperature.

In some materials, when hole-electron pairs are produced by quantum absorption as above, the carriers become trapped at lattice irregularities, at multiple microscopic *p-n* junction regions, or at impurity level trapping sites. Mobile carriers are present in increased numbers to maintain charge neutrality, and increased current flows until the excess trapped carriers recombine. Since many carriers can pass through the device for each quantum usefully absorbed, the quantum efficiency can be much higher than unity, but the same mechanism which provides additional sensitivity also decreases the speed of response.

Photovoltaic Cells. In photovoltaic cells the additional hole-electron pairs tend to be separated by the contact potential difference (usually about half the energy gap) across the junction. This produces an external potential difference at the terminals of the device which, operated properly, can recover part of the energy of the absorbed quantum as work done in the external circuit. The theoretically attainable efficiency of silicon using sunlight is less than 20%. Although this is partly due to internal losses, part is fundamental to the distribution of energy in the sun's spectrum, which includes short wavelength quanta with much more than enough energy to create a hole-electron pair, as well as long wavelength quanta which do not have enough energy to create hole-electron pairs. The former cannot be used efficiently since only one hole-electron pair is created per quantum, and the latter cannot be used at all. Even with these limitations, silicon junction "solar cells" are manufactured with efficiencies above 10%. Less efficient selenium cells which are less costly per watt of available power are also available.

8. TERMINOLOGY

Definitions of Semiconductor Terms, The Institute of Radio Engineers Standards on Electron Devices, 54 I.R.E. 7.S2, are reproduced here with the permission of the Institute of Radio Engineers (Ref. 1).

Acceptor *(in a Semiconductor)*—See *Impurity, Acceptor.*

Barrier *(in a Semiconductor)* (Obsolete)—See *Depletion Layer.*

Base Electrode *(of a Transistor)*—An *ohmic* or *majority carrier contact* to the *base region.*

Base Region—The interelectrode region of a *transistor* into which *minority carriers* are injected.

Boundary, *P-N*—A surface in the transition region between *P*-type and *N*-type material at which the *donor* and *acceptor* concentrations are equal.

Carrier—In a semiconductor, a mobile *conduction electron* or *hole.*

Collector *(of a Transistor)*—An electrode through which a primary flow of carriers leaves the interelectrode region.

Conduction Band—A range of states in the energy spectrum of a solid in which electrons can move freely.

Conductivity Modulation *(of a Semiconductor)*—The variation of the conductivity of a semiconductor by variation of the charge *carrier* density.

Conductivity, *N*-type—The conductivity associated with *conduction electrons* in a semiconductor.

Conductivity, *P*-type—The conductivity associated with *holes* in a semiconductor.

Contact, High Recombination Rate—A semiconductor-semiconductor or metal-semiconductor contact at which thermal equilibrium *carrier* densities are maintained substantially independent of current density.

Contact, Majority Carrier *(to a Semiconductor)*—An electrical contact across which the ratio of *majority carrier* current to applied voltage is substantially independent of the polarity of the voltage while the ratio of *minority carrier* current to applied voltage is not independent of the polarity of the voltage.

Crystal Pulling—A method of crystal growing in which the developing crystal is gradually withdrawn from a melt.

Depletion Layer *(in a Semiconductor)*—A region in which the mobile *carrier* charge density is insufficient to neutralize the net fixed charge density of *donors* and *acceptors.*

Diffusion Constant *(in a Homogeneous Semiconductor)*—The quotient of diffusion current density by the charge *carrier* concentration gradient. It is equal to the product of the *drift mobility* and the average thermal energy per unit charge of *carriers.*

Diffusion Length—In a homogeneous semiconductor, the average distance to which *minority carriers* diffuse between generation and recombination.

Diode, Semiconductor—A two-electrode *semiconductor device* having an asymmetrical voltage-current characteristic.

Donor *(in a Semiconductor)—See Impurity, Donor.*

Doping—Addition of *impurities* to a semiconductor or production of a deviation from stoichiometric composition, to achieve a desired characteristic.

Doping Compensation—Addition of *donor impurities* to a *P-type semiconductor* or of *acceptor impurities* to an *N-type semiconductor.*

Drift Mobility (*in a Homogeneous Semiconductor*)—The average drift velocity of *carriers* per unit electric field.

Note: In general, the mobilities of electrons and holes are different.

Electrode (*of a Semiconductor Device*)—An element that performs one or more of the functions of emitting or collecting electrons or *holes,* or of controlling their movements by an electric field.

Electrons, Conduction—The electrons in the *conduction band* of a solid, which are free to move under the influence of an electric field.

Element (*of a Semiconductor Device*)—Any integral part of the *semiconductor device* that contributes to its operation.

Emitter—See *Emitter, Majority* and *Emitter, Minority.*

Emitter, Majority (*of a Transistor*)—An *electrode* from which a flow of *majority carriers* enters the interelectrode region.

Emitter, Minority (*of a Transistor*)—An *electrode* from which a flow of *minority carriers* enters the interelectrode region.

Energy Gap (*of a Semiconductor*)—The energy range between the bottom of the *conduction band* and the top of the *valence band.*

Extrinsic Properties (*of a Semiconductor*)—The properties of a semiconductor as modified by *impurities* or *imperfections* within the crystal.

Fermi Level—The value of the electron energy at which the Fermi distribution function has the value one-half.

Forming, Electrical (*Applied to Semiconductor Devices*)—Process of applying electrical energy to a semiconductor device in order to modify permanently the electrical characteristics.

Generation Rate (*in a Semiconductor*)—The time rate of creation of electron-hole pairs.

Hall Constant (*of an Electrical Conductor*)—The constant of proportionality R in the relation

$\mathbf{E}_h = R\mathbf{J} \times \mathbf{H}$, where

$\mathbf{E}_h$ = Transverse electric field (Hall field)

$\mathbf{J}$ = Current density

$\mathbf{H}$ = Magnetic field.

Note: The sign of the *majority carrier* can be inferred from the sign of the Hall constant.

Hole—A mobile vacancy in the electronic valence structure of a semiconductor which acts like a positive electronic charge with a positive mass.

Imperfection (*of a Crystalline Solid*)—Any deviation in structure from that of an ideal crystal.

Note: An ideal crystal is perfectly periodic in structure and contains no foreign atoms.

Impurity, Acceptor (*in a Semiconductor*)—An impurity which may induce hole conduction.

Impurity (*Chemical*)—An atom within a crystal which is foreign to the crystal.

Impurity, Donor (*in a Semiconductor*)—An impurity which may induce electronic conduction.

Impurity, Stoichiometric—A crystalline imperfection arising from a deviation from stoichiometric composition.

Intrinsic Properties (*of a Semiconductor*)—The properties of a semiconductor which are characteristic of the pure, ideal crystal.

Intrinsic Temperature Range (*in a Semiconductor*)—The temperature range in which the electrical properties of a semiconductor are essentially not modified by *impurities* or *imperfections* within the crystal.

Junction (*in a Semiconductor Device*)—A region of transition between semiconducting regions of different electrical properties.

Junction, Alloy (*in a Semiconductor*)—A junction formed by alloying one or more *impurities* to a semiconductor crystal.

Junction, Collector (*of a Semiconductor Device*)—A junction normally biased in the high-resistance direction, the current through which can be controlled by the introduction of *minority carriers*.

Junction, Emitter (*of a Semiconductor Device*)—A junction normally biased in the low-resistance direction to inject *minority carriers* into an interelectrode region.

Junction, Fused (*in a Semiconductor*)—A junction formed by recrystallization on a base crystal from a liquid phase of one or more components and the semiconductor.

Junction, *N-N* (*in a Semiconductor*)—A region of transition between two regions having different properties in *N*-type semiconducting material.

Junction, *P-N* (*in a Semiconductor*)—A region of transition between *P*- and *N*-type semiconducting material.

Junction, *P-P* (*in a Semiconductor*)—A region of transition between two regions having different properties in *P*-type semiconducting material.

Junction (Semiconductor), Diffused—A junction which has been formed by the diffusion of an *impurity* within a semiconductor crystal.

Junction (Semiconductor), Doped—A junction produced by the addition of an *impurity* to the melt during crystal growth.

Junction (Semiconductor), Grown—A junction produced during growth of a crystal from a melt.

Junction (Semiconductor), Rate-grown—A *grown junction* produced by varying the rate of crystal growth.

Lifetime, Volume—The average time interval between the generation and recombination of *minority carriers* in a homogeneous semiconductor.

Majority Carrier (*in a Semiconductor*)—The type of *carrier* constituting more than half of the total number of *carriers.*

Minority Carrier (*in a Semiconductor*)—The type of *carrier* constituting less than half of the total number of *carriers.*

Mobility—See *Drift Mobility.*

Mobility, Hall (*of an Electrical Conductor*)—The quantity μ_H in the relation $\mu_H = R\sigma$, where R = *Hall constant* and σ = conductivity.

Ohmic Contact—A contact between two materials, possessing the property that the potential difference across it is proportional to the current passing through.

Photovaristor—A varistor in which the current-voltage relation may be modified by illumination, e.g., cadmium sulphide or lead telluride.

Point Contact—Pressure contact between a semiconductor body and a metallic point.

Primary Flow (*of Carriers*)—A current flow which is responsible for the major properties of the device.

Recombination Rate, Surface—The time rate at which free electrons and *holes* recombine at the surface of a semiconductor.

Recombination Rate, Volume—The time rate at which free electrons and *holes* recombine within the volume of a semiconductor.

Recombination Velocity (*on a Semiconductor Surface*)—The quotient of the normal component of the electron (*hole*) current density at the surface by the excess electron (*hole*) charge density at the surface.

Semiconductor—An electronic conductor, with resistivity in the range between metals and insulators, in which the electrical charge *carrier* concentration increases with increasing temperature over some temperature range. Certain semiconductors possess two types of *carriers,* namely, negative electrons and positive *holes.*

Semiconductor, Compensated—A semiconductor in which one type of *impurity* or *imperfection* (e.g., *donor*) partially cancels the electrical effects of the other type of *impurity* or *imperfection* (e.g., *acceptor*).

Semiconductor Device—An electron device in which the characteristic distinguishing electronic conduction takes place within a semiconductor.

Semiconductor Device, Multiple Unit—A semiconductor device having two or more sets of electrodes associated with independent *carrier* streams.

Note: It is implied that the device has two or more output functions which are independently derived from separate inputs, e.g., a duo-triode transistor.

Semiconductor Device, Single Unit—A semiconductor device having one set of electrodes associated with a single *carrier* stream.

Note: It is implied that the device has a single output function related to a single input.

Semiconductor, Extrinsic—A semiconductor with electrical properties dependent upon *impurities.*

Semiconductor, Intrinsic—A semiconductor whose electrical properties are essentially characteristic of the pure, ideal crystal.

Semiconductor, N-type—An *extrinsic semiconductor* in which the *conduction electron* density exceeds the *hole* density.

Note: It is implied that the net ionized *impurity* concentration is *donor* type.

Semiconductor, P-type—An *extrinsic semiconductor* in which the *hole* density exceeds the *conduction electron* density.

Note: It is implied that the net ionized *impurity* concentration is *acceptor* type.

Space Charge Region (*Pertaning to Semiconductor*)—A region in which the net charge density is significantly different from zero. See also *Depletion Layer*.

Thermistor—An electron device which makes use of the change of resistivity of a semiconductor with change in temperature.

Transistor—An active *semiconductor device* with three or more electrodes.

Transistor, Conductivity Modulation—A *transistor* in which the active properties are derived from *minority carrier* modulation of the bulk resistivity of a semiconductor.

Transistor, Filamentary—A *conductivity modulation transistor* with a length much greater than its transverse dimensions.

Transistor, Junction—A *transistor* having a *base electrode* and two or more *junction* electrodes.

Transistor, Point-contact—A *transistor* having a *base electrode* and two or more *point-contact* electrodes.

Transistor, Point-junction—A *transistor* having a *base electrode* and both *point-contact* and *junction* electrodes.

Transistor, Unipolar—A *transistor* which utilizes charge *carriers* of only one polarity.

Transition Region—The region, between two homogeneous semiconductor regions, in which the *impurity* concentration changes.

Valence Band—The range of energy states in the spectrum of a solid crystal in which lie the energies of the valence electrons which bind the crystal together.

Varistor—A two-electrode *semiconductor device* having a voltage-dependent nonlinear resistance.

Zone Leveling (*Pertaining to Semiconductor Processing*)—The passage of one or more molten zones along a semiconductor body for the purpose of uniformly distributing *impurities* throughout the material.

Zone Purification (*Pertaining to Semiconductor Processing*)—The passage of one or more molten zones along a semiconductor for the purpose of reducing the *impurity* concentration of part of the ingot.

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Transistor Circuits

Arthur P. Stern

1. BASIC CIRCUIT CONSIDERATIONS AND SYMBOLS

Bias Polarities. In transistors used as amplifying devices, the emitter-to-base junction is biased in the *forward* direction and the collector-to-base junction in the *reverse* direction. Consequently, the polarities of the d-c voltages and currents of an *n-p-n* transistor are opposite to those of *p-n-p* transistors (see Fig. 1*a*,*b*). Otherwise, there is little difference between the general behavior of *p-n-p* and *n-p-n* transistors from the point of view of the circuit designer. In the following discussion of transistor circuits, the use of *p-n-p* transistors will usually be assumed unless otherwise stated. Symbolic representation of transistors in circuits is shown in Fig. 1*c*,*d*.

Characteristics and Configurations

The grid current of electron tubes is very small and can usually be neglected. Consequently, the operating condition of a tube can be char-

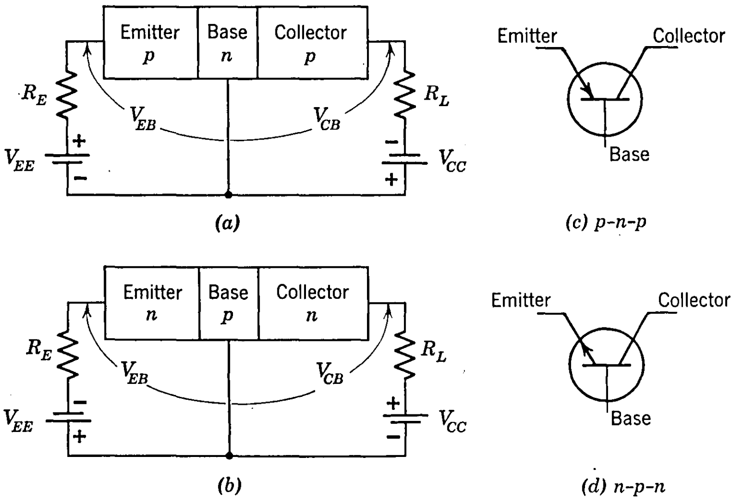


FIG. 1. Bias voltages applied to (*a*) *p-n-p* and (*b*) *n-p-n* transistors. R_E stabilizes the emitter current; R_L is the load resistor. Circuit symbols for transistors are (*c*) *p-n-p*, (*d*) *n-p-n*.

acterized at low frequencies by only three quantities: grid-to-cathode voltage, plate-to-cathode voltage, and plate current. The interdependence of these three quantities can be represented by a family of curves on a single graph, for example, by the *plate characteristics* of the electron tube, giving the plate current as a function of the plate-to-cathode voltage with the grid-to-cathode voltage as parameter. When describing the properties of transistors, four quantities must be taken into account, since the input current of a transistor is not negligible. This can be done conveniently by using two graphs, the *input* and *output characteristics* of the transistor.

Common Base. Transistors can be operated in three configurations: common base, common emitter, and common collector. In the *common base configuration* (i.e., using the emitter as the input, the collector as the output, and the base as the common terminal, as shown in Fig. 2) the emitter current I_E, the collector current I_C, the emitter-to-base voltage V_{EB}, and the collector-to-base voltage V_{CB} are four quantities suitable for the characterization of the transistor, two of which may be chosen arbitrarily (at least in principle).

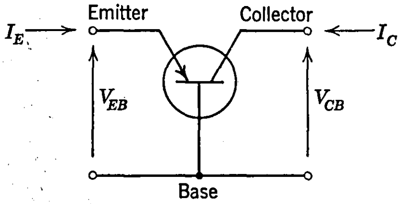


FIG. 2. A *p-n-p* transistor in common base configuration with positive directions of voltage and currents as shown. I_C and V_{CB} are negative.

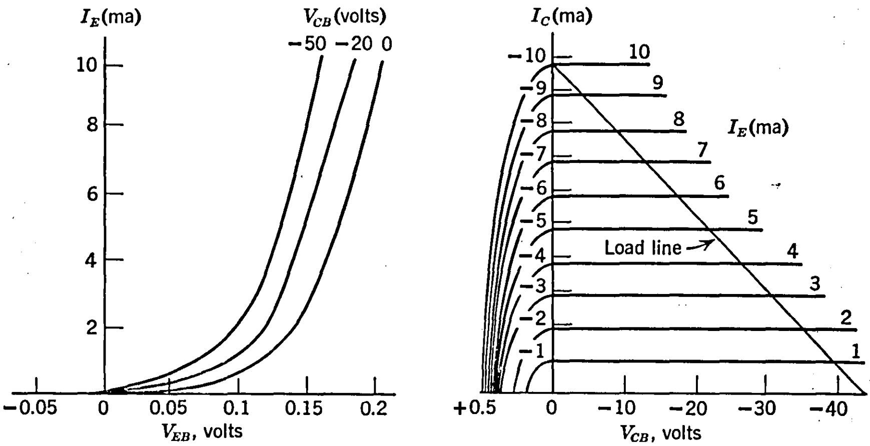


Fig. 3. Common base input and output characteristics.

The two graphs most often used to describe the behavior of the transistor are the common base input and output characteristics (see Fig. 3). Note that if properly biased, the signs of V_{CB} and I_C are negative for a *p-n-p* transistor.

Common Emitter. The behavior of the transistor in the *common emitter configuration* (Fig. 4) can be determined graphically from the common base characteristics of Fig. 3. However, owing to the importance of the common emitter configuration from the circuit point of view (this being the configuration most frequently used), the characteristics are often given in terms of quantities directly applicable to this mode of operation: base current I_B $(= -I_E - I_C)$, collector current I_C, base-to-emitter voltage V_{BE} $(= -V_{EB})$ and collector-to-emitter voltage V_{CE} $(= V_{CB} - V_{EB})$. Input and output characteristics of a transistor in the common emitter configuration are shown in Fig. 5. (If properly biased, the signs of I_B, I_C, and V_{CE} are negative for a *p-n-p* transistor.)

Common Collector. Transistors are often used in the common collector configuration (Fig. 6). Input and output characteristics appli-

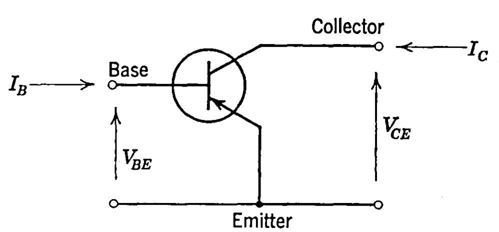


Fig. 4. Common emitter configuration.

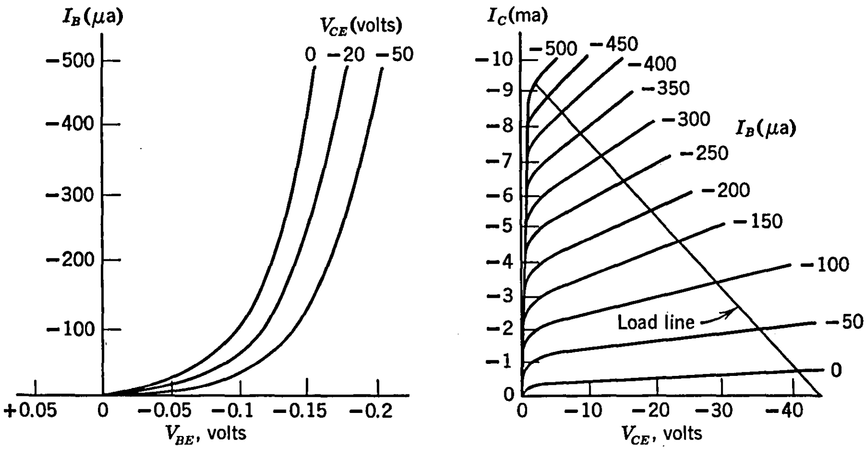


FIG. 5. Common emitter input and output characteristics.

cable to this mode of operation can be constructed graphically from either the common base or the common emitter characteristics.

Comparison with Electron Tube Circuits. It is sometimes useful to think of the common emitter transistor configuration as being analogous to an electron tube with grounded cathode. Then, the common base and common collector circuits correspond to the grounded grid and grounded plate cases respectively. These analogies must not be carried too far, since the behavior of transistors is in most respects different from that of electron tubes.

Tetrodes. Whereas the foregoing discussion deals with transistor triodes, the static input and output characteristics of transistor *tetrodes* are qualitatively not different from those of triodes. However, in the case of a tetrode, a bias current flows between the two base terminals as a result of the bias supply voltage $V_{BB'}$ (see Fig. 7). Varying $V_{BB'}$ results in a slight shift of the input and output characteristics.

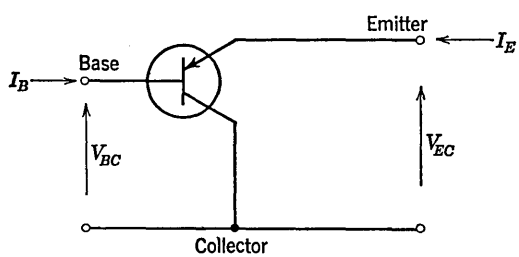


FIG. 6. Common collector configuration.

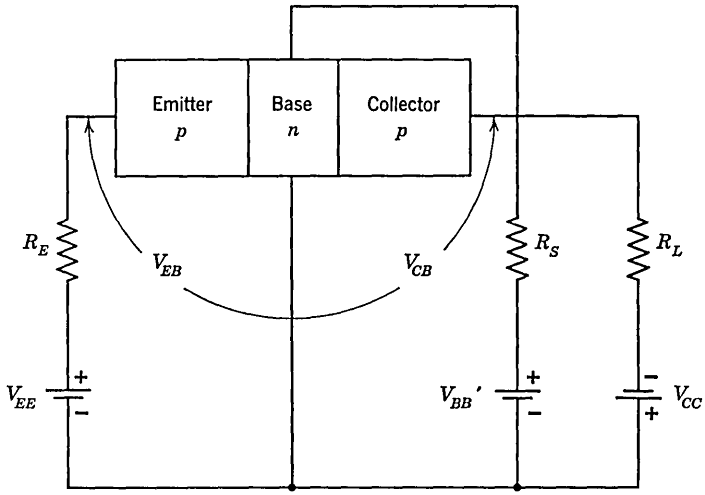


FIG. 7. Biased transistor tetrode.

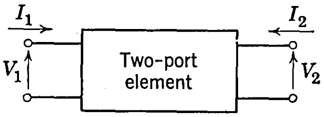
Small-Signal A-C Parameters

The *operating point* of a transistor used, for example, as an amplifier is defined by its d-c voltages and currents. (In principle, two of the voltages and currents can be chosen arbitrarily, and these determine the other d-c quantities.) The operating point is usually characterized by the emitter current I_E and the collector-to-base voltage V_{CB}. The low-frequency *small-signal a-c parameters* pertaining to a given operating point of the transistor in a given configuration can be determined (more or less accurately) from the static input and output characteristics. Alternatively they can be measured by using appropriate a-c methods. The small-signal parameters are: input (and output) impedances (or admittances) with output (input) terminals open- or short-circuited, forward and backward transfer impedances (or admittances), short-circuit current transfer ratios and open-circuit voltage transfer ratios.

The transistor is an active *two-port element* (four-pole, two-terminal-pair element). The electrical behavior of such an element is completely defined by its input voltage V_1, input current I_1, output voltage V_2, and output current I_2. These quantities can be related to each other by any one out of six pairs of two equations (see Table 1). The constants in each pair of equations constitute a *set of small signal parameters*. The parameters of a set can be calculated from the parameters of any other set (Tables 2 and 3).

Consequently, only one set of four small-signal parameters must be known to specify the low-frequency a-c properties of a transistor. (At

TABLE 1. REPRESENTATION OF TWO-PORT ELEMENTS



Six pairs of equations relating V_1, V_2, I_1 and I_2:

In Conventional Form	In Form of Matrix Equations	In Form of Symbolic Matrix Equations
$V_1 = z_{11}I_1 + z_{12}I_2$ $V_2 = z_{21}I_1 + z_{22}I_2$	$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11}; & z_{12} \\ z_{21}; & z_{22} \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$	$[V] = [z]\cdot[I]$
$I_1 = y_{11}V_1 + y_{12}V_2$ $I_2 = y_{21}V_1 + y_{22}V_2$	$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11}; & y_{12} \\ y_{21}; & y_{22} \end{bmatrix} \cdot \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$	$[I] = [y]\cdot[V]$
$V_1 = h_{11}I_1 + h_{12}V_2$ $I_2 = h_{21}I_1 + h_{22}V_2$	$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11}; & h_{12} \\ h_{21}; & h_{22} \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$	$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = [h] \cdot \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$
$I_1 = g_{11}V_1 + g_{12}I_2$ $V_2 = g_{21}V_1 + g_{22}I_2$	$\begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} g_{11}; & g_{12} \\ g_{21}; & g_{22} \end{bmatrix} \cdot \begin{bmatrix} V_1 \\ I_2 \end{bmatrix}$	$\begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = [g] \cdot \begin{bmatrix} V_1 \\ I_2 \end{bmatrix}$
$V_1 = a_{11}V_2 - a_{12}I_2$ $I_1 = a_{21}V_2 - a_{22}I_2$	$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} a_{11}; & a_{12} \\ a_{21}; & a_{22} \end{bmatrix} \cdot \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$	$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = [a] \cdot \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$
$V_2 = b_{11}V_1 - b_{12}I_1$ $I_2 = b_{21}V_1 - b_{22}I_1$	$\begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} b_{11}; & b_{12} \\ b_{21}; & b_{22} \end{bmatrix} \cdot \begin{bmatrix} V_1 \\ -I_1 \end{bmatrix}$	$\begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = [b] \cdot \begin{bmatrix} V_1 \\ -I_1 \end{bmatrix}$

higher frequencies, the knowledge of both real and imaginary components of four small-signal parameters is required, i.e., one needs altogether eight quantities.)

Common Base *h* Parameters. The set of small-signal parameters most often used to characterize the a-c behavior of transistors are the *common base h parameters:* h_{11b}, h_{12b}, h_{21b}, h_{22b}. (The h parameters are often designated as "hybrid" or "series-parallel" parameters.) These symbols have the following significance:

h_{11b} is the input impedance, the output terminals being short-circuited;
h_{12b} is the backward voltage transfer ratio, the input terminals being open-circuited;
h_{21b} is the forward current transfer ratio, the output terminals being short-circuited (i.e., the short-circuit current amplification);
h_{22b} is the output-admittance with open-circuited input terminals.

The advantage of the h parameters over other sets of parameters is due to the fact that their experimental determination calls for short-circuiting the high-impedance output terminals of the common base stage and open-circuiting its low-impedance input terminals, which can be easily achieved.

TABLE 2. INTERRELATIONS BETWEEN DIFFERENT SETS OF SMALL-SIGNAL PARAMETERS[a] (Ref. 22)

Parameters to Be Calculated	In Terms of z	y	h	g	a	b
z_{11}; z_{12}	—	$\frac{y_{22}}{\Delta^y}$; $\frac{-y_{12}}{\Delta^y}$	$\frac{\Delta^h}{h_{22}}$; $\frac{h_{12}}{h_{22}}$	$\frac{1}{g_{11}}$; $\frac{-g_{12}}{g_{11}}$	$\frac{a_{11}}{a_{21}}$; $\frac{\Delta^a}{a_{21}}$	$\frac{b_{22}}{b_{21}}$; $\frac{1}{b_{21}}$
z_{21}; z_{22}		$\frac{-y_{21}}{\Delta^y}$; $\frac{y_{11}}{\Delta^y}$	$\frac{-h_{21}}{h_{22}}$; $\frac{1}{h_{22}}$	$\frac{g_{21}}{g_{11}}$; $\frac{\Delta^g}{g_{11}}$	$\frac{1}{a_{21}}$; $\frac{a_{22}}{a_{21}}$	$\frac{\Delta^b}{b_{21}}$; $\frac{b_{11}}{b_{21}}$
y_{11}; y_{12}	$\frac{z_{22}}{\Delta^z}$; $\frac{-z_{12}}{\Delta^z}$	—	$\frac{1}{h_{11}}$; $\frac{-h_{12}}{h_{11}}$	$\frac{\Delta^g}{g_{22}}$; $\frac{g_{12}}{g_{22}}$	$\frac{a_{22}}{a_{12}}$; $\frac{-\Delta^a}{a_{12}}$	$\frac{b_{11}}{b_{12}}$; $\frac{-1}{b_{12}}$
y_{21}; y_{22}	$\frac{-z_{21}}{\Delta^z}$; $\frac{z_{11}}{\Delta^z}$		$\frac{h_{21}}{h_{11}}$; $\frac{\Delta^h}{h_{11}}$	$\frac{-g_{21}}{g_{22}}$; $\frac{1}{g_{22}}$	$\frac{-1}{a_{12}}$; $\frac{a_{11}}{a_{12}}$	$\frac{-\Delta^b}{b_{12}}$; $\frac{b_{22}}{b_{12}}$
h_{11}; h_{12}	$\frac{\Delta^z}{z_{22}}$; $\frac{z_{12}}{z_{22}}$	$\frac{1}{y_{11}}$; $\frac{-y_{12}}{y_{11}}$	—	$\frac{g_{22}}{\Delta^g}$; $\frac{-g_{12}}{\Delta^g}$	$\frac{a_{12}}{a_{22}}$; $\frac{\Delta^a}{a_{22}}$	$\frac{b_{12}}{b_{11}}$; $\frac{1}{b_{11}}$
h_{21}; h_{22}	$\frac{-z_{21}}{z_{22}}$; $\frac{1}{z_{22}}$	$\frac{y_{21}}{y_{11}}$; $\frac{\Delta^y}{y_{11}}$		$\frac{-g_{21}}{\Delta^g}$; $\frac{g_{11}}{\Delta^g}$	$\frac{-1}{a_{22}}$; $\frac{a_{21}}{a_{22}}$	$\frac{-\Delta^b}{b_{11}}$; $\frac{b_{21}}{b_{11}}$
g_{11}; g_{12}	$\frac{1}{z_{11}}$; $\frac{-z_{12}}{z_{11}}$	$\frac{\Delta^y}{y_{22}}$; $\frac{y_{12}}{y_{22}}$	$\frac{h_{22}}{\Delta^h}$; $\frac{-h_{12}}{\Delta^h}$	—	$\frac{a_{21}}{a_{11}}$; $\frac{-\Delta^a}{a_{11}}$	$\frac{b_{21}}{b_{22}}$; $\frac{-1}{b_{22}}$
g_{21}; g_{22}	$\frac{z_{21}}{z_{11}}$; $\frac{\Delta^z}{z_{11}}$	$\frac{-y_{21}}{y_{22}}$; $\frac{1}{y_{22}}$	$\frac{-h_{21}}{\Delta^h}$; $\frac{h_{11}}{\Delta^h}$		$\frac{1}{a_{11}}$; $\frac{a_{12}}{a_{11}}$	$\frac{\Delta^b}{b_{22}}$; $\frac{b_{12}}{b_{22}}$
a_{11}; a_{12}	$\frac{z_{11}}{z_{21}}$; $\frac{\Delta^z}{z_{21}}$	$\frac{-y_{22}}{y_{21}}$; $\frac{-1}{y_{21}}$	$\frac{-\Delta^h}{h_{21}}$; $\frac{-h_{11}}{h_{21}}$	$\frac{1}{g_{21}}$; $\frac{g_{22}}{g_{21}}$	—	$\frac{b_{22}}{\Delta^b}$; $\frac{b_{12}}{\Delta^b}$
a_{21}; a_{22}	$\frac{1}{z_{21}}$; $\frac{z_{22}}{z_{21}}$	$\frac{-\Delta^y}{y_{21}}$; $\frac{-y_{11}}{y_{21}}$	$\frac{-h_{22}}{h_{21}}$; $\frac{-1}{h_{21}}$	$\frac{g_{11}}{g_{21}}$; $\frac{\Delta^g}{g_{21}}$		$\frac{b_{21}}{\Delta^b}$; $\frac{b_{11}}{\Delta^b}$
b_{11}; b_{12}	$\frac{z_{22}}{z_{12}}$; $\frac{\Delta^z}{z_{12}}$	$\frac{-y_{11}}{y_{12}}$; $\frac{-1}{y_{12}}$	$\frac{1}{h_{12}}$; $\frac{h_{11}}{h_{12}}$	$\frac{-\Delta^g}{g_{12}}$; $\frac{-g_{22}}{g_{12}}$	$\frac{a_{22}}{\Delta^a}$; $\frac{a_{12}}{\Delta^a}$	—
b_{21}; b_{22}	$\frac{1}{z_{12}}$; $\frac{z_{11}}{z_{12}}$	$\frac{-\Delta^y}{y_{12}}$; $\frac{-y_{22}}{y_{12}}$	$\frac{h_{22}}{h_{12}}$; $\frac{\Delta^h}{h_{12}}$	$\frac{-g_{11}}{g_{12}}$; $\frac{-1}{g_{12}}$	$\frac{a_{21}}{\Delta^a}$; $\frac{a_{11}}{\Delta^a}$	

[a] Δ^z is the determinant of set z_{ij}: $\Delta^z = z_{11}z_{22} - z_{12}z_{21}$; etc.

TABLE 3. INTERRELATIONS OF DETERMINANTS OF DIFFERENT SETS OF SMALL-SIGNAL PARAMETERS (Ref. 22)

	In Terms of					
	z	y	h	g	a	b
Δ^z	—	$1/\Delta^y$	h_{11}/h_{22}	g_{22}/g_{11}	a_{12}/a_{21}	b_{12}/b_{21}
Δ^y	$1/\Delta^z$	—	h_{22}/h_{11}	g_{11}/g_{22}	a_{21}/a_{12}	b_{21}/b_{12}
Δ^h	z_{11}/z_{22}	y_{22}/y_{11}	—	$1/\Delta^g$	a_{11}/a_{22}	b_{22}/b_{11}
Δ^g	z_{22}/z_{11}	y_{11}/y_{22}	$1/\Delta^h$	—	a_{22}/a_{11}	b_{11}/b_{22}
Δ^a	z_{12}/z_{21}	y_{12}/y_{21}	$-h_{12}/h_{21}$	$-g_{12}/g_{21}$	—	$1/\Delta^b$
Δ^b	z_{21}/z_{12}	y_{21}/y_{12}	$-h_{21}/h_{12}$	$-g_{21}/g_{12}$	$1/\Delta^a$	—

Common Emitter and Common Collector *h* Parameters. The *common emitter h parameters* (h_{11e}, h_{12e}, h_{21e}, and h_{22e}) and the *common collector h parameters* (h_{11c}, h_{12c}, h_{21c}, and h_{22c}) can be computed from the common base h parameters (see Tables 4 and 5) or can be measured on these configurations. The common emitter h parameters are often used instead of the common base h parameters to describe the transistor.

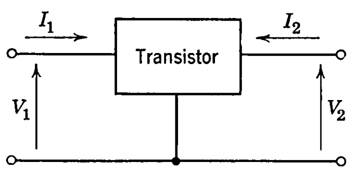


FIG. 8. Convention for positive directions of alternating currents and voltages.

The positive directions of alternating currents and voltages are, by convention, as shown in Fig. 8. The common base short-circuit current amplification h_{21b} has a negative sign. Its absolute value is often designated by α_{fb} or, simply, α_b.

$$h_{21b} = -\alpha_b.$$

For junction transistors, α_b is smaller than but close to unity (0.95–0.995).

The common emitter short-circuit current amplification h_{21e} is often designated by α_{fe} or, simply, α_e. This quantity is positive and can be expressed approximately as

$$h_{21e} = \alpha_e \cong -h_{21b}/(1 + h_{21b}) = \alpha_b/(1 - \alpha_b).$$

For junction transistors, typical values of α_e lie between 20 and 200.

The common collector short-circuit current amplification h_{21c} (α_{fc} or α_c) is also negative:

$$-h_{21c} = \alpha_c \cong -1/(1 + h_{21b}) = -1/(1 - \alpha_b).$$

The above expressions show that small variations of h_{21b} (between two transistors or within one transistor as a result of varying environmental conditions) result in considerable variations of h_{21e} and h_{21c}.

Short-Circuit Parameters. Another set of four small signal parameters, the *y parameters* (*short-circuit* or *admittance parameters*), are

TABLE 4. INTERRELATIONSHIPS BETWEEN h PARAMETERS OF DIFFERENT CONFIGURATIONS (EXACT FORMULAS) [a]

Parameters to Be Calculated	In Terms of Common Base Parameters	In Terms of Common Emitter Parameters	In Terms of Common Collector Parameters
h_{11b}; h_{12b} h_{21b}; h_{22b}	—	h_{11e}/A; $(\Delta_e^h - h_{12e})/A$ $-(\Delta_e^h + h_{21e})/A$; h_{22e}/A	h_{11c}/Δ_c^h; $(\Delta_c^h + h_{21c})/\Delta_c^h$ $-(\Delta_c^h - h_{12c})/\Delta_c^h$; h_{22c}/Δ_c^h
h_{11e}; h_{12e} h_{21e}; h_{22e}	h_{11b}/B; $(\Delta_b^h - h_{12b})/B$ $-(h_{21b} + \Delta_b^h)/B$; h_{22b}/B	—	h_{11c}; $1 - h_{12c}$ $-(1 + h_{21c})$; h_{22c}
h_{11c}; h_{12c} h_{21c}; h_{22c}	h_{11b}/B; $(1 + h_{21b})/B$ $-(1 - h_{12b})/B$; h_{22b}/B	h_{11e}; $1 - h_{12e}$ $-(1 + h_{21e})$; h_{22e}	—

[a] Definitions: $\Delta^h = h_{11}h_{22} - h_{12}h_{21}$

$A = 1 + h_{21e} - h_{12e} + \Delta_e^h$

$B = 1 + h_{21b} - h_{12b} + \Delta_b^h$

TABLE 5. INTERRELATIONSHIPS BETWEEN h PARAMETERS OF DIFFERENT CONFIGURATIONS (APPROXIMATE FORMULAS)

Parameters to Be Calculated	In Terms of: Common Base Parameters	Common Emitter Parameters	Common Collector Parameters
h_{11b}; h_{12b} h_{21b}; h_{22b}	—	$h_{11e}/(1 + h_{21e})$; $(\Delta_e^h - h_{12e})/(1 + h_{21e})$ $-h_{21e}/(1 + h_{21e})$; $h_{22e}/(1 + h_{21e})$	$-h_{11c}/h_{21c}$; $(\Delta_c^h + h_{21c})/h_{21c}$ $-(h_{21c} + 1)/h_{21c}$; $-h_{22c}/h_{21c}$
h_{11e}; h_{12e} h_{21e}; h_{22e}	$h_{11b}/(1 + h_{21b})$; $(\Delta_b^h - h_{12b})/(1 + h_{21b})$ $-h_{21b}/(1 + h_{21b})$; $h_{22b}/(1 + h_{21b})$	—	h_{11c}; $1 - h_{12c}$ $-(1 + h_{21c})$; h_{22c}
h_{11c}; h_{12c} h_{21c}; h_{22c}	$h_{11b}/(1 + h_{21b})$; 1 $-1/(1 + h_{21b})$; $h_{22b}/(1 + h_{21b})$	h_{11e}; 1 $-(1 + h_{21e})$; h_{22e}	—

$\Delta^h = h_{11}h_{22} - h_{12}h_{21}$

also frequently used to describe the a-c behavior of transistors. The common base short-circuit input and output admittances are, respectively, y_{11b} and y_{22b}. The common base short-circuit backward and forward transfer admittances are, respectively, y_{12b} and y_{21b}. The common emitter and common collector y parameters (e.g., y_{21e}, y_{12c}) can be computed from the common base y parameters. The various y parameters can, in turn, be expressed in terms of the h parameters (Table 2). The measurement of the common base y parameters at low frequencies is inconvenient because of the difficulty of short-circuiting the input terminal pair.

Open-Circuit Parameters. It may occasionally be useful to describe a transistor configuration in terms of its *z parameters* (or *open-circuit* or *impedance parameters*). The common base open-circuit input and output impedances are, respectively, z_{11b} and z_{22b}. The backward and forward transfer impedances are, respectively, z_{12b} and z_{21b}. The z parameters can be calculated from the h or y parameters (Table 2). The measurement of the z parameters calls for open-circuiting the output terminal pair. At low frequencies in common base configuration, this is not easily feasible.

General Equivalent Circuits. With the h, y, or z parameters corresponding to a given transistor configuration, the configuration can be represented by simple equivalent circuits (Fig. 9) consisting of two impedances or admittances and two voltage or current sources. At low frequencies, the parameters are real; at higher frequencies, imaginary components must be taken into account.

These circuits can be replaced by equivalent ones consisting of three impedances or admittances and a single (voltage or current) source. Such a circuit (Fig. 10), the *T type equivalent circuit,* is often used to represent the common base transistor configuration at low frequencies. At very low frequencies, the elements of the equivalent circuit are resistive and are simply r_e (*emitter resistance*), r_b (*base resistance*), r_c (*collector resistance*). The current source is proportional to the a-c emitter current I_e, the proportionality factor being $a \cong \alpha_b$. At low frequencies, α_b has no significant phase shift.

Already at higher audio frequencies the *collector capacitance* C_c (connected parallel to r_c in Fig. 10) may have to be taken into account. As the frequency increases, further reactive elements and the phase shift of α_b become important.

Letter Symbols for Semiconductor Devices

The letter symbols used in this chapter are (with a few exceptions) in accordance with the *IRE Standards on Letter Symbols for Semiconductor Devices* (Ref. 170). Excerpts from these Standards are given next.

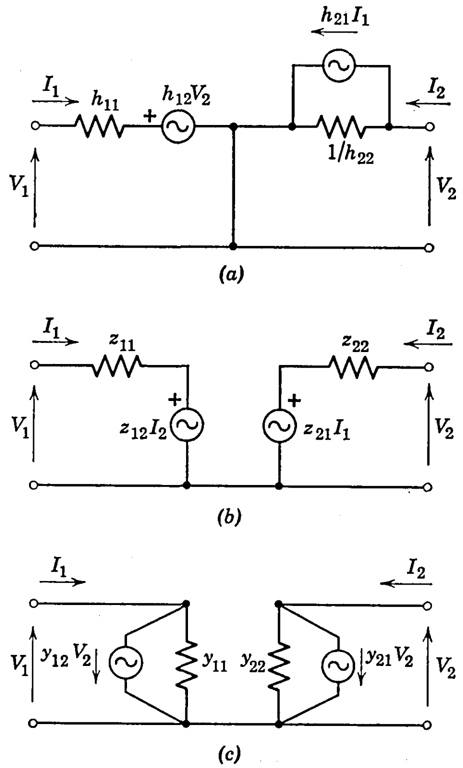


FIG. 9. (*a*) h, (*b*) z, and (*c*) y type equivalent circuits.

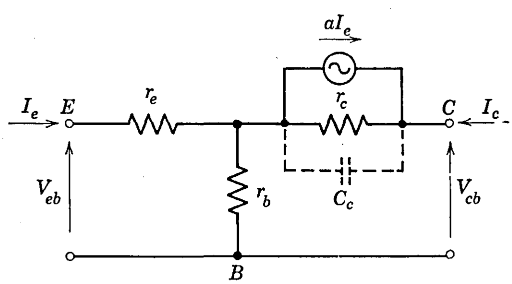


FIG. 10. Low-frequency T type equivalent circuit.

Electrical Quantities

Instantaneous values of current, voltage, and power, which vary with time, are represented by the lower-case letter of the proper symbol. *Examples:* i, v, i_e, v_{EB}.

Maximum, average (dc), and root-mean-square values are represented by the upper-case letter of the proper symbol. *Examples:* I, V, I_e, V_{EB}.

D-c values and instantaneous total values are indicated by upper-case subscripts. *Examples:* i_C, I_C, v_{EB}, V_{EB}, p_C, P_C.

Varying component values are indicated by lower case subscripts. *Examples:* i_c, I_c, V_{eb}, v_{eb}, p_c, P_c.

If necessary to distinguish between maximum, average, or root-mean-square values, maximum or average values may be represented by the addition of a subscript m or av. *Examples:* i_{cm}, I_{cm}, I_{CM}, I_{cav}, i_{CAV}.

Electrical Parameters

Values of four-pole matrix parameters, or other resistances, impedances, admittances, etc., *inherent in the device*, may be represented by the *lower-case symbol* with the proper subscripts. *Examples:* h_{ib}, z_{fb}, y_{oe}, α_{fb}, h_{IB}, α_{FB}.

Values of four-pole matrix parameters or other resistances, impedances, admittances, etc., in the *external circuits*, may be represented by the *upper-case symbols* with the appropriate subscripts. *Examples:* R_e, L_2.

Static values of parameters are indicated by the upper-case subscript. (The static value is the slope of the line from the origin to the operating point on the appropriate characteristic curve.) *Examples:* r_B, h_{IB}, α_{FB}.

Small-signal values of parameters are indicated by the lower-case subscript. *Examples:* r_b, y_c, h_{ib}, z_{ob}, α_{fb}.

The first subscript or subscript pair in matrix notation, identifies the element of the four-pole matrix: i or 11 = input; o or 22 = output; f or 21 = forward transfer; r or 12 = reverse transfer. *Examples:*

$$V_i = h_i I_i + h_r V_o \qquad V_1 = h_{11} I_1 + h_{12} V_2$$
$$I_o = h_f I_i + h_o V_o \qquad I_2 = h_{21} I_1 + h_{22} V_2$$

The second subscript or the subscript following the numeric pair identifies the circuit configuration. When the common electrode is understood, the second subscript may be omitted. *Examples:* e = common emitter; b = common base; c = common collector; j = common electrode, general. *Examples* (common base):

$$I_i = y_{ib} V_{ib} + y_{rb} V_{ob} \qquad I_1 = y_{11b} V_{1b} + y_{12b} V_{2b}$$
$$I_o = y_{fb} V_{ib} + y_{ob} V_{ob} \qquad I_2 = y_{21b} V_{1b} + y_{22b} V_{2b}$$

Electrical parameters characterizing the behavior of a device with associated circuitry are designated by upper-case symbols with an appropriate subscript. *Examples:* Z_i, Z_o.

Some Letter Symbols in Alphabetical Order

With the aid of the above rules, appropriate symbols can be chosen for quantities and parameters if need arises for new symbols. A list of often used symbols is given below.

α_F, α_{FB}, α_{FC}, α_{FE}. The static value of the short-circuit forward current transfer ratio.

α_f, α_{fb}, α_{fc}, α_{fe}. The small-signal short-circuit forward current transfer ratio.

α_r, α_{rb}, α_{rc}, α_{re}. The small-signal short-circuit reverse current transfer ratio.

(The algebraic sign of α for the common base configuration is taken as positive in accordance with established usage, therefore $\alpha_{fb} = -h_{fb} = -h_{21b}$.)

C_o, C_{ob}, C_{oc}, C_{oe}. The capacitance measured across the output terminals with the input open-circuited to ac.

f_α, $f_{\alpha b}$, $f_{\alpha c}$, $f_{\alpha e}$. The frequency at which the magnitude of the small-signal short-circuit forward current transfer ratio is 0.707 of the low-frequency value.

h_f, h_{fb}, h_{fc}, h_{fe}, h_{21}, h_{21b}, h_{21c}, h_{21e}. The small-signal short-circuit forward current transfer ratio.

h_i, h_{ib}, h_{ic}, h_{ie}, h_{11}, h_{11b}, h_{11c}, h_{11e}. The small-signal value of the short-circuit input impedance.

h_o, h_{ob}, h_{oc}, h_{oe}, h_{22}, h_{22b}, h_{22c}, h_{22e}. The small-signal value of the open-circuit output admittance.

h_r, h_{rb}, h_{rc}, h_{re}, h_{12}, h_{12b}, h_{12c}, h_{12e}. The small-signal value of the open-circuit reverse voltage transfer ratio.

y_f, y_{fb}, y_{fc}, y_{fe}, y_{21}, y_{21b}, y_{21c}, y_{21e}. The small-signal short-circuit forward transfer admittance.

y_i, y_{ib}, y_{ic}, y_{ie}, y_{11}, y_{11b}, y_{11c}, y_{11e}. The small-signal short-circuit input admittance.

y_o, y_{ob}, y_{oc}, y_{oe}, y_{22}, y_{22b}, y_{22c}, y_{22e}. The small-signal short-circuit output admittance.

y_r, y_{rb}, y_{rc}, y_{re}, y_{12}, y_{12b}, y_{12c}, y_{12e}. The small-signal short-circuit reverse transfer admittance.

z_f, z_{fb}, z_{fc}, z_{fe}, z_{21}, z_{21b}, z_{21c}, z_{21e}. The small-signal open-circuit forward transfer impedance.

z_i, z_{ib}, z_{ic}, z_{ie}, z_{11}, z_{11b}, z_{11c}, z_{11e}. The small-signal open-circuit input impedance.

z_o, z_{ob}, z_{oc}, z_{oe}, z_{22}, z_{22b}, z_{22c}, z_{22e}. The small-signal open-circuit output impedance.

z_r, z_{rb}, z_{rc}, z_{re}, z_{12}, z_{12b}, z_{12c}, z_{12e}. The small-signal open-circuit reverse transfer impedance.

I_{C0}, I_{CE0}, I_{CB0}. The collector current when the collector is biased in the reverse (high resistance) direction with respect to the reference electrode and the other electrode(s) is dc open-circuited (to the reference electrode).

I_{CS}, I_{CES}, I_{CBS}. The collector current when the collector is biased in the reverse (high resistance) direction with respect to the reference electrode and the other electrode(s) is dc short-circuited (to the reference electrode).

I_{E0}, I_{EB0}, I_{EC0}. The emitter current when the emitter is biased in the reverse (high-resistance) direction with respect to the reference electrode and the other electrode(s) is dc open-circuited (to the reference electrode).

I_{ES}, I_{EBS}, I_{ECS}. The emitter current when the emitter is biased in the reverse (high-resistance) direction with respect to the reference electrode and the other electrode(s) is dc short-circuited (to the reference electrode).

r_b. Resistance of the base branch of the low-frequency equivalent circuit shown in Fig. 10.

r_c. Resistance of the collector branch of the low-frequency equivalent circuit shown in Fig. 10.

r_e. Resistance of the emitter branch of the low-frequency equivalent circuit shown in Fig. 10.

r_m. The product of a and r_c of the low-frequency equivalent circuit shown in Fig. 10.

t_d. The ohmic delay time is the time interval between the rise of a pulse applied at the input terminals and the rise of the minority-carrier-generated pulse appearing at the output terminals.

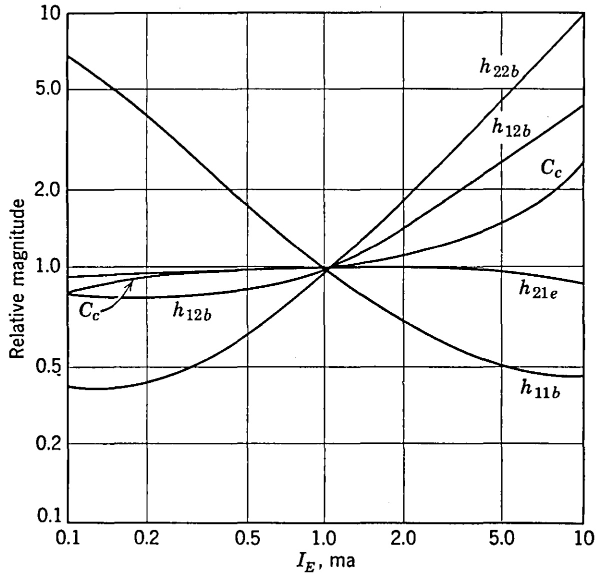
t_s. The storage time is the time interval between the fall of a pulse applied to the input terminals and the fall of the carrier-generated pulse at the output terminals.

BV_{C0}, BV_{CB0}, BV_{CE0}, BV_{E0}, BV_{EB0}, BV_{EC0}, BV_{B0}, BV_{BE0}, BV_{BC0}. The breakdown voltage between the electrode indicated by the first subscript when it is biased in the reverse (high-resistance) direction with respect to the reference electrode and the other electrode is open-circuited.

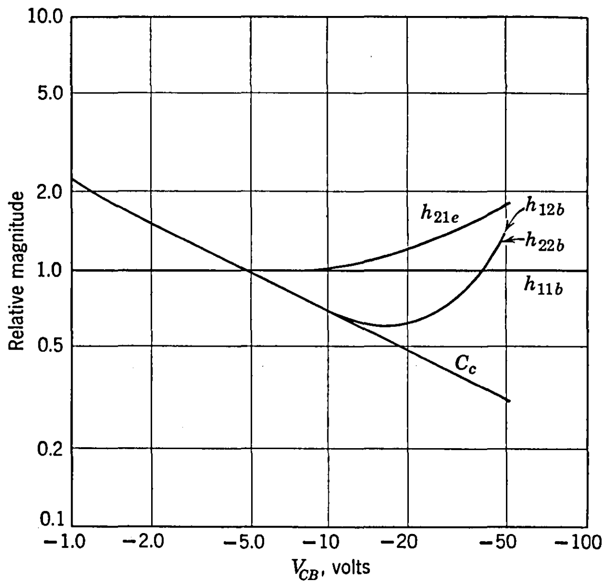
2. TEMPERATURE EFFECTS AND BIAS STABILIZATION

Transistor Parameters as Functions of Operating Point and Temperature. The small-signal parameters are functions of the operating point and of the temperature. The operating point is usually defined in terms of the emitter current I_E and the collector-to-base voltage V_{CB}. The emitter-to-base voltage V_{EB} being small, the difference between V_{CE} and V_{CB} can be neglected in most cases and one refers to the collector voltage V_C.

The common emitter and common base parameters most sensitive to emitter current variations are h_{11} and h_{22}: h_{11} decreases, whereas h_{22} increases with increasing I_E; h_{21b} varies only moderately with I_E, but h_{21e} is more sensitive to I_E-variations (Fig. 11a). Emitter current variations



(a)



(b)

FIG. 11. The h parameters of representative transistor as functions of (a) I_E and (b) V_{CB}.

can be minimized by feeding the emitter through a large resistor (current source).

Collector voltage variations have a considerable influence on h_{12b} and h_{22b} (and also on h_{12e} and h_{22e}). These parameters decrease as V_{CB} increases as long as V_{CB} is considerably smaller than the collector breakdown voltage. h_{21b} and h_{21e} vary only moderately with V_{CB} (Fig. 11*b*). In practical circuits, the problems caused by collector voltage variation are usually not too serious. If necessary, they can be minimized by using appropriate power supplies (for example, supply voltage stabilization with breakdown diodes).

The temperature dependence of transistor parameters is considerable. h_{22b} and h_{12b} increase strongly with increasing temperature. In most transistors, $|h_{21b}|$ increases moderately with increasing temperature (Fig. 12); but the increase of h_{21e} may be quite significant.

In addition to the small-signal parameters, the reverse saturation current I_{CBO} of the collector junction (measured with the emitter open-

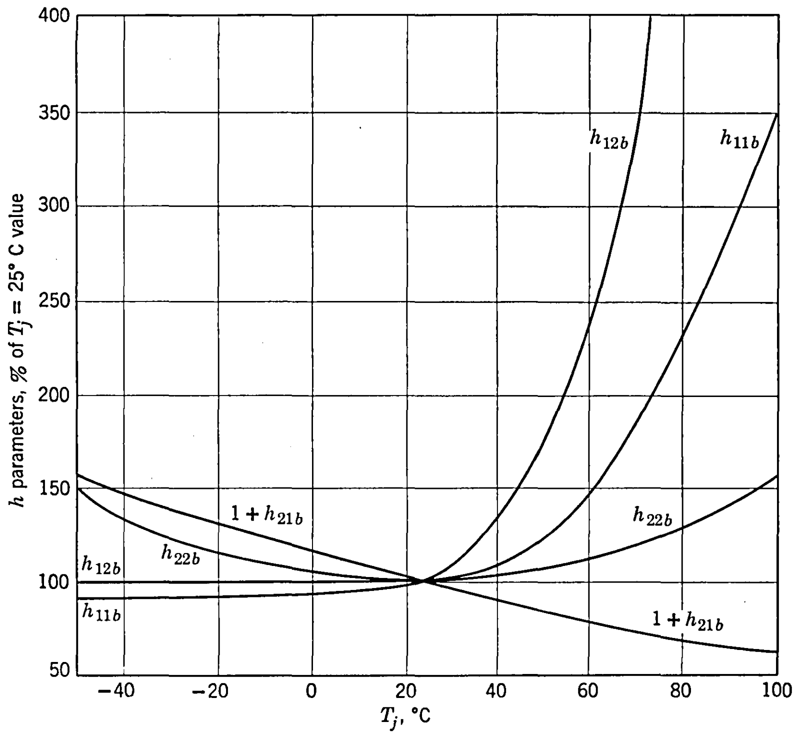


FIG. 12. The h parameters of representative germanium transistor as functions of the junction temperature, T_j.

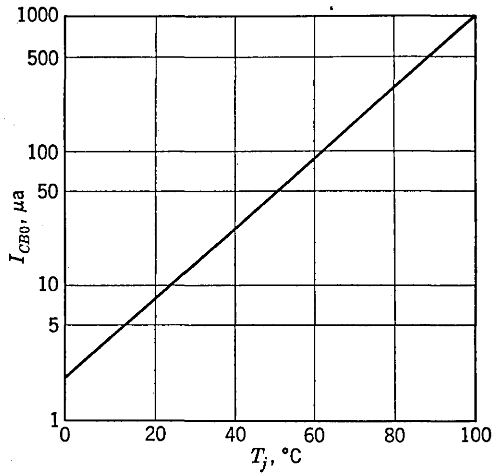


FIG. 13. The collector cutoff current I_{CBO} of germanium transistor as a function of the junction temperature T_j.

circuited) increases strongly (exponentially) with the temperature (Fig. 13):

$$I_{CBO} = (I_{CBO})_{T=T_1} e^{a(T-T_1)}$$

where $(I_{CBO})_{T=T_1}$ is the value of I_{CBO} at a given temperature T_1. The constant a is normally somewhat smaller than 0.1 per °C.

Bias Stabilization

Effect of Temperature. Since I_{CBO} contributes to the collector current I_C (the extent of this contribution depends on the transistor and its associated circuitry), increasing temperature leads to a usually undesirable shift of the operating point: (*a*) I_C increases and (*b*) if the load of the transistor is resistive, the increased I_C causes an increased voltage drop across the load and results in a reduction of V_C.

In some cases, where the load is such that V_C is not materially affected by increasing I_{CBO}, an increase in I_C may cause cumulative dissipation within the transistor, leading to *thermal runaway*. Thermal runaway occurs, when, as a result of increasing T, the dissipation increases. This results in a further increase of device temperature, which in turn causes the dissipation to increase further, etc., until the device is destroyed.

Stability Analysis. Since d-c bias can be applied to the transistor in many different ways, the importance of the effect of increasing I_{CBO} on circuit performance depends on the particular circuit used. For all biasing circuits, an analysis of the emitter current I_E yields expressions that can be written as the sum of a term proportional to I_{CBO} and of a temperature independent term, I_{E1}:

$$I_E = S_I I_{CB0} + I_{E1}.$$

S_I ($= dI_E/dI_{CB0}$) is the *current stability factor* and it is usually desirable that S_I be small. For practical biasing circuits $0 < S_I < 1/(1 - \alpha_b)$. S_I can be expressed in terms of the resistances composing the biasing circuit. Knowing the variation of I_{CB0} within the given temperature range, as well as the maximum tolerable variation of I_E, the value of S_I required for a certain application (using a given stabilizing circuit type) and the required circuit elements can be determined.

For different biasing circuits, the collector-to-base voltage V_{CB} can be written in a form similar to that used for I_E:

$$V_{CB} = S_V I_{CB0} + V_{CB1},$$

where S_V ($= dV_{CB}/dI_{CB0}$) is the *voltage stability factor* and V_{CB1} is a temperature independent term. S_V depends on the biasing circuit type and on the circuit elements used. If the required voltage stability and, consequently, S_V is known, the circuit elements can be chosen appropriately.

Linear Bias Stabilization. A biasing circuit for a common emitter amplifier using separate emitter and collector supplies is shown in Fig. 14.

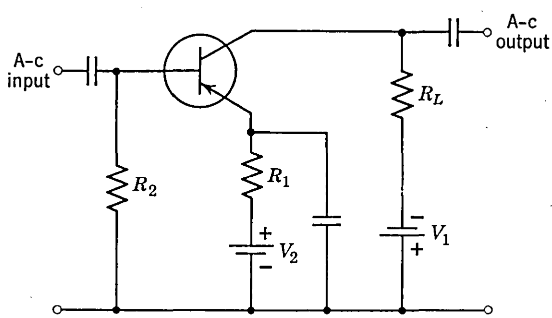


FIG. 14. Biasing circuit using two batteries (see Table 6a).

In this circuit large R_1 and small R_2 are desired for good current stability. R_1 and the supply V_2 are bypassed by a capacitor to prevent a-c feedback which would reduce the a-c gain of the stage.

A common emitter bias arrangement using a single power supply is shown in Fig. 15. Here, too, R_1 should generally be large, whereas the parallel combination of R_2 and R_3 should be small for good temperature stability.

The circuit of Fig. 16 using only one power supply and negative d-c feedback is also frequently used. A tap on resistance R_3 is grounded a-c wise to prevent a-c feedback and the resulting gain reduction.

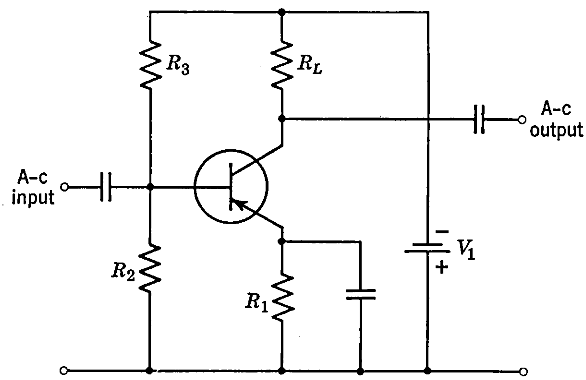


FIG. 15. Bias circuit using single battery (see Table 6*b*).

Table 6 gives formulas for S_I, S_V, I_E, and V_{CB} applicable to the three bias circuits of Figs. 14 to 16. With the aid of these expressions, bias circuits can easily be designed as indicated in the table. In each case, the design procedure involves *five* variables (V_1, V_2, R_1, R_2, and R_L for the circuit of Fig. 14; V_1, R_1, R_2, R_3, and R_L for the circuits of Figs. 15 and 16) with only *four* conditions given. Consequently, assuming that the desired values of S_I, S_V, I_E, and V_{CB} are known, one additional quantity (for example, R_1) can be chosen arbitrarily within reasonable limits (for details, see Refs. 22 and 23).

With the above methods of *linear bias stabilization*, the operating point can be kept constant within usually satisfactory limits. In some cases, where stable operation throughout a very wide range of temperatures or very precise stabilization is desired, circuits using special temperature compensating elements may be employed. In the circuit of Fig. 17 (suggested by E. Keonjian), R_T is a thermistor (temperature sensitive element having negative temperature coefficient) and is used to stabilize the operating

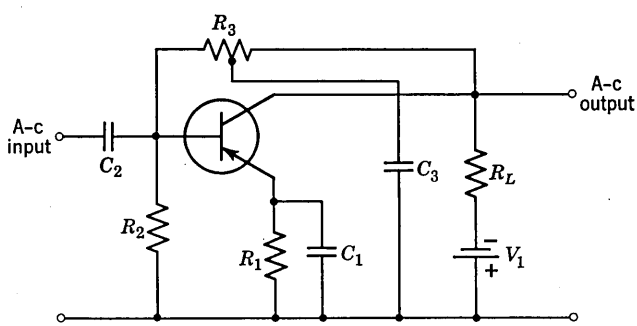


FIG. 16. Bias stabilization with d-c feedback (see Table 6*c*).

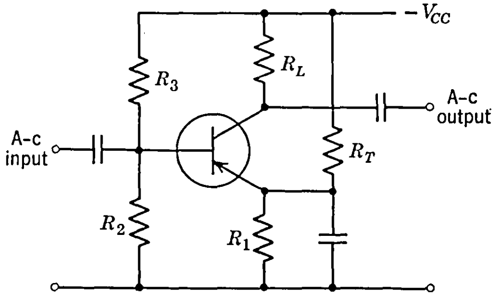


FIG. 17. Bias stabilization with thermistor R_T.

point. As the resistance R_T decreases, it "sucks away" an increasing fraction of the current passing through R_1 and thereby maintains practically constant emitter current.

TABLE 6

(a) Two-Battery Bias Stabilization (Fig. 14) Approximate Formulas (Ref. 22)

Absolute values of all quantities are used. (Signs depend on whether *p-n-p* or *n-p-n* transistors are used. S_I is always positive, S_V is always negative.) Approximations valid for: $\alpha_{b0} \cong 1$; $S_I I_{CBO} \ll I_E$; $S_I(1 - \alpha_{b0}) \ll 1$; and $(1 - \alpha_{b0})/R_1 \ll 1/R_2$. P_D = power dissipated in circuit.

Analysis Procedure

$$S_I \cong R_2/R_1.$$
$$S_V \cong S_I R_1 + R_L(1 + S_I).$$
$$I_E \cong V_2/R_1.$$
$$V_{CB} \cong V_1 + V_2 - I_E(R_1 + R_L).$$

Design Procedure

1. Select S_I, S_V, I_E, V_{CB}, and R_1.
2. $R_L \cong (S_V - S_I R_1)/(1 + S_I)$.
3. $V_2 \cong R_1 I_E$.
4. $V_1 \cong (S_I V_{CB} + S_V I_E - I_E R_L)/S_I - V_2$.
5. $R_2 \cong S_I R_1$.
6. $P_D \cong (V_1 + V_2) I_E$.

(b) Single-Battery Bias Stabilization (Fig. 15) Approximate Formulas (Ref. 22)

Absolute values of all quantities are used. (Signs depend on whether *p-n-p* or *n-p-n* transistors are used. S_I is always positive, S_V always negative.)

Approximations valid for: $\alpha_{b0} \cong 1$; $S_I I_{CBO} \ll I_E$; $S_I(1 - \alpha_{b0}) \ll 1$; $G_1(1 - \alpha_b) < (G_2 + G_3)$. $G_1 = 1/R_1$; $G_2 = 1/R_2$; $G_3 = 1/R_3$. P_D = power dissipated in circuit.

Analysis Procedure

$$S_I \cong G_1/(G_2 + G_3).$$
$$S_V \cong S_I R_1 + (1 + S_I) R_L.$$
$$I_E \cong S_I V_1/R_3.$$
$$V_{CB} \cong V_1 - I_E(R_1 + R_L).$$

TABLE 6.—(*Continued*)

Design Procedure

1. Select S_I, S_V, I_E, V_{CB}, and R_1.
2. $R_L \cong (S_V - S_I R_1)/(1 + S_I)$.
3. $V_1 \cong [S_I V_{CB} + I_E(S_V - R_L)]/S_I$.
4. $R_3 \cong S_I V_1/I_E$.
5. $G_2 \cong (G_1/S_I) - G_3$.
6. $P_D \cong I_E[V_{CB}(1 + S_I) + S_V I_E]/S_I$.

(*c*) *Single-Battery Feedback Bias Stabilization* (*Fig. 16*) *Approximate Formulas* (*Ref. 22*)

Absolute values of all quantities are used. (Signs depend on whether *p-n-p* or *n-p-n* transistors are used. S_I is always positive, S_V always negative.)

Approximations valid for: $\alpha_{b0} \cong 1$; $S_I I_{CB0} \ll I_E$; $S_I(1 - \alpha_{b0}) \ll 1$; $G_1(1 - \alpha_{b0}) \ll (G_2 + G_3)$. $G_1 = 1/R_1$; $G_2 = 1/R_2$; $G_3 = 1/R_3$. P_D = power dissipated in circuit.

Analysis Procedure

$$S_I \cong G_1/[G_2 + G_3 + G_3(G_1 + G_2)R_L].$$
$$S_V \cong (R_3 I_E - S_I V_{CB})/I_E.$$
$$I_E \cong S_I V_1/R_3.$$
$$V_{CB} \cong S_I V_1 R_1/R_2.$$

Design Procedure

1. Select S_I, S_V, I_E, V_{CB}, and R_1.
2. $V_1 \cong (S_I V_{CB} + S_V I_E)/S_I$.
3. $R_3 \cong S_I V_1/I_E$.
4. $P_D \cong I_E[V_{CB}(1 + S_I) + S_V I_E]/S_I$.
5. $R_L \cong V_1 I_E(S_V - S_I R_1)/P_D$.
6. $R_2 \cong S_I R_1 V_1/V_{CB}$.

Nonlinear Stabilization. In Fig. 18, a junction diode D is used for *nonlinear stabilization*. As the temperature increases, the diode resistance and, consequently, the base-to-emitter voltage is reduced. This stabilizes the emitter current which otherwise would tend to increase (see Ref. 24).

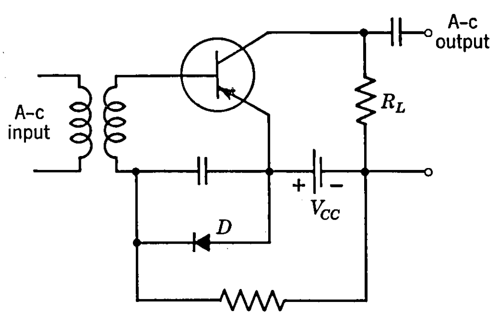


FIG. 18. Nonlinear bias stabilization with junction diode (Ref. 24).

3. LOW-FREQUENCY AMPLIFIERS

Low-Frequency Representation. Small-signal audio-frequency amplifiers can be analyzed and designed conveniently by using the h parameters. Typical values of the h parameters (at $I_E = 1$ ma and $V_C = 5$ volts) are within the ranges:

$$h_{11b} = 25\text{–}60 \text{ ohms} \qquad h_{11e} = 500\text{–}5000 \text{ ohms}$$
$$h_{12b} = (0.5\text{–}5) \times 10^{-4} \qquad h_{12e} = (0.5\text{-}5) \times 10^{-3}$$
$$h_{21b} = 0.95\text{–}0.995 \qquad h_{21e} = 20\text{–}200$$
$$h_{22b} = 0.1\text{–}1 \text{ micromho} \qquad h_{22e} = 5\text{–}100 \text{ micromhos}$$

At higher audio frequencies, the reactive component of h_{22} (and, in some cases, that of h_{12}) must be considered. Table 7 indicates the low-frequency h parameters of a representative transistor.

TABLE 7. h PARAMETERS OF REPRESENTATIVE TRANSISTOR

Parameter	Configuration: Common Base	Common Emitter	Common Collector
h_{11}	40 ohms	2000 ohms	2000 ohms
h_{12}	4×10^{-4}	16×10^{-4}	1
h_{21}	-0.98	49	-50
h_{22}	1×10^{-6} mho	50×10^{-6} mho	50×10^{-6} mho

The T type equivalent circuit of Fig. 10, which at very low frequencies becomes simply that of Fig. 19, is also frequently used to represent the common base stage in audio-frequency design. Although this circuit can also be used for the analysis and design of common emitter and common collector amplifiers, it is often convenient to use for these latter configurations a T type equivalent circuit in which the current generator is proportional to the input current (base current). Such an equivalent circuit is shown in Fig. 20. The constant b is equal to $a/(1 - a)$ and is approximately equal to α_e.

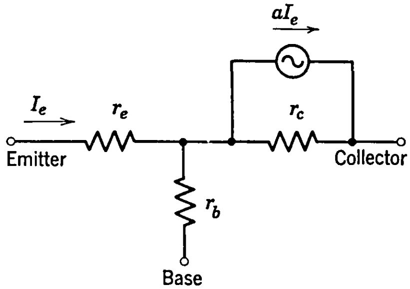


FIG. 19. Low-frequency T type equivalent circuit used for common base configuration.

In junction transistors typical values of the elements composing the equivalent circuits of Figs. 19 and 20 lie within the ranges (at $I_E = 1$ ma

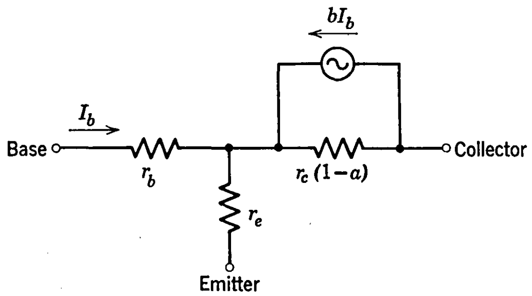


FIG. 20. Low-frequency T type equivalent circuit used for common emitter configuration.

and V_C = 5 volts):

$$r_e = 15\text{–}50 \text{ ohms}$$
$$r_b = 100\text{–}1000 \text{ ohms}$$
$$r_c = 1\text{–}10 \text{ megohms}$$
$$a = 0.95\text{–}0.995$$
$$b = 20\text{–}200$$

If r_e, r_b, r_c, and a are known, the low-frequency h parameters can be calculated and vice versa (see Tables 8 and 9).

At higher audio frequencies, the collector capacitance C_c must often not be neglected (see Fig. 10). Typical values for C_c are: 0.5 to 300 $\mu\mu$f, lower values being typical of high-frequency transistors and higher values of low-frequency high-power transistors. The collector capacitance can be taken into account in the common emitter and common collector equivalent circuit of Fig. 20 by shunting a capacitance $C_c/(1-a)$ across the resistance $r_c(1-a)$ as shown in Fig. 21. Of course, $C_c/(1-a)$ is considerably larger than C_c.

With the h parameters to represent the transistor, the effect of the collector capacitance can be considered by adding to the conductive component of h_{22b} the reactive component $(j2\pi fC_c)$.

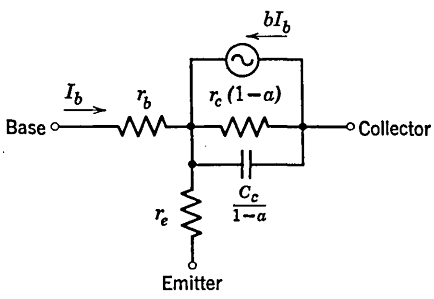


FIG. 21. Effect of collector capacitance on T type common emitter and common collector equivalent circuit.

TABLE 8. LOW-FREQUENCY PARAMETERS

T Equivalent Circuit Parameters	In Terms of: Common Base	Common Emitter	Common Collector
(a) Elements of T Type Equivalent Circuit in Terms of h Parameters (Exact Formulas) (Ref. 22)			
r_e	$(\Delta_b^h - h_{12b})/h_{22b}$	h_{12e}/h_{22e}	$(1 - h_{12c})/h_{22c}$
r_b	h_{12b}/h_{22b}	$(\Delta_e^h - h_{12e})/h_{22e}$	$(\Delta_c^h + h_{21c})/h_{22c}$
r_c	$(1 - h_{12b})/h_{22b}$	$(1 + h_{21e})/h_{22e}$	$-h_{12c}/h_{22c}$
a	$(h_{12b} + h_{21b})/(h_{12b} - 1)$	$(h_{12e} + h_{21e})/(1 + h_{21e})$	$(h_{12c} + h_{21c})/h_{21c}$
(b) Elements of T Type Equivalent Circuit in Terms of h Parameters (Approximate Formulas)			
r_e	$\dfrac{h_{11b}h_{22b} - h_{12b}(1 + h_{21b})}{h_{22b}}$	h_{12e}/h_{22e}	$(1 - h_{12c})/h_{22c}$
r_b	h_{12b}/h_{22b}	$\dfrac{h_{11e}h_{22e} - h_{12e}(1 + h_{21e})}{h_{22e}}$	$\dfrac{h_{11c}h_{22c} + h_{21c}(1 - h_{12c})}{h_{22c}}$
r_c	$1/h_{22b}$	$(1 + h_{21e})/h_{22e}$	$-h_{21c}/h_{22c}$
a	$-h_{21b}$	$h_{21e}/(1 + h_{21e})$	$(1 + h_{21c})/h_{21c}$

TABLE 9. h PARAMETERS IN TERMS OF ELEMENTS OF T TYPE EQUIVALENT CIRCUIT (APPROXIMATE FORMULAS)

h Parameters	Common Base	Common Emitter	Common Collector
h_{11}	$r_e + r_b(1 - a)$	$r_b + r_e/(1 - a)$	$r_b + r_e/(1 - a)$
h_{12}	r_b/r_c	$r_e/[r_c(1 - a)]$	1
h_{21}	$-a$	$a/(1 - a)$	$1/(1 - a)$
h_{22}	$1/r_c$	$1/[r_c(1 - a)]$	$1/[r_c(1 - a)]$

Properties of the Three Configurations

Circuit Performance. The performance characteristics and, consequently, the uses of the three transistor configurations differ widely. Expressions for input and output impedances (or admittances), current and voltage amplifications and power gain are given in Tables 10 and 11 in terms of the h parameters and in terms of r_e, r_b, r_c, a, the load impedance Z_L $(= 1/Y_L)$, and the generator impedance Z_G. (The expressions using the elements of the equivalent circuit of Figs. 19 and 20 are approximations based on inequalities usually fulfilled with junction transistors.)

TABLE 10. CIRCUIT PERFORMANCE IN TERMS OF h PARAMETERS

(Load admittance: $Y_L = G_L + jB_L$; generator impedance: $Z_G = R_G + jX_G$)
Input impedance: $Z_i = h_{11} - (h_{12}h_{21})/(h_{22} + Y_L)$.
Output admittance: $Y_o = h_{22} - (h_{12}h_{21})/(h_{11} + Z_G)$.
Current amplification: $A_i = (h_{21}Y_L)/(h_{22} + Y_L)$.
Voltage amplification: $A_V = (-h_{21})/(h_{11}h_{22} - h_{12}h_{21} + h_{11}Y_L)$.
Transducer gain (= output-power/available power of generator):

$$G_t = \frac{4|h_{21}|^2}{|(h_{11} + R_G)(h_{22} + G_L) - h_{12}h_{21}|^2}.$$

Power gain [a] (= output-power/input-power):

$$G_P = \frac{h_{21}^2}{(1 + h_{22}/G_L)(h_{11}h_{22} - h_{12}h_{21} + h_{11}G_L)}.$$

Maximum available gain [a] (output-power/input-power with both input and output matched):

$$G_{\max} = \frac{h_{21}^2}{(\sqrt{h_{11}h_{22}} + \sqrt{h_{11}h_{22} - h_{12}h_{21}})^2}.$$

Matched input resistance: [a]

$$R_{im} = \sqrt{h_{11}/h_{22}}\,\sqrt{h_{11}h_{22} - h_{12}h_{21}}.$$

Matched output conductance: [a]

$$G_{om} = \sqrt{h_{22}/h_{11}}\,\sqrt{h_{11}h_{22} - h_{12}h_{21}}.$$

[a] Valid at low frequencies only.

TABLE 11. CIRCUIT PERFORMANCE OF CONFIGURATIONS IN TERMS OF ELEMENTS OF LOW-FREQUENCY T TYPE EQUIVALENT CIRCUIT (APPROXIMATE FORMULAS) [a]

	Common Base	Common Emitter	Common Collector
Input resistance, R_i	$r_e + r_b(1 - a) + r_b R_L/r_c$	$r_b + \dfrac{r_e r_c}{r_c(1 - a) + R_L}$	$r_b + \dfrac{r_c R_L}{r_c(1 - a) + R_L}$
Output resistance, R_o	$r_c \cdot \dfrac{R_G + r_e + r_b(1 - a)}{R_G + r_e + r_b}$	$r_c(1 - a) + \dfrac{r_e(a r_c + R_G)}{R_G + r_e + r_b}$	$r_e + \dfrac{r_c(1 - a)(R_G + r_b)}{R_G + r_c}$
Current amplification, A_i	$a/(1 + R_L/r_c)$	$\dfrac{a}{1 - a + (R_L/r_c)}$	$\dfrac{1}{1 - a + (R_L/r_c)}$
Voltage amplification, A_V	$\dfrac{a r_c R_L}{r_c[r_e + r_b(1 - a)] + R_L(r_e + r_b)}$	$\dfrac{-a r_c R_L}{r_c[r_e + r_b(1 - a)] + R_L(r_e + r_b)}$	$\dfrac{R_L}{r_e + r_b(1 - a) + R_L}$
Power gain, G_P	$\dfrac{a^2 r_c R_L}{r_c[r_e + r_b(1 - a)] + R_L(r_e + r_b)}$	$\dfrac{a^2 r_c^2 R_L}{\left\{\begin{matrix}[r_c(1 - a) + R_L]\{r_c[r_e + r_b(1 - a)] \\ + R_L(r_e + r_b)\}\end{matrix}\right\}}$	$\dfrac{R_L r_c}{\left\{\begin{matrix}[r_c(1 - a) + R_L] \\ [r_e + r_b(1 - a) + R_L]\end{matrix}\right\}}$

[a] Approximations valid if: $r_e \ll R_L \ll r_c$; $r_e \ll (1 - a)r_c$; $r_b \ll r_c$.

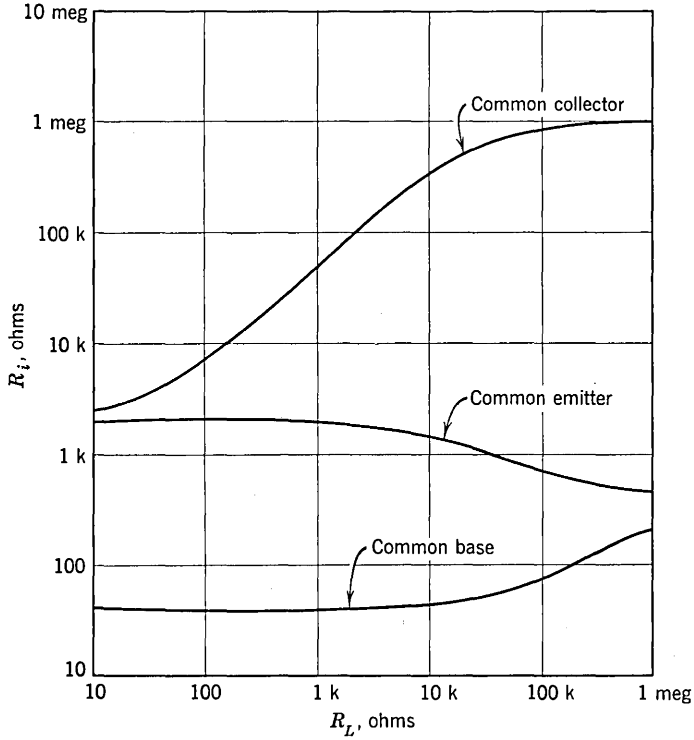


FIG. 22. Input resistance as function of load resistance.

Figures 22 to 25 show the input resistance R_i as function of load resistance R_L, the output resistance R_o as function of the generator resistance R_G, the current amplification A_i and the power gain as functions of R_L for the three configurations of the transistor whose h parameters are listed in Table 7.

Two factors are of importance:

1. The transistor is a *nonunilateral device.* In transistors there is no isolation between input and output terminals: as evident from Tables 10 and 11, input and output impedances are functions of load and source impedances respectively. Consequently, a transistor amplifier stage cannot be considered in an isolated manner and in designing a stage of a multistage amplifier, the effects of other adjoining stages must be taken into account. The procedure followed in the design of multistage amplifiers is to start by designing the last stage (the load impedance of which is usually given), then determining the input impedance of the last stage, which, in conjunction with biasing resistors, forms the load resistance of the previous stage, which now can be designed.

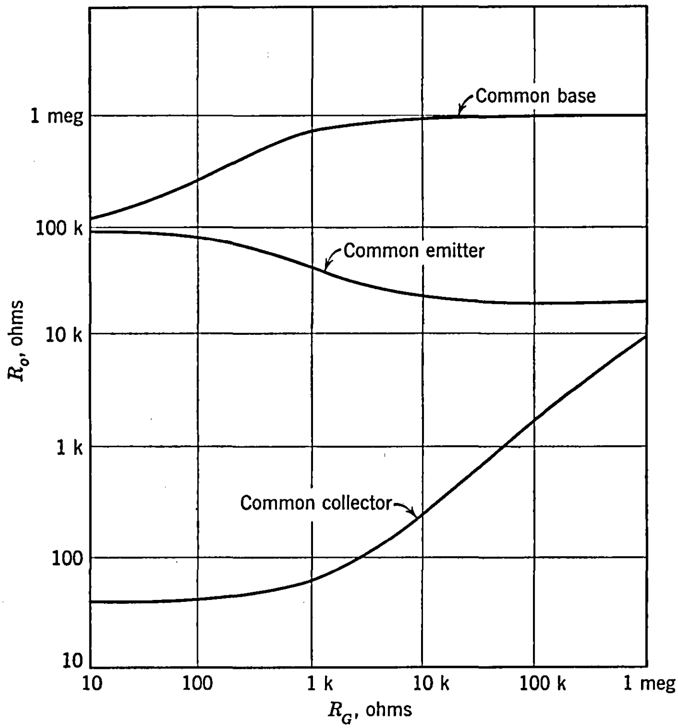


FIG. 23. Output resistance as function of the generator resistance.

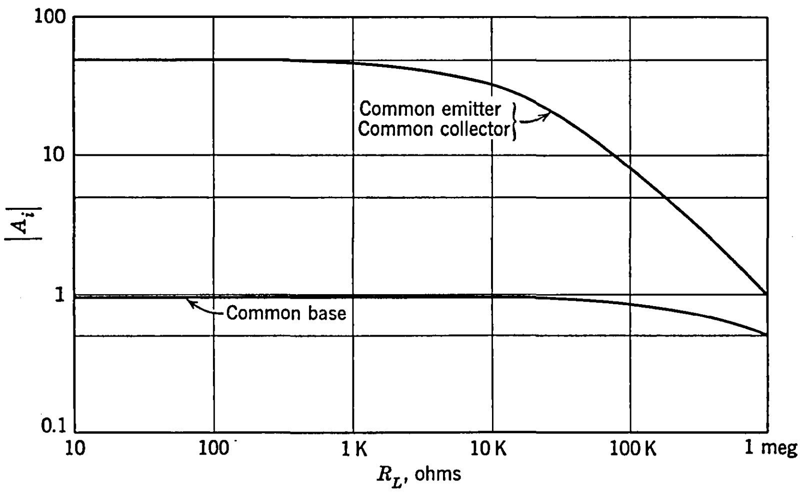


FIG. 24. Current amplification as function of load resistance.

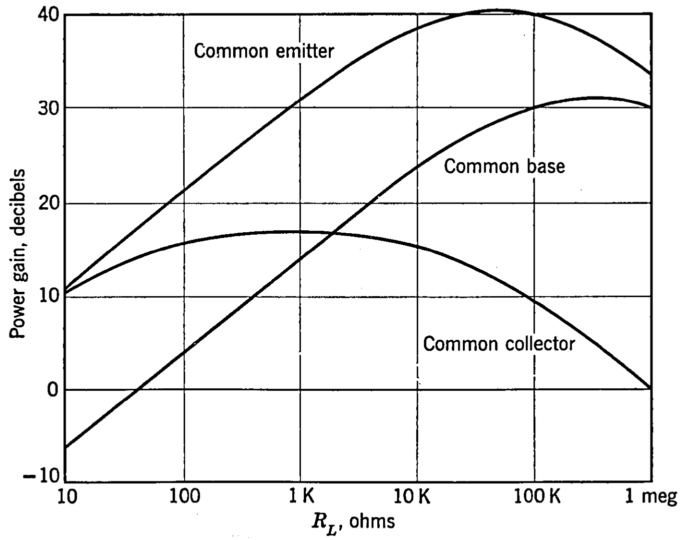


FIG. 25. Power gain as function of load resistance.

2. The concept of *voltage amplification* is usually less useful in the analysis and design of transistor amplifiers than the *current amplification* and the *power gain*. Both input and output impedances of transistors are finite (in common base and common emitter configurations, the input impedance is considerably smaller than the output impedance, whereas in electron tubes, the input impedance is very high and, in many cases, can be considered as infinite).

General Characteristics. The three configurations have the following general characteristics (only the order of magnitude of the numbers mentioned is significant). The *common base stage* has very low input impedance (100 ohms or less at $I_E = 1$ ma, depending on R_L), high output impedance (up to several megohms, depending on R_G), less than unity current amplification, and intermediate power gain.

The *common emitter configuration* has a higher input impedance (thousands ohms), lower output impedance (several ten thousand ohms), high current amplification (in excess of 20) and high power gain.

The *common collector configuration* has a high input impedance (up to several megohms, depending on the load impedance), low output impedance, high current amplification, close to unity voltage amplification and relatively low power gain.

Small-Signal Amplifiers

The fact that the three configurations have entirely different properties must be considered when cascading several amplifier stages to form a

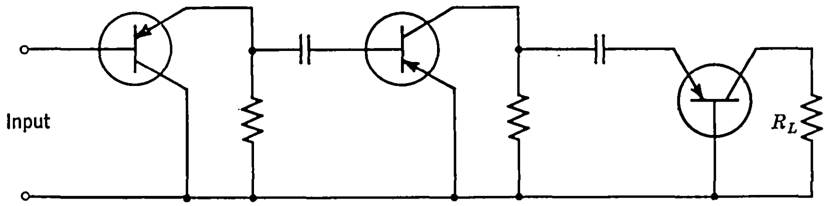


FIG. 26. RC-coupled amplifier (only a-c connections are shown).

multistage amplifier. Cascaded stages can be coupled to each other in several ways.

***RC* Coupling.** In this most common method used to couple cascaded stages, the intermediate stages are usually in common emitter configuration, yielding considerable current amplification. (Common base stages yield less than unity current amplification, whereas common collector stages yield less than unity voltage amplification. Consequently, neither of these yields power gain in intermediate stages, where the load impedance of a stage, i.e., the input impedance of the following stage, is on the same order of magnitude as the input impedance of the stage considered.) The first and last stages of multistage amplifiers do occasionally use common base or common collector stages, depending on the generator and load impedances from and into which the amplifier must work. A schematic example of an RC-coupled three-stage amplifier is shown in Fig. 26. This amplifier is required to have a high input impedance and uses a common collector circuit as its first stage. The output stage is common base to reduce distortion (common base characteristics are more linear than common emitter characteristics). The intermediate stage uses the common emitter configuration.

D-C Coupling. Transistor a-c amplifiers are sometimes *d-c-coupled* (Fig. 27). Such amplifiers can be designed to have improved tempera-

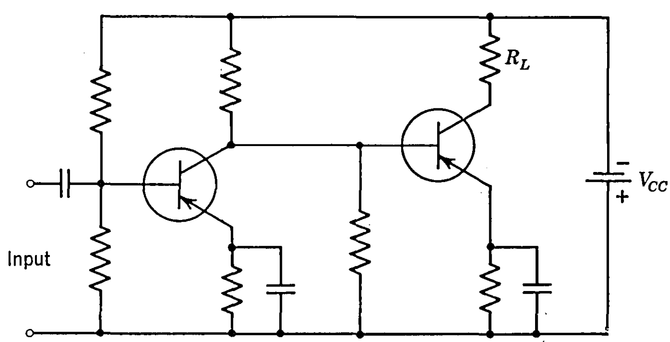


FIG. 27. D-c-coupled transistor amplifier.

ture characteristics and use fewer components. Here, too, common emitter is the preferred configuration, although the choice of the configuration may be influenced by d-c considerations.

Transformer coupling (Fig. 28) of stages may increase the stage gain (by improved impedance matching between stages) and is often used between driver and output stages of amplifiers. With coupling transformers, cascaded common base stages can provide considerable power gain and very low distortion. (This is a rather expensive design procedure and is almost never used.)

The effect of the *collector capacitance* C_c must be taken into account at relatively low frequencies—certainly at the upper end of the audio

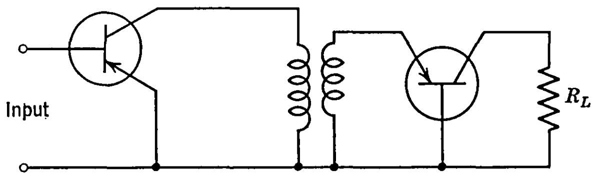


FIG. 28. Common base output stage transformer coupled to common emitter driver stage.

band. The cutoff frequency of the common base stage is usually determined by the product (C_cR_L) where R_L is the load resistance. In the equivalent circuit of Fig. 20, used for common emitter configuration, the capacitance is larger: $C_c/(1 - a)$. The cutoff frequency of the common emitter current amplification is determined partially by $[R_LC_c/(1 - a)]$ but is further influenced by the frequency dependence of $b = a/(1 - a)$. The constant $b \cong \alpha_e$ exhibits considerable phase shift and starts decreasing at relatively low frequencies, as will be discussed in the next section.

Other Circuit Factors. The frequency performance of an amplifier depends also on the nature of the coupling networks used and on the bypass capacitors used in conjunction with biasing resistors. For example, in the circuit of Fig. 26, the coupling capacitors must have an impedance substantially lower than the output impedance of a stage at all low frequencies at which "flat" frequency response is desired. The impedance of the bypass capacitors in the emitter circuits must be substantially lower than the impedance seen from the emitter at low frequencies where flat response is desired.

Transistor *feedback circuits* must be designed with consideration of the fact that transistor impedances are finite (see Refs. 37 and 38). Both series and shunt feedback are frequently used (see Fig. 29). The analytical treatment of these circuits is often considerably simplified by

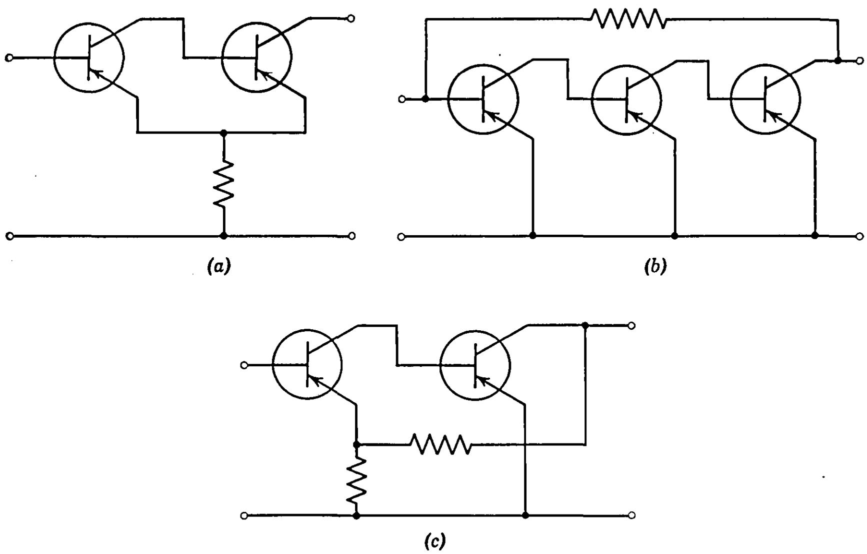


FIG. 29. Schematic examples of transistor feedback circuits.

the use of matrix algebra. The frequency dependence of transistor parameters starts at relatively low frequencies and this must be considered in feedback circuits.

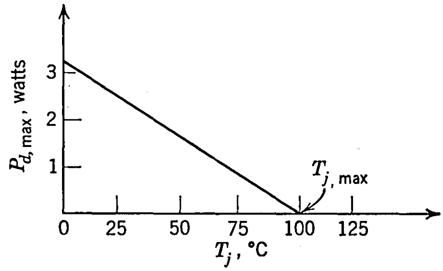
Power Amplifiers

The analysis and the design of power amplifiers differs from that of small-signal amplifiers in that the maximum ratings of the transistor and the nonlinearities of transistor characteristics must be taken into account.

Maximum Ratings. The maximum ratings of a transistor are usually: maximum collector voltage $V_{c,\,\max}$, maximum collector current $I_{c,\,\max}$ (or maximum emitter current, the difference being negligible) and maximum collector dissipation $P_{d,\,\max}$. (The collector dissipation usually accounts for almost the entire dissipation within the device. In some cases, however, the emitter dissipation may also be significant.) The maximum collector voltage is imposed in common base configuration by the breakdown voltage of the collector junction and in common emitter configuration by the collector-to-base punch-through voltage. The collector current should not exceed a certain maximum value because of the considerable decrease of the current amplification at high currents, which may result in excessive distortion and reduced gain at high levels.

The maximum permissible power dissipation is a function of the temperature of the collector junction. The junction temperature T_j must not exceed a certain maximum value $T_{j,\,\max}$. (For germanium transistors

$T_{j,\ \max}$ is usually on the order of 100°C.) The permissible power dissipation increases with decreasing temperature (along a straight line, see Fig. 30*a*). Since the permissible dissipation is a function of T_j and not directly of the ambient temperature T_{amb}, the relationship between permissible dissipation and ambient temperature is influenced by the nature of the heat dissipator on which the transistor is mounted (i.e., by the difference



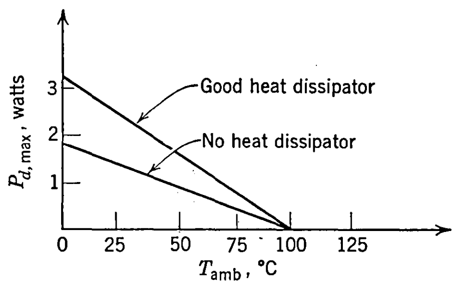


FIG. 30. Maximum permissible power dissipation of transistor as a function of (*a*) junction temperature, T_j and (*b*) ambient temperature, T_{amb}.

between ambient temperature and junction temperature). If no adequate heat dissipator is used, T_j will reach $T_{j,\ \max}$ at relatively low ambient temperatures. If a good heat dissipator is used, the difference between T_j and T_{amb} will be smaller and the transistor will be capable of larger power dissipation at a given ambient temperature. Figure 30*b* shows schematically $P_{d,\ \max}$ as a function of T_{amb} for two cases: without heat dissipator and with good heat dissipator.

Operating Ranges. The region of permissible operation of the power transistor is as shown in Fig. 31. (The graph shows common base type characteristics; the situation is, however, similar in the common emitter and common collector cases.) The region of permissible operation is bounded by $V_{c,\ \max}$, $I_{c,\ \max}$, and a hyperbola representing $P_{d,\ \max}$. Operation at currents below $I_{c,\ \min}$ and at voltages below $V_{c,\ \min}$ is usually un-

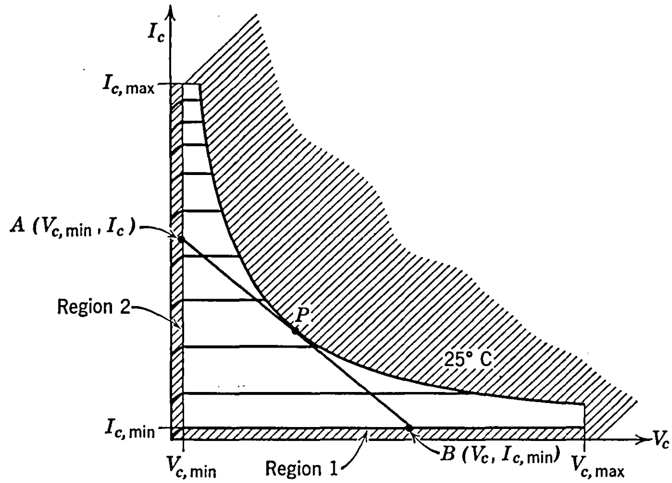


FIG. 31. Output characteristics of transistor indicating region of permissible operation.

desirable because of the nonlinearities of the output or transfer characteristics.

Design Procedures. The design procedures for transistor power amplifiers are in many respects quite similar to those used for electron tube power amplifiers.

Class A. In class A operation the collector efficiency cannot exceed 50%. In class B push-pull operation the maximum realizable efficiency is 78%. The smaller the output current for zero input current (region 1) and the smaller the output voltage at which the output current starts decreasing rapidly (region 2), the closer is the realizable efficiency to the theoretical maximum.

The distortion in power amplifiers can be attributed to several sources. The nonlinearity of the input characteristic (emitter current or base current versus emitter-to-base voltage) results in *input distortion,* which can be minimized by driving the transistor from a high impedance (= current) source. *Output distortion* may arise in the form of *hard clipping* of portions of the a-c signal for which the collector voltage is reduced to zero. Because of the nonlinearity of the current transfer characteristic (I_C versus I_E or I_C versus I_B) which manifests itself in the crowding of the output characteristics at high values of the output current, *soft clipping* may occur. This is common in transistors where α_b and, consequently, α_e are strongly decreasing functions of I_C (or of I_E, see Fig. 32). (This occurs for different transistors at different output current levels. In some transistors, α_b starts decreasing at emitter currents as low as 20 ma; in others, α_b is constant up to 5 to 10 amp.) In such transistors, the $I_C - V_C$ characteristics

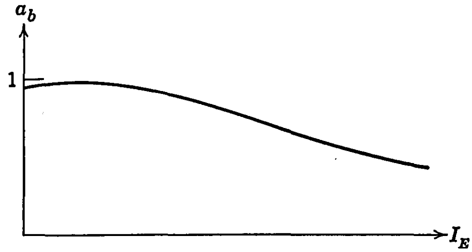


FIG. 32. Schematic response of a_b as function of the emitter current.

drawn with the *input-voltage* as parameter (instead of the input current) are more equidistant or, in other words, the I_C versus V_{BE} curve is more linear than the I_C versus I_B curve (Fig. 33). Consequently, distortion due to α_b decreasing with I_C can sometimes be reduced by driving the transistor from a low impedance (= voltage) source.

In class A amplifiers the operating point should be placed about midway between A and B (Fig. 31). This results in maximum efficiency, and, usually, in not excessive distortion. In class B amplifiers, using transistors where α_b decreases with I_C and which are consequently driven by a voltage source, considerable input distortion may arise at small-signal levels. This *crossover distortion* (Fig. 34) can be reduced by using class AB rather than pure class B operation, i.e., by applying a small forward bias to both push-pull transistors, as shown in Fig. 35. If a thermistor is used for R_2, the crossover distortion can be eliminated over a wide range of temperatures (see Ref. 2).

Class B. The choice of the transistor configuration to be used in class B amplifiers depends on many considerations. Distortion will be lowest in common base configuration, the gain, however, is relatively low. The common emitter circuit has higher gain but also higher distortion. Furthermore, the common emitter stage is more subject to thermal runaway than the common base circuit. The common collector gain may be larger than the common emitter gain for small load impedances. (The load impedance is small, whenever the collector voltage is small and the required power output is large.) For comparable conditions the common collector distortion is smaller than the common emitter distortion.

Since transistors have a finite gain, the input power required by high-power transistor amplifiers may be quite appreciable. The driver stage (which usually uses a transistor with smaller permissible dissipation) must sometimes also be designed as a power stage. Coupling between driver and class B output stages can be achieved by using center-tapped transformers or phase inverters. If the output terminals of the phase inverter are coupled to the input terminals of the push-pull transistors by

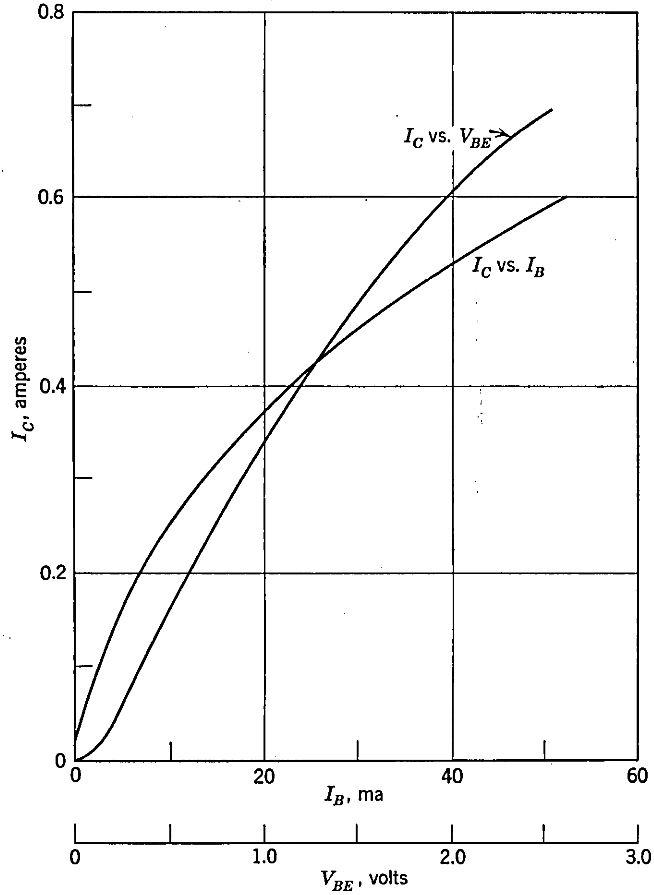


Fig. 33. Collector current vs. base current and base-to-emitter voltage.

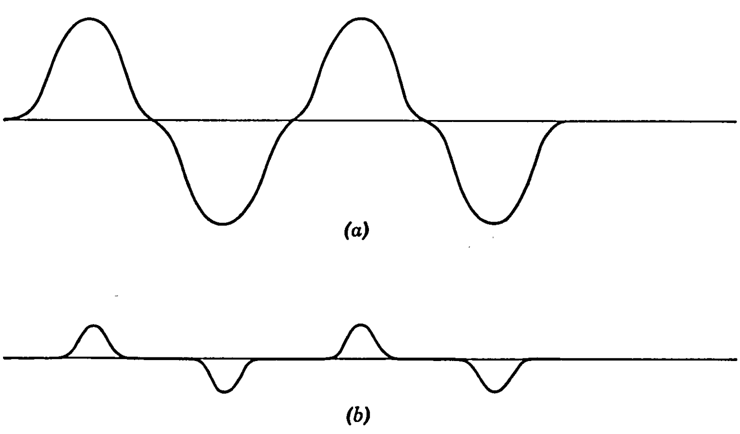


Fig. 34. Effect of crossover distortion on (*a*) large and (*b*) small sinusoidal signal.

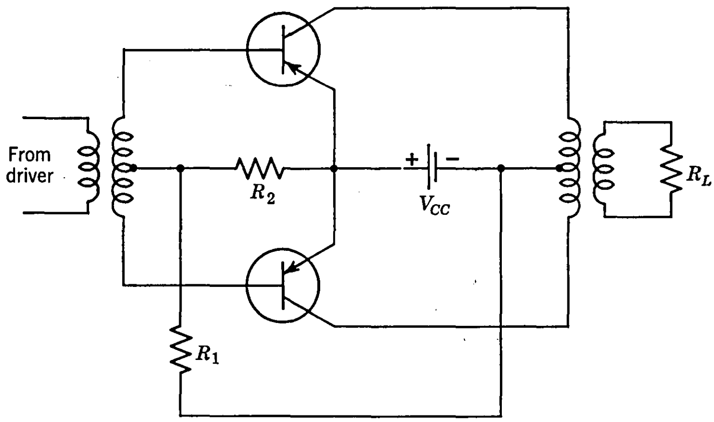


FIG. 35. Class AB operation of push-pull transistor stage.

means of capacitors, it is important to prevent the build up of charges on the capacitors which might drive the push-pull stage into class C operation. This can be achieved by the circuit of Fig. 36, using diodes (see Ref. 22).

Phase-inverting means (inverter stage or transformer) can be completely omitted if the class B output stage uses one *p-n-p* and one *n-p-n* transistor having similar characteristics (Fig. 37). In such arrangements the positive half-waves are amplified by one transistor and the negative half-waves by the other. Such *complementary symmetry* circuits do

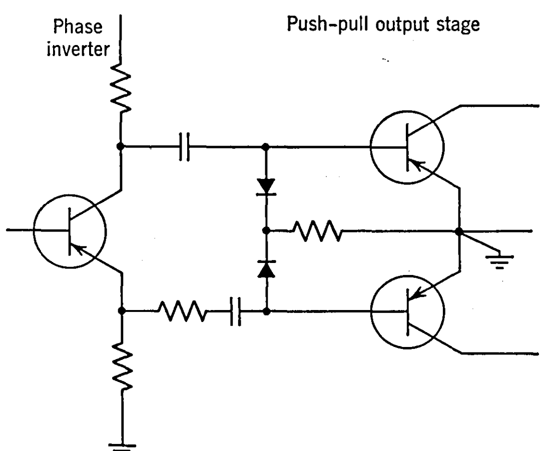


FIG. 36. Capacitor coupling of push-pull stage to phase inverter.

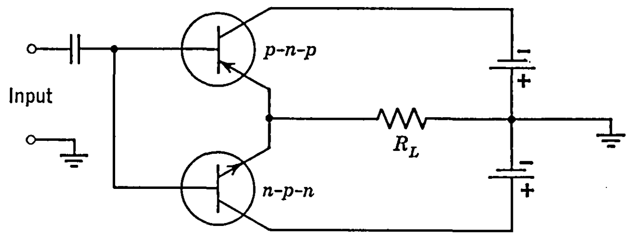


FIG. 37. Transistor push-pull stage using principle of complementary symmetry.

need, however, a center-tapped power supply (the supply voltage being twice that needed by an individual transistor).

Darlington Circuit. The behavior of the compound connection of two transistors shown in Fig. 38*a* (*Darlington circuit*) is similar to that of a transistor having α_b very close to unity. The transfer characteristic of the compound circuit in common emitter configuration (Fig. 38*b*) is more linear than that of an individual transistor. Class B stages using the compound connection in a complementary symmetry arrangement are useful as output stages of amplifiers having very low distortion (Fig. 38*c*).

Low-Noise Amplifier Stages

The noise properties of transistors are usually described in terms of the noise figure F:

$$F = P_{ni}/P_{ng},$$

where P_{ni} is the equivalent noise power referred to the input, and P_{ng} is the available noise power generated in the source resistance. The noise figure of modern junction transistors is 2 to 10 db at 1000 cps.

The frequency dependence of transistor noise is shown schematically in Fig. 39. In very low-frequency amplifiers, the $(1/f)$ portion of the noise vs. frequency characteristic is dominant, and the diagram of Fig. 40 can be used to determine the noise figure corresponding to a certain value of P_{ni} within a given passband (f_1 is the lower and f_2 the upper cutoff frequency of the amplifier).

The noise figure of a transistor can be minimized by using source resistances of the order of 100 to 1000 ohms and operating the transistor at low collector voltage (less than 1 volt) and low emitter current (less than 1 ma). In some cases, the collector voltage is even slightly reversed (see Ref. 55).

Low-Frequency Active Filters

Transistors can be used in the design of selective networks in several ways. Negative resistances produced with transistors can be combined

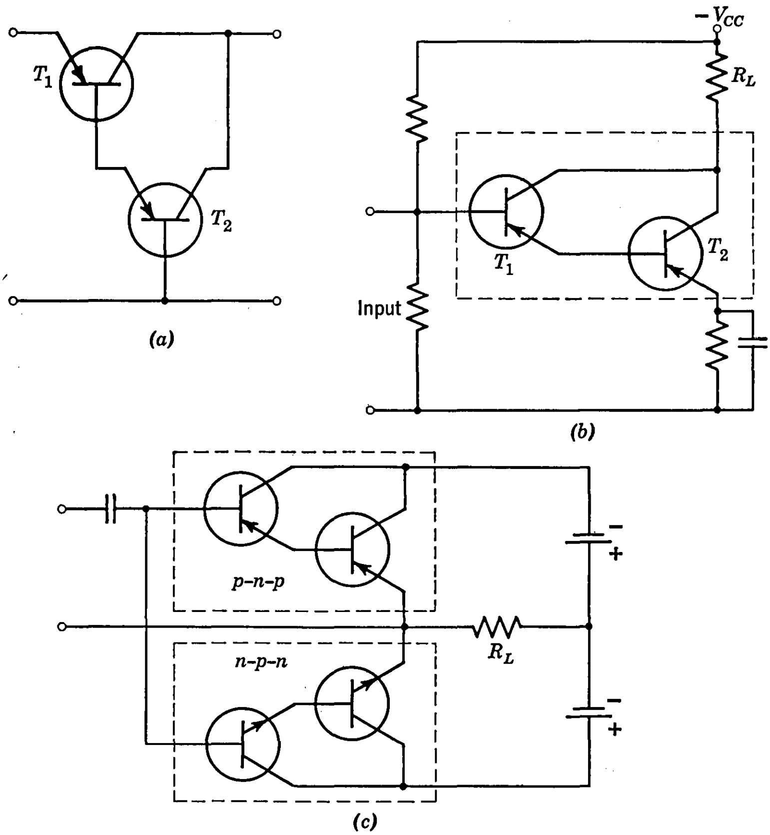


FIG. 38. Compound connection (Darlington circuit) and applications: (*a*) compound connection, (*b*) common emitter compound amplifier, (*c*) class B complementary symmetry compound output stage.

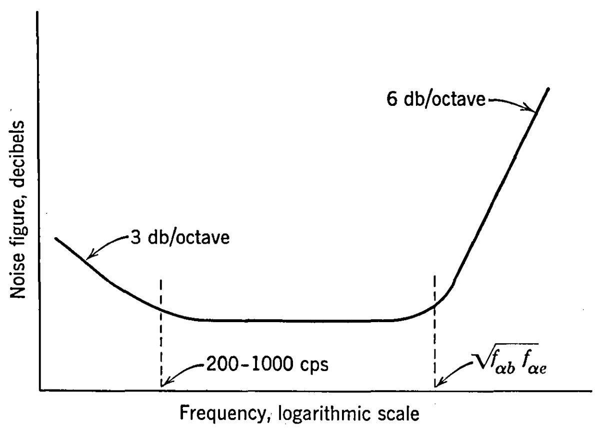


FIG. 39. Schematic frequency dependence of transistor noise figure.

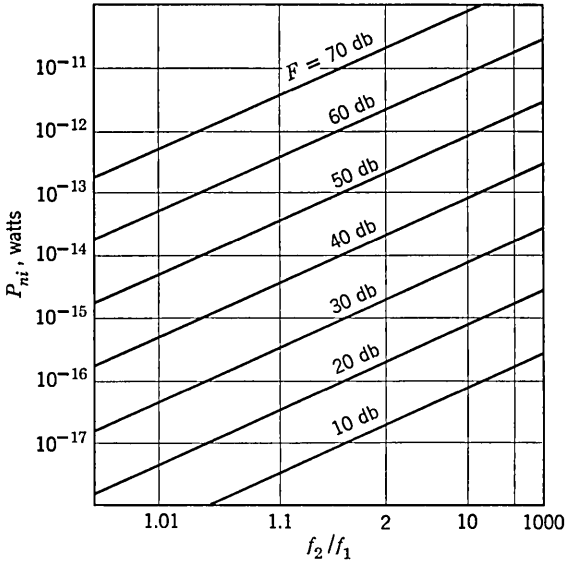


FIG. 40. Equivalent noise input power as function of the ratio of cutoff frequencies for low-frequency amplifiers (for constant noise figure).

with conventional inductances to improve their Q (*Q multiplier*). Small inductances in combination with transistors can be made to behave like considerably larger inductances (*L multiplier*). Networks consisting of resistances, capacitances, and transistors can be designed to exhibit properties usually achievable only with networks containing also inductances (Fig. 41). The subject is too complicated to be treated here (see Refs. 64–68).

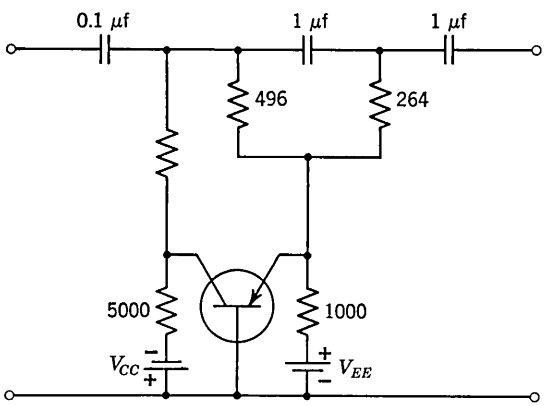


FIG. 41. Example of active high-pass filter using transistor.

4. HIGH-FREQUENCY AMPLIFIERS

High-Frequency Representation. At high frequencies, the small-signal parameters and the elements of the equivalent circuits are frequency dependent and complex, that is, they consist of both real and imaginary components. "Exact" transistor equivalent circuits which represent the transistor from very low up to very high frequencies contain many circuit elements, some of which are of distributed nature. Such accurate but very complicated circuits are not very useful in practical circuit design and will not be discussed here. An approximate equivalent circuit which is valid throughout a wide frequency range and is particularly useful in dealing with common base circuits is shown in Fig. 42.

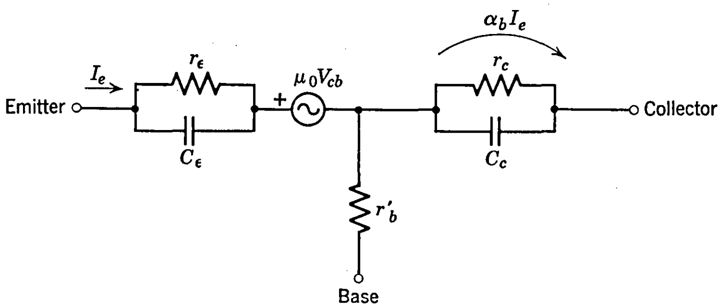


FIG. 42. Transistor equivalent circuit valid at low and high frequencies $[\alpha_b \cong \alpha_{b0}/(1 + jf/f_{\alpha b})]$.

The factor multiplying I_e in the current generator is substantially equal to the common base short-circuit current amplification α_b and is approximately the following function of the frequency:

$$\alpha_b \cong \alpha_{b0}/(1 + jf/f_{\alpha b}).$$

α_{b0} is the low-frequency value of α_b, and $f_{\alpha b}$ is the frequency at which α_b is 0.707 times α_{b0}. The term $f_{\alpha b}$ is usually called the α_b *cutoff frequency*. For modern transistors $f_{\alpha b}$ can be anywhere between 1 and 1000 mc.

The common emitter short-circuit current amplification is then approximately:

$$\alpha_e \cong \alpha_b/(1 - \alpha_b) \cong \alpha_{e0}/(1 + jf/f_{\alpha e}).$$

α_{e0} is the low-frequency value of α_e and $f_{\alpha e}$ is the frequency at which α_e is 0.707 times α_{e0}. It is often called the α_e *cutoff frequency*. It is important to note that $f_{\alpha e}$ is considerably smaller than $f_{\alpha b}$:

$$f_{\alpha e} \cong (1 - \alpha_{b0})f_{\alpha b}.$$

The *collector impedance* z_c consists of the collector resistance r_c in parallel with the collector capacitance C_c. This parallel connection defines the *collector impedance cutoff frequency.*

$$f_c = 1/2\pi r_c C_c.$$

At this frequency z_c has a 45° phase angle. The range of f_c is 10–100 kc.

The *extrinsic base resistance* r'_b (often also called *base-spreading resistance* or *high-frequency base resistance*) is different from r_b used in the low-frequency equivalent circuit of Fig. 10. They are related to each other by

$$r'_b + \mu_0 r_c \cong r_b.$$

Typical values of r'_b lie between 25 and 500 ohms.

If a voltage is applied between collector and base, a feedback voltage appears between emitter and base. This feedback is due to the existence of both μ_0 (*intrinsic feedback*) and r'_b. At higher frequencies, where C_c accounts for the collector impedance, the effect of μ_0 can be neglected.

The *emitter diffusion impedance* consists of r_ϵ (*emitter diffusion resistance*) and C_ϵ (*emitter diffusion capacitance*) in parallel. r_ϵ is a function of the d-c emitter current I_E ($r_\epsilon \cong 26/I_E$, where I_E is in milliamperes and r_ϵ in ohms). One has approximately:

$$1/2\pi r_\epsilon C_\epsilon \cong f_{\alpha b}.$$

Consequently, the effect of C_ϵ can be neglected up to relatively high frequencies. (Note that r_ϵ is different from r_e, which was used in the low-frequency T type equivalent circuit.)

The equivalent circuit of Fig. 42 is valid from very low frequencies up to approximately $0.3f_{\alpha b}$. At frequencies considerably larger than f_c but smaller than $f_{\alpha b}$, the equivalent circuit of Fig. 42 can be used in a simplified form shown in Fig. 43.

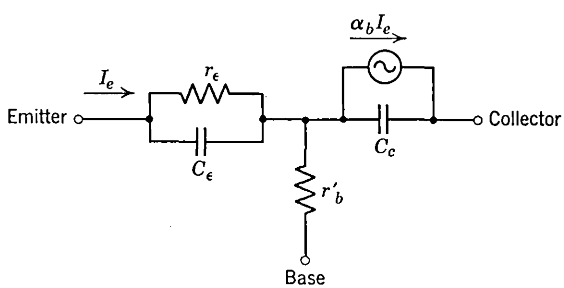


FIG. 43. Equivalent circuit useful for $f \gg f_c$.

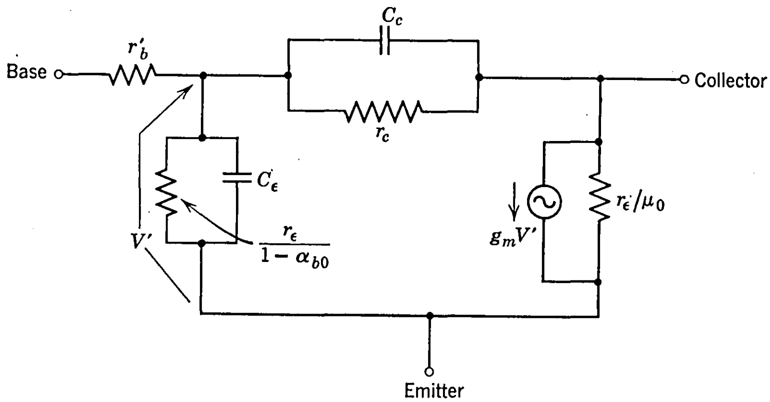


FIG. 44. Equivalent circuit often used to represent common emitter transistor configuration ($g_m \cong \alpha_{b0}/r_\epsilon$).

The common emitter configuration is often represented by the *hybrid* π type equivalent circuit of Fig. 44. (The elements of this equivalent circuit can be expressed in terms of the elements of the circuit of Fig. 42 as indicated.) This circuit is very similar in its structure to the equivalent circuit of an electron tube but has r'_b in series with the input of the π configuration. For frequencies such that $f_{\alpha e} < f < f_{\alpha b}$, this circuit becomes simply that of Fig. 45. The transconductance g_m is:

$$g_m \cong \alpha_{b0}/r_\epsilon.$$

Instead of using equivalent circuits in the solution of problems involving the application of transistors at high frequencies, the h parameters can be used directly in such calculations. Expressions for the h parameters can be derived from the equivalent circuits of Figs. 42 and 45. These expressions can be simplified considerably in certain frequency ranges. The frequency dependence of transistor parameters is determined by three fre-

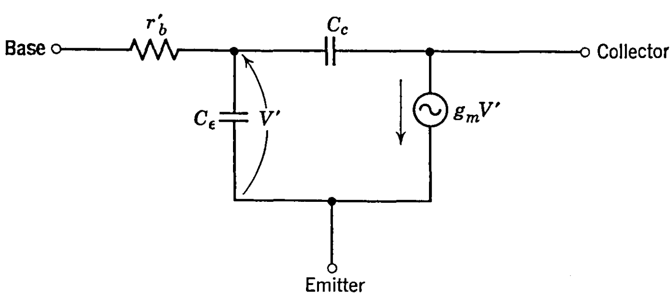


FIG. 45. Simplified version of the equivalent circuit of Fig. 44 applicable at $f_{ae} < f < f_{ab}$.

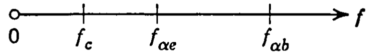


FIG. 46. Significant frequencies determining high-frequency behavior of transistor.

quencies: f_c, $f_{\alpha e}$, and $f_{\alpha b}$. The lowest of these frequencies is f_c, and $f_{\alpha e}$ is usually several times larger. The highest significant frequency is $f_{\alpha b}$ (Fig. 46). The approximate frequency response of the common base and common emitter h parameters is given in "broken line" representation in Figs. 47 and 48. Table 12 contains expressions for the h parameters of the three

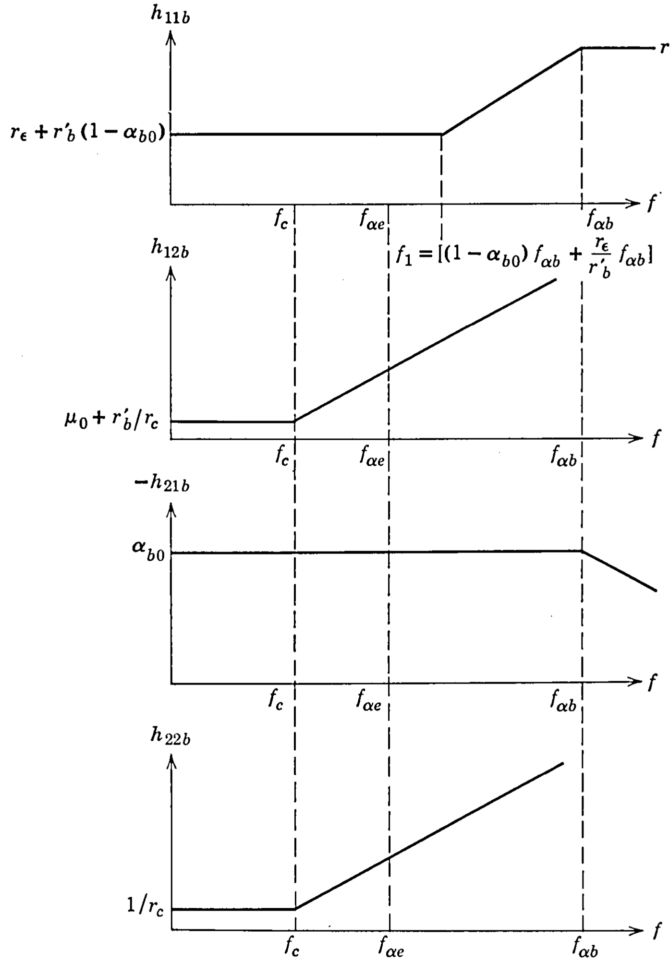


FIG. 47. Schematic frequency response of common base h parameters.

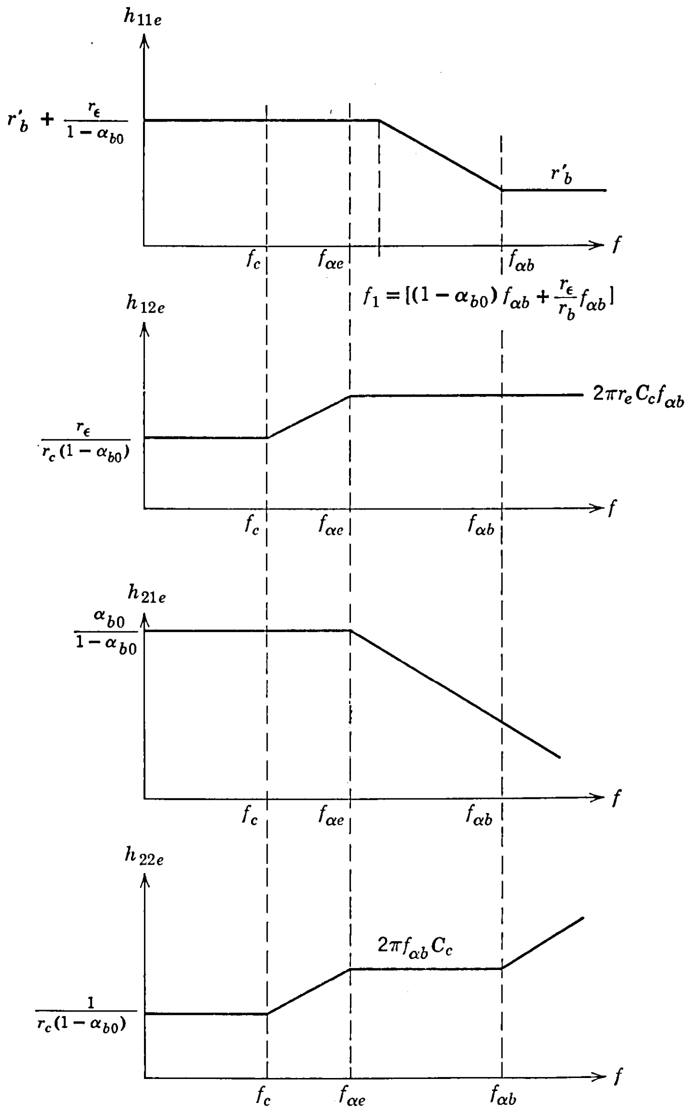


FIG. 48. Schematic frequency response of common emitter h parameters.

transistor configurations. Table 12a gives "complete" expressions valid from very low frequencies up to approximately $0.3f_{\alpha b}$. The expressions of Table 12b are valid at frequencies exceeding $f_{\alpha e}$ and smaller than $0.3f_{\alpha b}$.

The frequency range in which the transistor can be usefully applied is usually limited to frequencies well below $f_{\alpha b}$. In the relatively few cases where the transistor is operated at frequencies close to or slightly exceeding

$f_{\alpha b}$, a better approximation must be used for α_b, for example:

$$\alpha_b \cong \frac{\alpha_{b0}}{(1 + jf/f_{\alpha b})(1 + jf/4f_{\alpha b})}.$$

The equivalent circuits and h parameter expressions discussed do not include the effects of parasitic circuit elements which often are quite important. For example, when designing common emitter stages, the parasitic collector-to-base capacitance must sometimes be considered. In other cases, the ohmic resistances of the emitter and collector regions must be considered and added to h_{11b} and $1/h_{22b}$, respectively.

Wide-Band (Video) Amplifiers. The high-frequency response of transistor amplifiers starts decreasing at relatively low frequencies owing to both the reactive components of transistor impedances and the frequency variation of the forward characteristic.

The frequency response can be extended by methods similar to those used in vacuum tube amplifiers: series, shunt, or series-shunt peaking (Fig. 49). Another method leading to wider band is the use of frequency sensitive feedback (often in combination with peaking, see Fig. 50).

Video amplifier stages (like other RC-coupled stages) are usually of the common emitter type since both current and voltage amplification are needed as the signal progresses from one amplifier stage to the next

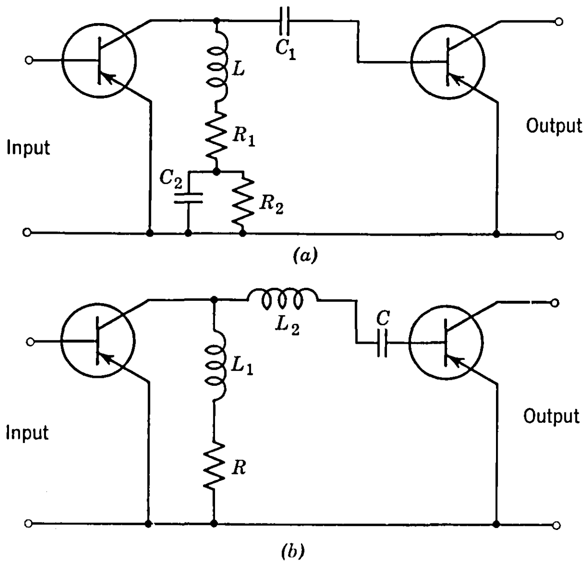


FIG. 49. Coupling of video amplifier stages: (a) shunt peaking and low-frequency compensation and (b) high-frequency shunt-series peaking.

TABLE 12

(a) Approximate h Parameters Valid Throughout the Low- and High-Frequency Range (up to Approximately 0.3$f_{\alpha b}$)

Parameter	Transistor Configuration: Common Base	Common Emitter	Common Collector
h_{11}	$\dfrac{r_\epsilon}{1 + j\omega/\omega_{\alpha b}} + [r'_b(1 - \alpha_{b0})]$ $\dfrac{1 + \dfrac{j\omega}{(1 - \alpha_{b0})\omega_{\alpha b}}}{[1 + j\omega/\omega_{\alpha b}]}$	$r'_b + \dfrac{r_\epsilon}{1 - \alpha_{b0}} \cdot \dfrac{1}{1 + \dfrac{j\omega}{(1 - \alpha_{b0})\omega_{\alpha b}}}$	$r'_b + \dfrac{r_\epsilon}{1 - \alpha_{b0}} \cdot \dfrac{1}{1 + \dfrac{j\omega}{(1 - \alpha_{b0})\omega_{\alpha}}}$
h_{12}	$\mu_0 + r'_b(g_c + j\omega C_c)$	$\dfrac{r_\epsilon(g_c + j\omega C_c)}{(1 - \alpha_{b0})\left[1 + \dfrac{j\omega}{(1 - \alpha_{b0})\omega_{\alpha b}}\right]} - \mu_0$	1
h_{21}	$-\alpha_{b0}/(1 + j\omega/\omega_{\alpha b})$	$\dfrac{\alpha_{b0}}{1 - \alpha_{b0}} \cdot \dfrac{1}{1 + \dfrac{j\omega}{(1 - \alpha_{b0})\omega_{\alpha b}}}$	$\dfrac{-1}{1 - \alpha_{b0}} \cdot \dfrac{1 + j\omega/\omega_{\alpha b}}{1 + \dfrac{j\omega}{(1 - \alpha_{b0})\omega_{\alpha b}}}$
h_{22}	$g_c + j\omega C_c$	$\dfrac{(g_c + j\omega C_c)(1 + j\omega/\omega_{\alpha b})}{(1 - \alpha_{b0})\left[1 + \dfrac{j\omega}{(1 - \alpha_{b0})\omega_{\alpha b}}\right]}$	$\dfrac{(g_c + j\omega C_c)(1 + j\omega/\omega_{\alpha b})}{(1 - \alpha_{b0})\left[1 + \dfrac{j\omega}{(1 - \alpha_{b0})\omega_{\alpha b}}\right]}$

(b) Approximate h Parameters in the Medium- and High-Frequency Range ($f_c < f < 0.3f_{ab}$)

Parameter	Transistor Configuration: Common Base	Common Emitter	Common Collector
h_{11}	$\dfrac{r_\epsilon}{1 + j\omega/\omega_{ab}} + [r'_b(1 - \alpha_{b0})]$	$r'_b + \dfrac{r_\epsilon}{1 - \alpha_{b0}} \cdot \dfrac{1}{1 + \dfrac{j\omega}{(1 - \alpha_{b0})\omega_{ab}}}$	$r'_b + \dfrac{r_\epsilon}{1 - \alpha_{b0}} \cdot \dfrac{1}{1 + \dfrac{j\omega}{(1 - \alpha_{b0})\omega_{ab}}}$
h_{12}	$\left[\dfrac{1 + j\omega/(1 - \alpha_{b0})\omega_{ab}}{1 + j\omega/\omega_{ab}}\right] j\omega C_c r'_b$	$\dfrac{j\omega C_c r_\epsilon}{1 - \alpha_{b0}} \cdot \dfrac{1}{1 + \dfrac{j\omega}{(1 - \alpha_{b0})\omega_{ab}}}$	1
h_{21}	$-\alpha_{b0}/(1 + j\omega/\omega_{ab})$	$\dfrac{\alpha_{b0}/(1 - \alpha_{b0})}{1 + \dfrac{j\omega}{(1 - \alpha_{b0})\omega_{ab}}}$	$-\dfrac{1}{1 - \alpha_{b0}} \cdot \dfrac{1 + j\omega/\omega_{ab}}{1 + \dfrac{j\omega}{(1 - \alpha_{b0})\omega_{ab}}}$
h_2	$j\omega C_c$	$\dfrac{j\omega C_c}{1 - \alpha_{b0}} \cdot \dfrac{1 + i\omega/\omega_{ab}}{1 + \dfrac{j\omega}{(1 - \alpha_{b0})\omega_{ab}}}$	$\dfrac{j\omega C_c}{1 - \alpha_{b0}} \cdot \dfrac{1 + j\omega/\omega_{ab}}{1 + \dfrac{j\omega}{(1 - \alpha_{b0})\omega_{ab}}}$

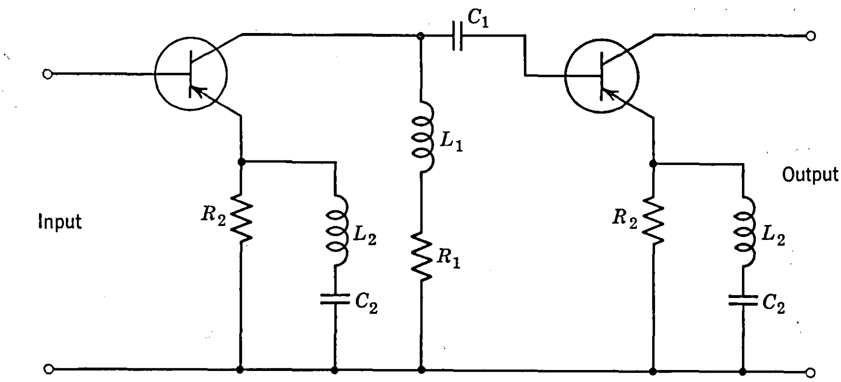


FIG. 50. High-frequency peaking of video amplifiers using combination of frequency sensitive feedback and ordinary shunt peaking.

one. The input impedance of a stage is usually considerably smaller than the output impedance of the preceding stage and, consequently, the power gain realized by a stage is approximately equal to the square of its current amplification (this assumes equal input impedances, i.e., *iterative impedances*). The maximum "flat" current amplification per video stage achievable in these circumstances is given by the current amplification of the transistor at the highest frequency where the response must be substantially equal to the low-frequency response. If the required cutoff frequency of a stage is f_v, the maximum flat current amplification obtainable per stage is:

$$A_{i,\ \max} \cong f_{\alpha b}/f_v.$$

Consequently, if, for example, a video amplifier stage must have a cutoff of 4 Mc and the transistors used have $f_{\alpha b} = 20$ Mc, the maximum flat current amplification is approximately $20/4 = 5$.

Video amplifiers may use occasionally common base or common collector configurations as input or output stages, wherever the impedance levels of these configurations are desirable. For example, when a video amplifier is used to drive the high input impedance of a cathode ray tube, the output stage will be preferably common base. The frequency response of the common base stage is superior to that of the common emitter configuration and is usually limited by the time constant (R_LC), where R_L is the load resistance and C is the sum of the collector, load, and stray capacitances.

Bandpass Amplifiers. Since transistors have finite input and output impedances and finite power gain, interstage coupling networks in bandpass (tuned) amplifiers must provide not only the desired selectivity but also adequate power transfer from driving to driven stage. This leads to high gain per stage and thereby minimizes the number of stages necessary to achieve the desired overall amplifier gain. Therefore, the resistive losses associated with essentially reactive coupling network elements must be small and the coupling network must provide an adequate impedance match between the output impedance of the driving stage and the input impedance of the driven stage.

Configurations. At all but the highest frequencies, the power gain of the common emitter circuit is superior to that of the common base circuit. Consequently, the common emitter circuit is used most often in tuned amplifiers. At "very high" frequencies the common base gain equals and may even slightly exceed the common emitter gain. The common collector circuit is used occasionally as the first or the last stage of a multistage amplifier for impedance transforming purposes. Since both common emitter and common base input impedances are smaller than the output impedances, the coupling network must transform voltages (impedances) in the downward direction.

Coupling. Depending on the desired selectivity, the center frequency of the amplifier and other specifications, various coupling networks can be used. Figure 51 shows several single- and double-tuned coupling networks. In Fig. 51*a*, *c* both transistors are in parallel with the tuned circuits. In Fig. 51*b*, *d* the second transistor is in series with a tuned circuit. The latter arrangement is preferable at high frequencies when it is difficult to achieve close coupling between primary and secondary windings if the ratio of output impedance to input impedance is large.

Power. The maximum achievable power transfer efficiency $\eta_{\max}$ is the ratio of the maximum power which can be delivered to the driven stage divided by the available power of the driving stage. $\eta_{\max}$ depends on the desired bandwidth and on the "unloaded" Q of the inductances used in the tuned circuits (Q_0). Considering the fractional "3-db bandwidth" B'_1, one finds for single-tuned circuits

$$\eta_{\max} = \left(1 - \frac{1}{B'_1 Q_0}\right)^2$$

and for transitionally coupled double tuned circuits

$$\eta_{\max} = \left(1 - \frac{\sqrt{2}}{B'_1 Q_0}\right)^2.$$

Consequently, good power transfer and narrow bandwidth can be achieved simultaneously only by using inductances having high values of Q_0. The required inductance values often are quite low. Since it may be difficult to build low-inductance coils with high Q_0, it is advantageous to make the inductance larger than required, in order to achieve good Q_0 and then to tap the inductance at an appropriate point, as shown in Fig. 52.

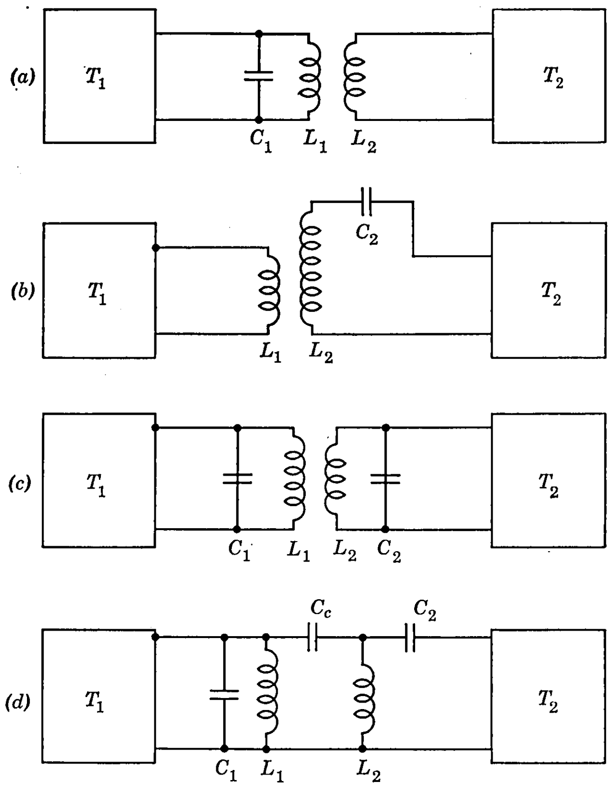


FIG. 51. Single- and double-tuned networks coupling transistor T_2 to transistor T_1.

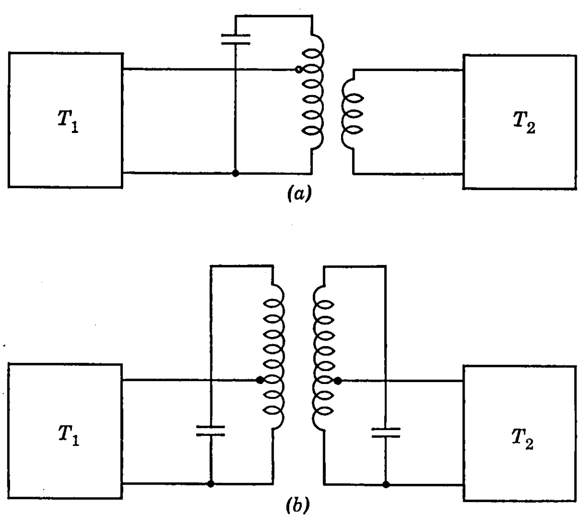


FIG. 52. Use of large tapped inductances to improve their effective Q.

Design. Table 13 contains design equations applicable to coupling networks of the types shown in Fig. 51*a*–*c*. It is assumed that, when designing an amplifier stage, the bandwidth B_n, the center frequency f_0, the Q_0 of available coils, as well as the input and output resistances (conductances), of the transistors to be used are known. Then the required inductance and capacitance values can be determined from Table 13.

Multistage synchronously tuned or staggered tuned amplifiers are designed by first considering the required overall gain and selectivity. The desired performance characteristics of an individual coupling network can then be determined from the overall requirements in a manner similar to that used in electron tube circuit design.

When designing narrow-band amplifier stages, one usually neglects the frequency dependence of transistor parameters within and in the neighborhood of the passband. Similarly, the effects of *reflected impedances* (i.e., the effect of the load and source impedances on input and output impedances respectively) can often be neglected. In wide-band amplifiers these effects must, however, sometimes be taken into account, and various methods compensating for both transistor parameter variation and reflected impedances can be used.

Stability. In addition to reflected impedances, the *nonunilateral* nature of transistors (i.e., the fact that in transistors there is no isolation between input and output terminals and a signal applied to the output will result in a response at the input) may result in instability (i.e., undesirable oscillations) of tuned amplifiers. This can be prevented by using relatively low values of source and load impedances (impedance mismatch) or by neutralization. In neutralized amplifiers (such as shown in Fig. 53), the feedback internal to the device is compensated by external feedback in a manner designed to make the overall circuit unilateral.

The circuit of Fig. 53*a* is usually used to neutralize the common base stage. Complete neutralization can be achieved at higher frequencies if

$$R = kr_c,$$

$$C = C_c/k,$$

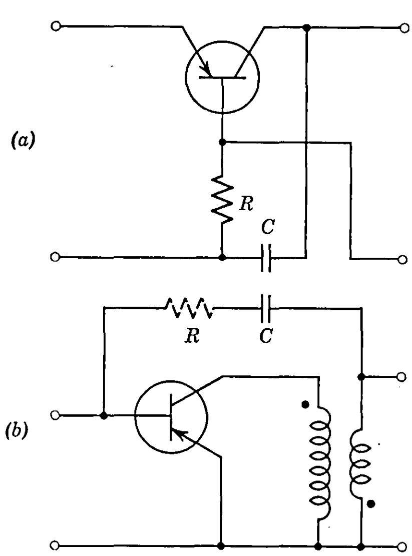


FIG. 53. Neutralized (*a*) common base and (*b*) common emitter stages.

TABLE 13. DESIGN OF COUPLING NETWORKS FOR TUNED AMPLIFIERS

	Single-Tuned Parallel-Parallel Circuit (Fig. 51*a*)	Single-Tuned Parallel-Series Circuit (Fig. 51*b*)	Double-Tuned Circuit with Critical Coupling (Fig. 51*c*)
L_1	$\dfrac{B'_nQ_0 - \sqrt{n}}{4\pi f_0 Q_0 g_{o1}\sqrt{n}}$	$\dfrac{B'_nQ_0 - \sqrt{n}}{4\pi f_0 Q_0 \sqrt{n}\, g_{o1}}$	$\dfrac{(Q_0B'_n/\sqrt{2}\sqrt[4]{n}) - 1}{2\pi f_0 Q_0 g_{o1}}$
C_1	$\dfrac{Q_0 g_{o1}\sqrt{n}}{\pi f_0(B'_nQ_0 - \sqrt{n})}$	None	$\dfrac{Q_0 g_{o1}}{2\pi f_0[(Q_0B'_n/\sqrt{2}\sqrt[4]{n}) - 1]}$
L_2	$\dfrac{B'_nQ_0 - \sqrt{n}}{4\pi f_0 Q_0 g_{i2}\sqrt{n}}$	$\dfrac{Q_0\sqrt{n}}{\pi f_0 g_{iS2}(B'_nQ_0 - \sqrt{n})}$	$\dfrac{(Q_0B'_n/\sqrt{2}\sqrt[4]{n}) - 1}{2\pi f_0 Q_0 g_{i2}}$
C_2	None	$\dfrac{g_{iS2}(B'_nQ_0 - \sqrt{n})}{4\pi f_0 Q_0 \sqrt{n}}$	$\dfrac{Q_0 g_{i2}}{2\pi f_0[(Q_0B'_n/\sqrt{2}\sqrt[4]{n}) - 1]}$
η_{max}	$(1 - \sqrt{n}/B'_nQ_0)^2$	$(1 - \sqrt{n}/B'_nQ_0)^2$	$(1 - \sqrt{2}\sqrt[4]{n}/B'_nQ_0)^2$
k	1	1	$B'_n/\sqrt{2}\sqrt[4]{n}$

Symbols. g_{o1} = parallel output conductance of first stage; g_{i2} = parallel input conductance of second stage; g_{iS2} = series input conductance of second stage; Q_0 = unloaded Q of inductance; f_0 = center frequency; B'_n = fractional bandwidth defined in terms of n. The bandwidth is defined as the interval separating two frequencies at which the power response is $1/(n + 1)$ times the center frequency response. ($n = 1$ corresponds to 3 db bandwidth; $n = 3$ to 6 db bandwidth, etc.)

where k is a constant. In principle k can be chosen arbitrarily but for high stage gain one usually has $k \cong 1$ (see Ref. 78).

The circuit of Fig. 53*b* is used frequently for common emitter neutralization. In this circuit R is often omitted and C is chosen such that

$$C = nC_c,$$

where n is the transformation ratio of the transformer; n is usually on the order of 10.

In common emitter configuration the danger of instability exists only at low frequencies when

$$f \leqq f_{\alpha b}(r_\epsilon/2r'_b).$$

The common base and common collector circuits are potentially unstable up to much higher frequencies: practically throughout the entire frequency range within which they can be usefully applied (see Ref. 103).

Gain Control of Amplifiers. Transistor small-signal parameters and, consequently, the power gain are functions of the d-c operating point. The gain can be decreased either by reducing the d-c emitter current or by reducing the d-c collector voltage of the stage which is to be controlled. Basic gain control circuits are shown in Fig. 54. In the circuit of Fig. 54*a* the gain control voltage V_{GC} changes the base potential and results in changes of emitter current. In the circuit of Fig. 54*b* a change in base potential results in change of the collector current, which, because of the gain control resistor R_{GC}, causes a change of the collector voltage. The gain control voltage V_{GC} is part of a feedback loop (V_{GC} may be derived from an amplitude detector) if *automatic gain control* is desired. Problems associated with transistor gain control are: (1) the control power needed is not negligible; (2) gain-controlled tuned amplifier stages are subject to changes of bandwidth and center frequency; (3) since, in order to achieve significant gain reduction, the emitter current must be reduced to the order of 100 μa or less or the collector voltage must be 100 mv or less, the signal may suffer considerable distortion (for details, see Refs. 96 and 97).

High-Frequency Power Gain. The power gain of transistors decreases with increasing frequency. If $f > f_{\alpha e}$ the maximum available power gain of the common emitter circuit using *alloy-junction transistors* is approximately:

$$G_{\max} \cong (0.035/f^2)(f_{\alpha b}/r'_bC_c).$$

The expression $(f_{\alpha b}/r'_bC_c)$ is often called the *high-frequency figure of merit* of the transistor. For good power gain at high frequencies one uses transistors with large $f_{\alpha b}$ and small (r'_bC_c) product.

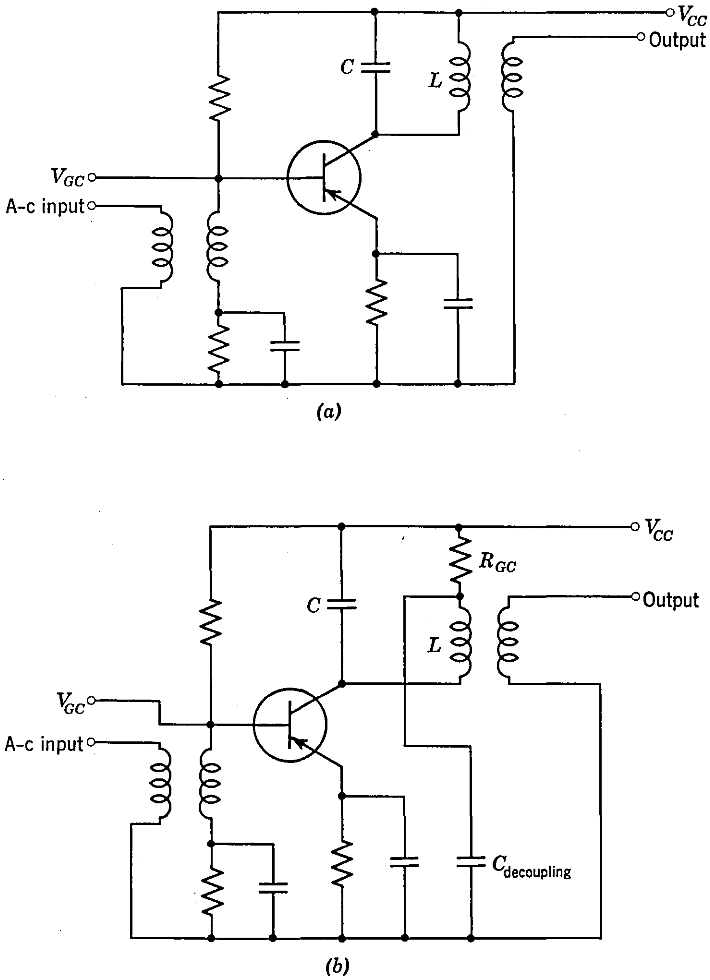


FIG. 54. Basic gain control circuits for (*a*) emitter current type and (*b*) collector voltage type gain control.

The formula just given is not valid for *grown junction transistors.* For such units we have (see Ref. 83):

$$G_{\max} \cong (0.03/f^{1/2})[f_{\alpha b}{}^{1/2}/C_c(r_\epsilon r'_b)^{1/2}].$$

5. D-C AMPLIFIERS

General Considerations. The small-signal parameters of a transistor (and, consequently, its impedances and gain) are functions of the temperature. Furthermore, as has been discussed, the collector reverse current I_{CBO} is strongly temperature-dependent. Therefore, a temperature-dependent drift component is associated with the output current of a transistor d-c amplifier. At low temperatures, the drift current is originated principally by varying small-signal parameters. At high temperatures, the drift is due mainly to variations of I_{CBO}. (Whether a given temperature range is "high" or "low" depends on whether germanium or silicon transistors are used, since the small-signal parameters vary in a similar manner in both types, but I_{CBO} of silicon transistors is considerably smaller than that of germanium transistors at the same temperature.)

The output drift current of d-c amplifiers can be reduced by using compensating temperature-sensitive elements, such as thermistors, junction diodes, and transistors (see Refs. 106–108). The degree of compensation depends on the accuracy with which the temperature sensitivity of the compensating element matches that of the transistor. Acceptable results can be achieved with simple circuits (for example, Fig. 55*a*), but excellent cancellation can be achieved with more complicated temperature sensitive networks with *shaped characteristics* (see, for example, Fig. 55*b*). An example for the use of an additional transistor as a

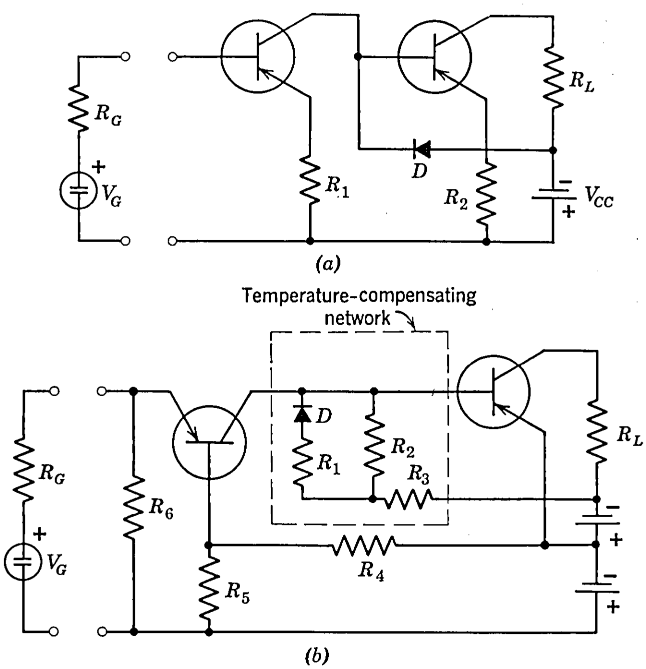


FIG. 55. Temperature compensation of d-c amplifier using (*a*) diode or (*b*) more complicated shaping network.

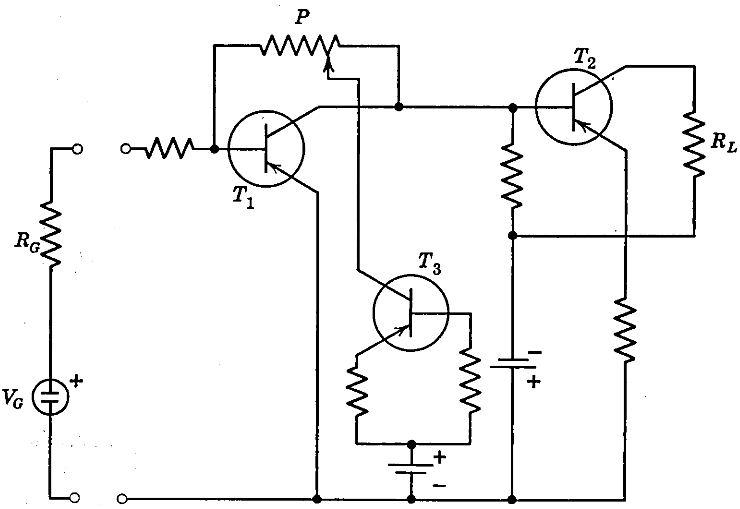


FIG. 56. Temperature compensation of d-c amplifier with auxiliary transistor.

temperature-compensating element is shown in Fig. 56, where the potentiometer P is adjusted in a manner that the drift component of the auxiliary transistor T_3 cancels the drift component of the amplifier proper.

Designing a sensitive d-c amplifier with constant performance throughout a wide temperature range is a particularly difficult problem with germanium transistors. Silicon transistors (having I_{CBO} of only a few millimicroamperes at room temperature) are, however, satisfactory for many purposes. Nevertheless, it is often necessary to use *chopper stabilization* (i.e., to convert the d-c signal to be amplified into a-c) and thereby to reduce the problem of d-c amplification into a much simpler one of amplifying a-c signals. The chopping device can be mechanical, in which case conversion from dc to ac is possible at extremely low d-c levels. Electronic chopping is also possible at moderately low d-c levels. The ring chopper circuit of Fig. 57 uses silicon diodes and permits the detection of direct currents of the order of 10^{-11} amp at room temperature. The sensitivity is reduced at higher temperatures. The diode must be matched over a significant temperature range to minimize the zero-drift current and its variations (see Ref. 106). The two-transistor chopper of Fig. 58 is also useful down to direct currents of the order of 10^{-8} amp at room temperature (see Refs. 111 and 112).

Operational Amplifiers. Operational amplifiers (of the summing, integrating, differentiating or high-gain type) are d-c amplifiers with strong feedback and are the basic building blocks of analog computers. The drift of d-c amplifiers affects the accuracy of operational amplifiers

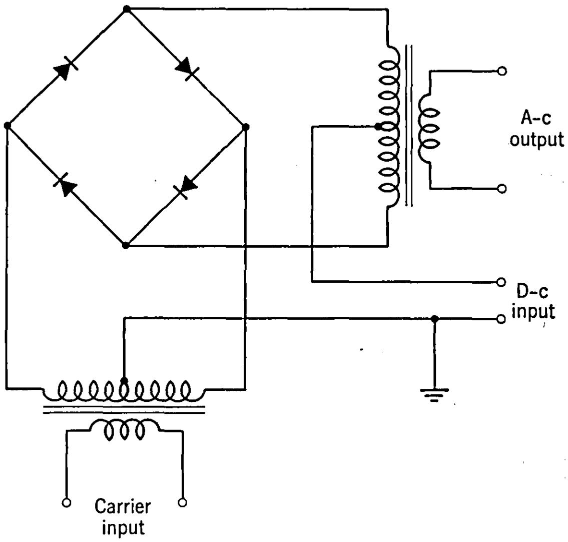


FIG. 57. Diode ring chopper circuit.

and, therefore, may limit the accuracy of the entire computing system. With transistors, the serious drift problem can be solved by applying the aforementioned or other usually more elaborate compensation methods.

The common emitter configuration is generally used because of its high-current amplification in direct-coupled stages. A problem associated with the common emitter circuit is that its frequency response starts decreasing at relatively low frequencies. This has an adverse affect on accuracy and response time and, in addition, may result in a-c instability. Frequency compensation may be achieved by applying appropriate feedback around individual stages, by designing suitable interstage coupling

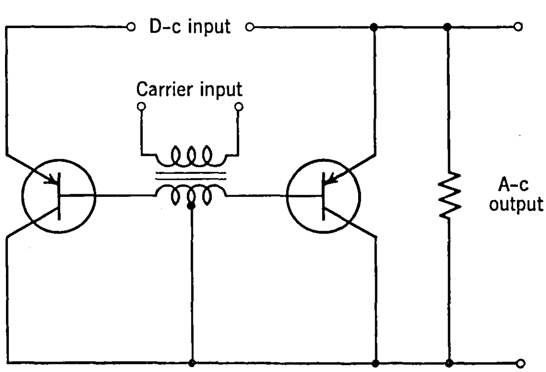


FIG. 58. Two-transistor chopper circuit.

networks, or by using frequency-dependent networks in the overall feedback path associated with the operational amplifier. In designing coupling and feedback networks, it is important to consider the frequency dependence of transistor input and output impedances which form part of these networks.

An example of a summing amplifier (see Ref. 113) is shown in Fig. 59.

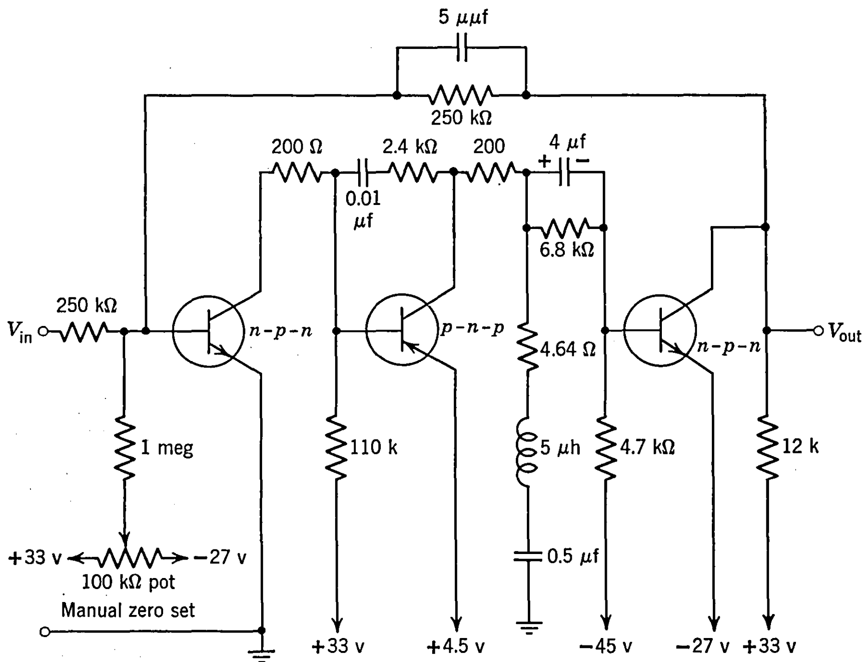


FIG. 59. D-c summing amplifier (see Ref. 113).

This amplifier uses three common emitter stages and combines the above features of frequency compensation (local feedback around stage 2, coupling network between stages 2 and 3, and a capacitive reactance in the overall feedback loop). A manual zero set is incorporated into the amplifier. Its temperature stability is adequate for moderate temperature variations with selected transistors having low I_{CBO} and using a low collector voltage in the first stage.

Further improvement from the drift point of view could be obtained by the use of temperature-sensitive compensating elements. However, in order to achieve really satisfactory accuracy, an *automatic zero set* circuit can be used. This principle is illustrated by the block diagram of Fig. 60. Since the summing amplifier itself has 180° phase shift, by

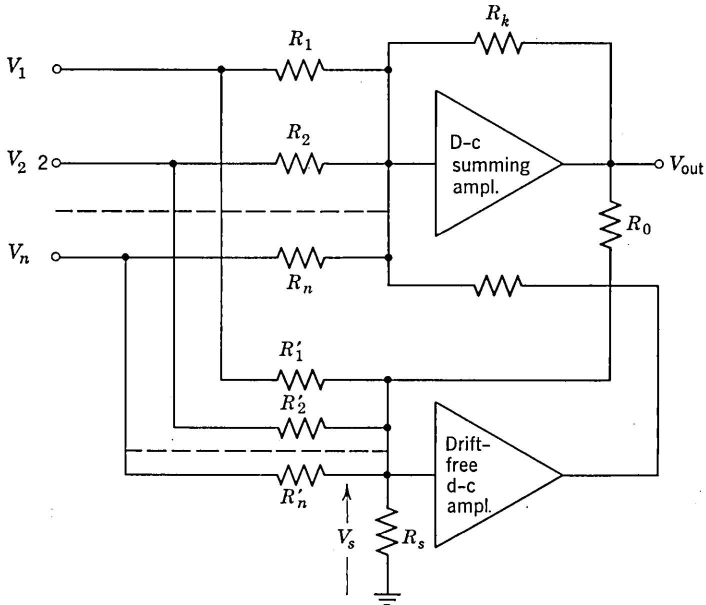


FIG. 60. D-c summing amplifier with automatic zero set (see Ref. 113).

proper choice of R'_0, R'_1, R'_2, etc., the voltage V_S is proportional to the drift voltage. This voltage can be amplified by a drift-free amplifier (using, for example, mechanical chopping) the output of which may be applied to the input of the summing amplifier. This achieves drift compensation.

Figure 61 shows an integrating amplifier (see Ref. 113) using frequency compensation of various kinds. A special coupling network is placed between stages 1 and 2; stage 1 uses local feedback and overall feedback is used from the output to the input. The diodes D_1 and D_2 prevent the integrator from overloading. Whenever the output voltage is outside the prescribed range, one of the diodes conducts. In order not to affect the accuracy of the amplifier, preferably silicon diodes having high back resistance (in excess of 1000 meg) are used.

6. OSCILLATORS, MODULATORS, MIXERS, DETECTORS

Sinusoidal Oscillators. The circuit diagrams of transistor oscillators are very similar to those of well-known electron tube oscillator types. The procedure used in designing transistor oscillators and analyzing their performance contains many new aspects because of the low-impedance levels of transistors and because of the frequency dependence of their small-signal parameters.

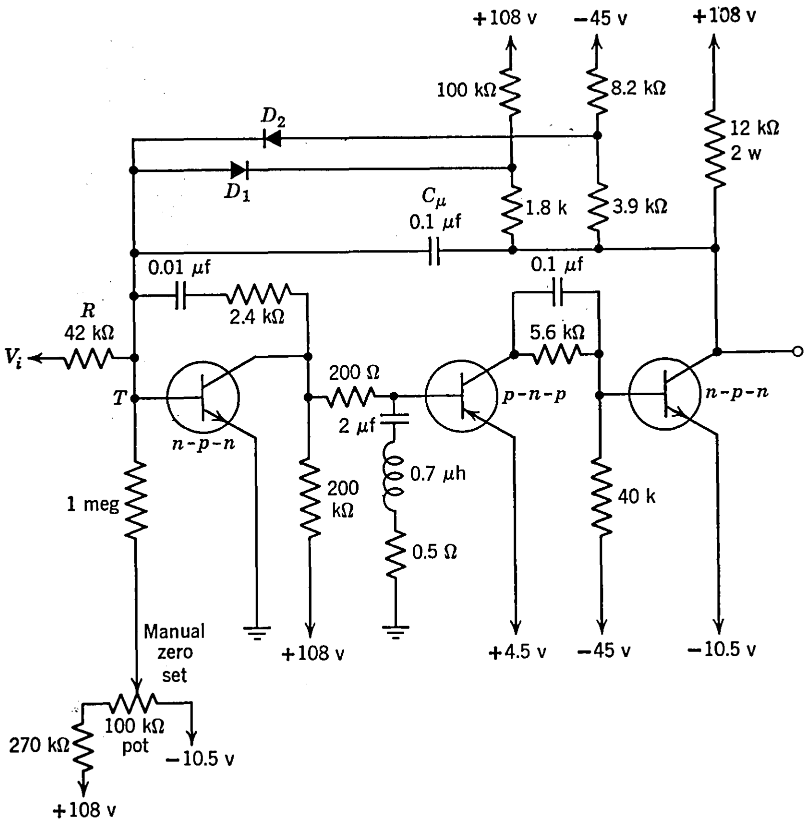


FIG. 61. Integrator (see Ref. 113).

The low-frequency *phase-shift oscillator* of Fig. 62 is capable of delivering considerable amounts of power. Conventional *Hartley* and *Colpitts type oscillators* are shown in Fig. 63. The push-pull Hartley oscillator of Fig. 64 can be used when large power output is desired. The *tuned base-tuned emitter oscillator* of Fig. 65 uses series resonant circuits and its operation is based on the fact that the input impedance (between points A and B) has a negative real component.

The frequency of oscillation of transistor oscillators is determined not only by the passive elements (L's, R's, and C's) used in the oscillator but also by the transistor impedances (especially their considerable reactive components), which are interconnected by the feedback circuit. Consequently, since transistor impedances are functions of the d-c operating point and of the temperature, the frequency stability of transistor oscillators constitutes a serious problem. The operating point (emitter current and collector voltage) can be stabilized in various ways (Sect. 2),

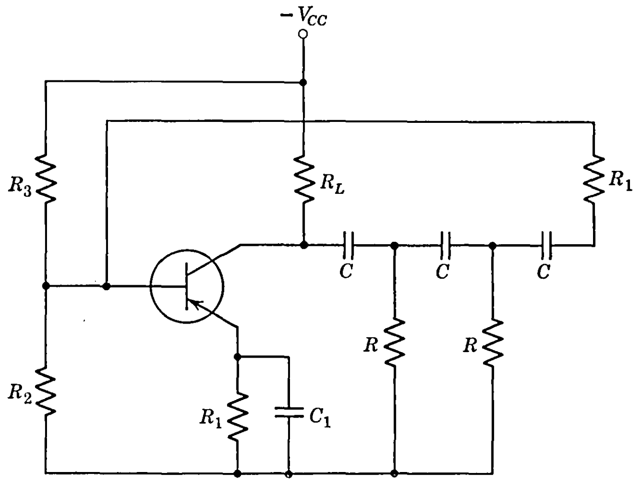


FIG. 62. Phase-shift oscillator.

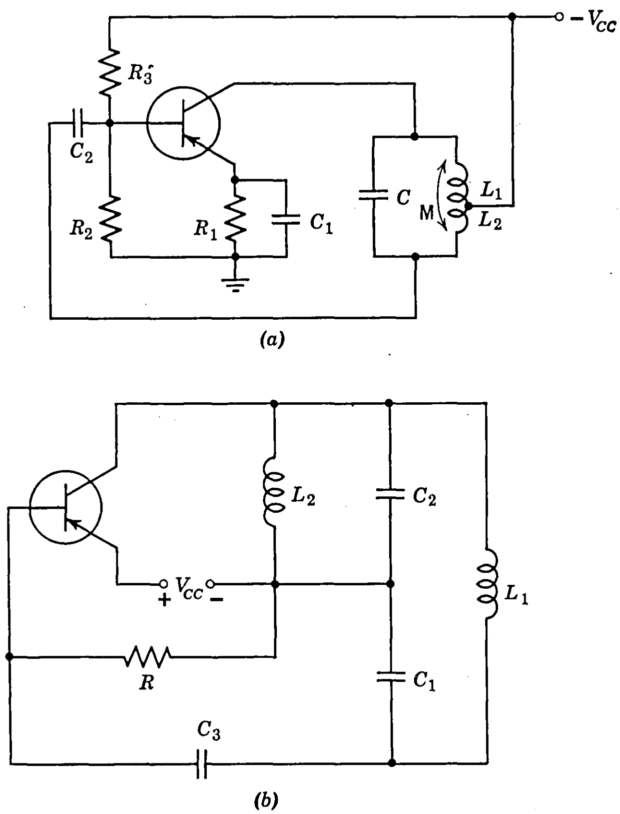


FIG. 63. (a) Hartley and (b) Colpitts type oscillators.

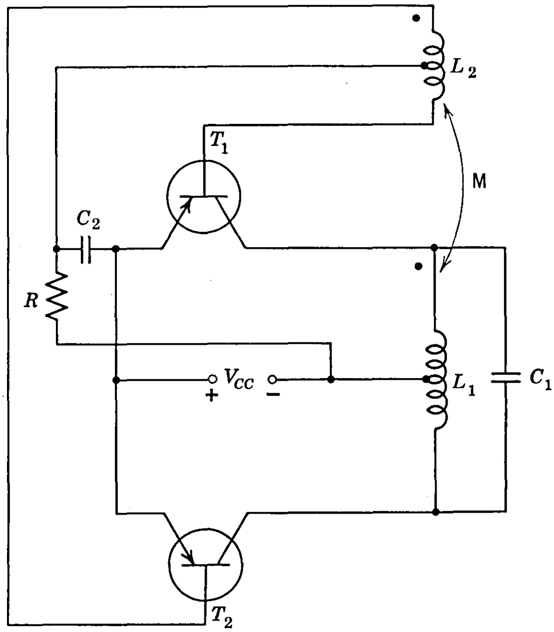


FIG. 64. Push-pull Hartley type oscillator.

but it is useful to use oscillator circuits which are inherently independent from transistor parameters. Such an oscillator—a modified *Clapp oscillator*—is shown in Fig. 66. Large values of C_1 and C_2 are required for good stability. The frequency stability of this arrangement can be improved further by using for R' a temperature-sensitive element with negative temperature coefficient (see Ref. 121).

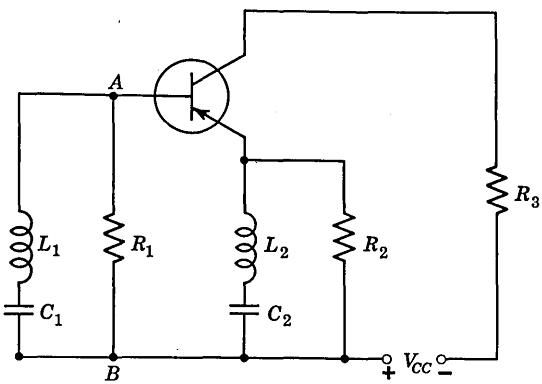


FIG. 65. Tuned base–tuned emitter oscillator.

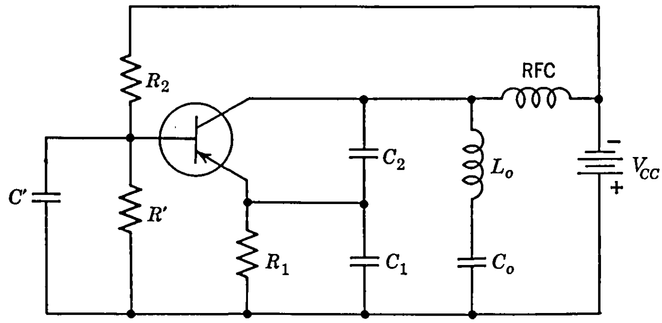


FIG. 66. Temperature stable LC oscillator.

A high degree of frequency stability can be achieved in *crystal oscillators.* An example of such a circuit is shown in Fig. 67 where the resonant circuit of Fig. 66 is replaced by a crystal (see Ref. 121).

Transistors have finite power gain and, consequently, the feedback circuit insuring oscillation must feed back part of the output power to the input. The highest frequency at which a transistor is capable of oscillating (without delivering power to an external load) is approximately equal to the frequency at which its maximum available power gain is unity. With the power gain expression applicable to alloy junction transistors mentioned in the section on bandpass amplifiers, one finds (see Ref. 123):

$$f_{\text{max}} \cong 0.2\sqrt{f_{\alpha b}/r'_b C_c}.$$

Nonsinusoidal Oscillators. Various oscillator circuits can produce waveforms different from a sine wave. The astable blocking oscillator of

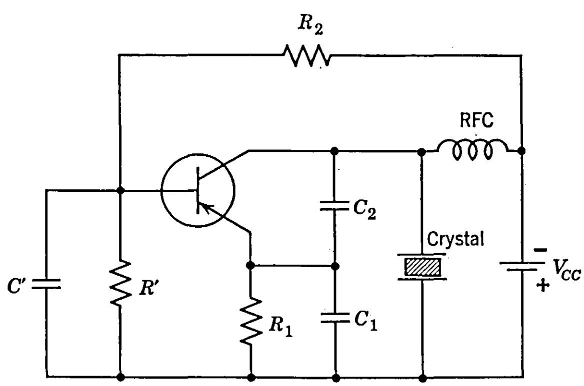


FIG. 67. Example of transistor crystal oscillator.

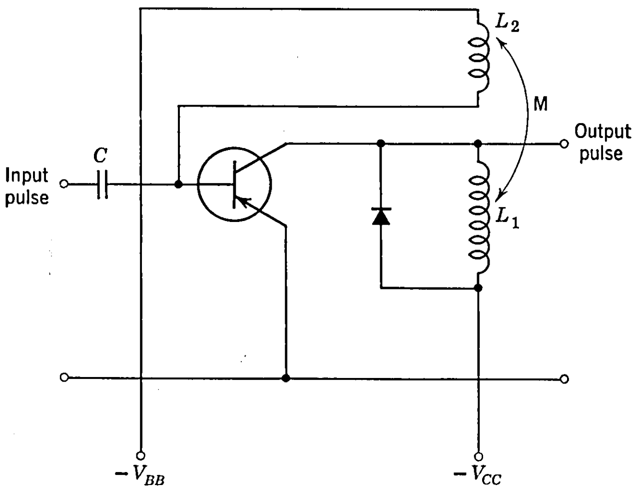


FIG. 68. Transistor blocking oscillator.

Fig. 68 is free running if a slightly negative d-c voltage is applied at V_{BB}. If input pulses, whose frequency is slightly larger than the free-running frequency of the oscillator, are applied, the frequency can be synchronized with that of the input pulses. The output of the oscillator is a sharp pulse which can be shaped by clipping (for details, see Ref. 124).

The astable multivibrator of Fig. 69 can be used to generate square waves. By introducing asymmetry in the circuit, one can obtain various

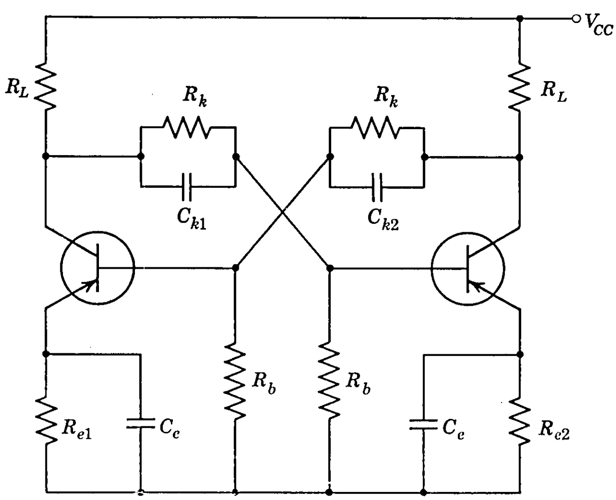


FIG. 69. RC-coupled multivibrator.

desired duty cycles. A design method for a symmetrical circuit is indicated in Table 14 (for details, see Ref. 125).

TABLE 14. DESIGN OF SYMMETRICAL MULTIVIBRATOR OSCILLATOR (FIG. 69) (Ref. 125)

Definitions

$$\begin{aligned}
\Delta V_c &= \text{collector voltage swing}\\
\Delta I_c &= \text{collector current swing}\\
f &= \text{frequency of oscillation}\\
V_{cc} &= \text{supply voltage}\\
V_{be} &= \text{base-to-emitter voltage}\\
A &= \Delta V_c/V_{cc}\\
K &= R_b/(R_k + R_b + R_L)\\
X &= R_L/R_e\\
u &= R_b/R_e\\
\psi &= (1/K)|V_{be}/V_{cc}|
\end{aligned}$$

Determine: R_L, R_b, R_K, R_e, C_k, C_e.

1. Select ΔV_c, ΔI_c, V_{cc}, and f.
2. Select K such that
$$(1 - A) \text{ and } A > K > |V_{be}/V_{cc}|$$
3. Select u:
For astability: $u > \dfrac{(1-K)^2}{1 - K - A/[(1-\psi)\alpha_{b0}]}$;
For I_{CB0} stability: $u \ll \Delta I_c/I_{CB0}$;
For realizability: $\alpha_{e0} \gg u > \dfrac{A}{\alpha_{b0}(1-\psi)(A-K)(1+K-A)}$
4. Calculate KX:
$$KX = u/2 - \sqrt{u^2/4 - uA/[\alpha_{b0}(1-\psi)]}.$$
5. Calculate R_e:
$$R_e = R_{e1} = R_{e2} = (\alpha_{b0}V_{cc}/\Delta I_c)(K - V_{be}/V_{cc}).$$
6. Calculate R_L: $R_L = XR_e$.
7. Calculate R_b: $R_b = uR_e$.
8. Calculate R_K: $R_K = (u/K - u - X)R_e$.
9. Calculate C_K:
$$C_K = C_{K1} = C_{K2} = \frac{-1}{2fR_KK(1 + X/u)\ln\{[(1-A)/A][(K + KX/u)/(1 - K - KX/u)]\}}.$$
10. Select C_e: $C_e \gg 1/2\pi f_{ab}R_e$.

The combination of a *p-n-p* and an *n-p-n* transistor in the circuit of Fig. 70 exhibits a negative resistance between terminals A and B and, with suitable terminations between these terminals (for example, a capacitor and a resistor), can be made to produce various waveforms.

Modulators, Mixers, Detectors. The processes of modulation, mixing, and detection have the common property that they make use of the non-

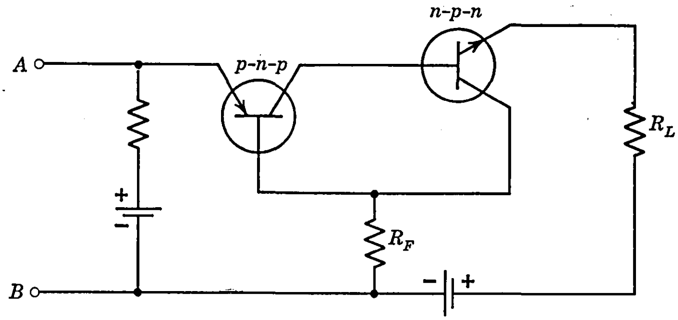


FIG. 70. Combination of *p-n-p* and *n-p-n* exhibiting negative resistance.

linear properties of the device used. The principal nonlinear characteristic of transistors is the curvature in the voltage vs. current characteristic of the emitter-to-base diode, but other nonlinear properties (for example, the nonlinear region of the common base and common emitter output characteristics at low collector voltages) can also be used.

With the emitter-to-base diode, the efficiency of all these nonlinear processes is affected at high frequencies by (1) the capacitance associated with this diode and (2) carrier storage effects taking place in the diode. The latter phenomenon has been described in the sections dealing with the physical properties of semiconductor devices.

Amplitude modulation can be achieved by injecting the modulating low-frequency signal directly into the carrier frequency oscillator or by using a separate modulating stage. The first method is usually undesirable since, due to variation of transistor reactances with the variation of the operating point (the operating point varies at the rate of the modulating signal), it is associated with considerable frequency modulation. With a separate modulating stage, the signals can be injected at different electrodes (see Fig. 71). If the modulating signal is injected at 1 or 2,

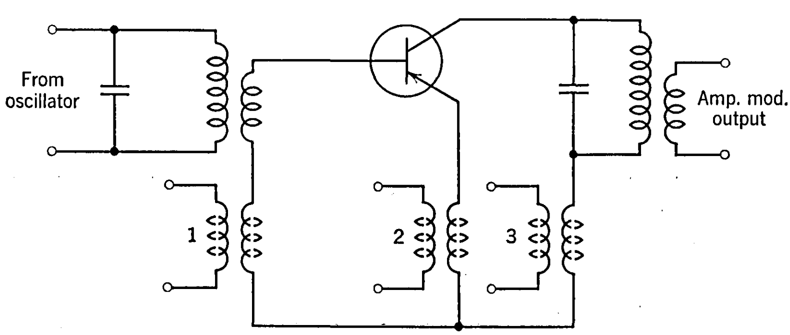


FIG. 71. Methods of injecting modulating signal into modulator.

the emitter diode nonlinearity is used (the impedance level of injection is higher in case 1 than in case 2). If injection method 3 is used, one relies on the nonlinearity of the collector characteristic (this calls for low collector voltages).

Frequency modulation may be obtained in one way by the circuit illustrated schematically in Fig. 72, which uses a *reactance transistor.* In other cases, the modulating signal is applied directly to the oscillator. This method usually leads to simultaneous amplitude modulation which can be removed by a subsequent limiting stage.

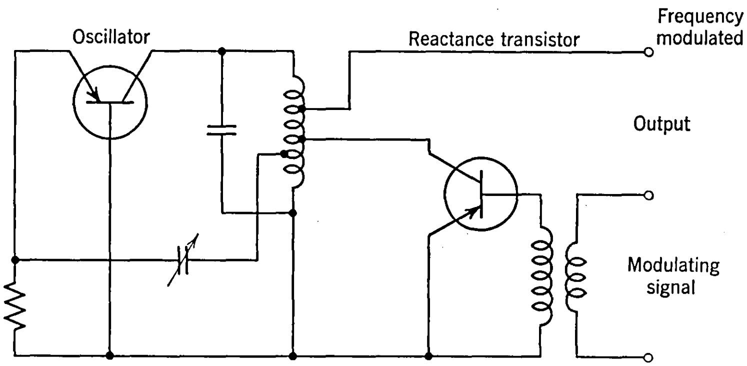


FIG. 72. Frequency modulation using reactance transistor.

Mixing action (i.e., frequency conversion) can be obtained in many ways. The modulated carrier and the local oscillator voltages can be applied to various electrodes, depending on the desired impedance level. If the conversion gain of the mixer is plotted against the emitter current or against the injected local oscillator level, the curves show a distinct maximum. A possible mixing arrangement is pictured in Fig. 73. Here, the modulated carrier and the local oscillator signal are applied to the same electrode, the base. This is not necessary; the signals can be applied to different electrodes. The functions of local oscillation and frequency conversion can be achieved with a single transistor, as shown in Fig. 74.

The emitter-to-base diode of the transistor biased approximately at cutoff can be used for *amplitude detection.* This can be achieved with the common base or common emitter stages. A common emitter detector is shown in Fig. 75. The advantages of transistor detectors over diode detectors are: (1) detection can be achieved at somewhat lower power level of the modulated signal; (2) the transistor delivers a considerably amplified signal; (3) the d-c component of the demodulated signal is often used to control the gain of preceding amplifier stages. Transistor detec-

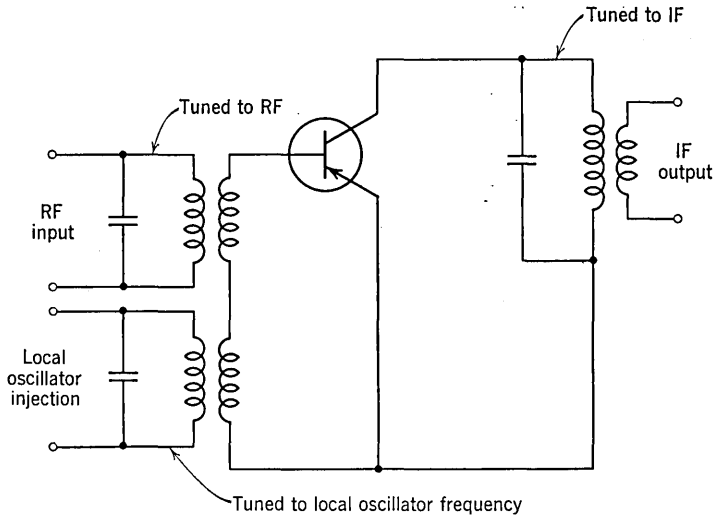


FIG. 73. Transistor mixer with both RF signal and local oscillator voltage applied in base circuit.

tors amplify the d-c component and, consequently, deliver adequate power for the control of several stages.

Regenerative detectors are used occasionally to obtain a large detected output signal while using a relatively small modulated signal at the input of the detector. A circuit of this type is shown in Fig. 76. Feedback from the collector to the emitter circuit achieves the regenerative action.

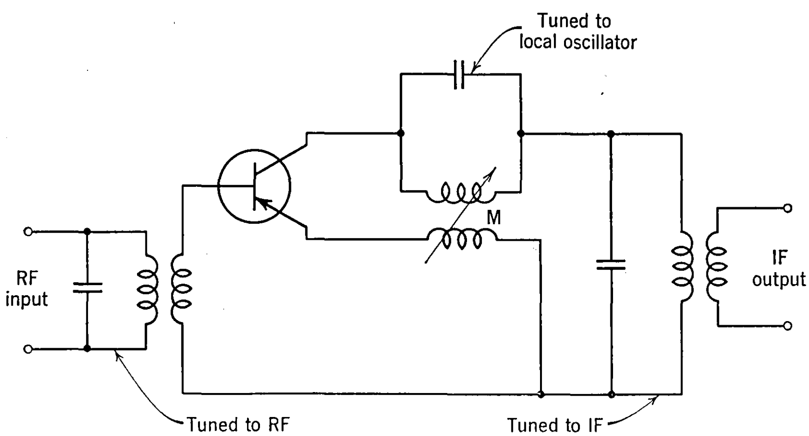


FIG. 74. Single transistor frequency converter.

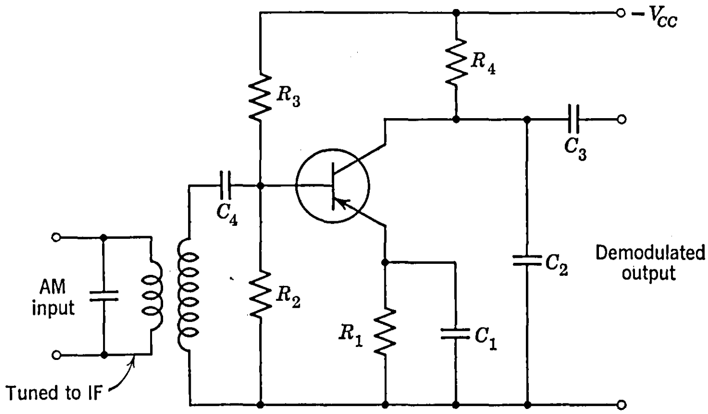


FIG. 75. Common emitter transistor detector.

7. SWITCHING CIRCUITS

Linear Pulse Amplifiers. Circuits designed to handle pulse type information are usually of the nonlinear type. Important exceptions to this rule are linear pulse amplifiers. These are small signal amplifiers handling pulse type signals and their design is, in principle, similar to that of video amplifiers.

The time constant of the response of a linear pulse amplifier stage to a step input depends on the transistor configuration, source and load impedances, and the quantities considered driving and responding variables respectively (source voltage or current, input voltage or current, output voltage or current). Even if simplified equivalent circuits are used, the computation of the transient response may call for the solution of higher

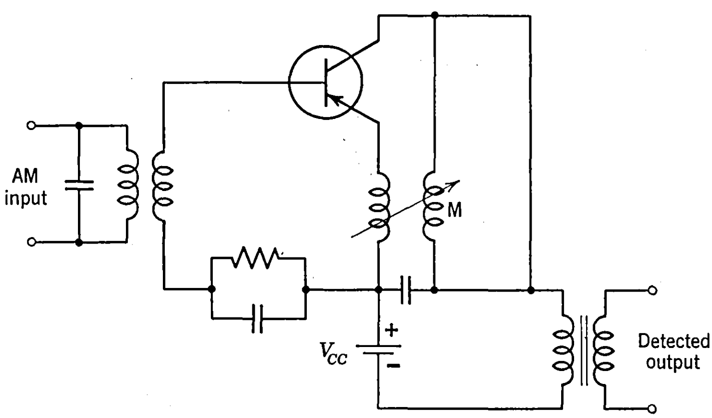


FIG. 76. Regenerative transistor detector.

order algebraic equations and may, therefore, present some difficulties. The results are, however, quite simple whenever one can assume (and this is frequently justified) that the load impedance of the linear pulse amplifier is very small compared to the output impedance of the stage. Then, with Laplace transformation, the current amplification $i_2(s)/i_1(s)$ is a suitable transfer function to be used. The Laplace transform of $v_1(s)/i_1(s)$ is also of interest if the driving quantity is a voltage. One has

$$i_2(s)/v_1(s) = [i_2(s)/i_1(s)]/[v_1(s)/i_1(s)],$$

the transfer function for the voltage-driven case. The transforms $i_2(s)/i_1(s)$ and $v_1(s)/i_1(s)$ of the three configurations are given in Table 15. Figures

TABLE 15. TRANSFER FUNCTIONS i_2/i_1 AND DRIVING POINT FUNCTIONS v_1/i_1 FOR THREE CONFIGURATIONS (ASSUMPTION: SMALL LOAD IMPEDANCE)

Common Base

$$i_C(s)/i_E(s) = -\alpha_{b0}/(1 + s/\omega_{ab}).$$

$$v_{EB}(s)/i_E(s) = [r_\epsilon + r'_b(1 - \alpha_{b0} + s/\omega_{ab})]/(1 + s/\omega_{ab}).$$

Common Emitter

$$i_C(s)/i_B(s) = \alpha_{b0}/(1 - \alpha_{b0} + s/\omega_{ab}).$$

$$v_{BE}(s)/i_B(s) = r'_b + r_\epsilon/(1 - \alpha_{b0} + s/\omega_{ab}).$$

Common Collector

$$i_E(s)/i_B(s) = (1 + s/\omega_{ab})/(1 - \alpha_{b0} + s/\omega_{ab}).$$

$$v_{BC}(s)/i_B(s) = r'_b + [r_\epsilon + R_L(1 + s/\omega_{ab})]/(1 - \alpha_{b0} + s/\omega_{ab}).$$

77 and 78 illustrate transient responses of the common base and common emitter circuits. The symbols used refer to the equivalent circuit of Fig. 42.

The equations of Table 15 can be used to include selected cases where the source impedance is finite. For example, if the common emitter stage is driven by a source having a resistance R_S, one can replace r'_b by $(r'_b + R_S)$.

Increasing the load impedance results in increasing time constants. Since $f_{\alpha e}$ is a relatively low frequency, the response time constant of the important common emitter circuit is large (usually several microseconds), and the circuit is not suited for the amplification of fast pulses. With video amplifier type compensation techniques, the time constant can be reduced (at the expense of the gain) (for more details, see Refs. 134 and 142).

Overdriven Pulse Amplifiers (Switches). Considerable improvement of the response time constant can be achieved by *overdriving* the transistor, i.e., by using it in a nonlinear manner as a switch. (This does, of course reduce the gain.) In this mode of operation the transistor is

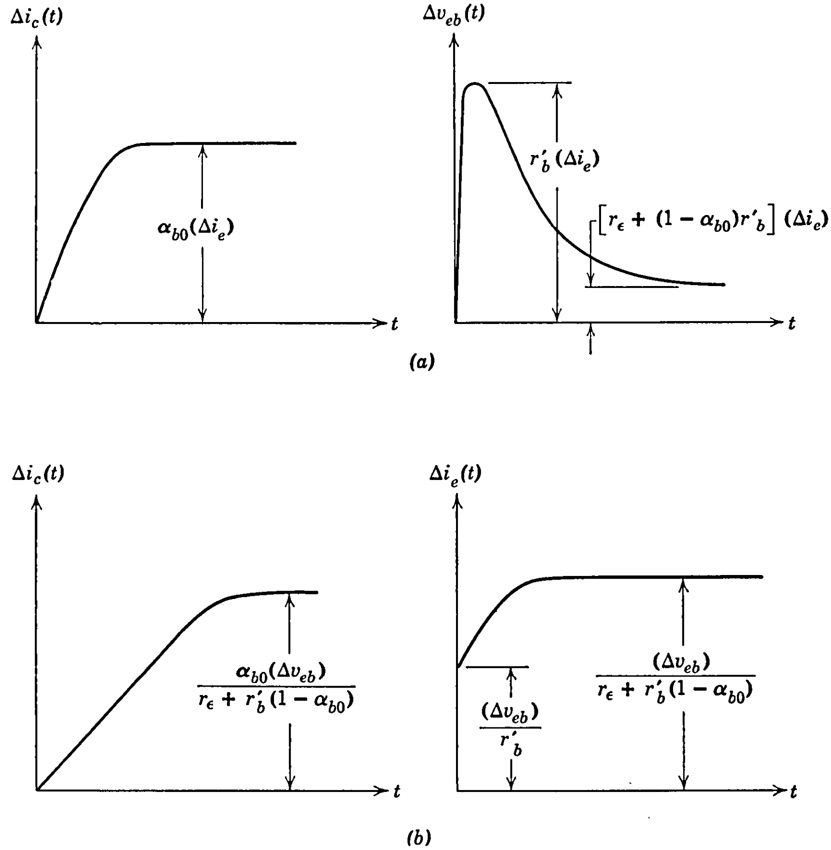


FIG. 77. Transient response for common base amplifier (Δ designates the increment of a quantity): (a) responses to step in emitter current; (b) responses to step in emitter-to-base voltage.

either conducting (*on,* large collector current and negligible collector voltage) or nonconducting (*off,* negligible collector current, the collector voltage being large and approximately equal to the supply voltage). The *on* and *off* conditions are indicated in Fig. 79 by points A and B respectively.

Since the transistor is switched from the *off* condition to the *on* condition or vice versa, in the course of such a large excursion of the operating point all transistor parameters are subject to considerable variations. If the transistor is driven by means of a current (i.e., if the driving source has a high impedance), the *large-signal transient response* can be understood by considering a situation such as the one shown in Fig. 80. Initially, v_{be} is zero or slightly positive, and the transistor is cut off. At

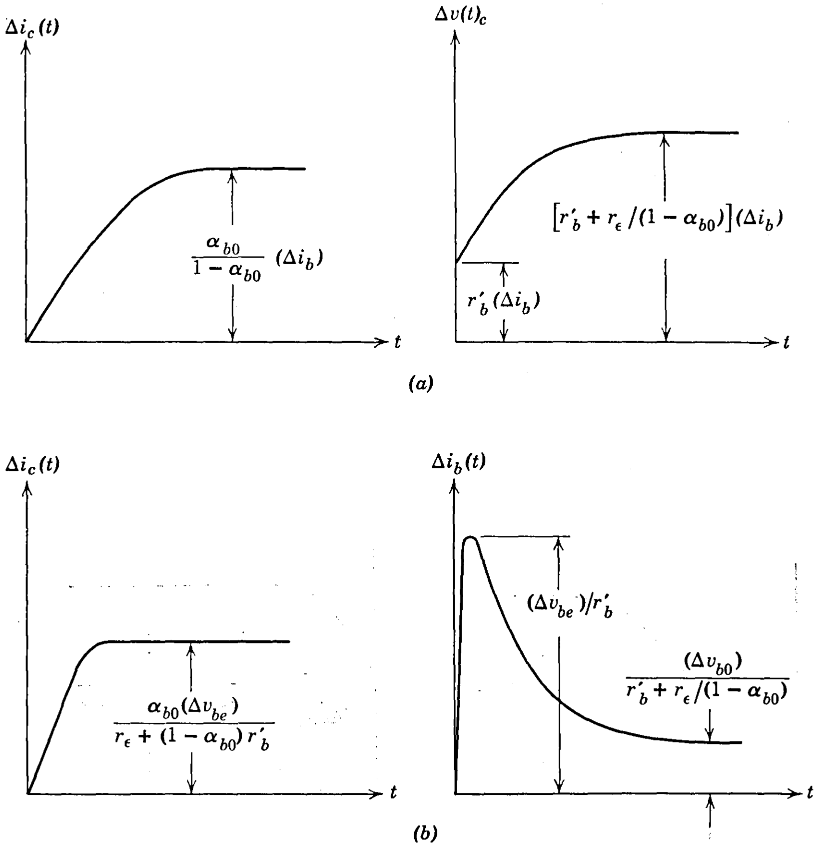


FIG. 78. Transient responses for common emitter amplifier (Δ designates the increment of a quantity): (*a*) responses to step in base current; (*b*) responses to step in base-to-emitter voltage.

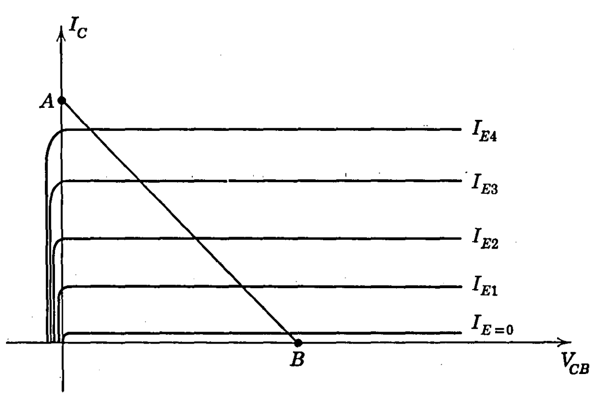


FIG. 79. *On* and *off* conditions of transistor.

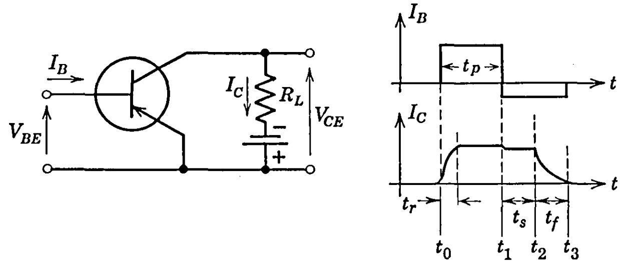


FIG. 80. Transient response of overdriven transistor.

time t_0 a large current pulse is applied to the base. The collector current increases rapidly in response to the base current pulse and develops a voltage drop across the load R_L. When the voltage drop across R_L becomes almost equal to the supply voltage, the collector-to-emitter voltage is practically zero and the collector current cannot increase further; the transistor is *saturated.* Only a fraction of the *small-signal rise time* is needed to complete the switching action from *off* to *on* since the base has been overdriven: the actual *rise time* is t_r. Having reached saturation, the transistor stays *on* as long as the base current pulse is maintained. When the input pulse is removed (or, as a matter of fact, reversed, i.e., overdriven in the opposite direction), the collector current first maintains itself: the carriers stored during the saturation period must be removed before a substantial decrease of the collector current can take place. After elapse of the *storage time* t_s, the collector is switched from *on* to *off* within the *fall time* t_f, which is small if the reverse current pulse applied to the base is large. Like t_r, t_f is usually only a fraction of the small-signal rise time.

The foregoing description of the switching action applies to the common emitter circuit. The process is, however, similar in the case of the common base or common collector configurations. For each configuration t_r, t_s and t_f depend on the transistor parameters and on the amplitude of the applied input current pulses. Table 16 gives expressions for t_r, t_s, and t_f for the three configurations.

It should be noted that the storage time t_s is function not only of the common base forward short-circuit current amplification α_{fb0} and its cutoff radian frequency $\omega_{\alpha fb}$ but also of these quantities measured in the reverse direction (α_{rb0} and $\omega_{\alpha rb}$), i.e., applicable to the case where the emitter is used as collector and the collector as emitter. In the expressions for t_s and t_f, the subscripts 1 and 2 refer to current values immediately before and immediately after the turnoff pulse is applied at the input terminal, respectively.

The expressions of Table 16 are based on the assumption that the collector capacitance does not significantly affect the rise and fall times, i.e.,

TABLE 16. RISE, STORAGE, AND FALL TIMES OF THREE CONFIGURATIONS FOR OVERDRIVEN CASE (Ref. 22)

Common Base Configuration

$$t_r = [1/\omega_{afb}] \ln [i_E/(i_E - 0.9i_C/\alpha_{fbb})].$$

$$t_s = \left[\frac{(\omega_{afb} + \omega_{arb})}{\omega_{afb}\omega_{arb}(1 - \alpha_{fb0}\alpha_{rb0})}\right] \ln \left[\frac{(i_{E1} - i_{E2})}{(i_{C1}/\alpha_{fb0}) - i_{E2}}\right].$$

$$t_f = \left[\frac{1}{\omega_{afb}}\right] \ln [(i_{C1} + \alpha_{fb0}i_{E2})/(0.1i_{C1} + \alpha_{fb0}i_{E2})].$$

Common Emitter Configuration

$$t_r = \left[\frac{1}{(1 - \alpha_{fb0})\omega_{afb}}\right] \ln \left[\frac{i_B}{i_B - (0.9)(1 - \alpha_{fb0})(i_C)/\alpha_{fb0}}\right].$$

$$t_s = \left[\frac{(\omega_{afb} + \omega_{arb})}{\omega_{afb}\omega_{arb}(1 - \alpha_{fb0}\alpha_{rb0})}\right] \ln \left[\frac{(i_{B1} - i_{B2})}{i_{C1}(1 - \alpha_{fb0})/\alpha_{fb0} - i_{B2}}\right].$$

$$t_f = \left[\frac{1}{(1 - \alpha_{fb0})\omega_{afb}}\right] \ln \left[\frac{i_{C1} - \alpha_{fb0}i_{B2}/(1 - \alpha_{fb0})}{0.1i_{C1} - \alpha_{fb0}i_{B2}/(1 - \alpha_{fb0})}\right].$$

Common Collector Configuration

$$t_r = \left[\frac{1}{(1 - \alpha_{fb0})\omega_{afb}}\right] \ln \left[\frac{\alpha_{fb0}i_B}{i_B - (0.9)(1 - \alpha_{fb0})i_E}\right].$$

$$t_s = \left[\frac{(\omega_{afb} + \omega_{arb})}{\omega_{afb}\omega_{arb}(1 - \alpha_{fb0}\alpha_{rb0})}\right] \ln \left[\frac{(i_{B1} - i_{B2})}{i_{B2} + i_{E1}(1 - \alpha_{fb0})}\right].$$

$$t_f = \left[\frac{1}{(1 - \alpha_{fb0})\omega_{afb}}\right] \ln \left[\frac{i_{E1} - (i_{B2})/(1 - \alpha_{fb0})}{0.1i_{E1} - i_{B2}/(1 - \alpha_{fb0})}\right].$$

Symbols. α_{fb0} and α_{rb0} are the low-frequency forward and reverse common base short-circuit current amplifications, respectively; ω_{afb} and ω_{arb} are the cutoff frequencies of α_{fb} and α_{rb}, respectively; subscripts 1 and 2 refer to currents immediately before and after the turnoff step is applied, respectively; t_r is the rise time, t_s the storage time, and t_f is the fall time.

that $R_LC_C\omega_{afb} \ll 1$ (this calls for a relatively small load impedance). In cases where this assumption is not true, the effect of the collector capacitance will be to increase the rise and fall times. In addition, in calculating t_s, it has been assumed that the duration t_p of the turnon pulse is sufficiently long for the carrier density in the base region to reach equilibrium condition:

$$t_p \geq 1/(1 - \alpha_{fb})\omega_{afb}.$$

If this assumption is not satisfied, the storage time will be shorter than t_s.

If the driving quantities are voltages rather than currents (i.e., if the impedance of the driving source is small), switching type operation is still possible and the qualitative response of the collector current is similar to that of the current-driven case, in the sense that one distinguishes rise, storage, and fall times. However, t_r, t_s, and t_f will be quantitatively quite different in the voltage-driven case and the shape of the transient response curves will also be different.

In practical cases the driving source impedance is finite and switching is neither of the purely current-driven nor of the purely voltage-driven type. However, the majority of practical circuits are driven from sources having relatively high impedance and in such cases the formulas of Table 16 are useful approximations (for details, see Refs. 2, 22, 136, and 137).

Clamping Circuits. Transistor saturation (carrier storage effects) may seriously limit the maximum repetition rate at which switching or gating circuits may be driven. A method for preventing saturation is the use of a diode clamp illustrated in Fig. 81*a*. When, as a result of an ap-

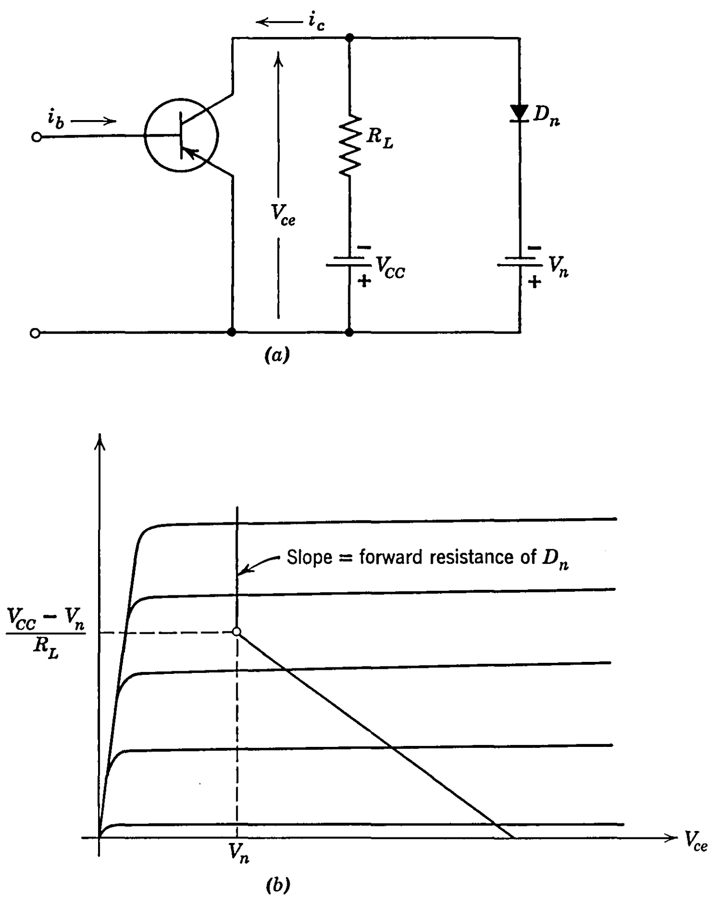


FIG. 81. Collector clamping to prevent saturation effects: (*a*) clamping circuit; (*b*) broken load line representation of clamp effect.

plied base drive, the collector-to-emitter voltage v_{CE} falls to the value of the clamping voltage V_n, the diode D_n begins to conduct and the collector voltage cannot fall below V_n. A broken load line representation of the clamping action is shown in Fig. 81*b*.

In order to maintain high switching efficiency, the clamping voltage V_n should be as low as feasible, i.e., as close to the saturation voltage as practicable. Since clamping voltages of less than 1 volt are difficult to obtain, a clamping circuit (developed by R. H. Baker, see Ref. 145) which uses the low voltage drop across a conducting diode is of considerable interest (Fig. 82). A requirement of this circuit is that the diode

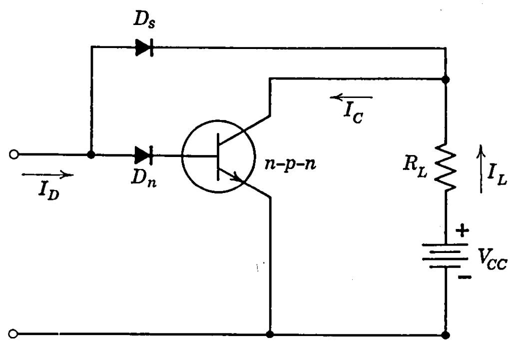


FIG. 82. Low dissipation, high-speed clamping circuit.

D_n have a higher voltage drop than the diode D_s. In practice, therefore, D_n is a silicon diode, whereas D_s is germanium. D_n is always conducting. When the collector potential reaches the clamping level, D_s, which is cut off at high collector voltage, starts conducting. Since $V_n > V_s$, the transistor cannot saturate.

Bistable Circuits. Many switching applications require *bistable* circuit configurations having memory in which the switching element is triggered *on* by a pulse and remains *on* after the pulse has been terminated.

A bistable Eccles-Jordan multivibrator (flip-flop) is shown in Fig. 83. The circuit has two stable states characterized by the conduction of transistor 1 or transistor 2, respectively. A trigger pulse applied at points A, B, or C switches the circuit from one stable state in the other. This *emitter coupled* flip-flop circuit can be designed for operation with or without saturation throughout a wide range of temperatures. A method of design for nonsaturating emitter-coupled flip-flops is given in Table 17 (see Ref. 125).

The *base return flip-flop* of Fig. 84*a* is also frequently used and has

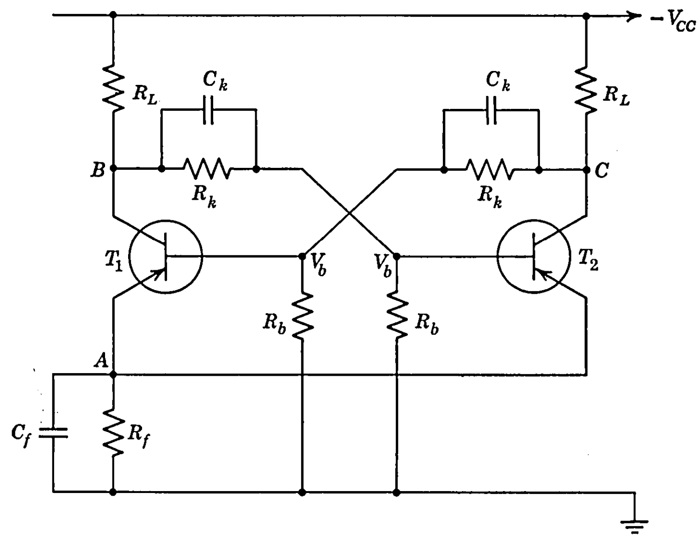


FIG. 83. Emitter-coupled bistable transistor flip-flop.

performance characteristics similar to those of the emitter-coupled circuit. It uses a separate power supply for base biasing.

The flip-flops of Figs. 83 and 84*a* are excellent if low output power is required but are of comparatively low efficiency. This low efficiency is due to the fact that the useful output power is usually *not* the power dissipated in the load resistances R_L, but the power transferred to flip-flops or other gating circuits activated by the flip-flop. Furthermore, additional loading may cause problems such as high driving power requirements and deterioration of frequency response.

It is possible to replace the resistances R_L by complementing transistors as shown in the *high-efficiency flip-flop* of R. H. Baker shown in Fig. 85 (see Ref. 145). Transistors T_1 and T_3 are *p-n-p* and form a conventional flip-flop and so do T_2 and T_4, which are *n-p-n*. One of the stable states of this flip-flop is indicated by the *on* and *off* notations in Fig. 85. Thus, when T_3 is supplying current to the load R_{L2} and V_{o2} is positive, T_2 is supplying R_{L1} and V_{o1} is negative. The load resistors are effectively connected between the collector points V_{o1} and V_{o2}, and the multivibrator may be thought of as a symmetrical bridge circuit in which the bridge arms are switches. The load resistors may assume a wide range of values, and efficiencies of the order of 90% can be achieved. Trigger pulses are applied at points P_1 and P_2. The capacitors C_k make sure that the trigger pulses reach two transistors, thereby turning one *on* and the other *off*.

High-speed operation of the emitter-coupled circuit of Fig. 83 can be

TABLE 17. DESIGN OF NONSATURATING EMITTER-COUPLED FLIP-FLOP (FIG. 83) (Ref. 125)

Definitions

V_{CC} = supply voltage
ΔV_c = collector voltage swing
ΔI_c = collector current swing
V_{be} = base-to-emitter voltage
t_{rn} = recovery time
$A = \Delta V_c/V_{CC}$
$K = R_b/(R_K + R_b + R_L)$
$X = R_L/R_f$
$u = R_b/R_f$
$\psi = (1/K)/|V_{be}/V_{CC}|$

Determine: R_L, R_b, R_K, R_f, C_K, C_f.

1. Select: V_c, I_c, V_{CC}, and t_{rn}.
2. Select K:
   (*a*) For bistability: $K > (1/A)|V_{be}/V_{CC}|$;
   (*b*) For realizability: $(1 - A)$ and $A > K \gg |V_{be}/V_{CC}|$.
3. Select u:

$$\alpha_{e0} \gg u > \frac{A}{\alpha_{b0}(1 - \psi)(A - K)(1 + K - A)}.$$

4. Calculate KX:

$$KX = u/2 - \sqrt{u^2/4 - uA/[\alpha_{b0}(1 - \psi)]}.$$

5. Calculate R_f:

$$R_f = (\alpha_{b0}V_{CC}/\Delta I_c)(K - |V_{be}/V_{CC}|).$$

6. Calculate R_L: $R_L = XR_f$.
7. Calculate R_b: $R_b = uR_f$.
8. Calculate R_K: $R_K = (u/K - u - X)R_f$.
9. Calculate C_K:

$$C_K = \frac{-t_{rn}}{R_K X(1 + X/u) \ln \{[(1 - A)/A][(K + KX/u)/(1 - K - KX/u)]\}}.$$

10. Select C_f: $C_f \gg 1/2\pi f_{\alpha b} R_f$ and $C_f < t_{rn}/\pi R_f$.

achieved and the reliability of switching can be improved by using the arrangement of Fig. 86. Diodes D_G are routing diodes which make sure that the negative trigger pulse reaches the collector of the nonconducting transistor, which is at a high potential, but does not reach the collector of the conducting one, the potential of which is low. Diode D_T allows the coupling capacitor C_T to discharge rapidly at the termination of the trigger pulse and thereby increases the trigger sensitivity of the circuit.

Frequency limitations due to saturation can be prevented by using the clamping techniques described in the previous section. These techniques are directly applicable to all the bistable flip-flops described above. How-

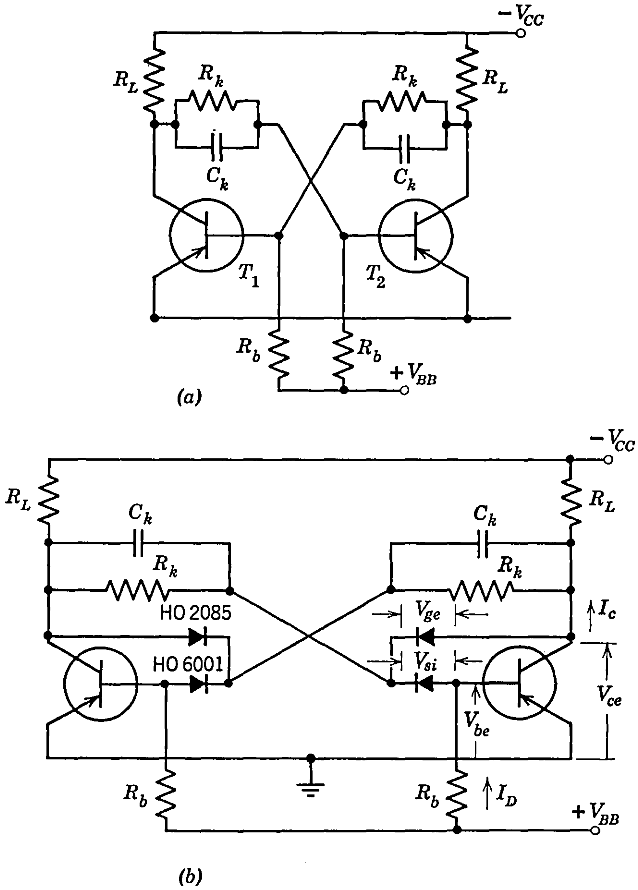


FIG. 84. (a) Unclamped and (b) clamped versions of base return flip-flop.

ever, it should be remembered that while clamping diodes prevent storage effects in transistors, storage effects in the diodes themselves must also be avoided. Consequently, the clamping diodes must be selected for short recovery time. A clamped version of the base return flip-flop is shown in Fig. 84*b*. A method of design for this circuit is given in Table 18. (The design method is due to E. A. Fisch, General Electric Company.)

A method developed by J. G. Linvill (see Ref. 143), which uses diodes for simultaneously routing trigger pulses and clamping, is illustrated by the circuit of Fig. 87. The diodes D_{ZK} and D_{ZS} are of the breakdown type. Diodes D_{ZK} are continuously broken down and maintain a constant voltage difference between the collector of one transistor and the base of the other. Diodes D_{ZS} are broken down when the voltage differ-

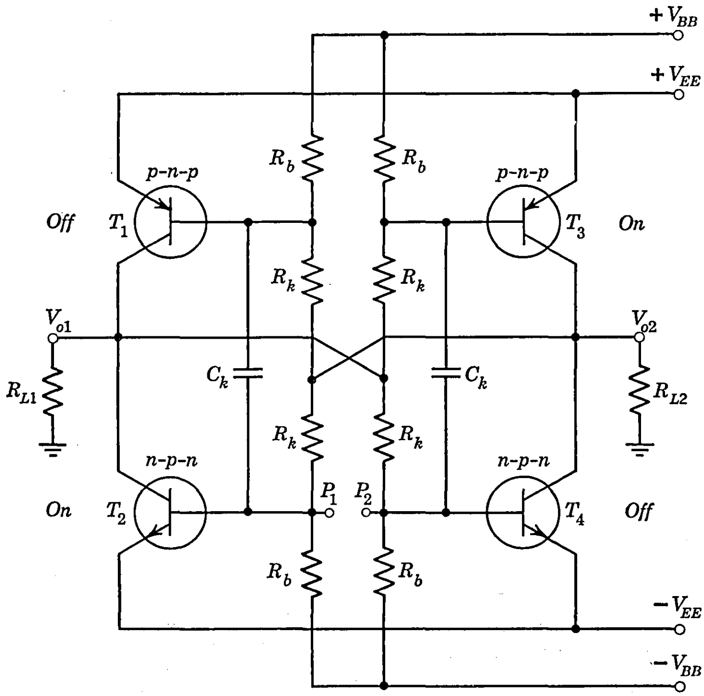


FIG. 85. High-efficiency flip-flop.

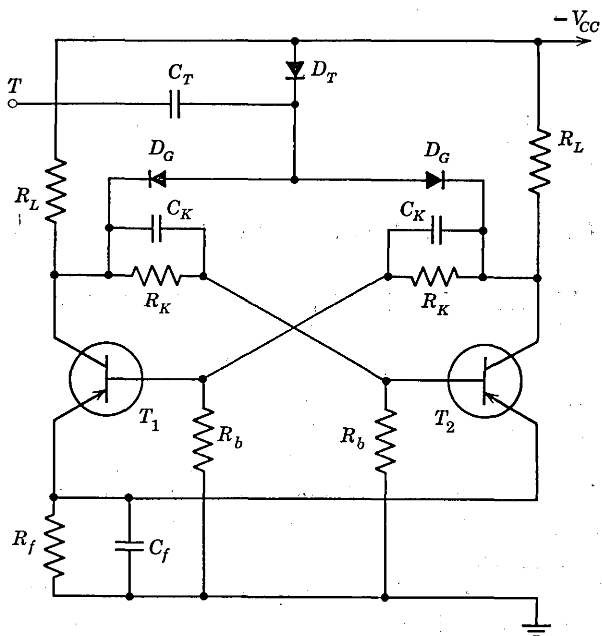


FIG. 86. Bistable multivibrator with steering diodes for high-speed triggering.

TABLE 18. DESIGN OF CLAMPED BASE RETURN FLIP-FLOP (FIG. 84b)
(From E. Fisch, General Electric Co., Syracuse, N.Y.)

Definitions

V_{CC} = collector supply voltage
V_{BB} = base supply voltage
ΔV_c = collector voltage swing
ΔI_c = collector current swing
I_D = current through voltage divider
V_{ce} = collector-to-emitter voltage of conducting transistor
V_{be} = base-to-emitter voltage of conducting transistor
V_{Si} = forward voltage of silicon diode
V_{Ge} = forward voltage of germanium diode

Determine: R_L, R_K, R_b, C_K, V_{BB}.

1. Select ΔI_c, V_{CC}, and t_{rn}.
2. Calculate V_{ce}:
   $V_{ce} = V_{be} + V_{Si} - V_{Ge}$;
   $V_{ce} < V_{CC}$; $V_{CC} = V_{ce} + \Delta V_c$.
3. Select V_{BB}: $V_{BB} > V_{ce}$.
4. Calculate R_L: $R_L = \Delta V_c / \Delta I_c$.
5. Select I_D: $I_D \gg I_{CB0}$.
6. Calculate R':
   $$R' = R_K + R_b = (V_{BB} + V_{CC})/I_D - R_L.$$
7. Calculate design parameters a, b, c:
   $a = V_{CC} - V_{be} - V_{Si}$;
   $b = V_{BB} + V_{be}$;
   $c = \Delta V_c / R_L - (V_{BB} + V_{ce} - V_{Si})/R'$.
8. Calculate design parameter m_0 by taking positive root of:
   $$m^2 + m\left[\frac{\alpha_{e0}(a+b)}{c} - \frac{R_L + R'}{R'}\right] - \frac{\alpha_{e0}b(R_L + R')}{cR'} = 0.$$
9. Calculate R_b: $R_b = m_0 R'$
10. Calculate R_K: $R_K = R' - R_b$
11. Calculate C_K:
   $$C_K = \frac{R_b + R_K + R_L}{R_K(R_b + R_L)} \cdot \frac{t_{rn}}{2.3}.$$

ence between the two collectors exceeds their breakdown voltage. Consequently, diodes D_{ZS} prevent collector saturation while the reference potentials established by diodes D_{ZK} fix the stable points of the circuit.

The *switching time* t_{sw} (i.e., the time within which the transient reaches 90% of its final value) of transistors in a nonsaturating flip-flop is given approximately by:

$$t_{sw} \cong 0.37/f_{\alpha b}.$$

The *maximum rate* at which a transistor flip-flop can be switched be-

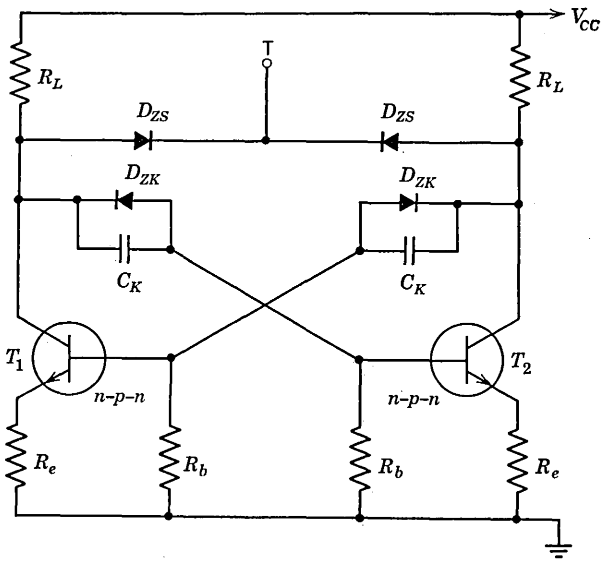


FIG. 87. Nonsaturating flip-flop using breakdown diodes.

tween its stable states is a complicated function of its circuit elements. For optimum design, neglecting the effect of the collector capacitance and that of the recovery time of the coupling network, the maximum switching rate is approximately:

$$f_{\max} \cong f_{\alpha b}\sqrt{1 - \alpha_{b0}} \cong \sqrt{f_{\alpha b} f_{\alpha e}}.$$

Flip-flops can be triggered from one stable state into the other stable state by applying a trigger pulse of sufficient amplitude and/or length. The required trigger charge (current multiplied by time) is approximately $Q_T = (i_{c,\ \max}/\pi f_{\alpha b})$, where $i_{c,\ \max}$ is the maximum current flowing in the collector of the conducting transistor (see Ref. 22).

Monostable Circuits. Monostable circuits have one stable state, the stable state being usually associated with zero output (or a nonconducting transistor). Application of a trigger pulse switches the circuit into a state where an output is obtained for a short time. The output may be considerably larger than the trigger input, and the shape of the output pulse may be considerably better than that of the input pulse. Consequently, monostable circuits can be used as *regenerative amplifiers.*

Many different circuits can perform this function. Eccles-Jordan type circuits can be made monostable by unbalancing a conventional bistable circuit. For example, by inserting a resistor R_{e1} in one of the emitter leads of the bistable flip-flop of Fig. 83, as shown in Fig. 88, the circuit becomes monostable. The multivibrator of Fig. 69 can also be made to

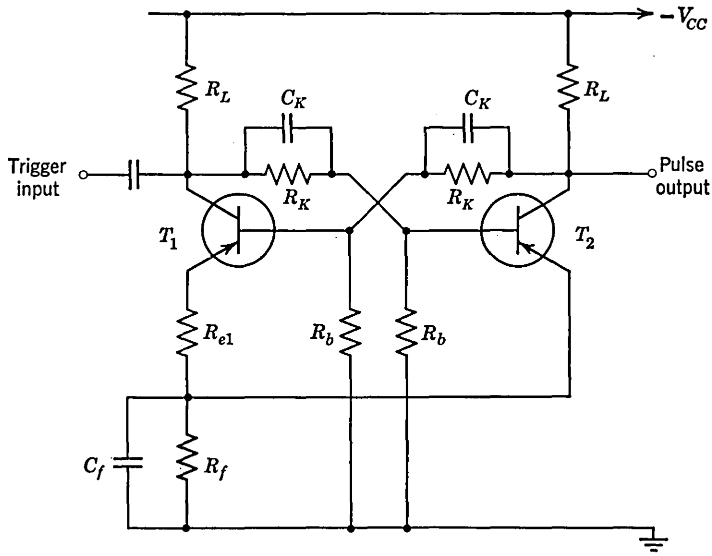


FIG. 88. Monostable multivibrator.

operate in a monostable manner. Table 19 gives a method of design for monostable multivibrators of this type (see Ref. 125).

Figure 89 shows a conventional transistor blocking oscillator. The circuit is freerunning (astable) if the emitter-base-diode is biased in the forward direction, i.e., if V_1 is negative. Then, the input signal serves to synchronize the freerunning circuit with the input. If the bias maintains the transistor in cutoff condition (V_1 being positive), the circuit is monostable and delivers an output provided an input trigger pulse is applied. The operation of the circuit can be improved considerably by using the arrangement of Fig. 90, suggested by Linvill and Mattson (see Ref. 124). Diode D_1 clamps the collector to potential V_1, thereby preventing collec-

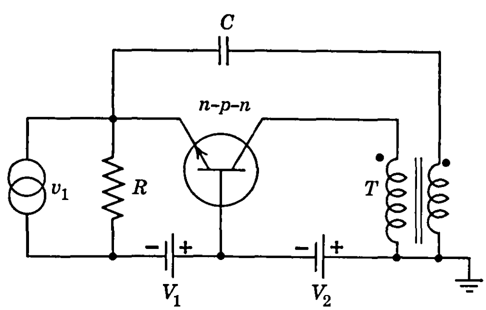


FIG. 89. Blocking oscillator type pulse generator.

TABLE 19. DESIGN OF MONOSTABLE MULTIVIBRATOR (FIG. 69) (Ref. 125)

Definitions

$$\begin{aligned}
V_{CC} &= \text{supply voltage}\\
\Delta V_c &= \text{collector voltage swing}\\
\Delta I_c &= \text{collector current swing}\\
V_{be} &= \text{base-to-emitter voltage}\\
t_{rn} &= \text{recovery time (pulse width)}\\
A &= \Delta V_c/V_{CC}\\
K &= R_b/(R_K + R_b + R_L)\\
X &= R_L/R_{e2}\\
u &= R_b/R_{e2}\\
\psi &= (1/K)/|V_{be}/V_{CC}|\\
m &= R_{e1}/R_{e2}
\end{aligned}$$

Determine: R_L, R_b, R_K, R_{e1}, R_{e2}, C_{K1}, C_{K2}, C_e.

1. Select ΔV_c, ΔI_c, V_{CC}, and t_{rn}.
2. Select K: $(1 - A)$ and $A > K \gg |V_{be}/V_{CC}|$.
3. Select u:

   For I_{CB0} stability: $u \ll \Delta I_c/I_{CB0}$;

   For realizability: $\alpha_{e0} \gg u > \dfrac{A}{\alpha_{b0}(1-\psi)(A-K)(1+K-A)}$.
4. Calculate KX:
$$KX = u/2 - \sqrt{u^2/4 - uA/[\alpha_{b0}(1-\psi)]}.$$
5. Select m to satisfy:

   (*a*) design realizability: $m \ll X$;

   (*b*) monostability:
$$m < KX\left\{\frac{1-\psi[1 + KX + K^2Xu/(u - KX)]}{1 - K(1 - KX\psi) + K^2Xu/(u-KX)}\right\}.$$
6. Calculate R_{e2}: $R_{e2} = (\alpha_{b0}V_{CC}/\Delta I_c)(K - |V_{be}/V_{CC}|)$.
7. Calculate R_{e1}: $R_{e1} = mR_{e2}$.
8. Calculate R_L: $R_L = XR_{e2}$.
9. Calculate R_b: $R_b - uR_{e2}$.
10. Calculate R_K: $R_K = [u/K - u - X]R_{e2}$.
11. Calculate C_{K2}:
$$C_{K2} = \frac{-t_{rn}}{R_K K(1 + X/u)\ln\{[(1-A)/A][(K + KX/u)/(1 - K - KX/u)]\}}.$$
12. Select C_{K1}. This is arbitrary but C_{K1} should be smaller than C_{K2}. For example:
$$C_{K1} = 0.1C_{K2}.$$
13. Select C_e:
$$C_e \gg 1/2\pi f_{\alpha b}R_{e2} \quad\text{and}\quad C_e < t_{rn}/\pi R_f.$$

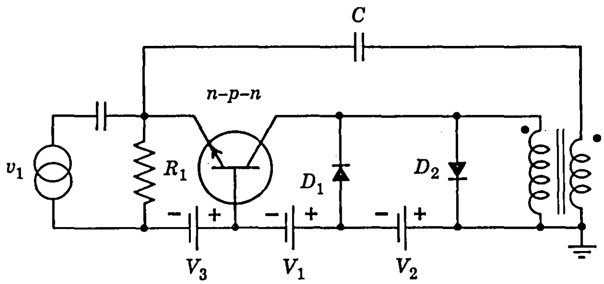


FIG. 90. Nonsaturating blocking oscillator.

tor saturation. Diode D_2 prevents the transformer primary voltage from being positive, thereby limiting the maximum collector-to-base voltage to $(V_1 + V_2)$, which must be smaller than the collector breakdown voltage of the transistor used.

Applications of Transistor Switching Circuits

Logical Switching Circuits. An important class of applications for switching circuits are logic circuits used in digital data processing systems. In general, a logic circuit has several input and output terminals with output signals appearing only for certain combinations of inputs. Complicated logical networks can be constructed by using *and, or* and *inhibit* circuits as building blocks. One advantage of transistor logic over the commonly used diode logic is the inherent amplifying capability of transistors which makes further amplification after a few logic stages unnecessary. Logic circuits of various types using transistors have been developed. One type (described by Bruce and Logue, see Ref. 154) uses the common collector stage as its basic switching element (Fig. 91).

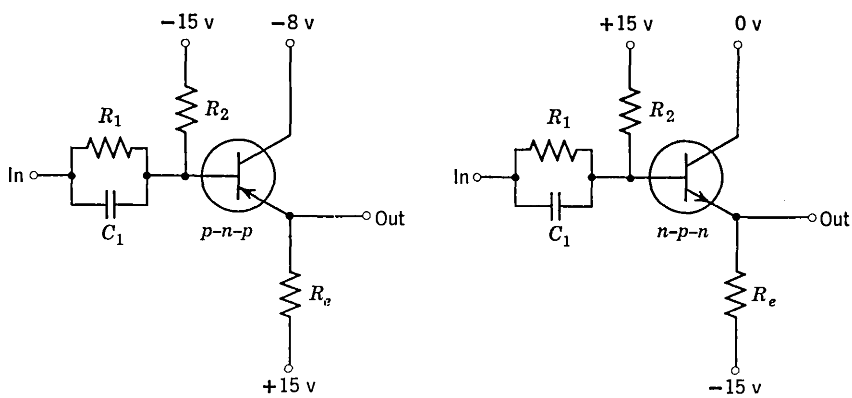


FIG. 91. Common collector stages *p-n-p* and *n-p-n* for logical functions.

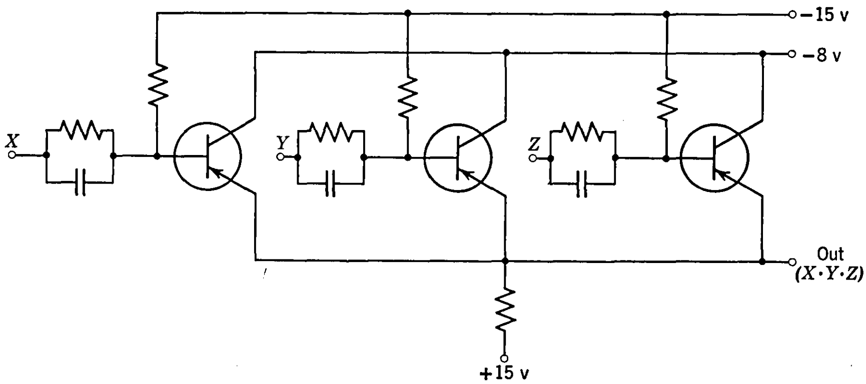


FIG. 92. *And* circuit.

Several such circuits having a common load resistor perform the *and* function for positive input pulses (Fig. 92). (For negative input pulses a similar circuit configuration acts as an *or*, provided the d-c potentials are appropriately changed.) With positive input pulses the *or* function can be performed using *n-p-n* transistors as shown in Fig. 93.

An *inverter* is shown in Fig. 94. If no input signal is applied, the transistor is nonconducting, the collector being a −5 volts potential due to the clamping action of diode *D*. When the input is driven negative (−5 volts), collector current flows and the collector becomes saturated (the collector potential being = 0 volts). After removal of the input, the charge accumulated on capacitor *C* flows into the transistor and turns it off.

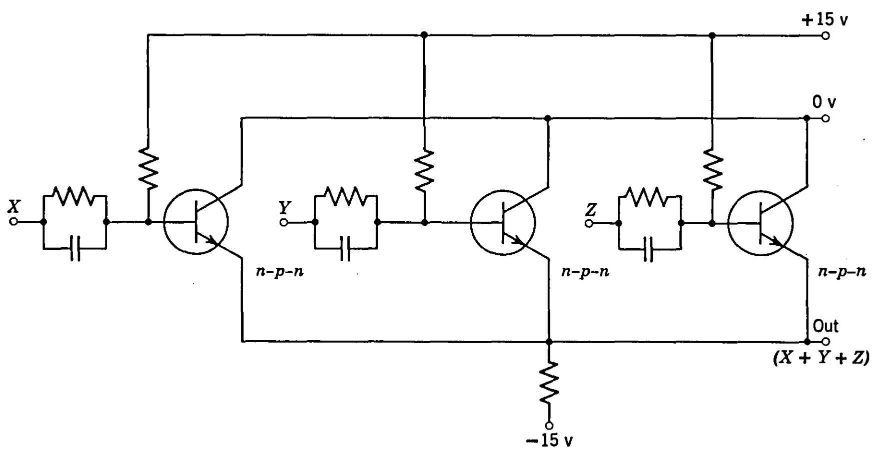


FIG. 93. *Or* circuit.

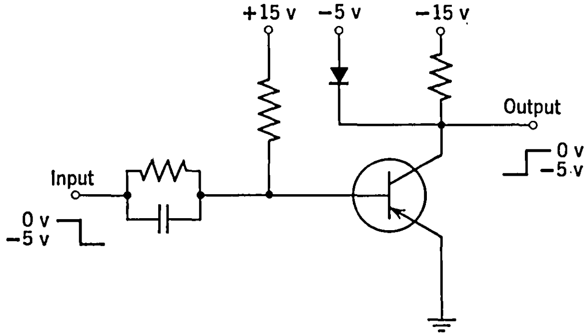


FIG. 94. Transistor inverter stage.

A transistor switching circuit which may be used to gate an input signal from a common input terminal to one of two output terminals is shown in Fig. 95. The input signal at A may be routed to the output of either transistor T_1 (B) or transistor T_2 (D), depending on the polarity of the gating signal applied at G. If a positive potential is applied at G, T_1 is biased into cutoff and a positive pulse applied at A will cause an output signal to appear at D only. If a negative potential is applied at G, the input pulse will cause T_1 to conduct, and the signal will appear at B but not D since the negative potential at G back-biases the emitter of T_2 but forward-biases the emitter of T_1. Similarly, once T_2 conducts, T_1 is clamped into cutoff by the positive potential at G.

The abovementioned logic circuits use several resistors and capacitors as biasing and coupling elements. Considerable circuit simplification can be achieved with the *direct-coupled* transistor logic circuits described in the following section.

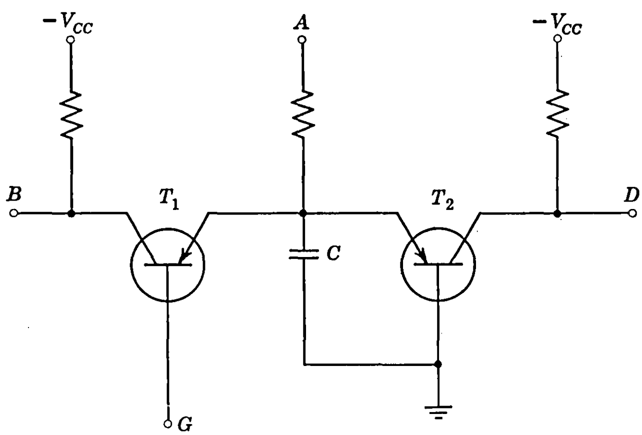


FIG. 95. Bidirectional transistor gate.

Direct-Coupled Switching Circuits. In digital circuitry a transistor is either conducting (*on*) or nonconducting (*off*). However, *on* and *off* are relative terms and imply only a significant difference in conduction. The difference must be sufficient for reliable operation. For example, if the collector of one transistor is coupled directly to the base of another transistor, as shown in the direct-coupled flip-flop of Fig. 96, satisfactory operation can be achieved, provided that the collector of the first (conducting) transistor is at a sufficiently low potential to permit only very little current flow in the other transistor and the collector of the second

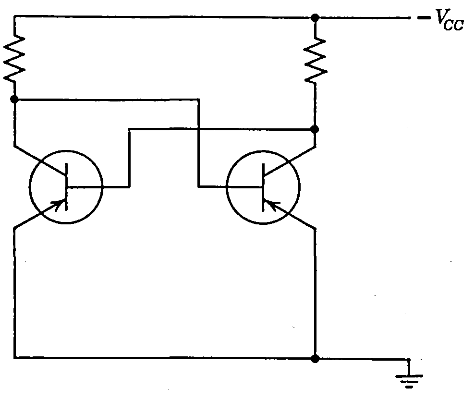


FIG. 96. Direct-coupled flip-flop.

nonconducting transistor is at a sufficiently high potential to cause considerable conduction of the first transistor. The input and output characteristics of many transistors satisfy these requirements and such transistors can be used in *direct-coupled transistor logic* (DCTL) circuitry. Such circuits result in considerable savings in component parts: capacitors and resistors (for details, see Vol. 2, Chap. 18, and Refs. 146 and 147).

Numerous logical and other switching functions can be achieved with direct-coupled circuits. An *or* and an *and* are shown in Fig. 97. The half-adder circuit of Fig. 98 has two output terminals: the "sum output" terminal is energized if only one input signal is present, whereas the "carry output" terminal is energized when both input signals exist. ($\bar{a}$ denotes the complement of a and $\bar{b}$ that of b.) The *odd parity checker* of Fig. 99 has four inputs. An output appears if any one or any three of the four inputs are present. There is no output for an even number of inputs.

Counters. An important class of application of bistable flip-flops (conventional or direct-coupled) are counting or frequency dividing circuits. Two cascaded bistable flip-flops, as shown in Fig. 100, give a *count by four.* In a similar manner, counting by numbers which are

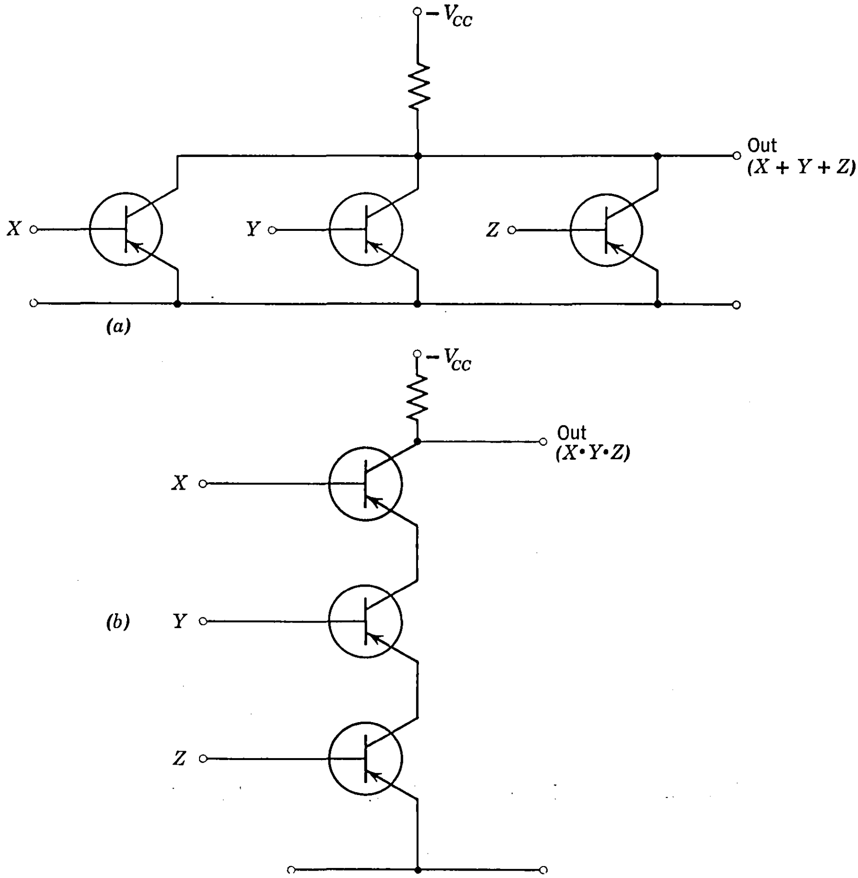


FIG. 97. Direct-coupled (*a*) *or* and (*b*) *and* circuits.

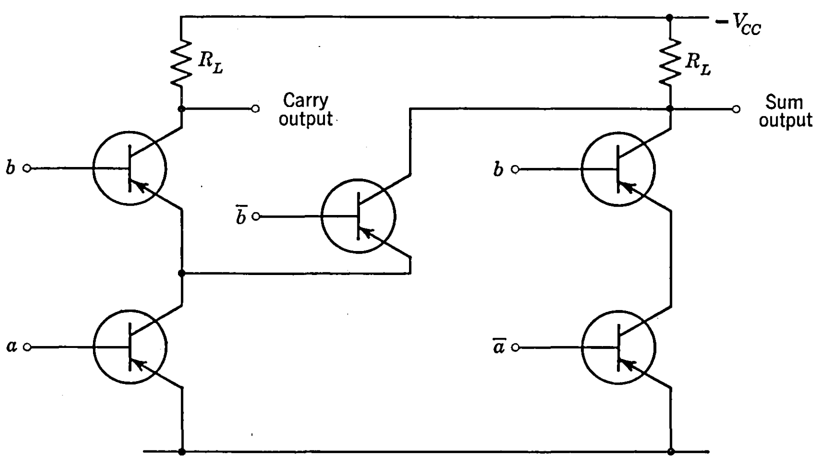


FIG. 98. Half-adder using direct-coupled logic.

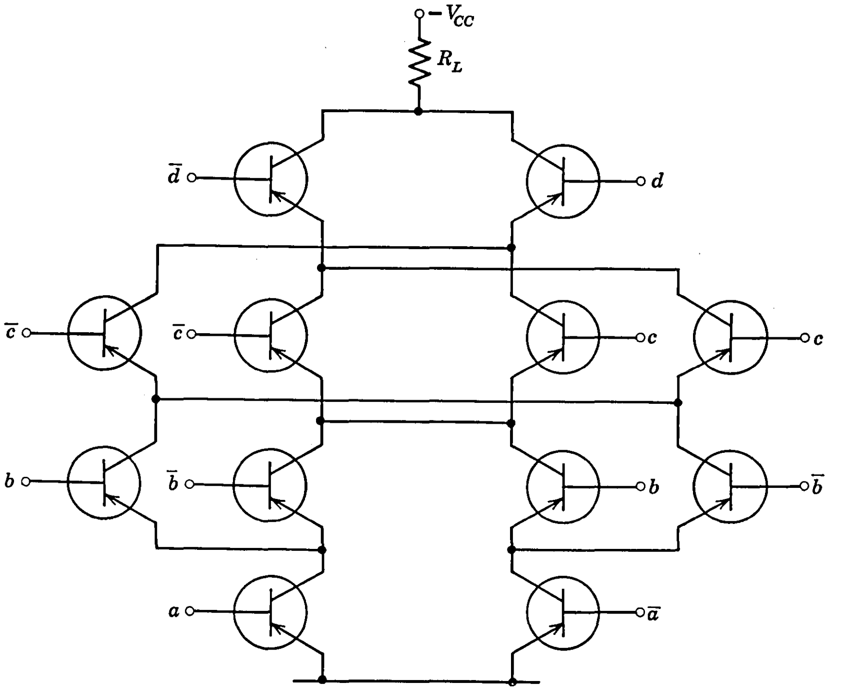


Fig. 99. Odd parity checker with direct-coupled transistors.

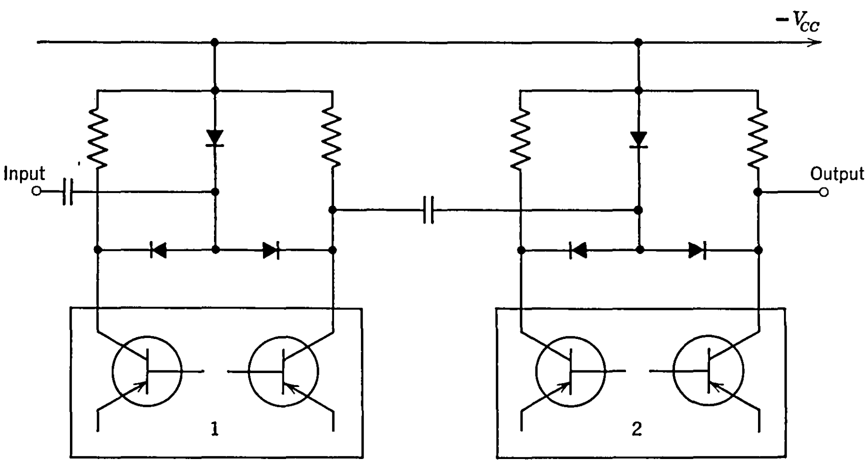


Fig. 100. Count-by-four circuit.

powers of two (16, 32, 64 ···), i.e., *binary counting,* can be achieved by cascading an appropriate number of flip-flops.

By using feedback paths between stages, counting ratios which are not powers of two can be obtained. For example, the "count-by-three" circuit of Fig. 101 consists of two binary stages with feedback (through a diode) from the second to the first stage in such a manner that every three input pulses generate one output pulse. The operation of this circuit can be understood by considering the waveforms of Fig. 102

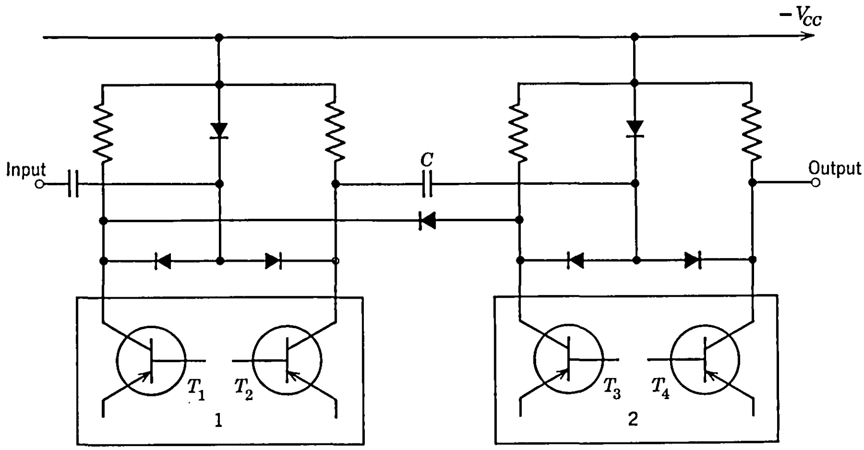


FIG. 101. Count-by-three circuit.

appearing at the collectors of the four transistors. The operation of the "count-by-five" circuit of Fig. 103 can be explained in similar manner taking into account the effect of the two feedback connections between the third and second stage (through diode D_2) and the first and third stage (through a diode D_1), respectively.

Shift Registers. A shift register consists of several binary storage elements (these are usually bistable flip-flops) arranged in such a manner that the application of a *shift pulse* transfers the state stored in one of the flip-flops to the flip-flop of next higher (or next lower) order. (A flip-flop can be in one of two states: transistor 1 *on* and 2 *off* or transistor 1 *off* and 2 *on.*)

The shifting operation can be performed in several different ways. One convenient method is represented by the circuit of Fig. 104, which uses diode gates. The state of the second flip-flop is changed as a result of an applied shift pulse whenever one of the diodes connects a high potential point of the first flip-flop to a low potential point of the second flip-flop. In other words, the first flip-flop controls the diode gate which steers the

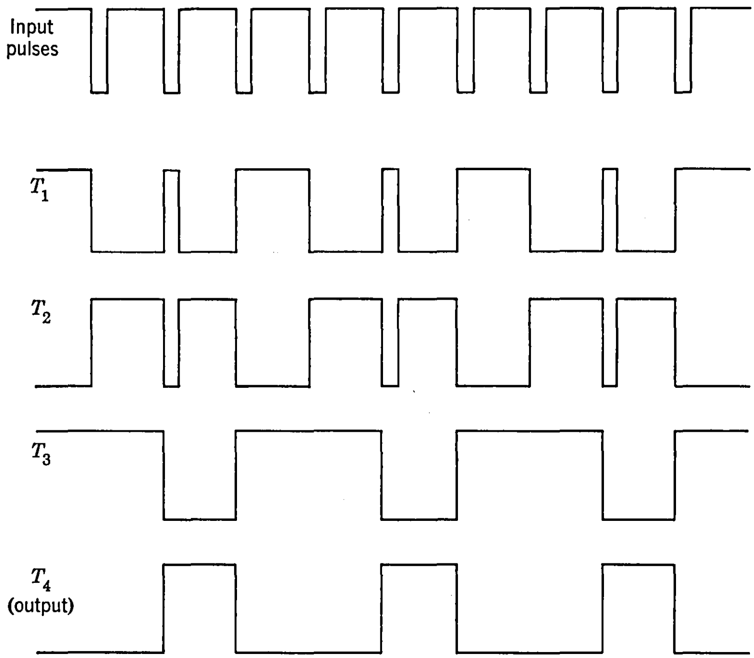


FIG. 102. Collector waveforms in ternary counter.

shift pulse so that the second flip-flop takes up the previous state of the first flip-flop.

The flip-flops are of conventional design. The function of the *RC* network associated with the gating diodes is to provide delay, since the diode gates must be controlled by the previous state of the first flip-flop and not by its new state (the first flip-flop may change state simultaneously with the second one).

Dynamic Circuits. Several of the digital circuits described in the paragraphs above apply to *d-c type data processing systems,* i.e., systems in which data are represented by suitable combinations of one out of two admissible discrete voltage levels. In *a-c systems* (often also *pulse systems*) data are represented by the existence or nonexistence of pulses at given time instants, these time instants being separated by intervals corresponding to the clock-frequency f_o of the system. For example, 1 may be represented by a pulse and 0 by the absence of a pulse. Since the pulse has a finite duration ($< \frac{1}{2}f_o$), it is necessary to ascertain whether one has 1 or 0 within a short time interval.

In pulse systems the pulses representing the digits are subjected to various operations (logic, storage, etc.). After the pulse has undergone

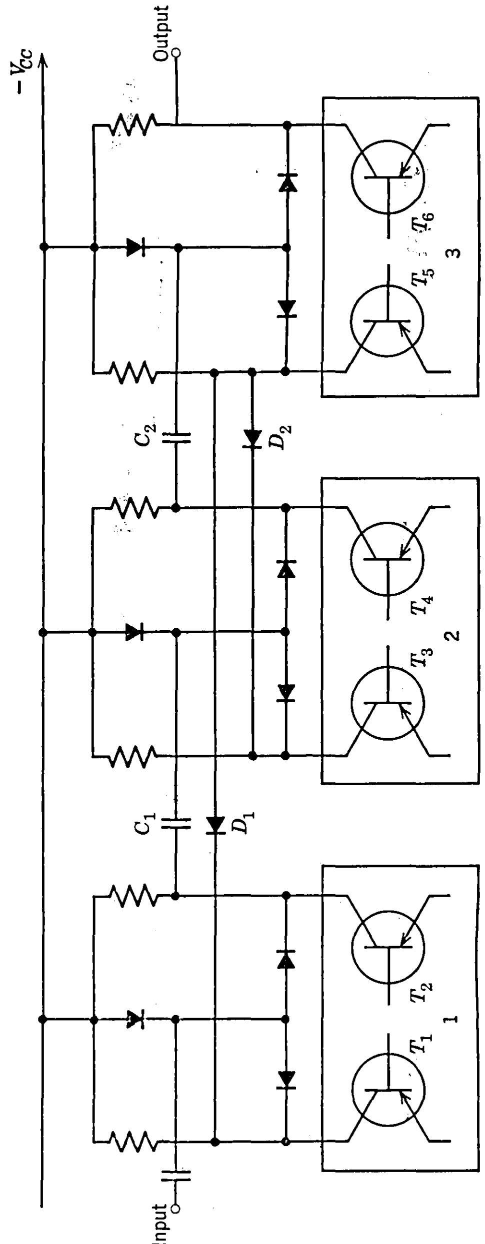


FIG. 103. Count-by-five circuit.

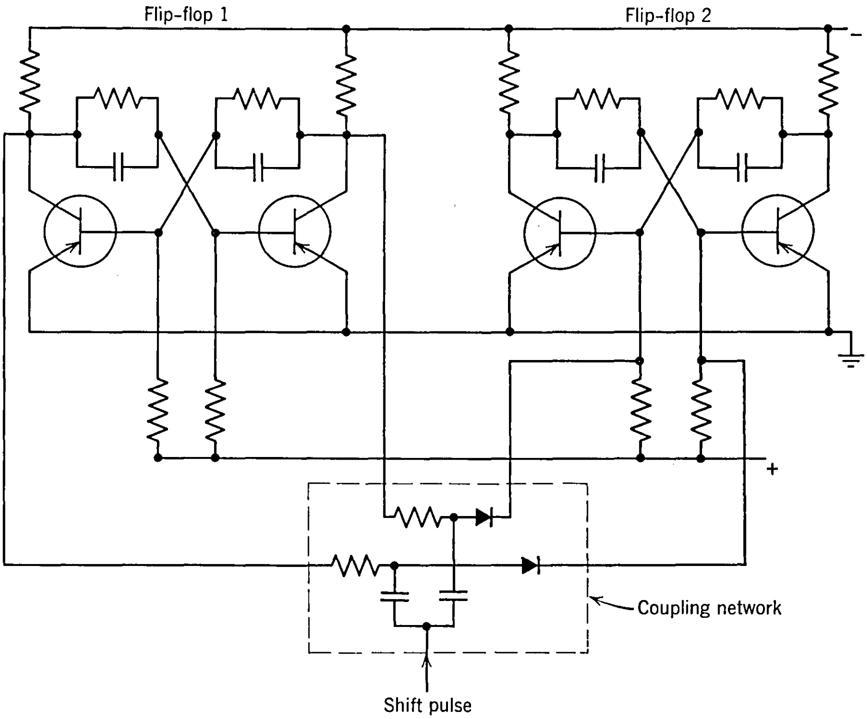


FIG. 104. A method of coupling shift-register stages.

a certain number of operations, it must be regenerated to reestablish its desired shape, length, timing, and amplitude. Furthermore, means permitting the temporary storage of pulses (or of pulse sequences, *words*) must be provided.

The block diagram of a circuit suitable for retiming, reshaping, and amplifying a deteriorated pulse is shown in Fig. 105. It is assumed that the word pulse starts somewhat before the beginning of the clock pulse. Coincidence of input and clock pulses activates the amplifier and results in an output pulse. Since the output pulse is fed back to the input, the output pulse is maintained until the clock pulse is removed. A circuit performing the functions of the block diagram is shown in Fig. 106 (see

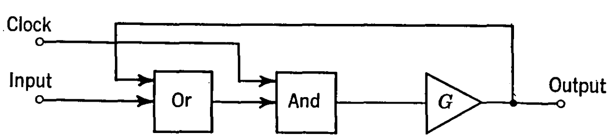


FIG. 105. Block diagram of pulse regenerating amplifier.

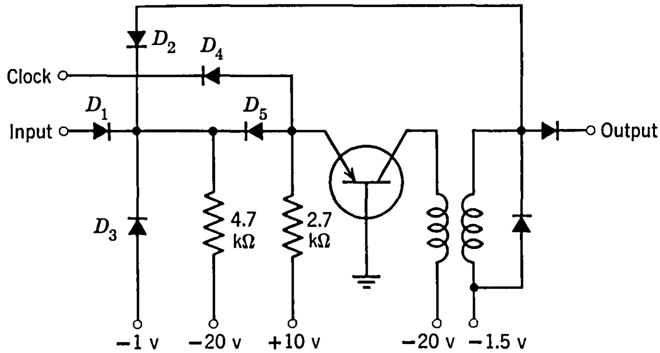


FIG. 106. Pulse regenerating amplifier.

Ref. 156). Diodes D_1 and D_2 form an *or* circuit feeding the *and* circuit composed of D_4 and D_5. The circuit uses a point contact transistor (biased into cutoff) in common base configuration but can be adapted to junction transistors in common base or preferably common emitter configuration (for details, see Ref. 156).

The block diagram of a *dynamic flip-flop* circuit capable of reshaping, retiming, amplifying and also of storing a pulse is shown in Fig. 107 along with the waveforms characterizing its operation. A three-phase clock is used: C_1 determines the time instant at which amplification of the deteriorated input pulse is started; C_2 (delayed by $\frac{1}{4}f_{CL}$) is used in the stages following the flip-flop considered; C_0 makes the operation of the *inhibit* gate more reliable. The input pulse is designated by W_1, the output pulse by W_2 and the pulse fed back through the delay line D by W_D. Amplification is started after arrival of W_1 as soon as C_1 starts and is maintained until C_1 ends. W_2 is delayed by D (the delay is somewhat less than $1/f_{CL}$ if one pulse is to be stored and somewhat less than n/f_{CL} if n pulses must be stored) and activates the amplifier again as soon as coincidence with C_1 exists. The pulse is recirculated in the circuit until I_1 inhibits the passage of W_2 and clears the circuit for new information. A complete circuit diagram (developed by G. A. Allard and S. K. Ghandhi) incorporating the functions of the block diagram of Fig. 107 is shown in Fig. 108. The circuit is suited for operation at 1-Mc rate.

8. POWER SUPPLIES

Regulated Power Supplies. Transistors can be used with great advantage for the design of regulated power supplies. Although transistors used in regulated power supplies are operated as d-c amplifiers, the problems often arising in connection with low-level d-c amplifiers (temperature stability) are of smaller significance in this type of application

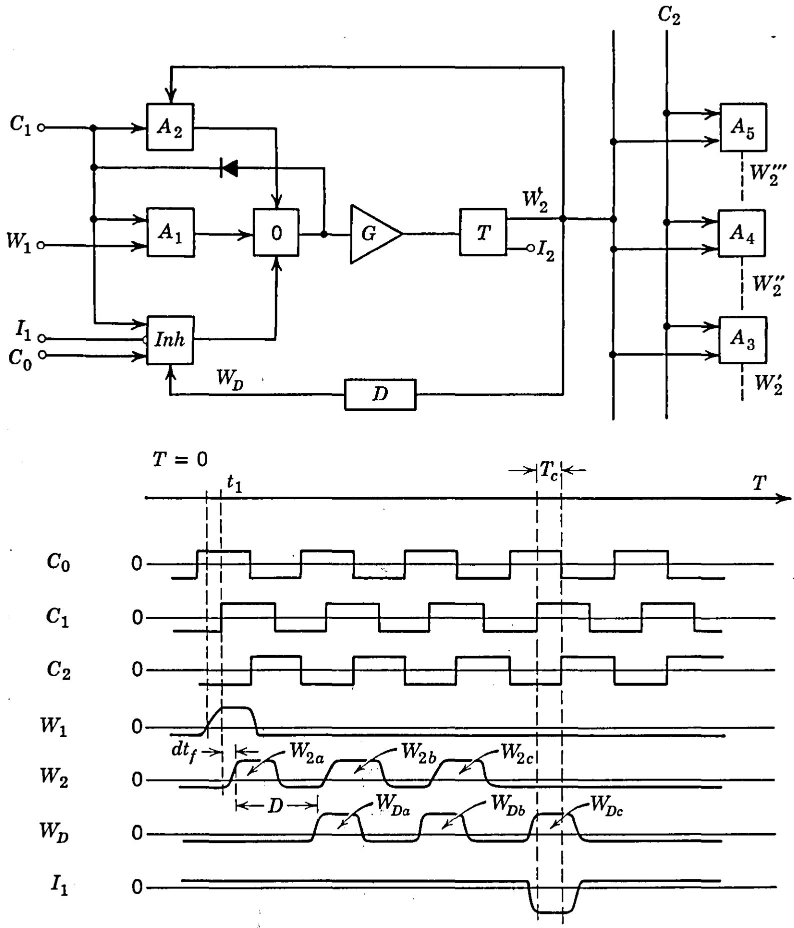


FIG. 107. Operation of dynamic flip-flop.

as a result of the higher voltage and current levels used and also because of the stabilizing effect of the feedback features often associated with these circuits.

A simple power supply circuit for low-level applications (Fig. 109) uses a breakdown diode for output voltage stabilization. The diode voltage is the breakdown voltage of the diode and the excess voltage (i.e., the difference $V_{\text{in}} - V_{\text{reg}}$) appears across R_S. The *shunt type transistor regulator* (Fig. 110*a*) is an extension of this circuit. V_{reg} is approximately equal to the diode breakdown voltage since the base-to-emitter voltage is very small for moderate currents. If the output voltage tends to increase, the excess voltage appears between the emitter and the base of transistor T.

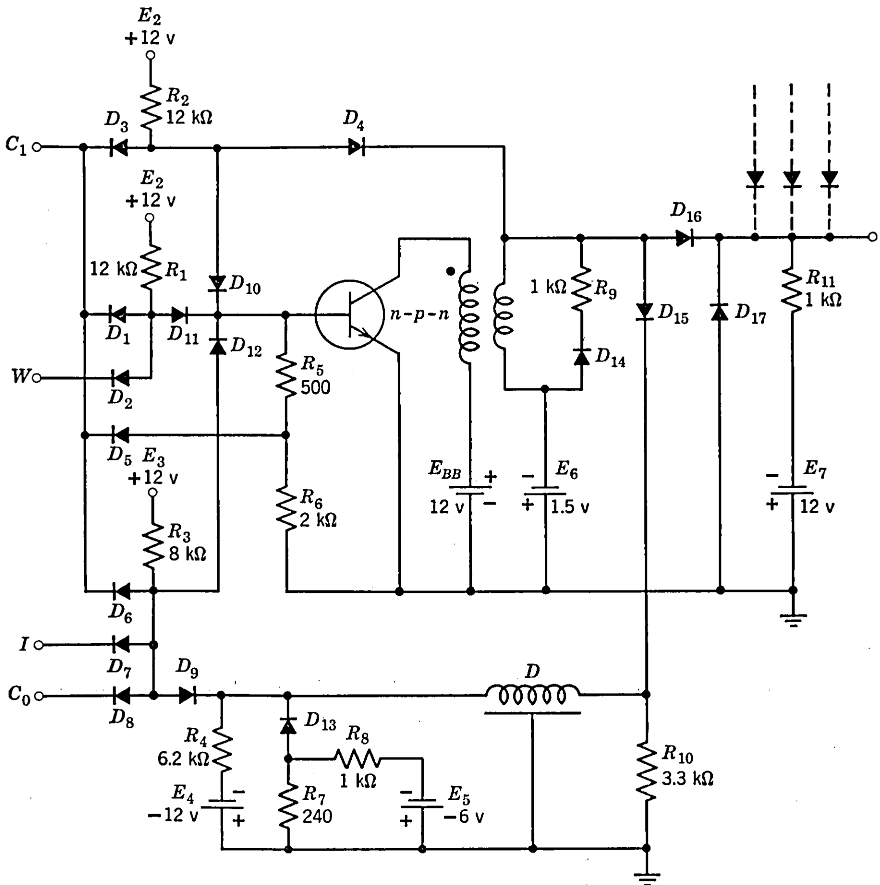


FIG. 108. Complete schematic of dynamic flip-flop.

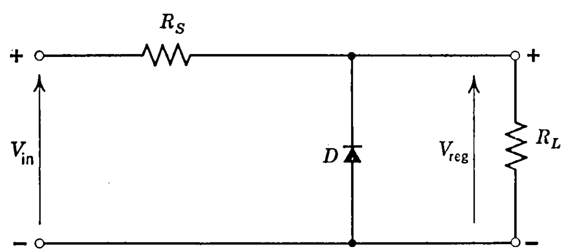


FIG. 109. Regulated power supply using breakdown diode.

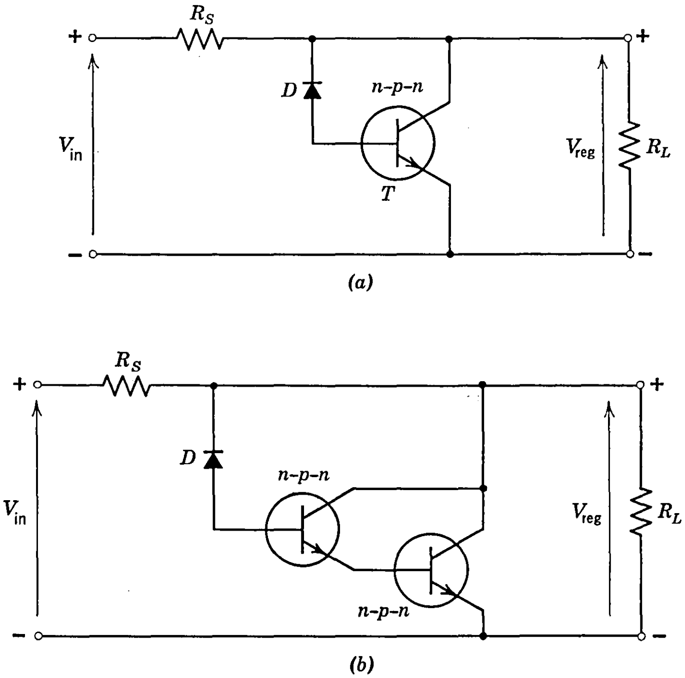


FIG. 110. Shunt type regulators using (*a*) single transistor and (*b*) compound transistor.

This causes the collector current to increase and increases the voltage drop across R_S thereby preventing the load voltage from increasing. The output resistance of the shunt regulator is

$$R_o \cong R_D(1 - \alpha_{b0}) + h_{11b},$$

where R_D is the a-c resistance of diode D. R_o is small (5 to 25 ohms, depending on the load current), implying good regulation for many applications. The circuit cannot be used for higher power outputs because of the limited current that can be passed through D. (This circuit can handle α_e times the current handled by the circuit of Fig. 109.) The performance of this circuit can be improved by replacing the transistor by a compound connection (Darlington circuit, see Sect. 3, Low-Frequency Amplifiers) of two transistors (Fig. 110*b*). This reduces the current through the voltage-reference (see Ref. 164).

The circuit of Fig. 111 shows a *series type transistor regulator*. A breakdown diode is used as voltage reference, but other voltage reference devices can also be used. Increasing output voltage causes the collector current of T_3 to increase and results in decreasing the collector current of T_2 and,

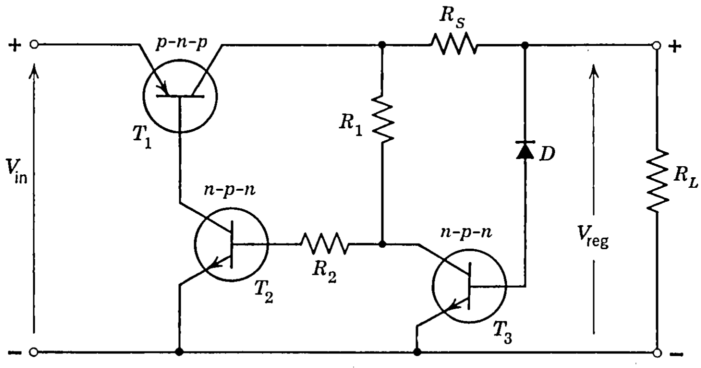


FIG. 111. Series type transistor regulator.

therefore, the base current of T_1. This decreases the collector current of T_1 and results in stabilization of V_{reg}. The voltage divider consisting of R_1 and R_2 has the effect of positive feedback and insures that the output voltage reaches the breakdown voltage level of D. This circuit has excellent regulation: output impedances of 1 ohm or less can be achieved in typical cases. The circuit also reduces the a-c ripple of V_{in} by a factor of 1000 or more.

The circuit of Fig. 112 is essentially a version of the series type regulator, the excess voltage ($V_{in} - V_{reg}$) lying across the collector junction. The reference voltage can be provided by a breakdown diode (Fig. 113) or other device, such as a VR tube. The output voltage is relatively independent of input voltage and load current variations because of the small voltage drop V_{BE} between base and emitter and the relatively small variations of V_{BE} with varying emitter current. In this configuration, too, the current through the voltage reference can be reduced considerably by using the Darlington circuit (Fig. 114, see Ref. 164).

In the circuit of Fig. 115, the output variation is fed back through T_2 to T_1 and improved regulation is achieved.

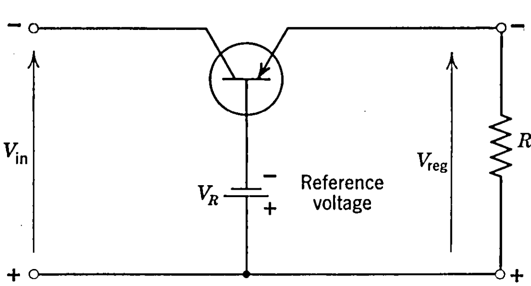


FIG. 112. Transistor-regulated power supply.

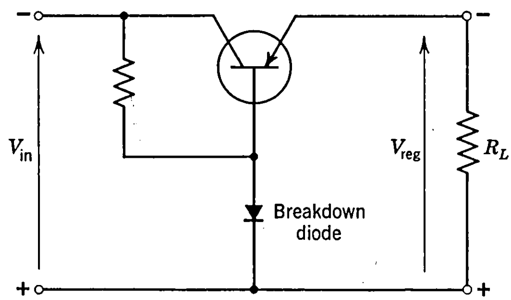


FIG. 113. Use of breakdown diode as voltage reference in circuit of Fig. 112.

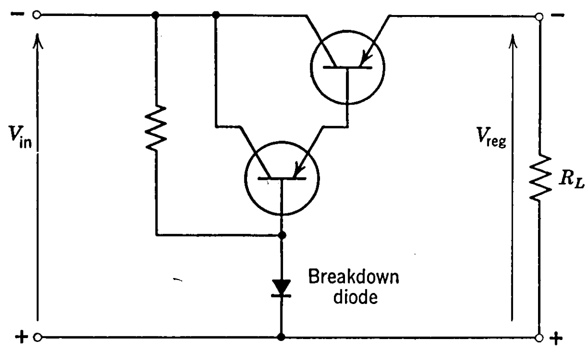


FIG. 114. Circuit of Fig. 113 using compound transistor.

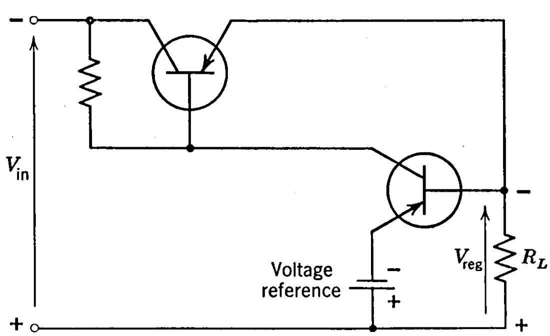


FIG. 115. Two-transistor regulated power supply.

A somewhat more elaborate circuit using a VR tube as voltage reference is shown in Fig. 116. An increase in output current increases the voltage drop across the small resistor R_3, thereby raising V_1 (D is a breakdown diode). This results in an increased current through the voltage reference which in turn increases the output voltage to compensate for the drop due to the load current. Resistors R_1 and R_4 act to reduce the effect of input voltage variations. If V_{in} rises, the emitter current of T_1 decreases, thereby decreasing the current through the voltage reference (for details, see Ref. 161).

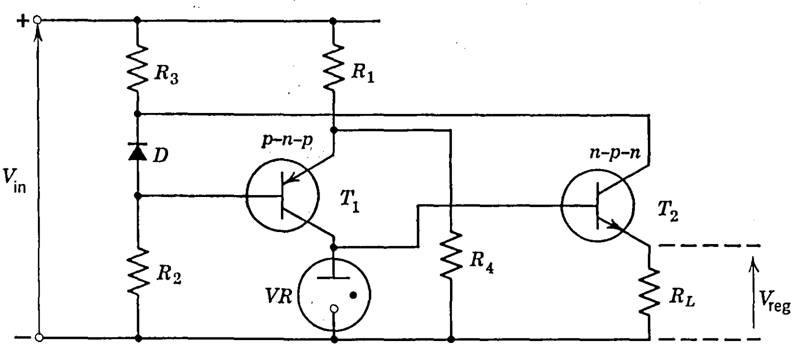


FIG. 116. Regulated power supply.

Power Converters. Transistor oscillators can be operated with low supply voltages. This can be used to design *power converters* for the conversion of low voltage d-c into higher voltage ac or dc. The diagram of Fig. 117 is that of a d-c to d-c converter. The two transistors operate as a push-pull oscillator with one of the transistors *on* and the other *off* alternatively. On account of saturation effects in the transformer which uses preferably a square loop material core, the voltage across the transformer is a square wave. This can be rectified if d-c output is desired. The square wave type operation results in very high efficiency (up to 95%), and dissipation in the transistors is small since the transistors are either nonconducting or saturated.

The two transistors are not necessarily identical. For example, a high-power transistor can be used in combination with a low-power transistor. (Such an arrangement is more economical and makes fuller use of the collector breakdown voltage of the high-power transistor.) In such cases, a half-wave rectifier is used in order not to extract excessive power from the low-power transistor which could lead to its destruction (for design details, see Refs. 166 and 167).

D-c to d-c converters can be designed for very high output voltages

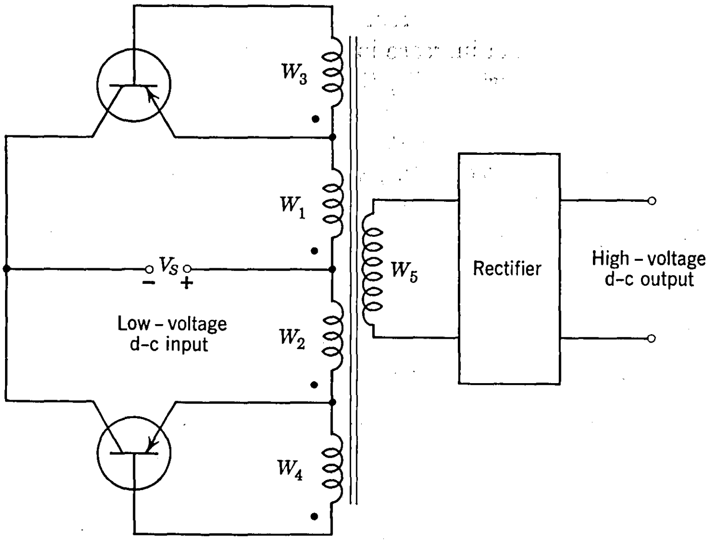


FIG. 117. Power converter.

and can be used as efficient high voltage supplies (for example, for application with cathode ray tubes).

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Gyroscopes

W. G. Wing

1. INTRODUCTION

Gyroscopes as Control System Components

The gyroscope has been an important element in certain types of automatic control systems for a great many years. Applications of the gyroscope greatly predate the relatively recent and extensive development of automatic control system theory. Generally speaking, the gyroscope is useful in automatic control systems as a reference with respect to which measurements of angular motions can be made. The importance of the gyroscope lies in the fact that it is a directional reference independent of the base on which it is carried, and hence is useful on moving vehicles where the vehicle frame itself is not a truly stable reference.

The Basic Gyroscope

A gyroscope can be described as a space reference, the direction of which can be changed by the application of appropriate torques; it is basically a spinning wheel. Useful properties are derived from the fact that the axis of such a wheel will maintain a fixed direction in space if no disturbing torques are present. If a torque is applied to such a wheel about an axis normal to the axis of the spin, the direction of the spin axis will change at a rate proportional to the applied torque; the rotation occurs about an axis which is normal both to the axis of spin and to the axis of the applied torque. This response to applied torques is known as *precession.*

The gyroscope will change its direction in space because of intentionally applied torques or because of torques which result from various instrument imperfections. Spin axis motions which result from such imperfections are generally referred to as *drift,* and the minimization of such imperfection torques is an important part of the design of all gyroscopic instruments.

A practical gyroscope design must provide means for spinning the wheel, for supporting its weight and at the same time providing the required degrees of angular freedom, for the application of such torques as may be desired, and finally for sensing the angular relationship between the spin axis of the wheel and such other element or elements as it is desired to control.

2. GENERAL DYNAMIC PRINCIPLES

Dynamic Characteristics

If it is assumed that the wheel shown in Fig. 1 has an angular velocity Ω about the axis 1–1 and that its polar moment of inertia (moment of inertia about axis 1–1) is J, then its angular momentum H is equal to $J\Omega$. The angular momentum is a vector and may be represented by an arrow

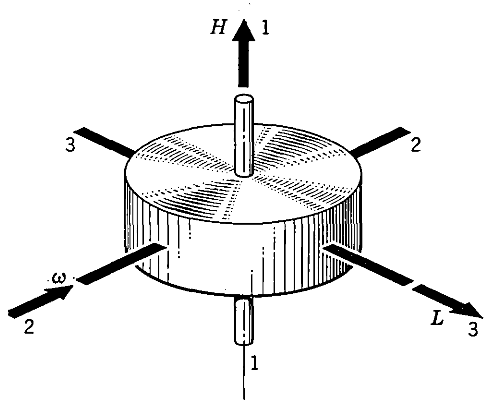

FIG. 1. Elementary gyroscope.

lying along the axis 1–1. If it is now assumed that an angular velocity ω exists about axis 2–2, it is evident that the direction of H is changing at this latter angular rate. A fundamental principle of dynamics states that if a body experiences a change in angular momentum, a torque is being applied; this torque must be coaxial with the angular momentum change and of a magnitude equal to the rate of change of angular momentum. Since the rotation of the H vector about axis 2–2 represents a rate of change of angular momentum about axis 3–3 of magnitude ωH it follows that a torque L of the same magnitude must exist about the axis 3–3.

If the angular velocities Ω and ω are expressed in radians per second, the units of torque L will depend on the units of J as indicated in Table 1.

TABLE 1. DIMENSIONS OF J, H, AND L

Units of J	Units of H	Units of L
gram-cm^2	gram-cm^2/sec	dyne-cm
slug-ft^2	slug-ft^2/sec	pound-ft
pound-ft^2	pound-ft^2/sec	poundal-ft

It is conventional in most current work to express gyro momentum in cgs units; hence the first line of this table is of greatest interest.

This discussion of precession is greatly simplified by the assumed steady-state condition. Because the gyroscope wheel has inertia about other axes than the polar axis, angular momentum changes exist which are not associated with changes in the direction of the spin. In practical cases additional masses may be coupled to the wheel, and springs and damping mechanisms may exercise restraints. In order to account for these various factors, motion equations may be written for the generalized device indicated in Fig. 2. Equations describing the behavior of this device can be applied to all the common types of gyroscopes.

Generalized Gyroscope Equations

The mechanism of Fig. 2 is a wheel mounted so as to have three degrees of rotational freedom. The wheel spins about axis 1–1 and has an angular momentum H which lies along this axis (assumed positive as shown). The following assumptions are also made:

1. Axes 1–1, 2–2, and 3–3 are mutually perpendicular. Rotations in space of the gyroscopic wheel about axis 2–2 are indicated by angle θ_2 which is assumed positive in the direction of the arrow. The total moment of inertia of the system about axis 2–2 is indicated by I_2 and any spring restraint and viscous damping between the inner and outer gimbals

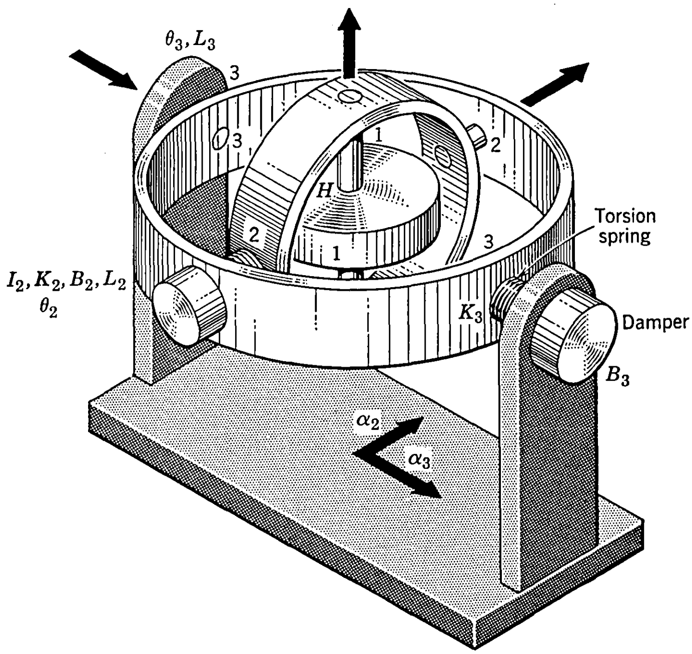

FIG. 2. Generalized gyroscope system.

are indicated by K_2 and B_2. Also, on axis 3–3 a space angle θ_3 (positive as shown), a moment of inertia I_3 and restraints (relative to the base) K_3 and B_3 are indicated.

2. Torques may be applied to the system and these are indicated on the figure as L_2 and L_3. The positive senses of L_2 and L_3 agree with the positive senses of θ_2 and θ_3.

3. Small space rotations of the base may occur. These are indicated as α_2 and α_3 and the positive senses are the same as those of θ_2 and θ_3.

If the spin axis is kept nearly perpendicular to 3–3 (i.e., if $\alpha_2 - \theta_2$ is always small), relatively simple transfer equations may be written which relate the various quantities of interest, when the system is disturbed. Table 2 lists these various equations in which s is the complex Laplace transform operator.

It should be noted that the expression for θ_2 as a dependent variable and θ_3 as an independent variable cannot be derived from that for θ_3 as a dependent variable and θ_2 as an independent variable. For this reason these equations must be used with some care.

A second point is that superposition principles may be used to determine the response of the system to several simultaneous disturbances.

These relatively few equations thus become adequate for analysis of complex applications.

3. TYPES OF GYROSCOPES

General

A number of variations on the simple gyroscope are used for various purposes. The dynamic characteristics of each may be determined by appropriate application of the equations of Table 2.

TABLE 2. TRANSFER EQUATIONS

Independent Variable	Transfer Function	Dependent Variable
L_2	$\theta_2 = \dfrac{(K_3 + B_3s + I_3s^2)L_2}{(K_3 + B_3s + I_3s^2)(K_2 + B_2s + I_2s^2) + H^2s^2}$	θ_2
L_2	$\theta_3 = \dfrac{-HsL_2}{(K_3 + B_3s + I_3s^2)(K_2 + B_2s + I_2s^2) + H^2s^2}$	θ_3
L_3	$\theta_2 = \dfrac{HsL_3}{(K_3 + B_3s + I_3s^2)(K_2 + B_2s + I_2s^2) + H^2s^2}$	θ_2
L_3	$\theta_3 = \dfrac{(K_2 + B_2s + I_2s^2)L_3}{(K_3 + B_3s + I_3s^2)(K_2 + B_2s + I_2s^2) + H^2s^2}$	θ_3
θ_2	$\theta_3 = \dfrac{-Hs\theta_2}{K_3 + B_3s + I_3s^2}$	θ_3
θ_3	$\theta_2 = \dfrac{Hs\theta_3}{K_2 + B_2s + I_2s^2}$	θ_2
α_2	$\theta_2 = \dfrac{(K_3 + B_3s + I_3s^2)(K_2 + B_2s)\alpha_2}{(K_3 + B_3s + I_3s^2)(K_2 + B_2s + I_2s^2) + H^2s^2}$	θ_2
α_2	$\theta_3 = \dfrac{-Hs(K + Bs)\alpha_2}{(K_3 + B_3s + I_3s^2)(K_2 + B_2s + I_2s) + H^2s^2}$	θ_3
α_3	$\theta_2 = \dfrac{Hs(K_3 + B_3s)\alpha_3}{(K_3 + B_3s + I_3s^2)(K_2 + B_2s + I_2s^2) + H^2s^2}$	θ_2
α_3	$\theta_3 = \dfrac{(K_2 + B_2s + I_2s^2)(K_3 + B_3s)\alpha_3}{(K_3 + B_3s + I_3s^2)(K_2 + B_2s + I_2s^2) + H^2s^2}$	θ_3

Two Degrees of Freedom Gyroscope

Description. The type of gyroscope which has been used in greatest numbers is the simple *free gyroscope* which consists basically of a wheel so supported as to have two degrees of rotational freedom (neglecting the spin axis freedom). This type of gyroscope utilizes directly the inherent fixity in space of such a spinning wheel and is thus conceptually the simplest of all types. Figure 3 illustrates such an instrument and includes gimbal rings for the support of the wheel. The device shown in Fig. 3 is much like that of Fig. 2 except that the springs and dampers are absent (i.e., K_2, K_3, B_2, and B_3 are equal to zero).

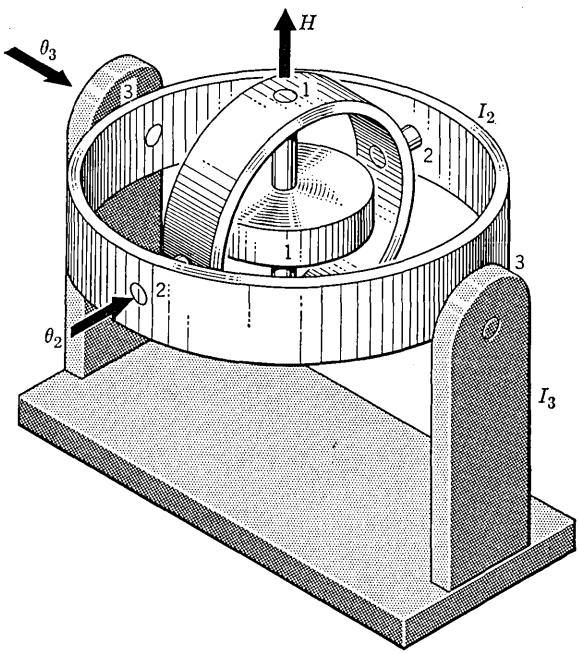

FIG. 3. Free gyroscope.

Dynamics. The dynamics of this device may be determined by application of the equations in Table 2 with K_2, K_3, B_2, and B_3 equal to zero. Thus

$$\theta_2 = \frac{L_3}{Hp}\left(\frac{1}{\dfrac{I_2 I_3}{H^2}s^2 + 1}\right),$$

$$\theta_3 = \frac{I_2}{H^2}L_3\left(\frac{1}{\dfrac{I_2 I_3}{H^2}s^2 + 1}\right).$$

Nutation Frequency. Inspection of these expressions reveals that if a torque L_3 is applied, the familiar gyroscopic precession results; in addition, an oscillatory response is superimposed which has an angular frequency equal to $H/\sqrt{I_2 I_3}$. This same oscillation frequency appears in the expression for θ_3 and further analysis reveals that a conical motion of the spin axis occurs. This motion is known as *nutation* and the angular frequency $H/\sqrt{I_2 I_3}$ is the nutation frequency.

Recalling that $H = J\Omega$, then the nutation frequency will be given by:

$$\omega_N = \frac{J}{\sqrt{I_2 I_3}}\,\Omega.$$

For the free gyroscope the nutation frequency is a constant, dependent on the various moments of inertia, multiplied by the spin frequency. For a thin disk with no coupled inertias $I_2 = I_3 = \frac{1}{2}J$, and $\omega_N = 2\Omega$; for a sphere, $I_2 = I_3 = J$ and $\omega_N = \Omega$.

The thin disk case represents the highest possible nutation frequency of a gyroscope; the case of the sphere illustrates that a sphere has no preferred axis of rotation (the nutation at spin frequency simply implies a shift of the axis of rotation). The normal free gyroscope with the coupled inertias of the spin motor, gimbals, pickoff elements, etc., usually has a nutation frequency lower than the spin frequency.

Suspension of Free Gyroscopes. Types of suspension other than the simple bearings and gimbals of Fig. 3 may be used for the free gyroscopes. Such suspensions are discussed further in Sect. 4 under Gyroscope Suspensions, but it should be noted that they may involve values of K_2, K_3, B_2, and B_3 which are not equal to zero and hence must be considered in the application of the Table 2 equations.

It is evident that suitable pickoffs may be mounted on axes 2–2 and 3–3 in Fig. 3 if the gyroscope is to be used in a control system. Such pickoffs are discussed in Sect. 4 under Pickoffs for Gyroscopes.

Rate Gyroscope

Description. Figure 4 illustrates the principle of the *simple rate gyroscope* which belongs to the general class known as *single degree of freedom* gyroscopes. Figure 4 is similar to Fig. 2 with K_2 and $B_2 \neq 0$ and θ as the independent variable. Using the appropriate equation from Table 2, θ_2 is given by the following:

$$\theta_2 = \frac{Hs\theta_3}{K_2 + B_2 s + I_2 s^2}.$$

In the steady state θ_2 is proportional to the rate of change of θ_3 but the transient response involves a damped oscillation. For some purposes

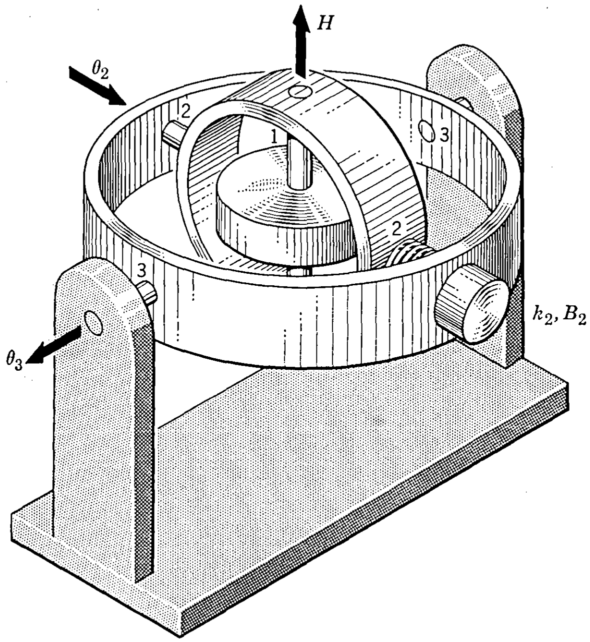

FIG. 4. Spring-restrained gyroscope.

the damping is obtained entirely from inherent losses (friction, spring hysteresis, air damping, etc.), and no actual damping mechanism is included.

Performance Limitations. Performance limitations exist which make the simple rate gyroscope unsuitable for many purposes.

1. The phase shift inherent in its response may be undesirable. This may be minimized by making the undamped natural frequency very high, but this technique results in a very small sensitivity (i.e., θ_2 is very small for a given rate of change of θ_3) with resultant difficulties in securing satisfactory pickoff performance.

2. When $\theta_2 \neq 0$ the effective input axis is no longer 3–3 but is actually the normal to axis 1–1. This condition may, in some cases, cause undesirable interaction among various axes in a control system, which can be minimized by making the undamped natural frequency very high; however pickoff difficulties (as in the above discussion) are created.

3. The device is sensitive to angular acceleration about axis 2–2, which can create undesirable interactions among the various axes of a control system. This interaction can be reduced by making I_2 very small compared with the angular momentum of the wheel (most easily done by spinning the wheel at a high speed).

Torque Generator. A large part of the pickoff problem which is created when an effort is made to achieve a high natural frequency in a rate gyroscope results from the difficulty of making the zero torque angle of the spring (K_2) coincide exactly with the zero output angle of the pickoff (this implies not only very careful initial adjustment but also very low spring hysteresis and good long-time stability in the spring and pickoff). If, instead of an actual mechanical spring, an electromechanical spring is used, this difficulty can be largely overcome. A torque generator is added to axis 2–2 and the pickoff signal is applied (through a suitable amplifier) to the torque generator; the sensing must be such as to cause centering to occur. In this arrangement the zero torque angle of the spring is inherently the same as the zero output angle of the pickoff. The undamped natural frequency is, in this case, limited only by (*a*) the dynamic stability of the closed loop, (*b*) the gain which can be desirably included in the amplifier, and (*c*) the electrical noise output of the pickoff. (The last may determine the amount of gain which can be effectively used.)

Rate Integrating Gyroscope

Description. Figure 5 illustrates the principle of the rate integrating gyroscope, the use of which has been pioneered by the Instrumentation

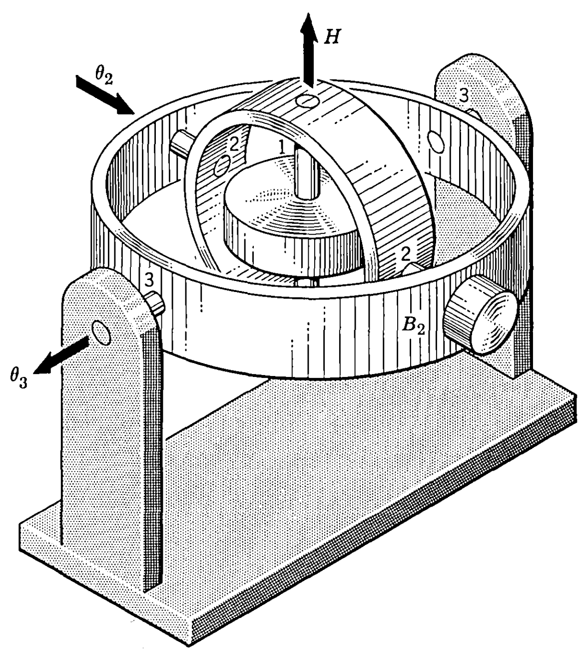

FIG. 5. Rate-integrating gyroscope.

Laboratory of the Massachusetts Institute of Technology. This is a single degree of freedom device and is conceptually similar to the rate gyroscope except that there is no restraining spring on the 2–2 axis (i.e., $K_2 = 0$). Application of the Table 2 equation with $K_2 = 0$ yields, for the response of primary interest, the following expression:

$$\theta_2 = \frac{H\theta_3}{B_2 + I_2 s}.$$

The angle θ_2 is thus found to be proportional to θ_3 in the steady state, but a simple first order time delay exists. This delay may be kept low (a few milliseconds) and is not normally troublesome in control systems.

Dynamics. The device is termed a rate integrating gyroscope because the rate of change of input angle (θ_3) is converted to a torque on the 2–2 axis (through the gyroscopic precession action), and this torque is integrated with respect to time by the damping mechanism.

This type of gyroscope is (for small angles) functionally similar to one axis of the two degrees of freedom type and is most useful where the angle θ_2 can be kept very small, as is the case where servo means are provided to control θ_3 in response to a signal proportional to θ_2.

Torque Summing Gyroscope

Description. The free gyroscope can be applied advantageously as a single degree of freedom device in some applications. Under Two Degrees of Freedom Gyroscope, the following equations were given for the response of the elementary two degree of freedom gyroscope (Fig. 3):

$$\theta_2 = \frac{L_3}{Hs}\left(\frac{1}{\dfrac{I_2 I_3}{H^2}s^2 + 1}\right),$$

$$\theta_3 = \frac{I_2}{H^2}\left(L_3 \frac{1}{\dfrac{I_2 I_3}{H^2}s^2 + 1}\right).$$

These equations indicate that if a torque L_3 is applied, the response on the corresponding axis (3–3) of the gyroscope is oscillatory in character; detailed analysis of practical devices reveals that the actual motion is usually of negligible amplitude. The chief response is the familiar precession on the 2–2 axis.

Use as One-Axis Reference Device. If the free gyroscope is used as a one-axis reference device, no important errors result from torques applied on the reference axis, provided these torques do not cause the precession on the remaining axis to result in an excessive angle. Torques applied on the 2–2 axis will cause a corresponding precession about the 3–3 axis, and the important consideration is thus the reduction of unwanted torques on the 2–2 axis.

When applying a gyroscope in this manner it is common practice to use the control loop indicated in Fig. 6. In this figure the output of a

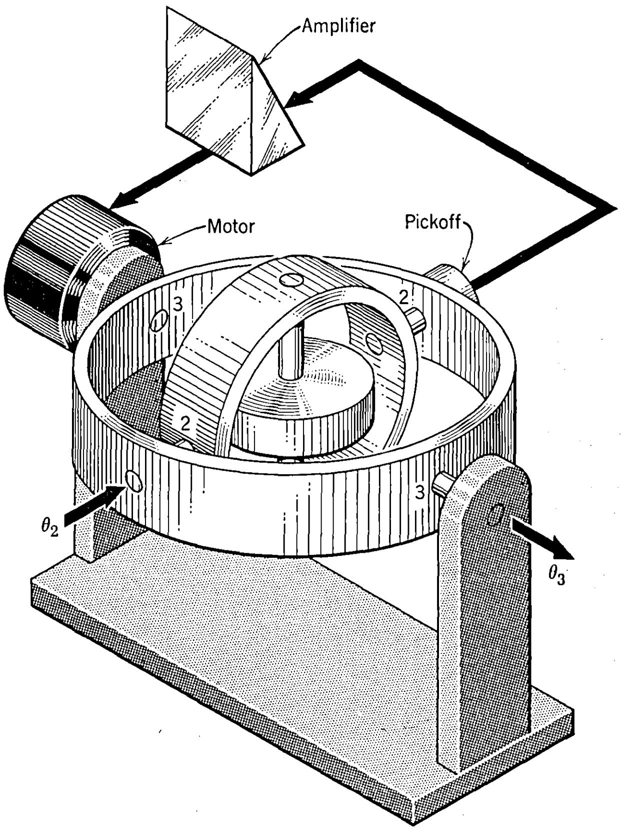

FIG. 6. Torque-summing gyroscope.

pickoff on the 2–2 axis is used to control power to a motor on the 3–3 axis. This loop serves to prevent excessive values of θ_2 even in the presence of a sustained torque on the 3–3 axis; only moderate gain is required in this control loop.

As shown in Fig. 6, no damping is present in the system and oscillation tends to occur at a frequency near the nutation frequency of the gyroscope. Damping can occur from inherent losses, from an intentionally included value of B_2 or B_3, or from appropriate networks in the control amplifier.

The Gyrotron Vibratory Gyroscope

Description. Although not a gyroscope according to the usual definition, the Gyrotron displays the characteristics of a rate gyroscope and can be used for the same purpose.

The Gyrotron vibratory gyroscope is basically a tuning fork kept in continuous, constant-amplitude vibration by a suitable driving system consisting of a pickoff, an amplifier, and a driving coil. Because the oscillatory motion of the fork tines is essentially radial (as referred to the axis of symmetry), the moment of inertia of the fork changes with time in phase with the tine motion. If an angular velocity ω is imposed about the axis of the fork, then a torque will be required about the base which is equal to the rate of change of angular momentum, i.e.,

$$L = \frac{dH}{dt} = I\frac{d\omega}{dt} + \omega\frac{dI}{dt}.$$

Because of the periodic variation of I (essentially sinusoidal) at the vibration frequency, a periodic variation in L is established at the same frequency. This can be detected by a suitable pickoff, resulting in a voltage which is essentially proportional to the applied angular velocity.

In the actually developed instrument the pickoff means is such that an appreciable time delay appears in the signal (the order of 5 msec). This time delay can be decreased at the expense of sensitivity.

Characteristics. The good features of the Gyrotron vibratory gyroscope include absence of wearing parts (such as the spin bearings in a gyroscope), a very wide range in terms of the ratio of maximum to minimum measurable rates, and good resistance to shock. In applications requiring a very good rate measuring instrument, the Gyrotron vibratory gyroscope is functionally superior to the spring-restrained rate gyroscope.

The undesirable characteristics of the Gyrotron vibratory gyroscope includes a very low-level output (requiring complex electronic equipment) and a relatively high level of drift. The drift (in the order of 50 degrees per hour) is very high compared with that of the best gyroscopes, but it is very low compared to the usual spring restrained rate gyroscope.

Stable Platform

Description. For many control purposes (particularly where control in three dimensions is involved) geometrical considerations make it highly desirable that a reference element be available which has zero (or at least very low or accurately controlled) angular motions about three mutually perpendicular axes. For such purposes a platform may be used which is supported to have three degrees of rotational freedom and is servo controlled in response to signals from two or more gyroscopes which are mounted on the platform. Figure 7 illustrates, in principle, such a platform.

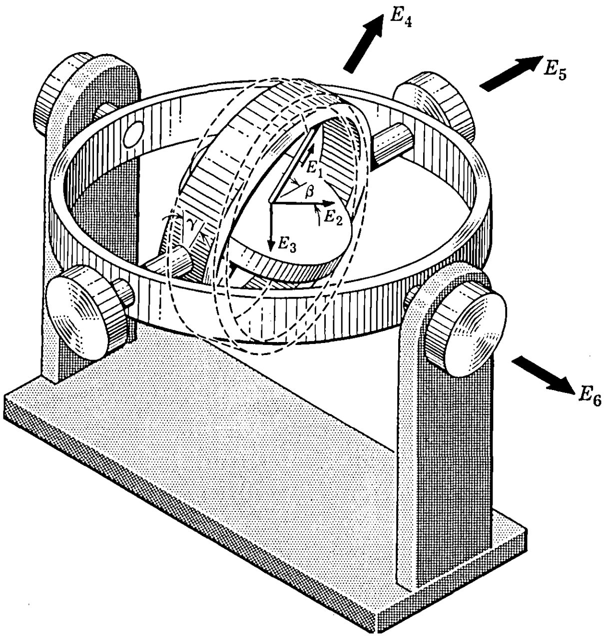

FIG. 7. Stable platform error conversion problem.

Signal Sources for the Platform. Any of the gyroscope types may be used as signal sources. For instance, two free gyroscopes may be used, which have their spin axes mutually perpendicular and hence become sources of error signals for motions about three mutually perpendicular axes in the stable platform. Three gyroscopes of any of the remaining types may be mounted on the platform instead of the two free gyroscopes and, provided their input axes are mutually perpendicular (usually mutually perpendicular, although any relative orientation may be used provided none of the input axes are coaxial), error signals will be provided for rotations about each of the platform axes.

When two free gyroscopes are used, redundant information will be available concerning platform rotations about one of the axes. This redundant error information may be used either to control a torque on one of the gyroscopes to keep it slaved (on one axis) to the stable platform, or it may be used in a more complex way to average the indications of the two gyroscopes and, hence, to improve to some degree the probable drift. If it is desired to accomplish the latter, then the sum of the error signals (redundant signals) is used for the control of that platform axis, and the difference is used to control torques on both gyroscopes. By this means the gyroscopes are, in essence, slaved to each other, and the resultant drift (about the redundant axis) is the mean of the drifts of the individual gyroscopes on that axis. A word of caution should be given to the effect that depending on the orientation of the gyroscopes, their relative quality and various other considerations, the average drift may not be lower than that of the better gyroscope alone and, hence, this may not be a useful procedure.

When three single degrees of freedom gyroscopes are used, no redundant information exists.

Servo Error Conversion. Figure 7 illustrates the use of gimbals to support the stable platform and to provide means for reducing the servo action to three single dimensional operations. If the rotations about the various gimbal axes are small, each of the three servos may be considered as being associated with one of the gyroscope input axes, and the control is straightforward. When the angles on these axes become large, however, means must be provided to convert the error information associated with the various platform axes to error information referred to the servo axes.

Based on the notation indicated in Fig. 7, the following equations are suitable for this error conversion. It is common practice to use electromagnetic resolvers to instrument these equations; the degree of accuracy required in the solution depends upon the servo characteristics and upon the magnitudes of the various angles. No general rule can be stated for determining the accuracy requirements, and the designer must decide on the basis of the individual requirements. It is usually true that errors of 5 to 10% are not objectionable.

$$E_4 = E_1,$$

$$E_5 = E_2 \cos \beta - E_3 \sin \beta,$$

$$E_6 = (E_2 \sin \beta + E_3 \cos \beta) \cos \gamma + E_1 \sin \gamma.$$

Study of Fig. 7 and the above equations shows that the loop gain is variable for the servo on the outermost axis (as a function of γ) and that if γ becomes equal to 90° the outermost axis becomes coaxial with the innermost axis and one degree of freedom is lost. In most applications it is possible to orient the gimbal system to prevent this angle from becoming excessive (for instance, in heavy aircraft the pitch angle is usually kept below about 15°); in cases where this cannot be done an additional gimbal may be used which is so controlled as to keep the angle γ near zero. This may normally be done quite roughly with adequate performance.

4. DESIGN CHARACTERISTICS

Gyroscope Spin Drives

Characteristics. Several characteristics should be considered in choosing a spin drive for a gyroscope. The following are among the more important:

1. *Mechanical Stability.* The design should be such that mass shifts are minimized.
2. *Heating.* Little heat can be removed from a gyroscope without use of means which may apply unwanted torques. Furthermore, in highly accurate gyroscopes only small temperature gradients are permissible within the device. For these reasons the heat produced by the drive should be kept low and as uniform as possible.
3. *Introduction of Power.* Because unwanted torques must be kept low, the drive should be such as to permit introduction of power to the rotor assembly by means which result in a minimum of torque.
4. *Weight.* For most purposes it is desirable that the ratio of wheel momentum to weight should be as high as possible. The drive should be such as to introduce a minimum of weight and to permit a large radius of gyration in the wheel.
5. *Constancy of Speed.* Constant speed is required in most rate-measuring applications.

The Air Turbine Drive. The air turbine once was almost universally used in aircraft instruments and torpedo controls but it is now becoming uncommon. When a suitable air supply is available, when the means of suspension allows air to be carried to the turbine, and when great accuracy of angular velocity is not required, then the air turbine offers certain advantages. These advantages include good momentum-to-weight ratio, relative simplicity, and absence of heat sources on the suspended element. In torpedoes the gyroscope is commonly brought quickly to speed by a turbine using high-pressure air and then is allowed to coast during the run.

The Oil Turbine Drive. The use of the oil turbine drive is so unusual as to require only brief comment. Its use implies many of the characteristics of the air turbine but introduces certain disadvantages because of oil leakage. Aside from the obvious disadvantages of leakage, the danger of unbalancing the suspended assembly as a result of oil drops is also involved.

The Spring Motor Drive. The spring motor is also uncommon but has been used where the operating time is short enough to allow the use of a coasting wheel. When this condition is fulfilled, it is possible to use a prewound spring motor which after bringing the wheel quickly to its operating speed is decoupled.

Electric Motor Drives. The great predominance of modern gyroscopes are driven by various types of electric motors. Even among electric motor drives a choice of type exists. Those which have been used include d-c motors, squirrel cage motors (both two-phase and three-phase) and hysteresis motors (again, both two- and three-phase). The various electric motor drives include:

1. *D-C Motor.* D-c motors should be considered only where a-c power is not available, and only for relatively low-accuracy applications of the gyroscope. The poor characteristics of the d-c motor for gyroscope use result from brush wear and loose parts inherent in the use of brushes (both are sources of balance change) and from the difficulties of maintaining satisfactory dynamic balance in a wound rotor. There is also difficulty in maintaining a highly accurate angular velocity.

2. *Squirrel Cage Motor.* The squirrel cage motor is the most commonly used type for gyroscope drives. It is relatively efficient (desirable to reduce heating) and can be designed to have a wide variety of characteristics. When operated from a closely controlled frequency source it will maintain reasonably constant angular momentum. Because of the high inertial load offered by the gyroscope wheel, special precautions must be taken in the design of the squirrel cage motor in order to limit the power input during starting; if this is not done, overheating will occur during starting.

High angular velocity is usually desirable in a gyroscope and this requires high-frequency operation of a squirrel cage motor. The 400-cps supply common in aircraft is satisfactory for most purposes, although higher frequencies are sometimes used. A two-pole squirrel cage moter operated on 400 cps will run at somewhat less than 24,000 rpm, a suitable speed for many purposes. Because bearing wear and input power both rise rapidly with speed, it is frequently desirable to use a four-pole motor which will operate at somewhat less than 12,000 rpm.

In order to maximize the radius of gyration of the wheel, it is common in gyroscope motors to invert the usual motor design; that is, the stator is commonly placed inside the rotor.

3. *Hysteresis Motor.* The hysteresis motor is very similar to the squirrel cage motor so that most of the previous comments concerning the squirrel cage types are applicable. Because the hysteresis motor is synchronous, wheel-speed accuracy is limited only by the accuracy of the supply frequency; for this reason this type of motor has become common where the greatest possible speed accuracy is required. The efficiency of

the hysteresis motor is somewhat less than that of the squirrel cage motor.

Practical Considerations of Electric Motor Drives. When electric motor drives are used, it is frequently desirable to minimize the required power by operating the wheel in helium or hydrogen at reduced pressure. This requires hermetic sealing but by reducing operating temperature can greatly increase life and reduce drift.

If angular displacements can be limited to a few degrees, then power for an electric motor drive can be introduced through flexible leads. Slip rings or point contacts at the center of rotation of the gimbal axes are used when large angular displacements must be accommodated.

Preloaded ball bearings are usually used for the spin axis bearings of gyroscopes. The magnitude of the preload is a compromise among the conflicting requirements of rigid positioning of the axis, low friction torque, and long operating life. Lubrication is usually permanent and may be a grease film, oil supplied by a saturated porous ball retainer or oil supplied by a wick. A minimum amount of lubricant must be used if it is not to be a potential source of mass shift.

Gyroscope Suspensions

General. For present purposes a gyroscope suspension will be defined as the means provided for supporting the wheel, its driving mechanisms, and any casing or other parts associated with the wheel. The suspension must provide angular freedom, but it must at the same time restrain linear motions and define axes of rotation. A suspension is normally chosen as a compromise among the following desirable characteristics: (*a*) low disturbing torques, (*b*) simplicity, (*c*) ease of manufacture, and (*d*) ability to withstand shock and vibration.

The following are among the more important methods of suspension which have been used: (*a*) ball bearings, (*b*) flexural devices, (*c*) fluid bearings, and (*d*) flotation.

Ball-Bearing Suspension. A ball-bearing suspension is usually the simplest and most easily manufactured. Ball bearings are only moderately good from the points of view of disturbing torques and ability to withstand shock and vibration. The latter statement should be qualified somewhat since the use of large enough bearings can result in excellent shock and vibration resistance but at the cost of frequently prohibitive torque levels. Ball bearings (simply applied) are generally useful for gyroscopes in the 10° per hour and up class, although they have been used in applications where drift requirements were in the region of 2° per hour.

Addendum to "Ball-Bearing Suspension." Great improvements in ball-bearing performance for gyro suspension can be achieved if provisions are made to keep the bearings always in motion. This causes their friction to be greatly reduced as compared to static friction. One approach to this is a moderate frequency angular oscillation of the bearings, induced by a suitable drive mechanism; this has been found to reduce friction levels by a factor of about three.

The greatest possible improvement is achieved by the Rotorace® (trademark of Sperry Gyroscope Co.) principle in which the two bearings of a suspension pair are rotated continuously in opposite directions for a short period of time and then both simultaneously reversed and driven in their new directions for a like period of time. This approach is found to reduce all bearing uncertainty torques by about one order of magnitude.

Flexural Suspension. Flexural suspensions usually take the form of crossed cantilever springs (see Fig. 8) or of wires in torsion. When used

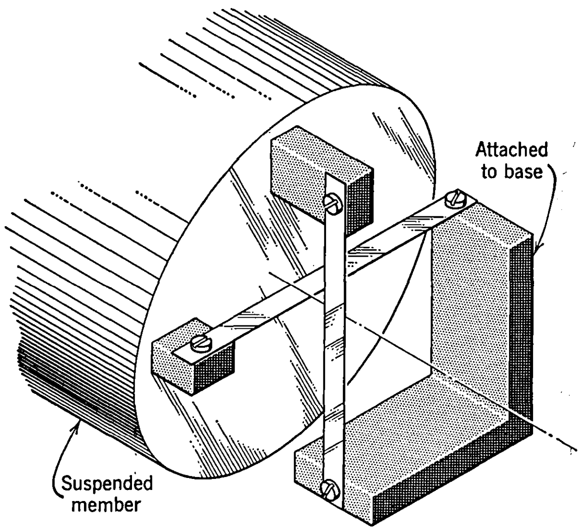

FIG. 8. Grossed spring suspension.

alone the stiffness is usually great enough to make them suitable only for rate gyroscope applications; in this service the suspension springs usually serve also as the restraining springs.

Fluid Bearing Suspension. For low torque, fluid bearings depend upon the maintenance of perfect lubrication which prevents all metal-to-metal contact. Because rotational velocities are necessarily quite low, a pump must be provided to produce a fluid flow; without such flow perfect lubrication is not possible. Such bearings may take a wide variety of forms and may use either a liquid or a gas for the lubricant.

Magnetic Suspension. Extremely low uncertainty torques may be achieved by the use of magnetic suspensions. In such suspensions the magnetic flux in a gap between a rotor element and a stator element (several elements are required to control the several degrees of freedom) is made to vary in accordance with the gap. If the flux density is made to increase with increasing gap, a stable suspension results.

Very good geometry is required if low uncertainty torques are to be achieved.

Electrostatic Suspension. Successful electrostatic suspensions have been applied which operate very similarly to the magnetic suspension just described but which depend on electrostatic attraction rather than magnetic. Relatively small forces can be achieved per unit area because of limitations in the breakdown potentials of suitable dielectrics.

Flotation Suspension. Flotation alone is not a complete gyroscope suspension because it can only support the weight and cannot define rotation axes or restrain linear motions. For this reason it is used in combination with another method. Such an application of flotation can reduce the weight carried by the suspension to nearly the vanishing point and hence can greatly reduce the suspension problem. This permits the use of jeweled bearings.

Flotation is equally effective against gravitation and against acceleration, with the result that it imparts good shock and vibration resistance.

It is impossible to achieve flotation without some damping of angular rotations; for this reason the dynamic response equations will always include the terms (refer to Table 2) containing damping coefficients. Advantage is taken of this inherent damping in the rate integrating form of floated gyroscope.

Pickoffs for Gyroscopes

General. Most control applications of gyroscopes involve the use of some type of pickoff (sometimes termed signal generator) for detecting deviations of the gyroscope frame from its normal angular position with respect to the spin axis or for transmitting to a remote location the actual relationship.

Fluid Valves. A great many types of pickoffs have been used although they may be classified into two groups as fluid valves or as electrical pickoffs. Fluid valves (pneumatic or hydraulic) were once widely used in aircraft automatic pilots and similar applications where pneumatic or hydraulic servos were used. Their use is now uncommon.

Electrical pickoffs for gyroscopes have included virtually all types which have been used for any other purpose, and because of their great variety, they will not be listed here.

Characteristics of Electrical Pickoffs. The requirements normally placed on electrical pickoffs for gyroscope use may be listed (more or less in order of importance) as follows: (*a*) low torque, (*b*) good resolution, (*c*) good linearity, and (*d*) low heating effect.

Variable Reluctance Pickoff. The commonest type of pickoff used is the variable reluctance (moving iron) device such as the *E transformer* or *microsyn.* Moving coil types have also been used even though they require additional electrical connections across the suspension bearings. In certain applications synchros may be useful for direct transmission of the angle between the gyroscope and its base.

Potentiometer Pickoff. Potentiometers have been widely used as pickoffs for gyroscopes which have relatively crude drift requirements (the order of 3° per minute) and where, for system reasons, a d-c output is desired. Such potentiometers are usually of a composition type for good resolution. Because the wiper pressure used must be a compromise between a very low value to minimize friction and a relatively high value to assure reliable contact (especially under vibration), the user should be careful to evaluate the applicability of the potentiometer pickoff to his particular needs.

Torque Generators for Gyroscope Use

General. Virtually all uses of gyroscopes require that means be available for the controlled application of torques. A gyroscope can be rotated in space by the application of torques and means for applying such torques must be provided if advantage is to be taken of this property. (Refer to Special Considerations in Gyroscope Applications, Sect. 5.) The spring restrained rate gyroscope does not require a means other than the restraining spring for the application of torque.

Many of the older designs of aircraft instruments used the reaction of air jets for the application of torques, but most modern designs include some type of electrical torque generator. Any type of electrical motor suitable for stalled operation can be used for a gyroscope torque generator but the actual choice will depend upon system consideration and upon the degree of refinement required in the torque application.

Characteristics. Among the factors which should be considered in the choice of a type of electrical torque generator are the following.

1. *Stability of Torque Zero.* The stability of torque zero includes such factors as hysteresis and effects of mechanical shifts (small air gap changes, etc.).

2. *Linearity.* Linearity is of secondary importance if the torque generator is to be used only for initial alignment, but it can assume great importance in uses where the relationship between electric input and precession rate must be known to high accuracy.

3. *Stability of Scale Value.* The comments in the above paragraph on linearity also apply.

4. *Range of Torque Requirements.* The torque generator chosen should be capable of covering the range of torque requirements of the application.

5. *Electrical Connections.* Moving coil types of torque generators will require electrical connections which cross the suspension axes; such connections can be a source of undesirable torques or of unreliability.

6. *Heating.* Minimum heat must be maintained.

Types of Torque Generators. The types of electrical torque generators which have been used on gyroscopes include the following.

1. *Induction Motors.* The induction motor can be quite satisfactory for moderate accuracy applications. Its major drawbacks include a need for relatively large power inputs (particularly at high frequency) when large torques are required and the need for the exact control of a-c voltages when good accuracy is needed.

2. *Moving Iron Devices* (Variable Reluctance). The moving iron type of torque generator is normally the most efficient and can have good linearity. As an a-c device it is limited by the need for accurate control of a-c voltages, and as a d-c device it suffers from hysteresis effects.

3. *Moving Coil Devices* (D'Arsonval). The moving coil torque generator is the best choice among available types when highest accuracy is required. This type usually uses a permanent magnet field which may include shunts for temperature compensation. In this form, accuracies of the order of 0.01% are possible.

Miscellaneous Design Factors

There are a number of gyroscope design factors which are not uniquely associated with any of the individual components (i.e., spin drive, suspension, pickoff, or torque generator) but which are vital to the best performance. Casual inspection of the design will seldom reveal how well these have been taken into account. Among the important ones are the following.

1. *Time Stability.* Time stability of the relationship between the locations of the center of gravity of the suspended assembly and the effective point of suspension.

2. *Ambient Temperature.* Effect of ambient temperature on drift performance. This is largely associated with degree of temperature control built into the gyroscope and the degree of symmetry.

3. *Ambient Temperature Gradients.* Effect of ambient temperature gradients on gyroscope drift.

4. *Isoelastic Construction.* Degree to which isoelastic construction has been achieved. In an ideal gyroscope the structure of the suspended assembly should be so designed that its spring rate is the same in all directions; if this is done, an applied linear acceleration will cause motion of

the center of gravity directly along the acceleration vector and no torque results. Failure to achieve isoelastic construction will result in drift due to accelerations, including vibratory accelerations.

5. GYROSCOPE APPLICATIONS

Vertical Gyroscope

General. There are many requirements in aircraft and on shipboard for a device that will give an accurate indication of the vertical. Under static conditions a simple pendulum will give such an indication, but because of the great similarity between the force of gravity on a mass and the force required to accelerate the mass, the simple pendulum does not give a good indication of the vertical when attached to an accelerating base. For this reason a simple pendulum is seldom a satisfactory indicator of the vertical for use on either an aircraft or a ship.

Fortunately, if the average motion of the base is unaccelerated, then the average position of the simple pendulum is a reasonably good indication of the vertical (this neglects nonlinearities due to large excursions). A pendulum combined with a suitable averaging device (or low-pass filter) is therefore a useful device for the indication of the vertical on moving bases.

Averaging Vertical Gyroscope. Figure 9 illustrates one possible mechanization of this principle. In this mechanization the gyroscope is used both as a stable reference against which to measure pendulum motions and an integrator in the smoothing circuit. If the various components are linear, the precession rate of the gyroscope toward alignment with the pendulum is always proportional to the difference angle. This results in a simple exponential response as the gyroscope approaches alignment with a fixed pendulum position. The frequency response of the device is such that relatively high-frequency motions of the pendulum are attenuated, and relatively slow motions of the pendulum are followed.

If, for an angle δ between the pendulum and the gyroscope, there results a precession rate of the gyroscope toward alignment with the pendulum equal to δ/τ, then the time constant for the exponential response of the gyroscope is equal to τ. If the pendulum has a sinusoidal motion with an amplitude θs and an angular frequency ω (radians per second), then the motion of the gyroscope, θg, in response is given by the following:

$$\theta g = \frac{\theta s}{\sqrt{1 + (\omega\tau)^2}}.$$

Rotation-of-Vertical Error. If there is to be a rotation of the indicated vertical (due to earth rotation or to own motion over the earth)

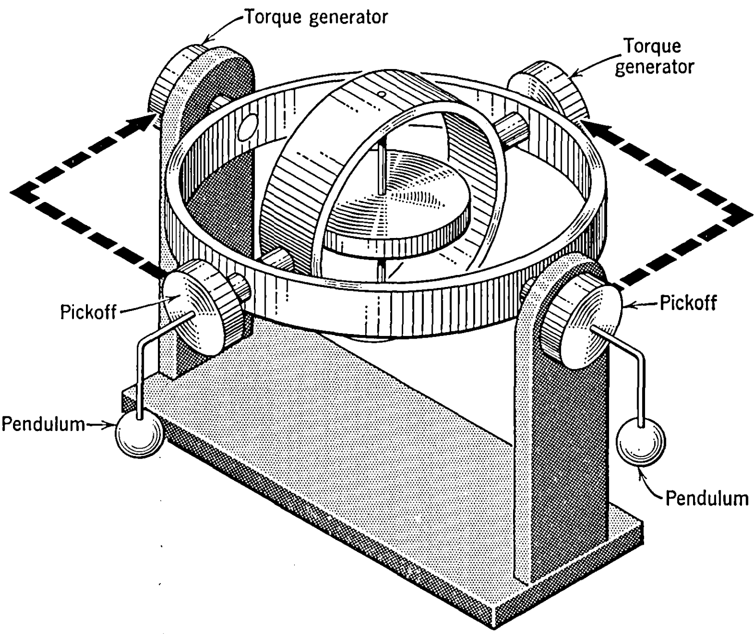

FIG. 9. Gyro vertical.

then a steady-state error will exist which is equal to the product of the angular velocity and the time constant τ. This error can be eliminated either by time integration of the signal representing displacement between pendulum and gyroscope or by the application of a computed correction. Such an error can be minimized by making τ small but at the expense of poor attenuation of pendulum swings. It should be noted that the rotation of the vertical due to earth rotation is from west to east and is of magnitude $\Omega_E \cos \lambda$ where Ω_E is the earth's angular velocity and λ is latitude.

Drift Error. If a steady gyroscope drift exists and its magnitude is ω_D, there will be a steady error in the indicated vertical equal to $\omega_D\tau$. This can be reduced by making τ small or by time integration of the signal representing deviations between pendulum and gyroscope. Actual gyroscopes will also exhibit random drift components and drift components resulting from vehicle accelerations. These drift components will also cause errors in the indicated vertical; accurate analysis of the resulting error requires good knowledge of the gyroscope characteristics.

Acceleration Error. A gyro vertical of the averaging type will have errors resulting from sustained accelerations; the error, in radians, is approximately equal to the acceleration imposed divided by the acceleration due to gravity. A common (and highly practical) way of minimizing such acceleration errors is to open the control loop any time a large acceleration occurs. This can be done on the basis of large pendulum swings or on the basis of an acceleration command (i.e., command for change of power setting or turn). If an independent measure of vehicle acceleration is available, direct acceleration corrections are possible. Such corrections should be applied either as a torque on the pendulum or as a correction to the measure of deviation between pendulum and gyroscope.

Coriolis Acceleration. One class of acceleration which affects a vertical gyroscope, but which is not completely familiar, is the Coriolis acceleration. This is an acceleration in space resulting from motions over the rotating earth (i.e., even when motions are unaccelerated with respect to the earth's surface, accelerations exist in space resulting from the interaction of earth rotation and vehicle velocity with respect to the earth). One component of Coriolis acceleration is that due to the horizontal component of linear velocity. It is always at right angles to the velocity vector and is directed toward the left in the northern hemisphere and toward the right in the southern hemisphere. The magnitude of this component of Coriolis acceleration is given by the following:

$$a = 2\Omega_E V \sin \lambda,$$

where Ω_E = earth's angular velocity (radians per second),
V = horizontal component of vehicle velocity (feet per second),
λ = vehicle latitude,
a = acceleration in feet per second^2.

A second component of Coriolis acceleration is that due to the vertical component of vehicle velocity. This acceleration is always directed toward the east when the velocity is upward; its magnitude is given by the following:

$$a = 2\Omega_E V \cos \lambda.$$

If desired, computed corrections can be made for the Coriolis accelerations.

Variations on the Simple Gyro Vertical. This discussion of the vertical gyroscope has been based on the particular instrumentation shown in Fig. 9. The single two degrees of freedom gyroscope can be replaced by two single degree of freedom gyroscopes (with appropriate servos to keep them operating in a nearly centered condition), and the pendulums can be replaced by accelerometers. The simple first order control system is also illustrative, and additional smoothing may be added by such means as damping the pendulums (or accelerometers) or the addition of a time delay in the external control amplifier. Departures from the simple first order circuit will result in higher order differential equations, and the response should be evaluated to avoid amplification of errors or actual instability.

It is usual practice to include an operating mode which allows for rapid initial alignment of the gyroscope to the vertical. Such a provision can be used at any time the vehicle motion is essentially unaccelerated.

Directional Gyroscope

General. In a directional gyroscope the inherent space rigidity of a gyroscope is used to maintain an azimuth orientation. It may be combined with a vertical gyroscope on a stable platform, or it may be a two degrees of freedom gyroscope similar to that shown in Fig. 10. In certain cases the gyroscope is used alone as a preset reference, whereas in others it is used as a smoothing device to average the indications of an unsteady reference. (This use is similar to that of the gyroscope discussed under Vertical Gyroscope in this section.) Devices are available which permit a

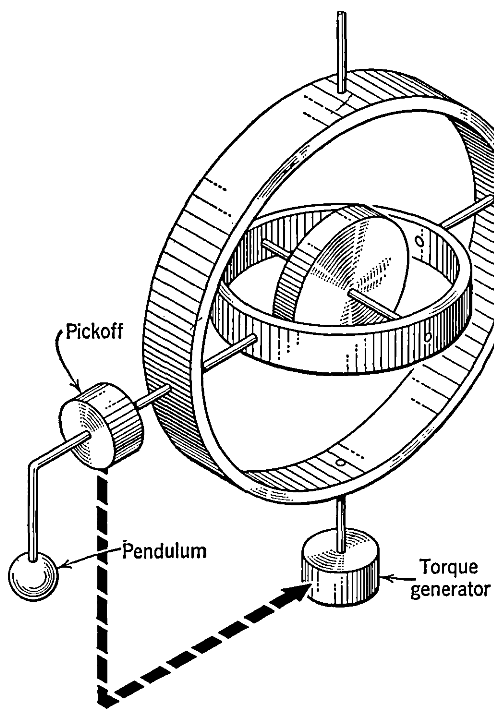

FIG. 10. Directional gyroscope.

gyroscope to be slaved to the direction of the horizontal component of the earth's magnetic field; the resultant mechanism combines the best features of a magnetic compass and of a gyroscope.

Basic Principles. In the mechanism of Fig. 10, a pendulum is shown which is used with a torque generator on the outer gimbal axis to keep the gyroscope spin axis horizontal. (This is similar to the operation of the vertical gyroscope discussed under Vertical Gyroscope in this section.) The gyroscope spin axis thus establishes an azimuth reference which lies in the horizontal plane; indications may be taken by direct mechanical means from the outer gimbal axis, or any suitable electrical transmission may be used.

In this device azimuth drift results only from torques about the inner gimbal axis, and moderate torques about the outer axis do not cause difficulty if they can be overcome by the torque generator on this axis.

Alternate Configuration. As an alternative to the use of a pendulum (or accelerometer) for the leveling of the gyroscope, a measure of the angle between the spin axis and the normal to the gimbal plane (i.e., rotation about the inner gimbal axis) is frequently used to control the torque applied to the outer gimbal axis. This is satisfactory if the average orientation of the vehicle is essentially level.

Auxiliary Reference. If it is desired to slave the gyroscope to an auxiliary reference, means must be included to measure the difference angle between the azimuth orientation of the gyroscope and that of the reference, and a torque generator must be included on the inner gimbal axis of the gyroscope.

North Indication. In many applications it is desired that a directional gyroscope indicate north. It should be remembered that the direction of north rotates in space because of earth rotation and vehicle motion over the earth. The component of this rotation caused by earth rotation is equal to $\Omega_E \sin \lambda$ and is counterclockwise (as seen from above) in the northern hemisphere. In this case Ω_E is the earth's rate of rotation and λ is the latitude.

The space rotation of the direction of north resulting from vehicle motion over the earth is equal to $\dot{L} \sin \lambda$, where $\dot{L}$ is rate of change of longitude (taken as positive for easterly motions). The direction of rotation is the same as that for the effect of earth rotation.

Both of the above rotation effects may be accounted for by means of computed corrections, or in the case of a slaved gyroscope, the slaving device may be used to supply suitable torque signals. In the latter case some standoff error may result.

Most directional gyroscope mechanizations include means for the rapid initial alignment of the gyroscope to the desired orientation.

Automatic Pilot

General. Automatic pilots are used in aircraft and ships to relieve the human pilot. In guided missiles and drone aircraft they are used to replace entirely the flight control functions of the human pilot. They may be simple devices intended only for the control of heading or more complex devices which control attitude about all three axes.

Basic Principles. Many aircraft automatic pilots make use of a vertical gyroscope (Vertical Gyroscope, this section) and a directional gyroscope (Directional Gyroscope, this section) with suitable signal pickoffs on the gimbal axes. The output signals from the pickoffs are proportional to angular deviations of the aircraft about the roll, pitch, and yaw axes. The aircraft control surfaces are moved by suitable servomechanisms in response to the angular deviation signals; the resultant aerodynamic forces cause the aircraft to rotate in such a direction as to reduce the angular deviation signals. The complete closed loop thus becomes quite complex and includes the gyroscope, the control-surface servomechanisms, and the entire airframe.

Some damping is available from the inherent airframe damping, but this is not sufficient to stabilize the control loop if high performance is desired. For this reason modern automatic pilots include suitable corrective networks in the control amplifiers or rate gyroscopes to generate damping signals.

Additional Requirements. In addition to the indicated design provisions needed for maintaining a fixed attitude, many additional requirements frequently must be met. These include the following.

1. Means may be required to permit large attitude changes under control of the automatic pilot.

2. Means may be required to coordinate turns automatically. This requires that the following relationship exist between bank angle and rate of turn

$$\text{Bank angle} \approx \tan^{-1}\left(\frac{\text{Rate of turn} \times \text{True air speed}}{\text{Acceleration due to gravity}}\right).$$

3. Means may be required to change parameters as flight conditions change to account for changes in aerodynamic response.

4. Means may be required to allow signals from auxiliary sources (altimeter, radio glide path receiver, etc.) to be superimposed on the automatic pilot control.

5. Means must usually be provided to reduce attitude error signals to zero before the automatic pilot is engaged. Failure to do this can result in dangerous (or at least uncomfortable) transients in the aircraft attitude.

Variations in Design. There are, of course, almost unlimited variations possible in the automatic pilot, but all are characterized by the use of suitable gyroscopes to measure attitude deviations of the vehicle and to control surfaces to restore the vehicle to the desired attitude. The possible variations include the following.

1. Elimination of monitoring controls on gyroscopes (i.e., pendulum for control of vertical gyroscope). This is satisfactory if relatively short time attitude control is satisfactory.
2. Use of single degree of freedom gyroscopes attached to vehicle frame instead of two degree of freedom gyroscopes previously indicated.

The complete dynamic analysis of an automatic pilot-aircraft combination is very complex and requires good knowledge of the aerodynamic characteristics of the particular airframe.

Schuler Pendulum

General. A method of determining the vertical in a moving vehicle was discussed which consists basically in averaging the indications of a simple pendulum. (See Vertical Gyroscopes in this section.) When the greatest possible accuracy is needed this method is unsuitable because of its sensitivity to sustained accelerations. One solution to the problem of the accurate determination of the vertical in the presence of accelerations is the use of what is variously known as "the Schuler pendulum," "the 84-minute pendulum," and "the earth's radius pendulum." This device was originally proposed by Schuler in 1923.

Basic Principles. Operation of the Schuler pendulum is philosophically different from that of the averaging vertical indicator. The basic difference is that the Schuler pendulum uses the measured vehicle acceleration to establish an appropriate angular acceleration of the indicated vertical (required because of the generally spherical shape of the earth), whereas the averaging vertical indicator attempts to ignore the vehicle accelerations. The Schuler pendulum principle may be readily understood from an inspection of Fig. 11.

Figure 11 shows an accelerometer held level at the surface of the earth. If the output of this accelerometer is integrated once with respect to time, the result is velocity. A second integration results in distance moved along the surface of the earth. Because of the curvature of the earth (assumed spherical, for the moment), when the accelerometer has been carried through a distance S along the surface, it must be rotated through an angle equal to S/R (in radians). A reference must be provided with respect to which the angle S/R can be measured, and a gyroscope is used for this purpose. If it is recalled that a gyroscope will integrate torques, then a suitable torque generator makes possible the second integration by the gyroscope. This variation is shown in Fig. 12.

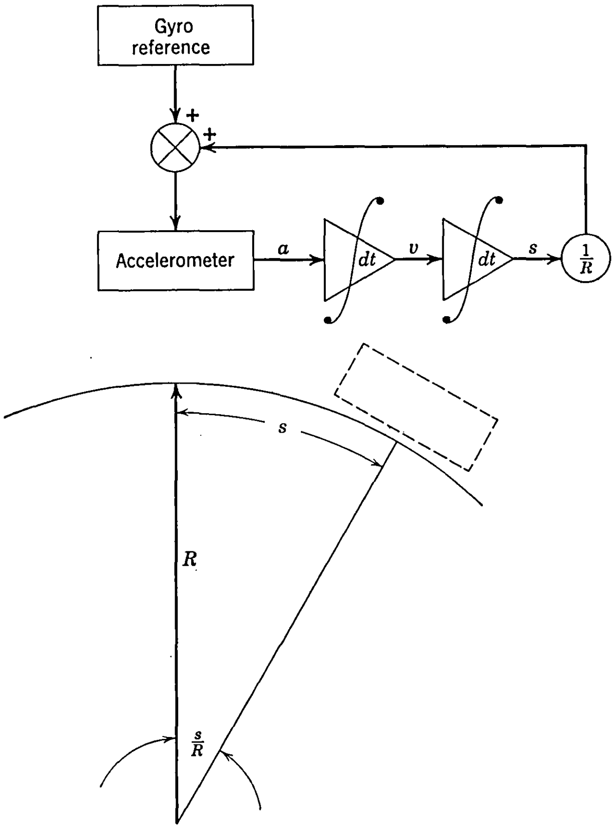

FIG. 11. Basic Schuler pendulum.

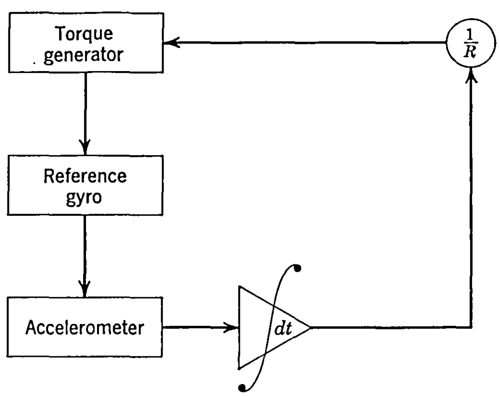

FIG. 12. Schuler pendulum using gyro as integrator.

Independence from Vehicle Acceleration. A device built according to the above principle will retain an accurate indication of the vertical (once established) regardless of vehicle accelerations. If any departure from the true vertical occurs, the accelerometer senses a component of gravity which it interprets as a vehicle acceleration; as a result, an angular acceleration of the indicated vertical is produced which, fortunately, is toward the true vertical. The net result is an undamped oscillation about the true vertical having a period $T = 2\pi\sqrt{R/g} \approx 84$ min. The name *earth's radius pendulum* derives from the fact that the above expression is that for the period of a simple pendulum having a length equal to the radius of the earth, and the name *84-minute pendulum* is of obvious derivation.

Damping. It is possible to introduce damping into the Schuler pendulum loop by any means which create a phase lead (one possible method is to bypass some signal around one of the two integrators). Generally speaking, any simple method of producing damping results in forced errors due to accelerations. Given an auxiliary measure of vehicle velocity it is possible to compare this measure of velocity with that resulting from the integration of accelerometer output and to use the difference for damping purposes; this eliminates the vehicle acceleration errors but may introduce errors due to errors in the auxiliary velocity measure. Figure 13 shows one possible loop utilizing this principle.

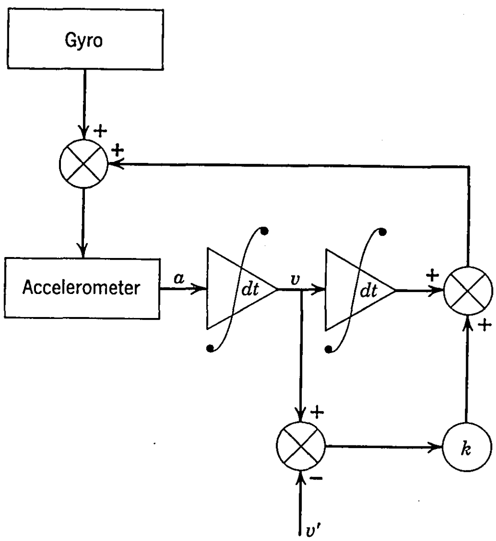

FIG. 13. Using auxiliary velocity measure for Schuler pendulum damping.

Practical Considerations. In actual operation it is necessary to use two Schuler pendulums which are physically orthogonal and to be careful that proper corrections are made for any acceleration errors which result from rotations about the vertical axis and for the effects of earth rotation.

Accurate operation of a Schuler pendulum requires that an accurate relationship be established between the measured linear acceleration of the vehicle and the generated angular acceleration of the indicated vertical. In the absence of damping, an error step of one knot of velocity introduced into the loop will produce an oscillation of 0.224 min of arc (the first error maximum occurs 21 min after the appearance of the velocity error). Because vehicle maneuvers are likely to be of very short duration compared with the period of the Schuler pendulum, a reasonable approximation to the oscillation produced by a maneuver, when the loop gain is not exactly correct, can be obtained for the undamped device from the relation that the oscillation (in minutes of arc) ≈ 0.224 ($0.01 \times$ % error in loop gain) times the velocity change in knots. When damping is present or when vehicle accelerations are expected to occur over long periods of time, a more complete analysis should be made of the actual loop used.

For the greatest possible accuracy, changes should be made in the loop gain of the Schuler pendulum to correct for altitude changes and for the oblateness of the earth.

Inertial Navigation

General. The general field of inertial navigation is an increasingly important application of the gyroscope. A complete discussion of the subject is beyond the scope of this chapter, but its basic principles are within the scope.

Basic Principles. Inertial navigation depends for its operation on one of two processes: (*a*) the measurement and double integration (with respect to time) of vehicle accelerations in a coordinate system established by suitable gyroscopes; (*b*) the measurement of the orientation of an indicated vertical (usually maintained by a Schuler pendulum as discussed under Schuler Pendulum above) in a coordinate system established by suitable gyroscopes. The first of these processes is usually used in vehicles which are essentially in free fall and the second in more conventional vehicles.

Inertial navigation requires gyroscopes of a very high accuracy since in the second type of system discussed above, a one minute of arc drift of the reference gyroscopes causes a one nautical mile error on the surface of the earth (this is a direct consequence of the definition of the nautical mile).

Gyrocompass

General. The gyrocompass was one of the earliest practical applications of the gyroscope, and the first U.S. instrument was installed on a Navy ship in 1911. The importance of the gyrocompass lies in its ability to indicate true north without the errors to which the magnetic compass is subject.

Basic Principles. The gyrocompass acts to align itself with the horizontal component of the earth's angular velocity. Since this lies in a precisely north-south direction, it provides an accurate north reference.

Basically the commonest form of gyrocompass consists of a gyroscope suspended in such a fashion as to reduce to an absolute minimum all disturbing torques about a vertical axis. This gyroscope has its spin axis lying in a horizontal plane. When the spin axis points north, the horizontal component of the earth's angular velocity lies along the spin axis and does not produce any disturbance. If the spin axis does not point north, then a component of the horizontal component of the earth's angular velocity appears along the horizontal axis which is normal to the spin axis. Because of this component of angular velocity, the gyroscope spin axis departs from the level condition at this same rate.

The gyroscope has attached to it a mechanism (in the simplest case, a pendulum) which, under the influence of gravity, produces (about the horizontal axis normal to the spin axis) a torque proportional to the departure of the spin axis from the horizontal. This torque causes precession about the vertical axis toward the north.

Damping. The form of device discussed above is inherently oscillatory and is normally artificially damped. It is usually adjusted to result in a period of about 84 min.

A gyrocompass is unable to distinguish between the angular velocity resulting from earth rotation and that resulting from vehicle velocity about the earth. For this reason it aligns itself with the vector sum of these angular velocities. A correction is thus required which is a function of vehicle speed, heading, and latitude. This correction is usually made automatically by the instrument itself. A gyrocompass having an 84-min period uses the accelerations accompanying vehicle maneuvers to precess the gyroscope instantaneously into alignment with the changing total angular velocity vector; for this reason 84 min is the optimum period for the simple compass.

Accuracy. There have been many variations on the gyrocompass but all operate on principles which are close to that described. The inherent accuracy of a gyrocompass depends largely on the success achieved in reducing disturbing torques about the vertical axis of the gyroscope. A steady torque about this axis sufficient to cause a precession rate equal to ω_D will cause a north pointing error given by the following:

$$\delta \approx \frac{\omega_D}{\Omega \cos \lambda} \text{ radians,}$$

where Ω is the angular velocity of the earth and λ is the latitude. A steady torque sufficient to produce 0.1° per hour precession rate will, therefore, result in an error of about $\frac{3}{4}$° at a latitude of 60°.

Fire Control

General. Accurate control of gunfire against moving targets requires that the position of the target at the time of projectile impact be predicted. Because target locating methods usually involve angular measurements, target velocity computations usually involve angular rate measurements. A gyroscope is a highly desirable reference against which to make these angular rate measurements, especially on a moving vehicle, the frame of which is not highly stable.

Basic Configurations. There are two common approaches to this application: (*a*) the use of a pair of rate gyroscopes connected to the sighting devices to give as outputs the two components of angular velocity and (*b*) the use of gyroscopes in the positional stabilization of the line of sight. Torques are applied to the stabilizing gyroscopes to cause the sighting device to stay on target. A measure of the torque applied to the gyroscope becomes a measure of the angular velocity of the line of sight.

Because this application is a rate measurement operation, it is necessary that gyroscope wheel speed be accurately controlled and that the torque application method be accurately calibrated.

Other Applications

There are many applications of the gyroscope in addition to those discussed in previous paragraphs. A partial list of these follows.

1. *Aircraft Stability Augmentation.* Many high performance aircraft are nearly impossible to fly manually because of low aerodynamic damping. In such aircraft it is common practice to use rate gyroscopes and servo means to add damping deflections to the control surfaces.

2. *Ship Roll Stabilization.* To increase passenger comfort or to improve fire control accuracy, ships may be roll stabilized by use of gyroscopic equipment. The presently favored method is the use of control fins moved by servo drives as a result of signals from gyroscopes. In the past, stabilization has been accomplished by the application of the precession torques of large gyroscopes to the ship hull.

3. *Radar Antenna Stabilization.* On moving vehicles there is frequently incompatibility between the high servo loop gain required to reduce the effects of vehicle angular motions and the low servo loop gain required to filter radar noise. A gyroscope may be incorporated that gives the high attenuation of vehicle motions; the radar error signal may be used to precess the gyroscope under low gain.

Special Considerations in Gyroscope Application

General. Because each application of the gyroscope has its own special problems, there is no universal approach to proper application but certain points should be considered in selecting a gyroscope and method of application.

Warmup. Most gyroscopes are more stable after a warmup period than they are immediately after starting. The requirements of the application should be examined to determine whether such a period can be allowed or whether it will be necessary to choose a gyroscope capable of meeting the performance requirements immediately after starting. In the case of the very highest accuracy needs (such as inertial navigation) it may not be possible to obtain the required performance without a warmup period.

Drift Correction. Any gyroscope will have as a substantial part of its drift a rate which changes very slowly with time. If the time of intended use is short as compared with the time during which the drift rate changes, a drift correction (by the application of a torque, for instance) can be very helpful. A careful study should be made of the drift characteristics of the particular gyroscope to be used in order to optimize the observation period on which the correction is based and to determine the worth of such a correction in light of the actual application. This is an area which should be approached with caution to make certain that the factors for which corrections are made do not change in actual use. Illustratively, a correction made on the ground for torques due to mass unbalance would not be proper if the vehicle is later in free fall.

Gyroscope Orientation. It is frequently possible to improve the performance of a gyroscope by choice of an optimum orientation. Illustratively, single degree of freedom gyroscopes will not be subject to disturbing torques as a result of the interaction of mass unbalance and gravity if the suspension axes are vertical. Even with such an orientation, they will, of course, be disturbed by accelerations in the horizontal plane.

Effects of Vibration. Quite aside from the destructive effects of vibration there are performance-degrading effects of vibration on gyroscopes. Linear vibration along nonprinciple axes will interact with nonisoelasticity effects to cause drift. Angular vibrations along nonprinciple axes can interact with cross-coupling effects to cause drift. Unfortunately, only rough estimates of these effects are usually possible by theoretical means; these effects should be considered, however, and such provisions as are practical should be made to eliminate forcing vibrations. Testing of the completed system should be considered to determine whether these are serious effects.

General Ambient Conditions. Because of the extremely low torques required to cause significant drift of a gyroscope, virtually no ambient condition should be neglected as a possible source of difficulty. When highest performance is needed, it may prove necessary to control the temperature and pressure and also to maintain very low thermal gradients in the vicinity of a gyroscope. Magnetic fields may cause gyroscope drift.

6. GYROSCOPE TESTING

General

Performance Testing. This section is concerned entirely with the performance testing of gyroscopes and will not discuss relatively routine testing which would be required on any component to determine its ability to withstand various ambient conditions. Performance tests should be repeated after exposure to various ambients, even when no obvious damage has resulted.

General Categories. Gyroscope performance tests may be reduced to four general categories: (*a*) electrical noise, (*b*) random drift, (*c*) effect of operating conditions on drift, and (*d*) determination of response to inputs.

Tests should be run in the order indicated. Unless the random drift magnitude is known, it is very difficult to assess the reliability of measurements of the effect of operating conditions on drift.

Electrical Noise Testing

Procedure. Because there is nothing specialized in the procedure of making electrical noise measurements on gyroscopes, no discussion of measurement techniques will be undertaken; an indication of the tests which should be made will suffice.

Tests.

1. Radio noise tests will frequently be required, particularly if the gyroscope has a heater with on-off control.
2. Tests should be made of electrical noise in the pickoff output. For a-c pickoffs this will include quadrature and harmonic voltages, and for potentiometer pickoffs the test should include the effect of contact discontinuities as the wiper is moved.

3. Tests should also be made on the interaction with the pickoff of various other circuits in the gyroscope. It is not uncommon to find that the gyroscope spin motor, torque generator, and heater may produce pickoff outputs when excited. In some gyroscopes, significant pickoff outputs may be generated by wheel rotation even when the drive motor is unexcited.

Random Drift Testing

Purpose. The purpose of random drift tests is to determine the effects of various internal gyroscope imperfections which may cause a time-varying precession rate. These effects may include friction, mass shifts (interacting with gravity), coercion of power leads, and various other effects. Random drift tests on rate gyroscopes may be made simply by recording the output signal of the gyroscope over a period of time.

Equipment. The chief tool in the random drift testing of displacement gyroscopes is a turntable on which the gyroscope can be operated in, as nearly as possible, its normal mode of operation. This turntable should, further, be equipped with means for accurately determining its angular position.

Procedure. When a single degree of freedom gyroscope, such as the rate integrating type, is operated on the turntable, it should be mounted with its input axis accurately parallel to the turntable axis, and its output signal should be used to control a servo driving the turntable. Two degree of freedom gyroscopes may be tested on a single axis turntable by using one pickoff output to control the turntable and the other pickoff output (through a suitable amplifier) to apply a slaving signal to the proper gyroscope torque motor. One axis of the gyroscope thus controls the turntable, and the second is kept in a centered condition.

Drift testing reduces itself to the determination of the time-varying angle of the turntable when under control of the gyroscope. Adequate drift testing, however, requires that several matters be considered.

1. Gyroscope drift is statistical in nature and large amounts of data are required to reach any reasonable conclusions. A number of runs should usually be made on a particular unit and, if possible, a number of units should be tested.

2. Care must be taken to hold conditions as nearly constant as possible during random drift runs; temperature, pressure, voltage levels, and supply frequencies are included.

3. In a well-designed gyroscope, the predominant component of drift will result from very slowly changing torques, and consequently extrapolation of short time runs to longer times is dangerous. For this reason, drift testing is a time-consuming operation.

4. Most gyroscopes will exhibit constant bias rates superimposed on the random drift. The bias rate for a particular gyroscope will commonly be different for each warmup. For this reason a random drift test run must be continuous.

5. Random drift tests should be run with the gyroscope in various orientations with respect to the gravity field.

6. Earth rotation corrections must be made to drift data.

Presentation of Results. Gyroscope random drift tests results may be presented in a number of ways. When results are to be presented on only a single unit (or a representative sample of a design), the actual data may be shown on a curve of drift angle vs time, or of instantaneous drift rate vs time.

Because of the statistical nature of random drift data, it is usually desirable to present a large quantity of data in an easily assimilated form. These data may be the result of a large amount of testing of a single gyroscope or of the testing of a number of samples. Figure 14 illustrates

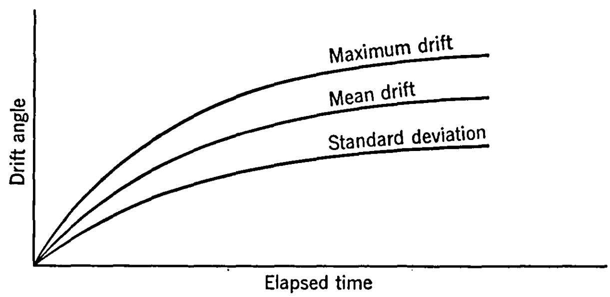

FIG. 14. Possible presentation of gyro drift data.

one form of data presentation which meets this need. This is a plot of the mean drift angle, the maximum drift angle, and the standard deviation of the data as functions of elapsed time from the start of the run. (Runs may be considered as starting at various arbitrary times within a longer run, thus giving a large group of numbers for one actual run.)

When it is desired to investigate the response of a system to the effects of gyroscope drift, it is very useful to have random drift data in the form of its power spectral density.

The presentation of random drift data may be varied by the inclusion or exclusion of the warmup period and by the inclusion or exclusion of a correction for any bias rate. The intended usage of the gyroscope as well as its characteristics will dictate the desirability of these variations.

Tests on Effect of Operating Conditions on Drift

Purpose. Because of the very low torque levels which can cause significant drift rates, almost all operating conditions may be considered as possible causes of drift. It is possible to lump all drifts into one category and to run drift tests under expected conditions of use; doing this will give the total drift, including the effects of the expected variations of conditions. It is highly desirable, however, to know the amount of contribution of a change of a particular condition to the drift of the gyroscope. With this information available an intelligent approach can be made to the control (where practical) of operating conditions.

Procedure. With the same turntable required for random drift tests, it is relatively easy to run tests in which the rate change is observed, as various excitation voltages and frequencies are changed. Part of the effect of a voltage or frequency change may be expected to be magnetic and result in an immediate rate change; part may result from thermal effects and may require a substantial period of observation.

The effects of ambient pressure, ambient temperature, air blasts, and nearby hot or cold surfaces may all be evaluated either by placing the entire turntable in a suitable test chamber or by placing a small test chamber on the turntable. The effects of thermal conditions should be observed over substantial periods of time.

As a generality, the effects of acceleration are the most difficult to determine, with the exception of those of a 1-g sustained acceleration. Higher sustained accelerations can be developed in the laboratory only by use of a centrifuge, and the rotational input of the centrifuge may be expected to mask any drifts caused by the acceleration. Testing under vibratory accelerations is also difficult because of the problem of measuring small angular motions in the presence of the vibration.

Tests. Despite the above difficulties there are a number of useful tests which can be made.

1. Test runs may be made in which the gyroscope rotates slowly through the gravity field while drift data are taken. This may be done by placing the turntable axis parallel to the earth's axis and allowing earth rotation to cause the relative rotation.

2. Test runs may be made on a centrifuge with the gyroscope nonrotating. In this case the gyroscope pickoff output is applied, through a suitable amplifier, to the torque generator to keep the instrument in a nulled condition. The torque generator current required is then taken as a measure of the disturbing torque.

3. Gyroscope drift under conditions of vibration may be best determined by observing the rotation optically. This permits the gyroscope under test to be vibrated while the measuring device remains steady;

observations may have to be intermittent, even with such an arrangement. Sufficient data should be collected on the effects of operating conditions to allow reasonable separation of uncertainties caused by random drift.

Determination of Response to Inputs

Purpose. The ideal gyroscope should have only two kinds of response to inputs. First, it should give an accurate and instantaneous indication of angular motions about either one or two axes (depending on the number of degrees of freedom). Secondly, it should respond instantaneously and accurately to precession command signals, again on either one or two axes. Further, there should be no interaction among the various axes. Gyroscope response measurements should be planned which demonstrate how well the actual instrument approaches the ideal.

Tests. The following tests should be made to determine the response of the gyroscope.

1. Tests should be made to determine over what range of angular velocities (about the input axis or axes) the output is a good measure of the input. If this range is very wide it may suffice to determine that the instrument actually covers the user's needs.

2. Tests should be made to determine whether the angular displacement resolution is low enough. This test will apply only to displacement types of gyroscopes and is particularly applicable to those having potentiometer pick-offs.

3. Tests should be made in which the input to the torque generator is accurately varied while the resultant precession rate is determined.

4. Tests should be made to determine what, if any, time delays are present. These may best be made by applying sinusoidally varying inputs (both angular and precession command) and observing the phase shift and magnitude of the output. An alternative to such testing is the use of the theoretical response equations of Table 2.

5. Tests should be made of cross-coupling effects. For instance, in a rate-integrating gyroscope, rotation of the instrument about the spin axis (after nulling the pickoff output) should result in no change in the output; any axis misalignment will cause such an output. Rotation of the rate-integrating gyroscope about its output axis should result in a change in pickoff output proportional to the angular velocity about the output axis. No change in output should occur which is proportional to the angle through which the instrument is rotated.

Cross-coupling effects will be the result either of imperfect axis alignment or of the instrument dynamics. The first class must be determined by test, while the second class may be determined either by test or by application of the Table 2 equations.

In testing to determine the response to inputs, care should be taken to hold the operating conditions constant (to the degree found necessary in the earlier testing), and sufficient data should be taken to allow separation of the random drift effects.

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