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THE COMPUTATION OF CERTAIN COMMUNICATION CHANNEL ERROR PROBABILITIES BY AN APPLICATION OF DIFFERENCE EQUATION METHODS

JULY 1966

S. Berkovits
E. L. Cohen

Prepared for

DEPUTY FOR COMMUNICATIONS SYSTEMS
ELECTRONIC SYSTEMS DIVISION
AIR FORCE SYSTEMS COMMAND
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L. G. Hanscom Field, Bedford, Massachusetts

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Project 7560
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ABSTRACT

A model for a channel is given. For this model, the recursive method is presented in order to calculate the probability of K symbol errors in a block of n m-bit symbols. The blocks can be interleaved or not.

REVIEW AND APPROVAL

This technical report has been reviewed and is approved.

Edgar A. Grabhorn

EDGAR A. GRABHORN, Lt. Colonel, USAF
Director of Communications Development
Deputy for Communications Systems

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SECTION I

THE MODEL

In estimating the performance of an error-correcting device on a specific communication channel, it is necessary to find a meaningful, yet tractable, mathematical model for that channel. An examination of data from real channels suggests that most channels pass through three distinct phases. The first phase, which nearly any error-correcting scheme can handle successfully, is that of long periods of practically error-free transmission. The second phase, which is the antithesis of the first phase in that no scheme can expect to correct it, is that of complete loss of signal for substantial periods of time. The third phase might be described as a generalized sputter or bursts of error bursts, and it is this phase, if it occurs frequently, for which error correctors should be designed. This third phase may sometimes be described by means of a two-state model. The three phases suggest a Markov process with four states, but such a process is mathematically unwiedly. However, when the three phase picture is reasonably correct and the sputter phase is accurately modeled for purposes of estimating coder performance, it is a simple matter to make corrections in such estimates for those periods when transmission is nearly error free or when the signal is lost.

The use of such a two-state model was suggested to us by some work by Gilbert [1]. Gilbert describes a model for binary error distributions in channels subject to noise bursts. Let $\{x_i\}$ be the error process with $x_i = 1$ for an error in the i^{th} demodulated bit, $x_i = 0$ for no error. Two states, designated G and B, of the channel are postulated such that at the i^{th} bit the state S_i is G or B, and the state S_{i+1} at bit i + 1 depends only on S_i. Thus, the state sequence $\{S_i\}$ is a simple Markov chain described by the two transition probabilities, P: G → B and p: B → G. We also use Gilbert's notation Q = 1 - P and q = 1 - p for the G → G and B → B transitions respectively. Let h,k denote respectively the probabilities of a correct bit in B and in G. Then $P\{x_i=1\} = 1 - h = h'$ if $S_i = B$ and $P\{x_i=1\} = 1 - k = k'$ if $S_i = G$.

Using this model with k = 1, Gilbert obtained a good fit to certain phone line error data. The statistic fitted was the probability of occurrence of zero (= error free) runs of length at least K. (The use of k < 1 has also been considered by Elliott [5].)

At MITRE, we have found sets of values for the model parameters P, p, h, k, which yield good fits to error data for several different types of communication media. One important statistic involved is the probability of specific error densities in various length blocks.

Given that we have p, P, h,k, [2,3,4], we present the recursive method used to calculate the probability of K symbol errors in a block of n symbols where each symbol consists of m bits.

SECTION II

RECURSIONS - PART I

In Appendix II of [2] (or Appendix B of [3] or [4]) we presented a brief outline of the recursive method used to calculate the probability of K symbol errors in a block of n symbols where each symbol consists of m bits. Now we present the outline in full. Since the last documentation, the technique has been extended to permit the n-symbol blocks to be interleaved or time spread. (To interleave s blocks means to transmit sequentially the first symbol of each of s blocks followed by the second symbol of each of those blocks, etc. Thus, a given m-bit symbol is transmitted s symbol times after the symbol preceding it in its block.)

Letting T and U represent either of the states G and B, we define

$$TOU(t) = P(x_1 = \cdots = x_t = 0 \text{ and } S_t = U | S_0 = T)$$

$$T1U(t) = P(\text{for some } i \leq t,\ x_i = 1 \text{ and } S_t = U | S_0 = T)$$

Then

$$GOG(1) = Qk \qquad GOB(1) = Ph$$

$$G1G(1) = Qk' \qquad G1B(1) = Ph'$$

$$BOB(1) = qh \qquad BOG(1) = pk$$

$$B1B(1) = qh' \qquad B1G(1) = pk'$$

and

$$GOG(t) = [GOB(t-1)\ p + GOG(t-1)\ Q]\ k$$

$$G1G(t) = [GOB(t-1)\ p + GOG(t-1)\ Q]\ k' + G1B(t-1)\ p + G1G(t-1)\ Q$$

$$G0B(t) = [G0B(t-1)\ q + G0G(t-1)\ P]\ h$$

$$G1B(t) = [G0B(t-1)\ q + G0G(t-1)\ P]\ h' + G1B(t-1)\ q + G1G(t-1)\ P$$

$$B0B(t) = [B0B(t-1)\ q + B0G(t-1)\ P]\ h$$

$$B1B(t) = [B0B(t-1)\ q + B0G(t-1)\ P]\ h' + B1B(t-1)\ q + B1G(t-1)\ P$$

$$B0G(t) = [B0G(t-1)\ Q + B0B(t-1)\ p]\ k$$

$$B1G(t) = [B0G(t-1)\ Q + B0B(t-1)\ p]\ k' + B1B(t-1)\ p + B1G(t-1)\ Q$$

Shortly after the program was written, we discovered that G0G(m), G0B(m), G1G(m), G1B(m), B0G(m), B0B(m), B1G(m), B1B(m) could be obtained from a difference equation in powers of J and L (see below). Since on our computer (IBM 7030), it took under a second to compute all eight quantities, we decided not to use the difference equation. However, we work out two, and give all eight results.

$$G0G(t) = \{G0G(t-1)\ Q + G0B(t-1)\ p\}\ k$$

$$G0B(t) = \{G0B(t-1)\ q + G0G(t-1)\ P\}\ h$$

The eigenvalues come from the 2nd order linear difference equation:

$$f_{t+1} - (Qk + qh)\ f_t - (p-Q)\ f_{t-1} = 0.$$

that is, $$2J = Qk + qh + \sqrt{(Qk + qh)^2 + 4hk(p-Q)}$$

and $$2L = Qk + qh - \sqrt{(Qk + qh)^2 + 4hk(p-Q)} .$$

Thus we have

$$G0G(t) = \alpha_1\ J^t + \alpha_2\ L^t ,$$

$$G0B(t) = \beta_1\ J^t + \beta_2\ L^t$$

and we get α_1, α_2, β_1 and β_2

from the initial conditions

$$GOG(0) = 1, \; GOG(1) = Qk$$

and $$GOB(0) = 0, \; GOB(1) = Ph \, .$$

So $\alpha_1 + \alpha_2 = 1$, $\alpha_1 J + \alpha_2 L = Qk$, which yields

$$GOG(t) = \{(Qk - L)/(J - L)\} \, J^t + \{(J - Qk)/(J - L)\} \, L^t$$

Also, $\beta_1 + \beta_2 = 0$, $\beta_1 J + \beta_2 L = Ph$, which yields

$$GOB(t) = \{Ph/(J - L)\} \quad (J^t - L^t)$$

All eight solutions are as follows:

$$GOG(m) = \{(Qk - L)/(J - L)\} \, J^m + \{(J - Qk)/(J - L)\} \, L^m$$

$$GOB(m) = \{Ph/(J - L)\} \, (J^m - L^m)$$

$$BOG(m) = \{pk/(J - L)\} \, (J^m - L^m)$$

$$BOB(m) = \{(qh - L)/(J - L)\} \, J^m + \{(J - qh)/(J - L)\} \, L^m$$

$$G1G(m) = p/(p + P) + \{P/(p + P)\} \, (Q-p)^m - \{(J - qh)/(J - L)\} \, J^m - \{(qh - L)/(J - L)\} \, L^m$$

$$G1B(m) = \frac{P}{p+P} \, [1 - (Q-p)^m] - \frac{Ph}{J-L} \, (J^m - L^m)$$

$$B1B(m) = \frac{P}{p+P} + \frac{p}{p+P} \, (Q-p)^m - \frac{1}{J-L} \, [\{J-Qk\} \, J^m + \{Qk-L\} L^m]$$

$$B1G(m) = \frac{p}{p+P} \, [1 - (Q-p)^m] - \frac{pk}{J-L} \quad (J^m - L^m)$$

SECTION III

RECURSIONS - PART 2

Again letting T and U represent either of the states G and B, we define

$$TOUI(s) = P(x_{(s-1)m+1} = x_{(s-1)m+2} = \cdots = x_{sm} = 0, S_{sm} = U|S_0 = T)$$

= P(m-bit symbol after s symbol times is correct and ends in state U|state T)

$$T1UI(s) = P(\text{for some } 1 \le i \le m,\ x_{(s-1)m+i} = 1,\ S_{sm} = U|S_0 = T)$$

= P(m-bit symbol after s symbol times has at least one bit error and ends in state U|state T)

Let GXG = G0G(m) + G1G(m), GXB = G0B(m) + G1B(m), BXG = B0G(m) + B1G(m), and BXB = B0B(m) + B1B(m).

Then G0GI(s) = GXG · G0GI(s-1) + GXB · B0GI(s-1) and B0GI(s) = BXG · G0GI(s-1) + BXB · B0GI(s-1).

(There will be similar equations in G1BI, B1BI and G0BI, B0BI, and G1BI, B1BI, but they have the same eigenvalues and will be omitted.)

$$T0GI(s) - [GXG + BXB]\ T0GI(s-1) + [GXG \cdot BXB - GXB \cdot BXG]\ T0GI(s-2) = 0$$

This yields the eigenvalues:

$$2\ \sigma = GXG + BXB + \sqrt{[GXG + BXB]^2 - 4\ [GXG \cdot BXB - GXB \cdot BXG]}$$

$$2\ \tau = GXG + BXB - \sqrt{[GXG + BXB]^2 - 4\ [GXG \cdot BXB - GXB \cdot BXG]}$$

Since GXG + GXB = BXB + BXG = 1,

$$\sigma \tau = GXG \cdot BXB - GXB \cdot BXG = (1-GXB)(1-BXG) - GXB \cdot BXG = 1-GXB-BXG$$

$$\sigma + \tau = GXG + BXB = 2 - GXB - BXG = 1 + \sigma \tau .$$

Hence $\sigma = 1$, and $\tau = 1 - GXB - BXG = GXG + BXB - 1$.

Thus we have

$$GOGI(s) = \lambda_1 \cdot 1^s + \lambda_2 \tau^s$$

and

$$BOGI(s) = \mu_1 \cdot 1^s + \mu_2 \tau^s$$

and we get λ_1, λ_2, μ_1 and μ_2 from the initial conditions.

$$GOGI(1) = GOG(m),\ GOGI(2) = GXG \cdot GOG(m) + GXB \cdot BOG(m)$$

$$BOGI(1) = BOG(m),\ BOGI(2) = BXG \cdot GOG(m) + BXB \cdot BOG(m).$$

Hence

$$\lambda_1 \cdot 1 + \lambda_2 \cdot \tau = GOG(m),\ \lambda_1 \cdot 1^2 + \lambda_2 \cdot \tau^2 = GXG \cdot GOG(m) + GXB \cdot BOG(m).$$

Solving, $\lambda_1 = [GOG(m)\ (1 - BXB) + BOG(m) \cdot GXB] / (1 - \tau)$

$$\lambda_2 = [GOG(m)\ BXB - BOG(m)\ GXG] / (1 - \tau) = [GOG(m) - \lambda_1] / \tau .$$

Also,

$$\mu_1 \cdot 1 + \mu_2 \cdot \tau = BOG(m),\ \mu_1 \cdot 1^2 + \mu_2 \cdot \tau^2 = BXG \cdot GOG(m) + BXB \cdot BOG(m).$$

Solving, $\mu_1 = [BOG(m)\ (1 - GXG) + BXG \cdot GOG(m)] / (1 - \tau)$

$$\mu_2 = [BOG(m) - \mu_1] / \tau$$

Therefore, $GOGI(s) = \lambda_1 \cdot 1^s + \lambda_2 \cdot \tau^s$, and $BOGI(s) = \mu_1 \cdot 1^s + \mu_2 \cdot \tau^s$, where λ_1, λ_2, μ_1, μ_2 are given above. Since s is fixed for any given application, we will refer to GOGI(s) as GOGI, and to BOGI(s) as BOGI.

Consider the first i m-bit symbols of a random interleaved block. Let $GB(i,j) = P(j \text{ symbol errors in } i \text{ symbols and } S_{ism} = B | S_0 = G)$. Similarly, we define GG(i,j), BB(i,j) and BG(i,j).

Then

$$GG(1,0) = GOGI \qquad GB(1,0) = GOBI$$

$$GG(1,1) = G1GI \qquad GB(1,1) = G1BI$$

$$BG(1,0) = BOGI \qquad BB(1,0) = BOBI$$

$$BG(1,1) = B1GI \qquad BB(1,1) = B1BI$$

Finally, for $i = 2, \cdots, n$ and $j = 0, \cdots, i$

$$GG(i,j) = GG(i-1,j)\ GG(1,0) + GB(i-1,j)\ BG(1,0) + GG(i-1,j-1)\ GG(1,1) + GB(i-1,j-1)\ BG(1,1)$$

$$GB(i,j) = GG(i-1,j)\ GB(1,0) + GB(i-1,j)\ BB(1,0) + GG(i-1,j-1)\ GB(1,1) + GB(i-1,j-1)\ BB(1,1)$$

$$BG(i,j) = BG(i-1,j)\ GG(1,0) + BB(i-1,j)\ BG(1,0) + BG(i-1,j-1)\ GG(1,1) + BB(i-1,j-1)\ BG(1,1)$$

$$BB(i,j) = BG(i-1,j)\ GB(1,0) + BB(i-1,j)\ BB(1,0) + BG(i-1,j-1)\ GB(1,1) + BB(i-1,j)\ BB(1,1).$$

Finally,

$$P(\text{random bit is in } G) = \alpha = \frac{p}{p+P} \quad \text{and hence}$$

P(K symbol errors in n symbols with s blocks interleaved)

$$= \alpha \left[GG(n,K) + GB(n,K)\right] + (1 - \alpha) \left[BG(n,K) + BB(n,K)\right]$$

SECTION IV

INPUT FOR THE PROGRAM

XM = No. of bits/symbol

XN = No. of symbols/block

XNEST = Largest number of symbol errors to be considered (if this field is blank, XNEST = XN)

XIPER = Number of interleaved symbols (0 or 1 means 1)

IK = 0 or not equal to 0 (0 means continue with CP, SP, H, SK; not equal to 0 means read new parameters)

CP = P

SP = p

H = h

SK = k

SECTION V

OUTPUT OF THE PROGRAM

TIMEX = A8 representation of the time read from IBM 7030 Time Clock by a STRAP coded routine (one can call his routine or omit it altogether)

CP, SP, H, SK as above

ALPHA = P (random bit is in G)

M (or XM) as above

N (or XN) as above

NS = No. of recursion terms to be attempted

WPI = P (symbol error)

WY155 = P (no errors when s = 1)

WPMU2 = mean number of errors in a block

FCMEAN = mean number of errors given an error occurred (when s = 1)

For J, L, A, B see [2,3,4]

IS = number of errors

P = P (IS symbol errors in a block)

R = P (IS symbol errors in a block|error occurred)

Q = P (≤ IS symbol errors in a block)

QH = P (> IS symbol errors in a block)

S = mean number of bits between blocks with > IS symbol errors

PBAR = contribution to mean number of errors per block made by probabilities actually calculated (if one wants the whole mean, then NEST = 0 or N; the same applies to VAR, SVAR, and CMEAN)

VAR = contribution to variance about PBAR made by probabilities actually calculated by the recursion

SVAR = approximate standard deviation

CMEAN = contribution to mean number of errors given an error occurred in the block made by probabilities actually calculated by recursion

APPENDIX

PROGRAM

```
FORTRAN SYSTEM -- VERSION  03/28/65 - CORRECTION LEVEL  03/28/65
      C...TO CALCULATE THE RECURSION PROBABILITIES
            COMMON  /ZZZ/ TIMEX
            DIMENSION  GG(515), GB(515), BG(515), BB(515)
            DIMENSION  BOB(205), B1B(205), BOG(205), B1G(205)
            DIMENSION  GOG(205), G1G(205), GOB(205), G1B(205)
            INTEGER XM(50),XN(50),XNEST(50),              XIPER(50)
       404  READ1701, LK
      1701  FORMAT (I2)
      C...READ IN THE NO. OF BITS PER SYMBOL, THE NO. OF SYMBOLS,
      C          HOW MANY SYMBOLS TO GO THRU, HOW MANY SYMBOLS TO INTERLEAVE
            READ  701,(XM(L), XN(L), XNEST(L), XIPER(L),  L=1,LK)
       701  FORMAT (I3,1X, I6, 2X, I3,                  1X, I4)
      C...READ IN CP, SP, H, SK
       604  READ 702, IK, CP, SP, H, SK
       702  FORMAT (I1, E17.10, 3(E18.11))
            DO 2000 K=1,LK
            M = XM(K)
            N = XN(K)
            NEST = XNEST(K)
            IPERD =XIPER(K)
            IF(NEST.EQ.0) NEST  =  N
            NS  =  NEST  +  1
            CALL TIME
            PRINT 300, TIMEX
       300  FORMAT(1H1,A8,21X, 77H*CALCULATION OF GILBERT CHANNEL ERROR PROBAB
           1ILITIES USING RECURSION FORMULAS*/////)
            MUD  =  M*N
            FMUD  =  FLOAT(MUD)
            MUD1  =  MUD - 1
            HCOMP  =  1. - H
            SKCOMP  =  1. - SK
            CQ  =  1. - CP
            SQ  =  1. - SP
            GAMMA  = 1./(CP+SP)
            ALPHA  = SP*GAMMA
            BETA   = CP*GAMMA
            P1 = (SP*SKCOMP + CP*HCOMP)*GAMMA
            PRINT 301, CP, SP, H, SK, ALPHA
       301  FORMAT  (5X, 27HCHANNEL PARAMETERS    CP = ,E11.6, 5X, 5HSP = ,
           1E11.6,5X, 5H H = , F8.6, 5X, 5H K = ,E16.10,5X, 8HALPHA = , F8.6)
            PRINT 302, M, N, NS, P1,        IPERD
       302  FORMAT  (8X, 24HCODE PARAMETERS      M = , I4, 13X, 4HN = , I5,
           112X, 35HNO. OF RECURSION TERMS REQUESTED = ,  I5/ 7X, $P1 = $,
           2 E13.7, 75X,                             I4, $ BLOCKS INTERLEAVED
           3 $,/)
```

```
      WN   =   SP - CQ
      WT   =   SK*CQ + H*SQ
      WR   =   WT**2 + 4.*H*SK*WN
      WSQRT   =   SQRT(WR)
      WJ   =   (WT + WSQRT)/2.
      WL   =   (WT - WSQRT)/2.
      WA   =   WJ +     WN   *(CP*SK*HCOMP   + SP*H*SKCOMP ) /
     1        (CP*HCOMP   + SP*SKCOMP    )
      WA   =   WA / WSQRT
      WB   =   WA - 1.
      WX2 = (WA*(1.-WJ**M)/(1.-WJ))   -   (WB*(1.-WL**M)/(1.-WL))
      WPI   =   PI   * WX2
      WPMU2   =   FLOAT(N) * WPI
      RJ   =   FLOAT(MUD) * ALOG10(WJ)
```

```
      RL   =   FLOAT(MUD) * ALOG10(WL)
      IF(RJ. LT. -50.)  F = 0.
      IF(RJ. GE. -50.)  F = WJ**MUD
      IF(RL. LT. -50.)  G = 0.
      IF(RL. GE. -50.)  G = WL**MUD
      WY155   =   1. -((WA*(1.-F)/(1.-WJ))-WB*(1.-G)/(1.-WL))*PI
      FCMEAN   =   WPMU2 / (1. - WY155)
      PRINT 806, WPI, WY155, WPMU2,FCMEAN,WJ, WL, WA, WB
  806 FORMAT  (1X, 20HPROB SYMBOL ERROR = ,E12.6, 1H*, 2X, 17HPROB NO ER
     1RORS = ,E12.6, 1H*, 2X, 8H MEAN = ,E12.6, 1H*, 2X, $CONDITIONAL ME
     2AN = $, E12.6, $*$ / 10X, 4HJ = , 3X, E16.10, 1H*,
     315X, 4HL = , 3X, F9.6, 1H*, 6X, 4HA = , 3X, F9.6, 1H*, 20X, 4HB =
     4,F9.6, 1H*///)
      GOG(1) = CQ*SK
      GIG(1) = CQ -    GOG(1)
      GOB(1) = CP*H
      GIB(1) = CP -    GOB(1)
      BOB(1) = SQ*H
      BIB(1) = SQ -    BOB(1)
      BOG(1) = SP*SK
      BIG(1) = SP -    BOG(1)
      IF(M.EQ.1) GO TO 80
      DO 50  J=2,M
      J1 = J-1
      TA     = GOB(J1)*SP + GOG(J1)*CQ
      GOG(J) = TA*SK
      GIG(J) = TA - GOG(J) + GIG(J1)*CQ + GIB(J1)*SP
      TB     = GOB(J1)*SQ + GOG(J1)*CP
      GOB(J) = TB*H
      GIB(J) = TB - GOB(J) + GIB(J1)*SQ + GIG(J1)*CP
      TC     = BOB(J1)*SQ + BOG(J1)*CP
      BOB(J) = TC*H
      BIB(J) = TC - BOB(J) + BIB(J1)*SQ + BIG(J1)*CP
      TD     = BOG(J1)*CQ + BOB(J1)*SP
      BOG(J) = TD*SK
   50 BIG(J) = TD - BOG(J) + BIG(J1)*CQ + BIB(J1)*SP
   80 PRINT 303, GOG(M), GIG(M), GOB(M), GIB(M)
```

```
  303 FORMAT  (5X,  6HGOG = , E12.5, 5X, 6HG1G = , E12.5, 5X, 6HGOB = ,
     1E12.5, 5X, 6HG1B = , E12.5)
      PRINT 304, BOG(M), B1G(M), BOB(M), B1B(M)
  304 FORMAT  (5X,  6HBOG = , E12.5, 5X, 6HB1G = , E12.5, 5X, 6HBOB = ,
     1E12.5, 5X, 6HB1B = , E12.5//)
      RPI1  = ALPHA*(G1B(M) + G1G(M))  +  BETA*(B1B(M) + B1G(M))
      PRINT 505, RPI1
  505 FORMAT  (1X, 30HPROB RANDOM SYMBOL IN ERROR = , E12.6, 1H*///)
      CALL TIME
      GG(1)  = GOG(M)
      GG(2)  = G1G(M)
      GB(1)  = GOB(M)
      GB(2)  = G1B(M)
      BG(1)  = BOG(M)
      BG(2)  = B1G(M)
      BB(1)  = BOB(M)
      BB(2)  = B1B(M)
      IF (IPERD .NE. 0) GO TO 9001
      GOGI = GOG(M)
      G1GI = G1G(M)
      GOBI = GOB(M)
      G1BI = G1B(M)
      BOGI = BOG(M)
      B1GI = B1G(M)

      BOBI = BOB(M)
      B1BI = B1B(M)
      GO TO 9002
 9001 GXG = GG(1) + GG(2)
      GXB = GB(1) + GB(2)
      BXG = BG(1) + BG(2)
      BXB = BB(1) + BB(2)
      SIG = 1.
      TAU = GXG + BXB - 1.
      DEN = 1. - TAU
      GT  = 1. - BXB
      BT  = 1. - GXG
      G   = 0.
      PERD = FLOAT(IPERD)
      IF (PERD * ALOG10( TAU ) .GE. -300.) G = TAU ** PERD
      PRINT 9003, SIG, TAU
 9003 FORMAT (8X, $SIGMA = $, E20.12, 10X, $TAU = $, E20.12/'
      AG  = (GG(1) *GT  +  GXB * BG(1)) / DEN
      CG  = (GG(1) - AG) / TAU
      AB  = (BXG * GG(1)  +  BG(1) * BT) / DEN
      CB  = (BG(1) - AB) / TAU
      GOGI = AG      +  CG * G
      BOGI = AB      +  CB * G
      AG  = (GG(2) *GT  +  GXB * BG(2)) / DEN
      CG  = (GG(2) - AG) / TAU
      AB  = (BXG * GG(2)  +  BG(2) * BT) / DEN
```

```
      CB   = (BG(2) - AB) / TAU
      G1GI = AG       +  CG * G
      B1GI = AB       +  CB * G
      AG   = (GB(1) *GT  +  GXB * BB(1)) / DEN
      CG   = (GB(1) - AG) / TAU
      AB   = (BXG * GB(1)   +   BB(1) * BT) / DEN
      CB   = (BB(1) - AB) / TAU
      GOBI = AG       +  CG * G
      BOBI = AB       +  CB * G
      AG   = (GB(2) *GT  +  GXB * BB(2)) / DEN
      CG   = (GB(2) - AG) / TAU
      AB   = (BXG * GB(2)   +   BB(2) * BT) / DEN
      CB   = (BB(2) - AB) / TAU
      G1BI = AG       +  CG * G
      B1BI = AB       +  CB * G
      PRINT 9005,GOGI, G1GI, GOBI, G1BI, BOGI, B1GI, BOBI, B1BI
 9005 FORMAT (/5X, 7HGOGI = ,E12.5, 4X, 7HG1GI = , E12.5, 4X, 7HGOBI = ,
     1 E12.5, 4X, 7HG1BI = , E12.5/ 5X, 7HBOGI = , E12.5, 4X, 7HB1GI = ,
     2 E12.5, 4X, 7HBOBI = , E12.5, 4X, 7HB1BI = , E12.5//)
 9002 IF(NEST.EQ.1) GO TO 90
C...CALCULATE THE RECURSION PROBABILITIES
      DO 51  L1=2,N
      LZ  =  MINO(L1,NEST)
      LY  =  L1
      IF(L1 .GT. NEST) LY = NEST + 1
      IF(L1 .GT. NEST) GO TO 97
      N1 = L1 + 1
      GG(N1)   =   GG(L1)*G1GI    +  GB(L1)*B1G(
      GB(N1)   =   GG(L1)*G1BI    +  GB(L1)*B1BI
      BG(N1)   =   BG(L1)*G1GI    +  BB(L1)*B1GI
      BB(N1)   =   BG(L1)*G1BI    +  BB(L1)*B1BI
   97 CONTINUE
      DO 52  L2=2,LY
      N2  =  LY - L2 + 2
      N3 = N2 - 1

      YAH1  =GG(N2)*GOGI  +GG(N3)*G1GI  +GB(N2)*BOGI  +GB(N3)*B1GI
      GB(N2)=GB(N2)*BOBI  +GB(N3)*B1BI  +GG(N2)*GOBI  +GG(N3)*G1BI
      GG(N2)  =  YAH1
      YAH2  =BG(N2)*GOGI  +BG(N3)*G1GI  +BB(N2)*BOGI  +BB(N3)*B1GI
      BB(N2)=BB(N2)*BOBI  +BB(N3)*B1BI  +BG(N2)*GOBI  +BG(N3)*G1BI
   52 BG(N2)  =  YAH2
      YAH3   = GG(1)*GOGI   + GB(1)*BOGI
      GB(1)  = GB(1) *BOBI   + GG(1) *GOBI
      GG(1)  =  YAH3
      YAH4   = BG(1) *GOGI   + BB(1) *BOGI
      BB(1)  = BB(1) *BOBI   + BG(1) *GOBI
      BG(1)  =  YAH4
   51 CONTINUE
   90 CONTINUE
      IF(NEST.GT.35) PRINT 503
```

```
      503 FORMAT(///////////////////50X, 33H(SYMBOL DATA STARTS ON NEXT PAGE))
          IF(NEST.GT.35) PRINT 502
      502 FORMAT  (1H1)
          PRINT 500, TIMEX
      500 FORMAT(1X,A8/ 1X, $SYMBOL NUMBER$10X, 15HRECURSION PROBS, 8X,
         1 17HCONDITIONAL PROBS, 15X, 13HCUM RECURSION, 12X, $1. - CUM$,
         2 9X, $MEAN 'TWEEN$)
          P  =  ALPHA*(GG(1) + GB(1))  +  BETA *(BG(1) + BB(1))
          Q  =  P
          R1  =  1. - P
          S1  =  Q * FMUD / R1
          PRINT 507, P, Q, R1, S1
      507 FORMAT (12X,$0*$,     8X, E16.9, 1H*, 34X, E18.12, 1H*,  9X,
         1 E10.4, $*$, 9X, E10.4, $*$)
          PBAR  =  0.
          PVAR  =  0.
          DO 53 I=2,NS
          IS  =  I - 1
          P  =  ALPHA*(GG(I) + GB(I))  +  BETA *(BG(I) + BB(I))
          Q  =  P  +  Q
          QH  =  1. - Q
          R  =  P / R1
          S  =  Q * FMUD / QH
          FIS = FLOAT(IS)
          FIST= FIS**2
          PBAR  =  PBAR  +  P * FIS
          PVAR  =  PVAR  +  P * FIST
          PRINT 501, IS, P, R, Q, QH, S
       53 CONTINUE
      501 FORMAT ( 8X, I5, 1H*, 2( 9X, E15.8; 1H*),  9X, E18.12, 1H*,  9X,
         1 E10.4, 1H*, 9X, E10.4, $*$)
      606 CONTINUE
          VAR   =  PVAR - PBAR*PBAR
          SVAR  =  SQRT(VAR)
          CMEAN  =  PBAR / R1
          CALL TIME
          PRINT 305,  PBAR, VAR, SVAR, CMEAN, TIMEX
      305 FORMAT  (//  25X, $PARTIAL MEAN              = $, E13.6/
         1             25X, $APPROXIMATE VARIANCE      = $, E13.6/
         2             25X, $APPROXIMATE ST. DEV.      = $, E13.6/
         3             25X, $PARTIAL CONDITIONAL MEAN = $, E13.6//1X, A8//)
     2000 CONTINUE
          IF(IK  .EQ. 0) GO TO 604
          GO TO 404
          END
```

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* References 2,3,4 are essentially the same, 2 is more accurate than 3, and 3 is more accurate than 4.

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A model for a channel is given. For this model, the recursive method is presented in order to calculate the probability of K symbol errors in a block of n m-bit symbols. The blocks can be interleaved or not.

14 KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Channel Modelling						
Communications						
Error Control						

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