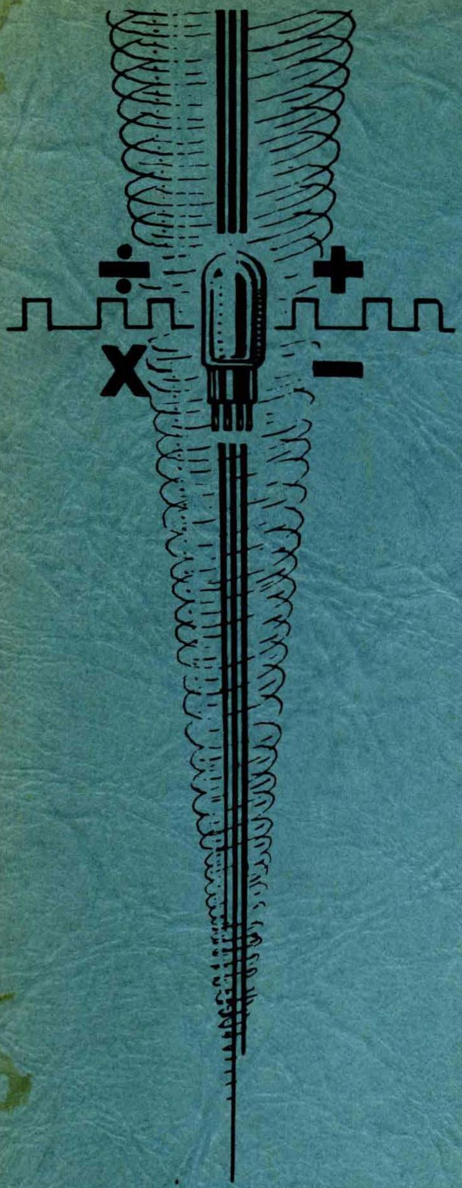
PROJECT WHIRLWIND



SUMMARY REPORT NO. 38
SECOND QUARTER 1954

DIGITAL COMPUTER LABORATORY
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

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Submitted to the

OFFICE OF NAVAL RESEARCH
Under Contract N5ori60
Project NR 048-097

DIGITAL COMPUTER LABORATORY
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Cambridge 39, Massachusetts

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1. QUARTERLY REVIEW AND ABSTRACT

Computer time allocated to the Scientific and Engineering Computation (S&EC) Group was divided among 48 problems. Previously developed computer logging facilities, now tested, are adequate; henceforth, WWI will process the logging tapes for the biweekly and quarterly reports of S&EC computer time. And finally -- a statistical study of tape preparation was completed.

The systems group, eliminating system weaknesses, replaced power-distribution panels with more reliable plug-in-type relays and revised the central d-c power-supply system for better regulation on transients.

The magnetic-tape system, now maintained by the systems group, has been given increased flexibility by the addition of a second delayed-printout typewriter.

To facilitate maintenance and to increase the reliability of terminal equipment, marginal-checking facilities have been improved, the file of test programs has been expanded, and checking and monitoring equipment has been added.

Pending engineering analysis, varying vacuum-tube heater voltage in conjunction with standard d-c marginal checking appears valuable in locating certain classes of tube faults.

Continued investigation indicates that previous results in the problem of transient changes in oxide cathodes are faulty; revised results are now being prepared.

Three seminars were held, one on advanced programming techniques, two on computing-machine methods. A number of papers were presented by staff members, the largest number at the Ann Arbor meeting of the ACM (at which there was also a showing of the recently completed Digital Computer Laboratory movie, "Making Electrons Count").

Field	Description	Problem Number	% WWI Time	Source
Aeronautical Engineering	Construction and testing of a delta-wing flutter model	166.C	1.33	MIT
	Determination of the downwash at the tail of an airplane due to the lift response of the wing to a sharp-edged gust	168.C	0.26	MIT
	Low-aspect ratio flutter	177.D	0.04	MIT
	Transient temperature of a box-type beam	179.C	0.35	MIT
	Blast response of aircraft	183.D	0.67	MIT
Chemical Engineering	Transient effects in distillation	*167.D	2.35	MIT
	Effect of gravity on relative water production in oil reservoirs	188.C	0.76	Atlantic Refining Co.
Chemistry	Optical properties of thin metal films	101.C	0.91	MIT
Civil Engineering	Analysis of reinforced concrete walls	*113.C	0.93	MIT
	Study of shock waves; vibration problems in solid bodies	*142.D	2.14	MIT
	Response of cohesive and cohesionless soils to transient loading	*161.C	0.32	MIT
Dynamic Analysis & Control Lab	Response of a fuel-flow controller	187.C	0.20	MIT
Electrical Engineering	Switching-circuit design	*169.B	0.17	MIT
	Connector provision in automatic telephone exchanges	176.B	1.60	MIT
	Crosscorrelation of blast furnace input-output data	180.B	1.22	MIT
Geology & Geophysics	Geophysical data analysis	106.C	3.15	MIT
	Interpretation of earth-surface resistivity measurements	*123.C	0.57	MIT
	Frequency and phase spectrum analysis of seismograms	192.D	1.05	MIT
Instrumentation Lab	An interpretive program to accept mathematical symbols	108.C	0.13	MIT
	System of nonlinear differential equations	109.C	0.18	MIT
	Trajectory study against an evading target	178.D	0.29	MIT
Insulation Research	Crystal structures	182.C	0.36	MIT
Lincoln Lab	Digital methods of detecting signal from noise	149.C	0.25	MIT
	Transmission in a rectangular waveguide containing a single ferrite slab	163.C	0.53	MIT
	Tracking response characteristics of the human operator	186.C	0.05	MIT
Mathematics	Spherical wave propagation	*119.C	0.40	MIT
Mechanical Engineering	Turbine design (aerothermopressor)	120.D	1.28	MIT
Meteorology	Synoptic climatology	155.D	2.27	MIT
Physics	Vibrational frequency spectrum of a copper crystal	143.D	0.63	MIT
	Self-consistent molecular orbitals	144.C	0.23	MIT
	Energy bands in crystals	147.C	3.10	MIT
	Evaluation of the reflection coefficient in a semi-infinite open rectangular waveguide	156.A	0.04	MIT
	Use of water storage in a hydroelectric system to minimize the expected operating cost	159.D	1.15	MIT
	Determination of phase shifts from experimental cross-sections	162.C	0.11	MIT
	Inverse and inverse square root of a real symmetric matrix	170.C	0.01	MIT
	Overlap integrals of molecular and crystal physics	*172.B	2.66	MIT
	Tight-binding calculations in crystals	174.C	0.32	MIT
	Impurity levels in crystals	175.C	0.41	MIT
	Perturbed coulomb wave functions	181.C	0.08	MIT
	Scattering electrons from hydrogen	184.D	0.18	MIT
Servomechanisms Lab	Autocorrelation and Fourier transform calculation	107.C	0.71	MIT
	Subroutine for the numerically controlled milling machine	132.C	0.39	MIT
	Data-reduction program; polynomial fitting	126.C	0.40	MIT
	Improved power-spectra calculations	171.C	1.30	MIT
Miscellaneous	Comprehensive system of service routines	100	61.15	MIT
	Special problems (staff training, etc.)	131	1.64	MIT
	Library of subroutines	141	0.40	MIT
	Course 6.537: digital computer application practice	173	1.33	MIT

Table 2-I. Current Problems Arranged According to Field of Application (* MIT Project on Machine Methods of Computation)

2.2 Problems Being Solved

100 COMPREHENSIVE SYSTEM OF SERVICE ROUTINES

The comprehensive system of service routines has been developed by the Scientific and Engineering Computation (S&EC) Group to simplify the process of coding. The system in use since the fall of 1952, described in Summary Reports 32-35, is now called CS I. A new system, called CS II, which is based on CS I, was described in Summary Reports 36 and 37. CS II has been thoroughly tested during the past quarter and is now being used on a routine basis.

Since the reader will find references in some of the reports below to the number system used in CS II, the following brief description is included here for the reader's convenience.

(m, n) numbers shall mean numbers which are of the form $z = x \cdot 2^y$ where x is an m-binary-digit number and y is an n-binary-digit number. For example, (24, 6) signifies a two-register floating-point system dealing with numbers of 24 significant binary digits (roughly 7 decimal digits) with magnitudes between 2^{63} and 2^{-64}.

Arithmetic involving these (m, n) numbers is carried out bymeans of (m, n) interpretive subroutines. These subroutines enable the programmer to write coded programs using (m, n) numbers as easily as, or even more easily than, he might write programs in the single-length fixed-point (15, 0) number system which is built into Whirlwind I.

The Comprehensive System of Utility Programs

CS I and CS II form part of a library of utility programs which are used daily by the S&EC Group. This library includes other conversion systems, post-mortem programs, input programs, and equipment programs. The major part of the library has been semipermanently recorded on either magnetic tape or the magnetic drum. A program called the utility control program, which arranges for the automatic selection of utility programs from the library, has been semipermanently recorded on the magnetic drum. The over-all system (the control program plus the library) is called the comprehensive system of utility programs.

The Utility Control Program

Normal operation of the computer is initiated by pressing the read-in button. This transfers computer control to a short program in test storage which records the contents of magnetic-core memory on drum group 0 and reads the utility control program from the drum into core memory. Computer control is then transferred to the utility control program.

Contents	Utility Program
1	Read in an fc tape and produce a binary tape. Record conversation results (title, floating-address table, binary tape, conversion post-mortem) for delayed printing and punching.
2	Record a Flexo stop character for delayed printing.
4	Display and photograph the contents of core memory.
5	Perform a programmed-arithmetic post-mortem, and record the results for delayed printing.
6 q	Perform a programmed-arithmetic post-mortem and record the results on the direct printer.
7	Record the title of the tape last read in for delayed printing.
8	Read in an fc tape and do not produce a binary tape. Record conversion results on the direct printer and punch.
15	Record the title of the tape last read in on the direct printer.
16, 17	Select CS I, convert the Flexo tape in the photoelectric tape reader, and record the binary tape for delayed punching.
24	Read in the magnetic-drum checking program.
32	Read in the magnetic-tape checking program.
40	Read in the scope-calibration program.
48	Read in the photoelectric-tape-reader checking program.
49	Terminate the paper-tape log.
62	Initiate the paper-tape log.
64	Perform a normal read-in procedure using the mechanical tape reader for input instead of the photoelectric tape reader.

101 OPTICAL PROPERTIES OF THIN METAL FILMS

The present stage of research in electronic digital computation applied to the optical and electrical properties of thin metal films was completed by A. Loeb of the MIT Chemistry Department during the past month. The final production runs have taken place, and the next month will be devoted to the analysis and publication of the results. While there is a possibility that some short runs may be desirable, this seems to be the appropriate time for a final summary.

Six modes of operation have been programmed, developed, tested, and used to deal with the functional relationships between the variables of a physical system consisting of a thin metal film on a nonabsorbing, thick backing. (The expressions thin and thick are used relative to the wavelength of incident radiation.) These variables are: film thickness, index of refraction of film, absorption coefficient of film, electrical conductivity of film, dielectric available was checked on WWI. The computer has been intimately integrated into the laboratory program. Since the density of thin metal films is not easily determined, the rapid calculations made it possible to repeat the calculations assuming different densities. Also, a quality check on some of our measurements was obtained because an underestimate of the reflectance, which is likely to occur with transparent films, was spotted by WWI. This led to the discovery of a lower limit of our film systems, which could be derived from the newly tested approximate equations.

The analysis of the results obtained from WWI has enabled us to calculate the relaxation time of electrons in metal blacks and metal brights. Also it enabled us to explain the abnormally high d-c resistance of some films, for the variation with wavelength of the optically evaluated conductivity of such films indicates that imperfections in the films act as condensers. The optical constants calculated from results obtained with radiation incident on the film differ in some cases from those calculated with radiation incident on the backing. This discrepancy has been blamed on the condition of the surface of the backing; improved cleaning methods for the backing in our own laboratory have decreased the discrepancies.

During the next year no new programs will be developed; whatever calculations will be performed will use one of the modes C_1, C_2, and D. It is hoped that in the future this research will be continued to encompass systems containing more than two films, each of which may be absorbing. To do so it would be most practical to develop a system analogous to the CS which can deal directly with complex numbers. This would then be used to solve Maxwell's equations directly on WWI for a system of various media separated by a series of parallel walls, each medium being characterized by its thickness and two optical or electrical constants.

The following publications include results of calculations made on WWI:

"Conductance and Relaxation Time of Electrons in Gold Blacks from Transmission and Reflection Measurements in the Far Infrared," Louis Harris and Arthur L. Loeb, J. Opt. Soc. Am. 43, 1114-1118.

"The Preparation and Infrared Properties of Aluminum Oxide Films," Louis Harris. Submitted to J. Opt. Soc. Am.

"The Evaluation of Optical and Electrical Constants of Thin Metal Films from Reflectance and Transmission Measurements by Electronic Digital Methods," Louis Harris and Arthur L. Loeb. To be submitted to J. Opt. Soc. Am.

"Electronic Digital Computation of Optical and Electrical Constants of Thin Films" (tentative title), Louis Harris, Harry H. Denman, and Arthur L. Loeb. To be submitted to J. S. I. A. M.

history; an analysis of their statistics is therefore most meaningful. The spectrum of the sunspots is the simplest statistic related to predictability. Its possible application to meteorology is clear.

The spectrum of the mean monthly sunspot numbers was completed. This function was desired for its possible applications in predictions where solar effects on the atmosphere would be significant, e.g., radio reception and meteorological conditions. The work is now completed, and a report is being prepared by A. Fleischer of the MIT Meteorology Department.

2. Investigation of Turbulent Flow

This project is being carried out by F. Raichlen of the MIT Hydrodynamics Laboratory. It deals mainly with the investigation of turbulent flow by means of a Pitot-tube pressure-cell combination. The velocity fluctuations which are recorded by means of an oscilloscope motion-picture-camera arrangement are of a random nature.

From these records 1800 points with time spacings of 0.001182 second were then sent to the Digital Computer Laboratory, and an autocorrelation curve was obtained to determine the relative importance of the signal distortion caused by the resonant frequency of the pressure cell. This was found to be negligible in comparison to the true signal.

A number of runs were then made at different levels in a high-velocity open-channel flow. From these records 1800 points were taken at 0.00151-second intervals and submitted to the Digital Computer Laboratory for correlation. The autocorrelation curves obtained showed in addition to the 180 cps component, caused by the natural frequency of the gage, a low-frequency (3.5 cps) component which is believed to be inherent to the flow. Mean-intensity spectra were then obtained from these autocorrelation curves for the frequency ranges 0 to 25 cps and 0 to 250 cps. From the 0 to 250 cps mean-intensity spectrum the fact that the distortion of the signal caused by resonance was negligible was again confirmed.

The program used for obtaining the mean-intensity spectrum consisted of convolving the autocorrelation function with the unit step function.

$$W(f) = \int_o^{\tau_o} R(\tau) \cos \omega\tau d\tau \quad \int_o^{\tau_o} \cos \omega\tau d\tau$$

$$W(f) = \left[\int_o^{\tau_o} R(\tau) \cos \omega\tau d\tau\right] \left[\frac{\sin \omega\tau_o}{\omega}\right]$$

Wherever there is a periodic component the integral $\int_o^{\tau_o} R(\tau) \cos \omega\tau d\tau$ will give a spike, and $\frac{\sin \omega\tau_o}{\omega}$ will give peaks and troughs on either side of the spike. For a maximum delay time of $t_o = 0.45$ second these peaks and troughs were subtracted from the results obtained from the Digital Computer Laboratory. Mean-intensity spectra showing 90 per cent of the

108 AN INTERPRETIVE PROGRAM

This program will accept as input algebraic equations, differential equations, etc., expressed on Flexowriter punched paper tape in conventional mathematical notation (within limits imposed by the Flexowriter) and automatically provide the desired solution.

For information concerning the use of this program, a programmer's manual (E-364) is available at the Instrumentation Laboratory Library.

The work done on this program during this quarter was that of combining all the tapes used in developing the program into one master tape and then making tests on this combined tape.

109 AN AIRPLANE PURSUIT-COURSE PROGRAM

An airplane pursuit-course program has been written which restrains the airplane and target to the same horizontal plane. This program computes prediction times for a two-gyro gunsight and a three-gyro gunsight. A slant-airplane pursuit-course program has also been written which computes prediction times for a two-gyro gunsight and a three-gyro gunsight and correction-time ratios for the latter gunsight. Each of these programs includes the effects of airplane dynamics and projectile ballistics.

A test problem has been run using these two programs with comparable initial conditions for each. The results of these runs are being analyzed and will be included in the final report on this problem.

It is anticipated that a number of runs will be made using the slant-plane pursuit-course program with the airplane and target initially in a slant plane. From the results of these runs it will be possible to evaluate previous methods used to calibrate two-gyro and three-gyro gunsights.

This study is being carried out by M. H. Hellman of the MIT Instrumentation Laboratory.

113 STRESS ANALYSIS OF A THROATLESS PRESS

The results of the Whirlwind calculations for the stress analysis of the throatless press have been compared with an experimental photoelastic analysis. The two solutions are in fairly close agreement. One noticeable difference in the results is the stress values at reentrant corners of the structure. These values are higher in the photoelastic analysis. If a finer network of bars was used to approximate the plate, more accurate values would be obtained at the points of high stress concentration. The indeterminacy of the structure increases, however, and the computation time required for a solution also increases as the network of bars is made finer.

traveling inwards. Thus the center of the sphere becomes recompressed. From this point on the two solutions differ. In Case A the center is recompressed to a maximum density only slightly above atmospheric. Therefore, the second outward-going wave that forms is very weak, so weak in fact that no discernible second inward wave forms. By this time conditions are close to atmospheric everywhere, and what motion is left dies out rapidly. These results differ not only with those of Case B but also with the results of J. J. Unwin* who considered precisely the problem of Case A. His first recoil wave was of such strength that the center of the sphere was recompressed to a density substantially above that of the original compression. This was naturally followed by a second strong outward wave followed by a second recompression. Unwin was forced to stop his calculations before reaching the maximum of the second recompression because of numerical difficulties in his numerical process, which is different in various respects from the one used here. Physical checks applied to the results indicate that the solution obtained here is very accurate and more accurate than that obtained by Unwin.

In Case B the first recompression is markedly stronger than that in Case A as indicated by the velocities with which the particles move toward the center. It even appears, in contrast with Case A, that a shock forms near the center of the sphere. However, before the calculations could be carried to the shock, two major numerical difficulties were encountered. The first is that near the shock the extrapolation method used to estimate the indeterminate quantity $2u/r$ on the t-axis becomes very inaccurate and causes serious numerical errors. The second difficulty is caused by the inherent numerical error in the difference process being used to solve the problem. The process used is second order with a general per-step error of $0(h^3)$. However, for small r the error tends to be of the order of h^2 because of the r in the denominator of the quantity $2u/r$.

Before any further work can be done on the problem it will be necessary to determine, first, the relative significance of the two errors and, second, some method of reducing the effect of the errors.

A complete discussion of this problem and the two sources of error may be found in a final report written by A. Ralston for the MIT Committee on Machine Methods of Computation.

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* Unwin, J. J., "The Production of Waves by the Sudden Release of a Spherical Distribution of Compressed Air in the Atmosphere," Proc. Roy. Soc. (London), Ser. A, Vol. 178

One important variation was necessary. It was found that the full values of the calculated parameter changes could not be applied because of the nonlinearity of the kernel function. The resultant parameters would oscillate wildly. At the suggestion of T. R. Madden, a parameter-change multiplier was introduced, so that a fraction of the calculated change could be used each time. To expedite convergence, this fraction should not be made too small. It is expected that the maximum allowable value of the parameter will approach unity as the true solution is neared. In the cases tested, 0.05 has been the largest allowable fraction at the beginning of a calculation, but 0.4 has been used later in the calculations.

Of the four kernels tried, two runs were stopped after two iterations. This was caused by a provision for automatic stopping when the sum of the squared errors increases. The error increase resulted from parameter oscillation. These two kernels have not yet been retested. The third case, a theoretical kernel for the case of thicknesses $1/1/\infty$ and resistivities $1/10/0.1$ was run through 25 iterations. The parameters were approaching asymptotes of about the right values. The sums of squares of errors decreased by 81 per cent in the 25 iterations.

The only kernel which it is believed was iterated a sufficient number of times had thicknesses $1/1/\infty$ and resistivities $1/0.1/1$. This case was iterated 91 times, increasing the parameter-change multiplier every 20 iterations, from 0.05 to 0.4. Automatic cutoff was taken out of the program. After 40 steps, the correct solutions were obtained, but the calculations were continued and the errors continued to decrease. After 50 steps, the thickness and resistivity of the second layer went negative, but the error increased slightly. Beyond this point, the first-layer thickness oscillated about a value 3 per cent too large, the second-layer thickness and resistivity oscillated about zero, and the error continued to decrease very slowly. At the end of the calculation the error was 0.2 per cent of the original error, whereas at the correct solution (50 iterations before the end) it had already decreased to 0.5 per cent of the original error. There is no question of the validity of this solution, and the question of how accurate a solution to expect arises. It would not, however, seem proper to cease calculation when the error reaches some predetermined value, because all solutions beyond this point are equally valid.

The particular case considered here is a difficult one in that the derivatives and the parameters do not each have a range (in λ) of predominance. It would be expected that an increase in the accuracy of the kernel data would help. However, the data used here were accurate to five places, two more than could reasonably be expected from field data. This then implies a fundamental restriction on the whole approach of studying kernels. Significantly, this restriction is of the sort required for the use of the Pekeris method of interpretation (see Summary Reports No. 9, 10, and 11). It may be that the potentials themselves be found in the interval 3, 662, 563, 604, 200 ± 15, 000 whose largest prime factor did not exceed 101. For this case factors were given for certain numbers outside the interval. Also for one number in the interval prime factors not exceeding 173 were given.

These results will be used in the design of the gear chains in the planetarium of the Boston Museum of Science.

132 SUBROUTINES FOR THE NUMERICALLY CONTROLLED MILLING MACHINE

During this quarter, four new or revised subroutines were added by J. Runyon of the Servomechanisms Laboratory to the library of routines for computations for the MIT numerically controlled milling machine. Among these are two routines for use on jobs requiring interpolation. One of these is for selecting a set of points for the interpolation by means of the Lagrangian formula.

When it became necessary to check several very long milling-machine tapes, a routine for reading these tapes into the computer was brought up to date. The Ferranti reader facilitates the process since no feedout is required where the tape must stop. The revised routine has been incorporated in a program for printing out in decimal form the numbers on the tape or for finding and printing out cumulative totals of milling-machine orders. A program for preparing tape for series-16 wing templates has been run several times, but the major portion of it has not performed satisfactorily. Error diagnosis is continuing.

Six test pieces were cut in wood from tapes prepared on WWI in the preceding quarter. These are cones with sinusoidal cross sections. The pieces illustrate two different methods of cut spacing and three degrees of approximation to the surface. Since the results of this machining were satisfactory, a program for preparing tape for a series-16 conic wing section has been obtained by modifying the sinusoidal-cone program. It will be run sometime in the future.

141 S&EC SUBROUTINE STUDY

A list of all available and tested library subroutines, including the storage requirements for each, has been added to the CS programming manual to inform programmers of the subroutines currently available.

A new subroutine for calculating the principal value of $\tan^{-1}X$ has been written, tested, and added to the subroutine library under the title, FU 7: $\tan^{-1}X$, (30-j, j), j=1, 2, ... 15. This closed subroutine, written in CS code, is entered with the value of X in the MRA (where $-\infty < X < \infty$, within the limits of the number system used). The subroutine makes use of two approximations; the value of X determines which is used.

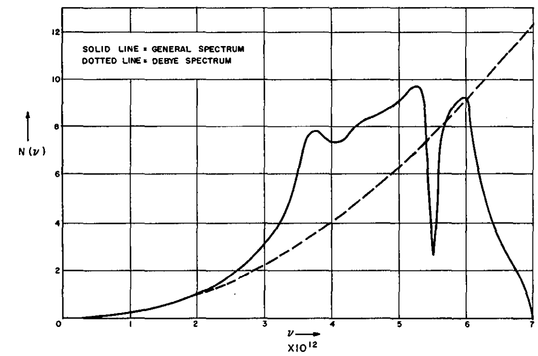

Fig. 2-2
Composite Spectra

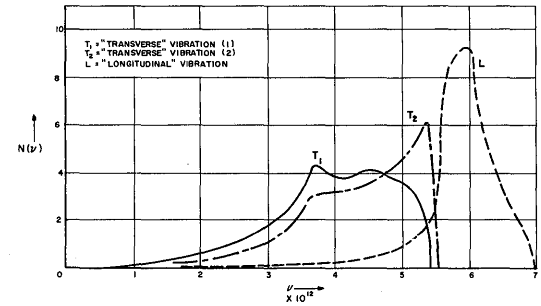

Fig. 2-3
Decomposition of Vibrational Spectrum into Branches

$$\alpha_1' = \alpha_1 + 2\,\alpha 3 \quad , \quad \beta_1' = \beta_1 + 2\,\beta\, 3$$

$$\alpha_1'' = \alpha_1' + 2\,\beta_1' \qquad \gamma_1' = \gamma_1 + 2\,\gamma 3$$

$\alpha_1 = 0.48 \times 10^4$ dy/cm

$\beta_1 = 0.87 \times 10^4$ dy/cm

$\gamma_1 = 1.25 \times 10^4$ dy/cm

$\beta_2 = 0.35 \times 10^4$ dy/cm

$\alpha_2 = -\ 0.072 \times 10^4$ dy/cm

$\alpha_3 = 0.09 \times 10^4$ dy/cm

$\beta_3 = -\ 0.022 \times 10^4$ dy/cm

$\gamma_3 = -\ 0.015 \times 10^4$ dy/cm

$\epsilon_3 = 0.06 \times 10^4$ dy/cm

The dotted line of Fig. 2-2 is the Debye spectrum adjusted to the cut-off frequency of the secular determinant which is 7.01×10^{12} cps. It is well known that the simple Debye spectrum does produce a specific heat in close agreement with experiment at medium low temperatures. The reason is obvious from Fig. 2-2 wherein it will be noted that the two curves coincide over about 1/3 of the spectrum. This is the most important region at low temperatures. Also, it will be noticed that the high-frequency end of the spectrum is sharply peaked due entirely to "longitudinal" waves. This result also occurs in the simple lattice models where exact frequency spectra can be easily calculated by analytic means and is of interest when considering electron conduction phenomena wherein the scattering of electrons by lattice waves is caused primarily by the longitudinal waves. This part of the spectrum can be approximated by a delta function or a rectangle in such cases.

has been used to solve 52 such equations, of order up to twenty, for Dr. F. Herman of RCA, Princeton; satisfactory results were obtained in all cases.

149 DIGITAL METHODS OF DETECTING SIGNAL FROM NOISE

During this quarter the study of digital methods of detecting signal from noise has been essentially completed by Dr. G. P. Dinneen of the Lincoln Laboratory.

Programs have been written which successfully generate a sequence of zeros and ones which simulate both signal and noise. The noise and signal regions, which differ only in the density of ones, have been so generated that the necessary records for a large number of trials may be kept. Simple statistical tests have been made to test the accuracy of this model.

Quite a large number of detectors of three different classes have been programmed and tested. The three classes are the sequential-observer detector described in Lincoln Laboratory Technical Report No. 20; the success-run observer based on the theory of success runs; and the density detector which consists of a moving slot inside of which the density of ones is measured. Data has been obtained on the operation of these detectors for several different signal-to-noise ratios and parameter settings. A report is now being prepared to present the results of this study.

Reference: William Feller, An Introduction to Probability Theory and Its Applications, John Wiley and Sons, Inc. 1950.

155 SYNOPTIC CLIMATOLOGY

The objectives of the synoptic climatology project under Prof. T. F. Malone of the MIT Meteorology Department have been stated in Summary Report No. 37. The five basic questions to be answered will be restated here along with the progress made toward their solution during the past quarterly period.

Questions:

1. How large a sample must be taken to insure stability of the prediction operators?
2. Over how large an area must the circulation pattern be considered?
3. How far back in time must the autocorrelation and crosscorrelation be extended before the point of diminishing returns on information is reached?
4. How much independence really exists between the several levels in the atmosphere?
5. How precisely must the circulation pattern be specified?

based on the 700-mb standardized height appear to be superior to the 700-mb height patterns for use as objective specifiers of temperature and precipitation.

The problem on Whirlwind involved computing fourteen coefficients of Tschebyscheff orthogonal polynomials fit to:

1. sea-level pressure,
2. 700-mb height,
3. 700-mb standardized height,
4. 700-mb cumulative-probability-of-height

for each day of five Januarys (1948-1952). Ninety-one sampling points were taken to represent the circulation over the region for each representation. Sums of cross-products of the various combinations of the orthogonal polynomial coefficients and the weather data were needed to supply the initial data for the multiple linear regression analysis. These cross-products were calculated on Whirlwind. Very little difficulty was encountered in the solution of these problems.

156 THE EVALUATION OF THE REFLECTION COEFFICIENT IN A SEMI-INFINITE OPEN RECTANGULAR WAVEGUIDE

The evaluation of the reflection coefficient in a semi-infinite open rectangular waveguide is obtained approximately by using Fourier transform techniques on the integral equations of the Wiener-Hopf type. The integrals are to be evaluated by the trapezoidal rule.

After making suitable checks, it was decided that the results from the test run for $0 \leq a < \pi^2$ were correct for the interval used in the trapezoidal rule but that this was not sufficiently accurate.

To obtain the additional accuracy desired, the intervals used in the trapezoidal rule were taken smaller around the singularity of the integrand. Corrections were made in the program to introduce these changes, and, so far, a successful run has not been obtained.

Once a successful run is completed on the above, the final section of the problem will remain to be done, i.e., the section for $-30 \leq a \leq 0$.

A successful run was made for $200 \leq a \leq 300$ in steps of 5.

This work has been carried out by A. Balser of Columbia University. Work was delayed during this quarter because of Mrs. Balser's absence from Cambridge.

159 WATER USE IN A HYDROELECTRIC SYSTEM

The following programs have been written and appear to be working satisfactorily:

1. A data program takes 40 years of weekly flows of the Columbia River at Grand Coulee, processes them, and stores them on the drum.

A designer, in using these results, would first establish the maximum allowable transient settlement for the building and then enter these charts to obtain the required footing size under the specified blast-loading condition.

162 DETERMINATION OF PHASE SHIFTS FROM EXPERIMENTAL CROSS SECTIONS

This problem, undertaken by Dr. F. J. Eppling of the MIT Laboratory for Nuclear Science, is concerned with the analysis of a nuclear-scattering experiment, the elastic scattering of protons by 0^{16}, over a range of bombarding energies from about 0.5 Mev to 4.6 Mev. Cross sections were measured at eight scattering angles from 168.0° to 90.4° (in the center of the mass system). From the experiment a series of eight curves is obtained in which the absolute differential cross section, d /d , is plotted as a function of the bombarding energy of the incident protons at each of the eight scattering angles. From these curves it is then possible to obtain d /d as a function of scattering angle at any given bombarding energy.

For a reaction such as the one above the theoretical expression for d /d is given as follows:

$$\frac{d\sigma}{d\omega} = ƛ^2 \left| \frac{-\eta}{2} \csc^2 \frac{\theta}{2} e^{i\eta \ell \eta \csc^2 \frac{\theta}{2}} + \sum_{\ell=o}^{\infty} (\ell + 1) P_\ell e^{i(\alpha_\ell + \delta_\ell^+)} \sin \delta_\ell^+ + \sum_{\ell=1}^{\infty} \ell P_\ell e^{i(\alpha_\ell + \delta_\ell^-)} \sin \delta_\ell^- \right|^2 + ƛ^2 \sin^2\theta \left| \sum_{\ell=1}^{\infty} P'_\ell e^{i\alpha_\ell} \left\{ e^{i\delta_\ell^-} \sin \delta_\ell^- - e^{i\delta_\ell^+} \sin \delta_\ell^+ \right\} \right|^2$$

where:

$ƛ = \frac{\hbar}{\mu v}$,

$\eta = \frac{zZ\epsilon^2}{\hbar v}$,

θ = the scattering angle in the center of mass system,

$P_\ell (\cos\theta)$ = Legendre polynomial or order ℓ,

$P'_\ell (\cos\theta) = \frac{d}{d(\cos\theta)} P_\ell (\cos\theta)$,

$\alpha_\ell = 2 \sum_{x=1}^{\ell} \tan^{-1} \frac{n}{x}$; and $\alpha_o = 0$.

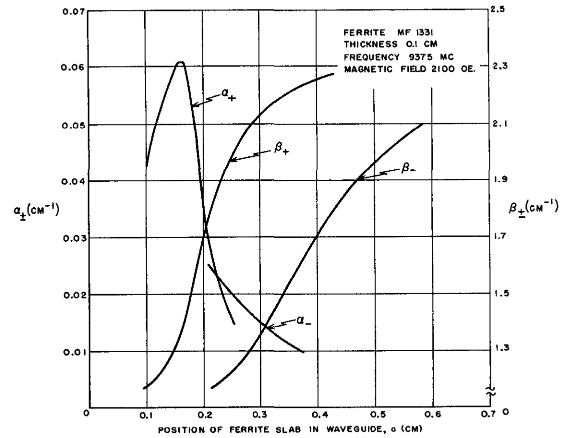

Fig. 2-4

The Attenuation Constant and Phase Constant for the Positive and Negative Directions of Propagation as a Function of Ferrite Slab Position

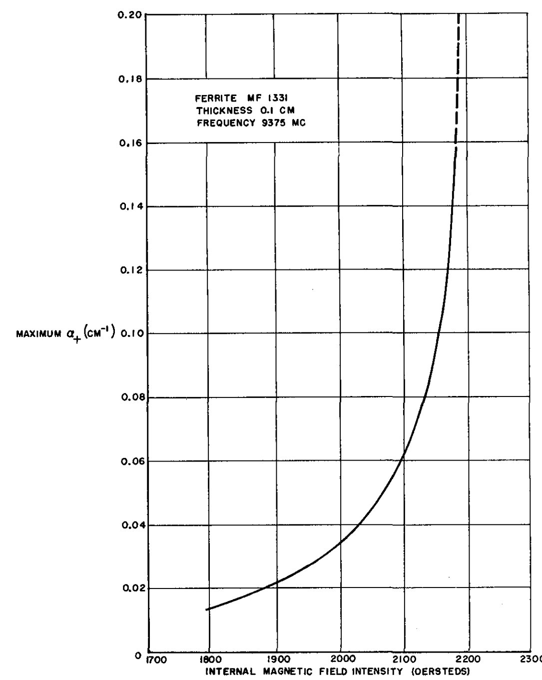

Fig. 2-5

Maximum Value of Attenuation Constant for Forward Propagation as a Function of Internal Magnetic Field Intensity

both directions of propagation as a function of the position of the ferrite in the waveguide for a constant value of the magnetic field of 2100 oersteds. A different value of magnetic field intensity was used for each set of solutions.

The dependence of the propagation constants upon the magnetic field intensity within the ferrite material has been determined in a region below ferromagnetic resonance. As the internal field was increased in an effort to look further into the resonance region, the peak value of the α_+ curve (see Fig. 2-4) increased sharply as expected. This indicated a strong absorption of the signal power by the ferrite material near the ferromagnetic-resonance condition. A plot of the maximum value of the attenuation constant for forward propagation is shown as a function of field intensity in Fig. 2-5.

At 2200 oersteds, the CS program was found to be incapable of providing values of α_+ larger than 0.2. Above this value the individual solutions of the two transcendental equations did not converge to a simultaneous solution. This indicates that the behavior of one or more of the transcendental factors as parameters are changed was not predicted properly. Several methods of analysis and approximation have been attempted without success. It has been established that an analysis involving extensive hand computation will be required in order to study the behavior of the rapidly changing transcendental factors which contribute

When the basic program was completed it was possible to utilize it to learn more about the physical aspects of the lattice-network design problem. On the basis of the runs that were made, the following conclusions were made evident:

1. Using simplifying assumptions, having perhaps questionable physical validity, it is possible to quickly and easily arrive at first trial stiffnesses which yield influence coefficients which match the given coefficients to well within 100 per cent on the average.

2. When it is necessary to eliminate an unknown stiffness, it is more satisfactory, mathematically, to assume a reasonable member cross section, thus defining the ratio of torsional to bending stiffness, rather than to equate two unknown stiffnesses. Although both types of assumption are reasonable from the physical point of view, the latter type leads to ill conditioning of the final matrix used to evaluate the X's.

The agenda for the immediate future will include constructing and testing a three-bay lattice structure of the physical dimensions incorporated in the present program. By co-ordinating the testing between the computer and structures laboratory phases, it is expected that a satisfactory conclusion to the problem can be obtained.

2.3 Tape-Preparation Statistics

During the past six months time records have been kept on all punched paper tapes prepared for Whirlwind I. These records have distinguished among three types of tapes because of the difference in procedures involved.

1. Original complete tapes are prepared directly on standard Flexowriter equipment from a coded manuscript submitted by the programmers.

In order to catch undetected mistakes the tape is verified by having a second person type the manuscript independently on a verifier. As the tape is typed, the first tape is compared line by line with the newly punched one. Disagreements in tapes will cause the verifier keyboard to lock while the typist determines which tape is correct.

Library subroutines can be inserted anywhere in the program by reproducing the subroutine tape onto the program tape. A comparator is used to detect errors in the duplication by comparing the original tape with the copy. If any difference is detected the comparator will stop.

167 TRANSIENT EFFECTS IN DISTILLATION

Work has continued under the direction of J. F. O'Donnell of the MIT Chemical Engineering Department on three basic problems which have been mentioned in previous reports. These problems all involve transient effects of holdup in binary distillation. They are: product takeoff in batch distillation, equilibration in batch distillation, and transients in continuous distillation.

For each case two types of programs have been written for Whirlwind. The first program finds the set of steady-state conditions (initial or final) for a specific set of parameters. This involves solving a set of simultaneous nonlinear algebraic equations and usually takes about one minute of machine time. The second program determines the variation of the compositions in the system as a function of time for a set of parameters during a transient period. These programs require the solution of sets of simultaneous nonlinear differential equations and usually have taken from five to twenty minutes of Whirlwind time each.

A decision was made to solve the differential equations by a Runge-Kutta method. When some trials were made using a fourth-order subroutine, they showed that the maximum permissible interval size was limited by instability rather than by convergence problems. Consequently, solutions were tried using a second-order routine and were found to give adequate results. This method was used henceforth.

The above-described programs are working satisfactorily and have been used to obtain considerable data. Much valuable information has been gained by studying the data, and the study is continuing. Sufficient data is now on hand for some phases, and further data will be taken where they are necessary.

In most cases qualitative relationships have already been established. Now the emphasis is on obtaining quantitative relationships for use by designers. For the case of product takeoff in batch distillation this has been difficult. At present a supplementary program is being written for Whirlwind which will calculate the product composition as a function of time for any set of parameters when the holdup in the system is taken to be zero. This is a limiting case which the present program will not solve. Comparisons of results with holdup to results without holdup, other conditions being the same, appear promising. Again a Runge-Kutta integration will be done of the nonlinear differential equation.

For the case of equilibration in batch distillation much of the work thus far has been done by Myers and has been described in detail in his thesis.[1]

1. Myers, H. J., "Equilibration Time in a Batch Distillation System," S. M. Thesis, Department of Chemical Engineering, MIT (1954).

The reduction of the function is performed by the following rules (note: all operations and variables are Boolean; let x be a "single variable"[1] and A and B be the products of any number of single variables):

1. AX + AX' = A
2. XB + X'AB = XB + AB
3. A + AB = A (this includes A + A = A)
4.[2] $X \underbrace{\text{(all or any part of A)}}_{f_1\{A\}} + X' \underbrace{\text{(all or any part of A)}}_{F_2\{A\}} + A = X f_1\{A\}) + X' (f_2\{A\})$

Note that an appearance of X indicates a lack of generality of the rule involved. Whether or not this is a major shortcoming remains to be seen. Spatial and temporal sequence applications of the rules have been found to be important; these points must be investigated further. The present system of applying the first three rules in order (1, 2, and 3) to each pair of terms and then applying rule 4 has led to satisfactory solutions in trial runs but is not always the best answer to the problem. It does not lead to the (or a) minimum solution for each arbitrary function. Several ideas for obtaining solutions that can be proved to be minimal in every case have been tried, and a fresh approach is being sought. If a more suitable solution is found, it will be programmed.

170 INVERSE AND INVERSE SQUARE ROOT OF A REAL SYMMETRIC MATRIX

A routine has been written by Dr. A. Meckler of the Solid State and Molecular Theory Group for the library of subroutines which calculates the inverse or the square root of the inverse of a real symmetric matrix. The routine has as its core the matrix-diagonalization routine developed under Problem 134. The given matrix is first diagonalized, the eigenvalues remaining in fast storage, the transformation matrix on a drum group. The reciprocals or the square roots of the reciprocals are then taken, and this diagonal matrix is then undiagonalized by the transposed use of the original transforming matrix.

The parameters to be planted in the routine are the order of the matrix, the starting address of the matrix, the criterion for diagonalization, and a drum group. This preparation is exactly like that necessary for straight diagonalization. The order of the matrix is again restricted to be less than or equal to 32 because of drum-group capacity.

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1. i.e., one of the relay-operation variables that form the group of independent variables for the E. C. Hoy prescribed switching function.

2. "All or any part of A" is to mean a product of any number of the single variables composing A (the product of no variable to be construed as "1"), e.g., if A is xyz, a product of single variables, a part of A would by xy; this explanation is necessary because when considering the origins of xyz and xy, xy actually includes xyz.

tions of the various wave functions and potentials involved. These programs have been written, tested, and combined along with an automatic "assembly" routine into one main tape. In operation, a toggle-switch setting allows any of six types of integrals to be obtained from the one tape.

At present these programs are being used to make calculations involved in the author's thesis in which the energy bands of graphite are being investigated by means of the tight-binding method.

173 COURSE 6.537 -- DIGITAL COMPUTER APPLICATION PRACTICE

Twelve students enrolled in the second semester Electrical Engineering course 6.537, entitled "Digital Computer Applications Practice," given by Professor C. W. Adams. The purpose of this course was to study the advanced preparation of coded programs for automatic electronic digital computers, particularly for Whirlwind I.

Each student programmed, prepared on punched tape, and executed on Whirlwind one problem of his own choosing, making use of CS II. The problems undertaken included: the simultaneous solution of a set of linear algebraic equations by the Crout method, the computation of a double integral of a probability equation, the solution of a delay-line spurious-noise problem, the formation of the Fourier components for discrete data taken over a period, the computation of the time of occurrence and the location of a quake by a least-square fit of empirical distance time curve, the finding of the roots of a polynomial by Newton's method, the solution of a set of linear equations by the Gauss-Jordan reduction method, the solution of a matrix equation by the Crout method, and the programming of an arc tangent subroutine.

174 TIGHT-BINDING CALCULATIONS IN CRYSTALS

The unperturbed wave function for a crystal is approximated by a linear combination of atomic functions. A Hamiltonian matrix is set up between these functions; the eigenvalues of this matrix are the energy levels of the electrons. The diagonalization of the Hamiltonian would be done on WWI.

A program has been written which computes the Hamiltonian matrix in the machine for the calculation of the energy levels of the electrons in nickel. Each element in the Hamiltonian matrix is a trigonometric polynomial of low order. After calculating the matrix the matrix is then diagonalized. This program has been tested, and the complete calculation of the energy levels of nickel is completed within this approximation of tight binding. The results are now being studied by Dr. G. Koster of the MIT Solid State and Molecular Theory Group.

tained, no endeavor has been made to allow for this interaction of sources and sinks. Consideration of the interaction of sources and sinks produces the following equations.

$$k P_k = \frac{p}{1 - p} P_{k-1} \sum_{w=0}^{S-k} (S - w - k + 1) Q_w \tag{1a}$$

$(k = 1, 2, 3 \ldots, S)$

$$\sum_{w=0}^{S} P_w = 1 \tag{1b}$$

where P_w is the probability that exactly w circuits will be simultaneously engaged as sinks,

S is the number of terminals which may serve either as a source or sink,

p is the probability of any given terminal being in use as a source at any given instant (assumed constant for all terminals),

and Q_w is the probability that exactly w circuits will be simultaneously engaged as sources (assumed equal to P_w).

From solutions to the above equations it is then required to find the realtionship between A = pS and N for a given B in the following equation.

$$B = \frac{1}{S(1 - p)} \sum_{k=N}^{S-1} \sum_{w=0}^{S-k-1} (S - k - w) Q_w P_k \tag{2}$$

A simple iterative procedure in which each P_k was calculated in turn as a function of the best available approximation of all the other P_k was first utilized. Although the method converged for small values of p, it diverged for large values of p.

A second procedure in which Equations (1a) and (1b) were reduced to a set of linear simultaneous equations was then devised. These equations were then solved by conventional means, and iteration of the process was found to converge over all values of p required.

A study has been completed using this second method over the following ranges of values:

Equation (1) $p = 0.02$ to 0.20
$S = 25, 50, 100, 200.$
Equation (2) $N = Sp, Sp + 1, Sp + 2, \ldots$ until $B \leq 0.001$.

Allowance for a typical distribution of calling rates produces the following equations:

able rank of the present program, now limited by high-speed storage space, then to program the complex eigenvalue problem for the machine.

This study is being conducted by H. M. Voss of the MIT Aero-Elastic and Structures Research Laboratory.

178 TRAJECTORY STUDY AGAINST AN EVADING TARGET

This problem has been concerned with a trajectory study of a missile seeking an evading target. It was desired to determine the minimum passing distance of these two objects for a variety of missile velocities, target velocities, and evasive courses. For each case it was assumed that the target evaded by flying a circular course instead of a predicted linear one. The procedure was to compute the distances between the missile and the target with reference to a certain coordinate system for some initial guess at the independent variable time. The rate of change of the distance was calculated. When this rate came sufficiently close to zero the corresponding value of the distance was considered to be the minimum passing distance.

The precedure was coded by C. Block of the MIT Instrumentation Laboratory. About 2000 runs were made and the results have been presented in tabular and graphic forms in Instrumentation Laboratory Report T-50. This report has been submitted as an M. S. thesis by R. Hansen and R. B. Doane to the Electrical Engineering Department.

179 TRANSIENT TEMPERATURE OF A BOX-TYPE BEAM

The transient temperature response of a box beam due to a time varying heat-flux input on one flange has been calculated by L. A. Schmit of the MIT Aero-Elastic and Structures Research Laboratory making use of an uncoupled finite-difference procedure. The system was treated as two dimensional (of unit length). Reradiation and convection losses were neglected, and the thermal properties of the structure were assumed constant. The calculated results are being compared with experimental test data.

As a result of this study two factors affecting the temperature distribution in this type problem come ot light.

1. The absorptivity of the irradiated surface as measured in the laboratory prior to field test may be considerably (as much as 30 per cent) different from the absorptivity of the exposed surface at the time of the field test. The full explanation of this substantial difference is not known, but a partial explanation is thought to lie in the accumulation on the exposed surface of highly reflective sand and dust between the time of laboratory measurements of absorptivity and the time of exposure in the field test. This difficulty is overcome by calculating the effective absorptivity from calorimeter and temperature data obtained during the field test. The final calculations (Tape 179-86-8) are based on a calculated

were entirely satisfactory for the purposes of the thesis, but, because of the several stages of intermediate hand computation and tape preparation which were required, such a system is valueless for production work and does not make efficient use of computer time. Required output time, in particular, is excessive.

In order to render the program more generally useful, the correlation portion has been rewritten in CS II and combined with the transform portion in such a way that tape preparation is greatly simplified, and computer and output time is minimized. The program handles functions having up to 250 values; it may be used whenever it is desired to cross-correlate two functions and evaluate the cosine and sine transforms of the resulting function or to autocorrelate a single function and transform the result (since only the cosine transform is required in this case, the program may be stopped after this has been obtained, thus reducing the required time further).

Output is obtained in the form of oscilloscope plots of (1) the even part of the correlation function, (2) the measure of indecision of the cosine transform, and (3) & (4) the cosine transform of the even part of the correlation function <u>before</u> (3) and <u>after</u> (4) a smoothing process which is written into the transform program. When the sine transform is desired four additional frames are plotted, similar to the above, except (1) shows the <u>odd</u> part, and (2), (3), and (4) involve the sine transform of the <u>odd</u> part of the function. The scope and camera are the only output devices used by the program, which considerably simplifies operation, especially when it is run with other programs which make use of the delayed-output equipment (magnetic tape).

181 PERTURBED COULOMB WAVE FUNCTIONS

Nuclear scattering in general remains the most direct way to get information about nuclear forces. Of particular interest is the comparison between neutron-deuteron scattering and proton-deuteron scattering, because it may be assumed that the forces involved in the two cases differ only by the addition of an electrostatic interaction between the charged proton and deuteron. The validity of this assumption requires that the specifically nuclear force between two protons be the same as that between two neutrons, and its proof or disproof is of basic interest.

An analysis of the nuclear-deuteron scattering data requires knowledge of the scattering that would result with only an electrostatic interaction between proton and deuteron. In the case of two point charges, this is given by the so-called Coulomb wave functions, tabulated in some detail by the U.S. Bureau of Standards. For proton-deuteron scattering, the charge distribution of the deuteron cannot be considered at a point, and the corresponding functions must come directly from the linear second-order differential equation by a numerical solution.

182 CRYSTAL STRUCTURES

In solving a crystal structure, the corrections to the atomic parameters are obtained by a least-squares method as a set of n simultaneous linear equations in n unknowns; n may be 30 or more. It is required to solve this system.

This problem is a continuation of Problem 105 to make use of routines developed by Dr. A. Meckler under Problem 134.

This problem involves the inversion of a 30th-order matrix with an eigenvalue ratio of about 1000 : 1. The only WWI routine currently available for the inversion of a matrix of this order proved to converge too slowly to be feasible. This routine is being rewritten in a form which will operate at least five times as rapidly as the former version. Consideration is being given to the development of other routines for handling matrices too large to be stored in rapid-access memory.

183 BLAST RESPONSE OF AIRCRAFT STRUCTURES

The present program is the first phase of a study by H. Lin of the MIT Aero-Elastic and Structures Research Laboratory attempting to establish a lethal criterion for airplane structures upon encountering a blast gust. For the purpose of examining some of the simplifying assumptions necessarily made in actual aircraft structures and of investigating the feasibility of a step-by-step solution in solving a nonlinear dynamic problem, this first phase treats the simple case of a cantilever, uniform, weightless beam subject to a triangular pulse at its tip mass. The structural characteristics of the beam are assumed to be such that its moment-curvature curve is bilinear. The resulting equation is a second-order nonlinear ordinary differential equation.

All quantities in the governing nonlinear equation have been nondimensionalized. The parameters of each case to be computed by WWI are:

K = buckled slope of the moment-curvature curve,

ϵ_y = yield curvature of the beam, and

τ_f = duration of triangular pulse.

The ultimate quantities desired from the present program are:

ϵ_a = maximum curvature at the root (the fixed end),

R' = ratio of maximum curvature to yield curvature, and

$Z_a \Delta \tau$ = time to reach maximum root curvature.

For a given beam of known ductility factor R defined by

$$R = \frac{\epsilon_c}{\epsilon_y},$$

accuracy on WWI for certain trial functions. If the calculation is successful, it is expected that the method will prove important in the calculation of the cross sections for scattering from more complicated atomic and nuclear systems.

An expression for the differential amplitude for the scattering of electrons from hydrogen atoms has been devised and has the property of being stationary with respect to variations in the wave function. The integrals in this expression have been reduced, for a given trial wave function, to a two-dimensional form. These two-dimensional integrals are to be evaluated by Simpson's rule for various values of angle of scattering and incident electron energy.

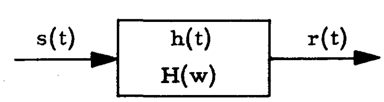
Programs have been written and tested for various forms of the integrand. A program to evaluate a two-dimensional integral, for a general integrand, has been written and tested for various mesh sizes.

186 TRACKING RESPONSE CHARACTERISTICS OF THE HUMAN OPERATOR

In many control systems (fire control, missile control, aircraft flight controls, etc.) the human operator is an important component of the system. To design such systems for optimum performance we must know the characteristics for all important components including the human operator.

The object of this investigation by J. I. Elkind of the Lincoln Laboratory at MIT is to measure the system characteristics of the human operator. To do this we determine the best linear representation for the human operator as well as try to measure his nonlinear characteristics. It is known that the human operator's system characteristics will change when some of the parameters of the control system of which he is a part are varied. His system characteristics tend to vary with time and also have been shown to be a function of the display, input signals, and output devices used. It is planned to explore these variations so as to be able to predict how the human operator will behave in a particular situation. Such information should be of considerable aid in the design of nonmachine systems.

In the experimental part of this study s randomly moving target is displayed, and the subject is asked to move a control to track the target movements to the best of his ability. In the simplified case we can represent the human operator by a linear filter with impulse response h(t). This simulation is shown in the figure below. The input stimulus is s(t); the operator's response is r(t).



188 EFFECT OF GRAVITY ON RELATIVE WATER PRODUCTION IN OIL RESERVOIRS

In a large number of oil reservoirs water, which underlies the oil, is produced along with the oil. The relative amount of water production depends for a given geometrical configuration on the effect of gravitational forces relative to the imposed pressure forces. In order to enhance the effect of gravity, the common practice is to plug the bottom part of the well bore so that fluid is produced from near the top of the formation; in some instances the imposed pressure forces are held to a minimum.

This problem has been undertaken in an effort to gain quantitative information concerning the effect on relative water production of partially penetrating wells and reduced pressure forces.

The interesting feature of this problem is the demarcation surface separating the oil and the water. This is a free surface in the sense that its location is not known beforehand but must be determined in the course of the computation.

The pressure p in either the oil or water zones obeys the following equation:

$$\frac{1}{r}\frac{\delta}{\delta r}\left\{r\frac{\delta p}{\delta r}\right\} + \frac{\delta^2 p}{\delta z^2} = 0 \tag{1}$$

where r is the radial distance from the center of the well bore and z is the vertical depth below the top of the reservoir.

Since no fluid moves across the upper and lower boundaries of the reservoir nor across the cylindrical surface where the well bore is plugged, simple boundary conditions in terms of the normal derivative of p are obtained at these boundaries. At the well bore and the outer radius of the formation the pressure is a predetermined function of depth. At the interface between the oil and the water the pressure in the water zone must equal the pressure in the oil zone; furthermore, since no fluid moves across the interface, the normal derivatives of water and oil pressures are simply related.

The proposed method of solution is to assume a fixed interface position and solve Equation (1) for the water and oil zones independently. The interface position and the pressure difference between the well bore and the outer radial boundary are then adjusted so that water and oil pressures at the interface are equal in the least-squares sense. The procedure is then repeated with the new interface position and new boundary pressures.

In the actual calculation the procedure has been simplified by making a linear transformation on pressure; a logarithmic transformation has been made on r to increase accuracy near the well bore.

nature and origin of these waves. Thus far, analysis has shown them to be Rayleigh waves of high frequency generated by standing ocean waves.

For the past few weeks amplitude spectra of two earthquake seismograms were computed and plotted according to a density-plot routine as previously described. Both analyses displayed visually many of the reflected and refracted body waves as well as dispersive characteristics in the surface waves which have been predicted from theory.

In addition to this, several power spectra of microseism trains were computed. These computations afforded additional evidenceffor the origin and nature of microseisms and also indicated a resonance phenomenon existing when the origin of same has moved to deeper regions of the ocean.

In the future a similar analysis will be made on those waves which arrive first from the source of an earthquake.

3.1 Systems Engineering

3.1.1 System Performance

The computer performance record for this quarter was essentially the same as for the previous quarter. During the 1026 hours assigned to computer applications, operators' reports showed an average reliability figure of 90.8 per cent. Approximately 22 hours (or 25 per cent) of the time lost were traced to sudden tube failures. In one particular instance, an intermittent short in a buffer amplifier of clock-pulse control which provides synchronizing pulses for the input-output devices resulted in the loss of 12 hours. As a consequence, additional checking facilities have been subjoined to the input-output-control marginal-checking test program to provide a more comprehensive test for such failures.

3.1.2 Replacement of Power-Distribution Panels

Within the past quarter the entire complement of "P" Row power-distribution panels has been replaced with units of new design. The new panels contain plug-in-type relays that should be more reliable and should reduce maintenance time considerably. After installation was started it was discovered that approximately 5 per cent of the new relays were defective The wires connecting the relay internally with the octal base plug were improperly soldered. A thorough inspection of all relays of this type, therefore, was carried out to select satisfactory units. The manufacturer has been notified and has assured us that he will modify his soldering process and institute an adequate final inspection to eliminate the defects.

3.1.3 Magnetic-Tape Facilities

The magnetic-tape units functioned with improved reliability. A second delayed-printout system has been installed and debugged. All of this equipment can be combined in a very flexible manner. Three of the 5 magnetic-tape units can be used interchangeably with either printout system, and both systems can operate simultaneously.

A design change in the head circuits of the read-record switch and reading amplifiers decreased the voltage level on the head windings from 150 to 0 volts The result diminishes the probability of insulation breakdown in the read-record heads.

3.1.4 Power-Supply Modifications

All of the original WWI d-c power supplies have now been replaced or modified to improve their operation. The replacement program was started about a year ago as a result of a Master's thesis research by J. J. Gano on power-supply and regulator design. Some of the improvements over the old supplies are faster response to the transients of

The disadvantages of increasing the electronics in the system are offset by reduced maintenance time and increased operating speed.

The time required for all the bay wiring necessary to install the new writers was comparable to the maintenance time already spent on present relay chassis (even though no attempt had been made to keep them operating at their original speed); a negligible amount of time was spent in maintaining the writers themselves. The number previously used in both drum systems is about equal to the number to be added.

With electronic group switching it is estimated that the group-switching operation will take about 128 microseconds as compared to 32 milliseconds which was allowed for the relays to operate.

At present all the bay wiring for the changeover has been completed (Fig. 3-2). One digit at a time is being removed from the relays and connected to the electronic write-group-selection circuits. Thus far, 8 digits have been converted. The additional 8 digits await the delivery of more writers from the shops. It is estimated that the writers will be received and the entire auxiliary-drum system converted to electronic write-group selection early in July 1954.

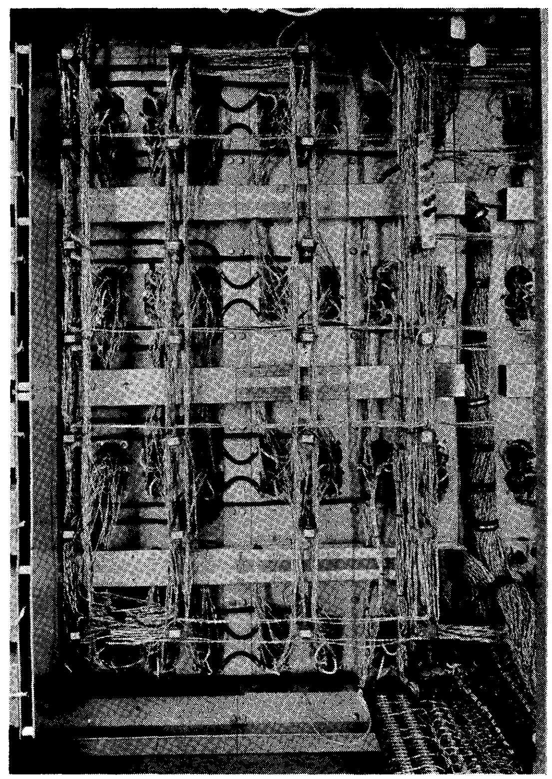


Fig. 3-2

Bay Wiring for Electronic Write-Group Selection in the WWI Auxiliary-Drum System. This bay has a capacity of 32 plug-in-type chassis, of which 27 are used for the new write switching. These 27 chassis replace 6 chassis used for relay switching.

3.2.2 Buffer-Drum System

A parity check has been added to the auxiliary-storage section of the buffer drum and has been of considerable assistance in detecting marginal operation of the buffer drum and in helping to localize system troubles.

Plans are presently being made to add 2 groups to the auxiliary-storage section of the buffer drum by utilizing presently unused tracks.

has prevented any further occurrence of the same trouble.

Initially, difficulty with the reader was expected from 2 apparently weak points in its design. The first of these was the brake-clutch mechanism. A high degree of performance is required of this device, since it must, if the reader is to operate properly, stop the paper tape from a speed of 20 inches per second in only 0.04 inch of tape travel. One instance of trouble occurred when mechanical adjustment loosened and caused the tape-drive to bind. By using lockwashers at those points which loosened, further trouble from this cause has been prevented. Some concern is still felt that wear may adversely affect future performance of the mechanism.

The second expected source of trouble was the exciter-lamp filament, the object actually focused onto the cathodes of the photoelectric tubes by the optical systems in the reader. Any motion of this filament causes its image to shift with respect to the photoelectric tube. (In fact, a shift of a sixteenth of an inch in the filament moves the image entirely off the cathode surface of the photoelectric tube.) The filament of the exciter lamp has proven sufficiently rugged to prevent this trouble.

It is now planned to install a second reader in the WWI system. The control for these 2 readers will be shared; the computer operator determines which reader is to be used by manually selecting one or the other. Both readers are kept on a standby basis with filament and plate voltages applied, allowing either to be selected for immediate use. Effectively, this provides a replacement reader that can quickly be put into service if one reader should fail. A further advantage is that the changeover can be made by the operator instead of a technician or an engineer. The design of the 2-reader system is complete, and the second unit should be installed during the next 3 months.

old 6Y6G tubes, and cathode followers using 5687 tubes. Engineering analysis of these tubes is not yet complete so that no firm data can be given on the condition of the troublesome tubes. Most of them were original tubes, which means that the 7AD7 and 6Y6G tubes had had about 25,000 hours of operation. The 5687 tubes were used in newer circuits; they had had only about 12,000 hours of operation. None of these tubes appears on the failure record for this quarter, since processing of the test information has been delayed by engineering tests.

First indications are that variation of heater voltage in conjunction with standard d-c marginal checking may be a valuable tool for locating certain classes of faults. However, the engineering analysis must be completed before conclusions can be drawn.

4.1.2 Vacuum-Tube Research

Continued analysis has shown that the analytical results presented in the last two Summary Reports are faulty. A requirement for electrical neutrality within the oxide layer was neglected. When this requirement is included in the analysis, the limiting value for iR_oq_o/kT (q_o assumed much less than the charge on 1 electron) is 2 rather than $2\sqrt{3}$. Experimentally, it appears that q_o, the average charge on a donor, is nearly that of 1 electron. Under such conditions, electron diffusion as well as electron mobility becomes important in the conduction process; then, the limiting value of iR_oq_o/kT is 4 rather than 2 as in the case when q_o is much less than 1 electron charge. The thesis study is now being written up, and the complete analysis, which is rather extensive, will be available in the thesis within the next several months. Thus, none of the details of the analysis will be included in this report.

This study of diffusion and electrolysis in oxide cathodes was conducted with the object of determining the reason for the "droop" frequently seen in the output of direct-coupled amplifiers when driven by a step function lasting for several seconds. In many cases this droop can be traced to the cathodes of various tubes in the amplifiers. As an original hypothesis it was assumed that the droop was caused by changes in the coating resistance of the oxide cathodes, following Nergaard's[1] depletion layer theory. The experimental results have shown fairly well that changes in the coating resistance are probably not the cause of the droop. In particular, it has been shown that the emission of an oxide cathode is strongly coupled to the direct current flowing through the cathode. When the direct current is sufficient to cause a voltage drop across the cathode of about 0.2 volt, the emission is frequently about equal to the direct current. This situation may correspond to a reduction in the zero-field emission of the cathode for direct current to as little as

1. Nergaard, L. S., RCA Rev. 13, 464 (Dec. 1952)

Type	Total in Service	Hours of Operation	Reason for failure; number failed: Change in Characteristics	Shorts, Opens,	Breakage	Burn-out	Gassy
6SH7	29	2000-3000	1				
6SN7	361	0-1000		1			
		7000-8000	3	1			
		11000-12000	1				
		23000-24000	3				
		24000-25000		1			
6V6GT	30	0-1000		1			
		2000-3000		1			
		23000-24000	3		1		
6X5GT	25	6000-7000		1			
		9000-10000		1			
6145/7AD7/ SR-1407	3118	10000-11000		1			
6145		no hours kept		1			
		0-1000	2	16		1	
		1000-2000	3	6			1
		2000-3000	2	7			
		3000-4000	1	9			
		4000-5000		3			
		5000-6000	1	4			
7AD7		7000-8000		2			1
		18000-19000		2			
		19000-20000		1			
		20000-21000	2	2			
		23000-24000		2			
		24000-25000	8	8	1		
		25000-26000		2			
7AK7	2592	0-1000	1	1		2	
		1000-2000	1	3			
		2000-3000	6	6			
		3000-4000	1				
		7000-8000		1			
		11000-12000		1			
		23000-24000	1	1			
		24000-25000			1		
12AU7/5963	402						
5963		1000-2000	1	2			
		2000-3000	8	1			
		7000-8000	3				
		8000-9000	2			1	
		10000-11000	1	1			
715C	124	no clock hours	1				
5651	36	4000-5000			1		
5687	105	0-1000		3			
		1000-2000		3			
		2000-3000	1	2			
		3000-4000	1	1			
		5000-6000	1				
		9000-10000	1				
		10000-11000		2			
5855	6	3000-4000	1				
5965	627	3000-4000		1			
		4000-5000		1			
		5000-6000		1			
6072	42	8000-9000	1				
		10000-11000	1				
6293	8	0-1000			1		
8008	30	4000-5000	1				

as small as possible. For a given minor diameter, the amount of thermal distortion is very nearly proportional to the square of this ratio.

These considerations indicate that tubes which can be expected to have very little drift for steps of several seconds probably will not have exceptional performances. Engineering compromises in practical design should allow tubes adequate for most purposes, however.

4.2 Component Replacements

Fig. 4-3 lists the replacements of components other than tubes during the second quarter of 1954.

5. ACADEMIC PROGRAM

5.1 Advanced Seminars on Computing

5.1.1 Seminar on Advanced Programming Techniques

These seminars provide an opportunity for the exchange of information on programming, components, logical design, and general developments in the computer field. The program was concluded during this quarter with the following talk:

9 April 1954	Recent Developments in the Digital Computer Field Including Current Trends and New Computers	C. W. Adams

5.1.2 Seminar on Computing Machine Methods

The seminars on Computing Machine Methods are arranged jointly by representatives of the MIT Committee on Machine Methods of Computation and the MIT Digital Computer Laboratory. The series was concluded during this quarter with the following two talks:

6 April 1954	An Electronic Analog Device for Delay Line Synthesis	Prof. James B. Reswick
20 April 1954	The Solution of Linear Programming Problems by the Simplex Method of Computation	Hrand Saxenian

5.2 MIT Courses

Twelve students enrolled in Electrical Engineering subject 6.537. Each student programmed, prepared on punched tape, debugged, and executed on Whirlwind I a problem of his own choice. The problems have been completed and are described under problem 173, above.

A series of three lectures was given on the subject of digital computers by J. D. Porter as part of Prof. T. Malone's course 119.33, Applied Climatology. A total of 23 students and staff members attended the lectures.

6. APPENDIX

6.1 Reports and Publications

Project Whirlwind technical reports and memorandums are routinely distributed to only a restricted group known to have a particular interest in the Project, and to ASTIA (Armed Services Technical Information Agency) Document Service Center, Knott Building, Dayton, Ohio. Regular requests for copies of individual reports should be made to ASTIA; emergency requests, to John B. Bennett, Lincoln Laboratory, P. O. Box 73, Lexington 73, Mass. Att: Code DCL-6. 1.

The following reports and memorandums were among those issued during the second quarter of 1954.

No.	Title	No. of Pages	Date	Author
SR-36	Summary Report No. 36, Fourth Quarter 1953	66		
R-221	Whirlwind I Operation Logic	109	5-1-54	M. Mann R. Rathbone J. Bennett
M-2728	Increased Facilities for Visual Display in the WWI Input and Output	18	3-17-54	G. A. Young
M-2729	Paper Tape Units and Printers in the WWI Input-Output System	19	3-15-54	G. A. Young
M-2834	Proposed Memory Address Selection System	3	5-26-54	D. Shansky
M-2868	Operation of the Magnetic Tape Printout Systems	2	6-16-54	A. Roberts J. Cahill R. Davis A. Favret C. Grandy
M-2870	Progress Report, May 17 through June 13, 1954	22	6-13-54	S & EC Group

MIT