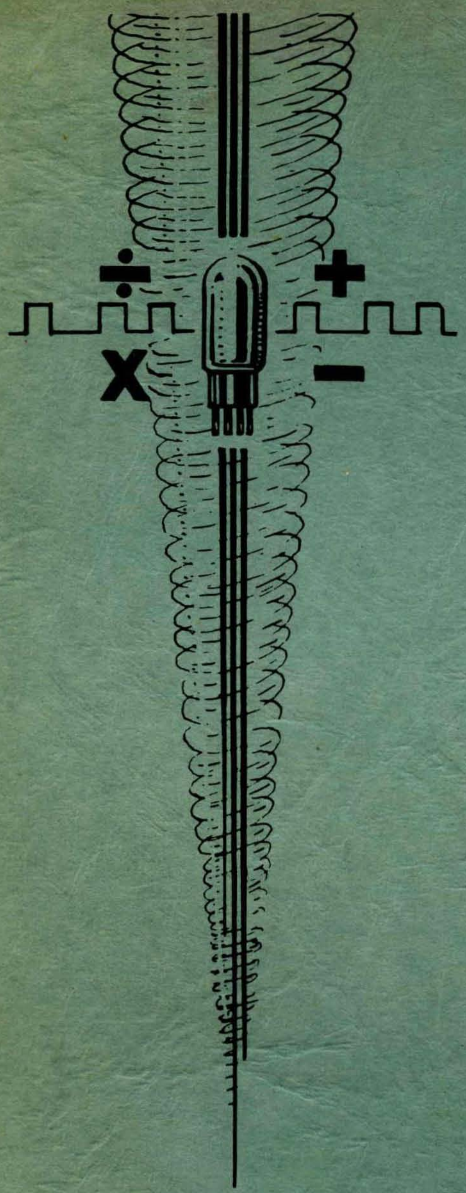

PROJECT WHIRLWIND

SUMMARY REPORT NO. 37
FIRST QUARTER 1954

DIGITAL COMPUTER LABORATORY
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

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Submitted to the

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DIGITAL COMPUTER LABORATORY
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Cambridge 39, Massachusetts

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FOREWORD

Project Whirlwind

Project Whirlwind at the Massachusetts Institute of Technology Digital Computer Laboratory is sponsored by the Office of Naval Research under Contract N5ori60. The objectives of the Project are (1) the application of an electronic digital computer of large capacity and very high speed (Whirlwind I) to problems in mathematics, science, engineering, simulation, and control, and (2) the study and development of component reliability in Whirlwind I.

The Whirlwind I Computer

Whirlwind I is of the high-speed electronic digital type, in which quantities are represented as discrete numbers, and complex problems are solved by the repeated use of fundamental arithmetic and logical (i. e., control or selection) operations. Computations are executed by fractional-microsecond pulses in electronic circuits, of which the principal ones are (1) the flip-flop, a circuit containing two vacuum tubes so connected that one tube or the other is conducting, but not both; (2) the gate or coincidence circuit; (3) the magnetic-core memory, in which binary digits are stored as one of two directions of magnetic flux within ferromagnetic cores.

Whirlwind I uses numbers of 16 binary digits (equivalent to about 5 decimal digits). This length was selected to limit the machine to a practical size, but it permits the computation of many simulation problems. Calculations requiring greater number length are handled by the use of multiple-length numbers. Rapid-access magnetic-core memory has a capacity of 32, 768 binary digits. Present speed of the computer is 40, 000 single-address operations per second, equivalent to about 20, 000 multiplications per second. This speed is higher than general scientific computation demands at the present state of the art, but is needed for control and simulation studies.

Reports

Quarterly summary reports are issued to maintain a supply of up-to-date information on the status of the Project. Detailed information on technical aspects of the Whirlwind program may be found in the formal R-series reports and the informal M-series memorandums that are issued to cover the work as it progresses. A list of selected publications, issued during the period covered by this Summary, appears in the Appendix.

1. QUARTERLY REVIEW AND ABSTRACT

Of the 36 problems which made use of computer time allotted to the Scientific and Engineering Computation Group during the quarter, 10 were new and 7 were completed. The revised version of the comprehensive system of service routines (CS II) is now in operation and is expected to come into every-day use shortly.

Computer reliability has continued high; 93 per cent of the 945 hours of assigned time was usable during the period. A reduction in maintenance time for the core memory has increased the number of hours which can be scheduled for operations. The consolidated test program mentioned in Summary Report No. 36 is in regular operation and will greatly increase the efficiency of the marginal-checking program. Terminal-equipment facilities have been improved and some new ones added, including a high-speed Ferranti photoelectric tape reader and a buffer-drum system.

Vacuum-tube life has been calculated for types 7AD7/SR1407/6145, 7AK7, 6080/6080WA/6AS7G, 5965, and 6BL7GT. A lower failure rate for the group 6080/6080WA/6AS7G is encouraging, since this tube is a vital part of the magnetic-core memory. Intensive experimental research has continued on transient changes in oxide cathodes as described in Summary Report No. 36.

Advanced seminars on computing were held regularly, providing an opportunity for the exchange of information on developments in the computer field. The 35 students enrolled in the DCL programming course represented the Departments of Electrical Engineering, Chemical Engineering, Mathematics, Geophysics, and Meteorology and the Aero-Elastic and Structures Research Laboratory, Instrumentation Laboratory, Dynamic Analysis and Control Laboratory, Lincoln Laboratory, Laboratory for Nuclear Science and Engineering, and the Solid State and Molecular Theory Group--all part of MIT.

2. MATHEMATICS, CODING, AND APPLICATIONS

2.1 Introduction

During the past quarter 36 problems made use of the computer time allotted to the Scientific and Engineering Computation (S&EC) Group. Progress reports on these problems are presented in numerical order in Section 2. 2. Of these, ten (158, 159, 160, 161, 163, 166, 168, 170, 171, and 172) represent new problems that are being described for the first time. Seven problems (108, 111, 112, 152, 153, 158, and 160) were completed during the period.

Tables 2-1 and 2-2 have been set up to provide the reader with a convenient reference to various interesting aspects of these problems. Table 2-1 lists the problems according to their fields of application. The column labeled "% WWI Time" has been included to indicate each problem's share of the WWI time allotted to the S&EC Group. The seven problems marked with asterisks represent work being performed by members of the MIT Project on Machine Methods of Computation, another phase of the work under contract N5ori60, which is reported more extensively in a separate report ("Machine Methods of Computation and Numerical Analysis," Quarterly Progress Report No. 11, Project DIC 6915, MIT, March 15, 1954).

The mathematical problems and procedures represented by the various current problems are tabulated separately in Table 2-2, because different problems in the same field frequently involve different mathematical methods, while problems in different fields sometimes are solved by identical methods.

As described in Summary Report No. 36, the letters A, B, C, and D are inserted after the problem numbers in these tables to indicate that a problem: (A) is neither for academic credit nor sponsored; (B) is just for academic credit, (C) is sponsored but not for academic credit, or (D) is both for academic credit and sponsored. The absence of a letter indicates that the problem originated within, and is being solved by, the S&EC Group.

The revised version of the comprehensive system of service routines (called CS II to distinguish it from the original system called CS I) is now in operation. It is expected that this system will rapidly come into routine use by all programmers. Some of the features of the new system are described below under Problem 100. Detailed descriptions will be provided in Digital Computer Laboratory memoranda. It is also planned to include these changes in the CS manual as soon as possible. Some of the progress reports given in Section 2. 2 include terms, such as floating addresses, number-system designations, and programmed arithmetic (PA), that refer to the CS system. For the reader's convenience, a brief description of these terms is repeated under Problem 100. For a more complete description of such terms, the reader is referred to Summary Report No. 32.

Field	Description	Problem Number	% WWI Time	Source
Aeronautical Engineering	Parametric study of the gust response of flexible swept-wing airplanes	153.C	1.53	MIT
	Construction and testing of a delta-wing flutter model	166.C	0.21	MIT
	Determination of the downwash at the tail of an airplane due to the lift response of the wing to a sharp-edged gust	168.C	0.79	MIT
Chemistry	Optical properties of thin metal films	101.C	1.09	MIT
Civil Engineering	Analysis of reinforced concrete walls	*113.C	7.64	MIT
	Study of shock waves; vibration problems in solid bodies	*142.D	0.01	MIT
	Response of cohesive and cohesionless soils to transient loading	*161.C	0.26	MIT
Electrical Engineering	Thermal and electrolytic diffusion in an oxide-coated cathode	152.D	4.09	MIT
Geology & Geophysics	Geophysical data analysis	106.C	8.52	MIT
	Interpretation of earth-surface resistivity measurements	*123.C	0.73	MIT
Instrumentation Lab	An interpretive program to accept mathematical symbols	108.C	3.54	MIT
	System of nonlinear differential equations	109.C	0.80	MIT
	Fourier analysis and autocorrelation calculations	111.C	0.00	MIT
Lincoln Lab	Digital methods of detecting signal from noise	149.C	0.51	MIT
	Transmission in a rectangular waveguide containing a single ferrite slab	163.C	0.25	MIT
Mathematics	Educational testing studies; Lawley's method of factor analysis	112.C	0.05	Outside
	Spherical wave propagation	*119.C	2.46	MIT
	Routine for multiplication of rectangular matrices of arbitrary order	157.C	0.04	MIT
Mechanical Engineering	Turbine design (aerothermopressor)	120.D	0.58	MIT
Meteorology	Synoptic climatology	155.D	6.00	MIT
Physics	Solution of Schrödinger equation for a coulomb potential	*122.B	0.26	MIT
	Eigenvalues of a real symmetric matrix	134.C	0.61	MIT
	Vibrational frequency spectrum of a copper crystal	143.D	2.13	MIT
	Energy bands in crystals	147.C	16.94	MIT
	Evaluation of the reflection coefficient in a semi-infinite open rectangular waveguide	156.A	0.66	MIT
	Use of water storage in a hydroelectric system to minimize the expected operating cost	159.D	0.32	MIT
	Similarity transformation of a matrix	160.C	0.47	MIT
	Overlap integrals of molecular and crystal physics	*172.B	0.47	MIT
Servomechanisms Lab	Autocorrelation and Fourier transform calculation	107.C	1.01	MIT
	Subroutines for the numerically controlled milling machine	132.C	0.83	MIT
	Data-reduction program; polynomial fitting	126.C	2.10	MIT
	Optimum response of an ideal, third-order relay mechanism	158.B	0.74	MIT
	Improved power-spectra calculations	171.C	0.66	MIT
Miscellaneous	Comprehensive system of service routines	100	29.36	MIT
	Library of subroutines	141	0.13	MIT

Table 2-1. Current Problems Arranged According to Field of Application (* MIT Project on Machine Methods of Computation)

Mathematical Problem	Procedure	Problem
1. Matrix algebra		
Matrix equation	Iteration involving Hotelling's method for solving the eigenvalue problem	112. C
System of fifteen linear equations	Relaxation	*113. C
Eigenvalues of a real symmetric matrix	Numerical diagonalization	134. C
Evaluation of elements and roots of a third-order secular determinant	Standard analytical solution	143. D
Matrix multiplication	Basic numerical	157. C
Similarity transformation of an n x n matrix	Matrix multiplication	160. C
Solution of a matrix equation	Iteration using Crout's method and matrix algebra	166. C
2. Transcendental equations		
Set of two transcendental equations	Iteration	101. C
System of three transcendental equations with six parameters	Iteration	158. B
Two simultaneous transcendental equations	Iteration	163. C
3. Ordinary differential equations		
General system	Gill's modified fourth-order Runge-Kutta	108. C
Fourteen simultaneous nonlinear	Fourth-order Runge-Kutta	109. C
Seven nonlinear first order	Step-by-step Euler method	120. D
Coulomb wave function equations	Series expansion	*122. B
Second order	Finite difference approximation	*142. D
Second-order linear with variable coefficients	Gauss-Jackson forward integration formula	147. C
Second-order nonlinear	Fourth-order Runge-Kutta	*161. C
4. Partial differential equations		
Nonlinear hyperbolic	Difference equations written along the characteristics and solved simultaneously by iteration	*119. C
Second-order nonlinear parabolic	Finite difference approximation	152. D
5. Integration		
Auto-, crosscorrelation and Fourier transform	Simpson's rule	107. C
Fourier analysis and autocorrelation	Trapezoidal integration	111. C
Integral evaluation	Trapezoidal rule	*123. C
Inversion of a Duhamel integral	Trapezoidal rule	153. C
Complex integral evaluation	Trapezoidal rule	156. A
Numerical integration	Trapezoidal rule	159. D
Numerical integration	Simpson's rule	168. C
Power spectra and confidence limits	Simpson's rule	171. C
Overlap integrals	Evaluation of analytic forms	*172. B
6. Statistics		
Multiple time series	Prediction by linear operators	106. C
Detecting signal from noise	Weighted counting	149. C
Calculation of the coefficients of a multiple regression system	Inner products	155. D
7. Data-reduction program	Polynomial fitting, etc.	126. C

Table 2-2. Current Problems Arranged According to the Mathematics Involved (* MIT Project on Machine Methods of Computation)

2.2 Problems Being Solved

100 COMPREHENSIVE SYSTEM OF SERVICE ROUTINES

The comprehensive system of service routines has been developed at the Digital Computer Laboratory to simplify the process of coding. The system in use since the fall of 1952, and described in Summary Reports No. 32 through 35, is now called CS I. A new system, CS II, based on CS I, was proposed in Summary Report No. 36. CS II is now in operation on a limited test basis and will soon be available on a routine basis. Some differences do exist between the interpretive routines of the two systems that make it impossible for all programs prepared for use with CS I to be used with CS II. For this reason CS I will also continue to be available on a routine basis as long as there is a demand for it.

Two of the many provisions included in both systems are referred to in some of the progress reports below. Consequently, brief descriptions of them are repeated here for reference.

1. Floating Address

A floating-address system enables a programmer to write his instructions so that they refer to the words of his program rather than to the location of those words in storage. The assignment of final storage locations is made by the computer as part of the conversion.

2. Number Systems and Programmed Arithmetic

(m, n) numbers shall mean numbers which are of the form $z = x \cdot 2^y$ where x is an m-binary-digit number and y is an n-binary-digit number. For example, (24, 6) signifies a two-register floating-point system dealing with numbers of 24 significant binary digits (roughly 7 decimal digits) with magnitudes between 2^{63} and 2^{-64}.

Arithmetic involving these (m, n) numbers is carried out by means of (m, n) interpretive subroutines. These subroutines enable the programmer to write coded programs using (m, n) numbers as easily as, or even more easily than, he might write programs in the single-length fixed-point (15, 0) number system which is built into Whirlwind I.

Purposes of CS II

The principal goals of CS II are to reduce (1) the time required to code and trouble shoot a program, (2) the amount of computer time required to run a program, and (3) the clerical work of the computer operator.

How these goals have been achieved is indicated in the following paragraphs.

The CS II Conversion Program

The CS II conversion program is similar to its counterpart in CS I. However, certain errors in Flexowriter program tapes are detected during a CS II conversion. These errors include: (1) illegal Flexo character, (2) unassigned floating address, (3) duplicate floating address, (4) overlapping by the program of the interpretive routine in high-speed storage, (5) improper floating-point number, (6) improper use of floating addresses in assigning words in the program, (7) too many floating addresses, and (8) too many output requests. When one of these errors is found during conversion, a short phrase describing the type of error and its location is recorded on a typewriter next to the operator's position, and the computer stops.

The CS II conversion program allows words to be assigned directly to any of the 20,480 available registers of auxiliary-drum storage. Future plans include a scheme for addressing the drum with floating addresses; only absolute addresses can be used at present.

A Flexo program tape may be read into the computer and operated immediately in CS II--production of an intermediate binary tape is optional. The detection of certain programming and Flexo tape errors before running an incorrect program and the elimination of the punching of binary tapes are expected to decrease the computer time required for running most CS programs.

The CS II Programmed-Arithmetic Routine

The CS II PA occupies about 20 per cent more high-speed storage than its CS I counterpart. However, it is about 10 per cent faster and also stops the computer immediately upon detecting certain logical and arithmetic errors while interpreting a CS II program. The computer stops in such a way that the operator, by a glance at the control panel, can deduce the type and location of the error. The errors so detected include: (1) attempting to execute an instruction not provided for in the particular PA routine being used, (2) referring to an improper floating-point number by an interpreted arithmetic instruction, (3) dividing by zero, (4) attempting to store an excessively large number, and (5) allowing the address section of an <u>isc</u> instruction to become too large.

The instruction <u>isc k,</u> select (cycle) counter k, is something new in CS II. Execution of an <u>isc k</u> instruction selects counter number k as the counter referred to by all chronologically subsequent interpreted instructions which use a counter. If k_{max} is the largest address section of an <u>isc k</u> instruction appearing in the original Flexo program, provision is made for k_{max} counters. Should the programmer mistakenly increase k beyond k_{max}, the PA routine detects the error and stops.

The buffer section of the CS II PA permits a programmer to obtain automatically multiple buffers. The multiple buffers are referred to by the addresses

b, 1b, 2b, ...

The actual storage addresses of the multiple buffers are assigned during the conversion process.

In case a CS II program must be stopped manually (because of a loop, for instance) the operator may press a button which will cause the interpretive routine to stop during the next interpreted transfer of control instruction.

In all these cases the operator then obtains semiautomatically (as discussed below) a so-called PA post-mortem which records enough information to allow diagnosis of the programming mistake.

CS II Post-Mortem Programs

CS II post-mortems are of two types: (1) PA post-mortem, and (2) storage print-outs.

The PA post-mortem is always obtained by the operator when a CS program stops unexpectedly. It is a typed record including the following information: (1) where the program stopped, (2) the interpreted instruction most recently executed, (3) contents of the interpretive arithmetic element, (4) contents of the index and criterion registers of all the counters called for in the program, and (5) the locations and present contents of the registers containing the five most recently executed interpreted transfers of control.

Storage print-outs in CS II are obtained by reading into the computer a short Flexo tape containing post-mortem requests. Post-mortem requests contain information about the output units to be used, whether the addresses of the registers to be printed are in octal or decimal, and how the words are to be recorded, e.g., as interpreted instructions, floating-point numbers, Whirlwind instructions, etc. The form of the request may be very general, and the rules for specifying the requests are extremely simple. For example, 200ii300 is a request to type the contents of registers 200 through 300 as interpreted instructions.

Utility Programs in CS II

Utility programs include the programs directly associated with CS I and CS II conversion and post-mortems and various programs for testing and calibrating in-out equipment. Other minor utility programs record a Flexo stop character on the delayed-printer equipment, allow examination of any register on the drum, etc. In CS II all these programs are literally as the operator's fingertips. Each utility program has a number corresponding to a labeled digit on a manual intervention register (MIR) at the operator's position. The operator selects the desired utility program by punching the corresponding button on the MIR. He then presses a starting button. This calls in a standard program which processes the information in the MIR and then brings in the requested utility program ready for operation.

Computer Logging

CS II provides for logging the read-in of each paper tape and of each utility program. The log is produced on a punched paper tape and on the scope. The problem number, programmer's number, and the tape serial number are recorded on the scope and photographed along with the time, date, and film index number. Thus all scope results are automatically identified, and the operator need not keep any record of camera operation.

The punched-paper-tape log contains the same information, except for the film index number. This tape therefore contains a complete record of computer operation. A program is being written which will process one or more of these paper-tape logging records and produce a typed summary of computer operation for any desired period of time up to several weeks. This summary will indicate primarily how much computer time was used by each problem in actual computation and in obtaining post-mortems.

101 OPTICAL PROPERTIES OF THIN METAL FILMS

The "Principal Program," described in Summary Report No. 35 as the "Main Program," was developed by Dr. A. L. Loeb of the MIT Chemistry Department for the evaluation of optical constants. This program processes data obtained for thin metal films on much thicker nonabsorbing backings. The input and output are as follows:

Input (on punched parameter tapes):

- Wavelength of incident radiation (λ)
- Thickness of metal film (a)
- Index of refraction of backing (n_b)
- Transmission of radiation normally incident on backing (T)
- Reflection of radiation normally incident on backing (R')
- Reflection of radiation normally incident on film (R)
- First estimate of index of refraction (n°)
- First estimate of absorption coefficient (k°)

Output (direct or delayed printer):

- Index of refraction of film (n)
- Absorption coefficient of film (k)
- Partial derivatives $\frac{\partial n}{\partial R}, \frac{\partial n}{\partial T}, \frac{\partial k}{\partial R}, \frac{\partial k}{\partial T}$
- Conductivity of film
- Dielectric constant of film

The "Inverse Program" modifies the Principal Program in that the index of refraction and the absorption coefficients are in the input instead of the reflection and transmission, while reflection and transmission are in the output instead of the index of refraction and the absorption coefficients. These last two coefficients are referred to as "Optical Constants."

At the end of the previous quarter a set of data reported by Krautkraemer was being evaluated. This evaluation has now been completed. The results of accurate computation on WWI confirm the validity of approximations made by Krautkraemer himself. We were able to compute separately the results for film incidence and for backing incidence, which Krautkraemer did not do. The discrepancy between these results indicates that Krautkraemer's films had rather irregular surfaces.

Woltersdorff has reported results of measurements with infrared radiation. The evaluation of these data led to rather new ideas and conclusions, and WWI was used as a laboratory instrument with various program modifications being based on the results of previous runs. The partial derivatives, $\frac{\partial n}{\partial R}$, etc., increase rapidly as the ratio of film thickness to radiation wavelength, $\frac{a}{\lambda}$, decreases. This indicates that the reflection and transmission are, in the infrared, not very sensitive to the optical constants, or, in other words, that an accurate determination of optical constants in the infrared requires extremely accurate reflection and transmission measurements. It also indicates that the iteration procedure used in the Principal Program should be used with great caution and that the first estimates of optical constants should be very good. Since such first estimates are particularly unavailable in the infrared, approximate methods were investigated and their validity tested by feeding the optical constants obtained approximately into the Inverse Program. Agreement between observed and calculated reflection and transmission was used as a test for the approximation. A first approximation used the assumption that the two optical constants are equal. This assumption is valid for massive metal and appeared to be fairly good for thin films, because the difference between observed and calculated reflection and transmission was not very large, though considerably larger than expected experimental errors. A better approximation was made and appeared to be excellent, because the difference between observed and calculated reflection and transmission was practically nil. The optical constants thus calculated differed from each other, though their product approximately equaled that of the optical constants calculated by the cruder approximation.

It appears that in the infrared R and T are sensitive to the product of the optical constants, nk, but not to individual variations in n and k made while keeping nk constant. The insensitivity of R and T to n and k had already been demonstrated by the size of the partial derivatives, $\frac{\partial n}{\partial R}$, etc. Therefore, a transformation of independent variables was made. One of the new independent variables is film conductivity, which is proportional to the product nk. The other independent variable is the dielectric constant, proportional to $(n^2 - k^2)$. It was then shown that in the infrared R and T are sensitive to the film conductivity but not to its dielectric constant. The crude approximation gave a fairly good conductivity (in other words, nearly the correct product, nk) but an entirely wrong dielectric constant (namely zero when $n \cong k$). Since R and T are very sensitive to the conductivity but not to the dielectric constant, the test calculation was very misleading. For thin films the optical constants do not equal each other.

The good approximation was combined with the Principal Program to find a first estimate for the optical constants, which can then be improved upon by iterations. It has been used to evaluate Woltersdorff's data and is now being used to compute the optical characteristics of transparent but slightly absorbing films made in our laboratory. Since this approximation neglects the backing materials in the infrared, it is not very good for the visible wavelength region. However, it has been shown that:

1. In the visible region the first estimate need not be very good,
2. The approximation is particularly valid when $\frac{a}{\lambda}$ is very small,
3. The influence of the backing is not very great when the film is relatively thick.

Therefore, in the visible wavelength region:

1. For the thinnest films the only appreciable error is that caused by the neglect of the backing, so that the approximation is not too bad,

2. For the thicker films the only appreciable error is that caused by the ratio $\frac{a}{\lambda}$ being large, so that the approximation is not too bad,

3. None of the first approximations is therefore so bad that it interferes with the iteration method.

106 MIT SEISMIC PROJECT

As discussed in previous reports, Problem 106 is concerned with the investigation of the use of test-analysis techniques in seismic-record interpretations, particularly in the separation of "reflections" from background interference on these records. More complete descriptions of the problem and the approaches used are contained in Summary Report No. 36 and in "Detection of Reflections on Seismic Records by Linear Operators" (Wadsworth, Robinson, Bryan, and Hurley--Geophysics, Vol. 18, No. 3, July 1953).

At the time of Summary Report No. 36 the Geophysical Analysis Group at MIT had arrived at a consideration of the problem in terms of the discrete-filtering properties of linear operators and had derived certain computational and theoretical results regarding the use of linear operators. The computational results included: (1) greater definition of the critical parameters in the operator structure, and (2) a separation of categories of record types for which the criterion used for operator choice was applicable and nonapplicable. The theoretical results included setting up procedures for approximating any desirable transfer function with linear operators.

Questions then raised were: (1) what further criteria could be used in selecting operators, and (2) what was the detailed statistical nature of the seismic records used. The answer to the second question depends largely on computational results and should be greatly influential in answering Question 1. For instance, if it should turn out that certain records exhibit a marked nonstationary character, it would be necessary to consider criteria involving filters which are nonlinear. From the standpoint of the linear operator this would mean the use of time-varying coefficients whose properties would vary corresponding to the nonstationary record itself.

Toward the solution of this second problem we have developed spectrum programs of a special nature. These perform spectrum analyses from highly overlapping portions of entire records. The variability of these results is an indication of the nonstationary character of the record. Because of the immense amount of output (4000 spectral estimates per seismic trace, 10 or more traces per record), the scope output is used, presenting the results in a density plot which allows direct analog-type interpretation. This program has just been completed, and the preliminary results are very satisfactory. It incidentally has had direct application to the problem of dispersed-wave propagation, permitting almost immediate determination of dispersion curves. Further programs along this line may show variability according to areal location of record, thus permitting estimates of the validity of extending methods from record to record.

The criteria being considered under Question 1 are, almost without fail, based on frequency information, and many of them involve a form of least squares. For this reason we have been improving our amplitude-phase and power-spectra programs during the last quarter as well as completing a matrix-inversion routine.

107 (a) AUTOCORRELATION AND (b) FOURIER TRANSFORM, INTEGRAL EVALUATION

This problem has made available routines for calculating autocorrelations, using rectangular rule or Simpson's rule for the evaluation of the integral, and for Fourier sine or cosine transforms, using Simpson's rule. Development work on this problem was completed by D. T. Ross of the MIT Servomechanisms Laboratory in the second quarter of 1953. However, this problem has been left open for users of WWI whose computation can be effected by these programs alone.

During the past quarter, these programs have been utilized by R. S. Tankin of the MIT Civil Engineering Department in the study of turbulence in water and by Dr. J. T. Farrar of the Massachusetts Memorial Hospitals in the analysis of human and animal intestinal-motility records. Descriptions of these two projects are included under this problem because of their general interest.

The project under R. S. Tankin is sponsored by the Office of Naval Research under Contract N5-ori-07874. It deals with the investigation of turbulence in water by means of a Pitot-tube pressure-cell combination. In turbulent flow, the velocity consists of a steady component and a fluctuating component which is of random nature.

Since the measuring instrument has a resonant frequency of 250 cps, it was decided to obtain an autocorrelation curve and find out the importance of this quantity. The autocorrelation function obtained from Whirlwind I indicated that the major portion of the signal had low-frequency components.

From the autocorrelation record, a mean intensity spectrum was obtained from Whirlwind I which bore out the fact that the mean power at 250 cps was small as compared to the mean power at 30 cps or less. However, this spectrum peaks at 3 cps and higher harmonics, indicating a nonlinear condition. This is believed to be in the flow itself, not in the instrument.

Dr. Farrar has submitted the following report describing his activities:

Existing methods of analyzing human and animal intestinal-motility records do not permit an objective, accurate measurement of the total amount of motor activity present, nor do they permit analysis of the component functions which make up the complex motility record.

Intestinal motility in man is composed of several different basic waveforms. These include: (1) periodic "segmental" waves having a frequency of 8-11 cpm, (2) large "peristaltic" waves of low frequency and questionable periodicity, and (3) random waveforms. Quantitative analysis of these components in a motility record has been impossible, as has been the evaluation of their relative contributions to the total motility record.

During the past year a cooperative program has been undertaken between the Gastroenterological Section of the Evans Memorial Hospital and MIT. The aim has been to apply analytical methods used in the study of sound to the interpretation of intestinal-motility records. The tracings of rabbit intra-enteric pressures and one very simple human balloon-kymographic record have been analyzed by generation of the autocorrelation function. This analysis has yielded numerical values for the frequency and amplitude of the various motility components, as well as their relative contribution to the total record. The autocorrelations were initially obtained by using the mechanical correlator in the Servomechanisms Laboratory. However, this machine proved unsatisfactory for the analysis of the more complex human records, since the wave excursions of the original data exceeded the narrow limits of the machine.

Generation of the autocorrelation function of a complex human motility record has been very satisfactorily achieved by the use of the Whirlwind I digital computer. Recordings are made originally on rolls of paper having a 1-mm grid. Numerical amplitudes are then read at each of the equally spaced (1-mm) points on the motility record. This data is prepared by Whirlwind personnel and the autocorrelation function calculated.

Four human records have been analyzed during the past two months. In all cases the autocorrelation functions appear superficially to reflect the periodicity and magnitude of the original records. However, the autocorrelation functions are too complex to permit accurate quantization of the amplitudes of the components. It is planned now to obtain Fourier transforms of these autocorrelations.

It is thought that these exploratory studies done on Whirlwind may form the basis for a more extensive study of intestinal motility by these analytical techniques.

108 AN INTERPRETIVE PROGRAM

A programmer's manual in the form of an Instrumentation Laboratory Engineering Memorandum (E-364) has been written by J. H. Laning and N. Zierler (see Summary Report No. 36). Copies of this manual are available upon request at the Instrumentation Laboratory Library.

The following major changes have been made in this program:

1. Instead of the problem tape being read in line by line on a mechanical tape reader, it is now inserted in the photoelectric tape reader (PETR) and read in completely and stored on the drum. This change speeds up the read-in phase considerably.

2. All the various tapes that make up this program have been combined onto one master tape, 108-60-30. Consequently, to make use of the interpretive program, a programmer simply writes the following instructions on his performance request (assume that the programmer's problem tape number is 178-63-14):

   a. E, 108-60-30 RI

   b. Insert 178-63-14 (STD) in PETR

   c. RS

The program will stop automatically on an $\underline{si}$ 1 in register 42.

All the routines described in E-364 have been tested and work correctly with the exception of the X|N routine.

A further report on the inner workings and programming methods used in this interpretive program has been started by N. Zierler. However, it will be some time before this report reaches completion, since the author transferred from the Instrumentation Laboratory to Project Lincoln.

109 AN AIRPLANE PURSUIT-COURSE PROGRAM

Two airplane pursuit-course programs have been developed by M. H. Hellman of the Instrumentation Laboratory at MIT which are based on the following assumptions. The first program restrains the airplane and target to the same horizontal plane. The second program restrains the airplane and target to a slant plane. Both of these programs include the effects of airplane dynamics and projectile ballistics.

Each of these programs is now being extended to compute prediction times for two-gyro and three-gyro gunsights. If comparable initial conditions are chosen for the horizontal and slant-plane programs, the prediction times computed from each will form a basis for comparing a gunsight calibration for a horizontal-plane pursuit course with a gunsight calibration for a slant-plane pursuit course. A satisfactory test run has been made with the modified horizontal pursuit-course program. A satisfactory test run with the modified slant-plane pursuit course is anticipated in the near future.

Upon completion of the above program, it is planned to make a number of slant-plane pursuit-course runs with the airplane and target initially in a slant plane. From the results of these runs it will be possible to evaluate the errors in the calibration of existing gunsights.

111 FOURIER ANALYSIS--AUTOCORRELATION PROBLEM

This study was undertaken by E. J. Frey of the Instrumentation Laboratory in the design and use of gyroscopes.

A function, $f(t)$, is given over the interval $(0, 12\pi)$ in the form of discrete values given at intervals of $\Delta t = \frac{2\pi}{360}$. The function is presumed to consist of a linear combination of the following periodic components: 1, sin t, cos t, sin 2t, cos 2t, and of a random noise, $n(t)$. The problem is to determine the Fourier coefficients a_0, a_1, a_2, b_1, b_2, and the autocorrelation function of $n(t)$.

For each interval $(2n\pi, 2(n+1)\pi)$ the Fourier coefficients are first calculated. A rectangular method of integration is used, i.e., ordinary summation. For a periodic function, this is the same as the trapezoidal method. In this case, where $f(t)$ is not strictly periodic, the only difference occurs in the evaluation of a_0, a_1, and a_2, and is equal to

$$\Delta a_0 = \frac{f(2(n+1)\pi) - f(2n\pi)}{720}$$

$$\Delta a_1 = \Delta a_2 = 2\Delta a_0$$

quantities which are negligible in the case considered. By a least-squares method, quantities $a_0+\alpha_0 t$, $a_1+\alpha_1 t$, $a_2+\alpha_2 t$, $b_1+\beta_1 t$, $b_2+\beta_2 t$ were fitted to the six sets of Fourier coefficients. (This last step was not done on WWI.)

The following was then computed:

$$g(t) = f(t) - (a_0+\alpha_0 t) - (a_1+\alpha_1 t)\cos t - (b_1+\beta_1 t)\sin t - (a_2+\alpha_2 t)\cos_2 t - (b_2+\beta_2 t)\sin 2t,$$

and an autocorrelation of g(t) was obtained. The method of computing the autocorrelation function was the following:

$$\phi\left(\frac{\pi}{180}n\right) = \frac{1}{2159-n} \sum_{K=0}^{2159-n} \phi\left(\frac{\pi}{180}K\right) \phi\left(\frac{\pi}{180}K + \frac{\pi}{180}n\right).$$

It was originally expected that the removal of the first two harmonics from the data would leave a stationary random process as the residual. However, the autocorrelation process indicated that the random process constituted a relatively small part of the residual and that the bulk of the residual consisted of combinations of analytic functions. Hence the original problem had to be attacked from this new standpoint.

No report was published on this work, since the results obtained were inconclusive as far as the physical equipment involved was concerned.

112 LAWLEY'S METHOD OF FACTOR ANALYSIS

This problem, undertaken in cooperation with Dr. F. M. Lord of the Educational Testing Service, Princeton, seeks to factor-analyze a 33 x 33 correlation matrix by Lawley's maximum-likelihood method successively under the several hypotheses that the number (m) of common factors is 4, 5, 6, . . . The number of common factors hypothesized is increased until a test of statistical significance shows that the data are in accord with the thus-modified hypothesis. For any given one of these hypotheses, the main problem from a mathematical point of view is to find the latent roots and vectors of a matrix whose diagonal elements are themselves functions of the latent vectors to be determined.

Complete factor analyses were run for m = 4, 5, . . ., 10. For m = 10, a statistical test showed that the data were in reasonable agreement with hypothesis. This completes the work on the Whirlwind computer. The latent vectors thus found were rotated to psychological meaningfulness on the matrix rotator at the Adjutant General's Office. Final results will be reported in the periodical literature.

113 A STRESS ANALYSIS OF AN L-SHAPED HOMOGENEOUS PLANAR STRUCTURE

A program has been prepared by S. Sydney of the MIT Civil Engineering Department for Whirlwind to analyze planar stresses in a rectangular plate which has a portion of its interior section removed. This plate simulates a two-dimensional slice of the throatless press (see Summary Report No. 35) which is being investigated by members of the Civil Engineering Department.

The convergence of the original program prepared for the analysis of the flat plate simulating a section of the throatless press is very slow because of the large number of panels that were used to approximate the plate. A new program has been prepared with a much "coarser" gridwork that will provide an initial set of values for the "fine" gridwork quite close to the final results.

The fine gridwork will now be used primarily for interpolation to obtain values of stresses at more closely spaced points.

119 SPHERICAL WAVE PROPAGATION

The propagation of spherical waves caused by the release of a compressed sphere of air, initially at rest in the atmosphere, is being studied. The problem is being solved by a numerical integration along the characteristic directions of the differential equations governing the flow. The problem has been solved for the case of an initial density distribution, $\rho = 1+2e^{-4r^2}$ (see Summary Report No. 36). In that case no shock was encountered. At present, work is proceeding on the case of an initial density distribution, $\rho = 1+5e^{-4r^2}$, for which it appears that a shock does form.

For this new case certain numerical difficulties were encountered which necessitated a good deal of reprogramming. This has now been completed, and a production run has been obtained using the new program. This computation takes the problem very close to the shock which appears to be forming near the center of the sphere at about time $t = 1.5$. Because of the nearness of the shock formation to the center of the sphere, some hand computation will be required to carry the computation up to the shock. An initial r-axis spacing of 0.06250 has been used for this computation.

A paper has been written by P. Fox and A. Ralston of the MIT Mathematics Department on the results described in Summary Report No. 36 which Sir Geoffrey I. Taylor will communicate to the Royal Society of London for publication.

120 THERMODYNAMIC AND DYNAMIC EFFECTS OF WATER INJECTION INTO GAS STREAMS OF HIGH TEMPERATURE AND HIGH VELOCITY.

This problem is connected with the development of a potential gas-turbine component, called an "aerothermopressor," in which a net rise in stagnation pressure of a hot gas stream is brought about by evaporative cooling associated with liquid injection into a high-velocity region of the flow. The term "stagnation pressure" of a compressible gas stream is analogous to the quantity "total head" in the more familiar problems of hydraulics. The role of the aerothermopressor in the gas-turbine cycle is itself analogous to that of the condenser in the steam power plant and would reduce both specific fuel and specific air consumption of the gas-turbine power plant. Further description of this device may be found in earlier reports, beginning with Summary Report No. 32.

The analytical work being carried out on Whirlwind I is concerned with the thermodynamic performance characteristics of the aerothermopressor; principal objectives are to obtain a better understanding of the complicated physical process and to obtain design data which may serve as a guide and foundation for future experimental work. During the past year, considerable information has been accumulated regarding the performance of constant-area aerothermopressors under various conditions of operation; methods for improving performance by means of suitable cross-sectional area variation were also studied. A complete report on this work, titled "A Theoretical Investigation of the Thermodynamic and Dynamic Effects of Water Injection into High-Velocity, High Temperature Gas Streams," has been submitted to the MIT Department of Mechanical Engineering in the form of a thesis for the degree of Doctor of Science. This thesis, written by Bruce D. Gavril and supervised by Professor Ascher H. Shapiro, is divided into three separate parts. Part I contains a survey of the historical development of the aerothermopressor together with a discussion of the fundamental concepts underlying its operation and application to gas-turbine power plants. Part II contains the derivations of the algebraic and differential equations of the one-dimensional aerothermopressor process, while Part III is devoted entirely to the programming procedures and problems involved in the numerical solution of the equations by means of Whirlwind I.

Because of its importance, this analytical work is being continued, and further calculations are currently being carried out with the primary purpose of developing a systematic calculation procedure for determining the cross-sectional area variation of the aerothermopressor evaporation section which will lead to optimum performance. Provision has also been made for treating aerothermopressors containing a variable-diameter plug within a concentric, constant-diameter cylindrical shell for purposes of supplying design data and subsequent interpretation of experimental results for the large aerothermopressor facility now under construction at the MIT Gas Turbine Laboratory.

The aerothermopressor development program is being carried out at MIT under the sponsorship of the Office of Naval Research.

122 COULOMB WAVE FUNCTIONS

A program has been written and tested by A. Temkin of the MIT Physics Department to compute the coefficients a_n^L in the recursion relation:

$$(n-L-1)(n+L)a_n^L = 2\eta a_{n-1}^L - a_{n-2}^L$$

$$n = -L,\ -L+1, \ldots$$

subject to the boundary conditions

$$a_{-L}^{L} = 1 \qquad a_{L+1}^{L} = 0$$

These coefficients are needed in an expansion, $\sum_{n=-L}^{\infty} \rho^{n} a_n^{L}$, which forms part of the irregular solution of the Coulomb wave function differential equation. Sufficient coefficients are assumed to be computed when $\rho^n a_n^L$ gets below a preassigned value.

For the values $\rho = 2$, $\eta = 0.5$, $L = 5$ the coefficients were found to converge very rapidly. For $\rho = 54$, $\eta = 15$, $L = 5$, however, the terms seemed to be actually divergent up to the 32 terms computed. This is an extreme combination of those parameters, and it may well be that it simply requires more terms before the convergence becomes apparent. We are, therefore, doing the calculation for $L = 5$, $\rho = 20$, $\eta = 20$, which is a more reasonable choice of these numbers.

123 EARTH RESISTIVITY INTERPRETATION

The problem under study is the determination of the resistivity distribution, $\rho(z)$, beneath the surface of the earth about a point source of current. This problem consists of two parts: first, the calculation of the Slichter kernel function from the observed potential data (see Summary Reports No. 35 and 36); and second, the analysis of the kernel function to yield the resistivity distribution.

The latter problem has been developed in the past quarter for the special case of three layers. The method used is the Newton method of solving sets of nonlinear algebraic equations, as described by von Sanden.* The Slichter kernel, $k(\lambda)$, was previously calculated for several values of λ from the observed potential about a point electrode situated on the surface of a semi-infinite medium containing two discontinuities (in resistivity) which are parallel to the surface. The parameters to be determined are d_1, d_2, and ρ_2, where d_1 and d_2 are the thicknesses of the first and second layers, and ρ_2 is the resistivity of the second layer. ρ_1 and ρ_3 are known from the asymptotic behavior of the potential function. It is known that, for a given distribution,

$$\frac{1}{1+k_{123}(\lambda)} = \frac{1}{2} + \frac{1}{2}\left[\frac{\rho_1-\rho_2 \quad \dfrac{1-\mu_{23}t_2}{1+\mu_{23}t_2}}{\rho_1+\rho_2 \quad \dfrac{1-\mu_{23}t_2}{1+\mu_{23}t_2}}\right] \qquad t_1 = f_{123}(\lambda) \tag{1}$$

* von Sanden, H., "Practical Mathematical Analysis," Methuen, London (1923), p. 163 ff.

where

$$\mu_{23} = \frac{\rho_2 - \rho_3}{\rho_2 + \rho_3}$$

$$t_1 = e^{-2\lambda d_1}$$

$$t_2 = e^{-2\lambda d_2}$$

An initial assumption of the values of the parameters must be made, and these are inserted into the expression for $f_{123}(\lambda)$ for several values of λ. For any value of $\lambda = \lambda_i$ chosen, let

$$\phi_i = f_{123}(\lambda_i) - f(\lambda_i)$$

where

$$f(\lambda_i) = \frac{1}{1+k(\lambda_i)}$$

Then,

$$\phi_{i1}\Delta\rho_2 + \phi_{i2}\Delta d_1 + \phi_{i3}\Delta d_2 - \phi_i = \delta_i \qquad (2)$$

where

$$\phi_{i1} = \frac{\partial\phi_i}{\partial\rho_1}\ , \quad \phi_{i2} = \frac{\partial\phi_i}{\partial d_1}\ , \quad \phi_{i3} = \frac{\partial\phi_i}{\partial d_2}$$

Equation (2) can be used to calculate δ_i for several values of λ. If three values are taken, there results a set of three linear equations to be solved for $\Delta\rho_2$, Δd_1, and Δd_2; consequently, δ_1, δ_2, and δ_3 can be reduced to zero.* If more than three values of λ are used, the system is overdetermined.

Then the values of $\Delta\rho_2$, etc., can be determined such that

$$S = \delta_1^2 + \delta_2^2 + \ldots + \delta_m^2$$

is a minimum. Carrying through the algebra, one arrives at the following set of equations:

$$A_{11}\Delta\rho_2 + A_{12}\Delta d_1 + A_{13}\Delta d_2 - B_1 = 0$$

$$A_{21}\Delta\rho_2 + A_{22}\Delta d_1 + A_{23}\Delta d_2 - B_2 = 0$$

$$A_{31}\Delta\rho_2 + A_{32}\Delta d_1 + A_{33}\Delta d_2 - B_3 = 0$$

$$A_{rs} = \sum_{k=1}^{m} \phi_{kr}\,\phi_{ks} = A_{sr} \qquad\qquad B_r = \sum_{k=1}^{m} \phi_k\,\phi_{kr}$$

* This is referred to as the three-point absolute-fit.

Thus the solution of the three-layer case again reduces to the solution of a 3 x 3 matrix which, in this case, is symmetric.

The entire procedure must then be repeated using the new values of the parameters

$$\rho_2^{(1)} = \rho_2^{(0)} + \Delta\rho_2, \text{ etc.}$$

and the repetition continued until S reaches a sufficiently small value or until the indicated changes become sufficiently small or random. The calculation for the case of more than three layers is entirely analogous and will be carried through if the results from this case seem to warrant it. However, it is not anticipated that any cases of more than six layers will be of any practical significance.

The three methods indicated above were applied to a three-layer case of parameters.

$d_1 = 1$	$d_1^{(0)} = 0.8$
$d_2 = 1$	$d_2^{(0)} = 1.2$
$\rho_1 = 1$	$\rho_2^{(0)} = 12$
$\rho_2 = 10$	
$\rho_3 = 0.1$	

The three-point fit converged in the first two steps, gave a wild value of d_2 in the third step, and then began to converge again.

The least-square fit, using 5-7 values of λ, was tried on the first step, and again using the results of the second step from the three-point method. Convergence here was considerably more rapid than in the previous method, but the same oscillation occurred in the third step. The reason was found to be a second derivative which, for the values concerned, was of the same order of magnitude as the first derivatives. This problem will be avoided by reducing the sizes of parameter changes, obtained from the solution to the normal equations, to some fraction of the indicated change.

Some investigation was made of the effect on the results of changing the values of λ_i used. A (logarithmic) plot of $k(\lambda)$ versus λ shows that it is asymptotic to constant values (determined by ρ_1 and ρ_3) for high and low λ, but in the intermediate range of λ, roughly $1 \leq \lambda \leq 0.001$, $k(\lambda)$ varies widely. The values to be used must be chosen from this range.

The program for the original problem, the iterative integration of the potential function to obtain the Slichter kernel, was completed but was found far too slow. A faster method was found and programmed and is being tested.

This work is being done by K. Vozoff of the MIT Geology and Geophysics Department with the assistance of Theodore Madden of that department.

126 A DATA-REDUCTION PROGRAM

In this problem a very large data-reduction program is being developed by D. T. Ross for use in the Servomechanisms Laboratory. The over-all problem is composed of many component sections which will be developed separately and then combined at a later date. Thus far, efforts have been focused on the development of utility-type programs. These programs, which have been described in previous Summary Reports, include a fully automatic program to fit polynomials to arbitrary empirical functions; a Mistake Diagnosis Routine, an automatic interruptive-checking routine; a general-purpose Lagrange interpolation program; and a flexible and fairly elaborate post-mortem routine.

During the present quarter all of the above programs have been changed to operate using the improved programmed-arithmetic routines of CS II. In addition, the post-mortem routine was expanded to include a format routine which will assemble information scattered throughout magnetic-core and drum storage, and type or punch the information in a format suitable for the particular type of information. Step 1 of the data-reduction procedure (previously described) was also changed to use CS II and appears to be operating correctly. Succeeding steps of the data-reduction procedure have been programmed, and testing will be started soon. A strong effort has been made to write the present version so that planned future expansions of the procedure can be made with almost no rewriting of existing sections.

132 SUBROUTINES FOR THE NUMERICALLY CONTROLLED MILLING MACHINE

The original objective of this problem was to write and test enough library-type subroutines to reduce the amount of programming connected with the preparation of milling-machine tapes for pieces bounded by plane curves to that necessary to allow the computer to find points on the boundary of the piece. This objective was attained by the end of the preceding quarter. Since many of the subroutines developed are applicable to data processing for any curve or surface, and since more experience in the use of the subroutines was desired in order to evaluate their usefulness, the objectives of the problem have been broadened to include the preparation of subroutines for several classes of surfaces in addition to the use of existing subroutines for tape preparation for several pieces. This work is being carried out by J. H. Runyon of the Servomechanisms Laboratory.

Testing of a set of subroutines needed for computing points on series-16 airfoil cross-sections was completed. These subroutines are being incorporated in a program for preparing tape for airfoil templates. Also tested successfully was a subroutine for determining parameter values along a curve such that a series of cuts connecting points on the curve corresponding to these values approximates the curve to within a given tolerance.

Subroutines necessary for tape preparation for cones were written and operated satisfactorily. Their functions include proper sequencing of cuts, cut spacing, and tool center offset computation. Several milling-machine tapes for a cone with a sinusoidal cross-section were prepared to illustrate methods of cut spacing and to show different degrees of approximation to the surface.

Two other subroutines were written and are now being tested. One of these is for selecting points for an interpolation routine. The other interchanges the roles of dependent and independent variables in a function by an iterative process.

134 NUMERICAL DIAGONALIZATION PROCEDURE

During the past quarter, a large number of 5 x 5 matrices which arose in the theory of electronic energy bands in crystals were diagonalized. The theory, sometimes called the tight-binding approximation, is described in reports of the Solid-State and Molecular Theory Group of MIT.* Basically, a one-electron wave function in a crystal is represented as a linear combination of atomic functions. The number of functions per atom is kept low, and it is this number which determines the maximum order of secular equations eventually to be solved. The reason why it is this number, and not it multiplied by the number of atoms in the crystal which determines the order of the secular equation, is that it is theoretically possible to eliminate at once the dependence of the coefficients of the atomic wave functions on the atomic position, leaving only as many independent coefficients as there are functions per atom.

Explicitly, the crystal wave function begins as an arbitrary linear combination of atomic functions

$$\psi = \sum_{\mu n} C_{\mu n} u_{\mu}(\vec{r} - \vec{R}_n)$$

where $u_{\mu}(\vec{r} - \vec{R}_n)$ represents the μth atomic function centered about the atom located at $\vec{R}_n$. Because of the translational symmetry of the crystal lattice, however, $C_{\mu n}$ must be of the form

$$C_{\mu n} = e^{i\vec{k}\cdot\vec{R}_n} d_{\mu}$$

where $\vec{k}$ itself can take on a latticed set of values, the resulting structure of quantum numbers being called the reciprocal lattice. Thus, only the d_{μ}'s remain to be varied.

* Quarterly Progress Report No. 9, July 15, 1953 and Technical Report No. 4, July 15, 1953, Solid-State and Molecular Theory Group, MIT.

The variation of the d_μ's is such as to make the energy stationary. If H represents the one-electron Hamiltonian operator, then the energy is given by

$$\int \psi^* H \psi dv = \sum_{\mu\nu} d_\mu^* d_\nu \sum_{nm} e^{i\vec{k}\cdot(\vec{R}_m - \vec{R}_n)} \int u_\mu^*(\vec{r}-\vec{R}_n) H u_\nu (\vec{r}-\vec{R}_m) dv$$

It is the matrix of H between the atomic functions (multiplied by the exponential phase factor) which is to be diagonalized for different values of $\vec{k}$. Details of the evaluation of the matrix elements as well as the discussion of the wave functions used, which are not strictly the atomic functions but orthogonalized combinations called Wannier functions, are given in the reports cited above.

This formalism was set up for a body-centered structure, and 220 5 x 5 matrices (corresponding to 220 geometrically independent $\vec{k}$ values) were diagonalized. The results are to appear in a paper in the Physical Review under the authorship of Prof. Slater and Dr. G. F. Koster. The preparation of the matrix elements, the actual problem, was done by Dr. Koster who will continue the work with a treatment of a face-centered structure under Problem 174.

138 SPHEROIDAL WAVE FUNCTIONS

Spheroidal wave functions are solutions of the scalar Helmholtz equation separated in spheroidal co-ordinates. A program has been developed for tabulating (1) the coefficients obtained by expanding the angular solutions of the first kind in associated Legendre functions and (2) the coefficients obtained by expanding the radial solutions of the first kind in spherical Bessel functions. These expansions are analytically substituted in the ordinary differential equations arising from the separation of

$$\nabla^2 \psi + k^2 \psi = 0$$

in spheroidal co-ordinates. After manipulation, 3-term recursion relations are obtained for the coefficients, and the radial and angular coefficients are found to be simply related. Both sets of coefficients are then determined by applying an iterative procedure to a continued-fraction equation derived from one of the 3-term recursion relations. The iteration proceeds until a value of the unknown separation constant of the differential equation which appears in the continued fraction is found such that it makes the coefficients compatible with their boundary conditions.

The above scheme has been programmed, tested, and all the computer production work done. The results are at present being prepared for publication as a book of over 500 pages. A significant feature of the programming was that a rather elaborate layout routine was programmed for the computer output, and, as a consequence, the results, automatically typed by the Flexowriters, are suitable for direct photo-offset reproduction without further transcription. For further details, of both the theory and the programming, the reader is referred to a more extensive report given in Quarterly Progress Report No. 11, Machine Methods of Computation and Numerical Analysis, MIT.

141 S&EC SUBROUTINE STUDY

The following subroutines have been added to the library of subroutines:

Tape No.	Title	Reference
LSR MA3	Rectangular Matrix Multiplication. Computes matrix "C" where $C=A^T B$, and A and B are recorded on the auxiliary drum. The calculated matrix C is also stored on the drum.	Summary Report No. 36 Problem 157
LSR MA4	Matrix Diagonalization. The diagonal form of the given matrix is computed and stored on the drum. The orthogonal matrix is also formed.	Summary Report No. 35 Problem 134
LSR SP2	Extract Integral Part of MRA. The integral part of the MRA is stored in the index register of the last selected counter. The fractional part remains in the MRA.	Utility program to be used with CS II

142 A STUDY OF SHOCK WAVES

Original calculations on the analysis of the propagation of a shock wave through an extended body were terminated temporarily in December and reported in Summary Report No. 36. An analysis of these results has been made, and it has been decided to continue this work. Several revisions are being made in the program by S. Sydney of the MIT Civil Engineering Department to facilitate the interpretation of the results.

143 THE VIBRATIONAL FREQUENCY SPECTRUM OF A COPPER CRYSTAL

The atomic constants have been determined for a copper crystal from X-ray measurements. These constants appear in a secular determinant relating frequency to wave vector for thermal waves propagating through the crystal. This equation has been solved for 3417 wave vectors from which a frequency spectrum has been constructed. This spectrum will be used to calculate the specific heat and zero-point energy of the lattice. Additional runs will be made in order to ascertain the effect of varying the atomic-force constants on the spectrum. Such information may be of interest in the case of superconducting metals where it is felt that the phenomenon of super-conductivity is closely connected to the lattice dynamics of the metallic crystal.

This problem is being carried out by E. H. Jacobsen of the MIT Physics Department.

147 ENERGY BANDS IN CRYSTALS

The previous quarterly progress report on this problem (see Summary Report No. 36) outlined the determination of the zeros of a function of two variables, $F(E, E_0)$. These are obtained by choosing a fixed value of the energy E of the augmented wave and determining the corresponding energy E_0 of the plane wave. All the production work necessary to complete this stage of the calculation for the particular metal under consideration, copper, has been carried out. Three separate potentials, V, have been considered, and, as an indication of the work involved, over 2000 numerical integrations of the second-order differential equation,

$$\frac{d^2P_\ell}{dr^2} = P_\ell \left(V - E + \frac{\ell(\ell+1)}{r^2}\right)$$

have been performed for various E, ℓ ; some 4000 roots of $F(E, E_0)$ have been obtained. These results show an encouraging agreement with theoretical predictions and lead one to hope that this method of determining electronic energies in solids will prove of real value, despite the large amount of computation involved.

The last part of the computation has been programmed by Dr. D. J. Howarth of the MIT Solid State and Molecular Theory Group and is now being tested. From the preceding work, a relationship between E and E_0 is obtained which determines wave functions of an electron, consisting of an expansion in spherical harmonics inside a sphere surrounding a nucleus, and a plane wave outside this sphere. Such wave functions possess a discontinuity of the derivative at the surface of the sphere. To account for this, a linear combination of such functions is used, the variational principle being used to determine the "best" combination and the associated energy. This involves the calculation of matrix elements of two wave functions and the solution of the resulting secular equation.

The matrix elements depend upon the functions

$$\frac{d}{dr}\left(\ell n\,\frac{P_\ell(E,r)}{r}\right) \quad \text{and} \quad \int_o^R P_\ell^2(E,r)dr.$$

The calculation of the former was programmed for the first stage of the work; the latter is obtained by use of Simpson's rule. The calculation of these functions in a form suitable for future use has been programmed, and most of the necessary results for copper have been obtained.

The matrix elements involve weighted sums of these functions, the weighting factors being products of spherical Bessel functions and Legendre polynomials; subroutines to calculate these, using recurrence relations, have been developed. The subsequent calculation of the matrix elements is elementary, though requiring care to allow for various special cases which can arise. Because of the length of the program, attention has been paid to designing programs to use the minimum amount of storage for the initial data, while simplifying the input requirements as much as possible.

The final secular equation to be solved is of the form

$$\det(H_{\mu\nu} - E\Delta_{\mu\nu}) = 0$$

where $H_{\mu\nu}, \Delta_{\mu\nu}$ are the matrix elements, and E is to be determined. Δ is not in diagonal form, so to reduce this to the normal form of secular equations with diagonal Δ, a technique due to Löwdin is used. We may write the equation as

$$\det(H'_{\mu\nu} - E\delta_{\mu\nu}) = 0$$

where $H' = \Delta^{-1/2}H\Delta^{-1/2}$, and H' may be diagonalized using the existing secular-equation subroutine.

The matrix $\Delta^{-1/2}$ is calculated by diagonalizing Δ, using the existing subroutine, forming Δ_d. $\Delta_d^{-1/2}$ is easily calculated, and $\Delta^{-1/2}$ is formed by a unitary transformation. Matrix multiplication then produces H'.

A program to solve such equations of order up to 30 x 30 has been programmed and tested and now exists in a production form, suitable for use by other programmers; several equations have been solved for Dr. R. McWeeny of the Solid State and Molecular Theory Group.

When the final errors have been eliminated in the program to calculate the matrix elements, the work will be completed for copper. If the convergence of the method is sufficiently good, and the results agree substantially with those obtained by use of other methods, it is hoped to repeat the calculations for several other materials; in order to investigate the method fully, many parameters have been varied in the case of copper, involving much additional computer time. To investigate other metals would require very much less computer time.

149 DIGITAL METHODS OF DETECTING SIGNAL FROM NOISE

The study of various digital methods for the detection of signals in noise where the message is a binary sequence is being continued by Dr. G. P. Dinneen of the Lincoln Laboratory. The sequential observer, which is discussed at length in Lincoln Laboratory Technical Report No. 20, has been simulated for various parameter settings. A stochastic process using a pseudorandom number generator has been used to generate signal and noise regions, and significant results have been obtained for this class of detector.

The success-run detector which is described in Summary Report No. 36 has been studied during this quarter. The equations for this detector have been solved to obtain the probability of detection for known signals. A program has been successfully operated for one set of reset conditions for the counter of this detector. Several other sets will be tried during the next quarter, and it is expected that this will complete the study.

152 DIFFUSION IN AN OXIDE-COATED CATHODE

This problem is being carried out by H. B. Frost of the Digital Computer Laboratory to determine the effects of combined thermal and electrolytic diffusion that occur in an oxide-coated cathode when current is caused to flow through the cathode. The results will be included in a doctoral thesis to be submitted to the MIT Electrical Engineering Department. The details of this problem have been described in Summary Report No. 36.

As this problem was originally coded, the two integrals which must be determined were evaluated by using trapezoidal integration. Critical evaluation of the results with large values of γ, corresponding to IRq/KT of greater than 2.75, indicated that appreciable errors were introduced by truncation errors in the integrals and by truncation in the step-by-step solution of the differential equation. The most serious error was introduced by a cumulative truncation error in the solution of the differential equation. Although the error did not amount to more than one part per thousand in the value of the integral

$$\int_o^a n(x, t)dx ,$$

the error in the integral

$$\int_o^a \frac{1}{\sqrt{n(x, t)}} dx$$

amounted to about 1.5 per cent for IRq/KT equal to 3.25. The first integral checks the truncation error in the solution of the differential equation, while the second integral corresponds to one of the desired results, the normalized resistance. In order to reduce errors, Simpson's rule integration was substituted for trapezoidal integration, and a running correction to the truncation error was performed by forcing the value of the first integral to unity, the correct value. The latter procedure was possible because the maximum error per step was quite small. The continuous correction was performed by dividing the integrand by the value of the integral. When these two changes were made in the coding of the problem, the error in the normalized-resistance integral was reduced to less than 0.5 per cent, and the error in the normalized emission was reduced from 0.5 per cent to less than 0.1 per cent. Better accuracy could be obtained at the expense of a finer net and longer running time, but further improvement did not seem justified. The running time varies as the cube of the number of points in the net.

The above solutions were made for constant cathode current. Experimental results dictated the need for a set of solutions at constant surface field. For this condition, the current varies as $\sqrt{n(0,t)}$. This set of solutions was obtained by making γ the product of an initial γ_0 and $\sqrt{n(0,t)}$. A set of solutions for this condition has also been completed.

Experimental confirmation of the theoretical results appears to have been obtained within the limits of the initial assumptions and the experimental accuracy.

153 GUST RESPONSE OF A FLEXIBLE SWEPT-WING AIRPLANE

This problem involves the solution of pairs of simultaneous integro-differential equations of the form:

$$\mu_1 q_0'' + \mu_2 q_1'' + \int_0^{s_o} q_0''(\sigma)\, I^{(0)}(s_o - \sigma)\, d\sigma + \int_0^{s_o} q_1''(\sigma)\, I^{(2)}(s_o - \sigma)\, d\sigma$$

$$+ a_1 \int_0^{s_o} q_1'(\sigma)\, I^{(1)}(s_o - \sigma)\, d\sigma = a_0 \int_0^{s_o} F'(\sigma)\, \psi_0(s_o - \sigma)\, d\sigma$$

One hundred and twenty-eight pairs of equations of this form were solved on the Whirlwind I computer, the solutions being desired for q_0'' and q_1'' as well as the particular value of $q_0''_{max}$, thus making the solutions involve numerical integration only, eliminating the difficulties of both differentiation and integration simultaneously. $q_0''_{max}$ represents the maximum acceleration of a flexible swept-wing airplane after entering a gust.

The functions I, F, and ψ are approximated by a series representing the true logarithmic and polynomial character of these functions to four significant figures, making calculation on Whirlwind I more direct. These functions are first evaluated and stored, then punched on paper tape for each set of parameters involved in a given equation-pair. The function sets are then read into the computer at a later time and the equations solved, results printed either as $q_0''_{max}$ values, or as a complete time-history of q_0''. The approximating functions all converged satisfactorily for the values of parameters under consideration. All 128 pairs of equations ran without developing singularities.

The programming was done by Kenneth Foss of the Aero-Elastic and Structures Research Lab and David Sternlight of the Mathematics Department. The program utilizes a punch routine made available by Harry Denman of the S&EC Group staff. Because of storage limitations, this program is stored in the magnetic drum. In the drum the original program is also kept available, and for resetting the computer for each calculation, the drum is merely mapped over magnetic-core memory, thus clearing all counter registers with a minimum of instructions.

A more detailed report, including physical and mathematical derivations, will be published under USAF Contract No. 33-(038)-7267, and copies will be available for limited distribution from the Aero-Elastic and Structures Research Lab, MIT. The problem is satisfactorily concluded.

155 SYNOPTIC CLIMATOLOGY

The purpose of this project, which is being carried out under the direction of Prof. T. Malone of the MIT Meteorology Department, is to seek a way of organizing climatic data in such a form that it will be directly applicable to the problem of the synoptic meteorologist, that is, to the problem of weather prediction. This problem is generally approached by the analysis of the circulation patterns at two or more levels in the atmosphere over areas ranging from the United States to an entire hemisphere. The analysis is, to a certain extent, subjective and is carried out by graphically representing the past, current, and anticipated circulation patterns on maps. The actual weather elements (temperature, rainfall, cloud cover, ceiling, visibility, etc.) are obtained from the circulation patterns by a highly subjective procedure.

A quantitative treatment is badly needed. Moreover, this treatment should be essentially statistical, because the very nature of the atmosphere makes it unlikely that precise, categorical forecasts will ever be feasible, while probability forecasts of high utility await only a proper analysis of available data. The overwhelming magnitude of the computational task required for a comprehensive analysis has been a decisive deterrent to any such program. The development and availability of a high-speed, large-capacity computer such as Whirlwind I has completely changed this picture.

The approach adopted in the present work has been to represent the circulation patterns by the coefficients of orthogonal polynomials fitted to atmospheric-pressure data summarized directly from the circulation patterns. The coefficients then constitute an ensemble of nonstationary time series. The temporal variation of a given weather element is another nonstationary time series. The mathematical problem here is the prediction of the latter from its past behavior and from the interrelationship it has with the ensemble of time series. The operator employed in this prediction is the multiple linear correlation which exists between the dependent and the independent variables.

In order to insure the most effective and the most efficient prediction, certain basic questions must be answered: (1) how large a sample must be taken to insure stability of the operator, (2) over how large an area must the circulation pattern be considered, (3) how far back in time must the autocorrelation and the crosscorrelation be extended before the point of diminishing returns on information is reached, (4) how much independence really exists between the several levels in the atmosphere, and (5) how precisely must the circulation pattern be specified by the orthogonal polynomials. The work during the present quarter has been directed toward answering these questions.

The cross products for a 14 x 14 matrix were computed for a series of twenty Decembers and compared with the results obtained by using the basic sample size of six Decembers. The results suggested that the sample size is adequate. More work is planned with the results of these computations to determine how well climatic regimes can be detected from the behavior of the per-cent reduction for each month.

A major undertaking has been to process the necessary data so that the relations established from years prior to 1940 could be extended to include the effects of upper-level circulation patterns. This involved computation of the coefficients for the surface and 700-mb levels for the five Januarys during 1949-1952, since adequate upper-air information is not available for the years before 1940. In order to test the relative efficacy of different ways of representing the upper-level flow patterns, coefficients were computed for three types of 700-mb representation: (1) conventional 700-mb contour charts, (2) standardized 700-mb anomalies, and (3) probability units. The cross products of these coefficients were computed on Whirlwind I, and the matrices were prepared by desk calculator. The results are now being analyzed and will constitute a substantial part of an Sc. D. thesis. It is anticipated that in further work of this kind the complete solution of the matrices will be carried out on Whirlwind I.

A program has been prepared by which the coefficients of orthogonal polynomials representing the circulation pattern over the entire western half of the Northern Hemisphere can be computed. The computations have been carried out and the results combined with contemporary and lag coefficients over North America to determine the predictability of sea-level pressure from the coefficients. This involved a 52 x 52 matrix. The cross products were computed on Whirlwind I, and the solution is progressing by hand. During the course of this work, an improved method of computing coefficients, giving considerably more precise specification, has been developed.

The techniques and data are now available for intensive work on Questions 2, 4, and 5 listed above, and this work will be given primary attention during the next quarter.

156 THE EVALUATION OF THE REFLECTION COEFFICIENT IN A SEMI-INFINITE OPEN RECTANGULAR WAVEGUIDE

The evaluation of the reflection coefficient in a semi-infinite open rectangular wave-guide is obtained approximately by using Fourier transform techniques on the integral equations of the Wiener-Hopf type. The integrals are to be evaluated by the trapezoidal rule. A detailed description of the problem was given in Summary Report No. 36. The investigation is being carried on by Dr. M. Balser of Project Lincoln at MIT.

The problem has been successfully run for the ranges

$\alpha = 9.9$
$10 \leq \alpha \leq 20$ in steps of 0.5
$20 \leq \alpha \leq 100$ in integral steps
$100 \leq \alpha \leq 300$ in steps of 5

A short test run for the range $0 \leq \alpha < \pi^2$ was made. (Since the integrals are complex-valued over this range, the calculation is far more involved here than for the previous ranges.) After making suitable checks, it was decided that the results were correct for the interval used in the trapezoidal rule but that this was not sufficiently accurate.

To obtain the additional accuracy desired, the intervals used in the trapezoidal rule were taken smaller around the singularity of the integrand. Corrections were made in the program to introduce these changes, and, so far, a successful run has not been obtained.

Once a successful run is completed on the above, the final section of the problem will remain to be done, i.e., the section for $-30 \leq \alpha < 0$.

157 RECTANGULAR MATRIX MULTIPLICATION

Rectangular matrix multiplication has been programmed successfully by Dr. A. Meckler of the MIT Solid State and Molecular Theory Group and now is part of the library of subroutines. The routine assumes that one matrix (A) of order kxm is on one drum group and another matrix (B) of order kxn is on another drum group. After the various matrix orders and drum groups have been planted within the routine, it will perform $A^T B$ (A^T = A transpose), leaving the result on a third drum group. The routine uses buffer registers and should, in general, be accurate up to the largest matrix it can handle which is one containing 1024 elements (half the registers on a drum channel).

158 RELAY SERVO RESPONSE

This problem considers the optimum response of an ideal, third-order relay servomechanism to the reference input

$$\theta_i = A_k + W_k t + \frac{1}{2} \alpha_k t^2 .$$

The system error and its derivatives must be brought to zero from their initial values (A_0, W_0, and α_0) with no overshoot in the minimum time, t_0. To do this, two reversals of the manipulated variable input, θ_f, are required at times t_x and t_y.

The critical times (t_x, t_y, and t_0) are related by the equations

$$(1+g)t_y = 2t_x - \beta_1 + (1-g)t_x - \beta_1^2 - (1+g)\frac{\beta^2}{2} + (1-g)\beta_2 \quad (1)$$

$$(1-g)t_0 = 2(t_y - t_x) + \beta_1 \quad (2)$$

$$(1-g)e^{at_0} = 1 + \frac{\alpha_0 - \alpha_k}{\theta_{fm}} a + 2(e^{at_y} - e^{at_x}) \quad (3)$$

where

$$\beta_1 = \frac{W_0 a}{\theta_{fm}} + \frac{\alpha_0}{\theta_{fm}}$$

$$\beta_2 = \frac{A_0 a}{\theta_{fm}} + \frac{W_0}{\theta_{fm}}$$

$$g = \frac{\alpha_k a}{\theta_{fm}}$$

and θ_{fm} and a are parameters of the system. The quantities to be found are the critical times as functions of the other parameters.

The method used was to find numerical values for the critical times for several different values of each of the 4 parameters, A_0, W_0, α_0 and α_k, which specify the initial error, and for each of the 2 system parameters, a and θ_{fm}. Calculations on the Whirlwind computer were made for about 250 sets of parameters. From these results an expression was derived for the critical times as a function of 6 parameters.

Hand calculations of 6-figure accuracy required about 30 minutes per set of parameters.

The procedure was to assume a value of t_x, solve Equations (1) and (2), and substitute values into Equation (3). The amount by which the two sides of (3) failed to agree was used as a basis for estimating the next trial value of t_x. This was continued for a given set of parameters until all three equations were satisfied.

The results of the calculations have been included as part of a thesis entitled "Phase-space Considerations in Performance of Relay Servo-mechanisms." The thesis has been submitted by John Wesley Stearns, Jr., to the MIT Department of Electrical Engineering in partial fulfillment of the requirements for the degree of Master of Science.

159 WATER USE IN A HYDROELECTRIC SYSTEM

The big storage reservoirs of a hydro system are used to store water during high flows for use during low flows. The exact amount to use or save at any given time depends on the amount of water in the reservoirs, the probability distribution of future river flows, the anticipated demand for power, and the cost of obtaining power from other sources. The problem is solved by calculating certain expected value functions and finding their minima. The program is fairly general, but specific examples are drawn from the Columbia River system.

The problem was started this quarter by J. D. C. Little of the MIT Physics Department, and programming is about one-third complete. Calculations have been performed for a model on which hand calculations had been made earlier. The next step is to take historical-flow data for the Columbia River at Grand Coulee and derive from it conditional probability distributions of river flow. Then, after the main program has been tested, these probability distributions will be inserted and optimum operation for the hydro system calculated.

160 SIMILARITY TRANSFORMATION OF A MATRIX

A matrix congruent transformation has been successfully programmed by Dr. A. Meckler of the Solid State and Molecular Theory Group. The program uses the matrix-multiplication subroutine (Problem 157), and the order of the matrices involved is therefore limited by that subroutine. The main program exists as a converted tape to be fed in at a fixed address. A matrix U and a matrix A are typed on separate tapes, and, after this is fed in, control is transferred to the main program which performs $U^T A^T U$ and displays the result on the scope. Control is then transferred back to the reader, and another set of U and A can be read in, and so on. The program was used on a set of 21 x 21 matrices and appeared very accurate.

161 BUILDING SETTLEMENT UNDER TRANSIENT LOADS

An analysis is being made by S. Sydney of the MIT Civil Engineering Department of the settlement of buildings when they are subjected to dynamic loads. The immediate practical goal of this analysis is to improve the existing concepts, relative to the proper choice of allowable bearing capacities for building footings, by obtaining relationships between settlements and various loads. The physical properties of various soils have been determined to some extent by empirical methods, and equations have been formulated to represent the soil action. Experimental work is being carried on to give some measure of confirmation to qualitative concepts and to provide numerical values for the parameters involved in the analytical studies.

The behavior of the soil-footing system has been approximated by the action of a single degree of freedom mass-spring system with nonlinear characteristics.

The properties of this system have been derived from established static-footing action concepts and a large amount of intuitive feeling for the manner in which the foundation soil acts under transient loadings. An approximation of this type has been utilized many times in connection with studies of soil action under periodic loadings, and, though this work is of limited quantitative value, it is of value in establishing qualitative concepts and in defining trends.

During settlement up to the yield point of the soil, the action of the soil is predicted by the equation

$$(M_F + M_{so} + \alpha y)\ddot{y} + \frac{1}{2}\frac{M_{sy} - M_{so}}{y_y}(\dot{y})^2 + By = F(t)$$

where M_F = the mass of the footing,

M_{so} = effective soil mass at initial static settlement,

M_{sy} = effective soil mass at yield point,

α = rate of increase of soil mass with settlement,

y_y = settlement at soil yield point,

B = slope of initial settlement curve.

After the yield point is reached, the following equation is used:

$$(M_F + M_{sy})\ddot{y} + By_y = F(t)$$

During unloading of the soil, the action of the system is predicted by the equation

$$(M_F + M_{so} + \alpha y_i + \alpha^1(y-y_i))\ddot{y} + \left(\frac{B^1}{2B}\ \frac{M_{sy} - M_{so}}{y_y}\right)(\dot{y}^2) + R_{sy} + B^1(y-y_i) = F(t)$$

where y_i = settlement value at which unloading cycle began,

B^1 = slope of unloading curve,

R_{sy} = maximum soil resistance at point of unloading.

The above equations are applicable only for cohesionless soils, such as sand and gravel, which do not exhibit a strain-rate effect. Cohesive soils which have a strain rate, such as some clays, will also be studied.

A Runge-Kutta fourth-order integration formula is being used in this analysis. A program has been written for Whirlwind and is being used at the present time to obtain analytical results for cohesionless soils.

163 FERRITE PHASE SHIFTERS IN RECTANGULAR WAVEGUIDE; TRANSCENDENTAL EQUATION

The analytical expressions obtained from the solution of the electromagnetic boundary-value problem dealing with a single ferrite slab in a rectangular waveguide show that such a transmission line is nonreciprocal. Under appropriately chosen conditions both the phase constant, β, and the attenuation constant, α, have different numerical values for the two directions of propagation.

Considerable simplification of the numerical work involved in extracting engineering-design data and scientific understanding has been obtained by choosing cases where α is known to be negligibly small.* This assumption leads to the expression (which will be called Part I of the problem):

$$a = \frac{L - \delta}{2} - \frac{1}{2ka} \cos^{-1} \left(\frac{-pr \pm q\sqrt{p^2 + q^2 - r^2}}{p^2 + q^2} \right) \qquad (1)$$

where a is the slab position, p and q are functions of β, and r is a complicated transcendental function of β.

It is necessary to consider cases for which α is not zero. A small α becomes significant in high power systems. A larger α will be encountered in the region of ferromagnetic resonance where its behavior in this configuration is expected to reveal significant scientific information concerning resonance effects.

Manual computing time is increased by a factor of 50 when attenuation is introduced. Two simultaneous equations of the form of Equation (1) must be solved. The symbols stand for very much more complicated transcendental expressions, and the two equations cannot be made completely explicit in a.

* Results published in Lincoln Laboratory Technical Memorandum No. 49 and accepted by the J. Appl. Phys.

Fortunately, the substitution of the approximate values obtained from Part I are very helpful in the numerical solution of Part II. A typical solution consists of four curves, α_+, α_-, β_+, and β_-, plotted as a function of slab position. Sets of curves must be obtained to establish dependence on frequency, on the physical parameters of the ferrite, and on the external magnetic-field intensity through the regions of ferromagnetic and dimensional resonance.

Basically, the programs for both Parts I and II were capable of calculating a single test point on each of the four curves. Some programming errors were revealed when attempts were made to calculate successive points for the construction of the four curves. As parameters changed to approach resonances, some factors eventually turned imaginary, and transcendental functions changed quadrant rapidly. Provisions were included in the original programs for most of these difficulties and, subsequently, instructions were inserted which provided for those which arose unexpectedly. Part I is now operating properly over a wide range, and it is believed that nearly all such difficulties have been remedied in Part II as well. The only problem not fully solved in programming is that of calculating the particularly sensitive endpoints of the curves.

This work is being carried out by K. J. Button of Lincoln Laboratory at MIT.

166 CONSTRUCTION AND TESTING OF A DELTA-WING FLUTTER MODEL

To solve the problem of designing, constructing, and testing a delta flutter model which simulates a given set of flexibility influence coefficients, the actual wing is replaced by a structurally equivalent lattice network, and the task becomes one of determining the bending and torsional stiffnesses of the component members. Employing the principle of minimum elastic-strain energy, and the elementary flexure and torsion theories, an analytical expression for the influence coefficients in terms of lattice geometry and the unknown stiffnesses of the system is set up, and an iteration procedure is evolved for obtaining the desired stiffnesses by successive error corrections which converge to zero.

The mathematical solution of the problem involves the evaluation of the matrix equations

$$C_{mn} = \alpha_{mn} - \alpha_{ms}\beta_{sn} \tag{1}$$

$$\Delta C = \Delta\alpha_{mn} - \Delta\alpha_{ms}\beta_{sn} - (\Delta\alpha_{ms}\beta_{sn})' + \beta'_{sn}\,\Delta\alpha_{rs}\,\beta_{sn} \tag{2}$$

where

$$\beta_{sn} = \alpha_{rs}^{-1}\,\alpha'_{ms} \tag{3}$$

Equation (1) reveals whether the trial estimate of the stiffnesses is accurate enough; if not, equation (2) yields corrections to the trial estimates, which then form the basis for a new estimate and the cycle is repeated.

To enable Whirlwind to handle the problem, the following operations have been evolved in subroutine form:

1. Matrix addition and subtraction.
2. Matrix multiplication.
3. Matrix inversion.
4. Combined performance of 3 and 2, above.
5. Solution of n simultaneous linear algebraic equations in n unknowns.
6. Transpose of a matrix.
7. Transpose of a matrix of unknowns.
8. Matrix addition and subtraction with each element a sum of known coefficients of unknown quantities.
9. Matrix multiplication with the premultiplier an unknown matrix.
10. Routine 9 for the case of the premultiplier being too large to fit wholly within high-speed storage.
11. Formation of a matrix with each element a sum of products.
12. Equating two unknown matrices term by term to form a set of simultaneous equations.

Operations 1 through 7 can be performed satisfactorily with matrices up to size 25 by 25. Operations 8 through 12 are limited to correspondingly smaller matrices, depending upon how many terms compose each element. Modification of the routines to take advantage of magnetic-drum storage can substantially increase the allowable sizes of the matrices to be handled.

Present efforts are being directed by S. I. Gravitz of the MIT Aeronautical Engineering Department toward integrating the component subroutines into a co-ordinated program to solve the wing-simulation problem.

168 INDICIAL DOWNWASH BEHIND A TWO-DIMENSIONAL WING

In the analysis of the response of an airplane to a sharp-edged gust, and particularly in the calculation of stresses in the horizontal tail, it is important to know the downwash at the tail caused by the lift response of the wing to the gust. In the present solution for the downwash behind a two-dimensional wing, something more than the "indicial downwash" is sought. This is effected by allowing the gust front itself to have an arbitrary horizontal velocity which, combined with the velocity of the airplane, u, results in the wing's penetrating the gust front at a velocity v.

The equation for the ratio of the downwash to the vertical gust velocity at the dimensionless time, s, and the dimensionless horizontal coordinate, x*, is

$$\frac{w_w^{(2)}(s, x^*)}{w_0} = \frac{-1}{\pi}\sqrt{\frac{x^*-1}{x^*+1}} \int_{-1}^{\frac{v}{u}s-1} \sqrt{\frac{1+Z}{1-Z}} \int_0^A \sqrt{\frac{A+2-S}{A-S}} \left[\frac{1}{x^*-Z} - \frac{1}{S+(x^*-1)-A}\right] \psi'(S)\, dZ ds \tag{1}$$

The upper limit on the first integral is to be taken as unity if

$$\frac{v}{u} s > 2$$

Also, in Equation (1)

$$A = s - \frac{u}{v}(1+z) \tag{2}$$

$$\psi'(S) = \frac{4}{\pi^2} \int_0^\infty \frac{J_0 - iJ_1}{[H_1^{(1)} + {}_iH_0^{(1)}]^2 + 4[J_0 - {}_iJ_1]^2} \; \frac{e^{-k(S-1)}}{k} \, dk \tag{3}$$

where the argument of the Bessel functions is (ik).

$\psi'(S)$ is the derivative of the Kármán-Sears function. Since the most accurate determinations of this function have been made numerically, and graphical solution for the derivative is much too inaccurate, it is desired to determine $\psi'(S)$ by numerical integration of Equation (3).

The middle integral may be split into two parts, such that

$$\frac{w_w^{(2)}(s, x^*)}{w_0} = -\frac{1}{\pi}\sqrt{\frac{x^*-1}{x^*+1}} \int_{-1}^{\frac{v}{u}s-1} \sqrt{\frac{1+z}{1-z}} \left[\frac{C_I(A)}{x^*-z} - C_2(A, x^*)\right] dz \tag{4}$$

where

$$C_I(A) \equiv \int_0^A \sqrt{\frac{A+2-S}{A-S}} \; \psi'(S) \, d\mathcal{S} \tag{5}$$

and

$$C_{II}(A, x^*) = \int_0^A \sqrt{\frac{A+2-S}{A-S}} \; \frac{\psi'(S)}{S+(x^*-1)-A} \, d\mathcal{S} \tag{6}$$

$C_I(A)$ and $C_{II}(A, x^*)$ essentially represent the indicial-downwash solution, except for a constant which can be determined analytically. For other values of $\frac{u}{v}$, the downwash may be found by numerical integration of $C_I(A)$ and $C_{II}(A, x^*)$.

The computation of $\psi'(S)$ has been successfully completed for seventy values of S. These seventy values permit interpolation for any other value of S to the desired accuracy.

$C_I(A)$ was computed and found to be identically unity. From this fact it may be deduced that $2\pi w_0\psi'(S)$ is exactly the vorticity distribution in the Wagner case; that is, the rate of growth of lift for an airfoil penetrating a sharp-edged gust is proportional to the vorticity distribution shed from an airfoil which has undergone a step-function change in angle-of-attack. It was subsequently discovered that Sears noted this rather surprising fact in the Journal of the Franklin Institute, July, 1940. Sears did not mention specifically, however, the interesting historical fact that Wagner, in his original work in 1925, inadvertently determined the Kármán-Sears function by calculating the circulation growth of an airfoil in response to a step-function change in angle-of-attack.

Since $C_I(A)$ is unity, the outer integral containing $C_I(A)$ can easily be handled analytically.

The remaining problem is to evaluate numerically $C_{II}(A, x^*)$ and the outer integral involving $C_{II}(A, x^*)$.

This study is being carried out by N. P. Hobbs of the MIT Aeronautical Engineering Department.

171 IMPROVED POWER SPECTRUM ESTIMATES

A major problem which arises in the calculation of power spectra for frequency analysis of an empirical function is the step of Fourier transformation of the autocorrelation function to give the power spectrum. (See Problem 107.) The use of a finite rather than infinite integral in the definition of the Fourier transform introduces a large amount of spurious ripple (Gibbs's Phenomenon) which is superimposed upon the correct answer and makes reasonable interpretation of results extremely difficult.

Problem 171 is concerned with the realization, as a WWI computer program, of techniques for minimizing this difficulty, which were derived by D. T. Ross in partial fulfillment of the requirements for a Masters Degree in the Department of Electrical Engineering.

The program has been written and appears to function correctly. It calculates a series of spectra, each closer to the correct spectrum than the previous one, and plots calibrated graphs of these functions on the scope along with a graph of the given function and a "Measure of Indecision" curve which shows, as a function of frequency, the relative confidence which can be placed in the series of spectra. Intensive testing, using an auxiliary program which generates test functions with known spectra, will now be undertaken to determine the limitations of the procedure. Theoretically, the programs should give the best possible spectra which can be obtained from the given finite section of function.

172 OVERLAP INTEGRALS OF MOLECULAR AND CRYSTAL PHYSICS

In the study of molecules or crystals one approach is to represent the unperturbed quantum states to a first approximation by linear combinations of atomic orbitals (i.e., wave functions); thus $\psi_j(\vec{r}) = \sum_k d_{jk}\phi_k(\vec{r})$ where the $\phi_k(\vec{r})$ are atomic orbitals (AO's), and the d_{jk} are dictated by the symmetry requirements of the specific cases. The AO's often desired are the self-consistent field orbitals of the free atom. The latter are given only in numerical form, but it is found that they usually can be fitted by two- or three-term sums of the more approximate, but analytic, Slater AO's, which when normalized have the form:

$$\phi_{n\ell m}(\vec{r}) \quad (n, \ell, m) = R_n(r) S_{m\ell}(\theta, \phi)$$

where

$$R_n(r) = (2\zeta)^{n+1/2} \left[(2n)!\right]^{-1/2} r^{n-1} e^{-\zeta r}$$

$$S_{m\ell}(\theta,\phi) = \left[\frac{2\ell+1}{2\pi(1+\delta_{om})} \cdot \frac{(\ell-|m|)!}{(\ell+|m|)!}\right]^{1/2} P_\ell^{|m|}(\cos\theta) \begin{cases} \cos|m|\phi & (m \geq 0) \\ \sin|m|\phi & (m < 0) \end{cases}$$

where ζ is a parameter which can be adjusted.

For a calculation of the energy states of a molecule or of the corresponding energy bands of a crystal, the perturbed or exact wave function, $\Psi_i(\vec{r})$, can be approximated by a linear combination of the unperturbed wave functions, $\Psi_j(\vec{r})$, such that $\Psi_i(\vec{r}) = \sum_j C_{ij}\Psi_j(\vec{r})$. The approximate energy value is then found by applying the variational principle to the expression:

$$E = \frac{\iiint \Psi_i^* H \Psi_i \, dv}{\iiint \Psi_i^* \Psi_i \, dv} \qquad H = \text{Hamiltonian Operator}$$

where the C_{ij} and C_{ij}^* are considered variational parameters. This leads to the secular equation, $\det|H_{ij} - ES_{ij}| = 0$, the solution[1] of which gives both the C_{ij} and the eigenvalue E. The elements H_{ij} and S_{ij}, of the Hamiltonian and overlap matrices, respectively, can be written, in view of the above, as particular combinations of the matrix elements formed from the Slater AO's. Furthermore, many of the major terms of the Hamiltonian operator matrix elements between Slater AO's can be resolved into combinations of two-center overlap integrals. Therefore, a large part of the solution of this formulation depends upon the ability to calculate the many specific overlap integrals required between Slater AO's.

Explicitly the overlap integrals have the form:

$$(n, \ell, m | n', \ell', m') \equiv \iiint \phi_{n\ell m}(\vec{r}-\vec{a}) \phi_{n'\ell' m'}(\vec{r}-\vec{b}) dv$$

where $\vec{a}$ and $\vec{b}$ are fixed vectors from the origin to the two centers, and the $\phi_{n\ell m}(\vec{r})$ are the normalized, real Slater AO's. These integrals, in principle, can be solved analytically by transforming to prolate spheroidal co-ordinates, and this has been done for the more common integrals;[2] the results are expressed in terms of two functions, $A_m(p)$ and $B_n(\sigma)$ (to be defined later). The latter auxiliary functions have been the object of a great deal of effort and have been tabulated to some extent (see bibliographies of references 2 and 3), but they are not very convenient when it is considered what a small role each individual overlap integral plays in the complete calculation. Recently Roothaan[3] has made a systematic study of these overlap integrals and, by substituting the known expansions of the $A_m(p)$ and $B_n(\sigma)$, has given explicit analytic forms in terms of powers and exponentials of the basic parameters. Unfortunately his results suffer from two serious defects: (1) the formulas are so complicated and involved that evaluation of the many integrals is a major problem, and (2) the expressions from a computational point of view are very poor since they not only can require subtractions of extremely large numbers with a small answer remaining, but special formulas are needed for limiting cases.

By a suitable reformulation of the $A_m(p)$ and $B_n(\sigma)$ functions, it is possible to remove the unnecessary singular behavior in Roothaan's results and at the same time arrive at a scheme which is computationally very good for a high-speed computer. This method involves the spherical Bessel functions of imaginary argument which have been previously investigated[4] by F. J. Corbató of the MIT Physics Department.

This scheme is to be embodied by Corbató in a subroutine capable of accepting the arbitrary parameters (within the regions of physical interest) and should partially eliminate some of the difficulties of one type of molecular and crystal calculation.

This program has already been written and tested. Thus there is now available a main subroutine for the 14 more-basic overlap integrals of Slater AO's. To make this subroutine more useful, several auxiliary routines are being written and tested. The latter routines are of three types:

1. Four different subcontrol programs each of which uses the main subroutine to give one of four kinds of integrals (overlap, kinetic energy, 2 classes of potential energy integrals), all of which are either identical or related to certain overlap integrals. These

four routines are to be arranged so that they will be interchangeable and will each have the same form of program entry.

2. A master-control program. This program is required when the numerically tabulated wave functions often used in a physical problem have been approximated by a finite series of Slater AO's. The program is to be arranged so that from a very general prescription of these approximated wave functions, the single desired integral of the tabulated wave functions is formed from a linear combination of the many integrals of the Slater AO's of the approximations. The combining process is entirely automatic and can use interchangeably any one of the four subcontrol programs of type 1. This master-control program will thereby eliminate the unnecessary output of intermediate results and, more important, will eliminate the rapidly increasing recombination labor involved in making higher-precision approximation series of Slater AO's.

3. Wave-function-fitting program. This program will be used to minimize the labor of fitting the tabulated wave functions by a finite series of Slater AO's. Current procedures often used have involved trial-and-error methods by means of a desk calculator. A semi-automatic scheme has been developed which offers an alternative means of fitting the tabulated function.

References:

1. A Whirlwind program which solves this type of equation has been developed by Drs. A. Meckler and D. Howarth and is available.
2. Mulliken, R. S., et al., Jour. Chem. Phys. 17, 1248 (1949).
3. Roothaan, C. C. J., Jour. Chem. Phys. 19, 1445 (1951).
4. Special problem done for Prof. J. C. Slater and Drs. Schweinler and Koster.

3. OPERATION OF WHIRLWIND I

Computer reliability has continued high. Operators' reports show that 93 per cent of the 945 hours of assigned computer time was usable in the period. The reduction in core-memory maintenance time, as compared to the requirements of electrostatic storage, has increased the number of hours which can be scheduled for operations. Fig. 3-1 gives a breakdown of the time schedule for WWI during the quarter.

	Hours Per Week													
	January					February				March				Total Hours
Activity	1-7	8-14	15-21	22-28	29-4	5-11	12-18	19-25	26-4	5-11	12-18	19-25	26-1	
Core Memory	1	2	2	2	2	3	2	2	2	0	0	0	0	18
Marginal Checking	3	5	4	4	4	4	4	3	4	4	4	4	5	52
Installation	7	7	8	7	8	8	7	0	7	8	8	8	8	91
Maintenance	21	16	20	21	34	24	20	17	23	22	30	34	32	314
Terminal Equipment Testing	17	29	31	29	32	36	34	23	26	28	27	31	31	374
Engineering & Scientific Computation	26	36	51	44	39	47	45	25	46	61	52	49	44	565
Other Applications	23	35	30	40	33	23	35	28	36	23	25	21	28	380
Total Hours	98	130	146	147	152	145	147	98	144	146	146	147	148	1794

Fig. 3-1. Allocation of Computer Time

Work of the Systems Section during the past quarterly period has been largely confined to a great number of small tasks, relatively unimportant individually but, taken together, valuable in achieving the long-range goal of system reliability and ease of maintenance. A relatively minor circuit change promises to reduce the already few core-memory failures. The consolidated test program mentioned in Summary Report No. 36 has now been written and tested and is in regular operation. This program will greatly increase the efficiency of the WWI marginal-checking program.

The In-Out Section continued the elaboration of terminal equipment. Old facilities have been improved, and new facilities have been added. The in-out control counter and real-time clock have both been replaced with units made up of standard WWI plug-in units. These new units have facilities for marginal checking which should increase their reliability. The noise in the display system has been reduced by a change of the circuit grounds to eliminate common tie points between the a-c and d-c circuits. New facilities include a high-speed Ferranti photoelectric tape reader and a buffer-drum system.

3.1 Systems Engineering

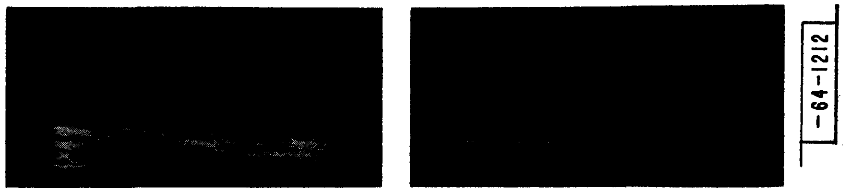
3.1.1 Matrix Cathode-Follower Troubles

Operation of the core memory has been very good. The only significant time lost due to core-memory failure was a period of 10 hours required to locate and replace several damaged crystals in the core-selection matrices. The crystals were destroyed by a gas arc in one of the cathode followers which drive the matrix. To prevent repetition of this failure, the high cathode-to-shield potential in the cathode follower has been eliminated (by returning the shields to near cathode potential), and the matrix inputs have been fused.

3.1.2 New Sensing Amplifier

After the contents are read out of a register of core storage, the information must be rewritten; the rewriting is followed by a post-write disturb. Both of these operations create transients which reach the core sensing amplifiers. In certain programming sequences the transient caused by the post-write disturb may be many times as large as a readout signal, and the present sense amplifier will not recover from this transient in time for the next read signal. A new sense amplifier has been designed with a more rapid recovery which eliminates the possibility of interference by the post-write-disturb transient with the following readout signal. The new unit is also more tolerant toward tube and component deterioration and provides an output signal which will make strobe timing less critical.

The signal out of the sense amplifier at the time of strobing inputs is shown in Fig. 3-2 for the old and new circuits. (Note the absence of base-line shift, the absence of the transient, and the broader top on the read signal out of the new amplifier. This broader top permits more latitude in the timing of the strobe pulse.)



(a) (b)

Fig. 3-2

The recovery time of the old core-memory sense amplifier was poor when it was subjected to the large transients which occurred in certain sequences of core-memory operation. The signal out of the old amplifier and the same signal out of a recently designed amplifier are shown above in (a) and (b), respectively. Note that, in addition to its faster recovery time, the new amplifier provides an output pulse with a broad top so that strobe timing is less critical.

3.1.3 Consolidated Test Programs

An important part of the preventive-maintenance program for WWI, routine daily marginal checking, has been speeded up by giving the computer a larger part of the job of checking itself. A comprehensive marginal-checking program is stored on magnetic tape. This includes all programs commonly used for marginal checking together with the necessary subroutines to select the appropriate lines for each program. To accomplish routine marginal checking, the tape is read onto the magnetic drum from which the individual test programs are read in as needed.

The Flexowriter printer records each test-program title and number and miscellaneous control information (for example, types and number of failures, the new program start-over point if required). The type of information which might be printed out during a typical marginal-checking period is shown in Fig. 3-3.

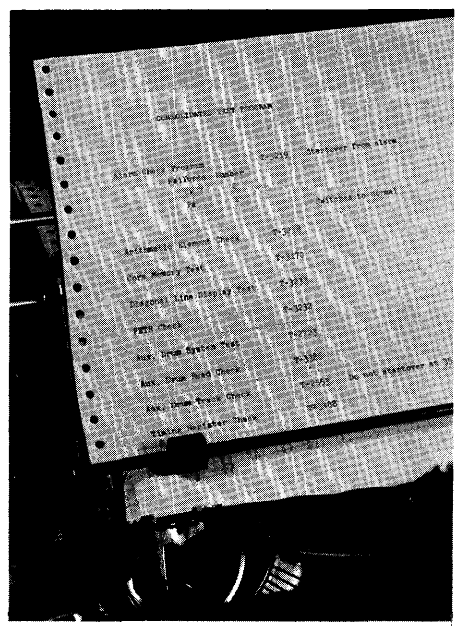


Fig. 3-3

A new consolidated test program, now in use, automatizes the routine marginal-checking procedures. While the program is in operation the computer types out pertinent data concerning the checking being done. The photo shows what might be typed out during a typical run.

3.2 Input-Output System

3.2.1 Timing Register

A new real-time clock (Fig. 3-4) has been installed in the computer room to take the place of the temporary one which had been used. The new clock has 10 digits instead of the former 15. There are provisions for reading out the 15 most significant or the 15 least significant digits, giving a coarse and a fine time reference. The coarse time reference will provide timing increments of 1.066 seconds with a capacity of about 9 hours and 43 minutes. The fine time reference will provide timing increments of 0.066 second with a capacity of about 36 minutes. The new clock has provisions for marginal checking which should improve reliability of operation.

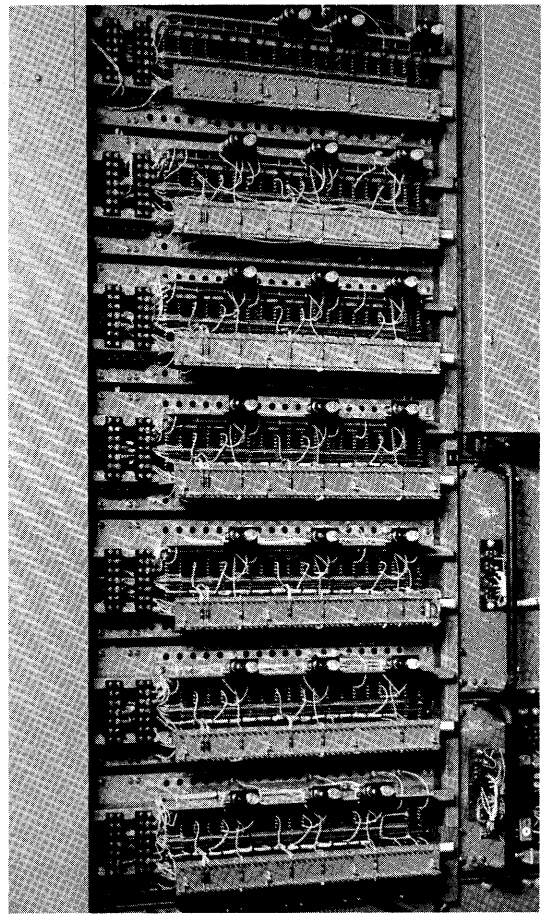


Fig. 3-4. New Real-Time Clock

3.2.2 Display Equipment

The annoying random variation in the position of displayed points and figures on the 16-inch display scopes has been greatly reduced by reconnections between the d-c and a-c power for the display systems. Small but significant 60-cycle currents from the a-c line flowed in the d-c ground line. Separation of the ground connections has resulted in a more stable display.

3.2.3 Ferranti PETR

A Ferranti photoelectric tape reader has been installed in the WWI system to replace the ERA (Engineering Research Associates) reader. The new reader is faster than the old ERA model both in running and in stopping and starting the tape. It also has an optical system which can distinguish between a hole and translucent tape. The old reader required opaque tape. The relative independence of tape translucence on the part of the Ferranti reader is credited with its excellent reliability.

The ERA reader moved the tape at 14 inches per second and required 2 inches for stopping. The Ferranti reader moves the tape at 20 inches per second (or 200 lines of information) and can stop the tape in about 0.04 inch, which permits line-by-line reading.

3.2.4 Magnetic Tape

Five magnetic-tape units are now in operation with WWI, each capable of storing over 100,000 words, each of 16 binary digits. Magnetic tape is used primarily for delayed print-out, for permanent storage of utility programs, for equipment testing, and for conversion of Flexowriter-coded tapes into binary-coded tapes.

A long-carriage Flexowriter to print or punch from magnetic tape has been installed (Fig. 3-5). The new equipment, modified to permit programmed selection of punched or printed output, is now used extensively in preference to the slower direct-output Flexowriter. The equipment has been enclosed in a soundproof cabinet to avoid annoyance to groups using the computer when the Flexowriter is typing out previously stored information.

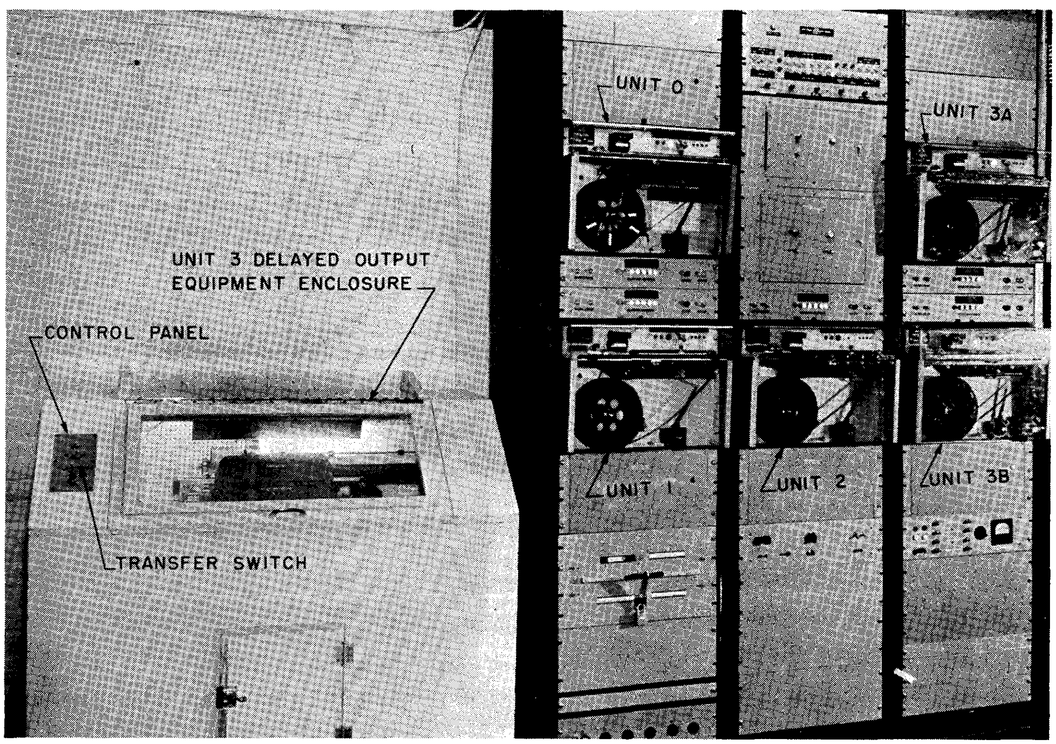


Fig. 3-5. Magnetic-Tape Units and Delayed-Output Equipment

Maintenance of the Flexowriter print-out system has been improved by the addition of marginal-checking facilities. Equipment has been added for remote control and testing of the tape-handling mechanism. It is now possible to detect and correct troubles before they cause errors in operation with the computer.

Reliability of the system has been improved by the substitution of mylar-base for acetate-base magnetic tape. The much stronger mylar tape has virtually eliminated the problem of tape breakage.

Investigation continues on the remaining reliability problems of insulation breakdown and surface wear of the read-record heads and unsatisfactory life of germanium diodes and vibrator relays in the magnetic-tape-mechanism circuitry.

3.2.5 Magnetic Drums

The WWI computer utilizes 2 separate drum systems. An auxiliary drum is used to increase the storage capacity of the computer. There is also available a buffer drum having a limited amount of auxiliary storage capacity which is used to increase further the storage capacity of the computer.

The auxiliary drum has been running with the computer for several months. The buffer drum has only recently been connected to the computer on a permanent basis.

Operation of the auxiliary-drum system with WWI has been quite reliable during the past quarterly period except for occasional failures caused by writing between the slots. Information pulses are spaced at 8-μsec intervals around the periphery of the magnetic drums in WWI. The current pulse which establishes the magnetic field patterns is of 2-μsec duration and is carefully timed to fall in the same physical location on successive write operations at the same address. Spurious writing which takes place "off time" (writing between the slots) alters the field pattern on the drum surface and can cause faulty readout from the drum. This spurious writing is caused by power-supply failures which remove bias from the circuits in control of the writing process. A temporary, unregulated power supply, used for the bias voltage, is believed to have been responsible for some trouble; this supply has now been replaced by a permanent, regulated supply. An additional source of trouble is the improper sequence of removing the supply voltages which occurs in some modes of power shutoff; power-supply control is currently undergoing revision to correct this.

A complete system of blown-fuse indication similar to that used in WWI has now been installed in the buffer-drum system. The buffer drum has also been connected to WWI supplies instead of to the separate supplies which were furnished by ERA. This change was made to simplify the WWI power system by reducing the number of separate power supplies and to provide blown-fuse indication and voltage interlocks which were similar to those in WWI. The buffer drum is now available at all times for the use of programmers.

4. CIRCUITS AND COMPONENTS

4.1 Vacuum Tubes

4.1.1 Vacuum-Tube Life

The WWI computer operated for 1670 hours during the first quarter of 1954; several weekend holidays reduced the total time somewhat.

Vacuum-tube life has been calculated for five different tube types as described in Summary Report No. 36. A summary of this information is shown below.

	FAILURE RATE, PER CENT PER 1000 HOURS		
Tube Type	1952	1953	First Quarter 1954
7AD7/SR1407/6145	2.00	3.3	1.75
7AK7	0.26	0.43	0.5
6080/6080WA/6AS7G		6.6*	1.1
5965		0.2*	0.4
6BL7GT		0.7*	0.3

* Last quarter 1953 only

The lower rate of failures of the group 6080/6080WA/6AS7G is particularly heartening, since this tube is a vital part of the magnetic-core memory now installed in WWI. The data for the last quarter of 1953 included a number of tubes which had failed on test for grid-to-grid shorts, which can cause no trouble in a majority of machine sockets.

Most of the sockets of the group 7AD7/SR1407/6145 are now filled by 6145 tubes. The figures here seem to indicate that a lower failure rate may be expected from the 6145 once the early-life failures are gone. These tubes are burned for 100 hours before installation, but there seems to be a residue of early-life failures. The majority of failures seems to occur during the first 2000 hours; not enough data has been gathered as yet to determine the exact behavior of the tube during this period.

Experience on the 5965 and 6BL7GT types continues to be quite satisfactory. If these preliminary indications are borne out by continued good life, both these types should be satisfactory for all but the most critical applications.

			Reason for failure; number failed			
Type	Total in Service	Hours at Failure	Change in Characteristics	Shorts, Opens	Breakage	Gassy
C16J	12	2000-3000			1	
C6J	8	0-1000	1			
		19000-20000	1			
		22000-23000	2			
OA3/VR75	4					
VR75		20000-21000	1			
OD3/VR150	23					
VR150		1000-2000	1			
		15000-16000	1			
2C51/5670	47					
5670		9000-10000	2			
		11000-12000	1			
2D21/5727	191					
2D21		no clock hours	2	1		
		1000-2000	2			
3E29/829B	141					
3E29		2000-3000	1			
		4000-5000	4			
		10000-11000	1			
		13000-14000	1	1		
		15000-16000	1			
		23000-24000		1		
5U4G	15	0-1000		1		
5Y3GT	6	0-1000	1			
6AG7	81	3000-4000		1		
		13000-14000	1			
		22000-23000		1		
6AK5/5654	13					
5654		0-1000			1	
6AS7G/6080/6080WA	718					
6AS7G		16000-17000	1			
		20000-21000		1		1
6080		0-1000	1	1		
		2000-3000		1		
		3000-4000	1			3
6080WA		0-1000	1			
		1000-2000	1			
		2000-3000		1		
6AU6/6136	244					
6AU6		2000-3000		2		
6BL7GT	473	2000-3000	2			
6L6G/5881	53					
6L6G		7000-8000	1			
		8000-9000	3			
		10000-11000	1			
		22000-23000	1	1		
		23000-24000	1			
5881		0-1000	3			
		1000-2000	2	1		
		4000-5000		1		
6SH7	29	0-1000	1			
		1000-2000	1			

Fig. 4-1. WWI Tube Failures

Type	Total in Service	Hours at Failure	Reason for failure; number failed: Change in Characteristics	Shorts, Opens	Breakage	Gassy
6SN7GT	363	17000-18000	1			
		22000-23000		1		
6V6GT	30	23000-24000	1			
6X5GT	25	18000-19000	1			
6Y6G	321	0-1000		1		
		5000-6000				1
		10000-11000		2		
		21000-22000	1	5		1
		22000-23000	2	1		
		23000-24000	2	2		
7AD7/6145/ SR-1407	3981					
7AD7		0-1000	1	1		
		1000-2000		1		
		2000-3000		1		
		3000-4000		1		
		6000-7000		1		
		8000-9000		2		
		10000-11000	2			
		11000-12000		2		
		12000-13000		1		
		15000-16000		3		
		16000-17000	1	12		
		17000-18000	1			
		18000-19000		2		
		21000-22000		7		
		22000-23000		10		
		23000-24000	3	12		
6145		0-1000	3	20		
		1000-2000	1	14		
		2000-3000	1	4		
		3000-4000		1		
		4000-5000		3		
		5000-6000	1	3		
		6000-7000		1		
7AK7	2838	0-1000	1			
		1000-2000				1
		2000-3000		1		
		3000-4000		1		
		8000-9000		1		
		9000-10000			2	
		14000-15000		1		
		16000-17000		1		
		20000-21000	1	1		
		21000-22000	2	4		
		22000-23000		5		
		23000-24000	1	1		
12AU7/5963	402					
12AU7		8000-9000	1	1		
5963		1000-2000		1		
		2000-3000	2			
		4000-5000	1			
		9000-10000	1			
		10000-11000	1	1		
715B-C	124					
715C		no clock hours	4	1		1
5651	36	0-1000	1			
		22000-23000	1			
5687	105	0-1000		1		
		7000-8000		2		
		8000-9000	2	1		
		9000-10000	1	2		
		10000-11000		1		
5965	606	no clock hours	1	1		
		1000-2000		2		
5998	34	0-1000		3		
		2000-3000		2		

4.1.2 Vacuum-Tube Research

During the past quarter intensive experimental research has been conducted on transient changes in oxide cathodes as described in Summary Report No. 36.

It has been possible to measure the pulsed and direct-current emission from oxide cathodes over a wide range of temperatures (determined by thermocouple); a Richardson plot summary of the data for one tube is shown in Fig. 4-2. The predicted value of d-c emission is shown superposed on the experimental plot. The magnitude of the predicted values is approximately correct (the prediction assumes that the donors carry a constant positive charge equal to that of one electron), but the slope of the predicted d-c emission does not correspond closely to the observed value. Very similar plots have been obtained for other tubes.

Very precise temperature control is required in order to obtain good measurements of coating resistance by the retarding-potential technique. The allowable deviation is less than 0.1 degree at 800 K. Such stability can be maintained adequately for pulsed measurements, but difficulties are introduced in d-c measurements by the power removed from the cathode by emitted electrons in overcoming the potential varrier of the work function. It has been possible to maintain adequate control for steady-state work; however, satisfactory transient measurements of the coating-resistance change have not been made because the control of temperature within necessary limits could not be accomplished.

Measurements of the changes in resistance in the transition pulsed to d-c conditions have been compared to the change in emission for the same transition. The two changes are not compatible with theory, as the resistance change is much smaller than predicted. The explanation for the discrepancy has not yet been found.

With very general assumptions it can be shown that the recovery (constant temperature) of cathode emission after direct current has passed through the cathode

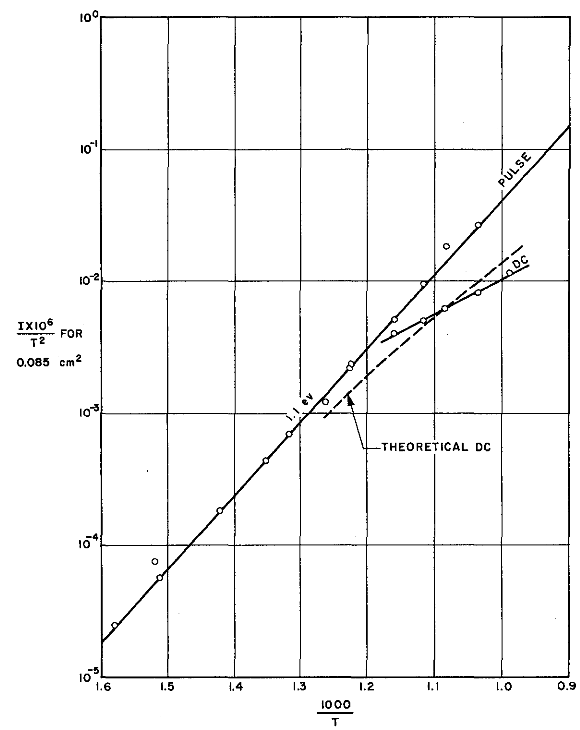


Fig. 4-2. Richardson Plot, RT 414

for some seconds follows an exponential, with some initial higher-order terms which die out quickly. The cathode emission must be expandible in a Taylor series as a function of the number of donor centers at the emitting surface. The time constant of the decay process after the higher-order terms died out is

$$\tau = \frac{d^2}{\pi^2 D} .$$

In this equation d is the coating thickness, and D is the diffusion constant. Using this equation and experimental decay curves, the diffusion constant for donors seems to be about 1.5×10^{-6} cm^2/sec at 850 K. The diffusion constant thus determined is 3×10^5 greater than the diffusion constant determined for barium in cathode at the same temperature.*
However, checks on this constant with three different cathodes give about the same results, indicating that the whole cathode is involved in the diffusion process.

4.2 Component Replacements

Fig. 4-3 lists the replacements of components other than tubes during the first quarter of 1954.

* Bever, R.S., Journal of Applied Physics 24 1008, 1953.

Component	Type	Total in Service	No. of Failures	Hours of Operation	Comments
Capacitor	0.01 ceramic disc	4525	1	2000-3000	Shorted
Crystal Diodes	D-358/1N38A	4617			
	D-358		1	11000-12000	Drift
			2	19000-20000	1 drift; 1 shorted
			2	21000-22000	Drift
			1	22000-23000	Unstable back resistance
			1	23000-24000	Unstable back resistance
	1N38A		2	0-1000	Low R_b
			1	3000-4000	Low R_b
			2	7000-8000	Low R_b
			1	16000-17000	Low R_b
			1	17000-18000	Drift
	D-357/1N34A	16469			
	1N34A		5	0-1000	Low R_b
			6	2000-3000	Low R_b
			4	3000-4000	Low R_b
	D-359/1N56A	466			
	D-359		2	20000-21000	1 shorted; 1 drift
	1N92	71	2	1000-2000	Drift
Potentiometers	25000-ohm 2-watt	36	1	1000-2000	Irregular taper
	1000-ohm 25-watt w/w ohmite	160	1	19000-20000	Open
Resistors	9000-ohm $\frac{1}{2}$-watt $\pm 1\%$ carbon deposited	842	2	0-1000	1 intermittent; 1 above tolerance
	2500-ohm $\frac{1}{2}$-watt $\pm 1\%$ carbon deposited	842	1	0-1000	Above tolerance
	220-ohm $\frac{1}{2}$-watt composition	9292	1	5000-6000	Above tolerance
	680-ohm 1-watt $\pm 5\%$ composition	40	2	0-1000	Burned out
Switch	SPST Part No. 2201	5	1	12000-13000	Defective
Timer	0-15 minute Cramer	3	1	22000-23000	Overheated
Transformer	Pulse, S-193-8 5:1	238	1	0-1000	Open primary

Fig. 4-3. WWI Component Failures January 1 - March 31, 1954

5. ACADEMIC PROGRAM

5.1 Advanced Seminars on Computing

These seminars provide an opportunity for the exchange of information on programming, components, logical design, and general developments in the computer field. The program during the last quarter was as follows.

Date	Title	Speaker
January 8	The Revised (30-j, j) Programmed Arithmetic Subroutine for the CS II Computer	F. C. Helwig
January 15	The Revised Conversion Program for the CS II Computer	J. M. Frankovich
February 19	Output for CS Computer	J. D. Porter
February 26	Multi-way Post-Mortems and Automatic Logging of Computer Operations	D. Combelic
March 5	Magnetic Core Memory	W. N. Papian
March 12	Magnetic Tape as Used for Auxiliary Storage and Delayed Printer	H. H. Denman
March 19	Memory Test Computer	P. R. Bagley

5.2 Programming Course

The DCL CS II programming course was given twice during this quarter. The course includes the following topics: relative addresses, temporary storage, floating addresses, preset parameters, programmed arithmetic, cycle counters, buffer storage, automatic output, post-mortems, and multi-pass conversion. The text for the course is a programmer's manual written by members of the S&EC Group.

The 35 students enrolled during this quarter represented the following groups: Aero-Elastic and Structures Research Laboratory, Instrumentation Laboratory, MIT Electrical Engineering Department, MIT Chemical Engineering Department, Dynamic Analysis and Control Laboratory, MIT Mathematics Department, Lincoln Laboratory, Geophysical Analysis Group, Laboratory for Nuclear Science and Engineering, Solid State and Molecular Theory Group, MIT Geophysics Department, and MIT Meteorology Department.

5.3 Visitors

During the evening of January 19, the Digital Computer Laboratory was host to 70 members of the American Society of Heating and Ventilating Engineers. The evening's program consisted of an after-dinner lecture on digital computers given by Edwin S. Kopley, a tour of the WWI installation, and demonstrations on the computer and Flexowriter equipment.

Tours of the WWI installation are conducted between 5 and 6 p.m. on the first Tuesday of each month. They include computer and Flexowriter demonstrations and an informal discussion of the major computer components. During the past quarter the following tours were conducted:

January 5	6 MIT students and staff members
February 2	9 MIT and 3 Los Alamos personnel
March 2	2 MIT students and 4 Project Lincoln personnel

5.4 Spring Term, 1954

Twelve students are enrolled in the Electrical Engineering subject 6.537. Each student has programmed, prepared on punched tape, debugged, and executed on Whirlwind one problem of his own choice. Two of these class problems are also to serve as senior theses, and one other will be continued this summer as part of a Master's thesis.

6. APPENDIX

6.1 Reports and Publications

Project Whirlwind technical reports and memorandums are routinely distributed to only a restricted group known to have a particular interest in the Project, and to ASTIA (Armed Services Technical Information Agency) Document Service Center, Knott Building, Dayton, Ohio. Regular requests for copies of individual reports should be made to ASTIA; emergency requests, to John B. Bennett, Digital Computer Laboratory, 211 Massachusetts Avenue, Cambridge 39, Mass. Att: Code DCL-6. 1.

The following reports and memorandums were among those issued during the first quarter of 1954.

No.	Title	Date	Author
SR-36	Summary Report No. 36, Fourth Quarter 1953		
M-2527 (Supplement 1)	Input and Basic Conversion Programs	1-18-54	P. R. Bagley
M-2603	Marginal Checking Breakdown of the Instruction Frame	1-6-54	R. J. Pfaff
M-2661	WWI Operation Times	2-1-54	H. H. Denman
M-2669	Use of Silver in Etched-Wiring Cards	2-3-54	A. L. Loeb B. B. Paine
M-2670	Logical Networks, II - Binary Network Algebra	2-3-54	R. C. Jeffrey
M-2728	Increased Facilities for Visual Display in the WWI Input and Output	3-17-54	G. A. Young
M-2729	Paper Tape Units and Printers in the WWI Input-Output System	3-15-54	G. A. Young

6.2 Professional Society Papers

At the 18th Symposium on Ceramic Dielectrics, held at Rutgers University on March 10, F. E. Vinal presented a paper entitled "Ferrites with Square Hysteresis Loops."

F. E. Vinal also spoke on "Preparation of Ferrites for Physical Measurements" at the New York Academy of Sciences on March 2.

W. N. Papian gave a talk on "Coincident-Current Magnetic-Core Memories" at the meeting of the Boston Section of the IRE, held on February 18 and 19.

At the Sixth Southwestern IRE Conference, held in Tulsa, Oklahoma on February 4-6, D. Combelic presented a paper entitled "Introduction to Electronic Digital Computers."

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