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Implementation of Numerical Weather Forecasting on a System 360, Model 65 with an Array Processor (2938, Model 2)

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Implementation of Numerical Weather Forecasting on a System 360, Model 65 with an Array Processor (2938, Model 2)

by L. E. Alsop
Washington Scientific Center

and R. J. Wepman
GEM Region

ABSTRACT

Timings for operations of the type used in numerical weather forecasting are obtained for the 360 Model 65 with an Array Processor. On the basis of these timings, it should be possible to make a 36 hour forecast in eight minutes. This is 18 times faster than on a 7094 II.

Index Terms for IBM Subject Index

05 - Computer Application

Meteorology

Differential Equations

Parallel Processing

08 - Earth and Atmosphere

Weather

The information in this report is based upon preliminary specifications of 2938 which are subject to change.

IBM Scientific Center
11141 Georgia Avenue
Wheaton, Maryland

Preface

The Array Processor is being constructed because of the needs of the Seismic Industry (Oil Exploration), but its capacity to do repetitive operations rapidly makes it valuable for other applications as well. The Washington Scientific Center is concerned with the environmental sciences, and, therefore, the Center was interested in finding other applications for the Array Processor within these fields.

Initially, numerical weather forecasting seemed to hold the most promise. Therefore, Mr. H. Hansen, the Account Representative, for the ESSA-Weather Bureau National Meteorological Center, arranged for a visit with Dr. F. G. Shuman, Director of the National Meteorological Center. Dr. Shuman was interested in the possibilities of the Array Processor, and accordingly he suggested a simple forecast model which would be suitable for timing studies, and from which he could make estimates of the time required to make a forecast for the actual forecast model he used. This report is primarily concerned with the timing results obtained for the model which Dr. Shuman proposed.

Mr. H. A. Bedient and Mr. A. Kneer of the National Meteorological Center were also most helpful. In particular, having seen our first attempt at solving the problem, which was I/O bound, Mr. Bedient then coded the Array Processor to agree with the method currently used at the NMC, and achieved a program which was no longer I/O bound. It is Mr. Bedient's program which is used in this report for timing purposes.

The Sample Problem

The difference equations of the simplified sample problem proposed by Dr. Shuman are the following:*

$$\overline{u}^{t}_{t} + \overline{\overline{u}^{xy}\,\overline{u_x}^{y} + \overline{v}^{xy}\,\overline{u_y}^{x} - f\,\overline{v}^{xy} + g\,\overline{h_x}^{y}}^{xy} = 0 \quad (1)$$

$$\overline{v}^{t}_{t} + \overline{\overline{u}^{xy}\,\overline{v_x}^{y} + \overline{v}^{xy}\,\overline{v_y}^{x} + f\,\overline{u}^{xy} + g\,\overline{h_y}^{x}}^{xy} = 0 \quad (2)$$

$$\overline{h}^{t}_{t} + \overline{\overline{u}^{xy}\,\overline{h_x}^{y} + \overline{v}^{xy}\,\overline{h_y}^{x} + \overline{h}^{xy}(\overline{u_x}^{y}+\overline{v_y}^{x})}^{xy} = 0 \quad (3)$$

where u and v are the horizontal components of the wind velocity, and h is the height of a constant pressure surface above some predetermined height, f and g are constants. These expressions are in Shuman's notation in which he defines

$$\overline{u}^{x}\,(i + 1/2) = 1/2\,(u\,(i) + u\,(i - 1)) \quad (4)$$

$$u_x\,(i + 1/2) = \frac{1}{\Delta x}\,(u\,(i) - u\,(i - 1)) \quad (5)$$

Here u is a field variable defined over the x field, which consists of N points each equidistantly spaced Δx apart. Expression (4) gives an averaged value of u midway between $x = i\Delta x$ and $x = (i-1)\Delta x$; while expression (5) is a difference for these two points placed midway between them. The extension to a two dimensional field is simple. The two dimensional average is equivalent to averaging or differencing first in one dimension and then in the other. In order to have the averaged and

* A discussion of the problem is contained in Appendix A.

differenced fields defined at the field points, it is necessary to perform a second average. In expressions (1) through (3) the first averages or differences are taken, then products are formed and summed and finally the second average is formed. Then, using (1) as an example, this quantity is set equal to $-\bar{u}_t^t$, where $\bar{u}_t^t$ is defined according to (4) and (5) but over a discrete time net. With $\bar{u}_t^t$ defined over the field at time t_o it is now possible to predict the value of u at a time $t_o + \Delta t$ later, where t is the interval between time points.

$$U\ (t_o + \Delta t) = u(t_o - \Delta t) + \frac{1}{2\Delta t} u_t^t\ (t_o + \Delta t) \qquad (6)$$

As proposed for this problem, the size of the x, y field is 50 x 50, i.e., there are 2500 field points. We will discuss timings involved for one integration step from the values of u, v, and h defined over the field at time t_o to their values at a time Δt later.

Timing for One Iteration on Array Processor (2938, Model 2)

It is extremely desirable to structure the problem in such a manner that the 2938 is always accepting 8 bytes from memory. This implies that the data string is located in contiguous core locations so that two single words may be accepted at any time.

The following method of solution was proposed by Mr. Bedient. Consider the field variables as an array. Read two columns of data into core. Perform a vector addition or subtraction on the two columns as the first part of the averaging and/or differencing operation. Each of the columns is a vector which is accepted 8 bytes at a time by the 2938. Store the intermediate output in contiguous positions in core. Complete the averaging and/or differencing operation by convolving the intermediate output with weights (1/4, 1/4) or $(\frac{1}{2d}, -\frac{1}{2d})$, where d is the distance between field points in the x or y direction. In this way $\overline{u}^{xy}$, $\overline{u_x}^{y}$, $\overline{u_y}^{x}$, $\overline{v}^{xy}$, $\overline{v_x}^{y}$, $\overline{v_y}^{x}$, $\overline{h}^{xy}$, $\overline{h_x}^{y}$, and $\overline{h_y}^{x}$ may be formed. The remaining operations require use of the vector addition and vector multiply operations of the 2938. A total of 33 operations on the Array Processor is required to obtain values of u,v, and h at time $t_o + \Delta t$ over one column of the field array. For maximum efficiency these operations should be chained so that only one Start I/O is required per column iteration. The actual code devised by Mr. Bedient is given in Appendix B.

The basic operations on the Array Processor may be broken down into three kernels for timing purposes:

Kernel 1: $A_i = B_i \pm C_i$

or $A_i = B_i * C_i$ $\quad i = 1, 50$

Kernel 2: $A_i = A_i + B_i * C_i$ $\quad i = 1, 50$

Kernel 3: $A_i = \sum_{j=0}^{1} D_j B_{i-j}$. $\quad i = 1, 50$

For operations of this type, the timings will depend on the time required for the 2938 to obtain the data and to store the result. Assuming a fetch or store time of 1.6 μ sec per double word, the timings per operation are:

Kernel 1 - 2.4 μ sec; Kernel 2 - 3.2 μ sec; Kernel 3 - 1.6 μ sec.

Finally multiplying by 50, we obtain kernel times of:

Kernel 1 - 120 μ sec.

Kernel 2 - 160 μ sec.

Kernel 3 - 80 μ sec.

The code in Appendix B contains 10 Kernel 1 operations, 10 Kernel 2 operations and 13 Kernel 3 operations. Thus the total time required per column is 3.84 m sec. To this must still be added the time required because of interference while obtaining and storing data on the drum plus overhead time.

For a problem of this sort the recommended storage device would be an IBM 2301 Drum, which requires a memory cycle approximately every 7 μ sec. per double word. The Model 65 has a memory cycle of .75 μ sec, so the drum would require approximately every tenth cycle. The 2938 requires about every second cycle. Assuming 100% interference between the drum and the Array Processor with the drum having priority, the time required for the Array Processor to complete the job would be increased by 10% or 0.38 m sec.

In order to perform an operation, the 2938 must first fetch four double words, one channel command word and three operand control words. Also the time given above for a convolution does not include the time required to obtain the one double word with the two filter weights. Finally, the pipe line must be filled and emptied during each operation, which adds approximately 2.5

u sec per operation. Therefore, a good estimate for overhead is 5% of the processor time of 0.19 m sec per column.

The timing for one column is thus:

Processor time	3.84 m sec
Drum Interference	.38 m sec
Overhead	.19 m sec
	4.31 m sec

The time for iteration over the entire field is 50 x 4.31 m sec = 215.5 m sec. Assuming an integration interval t = 10 min over a 36 hour forecast, the total forecast time would be 48 seconds. For the actual six-level model used by the weather bureau, the forecast time is about ten times greater than for the simple model considered here. Therefore, a Model 65 with a 2938 would take 8 minutes to make a forecast. Previously Dr. Shuman had estimated that a 7094 II would require approximately 150 minutes.

I/O Limitations

In the above timing estimates, we have not included the drum access time. It is assumed that the program used at the National Meteorological Center can be optimized with respect to the drum rotation rate.

As an example, for the sample problem considered above, it is necessary to write out three columns of results for the u, v, and h fields, and to send in six columns of stored u, v, and h data for previous times during a calculation of three columns of u, v, and h. This is a total of 450 words. The drum transfer time is 1.5 m sec. There is ample time for transfer.

In order to optimize for the drum rotation rate, four columns could be processed at a time. Then the total time required would be 17.24 m sec as compared with a drum rotation rate of 17.5 m sec. This would result in a change of the timing for processing one column from 4.31 m sec to 4.38 m sec. In any actual problem, of course, timings could never be this accurate, and a trial and error procedure would be required for optimization.

Conclusions

The Model 65 with a 2938 Model 2 is extremely well suited for numerical weather forecasting. This combination is 18 to 19 times faster than a 7094 II for problems of this sort.

Appendix A

Derivation of the Difference Equations

The problem as proposed by Dr. Shuman is an extremely simplified version of the basic primitive equations governing the flow of a fluid about a rotating gravitating earth. The equations take the form:

$$\frac{\partial u}{\partial t} + \frac{u \partial u}{\partial x} + \frac{v \partial u}{\partial y} - fv + g \frac{\partial h}{\partial x} = 0 \qquad (1')$$

$$\frac{\partial v}{\partial t} + \frac{u \partial v}{\partial x} + \frac{v \partial v}{\partial y} + fu + g \frac{\partial h}{\partial y} = 0 \qquad (2')$$

$$\frac{\partial h}{\partial t} + \frac{u \partial h}{\partial x} + \frac{v \partial h}{\partial y} + h \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) = 0 \qquad (3')$$

where u and v are the x and y components of the horizontal wind velocity and h is the height of a constant pressure surface above some arbitrary horizontal plane. For simplicity f and g are set constant, although in an actual problem, they are a function of position.

The first three terms of (1'), (2'), and (3') are just the total time derivatives of u,v, and h. In (1') and (2'), the resulting accelerations are set equal to the forces acting on the fluid namely Coriolis (the f term) and gravitational (the g term). The last two terms in (3') are recognizable as a divergence of the velocity. This is a continuity equation.

These three equations must now be put in a difference form to be solved numerically. The operation of equation (5) is to replace the partial derivatives of (1'), (2'), and (3') by differences. Equation (4), the averaging technique, may be thought of as a low pass filter. Perturbations acting over a small region are reduced in size to keep the difference system (1), (2), and (3) stable.

While the set of equations considered here are inadequate for numerical weather forecasting, they do illustrate the basic numerical methods used. Therefore, they are suitable for timing studies of the type contained in this report.

Appendix B

Code for Time Iteration of One Column*

Operation	Source of Intermediate Data
1. $\overline{u}^y = u_i + u_{i+n}$	
2. $\overline{u}^y_x = \left(\frac{1}{2d}, \frac{-1}{2d}\right) X \overline{u}^y$	result of 1
3. $\overline{u}^{xy} = \left(\frac{1}{4}, \frac{1}{4}\right) X \overline{u}^y$	result of 1
4. $A = \overline{u}^{xy} * \overline{u}^y_x$	result of 1,3
5. $u_y = u_{i+n} - u_i$	
6. $\overline{u}^x_y = \left(\frac{1}{2d}, \frac{1}{2d}\right) X u_y$	result of 5
7. $\overline{v}^y = v_{i+n} + v_i$	
8. $\overline{v}^y_x = \left(\frac{1}{2d}, -\frac{1}{2d}\right) X \overline{v}^y$	result of 7
9. $\overline{v}^{xy} = \left(\frac{1}{4}, \frac{1}{4}\right) X \overline{v}^y$	result of 7
10. $A = A + \overline{v}^{xy} * \overline{u}^x_y$	result of 4,6,9
11. $B = \overline{u}^{xy} * \overline{v}^y_x$	result of 3,8

*Based on example programmed by H.A. Bedient, ESSA-NMC.

12. $A = A - f * \overline{v}^{xy}$ result of 10,9

13. $B = B + f * \overline{u}^{xy}$ result of 11,3

14. $v_y = v_{i+n} - v_i$

15. $\overline{v}_y^x = \left(\frac{1}{2d}, \frac{1}{2d}\right) X_y^v$ result of 14

16. $B = B + \overline{v}^{xy} * \overline{v}_y^x$ result of 13,9,15

17. $\mathrm{div}V = \overline{u}_x^y + \overline{v}_y^x$ result of 2,15

18. $\overline{h}^y = h_{i+n} + h_i$

19. $\overline{h}_x^y = \left(\frac{1}{2d} - , \frac{1}{2d}\right) X\ \overline{h}^y$ result of 18

20. $C = \overline{u}^{xy} * \overline{h}_x^y$ result of 3,19

21. $A = A + g * \overline{h}_x^y$ result of 19

22. $h_y = h_{i+n} - h_i$

23. $\bar{h}_y^x = \left(\frac{1}{2d}, \frac{1}{2d}\right) X h_y$ result of 22

24. $C = C + \bar{v}^{xy} * \bar{h}_y^x$ result of 9,23

25. $B = B + g * \bar{h}_y^x$ result of 16,22

26. $\bar{h}^{xy} = \left(\frac{1}{4}, \frac{1}{4}\right) X \bar{h}^y$ result of 18

27. $C = C + \bar{h}^{xy} * \text{div}V$ result of 17,26

28. $\bar{C}^x = \left(\frac{\Delta t}{2}, \frac{\Delta t}{2}\right) X C$ result of 27

29. $\bar{B}^x = \left(\frac{\Delta t}{2}, \frac{\Delta t}{2}\right) X B$ result of 25

30. $\bar{A}^x = \left(\frac{\Delta t}{2}, \frac{\Delta t}{2}\right) X A$ result of 21

The $\bar{A}^x$, $\bar{B}^x$, $\bar{C}^x$ vectors are saved from row n-1 to row n

31. $u_{t+1} = u_{t-1} - \bar{A}^x_{n-1} + \bar{A}^x_n$

32. $v_{t+1} = v_{t-1} - \bar{B}^x_{n-1} + \bar{A}^x_n$

33. $h_{t+1} = h_{t-1} - \bar{C}^x_{n-1} + \bar{C}^x_n$

The operation (a,b) X u represents a convolution.