1620 GENERAL PROGRAM LIBRARY

The Numerical Calculation Of The Definite Integral Of A Real Function Using Simpson's Rule (Card)

7.0.043

DISCLAIMER

1620 USERS GROUP PROGRAM REVIEW AND EVALUATION

Program No. ____________________ Date ________________

Program Name: ____

1. Does the abstract adequately describe what the program is and what it does? Yes___ No___
   Comment ____

2. Does the program do what the abstract says? Yes___ No___
   Comment ____

3. Is the Description clear, understandable, and adequate? Yes___ No___
   Comment ____

4. Are the Operating Instructions understandable and in sufficient detail? Yes___ No___
   Comment ____
   Are the Sense Switch options adequately described (if applicable)? Yes___ No___
   Are the mnemonic labels identified or sufficiently understandable? Yes___ No___
   Comment ____

5. Does the source program compile satisfactorily (if applicable)? Yes___ No___
   Comment ____

6. Does the object program run satisfactorily? Yes___ No___
   Comment ____

7. Number of test cases run ____________________
   Are any restrictions as to data, size, range, etc. covered adequately in description? Yes___ No___
   Comment ____

8. Does the Program meet the minimal standards of the 1620 Users Group? Yes___ No___
   Comment ____

9. Please list any suggestions to improve the usefulness of the program. These will be passed on to the author for his consideration.
   Comment ____

Please return to:

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Marquette University
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1515 W. Wisconsin Avenue
Milwaukee 3, Wisconsin

Your Name ____________________

Company ____________________

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User Group Code ____________________

The Numerical Calculation Of The Definite Integral Of A Real Function Using Simpson's Rule (Card)

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<u>DECK KEY</u>

1. Valuation Routine 1 - 2 cards
2. Valuation Routine 2 - 2 cards
3. Example 1 Deck - 5 cards
4. Example 2 Deck - 5 cards
5. Source Deck
6. Object 1 Deck
7. Object 2 Deck

THE NUMERICAL CALCULATION OF THE DEFINITE INTEGRAL OF A REAL FUNCTION USING SIMPSON'S RULE (Card)

Donald S. Miller

Subject Classification 7.0

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Purpose/Description: This program will tabulate using Simpson's Rule, a preassigned number of definite integrals $F(\cdot, c_1, c_2, \cdots, c_m)$ defined by the relation

$$F(x, c_1, c_2, \cdots, c_m) = \int_a^x f(x, c_1, c_2, \cdots, c_m)dX$$

where 1) $a, c_1, c_2, \cdots, c_m$ are real numbers and 2) f is a real function whose evaluation at $X, c_1, c_2, \cdots, c_m$ may be represented in Fortran language by a single statement.

A real number $b > a$, and odd subdivision D: $a = X_0, X_0 + \Delta X_0, X_0 + 2\Delta X_0, \cdots, X_0 + 2n\Delta X_0 = b$, $\Delta X_0 = (b-a)/2n$, of the interval (a,b) and a range: $b, b + 2\Delta X_0, b + 4\Delta X_0, \cdots, b + 2(k-1)\Delta X_0$ for the variable x are associated with each $F(\cdot, c_1, c_2, \cdots, c_m)$.

The numbers $m, 2n+1, k, a, b, c_1, c_2, \cdots, c_m$ are specified in a series of control cards.

Method: Simpson's Rule is used for approximating all definite integrals. See Numerical Mathematical Analysis, Third Edition, by J. B. Scarborough, The Johns Hopkins Press, page 132, section 46 and page 174, section 57, (c).

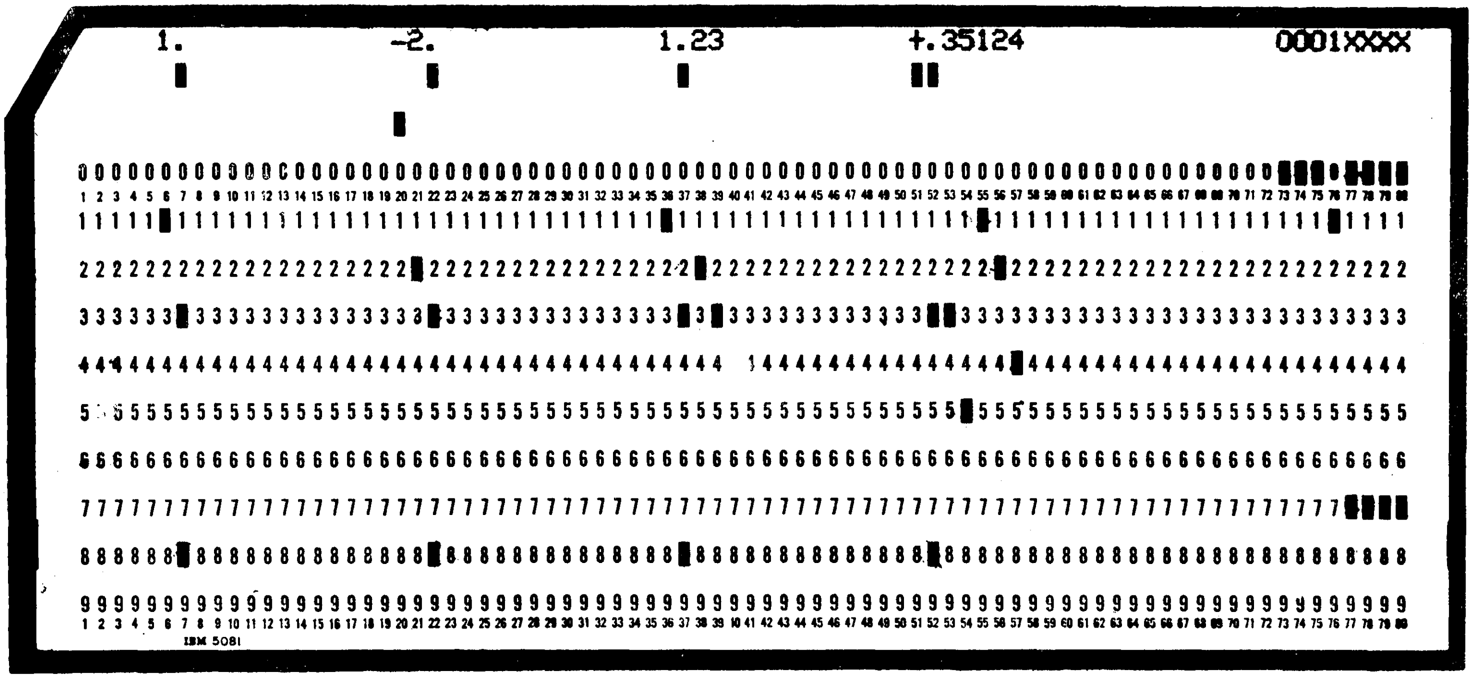
Restrictions/Range: All calculations are subject to the laws of rounding error and the precision of the Fortran system.

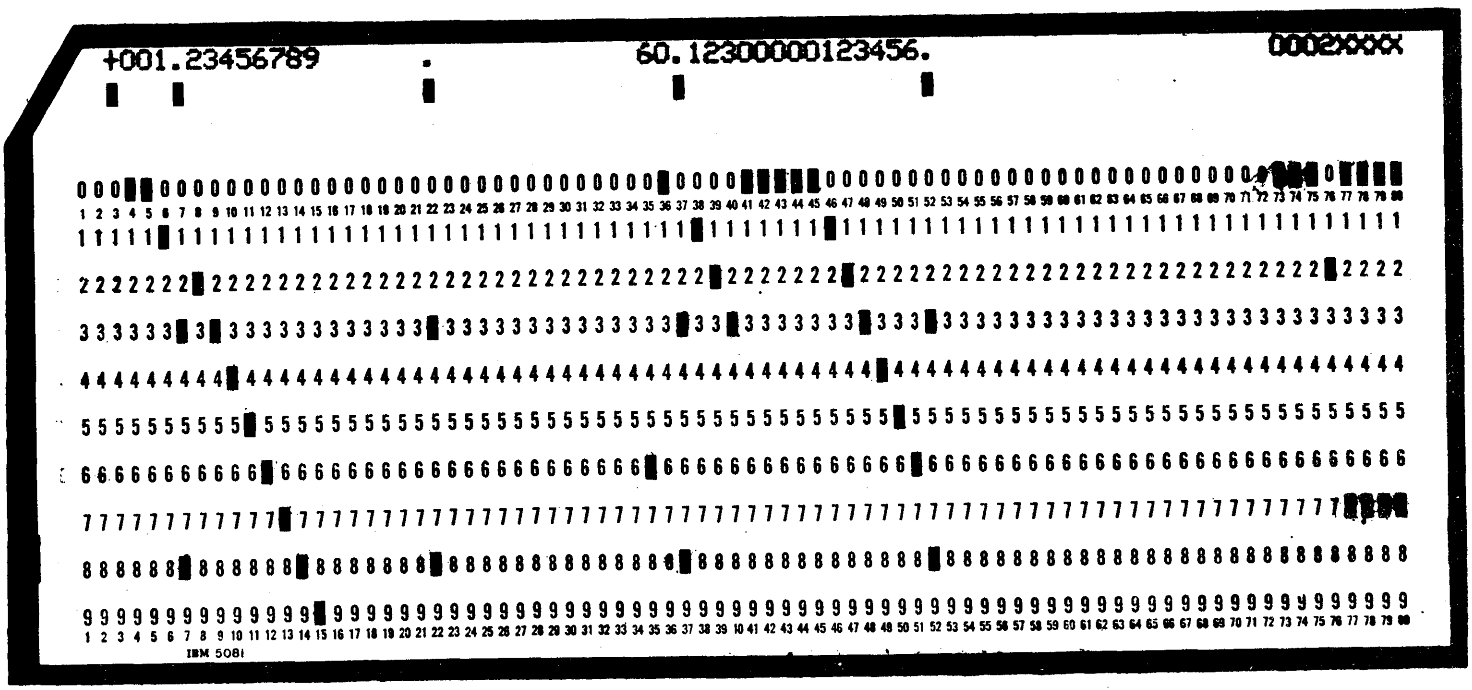
<u>Storage Requirements</u>: N/A

<u>Equipment Specifications</u>: N/A

<u>Additional Remarks</u>: This program was written in Fortran language.

REGULAR DATA CARDS





Program Description

Title THE NUMERICAL CALCULATION OF THE DEFINITE INTEGRAL OF A REAL FUNCTION USING SIMPSON'S RULE

Program Input Medium Except for control cards (which will be described in the Program Input section), all input to this program will be punched in regular data cards.

A regular data card is an 80-column, punched card subdivided into four, fifteen position fields, columns 1-15, columns 16-30, columns 31-45 and columns 46-60, where each sub-field is punched in a right justified manner with a decimal number having the form $\pm$ xxxxx.xxxxxxxx (x represents an arbitrary digit). Note that the decimal point is always punched in position seven of the corresponding sub-field and that the sign punch may be in any one of the positions 1-6. Insignificant zeros need not be punched. The omission of the sign punch automatically defines a non-negative number. Any blank sub-field will represent zero. Columns 61-80 may be punched in arbitrary fashion and so may be used for input sequencing and identification.

Examples of regular data cards are shown on page 3.

Program Input For $j = 1, 2, \cdots, s$ let it be required to approximate the definite integral

$$F_j(x, c_{1j}, c_{2j}, \cdots, c_{mj}) = \int_{a_j}^{x} f_j(X, c_{1j}, c_{2j}, \cdots, c_{mj})dX$$

where 1) $a_j, c_{1j}, c_{2j}, \cdots, c_{mj}$ are real numbers, 2) the subdivision D is defined by the points $a_j = X_{0j}, X_{0j} + \Delta X_{0j}, X_{0j} + 2\Delta X_{0j}, \cdots, X_{0j} + 2n_j\Delta X_{0j} = b_j$, $a_j < b_j$, $\Delta X_{0j} = (b_j - a_j)/2n_j$, 3) the evaluation of the real function f_j at X, $c_{1j}, c_{2j}, \cdots, c_{mj}$ may be expressed in Fortran language by a single statement and 4) the variable x ranges over the k_j points b_j, $b_j + 2\Delta X_{0j}$, $b_j + 4\Delta X_{0j}$, $\cdots$, $b_j + 2(k_j-1)\Delta X_{0j}$.

The input associated with the s functions $F_1, F_2, \cdots, F_s$ consists of the ordered collection C, $S_1, S_2, \cdots, S_s$ where C represents the Initial Control Card and for $j = 1, 2, \cdots, s$, S_j is the set of all input cards belonging to the j-th function F_j.

(1) The Initial Control Card C is punched according to the following table

Columns	Punch	Description
1-3	s	s is a three digit unsigned number.
4-6	m	m is a three digit unsigned number.

(2) S_1 consists of the ordered collection C_1 and S_{11} where C_1 is the initial Problem Control Card and S_{11} defines the input belonging to the first function f_1.

(a) The Problem Control Card C_1 is punched according to the following table

Columns	Punch	Description
1-3	$2n_1+1$	$2n_1+1$ is a three digit unsigned number.
4-6	k_1	k_1 is a three digit unsigned number.
7-21	a_1	a_1 is a thirteen digit, eight decimal digit signed number.
22-36	b_1	b_1 is a thirteen digit, eight decimal digit signed number.

(b) The ordered set of coefficients $c_{11}, c_{21}, \cdots, c_{m1}$ are punched continuously in regular data cards to form the input set S_{11}. Care should be taken to punch the first coefficient c_{11} in the first fifteen position field of the first regular data card in S_{11}.

(3) S_2 comprises the ordered collection C_2 and S_{21} where C_2 is the initial Problem Control Card and S_{21} defines the input belonging to the second function f_2.

The sets C_2 and S_{21} are defined as in (2), sections (a) and (b).

(4) The input is completed by punching the sub-inputs S_3, S_4, $\cdots$, S_s .

The ordered collection of sets C, S_1, S_2, $\cdots$, S_s will then define the total input.

The examples presented at the end of this report should clear up any difficulties.

Evaluation Routine It is necessary for the user of this program to construct the so-called Evaluation Routine which is to be inserted in a fixed location in the original Fortran source program.

As before, for $j = 1, 2, \cdots, s$ let it be required to approximate the definite integral

$$F_j(x, c_{1j}, c_{2j}, \cdots, c_{mj}) = \int_{a_j}^{x} f_j(X, c_{1j}, c_{2j}, \cdots, c_{mj})dX .$$

The Evaluation Routine is written in Fortran language and will compute the value of the function f_j at X, c_{1j}, c_{2j}, $\cdots$, c_{mj} . In writing this program one assumes the argument X to be stored at "Fortran" location X while the coefficients c_{1j}, c_{2j}, $\cdots$, c_{mj} are stored in the so-called C-array.

For simplicity suppose that the coefficients c_{1j}, c_{2j}, $\cdots$, c_{mj} are stored at locations C(1), C(2), $\cdots$, C(M) respectively.

The user must evaluate the given function f_j using the coefficients stored in the C-array and the argument stored at X.

The resultant functional value must be stored at "Fortran" location Y.

The ordered set of Fortran cards defined by the Evaluation Routine is then filed in the original Fortran source program immediately following the card with identification* 30011C50. (See Fortran Program Listing at the end of this report.)

The augmented Fortran source deck may then be used to compile the desired object program. After compilation, the user should remove the Evaluation Routine from the source program, which will then be ready to accept a different Evaluation Routine.

Restrictions Refer to the notation of Program Input section.

The number s must satisfy the inequality $1 \leq s \leq 999$.

The number of coefficients m belonging to the integrand f_j, $1 \leq j \leq s$ must presently satisfy the inequality $1 \leq m \leq 50$. The size of m is controlled by the single dimension statement with identification 01020C50. Consequently the range of m may be decreased or increased depending on the amount of computer memory available.

For $j = 1, 2, \cdots, s$, the number $2n_j+1$ of points of subdivision of the interval (a_j, b_j) and the number k_j of entries in the tabulation of the definite integral $F_j(\cdot, c_{1j}, c_{2j}, \cdots, c_{mj})$ must satisfy the inequality $1 \leq 2n_j+1, k_j \leq 999$.

All calculations are subject to the laws of rounding error.

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* The identification is punched in columns 73-80 of each Fortran source card.

Program Output

Switches All program switches may be set arbitrarily.

Content The notation of Program Input section will be used.

For $j = 1, 2, \cdots, s$, this program will record

(a) the number $2n_j+1$ of points of subdivision into which the interval (a_j, b_j) is subdivided,

(b) the number k_j of entries in the tabulation of the definite integral function $F(\cdot, c_{1j}, c_{2j}, \cdots, c_{mj})$,

(c) the interval limits a_j, b_j,

(d) the function coefficients $c_{1j}, c_{2j}, \cdots, c_{mj}$,

(e) the tabulation of the definite integral function $F(\cdot, c_{1j}, c_{2j}, \cdots, c_{mj})$.

Note that the columns headed by X, AREA and E respectively define the value of the independent variable x, the corresponding "Simpson's Rule" approximation to the definite integral $F_j(x, c_{1j}, c_{2j}, \cdots, c_{mj})$ and the error generated in using such an approximation.

Error Messages The notation of Program Input section will be used.

The message THERE IS AN INVALID PARAMETER will be recorded if for any j such that $1 \leq j \leq s$, one of the following error conditions occurs:

(a) the total number of points of subdivision N_j in subdivision D is even,

(b) the total number of points of subdivision N_j in subdivision D is less than 3,

(c) the total number of entries k_j in the tabulation of the j-th definite integral function $F_j(\cdot, c_{1j}, c_{2j}, \cdots, c_{mj})$ is negative.

Whenever one of the above error conditions arises, the given program will either proceed with the next numerical integration or will stop at end-of-job.

Numeric Example

1. Evaluate numerically the definite integral

$$\int_0^x X^2 \sqrt{1+6X} \, dX$$

for x = 1, 1.04, 1.08, ⋯, 1.76 .

2. Tabulate the function defined by the integral

$$\int_1^x X^2 \sqrt{7+9X} \, dX$$

over the range $5 \leq x \leq 6$.

3. Compute numerically the definite integral

$$\int_0^{2\pi} \frac{d\theta}{\frac{5}{4} + \sin\theta} .$$

4. Compute numerically the definite integral

$$\int_0^{\pi} \frac{d\theta}{\frac{3}{2} - \cos\theta} .$$

To apply the given program to examples 1 and 2 define the function f by the relation

$$F(X, c_1, c_2) = X^2 \sqrt{c_1+c_2X} .$$

A direct verification shows that

$$\int_0^x X^2 \sqrt{1+6X \cdot dX} = \int_0^x f(X, 1, 6)dX ,$$

$$\int_1^x X^2 \sqrt{7+9X \; dX} = \int_1^x f(X, 7, 9)dX .$$

Let it be assumed that the coefficients c_1 and c_2 are stored in the C-array at locations C(1) and C(2) respectively.

Evaluation Routine 1 for evaluation f at X, c_1 and c_2 is shown on page 18.

To apply the given program to examples 3 and 4 define the function g by the relation

$$g(X,c_1,c_2,c_3,c_4) = \frac{1}{c_1 + c_2\sin(c_3X + c_4)}.$$

A direct verification shows that

$$\int_0^{2\pi} \frac{d\theta}{\frac{5}{4} + \sin\theta} = \int_0^{2\pi} g(X,\tfrac{5}{4},1,1,0)dX \quad ,$$

$$\int_0^{\pi} \frac{d\theta}{\frac{3}{2} - \cos\theta} = \int_0^{\pi} g(X,\tfrac{3}{2},-1,-1,\tfrac{\pi}{2})dX \quad .$$

Let it be assumed that the coefficients c_1, c_2, c_3 and c_4 are stored in the C-array at locations C(1), C(2), C(3) and C(4) respectively.

Evaluation Routine 2 for evaluating the function g at X, c_1, c_2, c_3 and c_4 is shown on page 19.

Evaluation Routine 1 must then be inserted in the Fortran source program immediately after the card with identification (columns 73-80) 3011C50. After filing this evaluation routine, the resultant source program may then be compiled using the Fortran processor to give <u>Object Program 1</u>. A similar operation performed on Evaluation Routine 2 will generate <u>Object Program 2</u>.

Input 1 for examples 1 and 2 will be used with Object Program 1. s and m will both have value 2.

For example 1, set $a_1 = 0$, $b_1 = 1$, $2n_1+1 = 51^*$, $k_1 = 20$ and $S_{11} = (1,1)$.

For example 2, let $a_2 = 1$, $b_2 = 5$, $2n_2+1 = 151$, $k_2 = 25$ and $S_{21} = (7,9)$.

Input 2 for examples 3 and 4 will be used with Object Program 2. s and m will have values 2 and 4 respectively.

For example 3, set $a_1 = 0$, $b_1 = 2\pi$, $2n_1+1 = 75$, $k_1 = 1$ and $S_{11} = (\frac{5}{4},1,1,0)$.

For example 4, set $a_2 = 0$, $b_2 = \pi$, $2n_2+1 = 125$, $k_2 = 1$ and $S_{21} = (\frac{3}{2},-1,-1,\frac{\pi}{2})$.

The tabulations for the integrals

$$\int_0^X X^2 \sqrt{1+6X}\, dX \quad \text{and} \quad \int_1^X X^2 \sqrt{7+9X}\, dX$$

are shown in Output 1, Problems 1 and 2 respectively.

The values of the integrals

$$\int_0^{2\pi} \frac{d\theta}{\frac{5}{4} + \sin\theta} \quad \text{and} \quad \int_0^{\pi} \frac{d\theta}{\frac{3}{2} - \cos\theta}$$

are shown in Output 2, Problems 1 and 2 respectively.

---

* Note that $\Delta x_1 = .02$ and since $\Delta x_1 = \dfrac{b_1 - a_1}{2n_1}$, then $2n_1+1 = 51$.

INPUT 1

```
002002
051020     0.00000000     1.00000000                                            0001XXXX
     1.             1.             .              .                             0002XXXX
151025     1.00000000     5.00000000                                            0003XXXX
     7.             9.             .              .                             0004XXXX
```

OUTPUT 1

NUMBER OF COEFFICIENTS EQUALS 2

PROBLEM NUMBER 1

N	K	A	B
51	20	.00000000	1.00000000

FUNCTION COEFFICIENTS

COEFFICIENT NUMBER	COEFFICIENT
1	1.00000000
2	1.00000000

INTEGRAL FUNCTION

X	AREA	E
1.00000000	.44024182	.00000000
1.04000000	.49940039	.00000000
1.08000000	.56391849	.00000000
1.12000000	.63406824	.00000000
1.16000000	.71012593	.00000000
1.20000000	.79237193	.00000000
1.24000000	.88109052	.00000000
1.28000000	.97657012	-.00000000
1.32000000	1.07910290	-.00000000
1.36000000	1.18898490	-.00000000
1.40000000	1.30651620	.00000000
1.44000000	1.43200030	-.00000000
1.48000000	1.56574460	-.00000000
1.52000000	1.70806020	-.00000000
1.56000000	1.85926180	-.00000000
1.60000000	2.01966760	.00000000
1.64000000	2.18959970	-.00000000
1.68000000	2.36938330	-.00000000
1.72000000	2.55934720	-.00000000
1.76000000	2.75982400	-.00000000

PROBLEM NUMBER 2

N	K	A	B
151	25	1.00000000	5.00000000

FUNCTION COEFFICIENTS

COEFFICIENT NUMBER	COEFFICIENT
1	7.00000000
2	9.00000000

INTEGRAL FUNCTION

X	AREA	E
5.00000000	262.98258000	-.00000008
5.05333330	272.72274000	-.00000011
5.10666660	282.71626000	-.00000002
5.15999990	292.96706000	.00000002
5.21333320	303.47907000	-.00000002
5.26666650	314.25626000	.00000005
5.31999980	325.30256000	.00000008
5.37333310	336.62197000	.00000005
5.42666640	348.21849000	.00000002
5.47999970	360.09614000	.00000002
5.53333300	372.25898000	-.00000005
5.58666630	384.71105000	-.00000002
5.63999960	397.45644000	-.00000002
5.69333290	410.49927000	.00000002
5.74666620	423.84363000	-.00000017
5.79999950	437.49368000	-.00000017
5.85333280	451.45355000	-.00000002
5.90666610	465.72744000	.00000005
5.95999940	480.31954000	.00000026
6.01333270	495.23405000	.00000011
6.06666600	510.47519000	.00000014
6.11999930	526.04726000	.00000020
6.17333260	541.95448000	.00000038
6.22666590	558.20108000	-.00000029
6.27999920	574.79150000	.00000071

INPUT 2

```
002004
075001      0.00000000      6.28318531                                          0001XXXX
        1.25            1.00            1.00            0.00                    0002XXXX
125001      0.00000000      3.14159265                                          0003XXXX
        1.50000000      -1.00000000     -1.00000000     1.57079633              0004XXXX
```

OUTPUT 2

NUMBER OF COEFFICIENTS EQUALS 4

PROBLEM NUMBER 1

N	K	A	B
75	1	.00000000	6.28318530

FUNCTION COEFFICIENTS

COEFFICIENT NUMBER	COEFFICIENT
1	1.25000000
2	1.00000000
3	1.00000000
4	.00000000

INTEGRAL FUNCTION

X	AREA	E
6.28318530	8.37757770	-.00000002

PROBLEM NUMBER 2

N	K	A	B
125	1	.00000000	3.14159260

FUNCTION COEFFICIENTS

COEFFICIENT NUMBER	COEFFICIENT
1	1.50000000
2	-1.00000000
3	-1.00000000
4	1.57079630

INTEGRAL FUNCTION

X	AREA	E

3.14159260	2.80992480	-.00000000

EVALUATION ROUTINE LISTING 1

STAT NO.	FORTRAN STATEMENT	SER NO.	PRO NO.
C	EVALUATION ROUTINE 1	4001	0C50
	Y=X**2*SQRTF(C(1)+C(2)*X)	4002	0C50

EVALUATION ROUTINE LISTING 2

STAT NO.	FORTRAN STATEMENT	SER NO. PRO NO.
C	EVALUATION ROUTINE 2	40010C50
	Y=1./(C(1)+C(2)*SINF(C(3)*X+C(4)))	40020C50

FORTRAN PROGRAM LISTING

```
STAT                          FORTRAN                                   SER PRO
NO.                           STATEMENT                                 NO. NO.

  C      AREA UNDER CURVE USING SIMPSONS RULE                            00010C50
  C      DIMENSION STATEMENTS                                            01010C50
         DIMENSION C(50),S(6)                                            01020C50
  C      FORMAT STATEMENTS                                               02010C50
       1 FORMAT(2I3)                                                     02020C50
       2 FORMAT(1H ///38H         NUMBER OF COEFFICIENTS EQUALS I4)      02030C50
       3 FORMAT(1H ///30H                PROBLEM NUMBER I4)              02040C50
       4 FORMAT(2I3,2F15.8)                                              02050C50
       5 FORMAT(1H /49H         N         K              A            B) 02060C50
       6 FORMAT(5H      I4,4H      I4,5H      F15.8,5H       F15.8)      02070C50
       7 FORMAT(4F15.8)                                                  02080C50
       8 FORMAT(1H /35H            FUNCTION COEFFICIENTS)                02090C50
       9 FORMAT(44H     COEFFICIENT NUMBER          COEFFICIENT)         02100C50
      10 FORMAT(13H             I3,15H             F15.8)                02110C50
      11 FORMAT(1H ///33H               INTEGRAL FUNCTION)               02120C50
      12 FORMAT(1H /49H          X                   AREA             E) 02130C50
      13 FORMAT(F19.8,F19.8,F19.8)                                       02140C50
      14 FORMAT(1H /39H         THERE IS AN INVALID PARAMETER)           02150C50
  C      READ IN PROCESS NUMBER CONTROL CARD                             04010C50
     999 READ 1,LNOP,LCNO                                                04020C50
  C      PUNCH COEFFICIENT NUMBER                                        05010C50
         PUNCH 2,LCNO                                                    05020C50
  C      COUNT PROBLEMS                                                  06010C50
         DO 227 LPNO=1,LNOP                                              06020C50
  C      PUNCH PROBLEM NUMBER                                            07010C50
         PUNCH 3,LPNO                                                    07020C50
  C      READ IN PROBLEM CONTROL CARD                                    08010C50
         READ 4,N,K,A,B                                                  08020C50
  C      PUNCH PARAMETERS                                                09010C50
         PUNCH 5                                                         09020C50
         PUNCH 6,N,K,A,B                                                 09030C50
  C      READ IN COEFFICIENTS                                            10010C50
         DO 200 I=1,LCNO,4                                               10020C50
     200 READ 7,C(I),C(I+1),C(I+2),C(I+3)                                10030C50
  C      PUNCH COEFFICIENTS                                              11010C50
         PUNCH 8                                                         11020C50
         PUNCH 9                                                         11030C50
         DO 201 I=1,LCNO                                                 11040C50
     201 PUNCH 10,I,C(I)                                                 11050C50
  C      ERROR ANALYSIS                                                  12010C50
  C      ODD CHECK N                                                     13010C50
```

```
STAT               FORTRAN                                          SER PRC
NO.                STATEMENT                                        NO. NO.

           IF(N-(N/2))223,223,202                                   13020C50
 C         N IS ODD.NON-NEGATIVE CHECK N-3                          14010C50
      202  IF(N-3)223,226,203                                       14020C50
 C         N EXCEEDS 3.SET N=3 BIPASS SWITCH AND INC=-3             15010C50
      203  JTH1=1                                                   15020C50
           INC=-3                                                   15030C50
 C         NON-NEGATIVE CHECK N-5                                   16010C50
           IF(N-5)226,224,204                                       16020C50
 C         N EXCEEDS 5.SET N=5 BIPASS SWITCH                        17010C50
      204  JTH2=1                                                   17020C50
 C         POSITIVE CHECK K                                         18010C50
      225  IF(K)223,223,205                                         18020C50
 C         K IS POSITIVE.CALCULATE H                                19010C50
      205  AN M 1=N-1                                               19020C50
           H=(B-A)/AN M 1                                           19030C50
 C         POSITIVE TEST H                                          20010C50
           IF(H)223,223,206                                         20020C50
 C         H IS POSITIVE.PUNCH TABLE HEADINGS                       21010C50
      206  PUNCH 11                                                 21020C50
           PUNCH 12                                                 21030C50
 C         INITIALIZATION                                           22010C50
           JB=1                                                     22020C50
           JE=N+2                                                   22030C50
           X=A-H                                                    22040C50
           JTH=1                                                    22050C50
           YOD=0.0                                                  22060C50
           YEV=0.0                                                  22070C50
           JTH3=1                                                   22080C50
 C         COUNT NUMBER OF AREA VALUES TO BE COMPUTED               23010C50
           DO 211 I=1,K                                             23020C50
 C         COUNT NUMBER OF DEPENDENT FUNCTIONAL VALUES TO BE COMPUTED  24010C50
           DO 208 J=JB,JE                                           24020C50
 C         COMPUTE L                                                25010C50
           L=J-JE+6                                                 25020C50
 C         COMPUTE FUNCTIONAL VALUE AT X AND STORE AT Y.NOTE THAT THE  30010C50
 C         FUNCTION COEFFICIENTS ARE STORED IN C-ARRAY              30011C50
 C         BIPASS SWITCH                                            60010C50
           GO TO(207,218,222),JTH                                   60020C50
 C         STORE Y(1),Y(2) AND Y(3) AT S(1),S(2) AND S(3)           61010C50
      207  S(J)=Y                                                   61020C50
 C         3 TEST J                                                 62010C50
```

```
STAT                          FORTRAN                                      SER PRO
NO.                           STATEMENT                                    NO. NO.

            IF(J-3)208,216,223                                              62020C50
      C     J IS LESS THAN 3.INCREMENT X BY H                               63010C50
        208 X=X+H                                                           63020C50
      C     COMPUTE ERROR AND AREA                                          64010C50
      C     N=3 BIPASS SWITCH                                               65010C50
            GO TO(209,214),JTH1                                             65020C50
      C     N=5 BIPASS SWITCH                                               66010C50
        209 GO TO(210,212),JTH2                                             66020C50
      C     CALCULATE ERROR FOR N GREATER THAN 5                            67010C50
        210 ERROR=S(1)+S(6)-4.*(S(2)+S(5))+7.*(S(3)+S(4))-8.*(YOD-YEV)      67020C50
            ERROR=(-H/90.)*ERROR                                            67030C50
      C     CALCULATE AREA FOR N GREATER THAN 3                             68010C50
        213 AREA=(H/3.)*(S(2)+S(5)+4.*(S(3)+YEV+S(4))+2.*YOD)               68020C50
      C     PUNCH INDEPENDENT VARIABLE,AREA AND ERROR                       69010C50
        215 PUNCH 13,B,AREA,ERROR                                           69020C50
      C     INCREMENT B BY 2H                                               70010C50
            B=B+2.*H                                                        70020C50
      C     UPDATE YOD AND YEV                                              71010C50
            YOD=YOD+S(5)                                                    71020C50
            YEV=YEV+S(4)                                                    71030C50
      C     REPLACE Y(N-1) AT S(4) BY Y(N+1) AT S(6)                        72010C50
            S(4)=S(6)                                                       72020C50
      C     UPDATE JB AND JE                                                73010C50
            JB=JE+1                                                         73020C50
        211 JE=JE+2                                                         73030C50
            GO TO 227                                                       73040C50
      C     SET N=5 BIPASS SWITCH.CALCULATE ERROR FOR N=5                   74010C50
        212 JTH2=1                                                          74020C50
            ERROR=(-H/90.)*(S(1)+S(6)-4.*(S(2)+S(5))+7.*(S(3)+S(4))-8.*YOD) 74030C50
            GO TO 213                                                       74040C50
      C     SET N=3 BIPASS SWITCH.CALCULATE ERROR AND AREA FOR N=3          75010C50
        214 JTH1=1                                                          75020C50
            ERROR=(-H/90.)*(S(1)+S(6)-4.*(S(2)+S(5))+6.*S(3))               75030C50
            AREA=(H/3.)*(S(2)+4.*S(3)+S(5))                                 75040C50
      C     SET S(4)=0                                                      76010C50
            S(4)=0.0                                                        76020C50
            GO TO 215                                                       76030C50
      C     J=3.SET BIPASS SWITCH.O TEST L+INC                              77010C50
        216 JTH=2                                                           77020C50
        220 IF(L+INC)208,217,223                                            77030C50
      C     L+INC=0.SET BIPASS SWITCH                                       78010C50
```

```
STAT                 FORTRAN                                    SER PRO
NO.                  STATEMENT                                  NO. NO.

   217 JTH=3                                                    78020C50
       GO TO 208                                                78030C50
  C    YOD-YEV BIPASS SWITCH                                    79010C50
   218 GO TO(219,221),JTH3                                      79020C50
  C    SET YOD-YEV BIPASS SWITCH.UPDATE YOD                     80010C50
   219 JTH3=2                                                   80020C50
       YOD=YOD+Y                                                80030C50
       GO TO 220                                                80040C50
  C    SET YOD-YEV BIPASS SWITCH.UPDATE YEV                     81010C50
   221 JTH3=1                                                   81020C50
       YEV=YEV+Y                                                81030C50
       GO TO 208                                                81040C50
  C    STORE Y(N),Y(N+1) AND Y(N+2) AT S(4),S(5) AND S(6)       82010C50
   222 S(L)=Y                                                   82020C50
       GO TO 208                                                82030C50
  C    H IS NOT POSITIVE.THERE IS AN INVALID PARAMETER          83010C50
   223 PUNCH 14                                                 83020C50
       GO TO 227                                                83030C50
  C    N=5.SET N=5 BIPASS SWITCH                                84010C50
   224 JTH2=2                                                   84020C50
       GO TO 225                                                84030C50
  C    N=3.SET N=3 BIPASS SWITCH AND INC=-4                     85010C50
   226 JTH1=2                                                   85020C50
       INC=-4                                                   85030C50
       GO TO 224                                                85040C50
  C     CONTINUE STATEMENT                                      86010C50
   227 CONTINUE                                                 86020C50
  C    END OF JOB                                               87010C50
       PAUSE                                                    87020C50
       GO TO 999                                                87030C50
       END                                                      87040C50
```

General Remarks

1. Definite integrals may be approximated without using any coefficients in the C-array. For example, let it be required to tabulate

$$\int_0^x e^{-X^2} dX$$

for x = 1, 1.02, 1.04, 1.06, 1.08, 1.10.

The Evaluation Routine can be defined by the single Fortran statement

```
Y = EXPF(-X*X)   .
```

Referring to the notation of the Program Input section, the input for the corresponding object program would consist of (a) a Process Number Control Card C with m = s = 001, (b) a Problem Control Card with $2n_1+1 = 51$, $k_1 = 6$, $a_1 = 0$, $b_1 = 1$ and (c) a single blank card.

2. (a) The following referents must not be used in programming the evaluation routine

   1-14 inclusive,
   200-227 inclusive,
   999 .

   (b) The following variables must not be used in programming the evaluation routine

   A, ANMI, AREA, B, C, ERROR, H, I, INC, J, JB, JE, JTH, JTH1, JTH2, JTH3, K, L, LCNO, LNOP, LPNO, N, S, YEV, YOD.

3. If after tabulating a series of definite integrals, it is found necessary to make some changes or to investigate a new set of definite integrals (of the prescribed type), it is not required to re-read the object program into memory. Merely feed the new input into the card reader, press the console start key and then the card reader, reader-start key.

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