

# GENERAL PURPOSE ANALOG COMPUTATION

CHEMICAL/PETRO-CHEMICAL

APPLICATION STUDY; 6.4.7a

#### ANALOG COMPUTER STUDY OF A SEMI-BATCH REACTOR

INTRODUCTION: This paper considers a reactor design application of the PACE® TR-48 General Purpose Analog Computer, viz: the chlorination of benzene in a semi-batch reactor. The major objective of the study will be to relate the maximum production of intermediate chlorobenzenes to the chlorine feed rate and the time of reaction. Another objective consists of illustrating how experimental kinetic data can be fitted on such a computer to obtain reaction rate constants.

Since the majority of both inorganic and organic chemical products of the chemical industry are formed by one or more chemical reactions, the chemical engineer is continually faced with the problem of designing flow, batch, or semi-batch reactors. In recent years, the cost of designing and optimizing a reactor has been sharply reduced through the use of the general purpose analog computer. The computer is used to simulate the behavior of the process in question. Proposed changes in the process and its eventual optimization can then be accomplished quickly, and without expensive equipment modifications.

#### THE PHYSICAL SYSTEM

The chlorination of benzene ( $C_6H_6$ ) produces monochlorobenzene ( $C_6H_5C1$ ), dichlorobenzene ( $C_6H_4C1_2$ ), and trichlorobenzene ( $C_6H_3C1_3$ ) through successive, competing chemical reactions:

$$C_{6}^{H}_{6} + C1_{2} \longrightarrow C_{6}^{H}_{5}^{C1} + HC1$$
 (1)

$$C_6H_5C1 + C1_2 \longrightarrow C_6H_4C1_2 + HC1$$
 (2)

$$C_6H_4C1_2 + C1_2 \longrightarrow C_6H_3C1_3 + HC1$$
 (3)

These reactions are carried out in a lead-lined or iron vessel, which is shown in Figure 1, with ferric chloride (FeCl<sub>3</sub>) as a catalyst. The vessel is fitted with cooling coils, since the chlorination reactions are exothermic, and a reflux condenser. The purpose of the reflux condenser is to return vaporized chlorobenzenes to the system, while allowing the hydrogen chloride and excess chlorine vapor to leave the system. In order to maintain the reacting mixture at a uniform temperature and to minimize mass-transfer effects, the reacting mixture is well agitated.

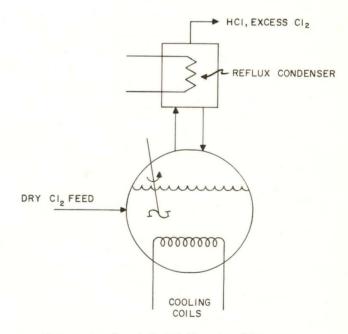


Figure 1. Semi-Batch Reactor Diagram

No chlorobenzenes can be formed until some of the chlorine gas, which is being bubbled through the liquid benzene, has gone into solution. The amount of chlorine which is capable of going into solution

is limited by the solubility of chlorine in the reacting mixture, which depends on the operating conditions of the reactor. Since the chlorobenzenes are formed successively, the concentrations of monoand dichlorobenzene will each pass through a definite maximum at some time during the process. Eventually, all of the benzene will have reacted to pure trichlorobenzene (and other chloro-hydrocarbons, which have been neglected).

The following restrictions can be made on the system:

- No liquid or vapor hold-up occurs in the reflux condenser.
- (2) The system operates under isothermic and isobaric conditions.
- (3) Changes in the volume of the reacting mixture are negligible.
- (4) Hydrogen chloride gas vaporizes and leaves the system.
- (5) Negligible mass transfer resistance occurs between the gaseous chlorine and the chlorine in solution (up to solubility limit of chlorine).

#### THE MATHEMATICAL MODEL

The equations defining the reaction kinetics of the system, given in several references (2,5), are

$$V \frac{dN_B(t)}{dt} = -k_1 N_B(t) N_C(t)$$
 (4)

$$V \frac{dN_{M}(t)}{dt} = k_{1} N_{B}(t) N_{C}(t) - k_{2} N_{M}(t) N_{C}(t)$$
 (5)

$$V \frac{d N_D (t)}{dt} = k_2 N_M (t) N_C (t) - k_3 N_D (t) N_C (t)$$
 (6)

and 
$$V \frac{dN_{T}(t)}{dt} = k_3 N_D(t) N_C(t)$$
 (7)

where

V = Volume of reacting mixture

t = Time

N(t) = Moles per unit volume

k = Rate constant in volume per unit mole per unit time.

and the subscripts represent the following:

C = Chlorine

M = Monochlorobenzene

D = Dichlorobenzene

T = Trichlorobenzene

B = Benzene

1 = Reaction producing monochlorobenzene

2 = Reaction producing dichlorobenzene

3. = Reaction producing trichlorobenzene

These equations have the initial conditions:

$$N_{R}(0) = N_{O}$$
 (8A)

$$N_{C}(0) = 0 \tag{8B}$$

$$N_{M}(0) = 0 (8C)$$

$$N_{D}(0) = 0 (8D)$$

$$N_{T}(0) = 0 (8E)$$

Equations 4-7 can be added and integrated to obtain the benzene material balance equation

$$N_{O} = N_{B}(t) + N_{M}(t) + N_{D}(t) + N_{T}(t)$$
 (9)

If the mass transfer effect of chlorine being transferred from the vapor to the liquid phase is neglected, the chlorine concentration in the system is described by the material balance equation

$$N_{C}(t) = \int_{0}^{t} dt - x(t)$$
moles of moles of moles of chlorine in chlorine solution supplied reacted (10)

where x(t) represents the moles of chlorine reacted and r denotes a constant chlorine feed rate in moles per unit time. The chlorine reacted can be obtained (in terms of the chlorobenzene concentrations) from the stoichiometry of the system, namely

$$x(t) = N_{M}(t) + 2 N_{D}(t) + 3N_{T}(t)$$
 (11)

The solubility of chlorine in the reacting mixture was obtained by considering the total pressure above the liquid,  $\pi$ , which is equal to the sum of the partial pressures, p. of the inidividual components in the vapor phase. Thus

$$\pi = p_{\mathbf{C}}(t) + p_{\mathbf{B}}(t) + p_{\mathbf{M}}(t) + p_{\mathbf{D}}(t) + p_{\mathbf{T}}(t)$$
 (12)

Since its tenure in the system is only momentary, the partial pressure of hydrogen chloride was neglected.\* By applying Raoult's Law, which states that the partial pressure is equal to the product of the mole fraction,  $\frac{N(t)}{N_O + N_C(t)}$ , and the vapor pressure, po, equation 12 becomes

$$\pi = \frac{N_{C}(t)}{N_{O}^{+}N_{C}(t)} p_{C}^{\circ} + \frac{N_{B}(t)}{N_{O}^{+}N_{C}(t)} p_{B}^{\circ} + \frac{N_{M}(t)}{N_{O}^{+}N_{C}(t)} p_{M}^{\circ} + \frac{N_{D}(t)}{N_{O}^{+}N_{C}(t)} p_{O}^{\circ} + \frac{N_{D}(t)}{N_{O}^{-}N_{C}(t)} p_{O}^{\circ} + \frac{N_{D}(t)}{N_{O}^{+}N_{C}(t)} p_{O}^{\circ} + \frac{N_{D}(t)}{N_{O}^{-}N_{C}(t)} p_{O}^{\circ} + \frac{N_{D}(t)}{N_{O}^{-}N_{C}(t)} p_{O}^{\circ} + \frac{N_{D}(t)}{N_{O}^{$$

This equation can be solved for the maximum chlorine concentration, N<sub>C</sub>(t), to obtain

$$\overline{N}_{C}(t) = \frac{N_{O}}{P_{C}^{\circ} - \pi} \left[ \pi - \frac{N_{B}(t)}{N_{O}} p_{B}^{\circ} - \frac{N_{D}(t)}{N_{O}} p_{D}^{\circ} - \frac{N_{D}(t)}{N_{O}} p_{D}^{\circ} \right]$$

$$- \frac{N_{T}(t)}{N_{D}} p_{T}^{\circ}$$
(14)

Since the vapor pressures are functions of temperature only and the system is isothermal and isobaric, the maximum chlorine concentration depends only on the chlorobenzene concentrations. Any excess chlorine supplied to the system leaves it through the reflux condenser.

A more convenient form\*\* of the system equations may be obtained by defining

$$h = \frac{k N_{O}}{V}$$
 (15)

$$\theta = b + (16)$$

$$C(\theta) = N_C(\theta) / N_O$$
 (17)

$$B(\theta) = N_B(\theta) / N_O \tag{18}$$

$$M(\theta) = N_{M}(\theta) / N_{O}$$
 (19)

$$D(\theta) = N_D(\theta) / N_O \tag{20}$$

$$T(\theta) = N_T(\theta) / N_O \tag{21}$$

$$X(\theta) = x(\theta) / N_{O}$$
 (22)

$$G = \frac{r}{N_0 h_1} \tag{23}$$

Using these definitions, equations 4 - 7, 9 and 11

$$\frac{dB(\theta)}{d\theta} = -B(\theta)C(\theta) \tag{24}$$

$$\frac{dM(\theta)}{d\theta} = B(\theta)C(\theta) - \left[\frac{k_2}{k_1}\right] M(\theta)C(\theta)$$
 (25)

$$\frac{dD(\theta)}{d\theta} = \left[\frac{k_2}{k_1}\right] M(\theta) C(\theta) - \left[\frac{k_3}{k_1}\right] D(\theta) C(\theta) \quad (26)$$

$$\frac{dT(\theta)}{d\theta} = \left[\frac{k_3}{k_1}\right]D(\theta)C(\theta) \tag{27}$$

$$I = B(\theta) + M(\theta) + D(\theta) + T(\theta)$$
 (28)

$$X(\theta) = M(\theta) + 2D(\theta) + 3T(\theta)$$
 (29)

Equation 10 can now be represented as

$$C(\theta) = \int_0^t G d\theta - X(\theta)$$
 (30)

which can be differentiated to obtain

$$\frac{dC(\theta)}{d\theta} = G - \frac{dX(\theta)}{d\theta}$$
 (31)

or, by substituting

$$\frac{d C(\theta)}{d \theta} = G - \left[ B(\theta) C(\theta) + \left( \frac{k_2}{k_1} \right) M(\theta) C(\theta) + \left( \frac{k_3}{k_1} \right) (32) \right]$$

$$D(\theta) C(\theta)$$

Equation 32 is the best form of the chlorine concentration equation for computational purposes.

The operating temperature and pressure used in this investigation are 55 deg. C--which is typical of the operating temperature used in industry for

<sup>\*</sup>System operated above critical temperature of HC1. \*\*The equations were written in terms of  $\theta$ ,  $\frac{k2}{k1}$ , and  $\frac{K3}{K1}$  because experimental data was available on the ratios of the rate constants; and k1 was

this system (4)--and 2 absolute atmospheres-which was assumed. Based on these operating conditions, the saturation concentration of chlorine,  $C(\theta)$ , can be assumed constant and equal to 0.120. The proof of this fact is shown in Appendix A. This means that the constraint on equation 32 is

$$C(\theta) \leqslant \bar{C} = 0.120 \tag{33}$$

#### PROGRAMMING THE COMPUTER\*\*\*

This problem, which was solved on a  $\pm 10$  volt PACE<sup>®</sup>, TR-10 General Purpose Analog Computer, presents no magnitude scaling problems. From the stoichiometry of the system it is obvious that B( $\theta$ ), M( $\theta$ ), D( $\theta$ ), and T( $\theta$ ) cannot exceed unity and that X( $\theta$ ) cannot exceed three. The chlorine concentration, C( $\theta$ ), is limited to 0.120; therefore its maximum value is obvious. These maximum values and their associated scale factors and computer voltages are tabulated in Table I. Table II lists the systems parametric data.

TABLE I. MAGNITUDE SCALING

Physical Variable (Dimensionless)	Estimated Maximum Value (Dimensionless)	Scale Factor Volts Per Dim. Unit	Computer Variable (Volts)
$\mathbf{B}$	1.0	10	[10 B]
M	1.0	10	[10 M]
D	1.0	10	[10 D]
${f T}$	1.0	10	[10 T]
x	3.0 < 4.0	2.5	[2.5 X]
С	0.12< 0.2	50	[50 C]

#### TABLE II. PARAMETER DATA

$$\frac{\frac{k_2}{k_1}}{k_1} = 0.125^{(2)}$$
 Dimensionless 
$$\frac{\frac{k_3}{k_1}}{k_1} = 0.00417^{(2)}$$
 Dimensionless

 $0 \le G \le 0.008$  (ASSUMED) Dimensionless

The scaled voltage equations, which are based on the information contained in Tables I and II, are

$$\frac{\mathbf{d}}{\mathbf{d}^{T}} \left[ \mathsf{IOB} \right] = -\mathsf{IO} \left( \frac{\mathsf{I}}{\mathsf{5OB}} \right) \frac{\left[ \mathsf{IOB} \right] \left[ \mathsf{5OC} \right]}{\mathsf{IO}}$$
(34)
$$\frac{\mathbf{d}}{\mathbf{d}^{T}} \left[ \mathsf{IOM} \right] = \mathsf{IO} \left( \frac{\mathsf{I}}{\mathsf{5OB}} \right) \frac{\left[ \mathsf{IOB} \right] \left[ \mathsf{5OC} \right]}{\mathsf{IO}} - \left( \frac{\mathsf{k}_{2}}{\mathsf{5k}_{1}} \right) \frac{\left[ \mathsf{IOM} \right] \left[ \mathsf{5OC} \right]}{\mathsf{IO}}$$
(35)

$$\frac{d}{d\tau} \left[ IOT \right] = \left( \frac{k_2}{5 \, k_1 \beta} \right) \frac{\left[ IOM \right] \left[ 5OC \right]}{IO} - \left( \frac{k_3}{5 \, k_1 \beta} \right) \frac{\left[ IOD \right] \left[ 5OC \right]}{IO}$$
(36)

$$\frac{d}{d\tau} \left[ IOT \right] = \left( \frac{k_3}{5k_1 \beta} \right) \frac{\left[ IOD \right] \left[ 5OC \right]}{IO}$$
 (37)

$$[2.5X] = 0.250[IOM] + 0.500[IOD] + 0.750[IOD]$$
 (38)

$$\frac{d}{d\tau} \left[ 50C \right] = \left( \frac{5G}{\beta} \right) \left[ 10 \right] - \left[ \left( \frac{1}{\beta} \right) \frac{\left[ 10B \right] \left[ 50B \right]}{10} \right]$$

$$+\left(\frac{\mathbf{k}_{2}}{\mathbf{k}_{1}\boldsymbol{\beta}}\right) \quad \frac{\left[\mathsf{IOM}\right]\left[\mathsf{5OC}\right]}{\mathsf{IO}} + \left(\frac{\mathbf{k}_{3}}{\mathbf{k}_{1}\boldsymbol{\beta}}\right) \quad \frac{\left[\mathsf{IO D}\right]\left[\mathsf{5O C}\right]}{\mathsf{IO}} \quad \left] \quad (39)$$

4

$$\begin{bmatrix} 50 & C \end{bmatrix} \le \begin{bmatrix} 50 & \overline{C} \end{bmatrix} = 6.00 \text{ volts}$$
 (40)

where machine time,  $\tau$ , is defined in terms of the time scale factor  $\beta$ , and  $\theta$  as

$$\tau = \beta \, \theta \tag{41}$$

The computer diagram implementing these equations is shown in Figure 2 and the components used are summarized in Appendix C. Potentiometer and amplifier assignments are tabulated in Tables III and IV. The time scale factor chosen for this problem was 0.05 (seconds of machine time per dimensionless unit). This choice was based on the fact that the feedback to the integrators had to be attenuated by responsible potentiometer settings in order to run the problem in a reasonable length of computer time (20  $\leq \tau \leq$  100 seconds).

When fitting experimental data to determine rate constants, it is advantageous to plot the results in time, t, rather than in the dimensionless variable,  $\theta$ , since the kinetic data will be in terms of time. Time, which is related to  $\theta$  by equation 16, can be obtained from the equation

$$\frac{dt}{d\theta} = \frac{1}{h_{\parallel}} = \frac{V}{k_{\parallel} N_{\Omega}} \tag{42}$$

<sup>\*\*\*</sup>It is assumed that the reader is familiar with fundamentals of analog computer programming.



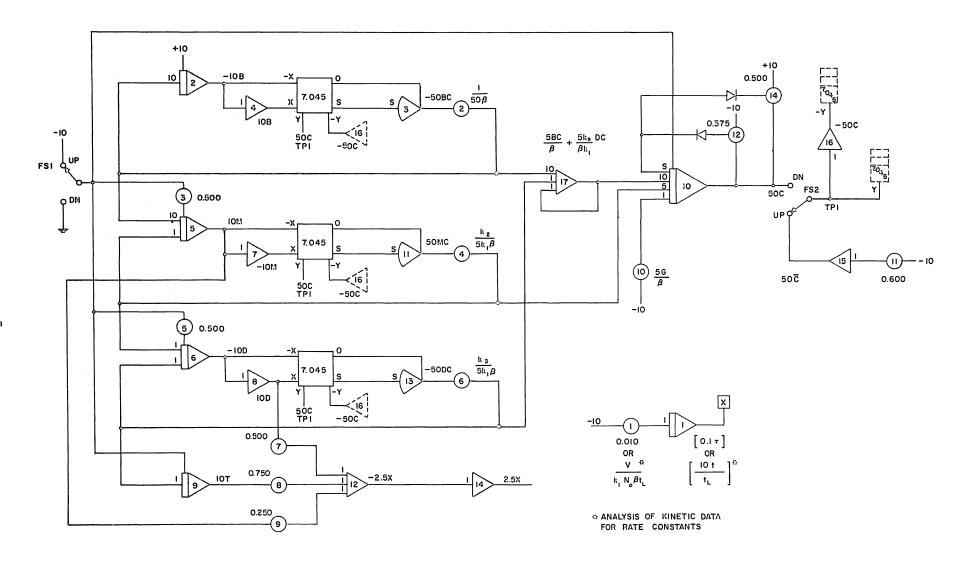


Figure 2. Computer Diagram

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# TR-IO POTENTIOMETER ASSIGNMENT SHEET

	7.11		7.00,0.1E.T.	0
PROBLEM	Semi-Batch	Reactor		
DATE				

POT NO.	PARAMETER DESCRIPTION	SETTING STATIC CHECK	STATIC CHECK OUTPUT VOLTAGE	SETTING RUN NUMBER I	NOTES	POT NO.
ı	CONSTANT OF V/K, Not B	0.010				1
2	1/50β	0,100	0.400			2
3	CONSTANT	0,500	,			3
4	ka/5k,B	0,500			<del>&gt;</del>	4
5	CONSTANT	0.500				5
6	k3/5k,B	0.500	0.017			6
7	CONSTANT	0.500			<del></del>	7
8	CONSTANT	0.750			>	8
9	CONSTANT	0.250				9
10	56/B	0.100	0.000			10
- 11	CONSTANT	0.600			<del></del>	
12	LIMIT	٥,375			→ APPROXIMATE	12
13						13
14	LIMIT	0.500				14
15						15
16						16
17						17
18						18
19						19
20						20
21						21
22						22
23	FS1	UP	DN —		<del></del>	23
24	FS2	UP	UP	PN -	<del>-</del>	24

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## TABLE IV

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TR-10 AMPLIFIER ASSIGNMEN SHEET PROBLEM Semi-Botch Reactor DATE\_\_\_\_

AMP		OUTPUT	STATIC CHECK				
NO. FE		VARIABLE	CALCUL INTEGRATOR INPUT SUM		MEAS INTEGRATOR INPUT SUM	URED	NOTES
ı	I	7/10 er 10t/t;	0.10	10.00	INPUT SUM	001701	
2	I	-10B	6.00 <sup>t</sup>	-10.00			
3	HG	-50BC		-6.00			
4	S	10B		10.00			
5	I	10M	4.50	5.00			
6	Ì	-10D	-3,cot	5,00			
7	S	-10M		-5.00			
8	S	100		-5.00			
9	I	IOT	-1,50 <sup>†</sup>	10.00			
10	I	50C	-3.00	10.00			LIMIT TO 6.00 volls
11	HG	50MC		3,00			
12	S	-2.5X		-6.25			
13	HG	-50DC		3.00			
14	S	2.5X		6.25			
15	S	50 C		6.00			
16	S	-50C		6.00			
17	5	5BC/p +5 & DC/k,p		2,38			
19		*ANALYSIS OF KINETIC	DATA F	OR RATE	CONSTI	\UTS	
19		TIOOK FEEDBACK IN	CHECK	AMPLIF	IER		
20		IOK FEEDBYCK IN	CHECK	AMPLIF	ER		

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TABLE V. TIME OF MAXIMUM PRODUCTION VERSUS CHLORINE FLOWRATE DATA

$\theta$ Max Dimensionless		G Dimensionless	1/G Dimensionless	
M	D			
22	264		0	
160	430	0.008	125	
210	520	0.006	167	
305	680	0.004	250	
585	1230	0.002	500	

whose scaled voltage representation is

$$\frac{d\left[10\uparrow/\uparrow_{L}\right]}{d\tau} = \left(\frac{V}{\beta k_{1} N_{0} \uparrow_{L}}\right) \left[10\right]$$
 (43)

The symbol  $t_{\rm L}$  is the maximum time required for the data analysis, and depends on the data being fitted.

A dynamic check of the computer circuit was made after completion of successful static check. The dynamic check consisted of checking the maximum concentration of monochlorobenzene and the time at which the maximum occurred, against the computer results for a special case of reactor operation. The special case (FSI - DN, FS2-UP) assumes that the chlorine flow rate is infinite and that the reaction mixture is always saturated with chlorine. Calculations made on this basis indicate the maximum value of  $M(\theta)$  will be 0.74 when  $\theta$  is 22 (Refer to Appendix B).

#### RESULTS

In addition to the dynamic check run, as shown in Figure 3, four additional runs were made at values of G ranging from 0.002 to 0.008. These results, which are plots of  $B(\theta)$ ,  $M(\theta)$ ,  $D(\theta)$ ,  $T(\theta)$ ,  $X(\theta)$  and  $C(\theta)$  versus  $\theta$ , are shown in Figures 4-7. From this information, it was possible to obtain the values of  $\theta$ ,  $\theta_{max}$  at which the maximum amount of

the intermediate products was present in the system. This data and the corresponding values of the flow rate variable, G, are tabulated in Table V. A plot of  $\log \theta_{\rm max}$  versus 1/G is shown in Figure 8 for each of the intermediate products.

Figure 9, which is a plot of B(t), M(t), D(t), and T(t) versus time for three values of  $k_3/k_1$ , serves to illustrate how experimental data can be fitted using the analog computer. By varying the reaction rate constants, which are unknown, the computer solution can be made to coincide with the experimental results to obtain these constants.

When this trial and error process is performed in real time, using an X-Y plotter, it is quite time-consuming. However, when high speed repetitive operation, which yields up to 60 solutions per second, is used in conjunction with an oscilloscope, a transparent overlay of the experimental data can be fitted in a matter of minutes, due to the fact that the effect of parameter variations on computer results can be observed instantly on the high persistence scope.

#### CONCLUSIONS

From Figure 3-7 it is obvious that the maximum reduced concentration of the intermediates is independent of the chlorine flow rate. These maximums, which occur at 0.74 and 0.88, can only be altered by changes in the reaction rate constants. This can be verified by consulting Figure 9. The chlorine feed rate, which controls the amount of chlorine available for reaction, affects the time at which the maximum concentrations occur. As shown in Figure 8, when the chlorine flow rate is infinite (1/G=0) and the reaction mixture is always saturated with chlorine,  $\theta_{\rm max}$  has its minimum values of 22 and 264. When the chlorine feed rate is reduced,  $\theta_{\rm max}$  increases. Obviously, if the feed rate of chlorine is zero  $(1/G-\infty)$ ,  $\theta_{\rm max}$  must be infinite.

The curve fitting technique mentioned in the results section, which is for the most part qualitative, serves only as an illustration of what can and has been done on the general purpose analog computer in regard to the analysis of kinetics data.

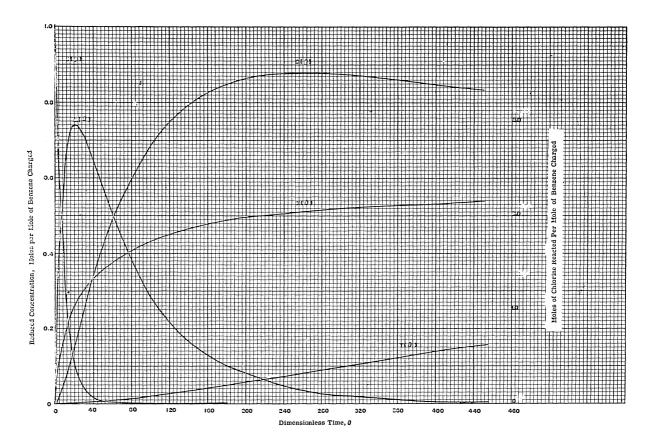


Figure 3. Reduced Concentrations Versus Dimensionless Time For Infinite Chlorine Feed Rate

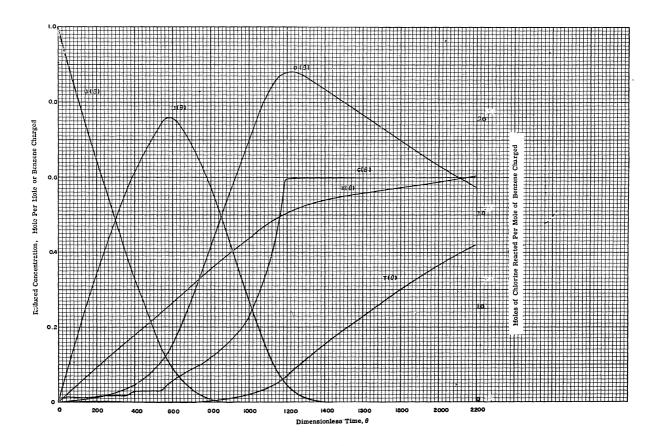


Figure 4. Reduced Concentrations Versus Dimensionless Time (G =  $2.0 \times 10^{-3}$ )

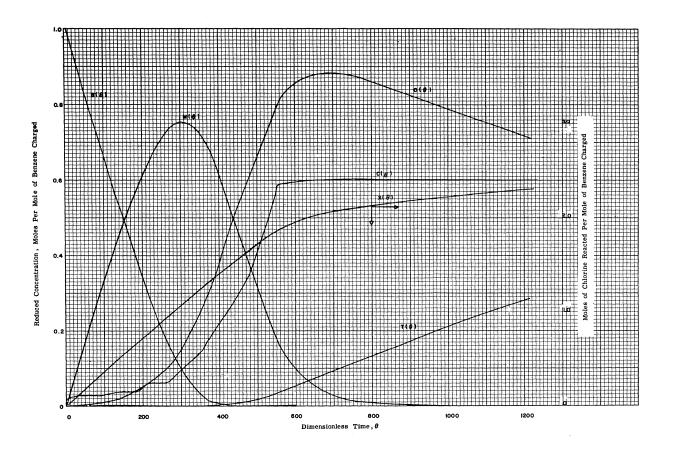


Figure 5. Reduced Concentrations Versus Dimensionless Time (G =  $4.0 \times 10^{-3}$ )

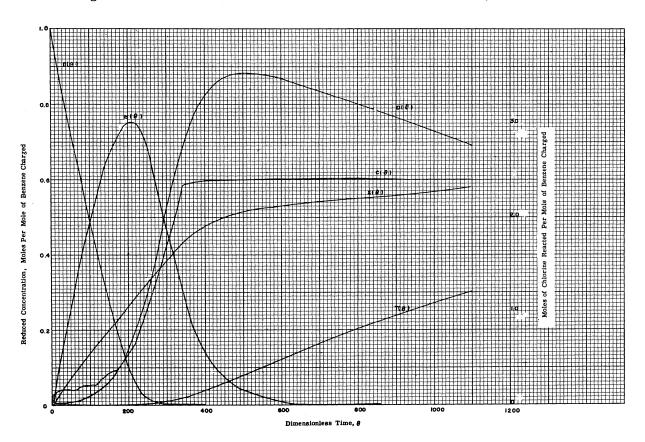


Figure 6. Reduced Concentrations Versus Dimensionless Time (G =  $6.0 \times 10^{-3}$ )

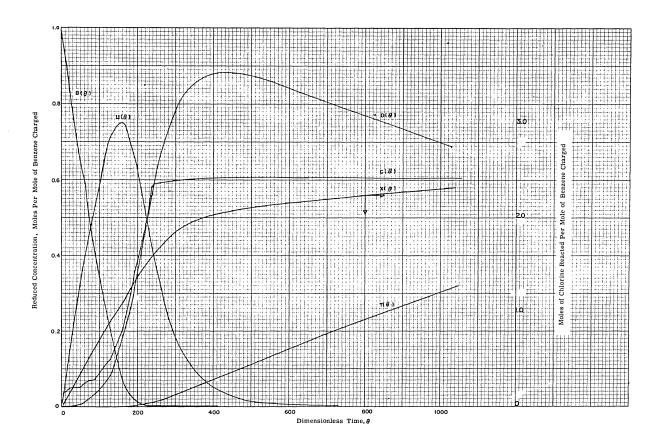


Figure 7. Reduced Concentration Versus Dimensionless Time (G =  $8.0 \times 10^{-3}$ )

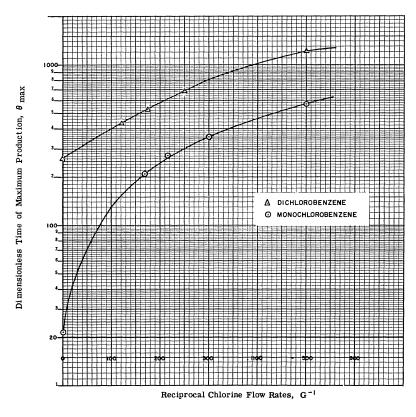


Figure 8. Dimensionless Time of Maximum Mono-and Dichlorobenzene Production Versus Reciprocal Chlorine Flow Rate

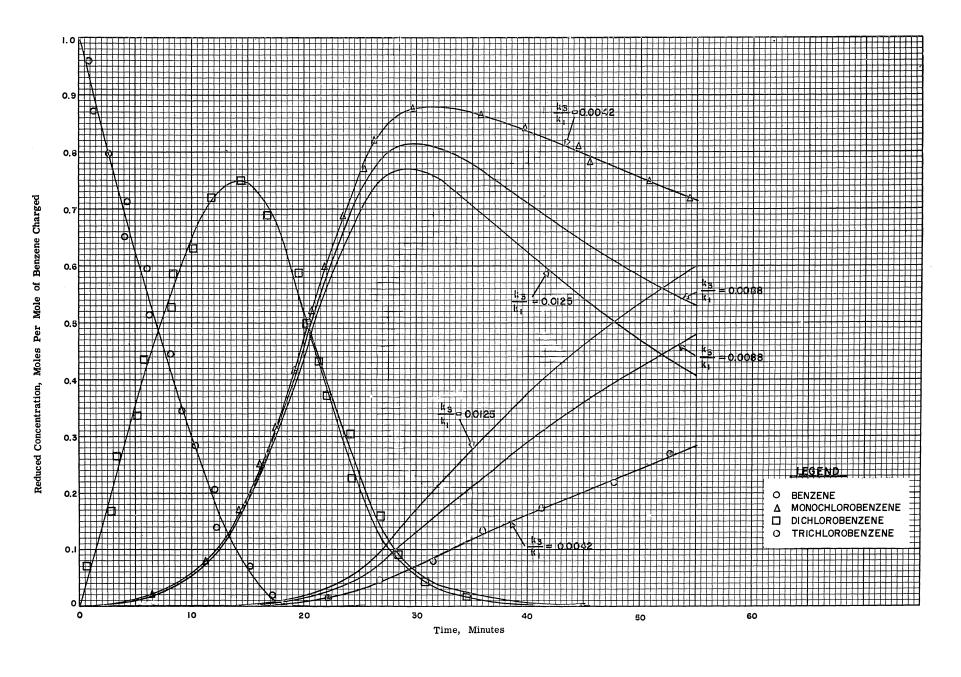


Figure 9. Reduced Concentrations Versus Time, Comparison of Experimental and Computer Results

#### APPENDIX A

#### SOLUBILITY CONSTRAINT CALCULATIONS

The vapor pressure data at the 55 deg. C operating temperature, necessary to solve the equation

$$\overline{C}(t) = \frac{1}{p_C^{\circ} - \pi} \left[ \pi - B(t)p_B^{\circ} - M(t)p_M^{\circ} - D(t)p_D^{\circ} - T(t)p_T^{\circ} \right]$$
(A-1)

is tabulated in Table VI.

TABLE VI. VAPOR PRESSURE DATA AT 55 C

Component	Vapor Pressure	Data Source
Benzene	327 mm of HG	3
Monochlorobenzene	48.8 mm of HG	3
Dichorobenzene	11.5 mm of HG	3
Trichlorobenzene	<5 mm of HG	3
Chlorine	16.9 ATM	. 1

If this data is substituted into equation A-1, it can be seen that for all practical purposes since the total pressure is two atmospheres (see Table II), and the concentration variables cannot exceed unity. Based on equation A-2 the range of C(t) is

$$\overline{C}(t) = \frac{1}{p \stackrel{\circ}{C}^{-\pi}} \qquad \left(\pi - B(t) p \stackrel{\circ}{B}\right) \tag{A-2}$$

$$0.105 < \overline{C}(\dagger) < 0.134$$
 (A-3)

which is very narrow; therefore, variations in C(t) will have little effect on the results of the problem. The average value of C(t), 0.120, was considered sufficiently accurate for this study.

#### APPENDIX B

#### DYNAMIC CHECK CALCULATIONS

If the reacting mixture is always saturated with chlorine, the equations describing the behavior of monochlorobenzene and benzene are

$$\frac{\mathsf{dB}}{\mathsf{d}\,\xi}\,(\xi) = -\,\mathsf{B}(\xi) \tag{B-1}$$

and

$$\frac{dM}{d\xi} (\xi) = B(\xi) - \frac{k_2}{k_1} M(\xi)$$
(B-2)

where

$$\xi = \overline{\mathsf{C}}\,\theta\tag{B-3}$$

These equations have the initial conditions

$$B(0) = I (B-4)$$

and 
$$M(0) = 0$$
 (B-5)

The solution to equation B-1 is

$$B(\xi) = \epsilon^{-\xi} \tag{B-6}$$

which can be substituted into equation B-2 to obtain

$$\frac{d M(\xi)}{d \xi} + \frac{k_2}{k_1} M(\xi) = \epsilon^{-\xi}$$
(B-7)

whose solution is

$$M(\xi) = \frac{1}{1 - \frac{k_2}{k_1}} \qquad \left[ -\frac{k_2}{\epsilon} \xi - \frac{-\xi}{\epsilon} \right]$$
(B-8)

The maximum concentration of  $M(\xi)$  occurs when

$$\frac{dM(\xi)}{d\xi} = 0 = \epsilon \xi' - \frac{k_2}{k_1} \epsilon'$$
(B-9)

where  $\xi^{\dagger}$  is the value of the argument at which the maximum occurs . . .

$$\xi' = \frac{1}{1 - \frac{k_2}{k_1}} \ln \frac{k_1}{k_2}$$
 (B-10)

This may be substituted back into equation B-8 to obtain the maximum value of M(ξ)

$$M(\xi') = \left(\frac{k_2}{k_1}\right)^{\frac{k_2/k_1}{1-k_2/k_1}}$$
(B-11)

Since  $\frac{k_2}{k_1}$  is one-eighth (see Table II) and C is 0.12 (see Appendix A)

$$M(\xi) = 0.74$$
 (B-12)

and 
$$\theta' = \frac{\xi'}{\overline{C}} = 22$$
 (B-13)

# APPENDIX C

# EQUIPMENT REQUIRED TO PERFORM SIMULATION

The equipment required to perform this simulation was:

- 1 X-Y Plotter
- 3 Quarter Square Multipliers
- 17 Operational Amplifiers
- 6 Integrator Networks
- 13 Potentiometers

and

2 - Function Switches

#### NOMENCLATURE

- B = Reduced Benzene concentration, dimensionless
- C = Reduced Chlorine concentration, dimensionless
- C = Reduced Maximum Chlorine concentration, dimensionless
- D = Reduced Dichlorobenzene concentration, dimensionless
- G = Modified chlorine flowrate, dimensionless
- M = Reduced Monochlorobenzene concentration, dimensionless
- N = Concentration variable, moles
- N<sub>C</sub> = Maximum chlorine concentration, moles
- T = Reduced trichlorobenzene concentration, dimensionless
- V = Volume of reacting mixture, cubic feet
- X = Reduced reacted chlorine variable, dimensionless
- h = Modified rate constant, seconds
- k = Reaction rate constant, cubic feet/mole second

- p = Partial pressure, atmospheres
- $p^{O}$  = Vapor pressure, atmospheres
- r = Chlorine feed rate, moles/second
- t = Time, seconds
- $t_{T_i}$  = Maximum time, seconds
- x = Moles of chlorine reacted
- $\pi$  = Absolute pressure, atmospheres
- $\theta$  = Dimensionless time variable, dimensionless
- T = Machine time, seconds
- eta = Time scale factor, seconds/dimensionless unit
- $\xi$  = Dimensionless time variable, dimensionless

#### SUBSCRIPTS

- B = Benzene
- C = Chlorine
- D = Dichlorobenzene
- M = Monochlorobenzene
- O = Initial Value
- T = Trichlorobenzene
- 1 = Reaction producing Monochlorobenzene
- 2 = Reaction producing Dichlorobenzene
- 3 = Reaction producing Trichlorobenzene

### REFERENCES

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