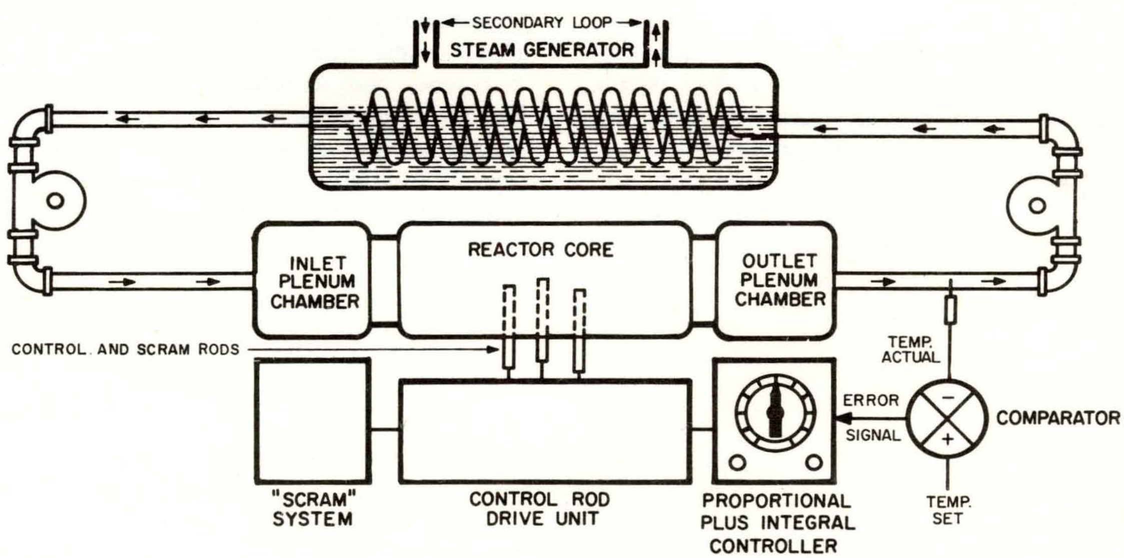
SIMULATION OF THE PRIMARY LOOP OF A NUCLEAR POWER PLANT WITH A SMALL, GENERAL PURPOSE ANALOG COMPUTER

INTRODUCTION

This study describes the simulation of the "primary loop" of a nuclear power plant on a twenty amplifier, general purpose analog computer. This simulation, which normally requires many more amplifiers, was performed using a passive network to simulate reactor kinetics and a capacitor memory to simulate transport delay. These special components did not impair the versatility of the general purpose computer since they are interchangeable with the computer's normal computing components.

In this simulation, the primary loop was considered to be operating under steady-state conditions at one-half of its maximum power. The objects of the simulation were: to determine system response to demand power changes and step inputs in reactivity; to observe reactor response to control system failure during a demand power change; to determine whether the reactor can be controlled manually; and to illustrate reactor behavior when "scram rods" are inserted in the system. The results are sufficiently accurate for an approximate analysis of the over-all behavior of the system and are useful in practice.

The simulation and special purpose equipment will be described after considering first the physical system and the mathematical model in detail. In this way, a more complete explanation of their function can be given. This will be followed by a discussion of the mechanization of the model and the results of the simulation.



Primary Loop of a Nuclear Power Plant

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PHYSICAL SYSTEM

The physical system to be simulated is shown schematically in Figure 1. The fission process, which takes place in the U^{235} fuel elements, generates a large amount of heat energy. This energy, which raises the temperature of the fuel elements, the moderator, etc., is transferred to a coolant flowing through the core of the reactor. This coolant, a pressurized liquid, leaves the reactor through an outlet plenum (mixing chamber) at an elevated temperature and is transported to a steam generator. In the steam generator, the coolant transfers heat to feedwater or low quality steam, converting it to high pressure steam, which flows in a secondary loop. This steam is then converted to electricity in the conventional manner or put to some other use. The coolant leaving the steam generator is transported back to the core of the reactor, entering through an inlet plenum chamber.

The control system of the reactor consists of a comparator, a controller, and a control rod drive unit.

The comparator, which compares actual reactor power level with desired power level, generates an error signal corresponding with any detected difference. This error signal is fed to a controller which in turn positions the reactor-control-rods to correct the difference. The position (depth of insertion) of the neutron-absorbing control rods, which are located adjacent to the fuel elements, governs the number of fissionable neutrons produced per fission. The neutron density in the fuel, in turn, governs the amount of energy released by the fission process. This energy release is the determining factor of the actual power level of the reactor.

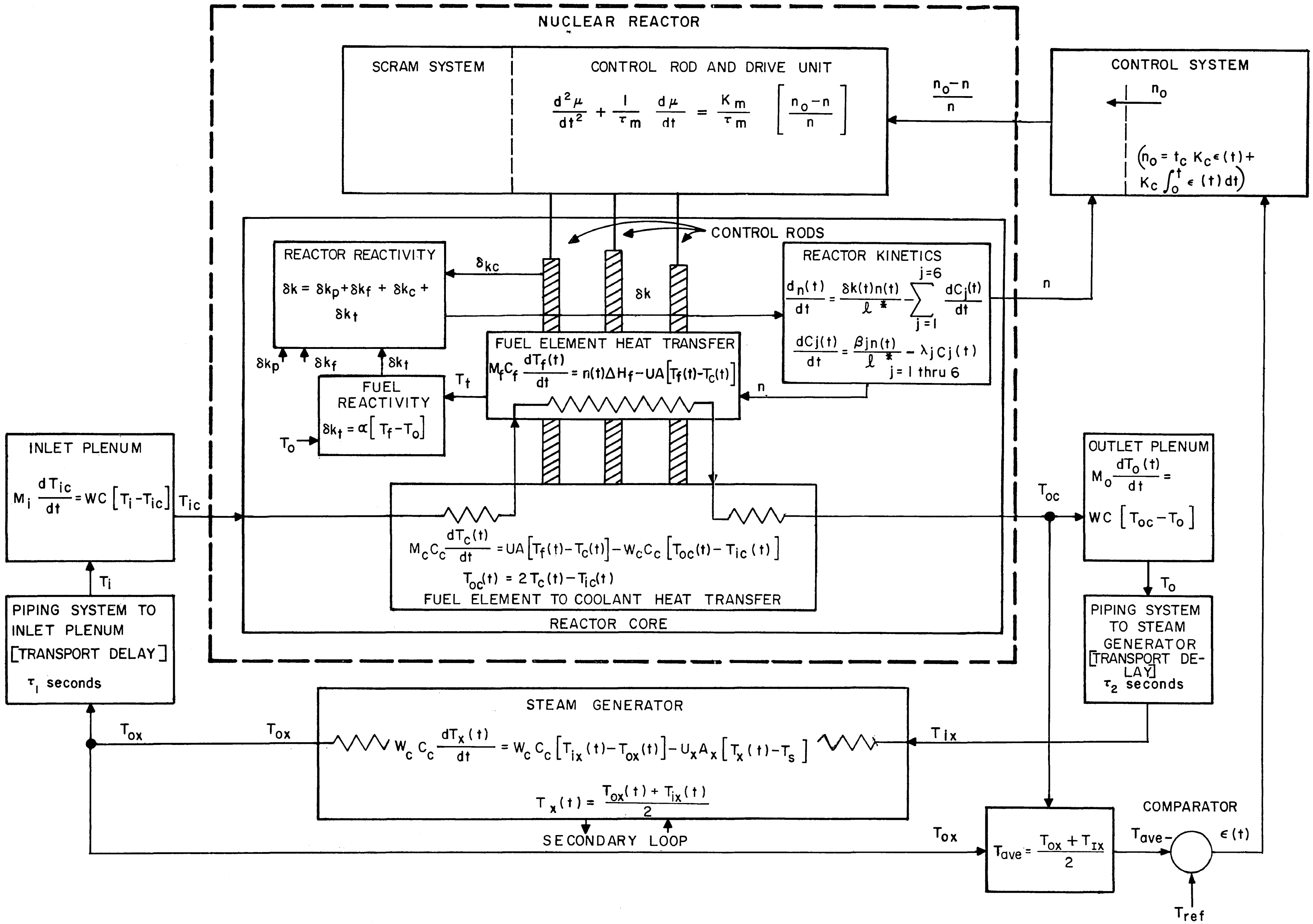


Figure 1. Primary Loop of a Nuclear Power Plant, Simplified Schematic

The purpose of the "scram" system is to position the neutron-absorbing rods in the reactor rapidly when the multiplication of neutrons becomes uncontrollable. These scram rods, which are spring-loaded or magnetically held, absorb enough neutrons to stop the fission process safely.

The various components of the "primary loop", and their interdependence, will be discussed in the formulation of the mathematical model.

Since this simulation dealt with the behavior of the loop for various disturbances, and not with equipment design, the mathematical model was derived in terms of average quantities. This permitted mathematical definition without the use of partial differential equations.

In addition, certain restrictions were placed on the system in order to stay within the limits of a twenty-amplifier computer. First, coolant flow rate was assumed constant during all transient periods of operation. Second, average steam temperature in the generator was assumed constant at all times. This restriction limited the simulation to the primary loop.

MATHEMATICAL MODEL

The following areas of the primary loop must be defined mathematically to achieve a successful simulation:

1. Reactor Kinetics — The kinetic equations for U^{235} fuel can be found in several references (1, 2, 3, 4, 6, 7). These equations, which take into account delayed as well as prompt neutrons, are

$$\frac{dn(t)}{dt} = \frac{\delta k(t) n(t)}{\ell^*} - \sum_{j=1}^{j=6} \frac{dC_j(t)}{dt} \quad (1)$$

Rate of Accumulation (or Depletion) of Neutrons	Prompt Neutrons	Delayed Neutrons

and

$$\frac{dC_j(t)}{dt} = \frac{\beta_j n(t)}{\ell^*} - \lambda_j C_j(t) \quad (2)$$

The reactivity of the system, $\delta k(t)$, is normally defined in terms of the multiplication factor, $k(t)$, by the equation

$$\delta k(t) = \frac{k(t)-1}{k(t)} \quad (3)$$

where $k(t)$ is the number of fissionable neutrons produced per fission. In order for the chain reaction to be self-sustaining, this multiplication factor must be unity and reactivity must be zero. When these conditions are met the reactor is said to be in a "critical" state, and will continue the chain reaction or fission process for long periods of time or until stopped. When reactivity is greater than zero $[k(t) > 1]$, the reactor is in a "supercritical" (disastrous) state; when less than zero $[k(t) < 1]$, the reactor is in a "subcritical" state and will eventually cease to produce neutrons and stop the process.

For computational purposes, the system reactivity is normally expressed as the linear combination of its contributing reactivities (3, 7), namely:

$$\delta k(t) = \delta k_f(t) + \delta k_p(t) + \delta k_t(t) + \delta k_c(t) \quad (4)$$

Since changes in $\delta k_f(t)$ with time depend upon fuel depletion, breeding, etc., it can be considered as constant for short transient periods. This is also true for neutron-absorbing reactor poisons such as Xe^{135}. Their concentration changes have a negligible effect on over-all reactivity compared to the effects of fuel temperature and control rod position. It should be noted, however, that the effect of poisons is quite important during simulations of start-up and shut-down procedures.

The reactivity contribution made by fuel temperature is represented mathematically by the Equations (3, 6, 7)

$$\delta k_t(t) = \alpha \left[T_f(t) - T_o \right] \quad (5)$$

In this study, the temperature coefficient of reactivity is negative (a U^{235} fuel characteristic) and the system is self regulating, i.e., as the reactor heats up, its reactivity is reduced. The contribution of reactor poisons and control rod position are also negative, and tend to reduce system reactivity and produce a "subcritical" state. The built-in reactivity of the fuel has a positive effect on reactivity.

Accordingly, Equation 4 may be rewritten in the form

$$\delta k(t) = k + \alpha T_f(t) + \mu(t) \tag{6}$$

Here

$$\mu(t) = \delta k_c(t) - \delta k_c(0) \tag{7}$$

and

$$k = \delta k_c(0) + \delta k_p + \delta k_f - \alpha T_o \tag{8}$$

Equation 6 is the most convenient form of the reactivity equation to mechanize.

2. Fuel Element Heat Trans — Considering reactor fuel elements, cladding, etc., as a whole, the differential equation governing heat transfer in the fuel elements is

$$M_f C_f \frac{dT_f(t)}{dt} = n(t)\Delta H_f - UA\left[T_f(t) - T_c(t)\right] \tag{9}$$

Heat Energy Accumulated in Fuel	Heat Generated in Fuel	Heat Removed from Fuel by Coolant

In deriving Equation 9, the rate of heat generation in the fuel was assumed proportional to neutron density, n (t). The proportionality constant, ΔH_f, is defined as the heat of fission.

3. Fuel Element, Coolant Heat Transfer — The equation describing transfer of heat energy between fuel elements and flowing coolant can be given, verbally, as: total heat energy accumulated by coolant in reactor equals the difference between heat input by fuel and total heat removed by coolant, or

$$M_c C_c \frac{dT_c(t)}{dt} = UA\left[T_f(t) - T_c(t)\right] - W_c C_c\left[T_{oc}(t) - T_{ic}(t)\right] \tag{10}$$

This transfer of energy is usually described by a partial differential equation; in the interest of simplicity, it was linearized to obtain Equation 10 (the mathematics involved are shown in Appendix A). A byproduct of this linearization is that inlet, average, and outlet temperatures of coolant flowing in core are related by the equation

$$T_c(t) = \frac{T_{ic}(t) + T_{oc}(t)}{2} \tag{11}$$

This equation may be combined with Equation 10 to obtain

$$M_c C_c \frac{dT_c(t)}{dt} = UAT_f(t) + 2W_c C_c T_{ic}(t) - \left[UA + 2W_c C_c\right] T_c(t) \tag{12}$$

or

$$\frac{dT_c(t)}{dt} + \left(\frac{UA + 2W_c C_c}{M_c C_c}\right) T_c(t) = UAT_f(t) + 2W_c C_c T_{ic}(t) \tag{13}$$

which is the equation for a first order lag.

4. Mixing in Plenum Chambers — These equations were derived by treating the chambers as stirred tanks. Thus

$$M_o \frac{dT_o(t)}{dt} = W_c\left[T_{oc}(t) - T_o(t)\right] \tag{14}$$

and

$$M_i \frac{dT_{ic}(t)}{dt} = W_c\left[T_i(t) - T_{ic}(t)\right] \tag{15}$$

For convenience, Equation 14 can be combined with Equation 11.

$$M_o \frac{dT_o(t)}{dt} = W_c\left[2T_c(t) - T_{ic}(t) - T_o(t)\right] \tag{16}$$

5. Transport Delay — Generation of this function is necessary to simulate flow in the piping sections between outlet plenum chamber and inlet to steam generator, and output of steam generator

and inlet to plenum chamber. Coolant will be delayed a given length of time, τ_o, as determined by its mean velocity, ν_o, and length of the piping section, ℓ_o. The function which must be generated to satisfy this requirement is

$$T_{ix}(t) = T_o(t - \tau_o) \tag{17}$$

and

$$T_i(t) = T_{ox}(t - \tau_i) \tag{18}$$

Here

$$\tau_o = \frac{\ell_o}{\nu_o} \quad \text{and} \quad \tau_i = \frac{\ell_i}{\nu_i} \tag{19}$$

6. Heat Transfer in Steam Generator — This equation is very similar to the one governing heat transfer between fuel and coolant in reactor core. That is, total heat energy accumulated by coolant in the steam generator equals difference between heat input and total heat transferred to secondary loop, or

$$M_x C_c \frac{dT_x(t)}{dt} = W_c C_c \left[T_{ix}(t) - T_{ox}(t)\right] - U_x A_x \left[T_x(t) - T_s\right] \tag{20}$$

This equation was linearized by the same method used to obtain Equation 10; therefore, the connecting relationship

$$T_x(t) = \frac{T_{ox}(t) + T_{ix}(t)}{2} \tag{21}$$

is valid in describing the operation of the steam generator. A more convenient form of Equation 20 is

$$M_x C_c \frac{dT_x(t)}{dt} = 2W_c C_c T_{ix}(t) - U_x A_x T_s - \left[U_x A_x + 2W_c C_c\right] T_x(t) \tag{22}$$

7. The Control System — Automatic control of power level is achieved by positioning the control rods in accordance with the error signal representing the difference between desired and actual power levels. Actual power output is proportional to neutron density and the power-demand signal is determined by a control program. A typical control program is to maintain constant average temperature (4). This program can be achieved by means of a proportional plus integral controller described mathematically by

$$n_o(t) = \tau_c K_c \, \epsilon(t) + K_c \int_0^t \epsilon(t) \, dt \tag{23}$$

and

$$\epsilon(t) = T_{ref} - T_{ave}(t) \tag{24}$$

The control system is made up of a comparator, a controller, and a control rod drive unit. The comparator, defined mathematically by Equation 24, compares a reference temperature to an average temperature to form an error signal. The reference temperature is a predetermined function of demand-power level. Average system temperature in this study

$$T_{ave}(t) = \frac{T_{ox}(t) + T_{oc}(t)}{2} \tag{25}$$

is an arithmetic average of high and low system temperature. If Equations 11, 21, 24, and 25 are combined, the error signal equation becomes

$$\epsilon(t) = T_{ref} - T_x(t) - T_c(t) + \frac{T_{ix}(t)}{2} + \frac{T_{ic}(t)}{2} \tag{26}$$

the most convenient form of the error equation to use in the simulation. The controller is defined by Equation 23 and is self-explanatory.

The control rod drive unit, which positions the control rods, can be defined by the transfer function

$$X = \frac{K'}{s(1 + \tau_m s)} \left(\frac{n_o - n}{n}\right) \tag{27}$$

The unusual forcing function, $(n_o - n)/n$, is used to compensate in part for the non-linearities of the reactor. In practice, the reactivity contribution of control rod position is a non-linear function. In the interest of simplicity, it was assumed proportional to control rod position. Therefore, Equation 27 becomes

$$\delta k_c = \frac{K_m}{s(1+\tau_m s)} \left(\frac{n_o - n}{n}\right) \quad (28)$$

where K_m is a modified gain constant. The most convenient representation of Equation 28 for simulation purposes is in terms of variable $\mu(t)$, as defined by Equation 7. By combining Equations 7 and 28 and converting the transfer function to a differential equation, the governing differential equation for control rod drive unit becomes

$$\frac{d^2\mu(t)}{dt^2} + \frac{1}{\tau_m}\frac{d\mu(t)}{dt} = \frac{K_m}{\tau_m}\left[\frac{n_o(t) - n(t)}{n(t)}\right] \quad (29)$$

The mathematical model for the entire system, which is summarized in Figure 1, has now been developed with the exception of the scram system. This system, which must be considered as manually controlled because of equipment limitations, is discussed in the mechanization section. No mathematical equations can be derived to define this system. Note that reactivity is highly negative when the scram rods are inserted in the reactor. Since these rods are spring-loaded or magnetically held, no time lag is experienced when they are inserted.

The quantities $n_o^*(t)$, n_m, K^*, and $n^*(t)$, which appear in the remainder of this paper, are the result of redefining the neutron density as

$$n^*(t) = \frac{n(t)}{n_m} \quad (30)$$

where n^* is reduced neutron density and n_m is the maximum practical neutron density. Neutron density was normalized to prevent scaling difficulties which could be encountered in mechanizing the mathematical model. This also aids in presenting the results of the simulation in more general terms.

MECHANIZATION OF MATHEMATICAL MODEL †

The mathematical model was magnitude-scaled for a ±10 volt, PACE® TR-10 General Purpose Analog Computer, in the conventional manner using the data contained in Tables I and II. Table I is a summary of the parameters encountered in this type of system. Table II is a summary of computer variables, initial values, and scale factors.

TABLE I. SUMMARY OF PARAMETERS

Fuel Elements, Etc.	**Reactor Core**
$n_m \Delta H_f / UA = 1000°F$	$UA/W_c C_c = 0.4$
$n_m \Delta H_f / M_f C_f = 200°F/sec$	$M_c C_c / UA + 2W_c C_c = 1/3$ sec
$M_f C_f / UA = 5$ sec	$UA/UA + 2W_c C_c = 1/6$
Plenum Chambers	**Transport Delays**
$M_o / W_c = 2$ sec	$\ell_o / v_o = 8$ sec
$M_i / W_c = 2$ sec	$\ell_i / v_i = 8$ sec
Steam Generator	**Control System**
$U_x A_x / U_x A_x + 2W_c C_c = 0.4$	$T_{ref} = 1000°F$ (MAXIMUM)
$U_x A_x / W_c C_c = 4/3$	$K_c^* = 10^{-4}$ (°F-sec)$^{-1}$
$M_x C_c / U_x A_x + 2W_c C_c = 1$ sec	$\tau_c = 10$ sec
$T_s = 350°F$	$K_m = 0.05$ sec^{-1}
	$\tau_m = 0.5$ sec
Reactivity	**Reactor Kinetics (U^{235} Fuel)**
$\alpha = -2 \times 10^{-5}$ °F^{-1}	$\ell^* = 10^{-4}$ sec
$\delta k_c = -0.005$ (MAXIMUM)	

TABLE II. SUMMARY OF VARIABLES

Variable	Initial Value	Maximum Value	Scale Factor	Scaled Voltage
T_f	1000°F	2000°F	1/200	$[T_f / 200]$
T_c	500°F	1000°F	1/100	$[T_c / 100]$
T_{oc}	600°F	1000°F	1/100	$[T_{oc} / 100]$
T_{ix}	600°F	1000°F	1/100	$[T_{ix} / 100]$
T_x	500°F	1000°F	1/100	$[T_x / 100]$
T_{ox}	400°F	1000°F	1/100	$[T_{ox} / 100]$
T_o	400°F	1000°F	1/100	$[T_o / 100]$
T_{ic}	400°F	1000°F	1/100	$[T_{ic} / 100]$
n^*	0.5	1.0	10	$[10\, n^*]$
n_o^*	0.5	1.0	10	$[10\, n_o^*]$
$n_o^* - n^*$	0	0.5	20	$[20(n_o^* - n^*)]$
ϵ	0	200°F	1/100	$[\epsilon / 100]$
δk	0	0.005	2000	$[2000\, \delta k]$
μ	0	0.005	2000	$[2000\, \mu]$

† *The subject of analog computer programming and scaling will not be discussed in this paper since several excellent references are available (3, 5).*

Scaled voltage equations, with the exception of reactor kinetics and transport delay equations, are summarized in Table III.

The equations require no explanation. It should be noted that none of these equations required time scaling. The system was simulated in "real time", therefore.

TABLE III. SCALED COMPUTER EQUATIONS

Control System

$$\left[\frac{\epsilon}{100}\right] = \left(\frac{T_{ref}}{1000}\right)[10] + \frac{1}{2}\left[\frac{T_{ic}}{100}\right] + \frac{1}{2}\left[\frac{T_{ix}}{100}\right] - \left[\frac{T_x}{100}\right] - \left[\frac{T_c}{100}\right]$$

$$\left[2\,(n_0^* - n^*)\right] = \left(2 \times 10^3 K_c^*\right)\int_0^t \frac{1}{10}\left[\frac{\epsilon}{100}\right]dt + \left(200 K_c^* \tau_c\right)\left[\frac{\epsilon}{100}\right] - 0.200\left[10 n^*\right]$$

$$\frac{d^2}{dt^2}\left[2000\mu\right] + \left(\frac{1}{\tau_m}\right)\frac{d}{dt}\left[2000\mu\right] = \left(\frac{K_m 10^3}{\tau_m}\right)\frac{\left[20(n_0^* - n^*)\right]}{\left[10 n^*\right]}$$

Reactivity

$$\left[2000\,\delta k\right] = 10\,(20k)[10] - 10\left(4 \times 10^4 |\alpha|\right)\left[\frac{T_f}{200}\right] + \left[2000\,U\right]$$

$$k = \delta k_f + \delta k_c(o) + \delta k_p - \alpha T_o$$

Fuel Element,
Coolant Heat Transfer

$$\frac{d}{dt}\left[\frac{T_c}{100}\right] = \left(\frac{2\mu A}{M_c C_c}\right)\left[\frac{T_f}{200}\right] + 10\left(\frac{W_c C_c}{5 M_c C_c}\right)\left[\frac{T_{ic}}{100}\right] - 10\left(\frac{UA + 2 W_c C_c}{M_c C_c}\right)\left[\frac{T_c}{100}\right]$$

Fuel Element
Heat Transfer

$$\frac{d}{dt}\left[\frac{T_f}{200}\right] = 0.1\left(\frac{n_m \Delta H_f}{200 M_f C_f}\right)\left[10 n^*\right] - \left(\frac{UA}{M_f C_f}\right)\left[\frac{T_f}{200}\right] + 0.1\left(\frac{5UA}{M_f C_f}\right)\left[\frac{T_c}{100}\right]$$

Plenum Chambers

$$\frac{d}{dt}\left[\frac{T_o}{100}\right] = \left(\frac{2 W_c C_c}{M_o C_c}\right)\left[\frac{T_c}{100}\right] - \left(\frac{W_c C_c}{M_o C_c}\right)\left[\frac{T_{ic}}{100}\right] - \left(\frac{W_c C_c}{M_o C_c}\right)\left[\frac{T_o}{100}\right]$$

$$\frac{d}{dt}\left[\frac{T_{ic}}{100}\right] = \left(\frac{W_c C_c}{M_i C_c}\right)\left[\frac{T_i}{100}\right] - \left(\frac{W_c C_c}{M_i C_c}\right)\left[\frac{T_{ic}}{100}\right]$$

Steam Generator

$$\frac{d}{dt}\left[\frac{T_x}{100}\right] = \left(\frac{2 W_c C_c}{M_x C_c}\right)\left[\frac{T_{ix}}{100}\right] + \left(\frac{U_x A_x T_s}{1000 M_x C_c}\right)10 - \left(\frac{U_x A_x + 2 W_c C_c}{M_x C_c}\right)\left[\frac{T_x}{100}\right]$$

NOTE: Quantities in [ ] = Scaled Variable, ( ) = Pot Setting
1/2, 1, 10, etc. = Input gain when appearing on left of pot setting, reference voltage when appearing on right of pot setting

The conventional PACE TR-10 General Purpose Analog Computer includes twenty amplifiers, twenty-four attenuators, and a variety of nonlinear equipment. In order to simulate the reactor kinetics and transport delays some of this equipment was exchanged for special purpose components specially developed for nuclear studies. The four wirewound, 10-turn attenuators and two function switches, normally located on the control panel, were removed, and replaced by a control panel for a Reactor Kinetics Network (RKN). This network was located in the rear of the computer. The control panel contained three two-position function switches (one labeled "scram"), and their patching terminations. An initial-condition potentiometer and input and output patching terminations were also mounted. This RKN simulated the six delayed-neutron-equations, using six "RC" passive element networks in the feedback loop of a high gain amplifier. When the RKN and the high gain amplifier are used in conjunction with a feedback capacitor, C_O, and an input resistor, R_O, Equations 1 and 2 are completely simulated. This circuit is shown in Figure 2. It accepts an input voltage, which is the negative product of the voltages

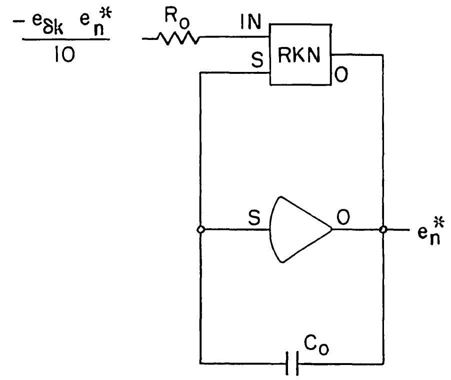
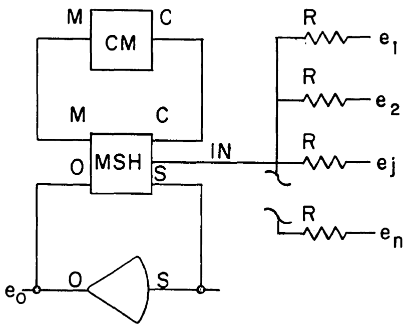


Figure 2. Reactor Kinetics Circuit

representing neutron density and reactivity, and has as its output the voltage representing the neutron density. It should be noted that seven amplifiers were saved by using the RKN. The values of the mean neutron lifetime and the maximum value of the reactivity determine the magnitude of R_O and C_O. The theory of the RKN is discussed in Appendix B.

Representation of transport delay was accomplished by storing discrete samples of a time-varying voltage in a capacitor memory, and subsequent readout of the stored voltages to generate a delayed "staircase" approximation of the input function. The switching and storing functions of the memory were performed by four components, which must be added to the conventional TR-10 Computer. These components are: a timing unit, a ring counter, capacitor memories, and multiple sample-hold units. The timing unit, which replaces the repetitive operation drive unit on the TR-10 control panel, generates a pulse train to the ring counter. The ring counter, located in the nonlinear row of the computer, drives the capacitor memory and multiple sample hold units. These are also located in the non-linear row. Each transport delay requires a high gain amplifier, a capacitor memory, and a multiple sample hold unit. The transport delay circuit is shown in Figure 3.



$$e_o(t) = \sum_{j=1}^{j=n} \frac{e_j(t-\tau)}{N}$$

Figure 3. Transport Delay Circuit

The reactor kinetics and transport delay circuits designed for the TR-10 computer also are available on the TR-20 unit so that the program can be run equally well on either machine.

The capacitor memory contains 16 storage capacitors, and relays associated with each capacitor. This unit, with the aid of the multiple sample-hold circuit and the timing equipment, is capable of sampling rates of 0.1 to 10 samples per second. Sampling rate is determined by a multiposition switch and vernier, located on the front of the timing unit. In the simulation under consideration, the time delay was 8 seconds; therefore, a sampling rate of 2 samples per second was required. Transport delay units are wired to begin sampling simultaneously with computer start-up. Provision is also made in this unit for obtaining an initial condition voltage at its output; however, the same voltage is stored on all sixteen capacitors.

Please note that when a fourth order Padé approximation is used to simulate a transport delay, it requires six amplifiers and six potentiometers.

The computer diagram for the simulation of the primary loop is shown in Figure 4. The potentiometer settings and amplifier outputs associated with this circuit are tabulated in Table IV. This table consists of the static check potentiometer settings and amplifier outputs, as well as the "steady-state check" potentiometer settings. The purpose of the steady-state check will be discussed in the results.

RESULTS

The first series of computer runs, performed after a successful static check, resulted in a family of curves of reduced neutron density versus time, for various reference temperature inputs. These runs were performed to make a steady-state check of the system. The system was designed for a 500°F reference temperature input to be compatible with the initial conditions and potentiometer settings. Therefore, if a reference temperature of 500°F was patched into the circuit, it should draw a horizontal line along the initial value of n*(5), which was one-half, or five volts. The steady-state check was successful as shown in Figure 5. The slight off-set in the steady state value is due to the accumulated error of the computer components, each of which is individually rated at 0.10%. This off-set is about 1.0%.†

Figure 5 shows that maximum reference temperature (or power level) of the system lies between 600°F and 700°F. During these runs, function switches in the computer circuit were in the "up" position, which meant that the control system was being used.

The same circuit was used also to obtain representative variations of $n^*(t)$, $\delta K(t)$, $\epsilon(t)$, $T_f(t)$, $T_c(t)$ and $T_x(t)$ versus time (for a reference temperature of 650°F). Typical results are shown in Figures 6, 7, and 8. A hazard evaluation of the control system is shown in Figure 6. This was accomplished by throwing the function switch feeding the final integrator of the control circuit (amplifier #18) to the down position after 11 seconds of operation. This grounded the integrator-input and fixed the position of the control rods, which is the output of amplifier #18. The system stabilized itself by virtue of the temperature feedback of the fuel, but not at the desired level. The noise on the $\epsilon(t)$ versus time curve, shown on Figure 7, is due to the switching in of the transport delay circuits as well as the quantitizing of the input function. This presents no problem if the output of the transport delay is fed to an integrator. The integrator, which acts as a smoothing device, produces a smooth curve. This can be proven by referring to the graph of T_x (t) versus time.

The third set of results, illustrated in Figures 9, 10, and 11, show the behavior of the system for a step change in reactivity(T_{ref} = 500°F), for manual control of the reactor, and for the effect of scramming, respectively. In the first case, coefficient potentiometer #1, which feeds amplifier #4, was set so that the voltage at the output of the amplifier

† Although the authors have not performed this simulation on the TR-20 computer as of this writing, it is likely that the greater component accuracy of the TR-20 machine would result in a substantially lower total error.

was -5 volts, which corresponds to a step change in reactivity of -2.5×10^{-3}. As shown in Figure 9, the control system returned the neutron density to the initial value rapidly and with typical response.

Manual control of the reactor was illustrated by removing the control system from the computer circuit. This was accomplished by putting both function switches in the "down" position. The computer then produced the initial value of n* (t) illustrated by the "steady-state check", and by the first 30 seconds of the plot shown in Figure 10. At this point, coefficient potentiometer #1 was varied manually so as to increase the power level of the reactor from a neutron density level of 0.50 to 0.80.

A situation calling for the use of the scram rods, which are assumed to be present in addition to the control rods, was simulated by setting the reference temperature to an arbitrary value greater than 700°F. As previously shown, 700°F is too large for normal reactor behavior; therefore, the flux level proceeded to a maximum level as shown in Figure 11. When the flux reached its maximum value, the two function switches and the "scram" switch (which feeds the reactiyity summer — amplifier #4), were thrown simultaneously. This removed the control system from the circuit, and forced the reactivity to its lower limit of -10 volts. This was accomplished using a diode limiter on amplifier #4, as shown in Figure 4, and placing a large voltage on its input via the scram switch. The neutron level responded as expected by dropping quickly toward zero, first rapidly and then gradually. The same procedure was repeated for the $T_x(t)$ and $T_c(t)$ versus time, and their behavior was as expected.

CONCLUSIONS

The results, as shown, are sufficiently accurate for an approximate analysis of over-all system behavior and are of use in practice.

This simulation was possible on a twenty-amplifier analog computer only through the use of the special Reactor Kinetics Network and Transport Delay Simulator components. Their use reduced the required number of amplifiers and associated potentiometers by at least seventeen, and provided the necessary special circuitry for reactor kinetics and time delay simulations.

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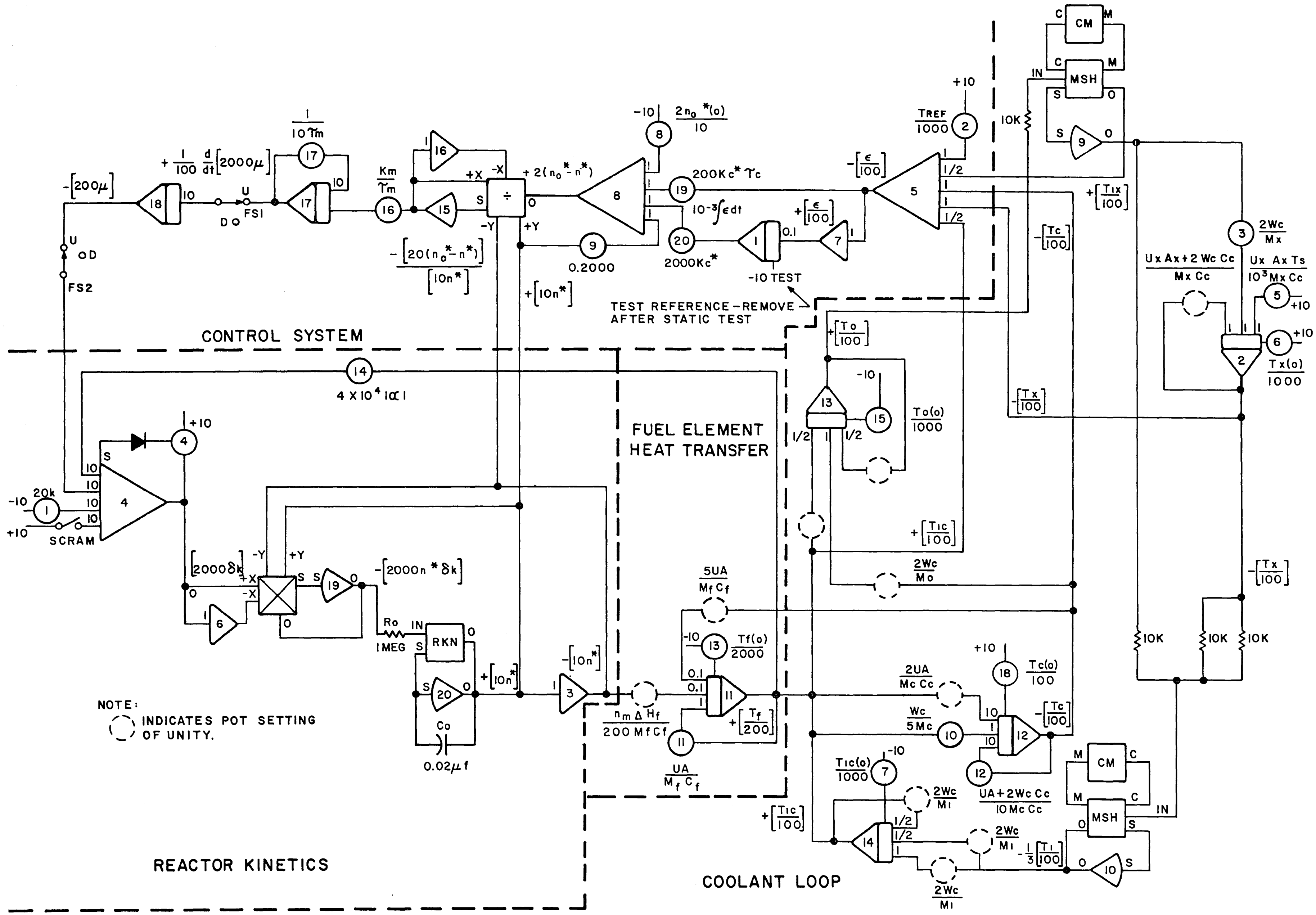


Figure 4. Computer Diagram

TABLE IV
TR-10 POTENTIOMETER ASSIGNMENT SHEET

PROBLEM NUCLEAR REACTOR

POT NO.	PARAMETER DESCRIPTION	SETTING STATIC CHECK	SETTING RUN NO.1 *	SETTING RUN NUMBER 1	SETTING RUN NO.	NOTES	POT NO.
1	20k	0.400	0.400			TRIM POT	1
2	$T_{REF}/1000$	0.550	0.500				2
3	$2W_C C_C / M_X C_C$	0.600	0.600				3
4	LIMITER	0.500	0.500			LIMIT TO -10V	4
5	$U_X A_X T_S / 10^3 M_X C_C$	0.140	0.140				5
6	$T_X(0)/1000$	0.800	0.500				6
7	$T_{IC}(0)/1000$	0.600	0.400				7
8	$2n^*(0)/10$	0.300	0.500				8
9	SCALING RATIO	0.200	0.200				9
10	$W_C C_C / 5 M_C C_C$	0.250	0.250				10
11	$UA/M_F C_F$	0.200	0.200				11
12	$(UA+2W_C C_C)/10 M_C C_C$	0.300	0.300				12
13	$T_F(0)/2000$	0.600	0.500				13
14	$4 \times 10^4 / \alpha /$	0.800	0.800				14
15	$T_O(0)/1000$	0.600	0.600				15
16	K_m / τ_m	0.100	0.100				16
17	$1/10\tau_m$	0.200	0.200				17
18	$T_C(0)/1000$	0.800	0.500				18
19	$200 K^* \tau_C$	0.200	0.200				19
20	$2000 K^*$	0.200	0.200				20
21	$n^*(0)$	0.500	0.500			RKN IC POT	21
22							22
23							23
24							24

*STEADY - STATE CHECK

TABLE V
TR-10 AMPLIFIER ASSIGNMENT SHEET

PROBLEM NUCLEAR REACTOR

AMP NO.	FB	OUTPUT VARIABLE	STATIC CHECK				NOTES
			CALCULATED		MEASURED		
			CHECK PT.	OUTPUT	CHECK PT.	OUTPUT	
1	I	$-10^{-3}\int_0^t \varepsilon\, dt$	-0.45	-10.00	✓	✓	
2	I	$-[T_X/100]$	-3.0	-8.0	✓	✓	
3	S	$-[10\, n^*]$		-5.0		✓	
4	S	$+[2000\, \delta k]$		-8.0		✓	
5	S	$-[\varepsilon/100]$		+4.5		✓	
6	S	$-[2000\, \delta k]$		+8.0		✓	
7	S	$+[\varepsilon/100]$		-4.5		✓	
8	S	$+[2(n_o^* - n^*)]$		-0.9		✓	
9	HG	$+[T_{IX}/100]$		+6.0		✓	
10	HG	$-\frac{1}{3}[T_I/100]$		-10/3		✓	
11	I	$+[T_F/100]$	-0.1	+6.0	✓	✓	
12	I	$-[T_C/100]$	-3.0	-8.0	✓	✓	
13	I	$+[T_o/100]$	-2.0	+6.0	✓	✓	
14	I	$+[T_{IC}/100]$	-2.0	+6.0	✓	✓	
15	HG	$-\left[2\, \frac{n_o^* - n^*}{n^*}\right]$		+1.8		✓	
16	S	$+\left[2\, \frac{n_o^* - n^*}{n^*}\right]$		-1.8		✓	
17	I	$+\frac{1}{100}\frac{d}{dt}[2000\, u]$	+21.8	+10	✓	✓	-10V TO I.C. IN STATIC CHECK
18	I	$-\frac{1}{10}[2000\, u]$					USED AS CHECK AMPL. DURING S.C.
19	HG	$-2000\, n^* \delta k$		+4.0		✓	
20	HG	$+[10\, n^*]$		+5.0		✓	

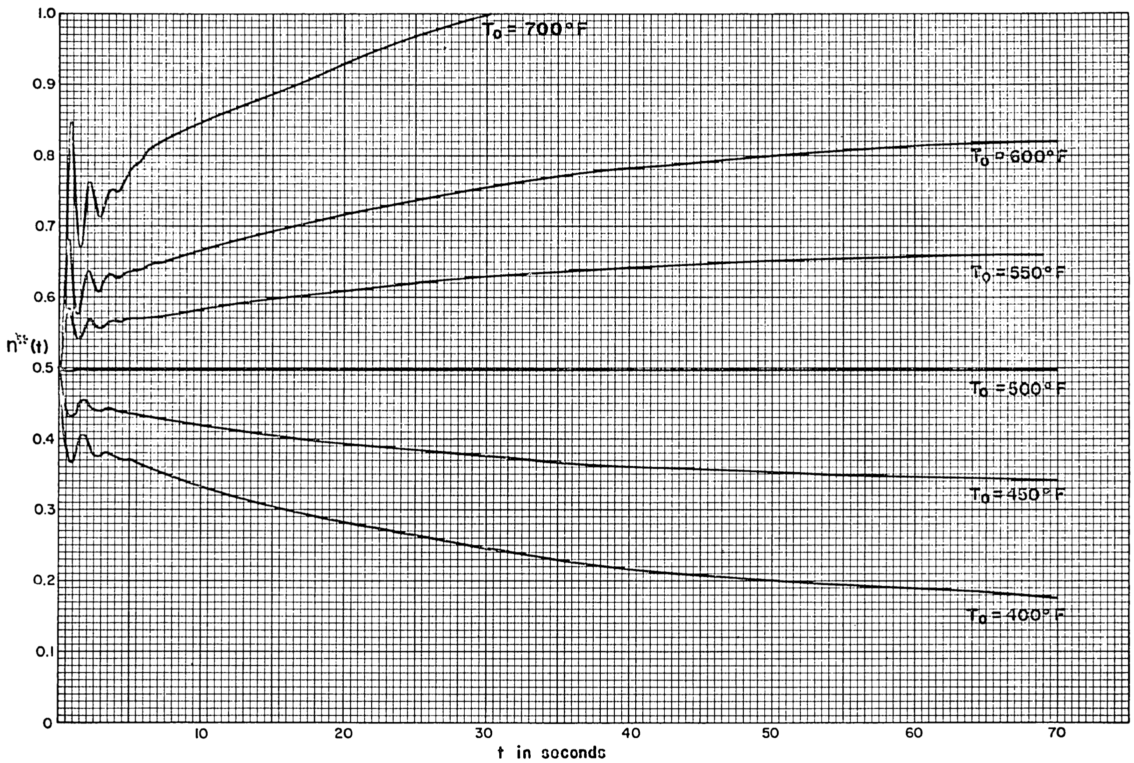


Figure 5. n*(t) Versus t for Various Reference Temperature (T_o) Inputs

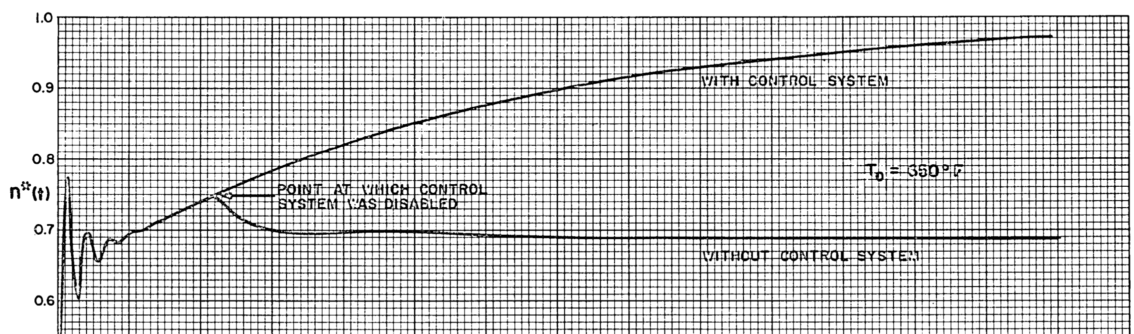


Figure 6. The Effect of Control System Failure on Reactor Operation

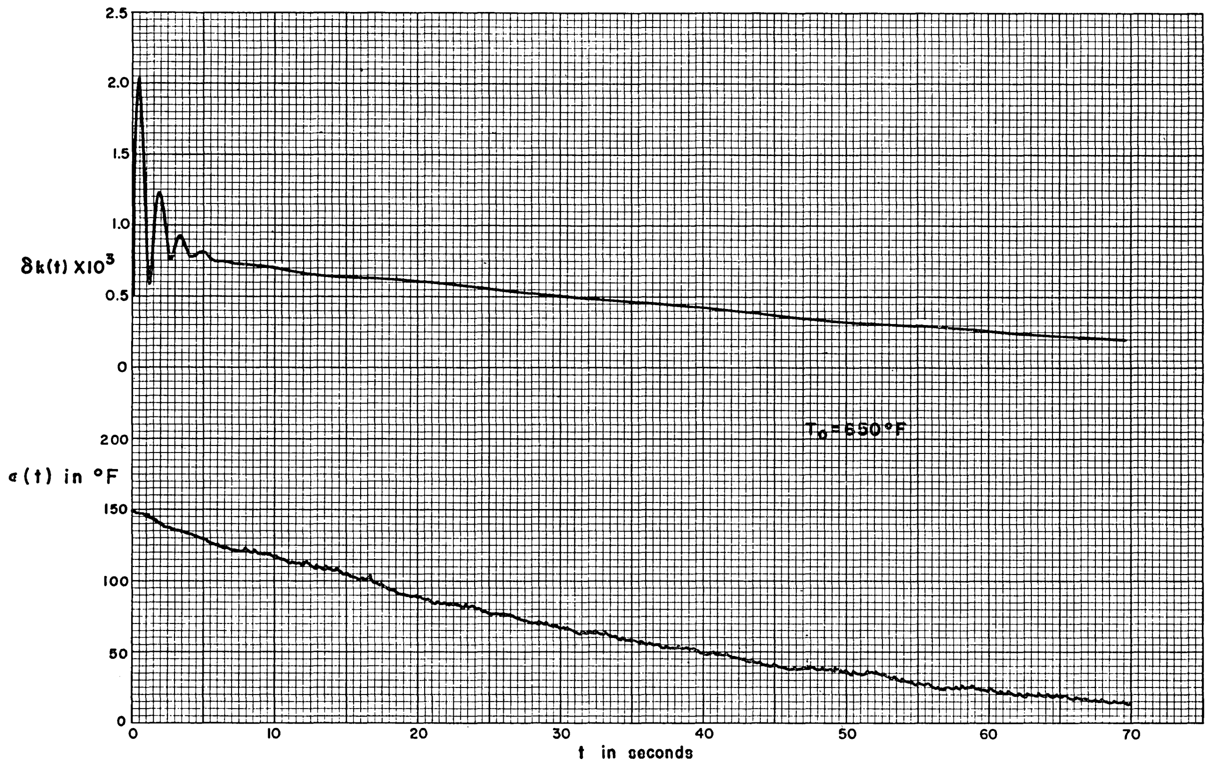


Figure 7. $\epsilon(t)$ and $\delta k(t)$ Versus t

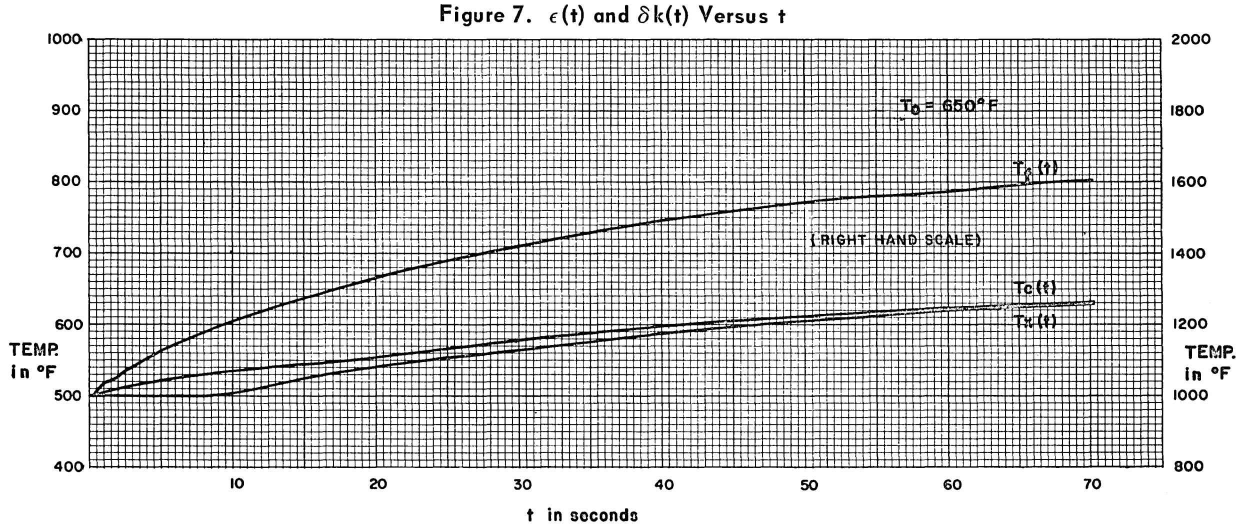


Figure 8. $T_c(t)$, $T_x(t)$, and $T_f(t)$ Versus t

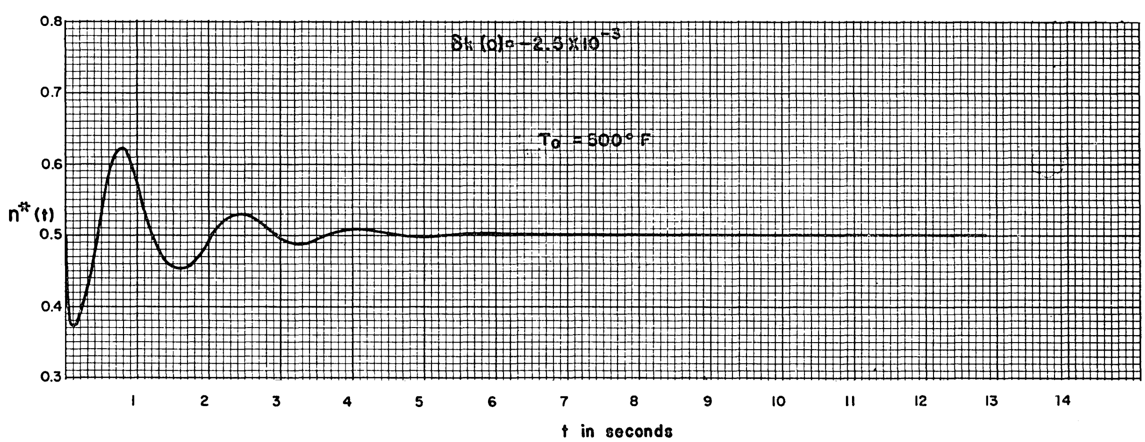


Figure 9. The Effect of a Negative Step Input in Reactivity on Reactor Operation

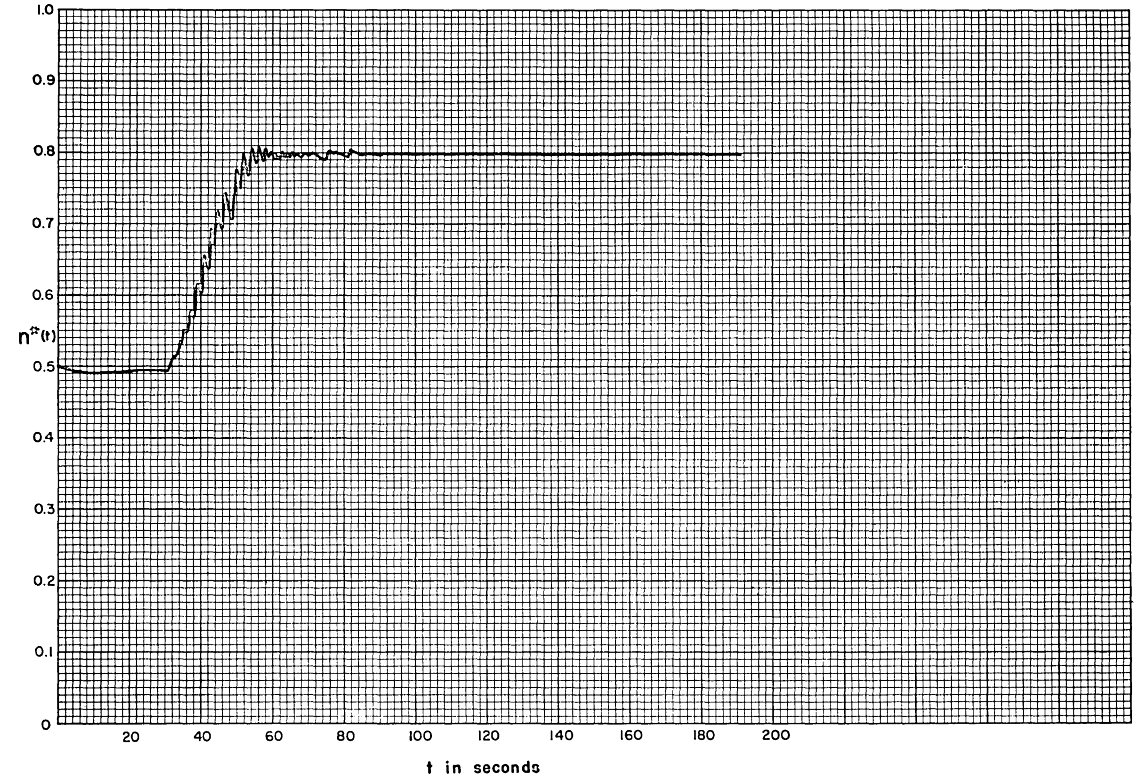


Figure 10. $n^*(t)$ Versus t for Manual Control of the Reactor

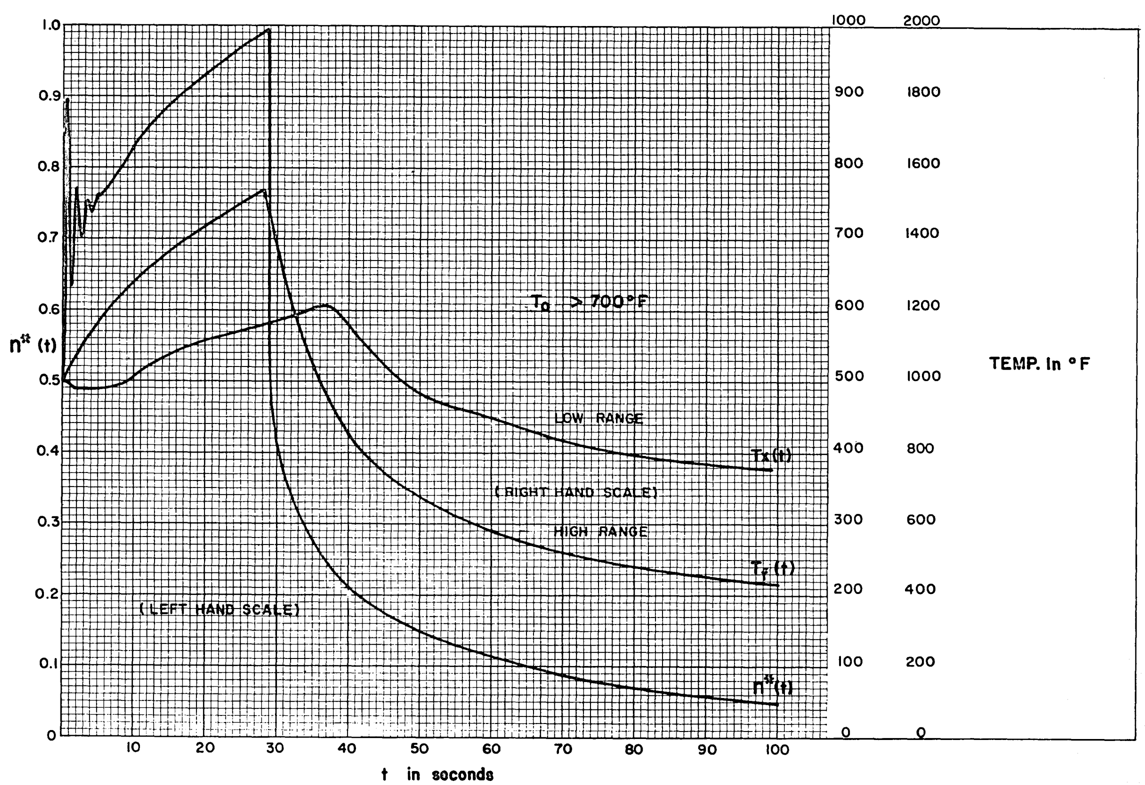


Figure 11. The Effect of "Scramming" on $n^*(t)$, $T_f(t)$, and $T_x(t)$

APPENDIX A

LINEARIZATION OF FLUID HEAT TRANSFER EQUATION

Consider a fluid flowing through a closed conduit which is gaining thermal energy from its surroundings. If it is assumed that the ambient temperature of the surroundings is not a function of conduit length, the temperature history of the fluid may be represented by the familiar equation

$$A\rho C \frac{\partial T(x,t)}{\partial t} = Ua\left[T_a(t) - T(x,t)\right] - WC\,\frac{\partial T(x,t)}{\partial x} \tag{A-1}$$

This equation is governed by the initial conditions

$$T(o,x) = f(x) \tag{A-2}$$

and

$$T(t,o) = g(t) \tag{A-3}$$

If one assumes that a linear temperature profile with respect to distance,

$$T(x,t) = T(t,o) + \Big(T(t,L) - T(t,o)\Big)\frac{x}{L} \tag{A-4}$$

exists at any time, t, the mean fluid temperature, $\overline{T}(t)$, is

$$\overline{T}(t) = \frac{\int_0^L T(x,t)\,dx}{\int_0^L dx} = \frac{T(t,o) + T(t,L)}{2} \tag{A-5}$$

This equation relates the inlet, outlet, and mean temperatures of the fluid which are all time varying functions.

Integrating Equation A-1 with respect to distance yields

$$A\rho C\int_0^L \frac{\partial T(x,t)}{\partial t}\,dx = Ua\int_0^L \left[T_a(t) - T(x,t)\right]dx - WC\int_0^L \frac{\partial T(x,t)}{\partial x}\,dx \tag{A-6}$$

or

$$LA\rho C\int_0^1 \frac{\partial}{\partial t}\left[T(t,o) + \Big(T(t,L) - T(t,o)\Big)s\right]ds = UaL\left[T_a(t) - \int_0^1\left[T(t,o) + \Big(T(t,L) - T(t,o)\Big)\right]x\,ds\right] - WC\left[T(t,L) - T(t,o)\right] \tag{A-7}$$

which reduces to

$$LA\rho C\,\frac{d\overline{T}(t)}{dt} = UaL\left[T_a(t) - \overline{T}(t)\right] - WC\left[T(t,L) - T(t,o)\right] \tag{A-8}$$

Note that

$$LA\rho = M \tag{A-9}$$

and

$$aL = A_h \tag{A-10}$$

where M is the mass of the fluid in the conduit and A_h is the heat transfer area. Substituting these expressions in Equation A-8, we obtain

$$MC\,\frac{d\overline{T}(t)}{dt} = UA_h\left[T_a(t) - \overline{T}(t)\right] - WC\left[T(t,L) - T(t,o)\right] \tag{A-11}$$

which, except for notation, is Equation 10.

APPENDIX B

REACTOR KINETICS NETWORK THEORY

The passive network for simulating the reactor kinetic equations is shown in simplified form in Figure B-1. If the input stage of the amplifier draws negligible current.Kirchoff's current law gives

$$i + i_f + \sum_{j=i}^{6} i_j = 0 \qquad \text{(B-1)}$$

If one further assumes that the summing junction is at zero potential, Equation B-1 can be written

$$\frac{e_i}{R_o} + C_o \frac{de_o}{dt} + \sum_{j=1}^{6} i_j = 0 \qquad \text{(B-2)}$$

If the voltage across capacitor C_j is called e_j, and its charge Q_j, then with sign conventions as shown in Figure B-1, we obtain

$$Q_j = C_j e_j \qquad \text{(B-3)}$$

$$e_j + i_j R_j = e_o \qquad \text{(B-4)}$$

$$i_j = \frac{dQ_j}{dt} \qquad \text{(B-5)}$$

In deriving B-4, we again used the assumption that the summing junction is at zero potential.

Eliminating e_j from B-3 and B-4:

$$i_j = \frac{e_o}{R_j} - \frac{Q_j}{R_j C_j} \qquad \text{(B-6)}$$

Substituting B-6 into B-2:

$$\frac{e_i}{R_o} + C_o \frac{de_o}{dt} + \frac{1}{R} e_o - \sum_{1}^{6} \frac{Q_j}{R_j C_j} = 0 \qquad \text{(B-7)}$$

where

$$\frac{1}{R} = \sum_{1}^{6} \frac{1}{R_j}$$

Solving B-7 for

$$\frac{de_o}{dt}$$

$$\frac{de_o}{dt} = \frac{-e_i}{R_o C_o} - \frac{1}{RC_o} e_o + \frac{1}{C_o} \sum_{1}^{6} \frac{Q_j}{R_j C_j} \qquad \text{(B-8)}$$

Eliminating i_j from B-5 and B-6, we obtain

$$\frac{dQ_j}{dt} + \frac{Q_j}{R_j C_j} = \frac{1}{R_j} e_o \qquad \text{(B-9)}$$

Comparing B-8 and B-9 with the reactor kinetic equations

$$\frac{dn}{dt} = \frac{n\,\delta k}{\ell^*} - \frac{\beta}{\ell^*} n + \sum_{1}^{6} \lambda_j C_j \qquad \text{(B-10)}$$

$$\frac{dC_j}{dt} + \lambda_j C_j = \frac{\beta_j}{\ell^*} n \qquad \text{(B-11)}$$

it becomes clear that if we make ($-e_i$) proportional to $n\delta k$, and e_o proportional to n, the equations will have the same form. The values of R_j, C_j, R_o, and C_o can be calculated in terms of the known constants λ_j, β_j, β, and ℓ^*, and the scale factors chosen for n and δk.

It turns out that the R_j and C_j values may be fixed if a given fuel is used, and C_o and R_o chosen as functions of ℓ^* (which may vary widely with reactor design) and the scale factor for δk (which depends on the estimated maximum value of δk). The six resistors and capacitors, R_j and C_j, are packaged within the reactor kinetics unit, while R_o and C_o are patched externally, so that various values may be used to suit a particular problem.

Typical values of ℓ^* and δk_m (the maximum reactivity for the simulation) are given in Table 1-B, along with the calculated values of R_o and C_o. The component values shown in this table are available from Electronic Associates, Inc. as plug-in units.

TABLE 1-B. SELECTED VALUES OF R_o AND C_o†

ℓ^*	$\delta k_m X10^2$	R_o	C_o^*
seconds	dimensionless	megohms	mfd
10^{-3}	0.05	10.0	0.200
10^{-4}	0.05	10.0	0.020
10^{-5}	0.05	10.0	0.002
10^{-3}	0.20	2.5	0.200
10^{-4}	0.20	2.5	0.020
10^{-5}	0.20	2.5	0.002
10^{-3}	0.50	1.0	0.200
10^{-4}	0.50	1.0	0.020
10^{-5}	0.50	1.0	0.002

† *Electronic Associates, Inc.; PIR 6239*

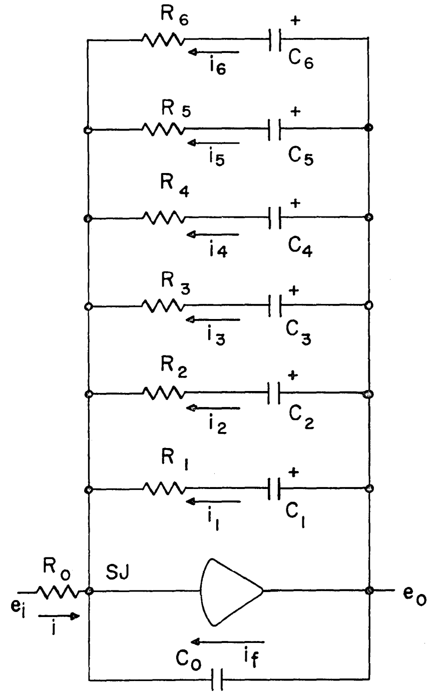


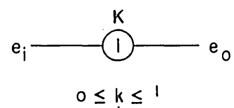
Figure B-1. Simplified Diagram of the Reactor Kinetics Network

APPENDIX C

SYMBOLS FOR COMPUTING CIRCUITS †

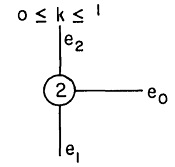
POTENTIOMETERS

GROUNDED



$e_o = k\ e_i$

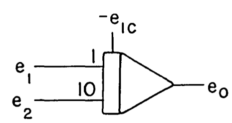
UNGROUNDED



$e_o = e_1 + k(e_2 - e_1)$

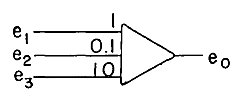
OPERATIONAL AMPLIFIERS

INTEGRATOR



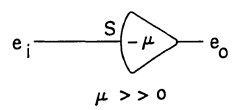
$e_o = e_{IC} - \int_0^t (e_1 + 10 e_2)\, dt$

SUMMER



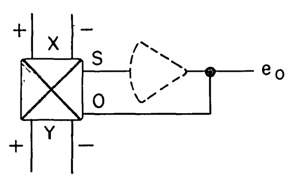
$e_o = -(e_1 + 0.1 e_2 + 10 e_3)$

HIGH-GAIN



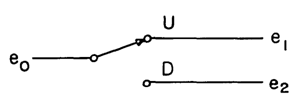
$e_o = -\mu\ e_i$

MULTIPLIERS



$e_o = \frac{-XY}{10}$

FUNCTION SWITCH



$e_o = e_1$ U-POSITION

$e_o = e_2$ D-POSITION

† *TR-10 Operators Manual*

APPENDIX D

TABULATION OF COMPUTER COMPONENTS

The following TR-10 Computing Components (which were housed in a standard TR-10 Console), patching accessories, and readout devices were required to carry out the simulation.

Computing Components

20 - Operational Amplifiers
8 - Integrator Networks
2 - Quarter-Square Multipliers
2 - Capacitor Memory Units
2 - Multiple Sample Hold Units
1 - Ring Counter
1 - Timing Unit
20 - Coefficient Setting Potentiometers
1 - Reactor Kinetics Network and Control Panel

Patching Components

122 - Patch Cords
- 22 - Black, 6 inch
- 67 - Brown, 12 inch
- 30 - Orange, 18 inch
- 3 - Blue, 30 inch
- 4 - Multiple Connectors

11 - Feedback Resistors
- 2 - 100K
- 9 - 10K

38 - Input Resistors
- 4 - 1 megohm, includes input resistor to RKN
- 4 - 200K
- 7 - 100K
- 27 - 10K

1 - Diode

1 --Capacitor, 0.02 fd feedback for RKN

Readout Devices

1 - X-Y Plotter (with time base generator)

LIST OF SYMBOLS

SYMBOL		MEANING	UNITS
A	=	Heat transfer area in the reactor	Sq Ft
A_x	=	Heat transfer area in the steam generator	Sq Ft
A'	=	Cross sectional flow area	——
C_c	=	Specific heat of the coolant	Btu/lb-°F
C_f	=	Specific heat of the fuel, moderator, etc.	——
C_j	=	Concentration of neutrons in delayed neutron group "j"	Neu/sec
C'	=	Fluid specific heat	——
k	=	Reactivity constant	Dimensionless
K	=	Control rod drive unit gain	Ft/sec
K_c	=	Controller gain	$\frac{\text{Neutrons}}{°\text{F-sec-cm}^2}$
M_c	=	Mass of coolant in the reactor	Lbs
M_f	=	Mass of fuel, moderator, etc.	Lbs
M_i	=	Mass of coolant in the inlet plenum chamber	Lbs
M_o	=	Mass of coolant in the outlet plenum chamber	Lbs
M_x	=	Mass of coolant in the steam generator	Lbs
S	=	Operator	Seconds^{-1}
T_a	=	Ambient temperature	——
T_c	=	Average coolant temperature in the reactor	°F
T_f	=	Average fuel temperature	°F
T_i	=	Coolant temperature at the input to the reactor inlet plenum chamber	°F
T_o	=	Coolant temperature at the outlet of the reactor outlet plenum chamber, or temperature coefficient of reactivity reference temperature (temperature at which temperature contribution is zero).	°F
T_s	=	Average temperature of secondary fluid in steam generator	°F
T_x	=	Average coolant temperature in the steam generator	°F
T_{ic}	=	Temperature of coolant entering the reactor	°F
T_{ix}	=	Temperature of coolant entering the steam generator	°F
T_{oc}	=	Temperature of coolant leaving the reactor	°F
T_{ox}	=	Temperature of coolant leaving the steam generator	°F
T_{ref}	=	Reference temperature	°F
T_{ave}	=	Average system temperature	°F
T	=	Fluid temperature	°F
U	=	Overall coefficient of heat transfer in the reactor core, or Control rod reactivity variable	$\frac{\text{Btu}}{\text{sec}-\text{Ft}^2}$ / Dimensionless
U_x	=	Overall coefficient of heat transfer in the steam generator	Dimensionless
U'	=	Overall coefficient of heat transfer	Dimensionless
V	=	Mean fluid velocity	Ft/sec
W_c	=	Mass flow rate of coolant	Lbs/sec
W	=	VA, mass rate of flow of the fluid	——
X	=	Control rod position	Dimensionless
a	=	Heat transfer area per unit length of conduit	——
j	=	Index for delayed neutron groups	Dimensionless
ℓ_i	=	Effective length of inlet piping system	Ft
ℓ_o	=	Effective length of outlet piping system	Ft
ℓ^*	=	Effective neutron lifetime	Seconds
n	=	Neutron density	Neutron/cu cm
n^*	=	Reduced neutron density	Dimensionless
n_m	=	Maximum practical neutron density	Neutron/cu cm
n_o	=	Demand-power level neutron density	Neutron/cu cm
t	=	Time	Seconds
ν_o	=	Mean velocity of coolant in outlet piping system	Ft/sec
ν_i	=	Mean velocity of coolant in inlet piping system	Ft/sec
x	=	Position along conduit	——
α	=	Temperature coefficient of reactivity	°F-1
β_j	=	Fraction of prompt neutrons appearing in delayed neutron group "j"	Dimensionless
δ_k	=	Reactivity	Dimensionless
δk_c	=	Reactivity contribution of control rod positions	Dimensionless
$\delta k_c(o)$	=	Initial reactivity contribution of control rods	Dimensionless
δk_f	=	Built-in reactivity of fuel	Dimensionless
δk_p	=	Reactivity contribution of reactor poisons	Dimensionless
δk_t	=	Reactivity contribution due to the fuel temperature	Dimensionless
ΔH_f	=	Heat of fission	$\frac{\text{Btu - cu cm}}{\text{Sec - Neu}}$
$\epsilon(t)$	=	Error signal	°F
λ_j	=	Decay constant associated with group "j"	Dimensionless
$\mu(t)$	=	Departure of control rod reactivity from its initial value	Dimensionless
τ_o	=	Outlet piping system delay time	Seconds
τ_c	=	Controller time constant	Seconds
τ_i	=	Inlet piping system delay time	Seconds
τ_m	=	Control rod drive unit time constant	Seconds
ρ	=	Fluid density	——