RECOMP II USER'S PROGRAM NO. 1111

PROGRAM TITLE: ROOT LOCUS

PROGRAM CLASSIFICATION: General

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PURPOSE: To produce a point by point root locus in the s-plane, when given a set of open loop poles and zeros. Up to 20 poles and 20 zeros may be accommodated.

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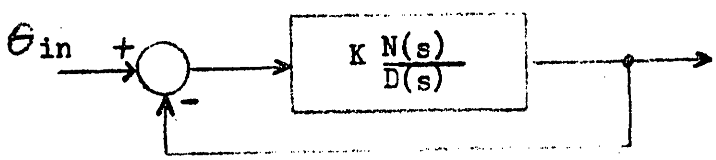
1.0 INTRODUCTION

In the design of the control systems, a method frequently used to determine system parameters and performance is the root locus method of W. R. Evans [(1)]. This method produces closed loop transfer functions when open loop functions are given (in Laplace transform notation).

2.0 METHOD

The assumed form of the closed loop is the following:



N(s) = polynomial in s
D(s) = polynomial in s

Any linear loop or series of loops may be reduced to a form similar to this. Solution for closed loop roots involves the solution of the equation

$$K\frac{N(s)}{D(s)} = -1 .$$

This equation is solved directly by the root locus program, where K, N(s), and D(s) are entered by the operator.

3.0 RESTRICTIONS

(a) The order of the numerator polynomial, N(s), must be less than or equal to the order of the denominator polynomial, D (s).

(b) The order of the denominator polynomial, D(s), must be less than 21, i.e. the open-loop system must have fewer than 21 poles.

(c) Limitations imposed by AN-007.1 and AN-014.

4.0 USAGE

4.1 Read in the "Root Locus" tape.

4.2 If all open loop poles and zeros are expressed in quadratic form, press "Start 1" and proceed to 4.3 below. If not, proceed at 4.12 below.

4.3 The computer will ask for the order of the numerator and denominator. Type these orders as they are asked for as two digit numbers (e.g. 07).

(1) Ref. Evans, W.R. "Control System Synthesis by Locus Methods" AIEE Preprint, 50-51, January 1950
Also see "Root Locus" in any recent servomechanisms text.

4.4 The computer will then ask for the coefficients in the numerator. The assumed form of the numerator is as follows:

$$(A_1s^2 + B_1s + C_1)\ (A_2s^2 + B_2s + C_2)\quad \text{etc.}$$

Therefore, the polynomial should be expressed as the product of quadratics, except that if one zero is left over, it may be entered as a linear factor, i.e. A = 0. The coefficients may be entered on the typewriter as the computer asks for them, in a number of formats. For example,

26.91 becomes

(a)	+26.91	(carriage return)
(b)	26.91	(carriage return)
(c)	+.2691+2	(carriage return)
(d)	+2691-2	(carriage return)
(e)	+.02691+3	(carriage return)
	etc.	

The carriage return may be replaced by a space or tab, if desired. A space, tab, or carriage return given by itself will yield a + zero.

4.5 The computer will then ask for the coefficients in the denominator. Type these as in 4.4 above.

4.6 The computer will ask for the initial value of K, the open loop gain and the value of the increment in gain, dK. Type these in the same format as in 4.4 above.

4.7 The computer will ask "ERROR?". If you have made an error in typing in the numerator coefficients, type an N followed by a two digit number representing the number of the coefficient in error. (For example, B_2 would be 05). After this, retype the coefficient as in 4.4 above. If the error is in the denominator, type a D, then continue as above. If no error has been made, type either a space or a carriage return.

4.8 Upon typing a space or carriage return in 4.7 above, the computer will halt in location 0003. You then have two options. You may:

(a) Press "Start 3", thereby starting the computations, or
(b) Turn sense switch D on and press "Start 1" to punch out the coefficients you have entered on a tape. Leader and tail are punched automatically. After punching, you may press "Start 3" and proceed as in 4.8 (a) above. See 4.13.

4.9 If, during computations, you desire to change K or dK, turn sense switch B on. The computer will ask for the new value of K and dK which you may enter as in 4.6 above.

4.10 The format of the output is as follows:

(a) K is typed out.
(b) The closed loop roots at that value of K are typed out. If two numbers are typed in one row, the roots are complex conjugates, the real part being typed first.
(c) K is incremented by dK and the cycle is repeated.

4.11 To stop the program, push "Stop". To begin a new locus, proceed as in 4.2 above.

4.12 If the factors in numerator or denominator are available, not as quadratics, but as linear or complex conjugate forms, these may be expanded by subroutines built into this program.

(a) Complex pairs. Turn sense switch C on and press "Start 1". The assumed form of the complex pair is

$$(s + A + jB)\ (s + A - jB).$$

Type in A and B as the computer asks for them. (For format of type-in, see 4.4 above). The computer will type out two numbers, M and N, which will be the coefficients in $(s^2 + Ms + N)$, the expansion of the complex pair. This may be repeated for each complex pair without pressing "start 1" again.

(b) Real pairs. Turn sense switch C on and press "Start 2". The assumed form of the linear pair is

$$(s + A)\ (s + B).$$

Type in A and B as the computer asks for them. (For format of type-in, see 4.4 above). The computer will type out two numbers, M and N, which will be the coefficients in $(s^2 + Ms + N)$, the expansion of the real pair. This may be repeated for each real pair without pressing "Start 2" again.

When all real and complex pairs have been converted to quadratics (except for a possible <u>single</u> linear factor left over), proceed as in 4.2 above.

4.13 To use previously punched coefficients (see 4.8.(b)), perform steps 4.1, 4.2, and 4.3 above. Following this, press "error reset" and read in the previously punched tape. Proceed from step 4.8 (a) above.

4.14 During the root extraction routine, an overflow halt may indicate a zero root which will be output if "error reset" and "start" are pressed in sequence. Incorrect order of denominator will also cause an overflow halt.

5.0 SAMPLE PROBLEM

PRINTOUT	EXPLANATION
ROOT LOCUS	
ORDER OF NUMERATOR 01 ORDER OF DENOMINATOR 04 NUMERATOR COEFFICIENTS A 0 B 1 C 2 DENOMINATOR COEFFICIENTS A 1 B 3 C 3 A 1 B 4 C 5	Transfer function is $K\dfrac{(s+2)}{(s^2+3s+3)(s^2+4s+5)}$
K 0 DK 1.75 0.25	Initial K and dK
ERROR? D 06 4.001 ERROR?	Change C_2 in denominator from 5 to 4.001
K +.000000 + 0 -.200000 + 1 +.316228 - 1 -.150000 + 1 +.866025 + 0	for K=0, roots are $(s+2\pm j.0316)$ $(s+1.5\pm j.866)$
K +.175000 + 1 -.200057 + 1 -.275299 + 1 -.112322 + 1 +.124629 + 1	for K=1.75, roots are $(s+2)(s+2.75)$ $(s+1.12\pm j1.25)$
K +.350000 + 1 -.200029 + 1 -.307863 + 1 -.960542 + 0 +.147078 + 1	for K=3.5, roots are $(s+2)(s+3.08)$ $(s+.961\pm j1.47)$
K 10 DK 15	← Sense switch B on. Change K and dK
K +.100000 + 2 -.373713 + 1 -.631387 + 0 +.197051 + 1 -.200010 + 1	for K=10, roots are $(s+3.74)(s+.631\pm j1.97)$ $(s+2)$
K +.250000 + 2	↓ etc.

6.0 CODING INFORMATION

Subroutines utilized

AN-007.1		Floating point input in 3000-3177
AN-014		Floating point output in 3200-3437
Data	storage	6000-6400
Program (including constants and subroutines)	storage	0000-4000

Root extraction routine is taken from one written by W. Beykirch of Autonetics.